Determining Magnification Factors from Luminosity vs FWHM Relations of CO Rotation lines in SMG Galaxies

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In this report we investigate CO line luminosities vs FWHM relations for SMG galaxies and determine their accuracy in deriving gravitational lensing factors. Analysis of CO J:1-2 and J:3-2 lines yield possible trends in accuracy of using such relations to determine magnification factors, by comparison to factors found in the literature. Power-law fits and derived magnification factors are compared with Harris et al (2012), highlighting the sensitivity of this method to small sample sizes. Further lines of analysis of CO vs FWHM plots are suggested and the need for collaboration to compile a larger sample of sources is highlighted.

1 Introduction

The line luminosity of a CO rotational line (L_{CO}) in a galaxy of redshift z, luminosity distance D_L, and with observed CO rotational line flux S_{CO} is given by equation (1).(Solomun et al 1997)

$$L_{CO} = \frac{c^2}{2k\upsilon^3}\frac{D_L^2}{1+z}\Delta V_{FWHM} S_{CO} \tag{1}$$

Where υ is the emitted frequency of the transition and V_{FWHM} is the Full Width Half Maximum of the CO rotational line.

The apparent luminosity observed L'_{CO} differs by a magnification factor of μ. This magnifaction is caused by gravitational lensing and the relation between the intrinsic CO line luminosity of the source and apparent line luminosity observed is illustrated by equation (2).

$$L'_{CO} = \mu L_{CO} \tag{2}$$

Many sources currently have no detailed lens model available to accurately predict μ (Harris at al 2012). Methods to determine magnification factors accurately from already ready known data (L_{CO},ΔV_{FWHM}) of sources, could prove a useful tool. To what extent the assumptions that this tool relies on will affect the accuracy, and whether this accuracy is dependant on the particular CO transition observed, is not known.

In this report the method followed by Harris et al (2012) is used to determine magnification factors. This method is tested for different CO transitions for SMG type galaxies, and derived magnification factors using this method are compared to magnification factors found in the literature.

2 Method

Initialy, python routines where used to systematically extract and plot L'_{CO} vs ΔV_{FWHM} relations for a range of different galaxy types, each for a range of different CO transitons, from a databse of sources. This systemiatic search of the data base yielded the most sources with a fair number of lensed and un-lensed sources for SMG galaxies with CO rotational transitions J=1-0 and J=3-2. Many of the other galaxy types had very few sources and the ones that did either had nearly all lensed or un-lensed sources.

The luminosity vs FWHM relations were plotted for the different transitions J=1=0,J=3-2, inlcuding lensed sources (of known magnification) with their CO line luminosity values not corrected for lensing.

A power-law fit of type, and identical to the one used in Harris et al (2012), shown in equation (4) was fitted to the unlensed sources.

$$L_{CO} = a(\Delta v_{FWHM})^b \tag{3}$$

Assuming that the gravitational lensing only affects the L'_{CO} value of the sources and the power-law fit is typical of all sources in the sample, a formula for the magnification factor (μ) can be determined by substituting equation (2) into (3).

$$\mu = \frac{L'_{CO}}{a(\Delta V_{FWHM})^b} \tag{4}$$

This power-law fit was calculated from the unlensed sources and then applied to the lensed sources to derive magnification factors. The derived magnification factors for the lensed sources were compared to literature values.

3 Results and Analysis

3.1 CO J:1-2

The first plot investigated was for SMG galaxies with CO rotational lines J=1-0. The plot shows of a modest number of sources, however it contains enough unlensed sources to obtain a power-law fit. This plot is shown below in fig 1.

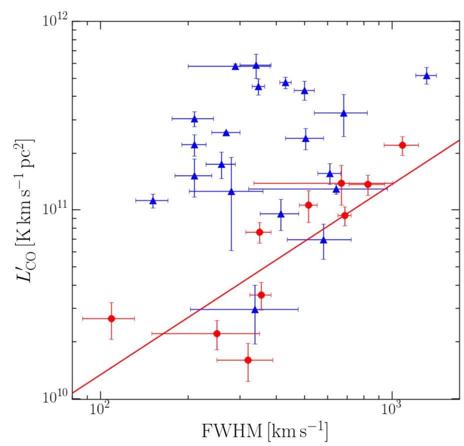

Figure1. Circular points indicate line luminosities L'_{CO} vs CO (J:1-0) FWHM line widths for 10 sources with magnification 1. The solid lines shows a power law fit to these data. Triangular points indicate L'_{CO} vs CO(J:1-0) FWHM line widths for 20 sources with magnification >1, which have L'_{CO} values un-corrected for lens magnification. Lens sources data from Riechers et al. (2011f), Harris et al. (2010), Riechers et al. (2011e), Riechers et al. (2011e), Thomson et al. (2012), Sharon et al. (2013), Fu et al. (2013), Aravena et al. (2013), Ivison et al. (2013), Harris et al. (2012), Frayer et al. (2011),Riechers et al. (2011d), Swinbank et al. (2010). Unlensed sources data from Carilli et al. (2010), Hainline et al. (2006), Ivison et al. (2011), Hodge et al. (2013), Ivison et al. (2013).

Calculating the parameters of the power-law fit of the unlensed data and arranging into a style similar to Harris et al (2012) as to allow for comparison, we obtain an expression for the magnification factor μ shown in equation (5).

$$\mu = 3.5\frac{L'_{CO}}{10^{11}Kkms^{-1}pc^2}\left(\frac{178kms^{-1}}{\Delta V_{FWHM}}\right)^{1.01} \tag{5}$$

Inital comparison with the expression derived in Harris et al (2012), in which a marginally larger sample of 15 sources was used, yields very different power-law fit parameters. Harris et al (2012) derived an exponent of 1.7 compared to the exponent seen in equation (5) of 1.01. Moreover, the constant within the exponent differs from $400km^{-s}$ in Harris et al (2012) to $178kms^{-1}$ derived in equation (5). These large differences could indicate the sensitivity of this method to modest sample sizes.

Table 1 shows the derived magnification factors from implementation of equation (5) (μ_r) compared with magnification factors found in the literature (μ_l). Bold ID entries indicate comparison with Harris et al (2012) where the values of u_l were obtained using the same method outlined in this report.

Table 1
Comparison of literature magnification factors μ_l with derived magnification factors μ_r for transition J:1-0

ID	μ_l	μ_r	$\frac{\mu_r - \mu_l}{\mu_l}$
SMMJ04135+1027	1.6	3.5±0.7	1.2
SMMJ04431+0210	4.4	1.7±0.4	-0.61
SMMJ09431+4700-H6	1.3	0.65±0.35	-0.50
SMMJ09431+4700-H7	1.2	0.88±0.28	-0.27
SMMJ14009+0252	1.5	1.5±0.8	-0.02
SMMJ14011+0252	5.0	5.5±0.8	0.12
HXMM01	1.54	2.9±0.4	0.87
SPT-S053816-5030.8blue	20	11 ±2	-0.46
SPT-S053816-5030.8red	20	3.3±2.0	-0.83
HATLASJ084933.4+021443-T	1.5	1.9±0.3	0.26
H-ATLASJ090302.9-014128-17b	18	7.1±0.8	-0.61
H-ATLASJ090311.6+003906	25	8.1±0.7	-0.68
H-ATLASJ091840.8+023047	5.0	3.5±1.2	-0.29
HLSW-01	10.9	9.6±1.1	-0.12
H-ATLASJ113526.3-014605	17	5.4±1.5	-0.68
H-ATLASJ115820.2-013753	13	5.0±1.0	-0.62
H-ATLASJ133649.9+291801	23	7.9±1.3	-0.66
H-ATLASJ141351.9-000026	10	6.4±0.9	-0.36
SMMJ2135-0102	32.5	14.8±4.6	-0.54
SPT-S233227-5358.5	15	13±2	-0.15

Notes. Entries in bold indicate sources compared with Harris et al (2012) where the literature magnification factors (μ_l) are derived by the method outlined in this report. Uncertainty values for μ_l are not available so uncertainty in the values of $\frac{\mu_r - \mu_l}{\mu_l}$ are shown on figure 3 are the same fractional uncertainty as is μ_l and should be treated as a minimum.

The average fractional disagreement between the power-law fit in this report compared with Harris et al (2012) was 0.56 in a sample of 4 sources with a standard deviation of 0.14. This again highlights the possible sensitivity this method has when using modest sample sizes.

The remainder of the sample generally showed a trend of underestimation of the literature magnification values μ_l, with the magnitude of the under estimation increasing with magnification factor size. Figure 2 shows a plot of derived magnification factors u_r against magnification factor found in the literature u_l and illustrates this trend in under estimation.

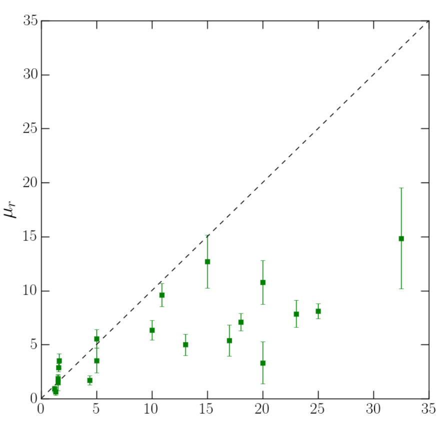

Figure2. Data points indicate a comparison between magnification factors derived using equation (5) μ_r and magnification found in the literature μ_l, for all sources listed in Table 1. The Dashed line indicates the line of direct correspondance between derived and liternature values of μ.

This trend is shown more clearly in fig 3, which plots the fractional difference in magnification against the μ_l. Figure 3 shows that, in the case of smaller magnifcation factors, there is a scatter around the actual value. However, as the magnification factor increases, there is a trend of increasing magnitude of underestimation. This could imply for CO J:1-0 line luminosity that the accuracy of the power-law fit method is dependant on the size of the magnification factor being derived.

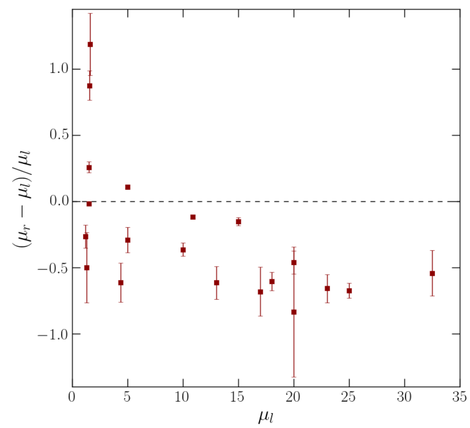

Figure3. Data points indicate the fractional difference between derived magnification factors the magnification factors found in the literature $(\mu_r - \mu_l)/\mu_l$ against the literature magnification factor μ_l. The dashed line indicates a fractional difference of zero.

3.2 CO J:3-2

The next set of sources investigated was a SMG sample with CO rotational lines J:3-2. This sample contained a greater number of unlensed sources of 21 when compared with the CO J:1-0 sample. However, it contained a smaller number of lensed sources. The same plot of L'_{CO} against ΔV_{FWHM} was carried with a power-law fit obtained for the un-lensed sources. This plot is shown in fig 4.

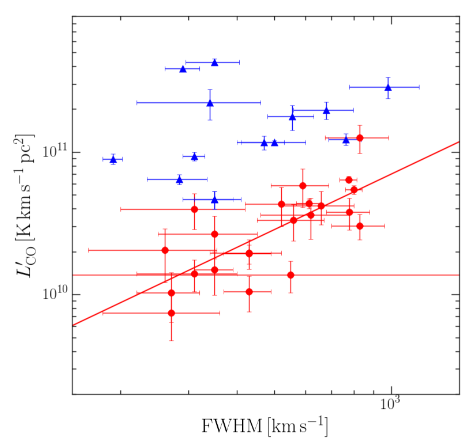

Figure4. Circular points indicate line luminosities L'_{CO} vs CO J:3-2 FWHM line widths for 21 sources with magnification 1. The solid lines

shows a power-law fit to these data. Triangular points indicate L'_{CO} vs CO J:1-0 FWHM line widths for 15 sources with magnification >1, which have L'_{CO} values un-corrected for lens magnification. Lensed source data from, Frayer et al. (1998), Hainline et al. (2004), Riechers et al. (2013b), Neri et al. (2003), Tacconi et al. (2006), Frayer et al. (2003), Weiss et al. (2009), Frayer et al. (1999), Knudsen et al. (2009), Sheth et al. (2004), Weiss et al. (2005b), Fu et al. (2013), Ivison et al. (2013), Riechers et al. (2011d), Swinbank et al. (2010). Unlensed data from, Tacconi et al. (2006) ,Bothwell et al. (2013), Coppin et al. (2008), Neri et al. (2003), Greve et al. (2005), Ivison et al. (2013).

Similarly to the J:1-0 sample we calculate the parameters of the power law fit of the unlensed data, and arranging into a style simlar to Harris et al (2012), we obtain an expression for the magnification factor μ shown in equation (6).

$$\mu = 3.5\frac{L'_{CO}}{10^{11}Kkms^{-1}pc^2}\left(\frac{13.6kms^{-1}}{\Delta V_{FWHM}}\right)^{1.29} \quad (6)$$

On examination of equation (6) the power-law fit for transition J:3-2 yields different parameters to the power law fit to the J:1-0 sources. The most notable difference is the parameter within the exponent which is $13.6kms^{-1}$, in comparison to $178kms^{-1}$ obtained from the J:1-0 sample. This large difference could imply a dependence on transition for the the parameter inside the exponent. The exponent itself is larger at 1.29 compared to 1.01 in the J:1-0 sample, this difference, however, could be caused by the larger sample of 21 sources available for J:3-2. Nonetheless, both parameters are different to the ones obtained in the power-law fit in Harris et al (2012), which again suggests that the parameters derived using this method are somewhat transistion dependant.

Table 2 shows a comparison for magnification factrs derived using equation (6) for the J:3-2 sample of sources and the factors found in the literature.

Table 2
Comparison of literature magnification factors μ_l with derived magnification factors μ_r for transition J:3-2

ID	μ_l	μ_r	$\frac{\mu_r-\mu_l}{\mu_l}$
SMMJ02399-0136	2.5	2.5±0.4	0.0
SMMJ04135+1027	1.6	13±6.6	7.0
SMMJ04135+1027	1.6	4.6±1.2	1.9
SMMJ04431+0210	4.4	2.6±0.7	-0.41
SMMJ04431+0210	4.4	2.6±0.5	-0.41
SMMJ04431+0210	4.4	2.7±1.8	-0.37
SMMJ14009+0252	1.5	4.4±0.9	2.0
SMMJ14011+0252	5.0	10±1	1.1
SMMJ163541.2+661144	1.7	4.7±1.1	1.8
SMMJ16359+6612-B	22	4.1±1.1	-0.81
SMMJ16359+6612-B	22	6.1±0.7	-0.72
HXMM01	1.54	4.2±1.3	1.7
HATLASJ084933.4+021443-T	1.5	5.4±1.4	2.6
HLSW-01	10.9	24 ±5	1.1
SMMJ2135-0102	32.5	27 ±4	-1.6

Note. Uncertainty values for μ_l are not available so uncertainty in the values of $\frac{\mu_r-\mu_l}{\mu_l}$ are shown on figure 6 are the same fractional uncertainty as is μ_l and should be treated as a minumum.

Fig 3 shows μ_r against μ_l for sources in the J:3-2 sample. Fig 3 shows no apparent trend in under or over estimation in the magnification factors. In contrast to the J:1-0 sample, more sources show an over-estimate than an under-estimate and, in the over estimate case, no apparent dependance of μ_l is observed. This could be attributed to the modest number of lensed sources in the J:3-2 sample. More data for lensed sources for the J:3-2 transitions would allow for better comparison between the J:1-0 and J:3-2 samples.

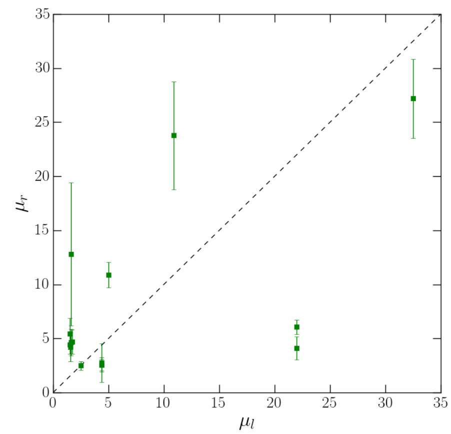

Figure5. Data points indicate a comparison between magnification factors derived using equation (6) μ_r and magnification found in the literature μ_l, for all sources listed in Table 2. The dashed line indicates the line of direct correspondance between derived and literature values of μ_l.

The absence of a trend in accuracy dependent on magnification factor is more clear in Fig 6, where the fractional change in magnification factor is plotted against the magnification factor. In contrast, Fig 6 actually shows some smaller magnification factors with the largest fractional deviation from the literature. However, these values have large associated uncertainties and the overall plot shows a scatter of over and under-estimation.

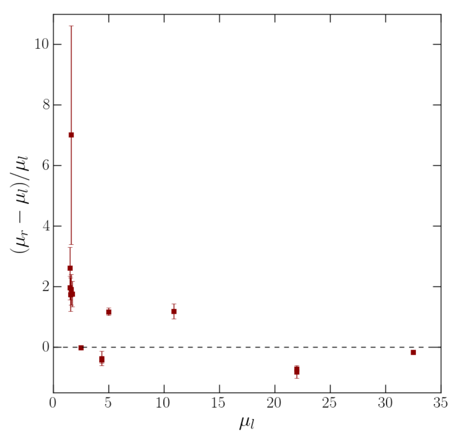

Figure6. Data points indicate the fractional difference between the derived and literature magnification values against the values of the literature magnification. The dashed line indicates a fractional difference of zero.

3.3 Uncertainty Calculations

The uncertainty values in Tables 1 and 2 calculated from propigation of the uncertainty in L'_{CO} and ΔV_{FWHM} through the power-law fits in equations (4) and (5). For a power-law fit of the type used in this report, shown in equation (3), uncertainty in the derived magnification factors ($\delta\mu$) is obtained via equation (7) shown below.

$$\frac{\delta\mu}{\mu} = \sqrt{\left(\frac{\delta L'_{CO}}{L'_{CO}}\right)^2 + \left(a\frac{\delta\Delta V_{FWHM}}{\Delta V_{FWHM}}\right)^2} \quad (7)$$

Equation (7) indicates a stronger dependance of the uncertainty in μ on ΔV_{FWHM} then on L'_{CO}, assuming that the exponent a is greater than unity, which it has been in all investigated cases. This implies that this method is sensitive to deriving magnification factors with accuracy from measurements with large ΔV_{FWHM} uncertainties, and less sensitive to large uncertainties in L'_{CO} measurements.

4 Discussion and Conclusions

Overall, it is difficult to assess the accuracy of predicting magnification factors from L'_{CO} vs ΔV_{FWHM} relations. This is predominantly due to the lack of data available in the literature. Moreover, the parameters of L'_{CO} vs ΔV_{FWHM} power-law fit are observed to be incredibly sensitive to the random scatter of data about them. This is illustrated by the difference in parameters calculated in this report and in Harris et al (2012).

In the J:1-0 sample of sources, the power-law fit in equation (5) yielded an under prediction, the magnitude of which appearing to depend on the size on the magnification factor (see fig 3). Conversely, in the J:3-2 sample of sources no such trend was observed (see fig 6), although the J:3-2 sample contained a greater number of unlensed sources from which the power-fit law was derived.

The scatter of accuracy seen for the J:3-2 sources could indicate the L'_{CO} or ΔV_{FWHM} values for those sources being affected by other, possibly random, factors. These could include galaxy inclination or dispersion in intrinsic galaxy properties for sources both lensed or unlensed. Systematic plotting of samples of lensed and unlensed sources with known magnificiation factors and comparision with magnification factors derived in the method outlined in this report could provide insight into how such samples are affected by these unknown factors.

The modest sets of sources investigated in this report suggest that the parameters of the power law fit for L'_{CO} vs ΔV_{FWHM} for SMG type galaxies are not constant and depend to a certain extent on the CO transition observed. This is shown by comparing equations (5) and (6) which yield very different parameters. A further possible dependency of the accuracy of this method, that has not been investigated in this report, could be redshift, and whether or not trends are seen across different galaxy types e.g. QSO, SFG... type galaxies.

In conclusion, more data across a greater number of transitions needs to be complied to allow for detailed comparison and analysis for SMG galaxies. However, certain trends mentioned in this report, e.g the under-estimation seen for the J:1-0 sample, could prove useful in evaluating the accuracy of this method and other possible uses it may have, if a large enough sample of sources is used.

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