OPTIMIZATION OF A ONE DIMENSIONAL HYPERTELESCOPE FOR A DIRECT IMAGING IN ASTRONOMY

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MOTIVATION

- Direct Imaging of Exoplanets.
- Challenges in Designing an Instrument for High Resolution Imaging.
- The Role of Optimization in Improving the Point Spread Function (PSF).

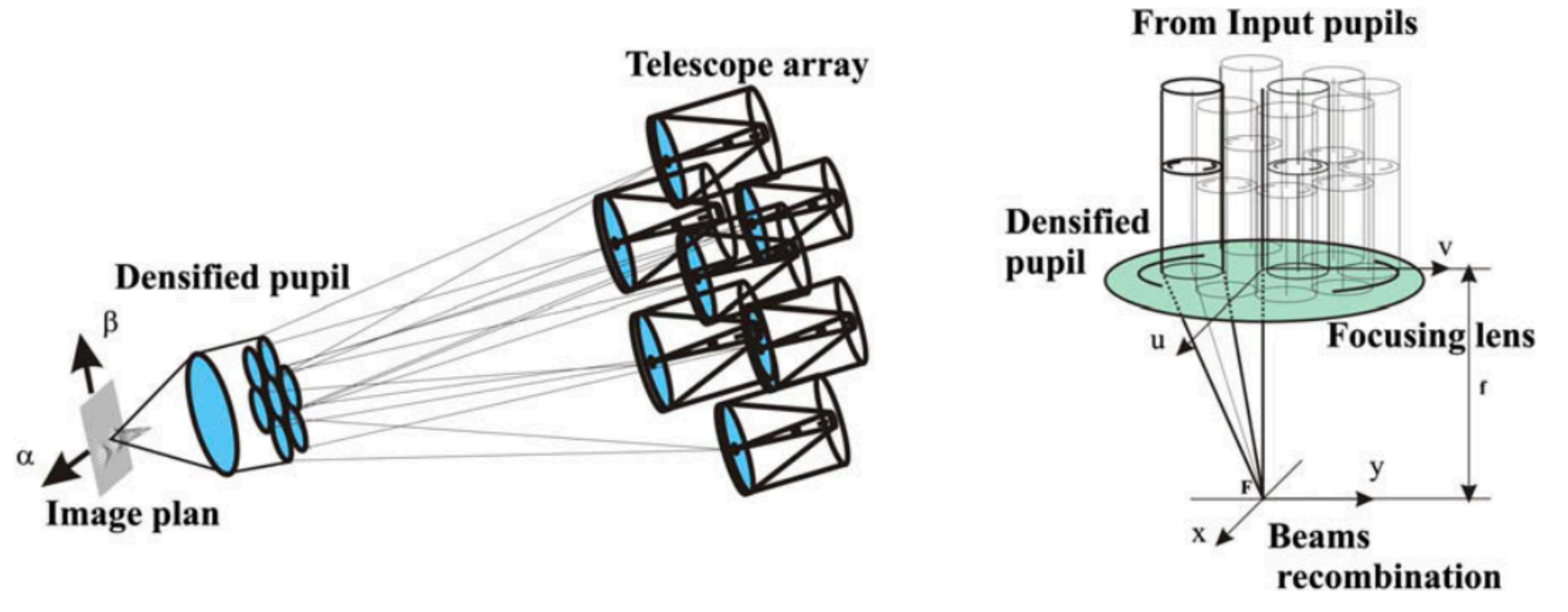


Fig. 1. Structure of a hypertelescope.

PROBLEM FORMULATION

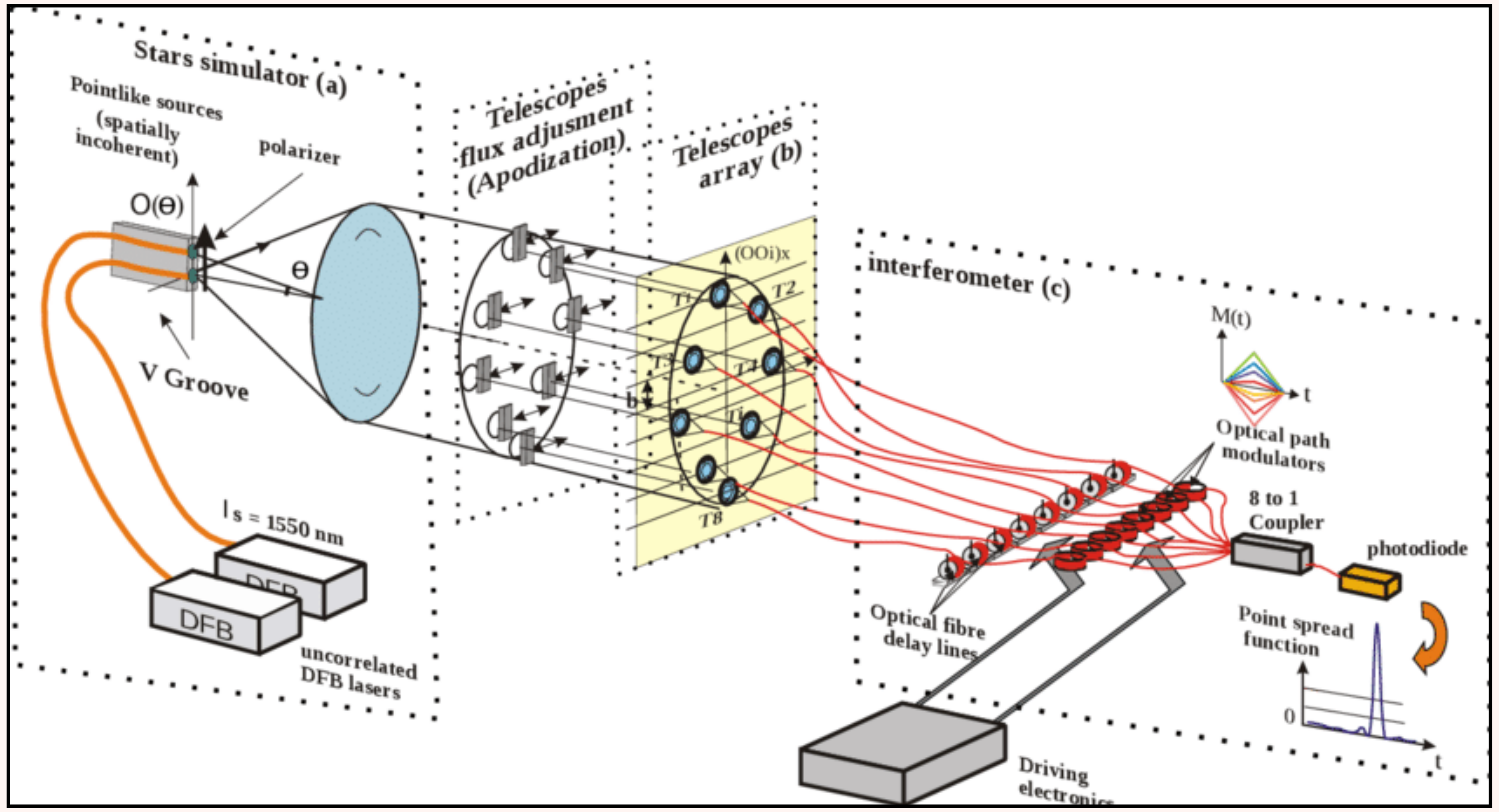
Point Spread Function

- The PSF represents the response of an imaging system to a point source, and its quality affects the system's ability to resolve celestial objects.
- In the context of a hypertelescope or an astronomical interferometer, the PSF represents the shape and intensity distribution of the image produced by the instrument when observing a point source.
- By doing Some rigorous Mathematics and taking the conditions suitable to our paper we arrive at a PSF function.
- The normalized PSF for hypertelescope is given by:

$$\Psi(\alpha) = |\frac{2}{\pi\alpha} J_1(\pi\alpha)|^2 \; |\Sigma_{k=1}^{n} a_k \, e^{-\frac{2i\pi u_k}{d}\alpha}|^2$$

We must find the moduli a_k and the positions u_k for which the PSF is as small as possible in a region close to the main central lobe. We define an interval $[\alpha_{min}, \alpha_{max}]$ of α values for which we want that the values $\Psi(\alpha)$ are as small as possible. It is in this interval that the main central lobe of a secondary source of light could be detected. We call it the clean field of view (CLF).

How Do we get PSF Actually ?



How Does a Normalised PSF Looks Like ?

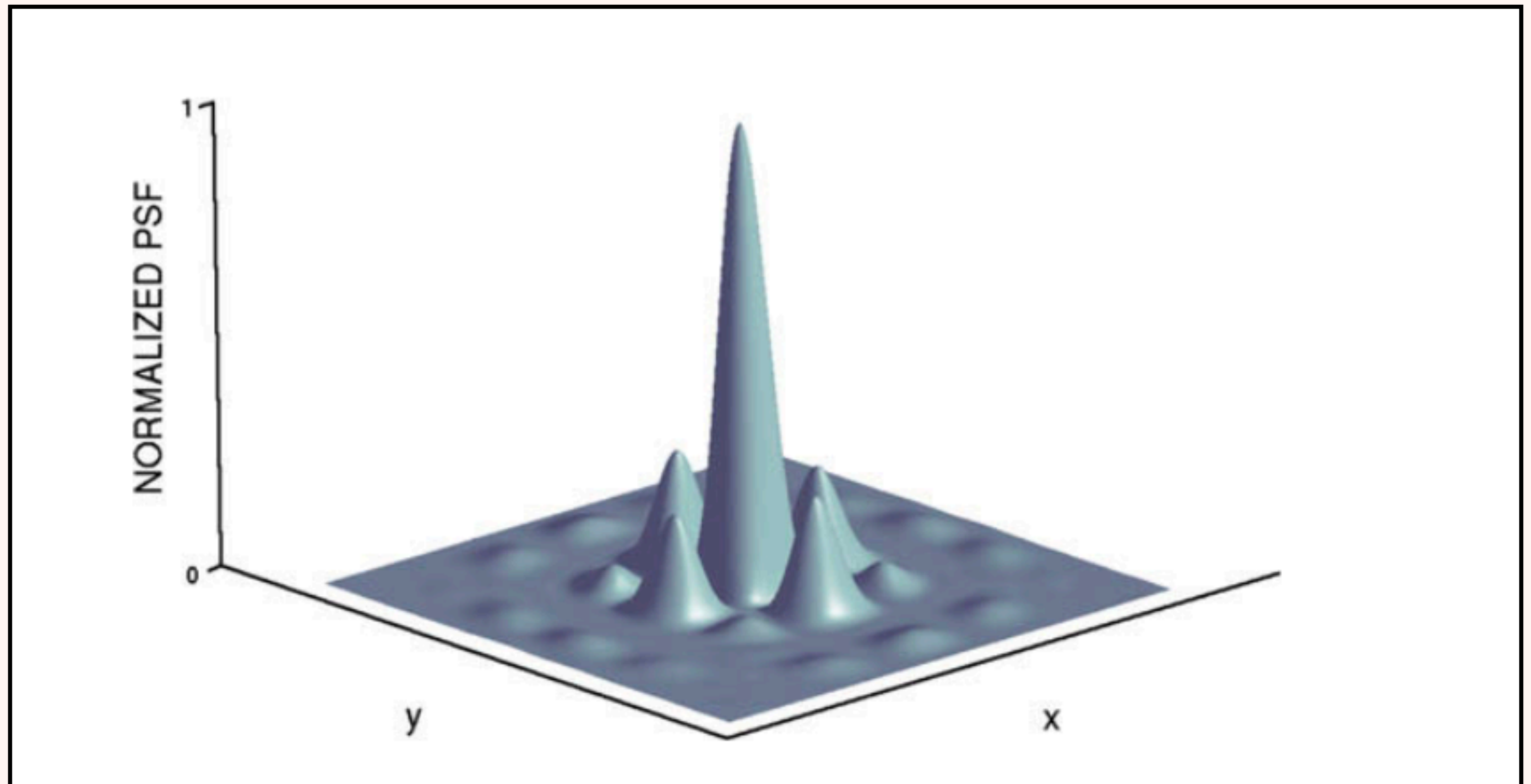


Fig. 2. Normalized PSF with four pupils.

How do we approach to the Problem?

1. Model the problem as a semi-infinite nonlinear minimization problem, which represents the initial complex and challenging nature of the problem.

2. Transform the problem into a more manageable form by applying discretization, resulting in a minimization problem with a finite number of constraints. Discretization is a process of converting a continuous problem into a discrete (finite) one by breaking it into smaller, finite components. In this case, the authors convert the semi-infinite problem into a minimization problem with a finite number of constraints, which is easier to handle computationally.

3. Use a minimization solver (an optimization algorithm) to find the optimal solution, which provides the desired moduli (amplitudes) and positions of the output pupils necessary for high-resolution and high dynamic range imaging in the hypertelescope

Minimize : Max{ $\Psi(\alpha)$: $\alpha_{min} \leq \alpha \leq \alpha_{max}$ }

Subject to : 1. $u_{k+1} - u_k \leq d$, for $k = 1,2,3,...(n-1)$,

2. $\Sigma_{k=1}^{n} a_k = 1$,

3. $a_k \geq 0$, for $k = 0,1,2,3,...,n$.

Semi-infinite nonlinear optimization problem

Minimize : t

Subject to : 1. $|\frac{J_1(\pi\alpha)}{\alpha} \Sigma_{k=1}^{n} a_k \, e^{-\frac{2i\pi u_k}{d}\alpha}| \leq t$, $\alpha_{min} \leq \alpha \leq \alpha_{max}$,

2. $u_{k+1} - u_k \leq d$, for $k = 1,2,3,...(n-1)$

3. $\Sigma_{k=1}^{n} a_k = 1$,

4. $a_k \geq 0$, for $k = 0,1,2,3,...,n$

Discretization method

Minimize : t

Subject to : 1. $-t \leq \frac{J_1(\pi\alpha_j)}{\alpha_j} \Sigma_{k=1}^{n} a_k \, cos(\frac{2\pi}{d} u_k \, \alpha_j) \leq t$ for $j = 1,2,...q$, $u_1 \geq \frac{d}{2}$,

2. $u_{k+1} - u_k \leq d$, for $k = 1,2,3,...(n-1)$

3. $\Sigma_{k=1}^{n} a_k = 1$,

4. $a_k \geq 0$, for $k = 0,1,2,3,...,n$

Starting Point Strategy

A difficulty when solving problem is to avoid to be trapped by a locally optimal point which is not a global minimizer, a common situation which is hard to deal within nonlinear optimization. To cope with this difficulty we considered to solve several occurrences of the same problem but with a random starting point for the solver.

1. An even number of pupils, denoted by n = 2m, is considered. The case of an odd number can be treated similarly.
2. The pupils are symmetrically placed around zero, with the first m pupils having negative positions and the last m pupils having positive positions.

To generate the starting points for the optimization process, the following procedure is used :

Calculate the initial positions (u_init) of the pupils:

1. For the positive side, the positions are determined by: $(k - 0.5) * d/(\alpha_{min} + \alpha_{max})$ for k in the range of 1 to m+1.
2. For the negative side, the positions are the negatives of the positive side positions, arranged in reverse order.
3. Combine both the negative and positive side positions into a single array.

Calculate the initial amplitudes (a_init) of the pupils:

1. Initialize an array of ones with a length equal to the number of pupils (n).
2. Normalize the array by dividing each element by the sum of all elements, ensuring that the sum of all amplitudes equals 1

Results Obtained for eight pupils (m = 4), d = 1, $[\alpha_{min}, \alpha_{max}]$=[0.25,0.75]

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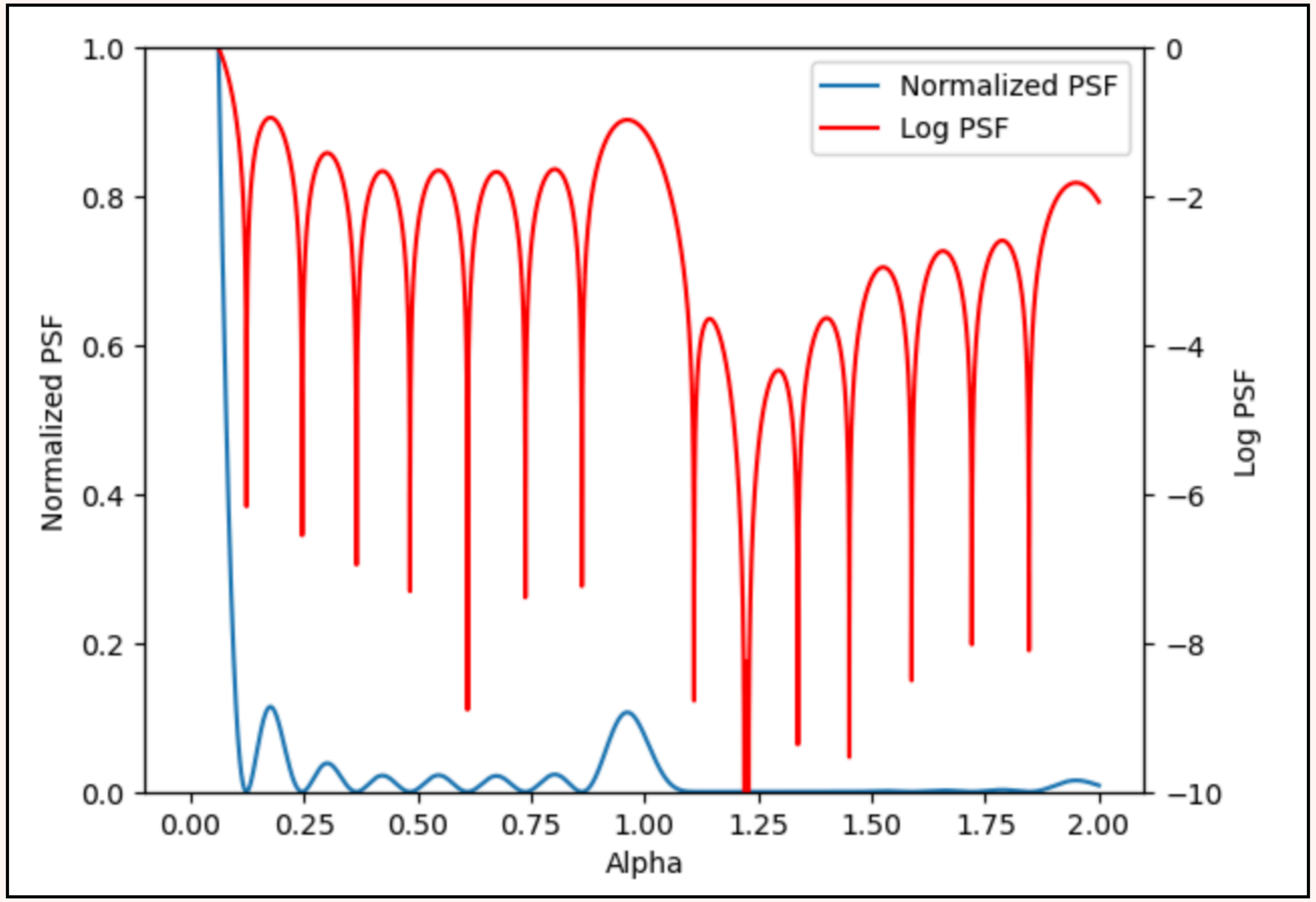
Optimal values of a_k: [0.12464567 0.12471124 0.12471702 0.124702   0.12471354 0.12471062
 0.12471698 0.12471734]
Optimal values of u_k: [-3.57926092 -2.52830733 -1.52745361 -0.50659417  0.50657642  1.52743999
  2.52829939  3.57924931]
Dynamic: 0.7240451356083052
Central flux: 0.8647771416715107
```

k	a_k	u_k	u_k - u_{k-1}
1	0.12464567	0.50657642	-
2	0.12471124	1.52743999	1.0208635760727653
3	0.12471702	2.52829939	1.0008593927626042
4	0.12470200	3.57924931	1.0509499208386166



Predicted for eight pupils (m = 4), d = 1, $[\alpha_{min}, \alpha_{max}]$=[0.25,0.75]

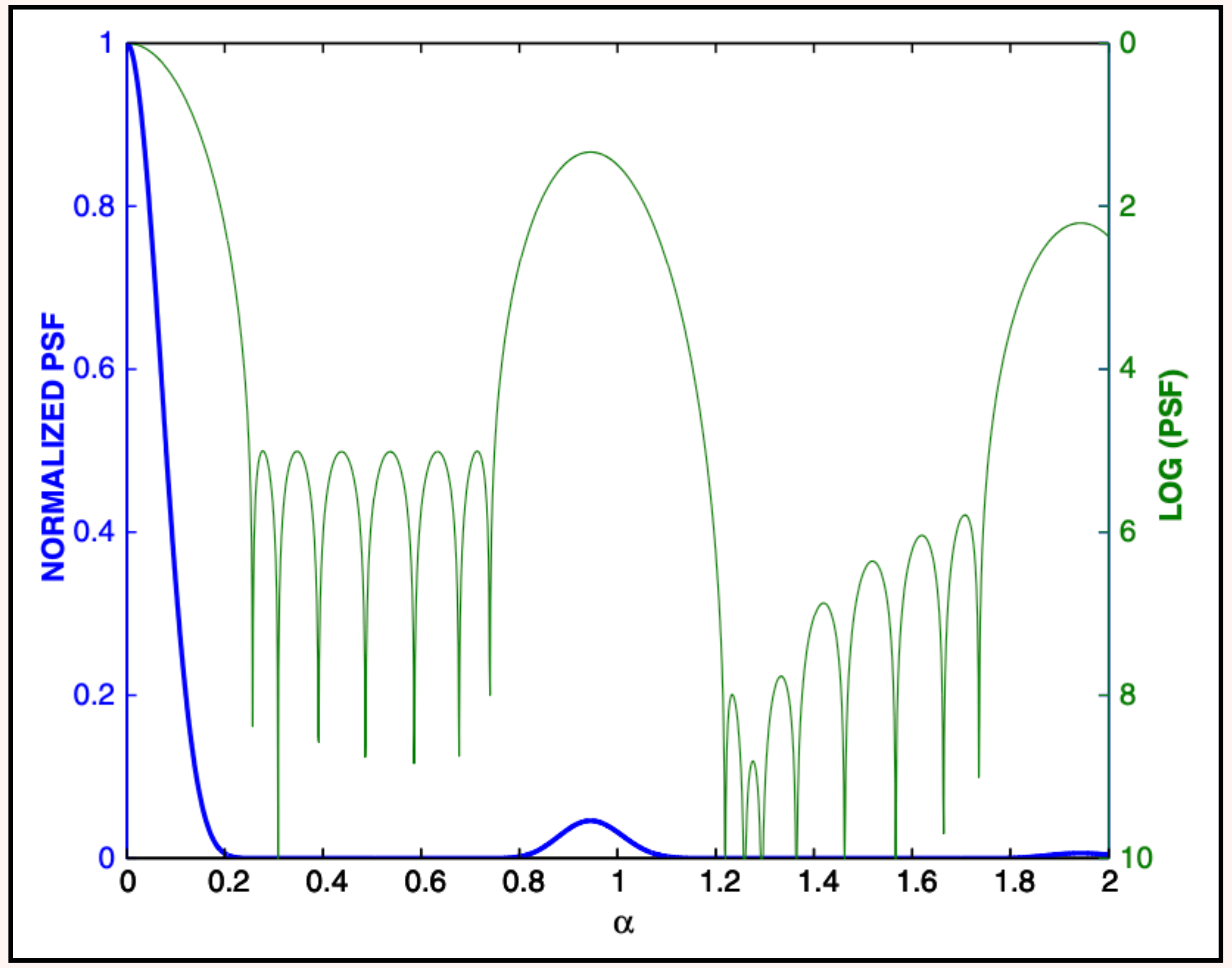
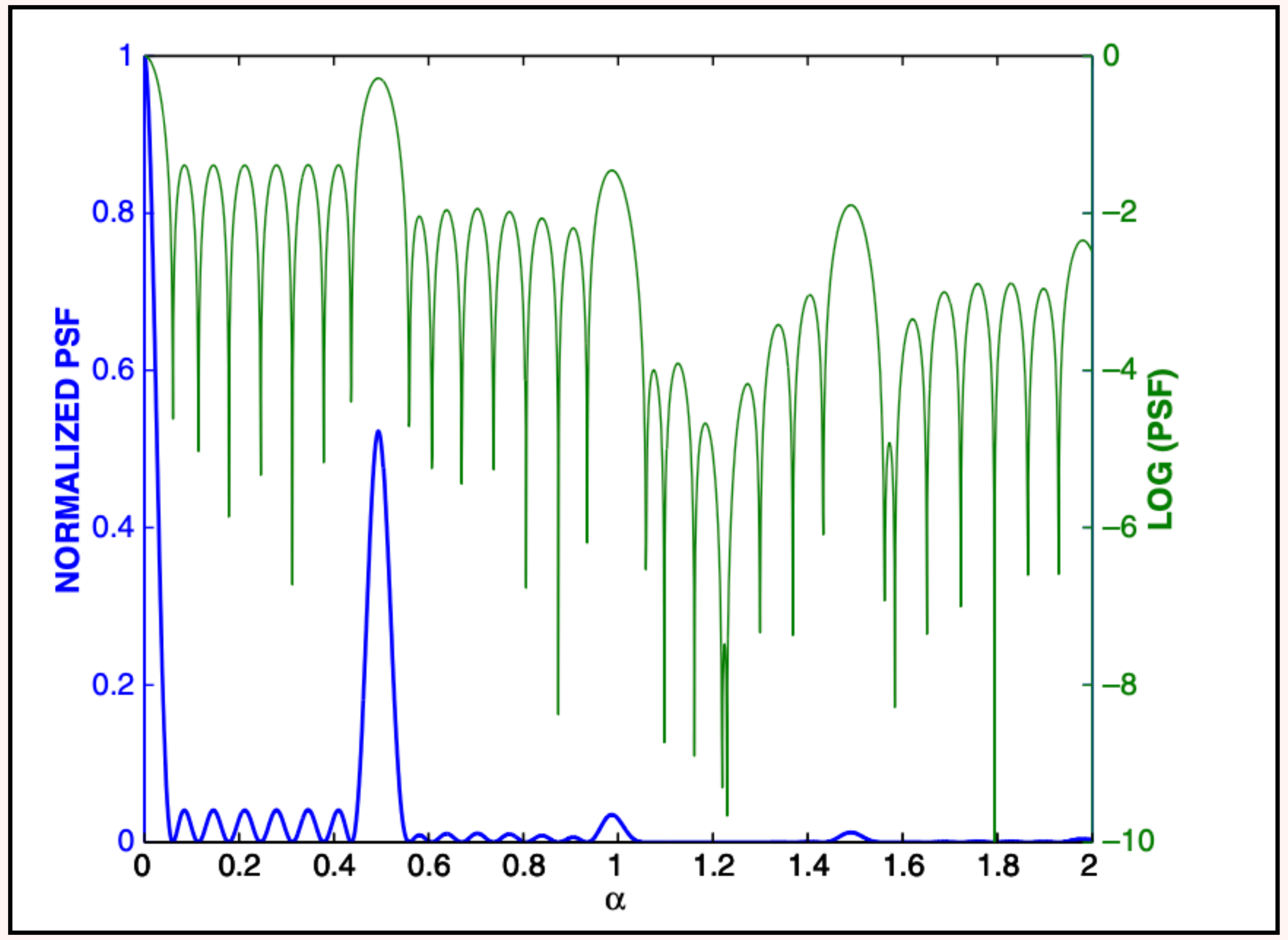


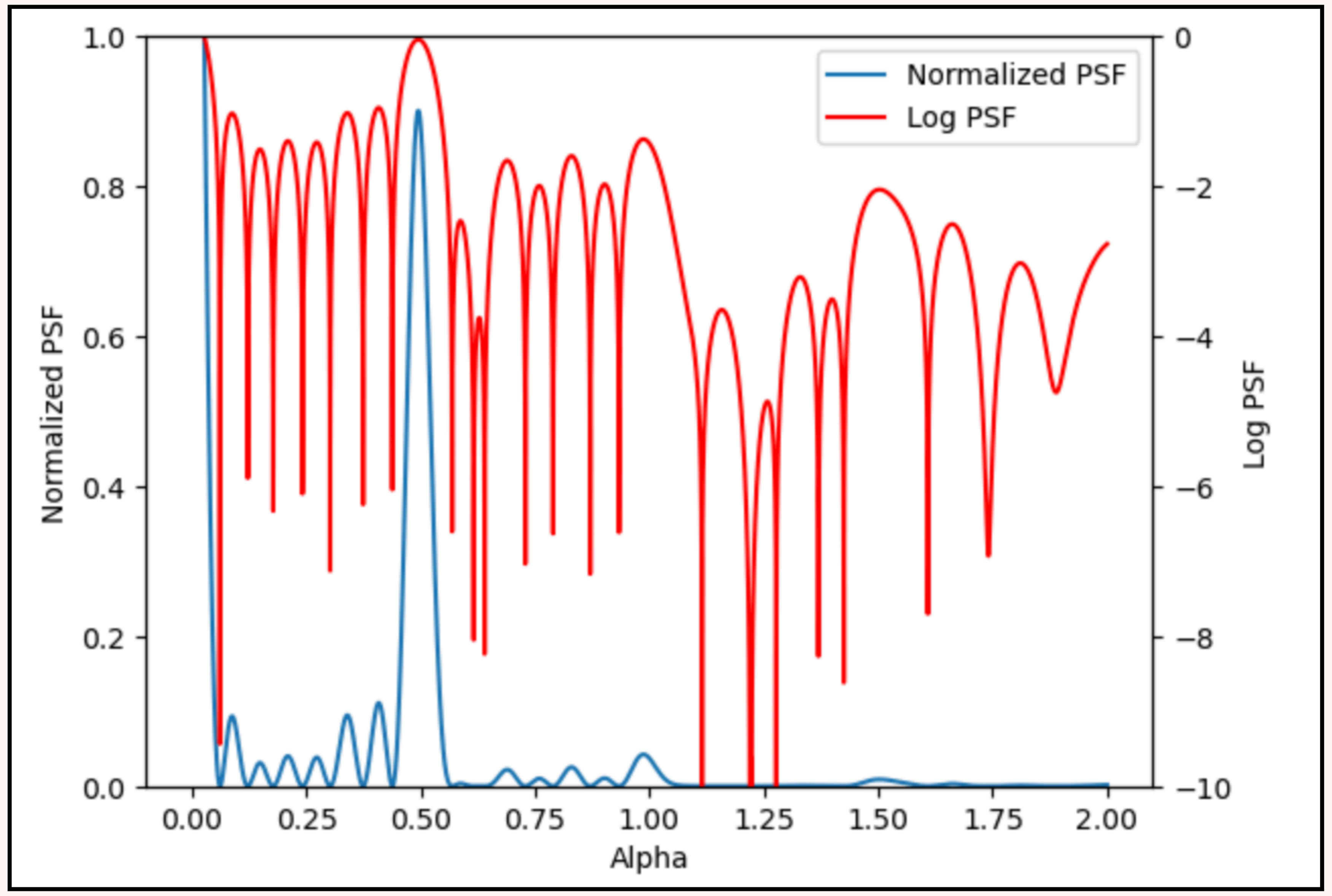
Table 1
Optimal values for the best solution

k	a_k	u_k	$u_k - u_{k-1}$
1	1.0000	0.5054	1.0108
2	0.7277	1.5167	1.0113
3	0.3644	2.5299	1.0132
4	0.1044	3.5510	1.0211

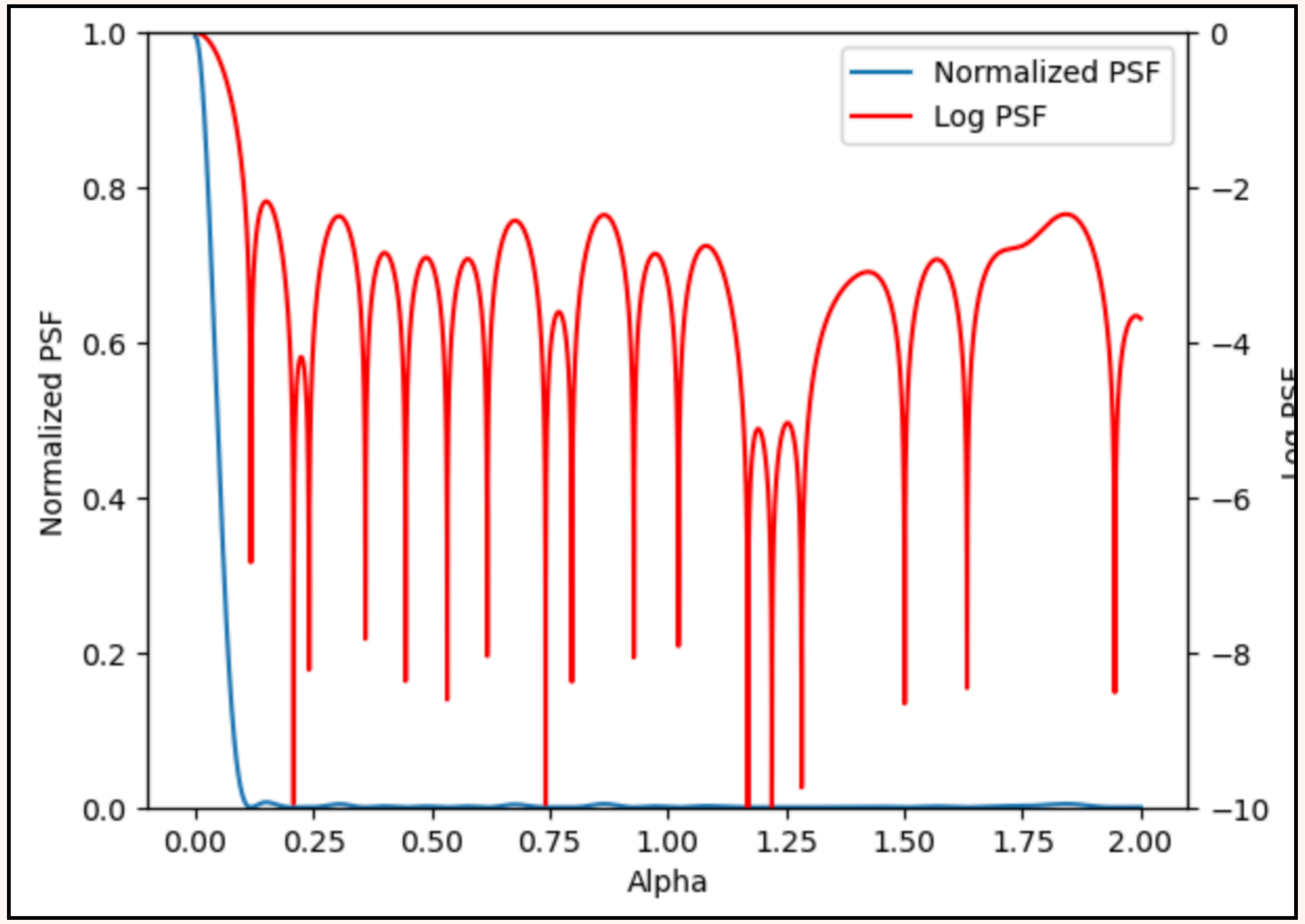
The value u_0 is set to the position of the pupil symmetric to the first one with respect to zero.

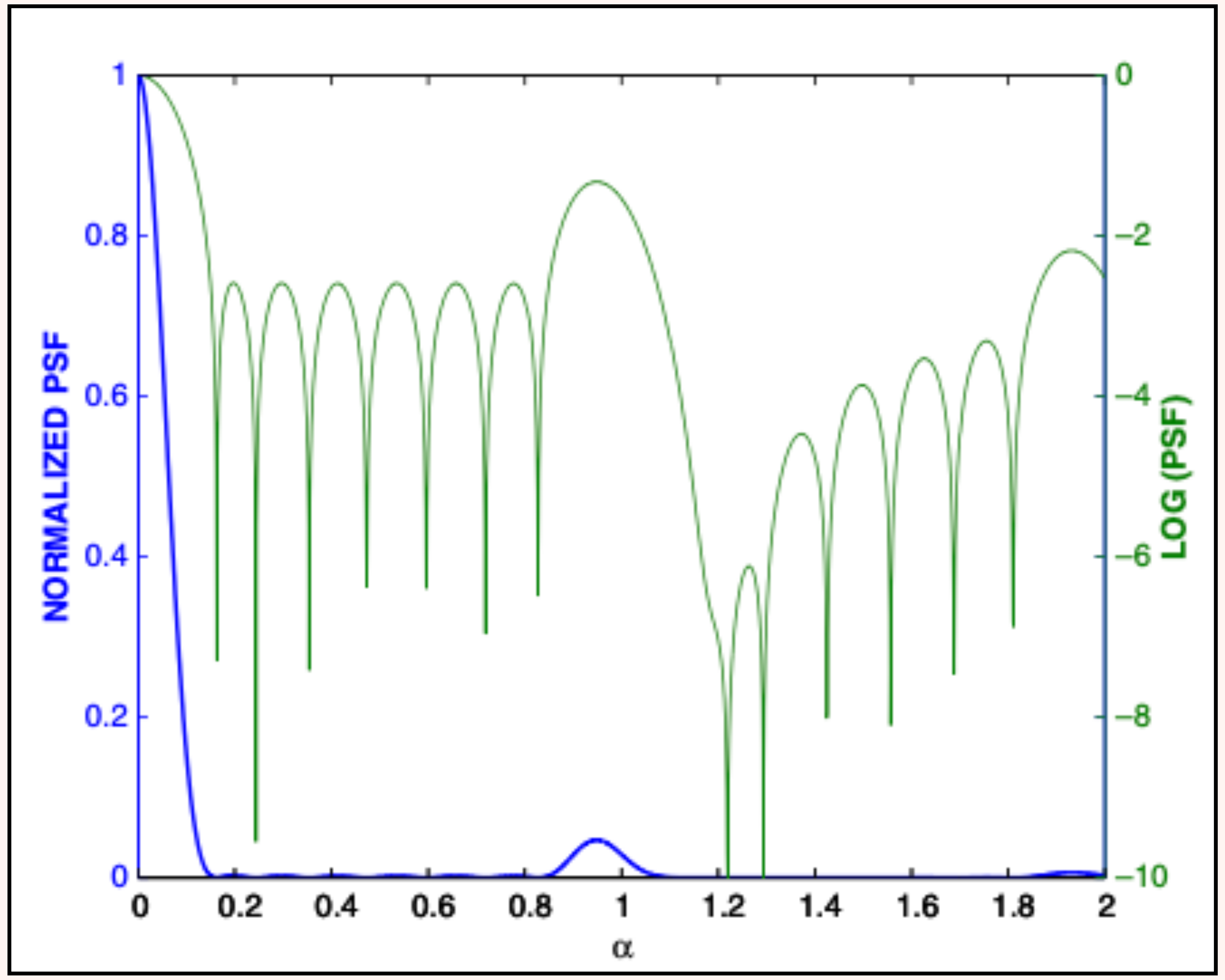
Predicted Vs Observed for eight pupils (m = 4), d = 1, $[\alpha_{min}, \alpha_{max}]$=[0.05,0.45]





Results Obtained for eight pupils (m = 4), d = 1, $[\alpha_{min}\ ,\ \alpha_{max}]$=[0.15,0.85]





k	a_k	u_k	u_k − u_{k−1}
1	0.06474477	0.37303218	−
2	0.08592511	1.37403218	1.00100000000168
3	0.08406073	2.44736806	1.0733358747385937
4	0.07044488	4.03130573	1.583937668334476

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Optimal values of a_k: [0.06474477 0.08592511 0.08406073 0.07044488 0.09450025 0.07951576
 0.08338714 0.07241683]
Optimal values of u_k: [-5.0662277  -3.17001423 -1.90083763 -0.85099288  0.37303218  1.37403218
  2.44736806  4.03130573]
Dynamic: 1.6394503730749206
Central flux: 0.9548497267723799
```

Conclusion and Future Work

In this study, we investigated the problem of optimizing the positions and amplitudes of an array of pupils to minimize the side lobes of the point spread function. We formulated the problem as a semi-infinite nonlinear optimization problem and proposed a discretization method to transform it into a finite nonlinear optimization problem. We used the interior-point algorithm to solve the optimization problem and implemented a starting point strategy to provide a good initial solution for the algorithm.

The results obtained showed the effectiveness of the proposed approach in minimizing the side lobes of the point spread function and maintaining a high central flux. The comparison between the paper's results and our implementation demonstrated the validity and replicability of the proposed method.

- We can extend the problem formulation to handle odd numbers of pupils and other configurations, such as non-symmetrical placements of the pupils.
- We can study the impact of different starting point strategies on the convergence and the quality of the final solution.