Appendix E
Metric convention conversion table

In this book we have systematically used the metric convention, $\eta_{\mu\nu} = \text{Diag}[-1, +1, +1, +1]$, the "Pauli," "East Coast," or "mostly plus" metric. The other common convention, the "Bjorken and Drell," "West Coast," or "mostly minus" convention, takes $\eta_{\mu\nu} = \text{diag}[+1, -1, -1, -1]$. The "mostly minus" metric convention is currently in more common use in the field of phenomenology. The "mostly plus" convention predominates in the general relativity, string theory, supersymmetry, and formal field-theory communities.

To make this book more useful to its intended audience, who primarily use the opposite metric convention, we describe in this appendix the differences between these conventions, culminating in a "translation table" between the conventions, which should ease the difficulty in hopping between our conventions and the conventions appearing in most of the relevant literature. There are other conventions besides the metric which must be decided on, and these are not uniform in either community; since it would be too complicated to discuss every possible set of conventions, we will focus only on the most common coherent set of "mostly minus" conventions, taken to be those of Peskin and Schroeder, "*An Introduction to Quantum Field Theory*," Westview, 1995.

Finally, we will end this section with an explanation of why we prefer the "mostly plus" metric. We postpone that discussion to the end because some physicists approach this issue with almost religious conviction, and it is important to us that you not slam this book shut before reading the rest of this appendix.

E.1 Propagation of the differences

In going between metric conventions, it is necessary to decide what is kept the *same*. Generally $x^\mu = (t, \vec{x})$ in both conventions, and similarly $p^\mu = (E, \vec{p})$. Therefore $\partial_\mu = (\partial_t, \partial_{\vec{x}})$ is also the same. This means that x_μ, p_μ, and ∂^μ are opposite. The relations for spinorial objects, gauge fields, and generators of transformations are more complicated and are discussed in turn.

E.1.1 Dirac algebra

The Dirac matrices γ^μ are required to obey the anticommutation relations (the Clifford algebra)

$$\left\{ \gamma^\mu, \gamma^\nu \right\} \equiv \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\eta^{\mu\nu} \tag{E.1}$$

The difference in sign in $\eta^{\mu\nu}$ then requires a difference in normalization for the Dirac matrices. We will only discuss the chiral basis for the Dirac matrices. In this case, the relationship is

$$\gamma^\mu_{\text{mostly plus}} = -\mathrm{i}\gamma^\mu_{\text{mostly minus}} \qquad \text{or} \qquad \mathrm{i}\gamma^\mu_{\text{mostly plus}} = \gamma^\mu_{\text{mostly minus}} \tag{E.2}$$

In the mostly plus metric, all gamma matrices are Hermitian except γ^0, which is anti-Hermitian. In the mostly minus metric, the reverse is true; γ^0 is Hermitian but the others are anti-Hermitian. Because of this change, it is necessary in the mostly plus case to introduce a new matrix $\beta = \mathrm{i}\gamma^0_{\text{mostly plus}} = \gamma^0_{\text{mostly minus}}$, which is Hermitian and which serves the role of relating $\bar{\psi}$ and $\psi^\dagger$. In the mostly minus case, this matrix is the same as γ^0 and is generally not given an independent symbol.

The factor of i changes the appearance of the fermionic kinetic term in the Lagrangian;

$$\mathcal{L}_{\text{fermion}} = -\bar{\psi}(\not{D} + m)\psi\,, \text{ “mostly plus ,”} \qquad \bar{\psi}(\mathrm{i}\not{D} - m)\psi\,, \text{ “mostly minus”} \tag{E.3}$$

This makes a corresponding change in the Dirac equation,

$$\text{“mostly plus”: } [\mathrm{i}\not{p} + m]u(p) = 0\,; \qquad \text{“mostly minus”: } [\not{p} - m]u(p) = 0 \tag{E.4}$$

We reiterate that these expressions look different, but they have exactly the same content; the different appearance is because the definition of the symbol γ^μ has changed, and because $\gamma^\mu p_\mu$ in one case is $-\gamma^0 p^0 + \vec{\gamma}\cdot\vec{p}$, while in the other it is $\gamma^0 p^0 - \vec{\gamma}\cdot\vec{p}$.

The sign of the matrix γ^5 is also convention dependent. The eigenspinors of γ^5 are the spinors of definite chirality. The old convention for γ^5 was that right-handed spinors should have eigenvalue 1 and left-handed spinors should have eigenvalue −1. The "mostly minus" literature has generally maintained this convention. However, this convention was established before the discovery of the weak interactions, which couple exclusively to left-handed particles; so it is in some ways more convenient to adopt the opposite sign convention. Weinberg does this and we have followed his convention. Therefore,

$$\begin{aligned} \gamma^5_{\text{mostly plus}} &= -\mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \\ \gamma^5_{\text{mostly minus}} &= \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3 = -\gamma^5_{\text{mostly plus}} \end{aligned} \tag{E.5}$$

We emphasize that this is *not* a consequence of our choice of metric, it is a separate choice to modernize the notation which we choose to make at the same time. Because of this choice, the ubiquitous $2P_{\mathrm{L}} = (1+\gamma^5)$ projectors which appear in this book would be $2P_{\mathrm{L}} = (1-\gamma^5)$ in the notation of almost all "mostly minus" authors.

In addition, there are differing conventions in the definition of the charge conjugation matrix C (which should be distinguished from the charge conjugation operator $\mathcal{C}$). We have followed the most common practice of defining C such that, for Majorana fermions, Eq. (1.97) holds: $\psi_{\mathrm{M}} = C\overline{\psi}^T_{\mathrm{M}}$. We cannot compare with Peskin and Schoeder because C is not defined there. Therefore we take the mostly minus convention also to give $\psi_{\mathrm{M}} = C\overline{\psi}^T_{\mathrm{M}}$, which requires $C = -\mathrm{i}\gamma^2\gamma^0$ (mostly minus).

Finally, because of the different factor of i in γ^μ, the behavior of fermion bilinears under Hermitian conjugation differs in the two conventions. In Problem 1.1, the first set of relations all hold unchanged in mostly minus, but in the second and third sets, the equations involving γ^μ and $\gamma^\mu\gamma^5$ have the opposite sign.

These differences are summarized in Table E.1.

E.1.2 Gauge fields and Poincaré generators

We follow the convention, $\mathrm{D}_\mu = \partial_\mu - \mathrm{i}gT^aA^a_\mu$ for gauge fields (with $T^a = q$ the charge for a $U(1)$ field). Peskin and Schroeder choose the opposite sign for $U(1)$ fields but the same sign for non-abelian fields. Therefore,

$$A^\mu_{\text{mostly plus}} = A^\mu_{\text{mostly minus}}, \quad \text{but} \quad G^\mu_{\text{mostly plus}} = -G^\mu_{\text{mostly minus}} \tag{E.6}$$

With this sign convention, $A^\mu = (\Phi, \vec{A})$, with Φ and $\vec{A}$ the conventional scalar and vector potentials, in both conventions. With this choice, the

Table E.1 *Metric-convention conversion table*

Equation	"mostly plus"	"mostly minus"
Clifford Algebra	$\left[\gamma^\mu, \gamma^\nu\right] = 2\eta^{\mu\nu}$	$\left[\gamma^\mu, \gamma^\nu\right] = 2\eta^{\mu\nu}$
Dirac Lagrangian	$-\overline{\psi}(\not{D}+m)\psi$	$\bar{\psi}(\mathrm{i}\not{D}-m)\psi$
Dirac Equation	$[\mathrm{i}\not{p}+m]u(p)=0$	$[\not{p}-m]u(p)=0$
Spinor bilinears	$\sum_\sigma u\overline{u}/v\overline{v}(p,\sigma) = -\mathrm{i}\not{p}\pm m$	$\sum_\sigma u\overline{u}/v\overline{v}(p,\sigma) = \not{p}\pm m$
γ Hermiticity	$\beta\gamma_\mu^\dagger = -\gamma_\mu\beta\,,\ \ \beta\equiv \mathrm{i}\gamma^0$	$\beta\gamma_\mu^\dagger = +\gamma_\mu\beta\,,\ \ \beta\equiv \gamma^0$
Charge conjugation	$\psi_{_M} = C\overline{\psi}_{_M}^{T},\ \ C^{T}=-C$	same
Gamma transposes	$\gamma_\mu^{T} C = -C\gamma_\mu$	same

relation between the field strength and the non-covariant field strength is opposite for "mostly plus" as for "mostly minus": the field strengths are

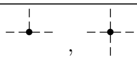
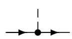
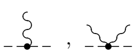
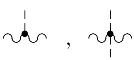
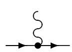
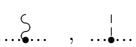
$$\begin{aligned}
\text{“mostly plus”}: \vec{E}_i &= F_{i0} = F^{0i}, \quad \vec{B}_i = \frac{1}{2}\epsilon_{ijk}\times(F_{jk}=F^{jk}) \\
\text{“mostly minus”}: \vec{E}_i &= F_{0i} = F^{i0}, \quad \vec{B}_i = \frac{1}{2}\epsilon_{ijk}\times(F_{kj}=F^{kj})
\end{aligned} \tag{E.7}$$

As for the generators of the Lorentz group, we adopt the convention that $\hat{P}^\mu$ acting on a state return its four-momentum p^μ. Therefore $P^\mu_{\text{mostly plus}} = P^\mu_{\text{mostly minus}}$. The lowered components are opposite, $P_\mu = -\mathrm{i}\partial_\mu$ in "mostly plus" whereas it is $\mathrm{i}\partial_\mu$ in "mostly minus." The phase $\mathrm{e}^{-\mathrm{i}\omega t+\mathrm{i}p\cdot x}$ becomes $\mathrm{e}^{\mathrm{i}p_\mu x^\mu}$ in "mostly plus," while it is $\mathrm{e}^{-\mathrm{i}p_\mu x^\mu}$ in "mostly minus." An active translation by ξ^μ is therefore implemented by the operator $\mathrm{e}^{-\mathrm{i}\hat{P}^\mu x_\mu}$. (To remember this, recall that an active transformation is one which changes the position or time of a particle. If a particle with wave function $\mathrm{e}^{\mathrm{i}\vec{p}\cdot\vec{x}}$ is shifted so what was its origin is now at $\vec{\xi}$, then the phase is 0 at $\vec{\xi}$ and must be $\mathrm{e}^{-\mathrm{i}\vec{p}\cdot\vec{\xi}}$ at the origin.)

In both conventions, $J^{\mu\nu} = x^\mu p^\nu - p^\mu x^\nu$ when acting on a scalar field. That is, $J^{\mu\nu}_{\text{mostly plus}} = J^{\mu\nu}_{\text{mostly minus}}$. The object with both lowered indices is also the same in the two conventions, but when one index is raised and one is lowered, the conventions differ. With these conventions, $\vec{J}_i = \frac{1}{2}\epsilon_{ijk}J_{jk}$ is the conventional angular momentum operator. Because of the opposite sign on the metric, the signs in Appendix C Eq. (C.25) and Eq. (C.26) are flipped between the two conventions.

Table E.2 *Metric-convention conversion table for Feynman rules*

Propagators Spin	symbol	"mostly plus"	"mostly minus"
0		$\dfrac{-\mathrm{i}}{p^2+M^2-\mathrm{i}\epsilon}$	$\dfrac{\mathrm{i}}{p^2-M^2+\mathrm{i}\epsilon}$
$\frac{1}{2}$		$\dfrac{-\mathrm{i}(-\mathrm{i}\not{p}+m)}{p^2+m^2-\mathrm{i}\epsilon}$	$\dfrac{\mathrm{i}(\not{p}+m)}{p^2-m^2+\mathrm{i}\epsilon}$
1 Massless		$\dfrac{-\mathrm{i}\eta^{\mu\nu}}{p^2-\mathrm{i}\epsilon}$	$\dfrac{-\mathrm{i}\eta^{\mu\nu}}{p^2+\mathrm{i}\epsilon}$
1 Unitary		$\dfrac{-\mathrm{i}\left(\eta^{\mu\nu}+\frac{p^\mu p^\nu}{M^2}\right)}{p^2+M^2-\mathrm{i}\epsilon}$	$\dfrac{-\mathrm{i}\left(\eta^{\mu\nu}-\frac{p^\mu p^\nu}{M^2}\right)}{p^2-M^2+\mathrm{i}\epsilon}$
1 R_ξ		$\dfrac{-\mathrm{i}\left(\eta^{\mu\nu}+\frac{(\xi-1)p^\mu p^\nu}{p^2+\xi M^2}\right)}{p^2+M^2-\mathrm{i}\epsilon}$	$\dfrac{-\mathrm{i}\left(\eta^{\mu\nu}+\frac{(\xi-1)p^\mu p^\nu}{p^2-\xi M^2}\right)}{p^2-M^2+\mathrm{i}\epsilon}$

Vertices type	symbol	"mostly plus"	"mostly minus"
Scalar		$-\mathrm{i}\lambda\nu$, $-\mathrm{i}\lambda$	$-\mathrm{i}\lambda\nu$, $-\mathrm{i}\lambda$
Yukawa		$-\mathrm{i}f_{mn}$	$-\mathrm{i}f_{mn}$
Gauge-scalar		$\mathrm{i}e(p-k)_\mu$, $-\mathrm{i}e^2\eta_{\mu\nu}$	$\mathrm{i}e(p-k)_\mu$, $+\mathrm{i}e^2\eta_{\mu\nu}$
Gauge-Higgs		$-\mathrm{i}e^2\nu\eta_{\mu\nu}$, $-\mathrm{i}e^2\eta_{\mu\nu}$	$\mathrm{i}e^2\nu\eta_{\mu\nu}$, $\mathrm{i}e^2\eta_{\mu\nu}$
$A\bar{\psi}\psi$		$-e\gamma_\mu$	$\mathrm{i}e\gamma_\mu$ [$-\mathrm{i}$, abelian]
A^3, A^4		$+gf_{abc}\ldots$, $-\mathrm{i}g^2\ldots$	$+gf_{abc}\ldots$, $-\mathrm{i}g^2\ldots$
ghost		$-gp_\mu$, $-\mathrm{i}\xi g^2 v$	$+gp_\mu$, $-\mathrm{i}\xi g^2 v$

We have taken these differences, the differences due to the Dirac algebra described above, and the differences due to the metric, and used them to find the changes to Feynman rules, summarized in Table E.2. Vertices involving gauge bosons are for non-abelian interactions; for QED, the sign of e in "mostly minus" must be switched.

E.2 Why we use "mostly plus"

There are two key advantages of the "mostly plus" metric.

First, it is much easier to go back and forth between covariant and non-covariant notation when using "mostly plus." This is because the non-covariant notation also involves a metric, δ_{ij}, the metric on the three spatial indices. It would be awkward to make this metric purely negative, and no one does. In the "mostly plus" convention, in passing to the non-covariant notation, the part involving a metric stays unchanged, only the temporal part, which is already being split off and treated differently, has its sign flipped. In the "mostly minus" convention, to pass to non-covariant notation, it is the part which is otherwise being treated as special which retains its sign, and the term which has a metric in it must have its sign flipped.

Second and even more important, most complicated calculations in quantum field theory involve Wick rotation, that is, continuation to an imaginary value of the time or momentum variable. Indeed, field theories are probably only formally well defined after such continuation. When using the "mostly plus" convention, this continuation is very simple; the negative term in the metric is switched to being positive. When using the "mostly minus" convention, if one merely continues the time or frequency coordinate to imaginary values, one is left with a totally negative metric. One either has to work with a totally negative metric, or flip the sign convention of the metric at the same time as analytically continuing. Either approach introduces extra opportunities for confusion and error, and neither one is very appealing.

In addition to these major advantages, there are a number of minor advantages to the "mostly plus" metric.

(i) Photon polarization vectors ϵ_μ have positive squares. Similarly, most components of the gauge field A^μ have positive norm. The unphysical, negative norm gauge field states which can arise in certain quantization procedures (Gupta–Bleuler) arise from the negative piece of the metric, rather than from the positive piece.

(ii) The sign on scalar and vector propagators is the same.

(iii) Most of the Dirac matrices are Hermitian.

To be fair, there are also advantages to the "mostly minus" metric. Indeed, if there were not, then everyone would agree by now on a metric convention. In our view, these advantages are,

(i) most four-momenta encountered in particle physics are timelike, and this convention gives them positive squares, $p^2 = m^2 > 0$;
(ii) the matrix β and the matrix $\gamma^0 = \gamma_0$ are the same;
(iii) the Dirac propagator involves $\not{p} + m$, which does not contain a relative factor of i between the two terms. In addition, because the γ matrices satisfy $\gamma_\mu^\dagger \beta = \beta \gamma_\mu$ without a minus sign, there are no "surprise minus signs" emerging from the complex conjugation of a matrix element.

Besides these technical reasons is the practical one: "mostly minus" users are in the majority in the phenomenology community and communication within the community is easiest if people converge on a set of conventions. The importance of this final reason should not be underestimated.

Based on our experience we prefer the "mostly plus" metric, and have written this book in that convention. We hope that this appendix, and in particular the two lookup tables it contains, eases the translation between the conventions in practical calculations and renders this book usable to both communities.