Notes on the partially resoleved astrometric microlensing shift with parallax

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Abstract

The astrometric deflection model for a source being gravitationally lensed in a partially resolved system is presented. Two regimes of this model are considered. Firstly, the regime where the parallax effects are not present in the deflection signal is presented. Then the regime where the parallax can be detected in the deflection signal is presented. We conclude that if a parallax signal is detected in the deflection, the lens mass can be directly inferred from the astrometric time series of the source. In the case where the parallax signal is not detected then auxiliary astrometric information about the lens is needed to constrain the lens mass.

1 Preliminaries

In this note, vectors are denoted $\vec{a}$, the magnitude of a vector is denoted $|a|$, the dot and cross products between two vectors are denoted $\vec{a} \cdot \vec{b}$ and $\vec{a} \times \vec{b}$ respectively.

Let a background source (the source) be located at distance D_S from an observer, and a lens with mass M be at distance D_L, where $0 < D_L < D_S$. Let $\vec{\psi}_L$ and $\vec{\psi}_S$ be the angular positions of the lens and source respectively. We then define the dimensionless vector,

$$\vec{u}(t) = \frac{\vec{\psi}(t)}{\theta_E} = \frac{\vec{\psi}_L(t) - \vec{\psi}_S(t)}{\theta_E}, \quad \theta_E = \sqrt{\frac{4GM}{c^2}\frac{D_L - D_S}{D_S D_L}}. \tag{1}$$

In the case where the lens and major source image can be resolved for the duration of the microlensing event, the apparent deflection of the source is given by the position of the major image by e.g Bramich [2018] as,

$$\vec{\delta}_+(t) = \frac{1}{2}\left(\sqrt{|\vec{u}(t)|^2 + 4} - |\vec{u}(t)|\right)\theta_E \frac{\vec{u}(t)}{|\vec{u}(t)|}. \tag{2}$$

$\vec{u}(t)$ is usually at least parameterised by the standard microlensing parameters (u_0, t_0, t_E) which we will define here, the specifics of the paramterisation will be addressed in the next section. u_0 is the minimum normalised lens source separation in the case of linear lens-source relative motion. t_0 is the time of minimum lens-source separation again in the case of linear lens-source relative motion. t_E is the Einstein time and is defined as the relative lens-source proper motion divided by the Einstien radius $|\mu_{\rm rel}/\theta_E|$. In the case of linear lens-source relative motion, this sets the timescale of the event.

2 Parameterisation of $\vec{u}(t)$

Equation (2) shows that the astrometric microlensing signal is completely defined by the direction and magnitude of $\vec{u}(t)$. In this section, we will outline its parameterisation in two regimes. Firstly, in the regime where the motion of the observer is negligible, then in the general case where the observer motion is included. In this section we will work with all vectors with the local unit north $(\hat{n})$ and east $(\hat{e})$ basis. Specifically we will use the notation $\vec{a} = (a_n, a_e) = a_n\hat{n} + a_e\hat{e}$. In the following sections I will be following the notation and conventions of Gould [2004].

2.1 Stationary observer

Let the relative lens-source vector be $\vec{u}(t) = -(\tau(t), \beta(t))$. In the case of linear lens-source motion and no parallax effects we have,

$$\tau(t) = \frac{t - t_0}{t_E}, \quad \beta(t) = \beta = u_0. \tag{3}$$

This leads to $\vec{u}(t) \rightarrow \vec{u}(t; u_0, t_0, t_E)$ and therefore by equation (2) $\vec{\delta}_+(t) \rightarrow \vec{\delta}_+(t; u_0, t_0, t_E, \theta_E)$.

2.2 Moving observer

To include the effect of a moving observer we introduce a small perturbation to the linear lens-source motion model. Specifically, we let the relative lens-source vector be $\vec{u}(t) = -(\tau(t), \beta(t))$, and with a small perturbation equation

(3) becomes,

$$\tau(t) = \frac{t - t_0}{t_E} + \delta\tau(t), \quad \beta(t) = u_0 + \delta\beta(t). \tag{4}$$

To find the forms of $\delta\tau(t)$ and $\delta\beta(t)$ let us consider the following. Let $\vec{s}(t)$ be the observer-sun vector in units of AU and in the heliocentric frame. Let t_p be some fixed reference (ideally close to the event maximum t_0) as seen by the observer. We evaluate the derivative of $\vec{s}(t)$ at this time,

$$\vec{v}_p = \left.\frac{d\vec{s}(t)}{dt}\right|_{t=t_p}. \tag{5}$$

The offset of the sun is then,

$$\Delta\vec{s}(t) = \vec{s}(t) - (t - t_p)\vec{v_p} - \vec{s}(t_p). \tag{6}$$

Consider now observations towards the event at some time with given celestial coordinates,

$$(s_n(t), s_e(t)) = (\Delta\vec{s}(t) \cdot \hat{n}, \Delta\vec{s}(t) \cdot \hat{e}) \tag{7}$$

We can now write the form of $\delta\tau(t)$ and $\delta\beta(t))$ as,

$$(\delta\tau(t), \delta\beta(t)) = |\vec{\pi}_E|\Delta\vec{s}(t) = (\vec{\pi}_E \cdot \Delta\vec{s}(t), \vec{\pi}_E \times \Delta\vec{s}(t)), \tag{8}$$

or more explicitly,

$$(\delta\tau(t), \delta\beta(t)) = (s_n(t)\pi_{EN} + s_e(t)\pi_{EE}, -s_n(t)\pi_{E,E} + s_e(t)\pi_{EN}) \tag{9}$$

where here we have introduced the microlensing parallax vector $\vec{\pi}_E = (\pi_{EN}, \pi_{EE})$. It is noted that,

$$|\vec{\pi}_E| = \frac{\pi_{\rm rel}}{\theta_E}. \tag{10}$$

Here, $\pi_{\rm rel}$ is the relative lens-source parallax. This leads to $\vec{u}(t) \to \vec{u}(t; u_0, t_0, t_E, \pi_{EN}, \pi_{EE})$ and therefore by equation (2) $\vec{\delta}_+(t) \to \vec{\delta}_+(t; u_0, t_0, t_E, \pi_{EN}, \pi_{EE}, \theta_E)$. Note that if you take $\pi_{\rm rel} = (1/D_S) - (1/D_L)$, then the mass of the lens can be directly obtained through θ_E and $|\vec{\pi_E}|$.

3 Deflection model of a lensed source

Now we write the full astrometric model for a source with a microlensing deflection term in both regimes. In both cases we assume the source has some initial reference position, $\vec{\xi}_{\text{ref}} = (\xi_{\text{ref,N}}, \xi_{\text{ref,E}})$ at reference time t_{ref}, and proper motion $\vec{\mu} = (\mu_N, \mu_E)$, and the parallax vector of the source is $\vec{\Pi}_S$, with the source parallax amplitude as π_S. We can write the apparent position of the source at some time t for the stationary observer model as,

$$\begin{aligned} \vec{\xi}(t; \xi_N, \xi_E, \mu_N, \mu_E, t_0, t_E, u_0, \pi_S, \theta_E) = \\ + \vec{\xi}_{\text{ref}} \\ + \vec{\mu}(t - t_{\text{ref}}) \\ + \vec{\Pi}_S(t; \pi_s) \\ + \vec{\delta}_+(t; t_0, t_E, u_0, \theta_E). \end{aligned} \tag{11}$$

For the moving observer regime we can write the apparent position as,

$$\begin{aligned} \vec{\xi}(t; \xi_N, \xi_E, \mu_N, \mu_E, t_0, t_E, u_0, \pi_{EN}, \pi_{EE}, \pi_S, \theta_E) = \\ + \vec{\xi}_{\text{ref}} \\ + \vec{\mu}(t - t_{\text{ref}}) \\ + \vec{\Pi}_S(t; \pi_s) \\ + \vec{\delta}_+(t; t_0, t_E, u_0, \pi_{EN}, \pi_{EE}, \theta_E). \end{aligned} \tag{12}$$

Note that if a parallax signal is detected in the deflection (we can fit the moving observer model) then the mass of the lens can be directly inferred from the astrometric time series. However, if we don't observe a parallax signal in the deflection (we are only able to fit the stationary observer model) then only θ_E can be inferred from the astrometric time series. Determination of the lens mass would require auxiliary information about the lens (the lens parallax) which could be obtained from Gaia Data Release 2 for example.

References

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