Quantum Mechanics

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★ Single Delta Function.

Lets define the wave functions in different regions :

$\psi_1(x) = A1\, e^{i k x} + B1\, e^{-i k x}$, $\psi_3(x) = A3\, e^{i k x} + B3\, e^{-i k x}$, at x = a ~ (delta potential)

<1> Calculating the **Transfer Matrix** using the wave equation continuity and Differentiability condition for delta potential :

```
In[ ]:= m1 = {{e^(i k a), e^(-i k a)}, {e^(i k a) (λ + i k), e^(-i k a) (λ - i k)}};
m2 = {{e^(i k a), e^(-i k a)}, {i k e^(i k a), -i k e^(-i k a)}};
m2inverse = 1/Det[m2] {{m2[[2, 2]], -m2[[1, 2]]}, {-m2[[2, 1]], m2[[1, 1]]}};
m2inverse // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2} e^{-i a k} & -\frac{i e^{-i a k}}{2 k} \\ \frac{1}{2} e^{i a k} & \frac{i e^{i a k}}{2 k} \end{pmatrix}$$

In[∘]:=
```
Msingledelta = m2inverse.m1 // Simplify
```
Out[∘]=

$$\left\{\left\{1-\frac{\mathrm{i}\,\lambda}{2\,k},\,-\frac{\mathrm{i}\,e^{-2\,\mathrm{i}\,a\,k}\,\lambda}{2\,k}\right\},\,\left\{\frac{\mathrm{i}\,e^{2\,\mathrm{i}\,a\,k}\,\lambda}{2\,k},\,1+\frac{\mathrm{i}\,\lambda}{2\,k}\right\}\right\}$$

In[∘]:=
```
Msingledelta // MatrixForm
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} 1-\frac{\mathrm{i}\,\lambda}{2\,k} & -\frac{\mathrm{i}\,e^{-2\,\mathrm{i}\,a\,k}\,\lambda}{2\,k} \\ \frac{\mathrm{i}\,e^{2\,\mathrm{i}\,a\,k}\,\lambda}{2\,k} & 1+\frac{\mathrm{i}\,\lambda}{2\,k} \end{pmatrix}$$

<2> Finding **S Matrix** from the Transfer Matrix.

In[∘]:=
$$\text{Ssingledelta} = \frac{1}{\text{Msingledelta[[2, 2]]}}$$
```
  {{-Msingledelta[[2, 1]], 1}, {Det[Msingledelta], Msingledelta[[1, 2]]}} // Simplify
```
Out[∘]=

$$\left\{\left\{\frac{e^{2\,\mathrm{i}\,a\,k}\,\lambda}{2\,\mathrm{i}\,k-\lambda},\,\frac{2\,k}{2\,k+\mathrm{i}\,\lambda}\right\},\,\left\{\frac{2\,k}{2\,k+\mathrm{i}\,\lambda},\,\frac{e^{-2\,\mathrm{i}\,a\,k}\,\lambda}{2\,\mathrm{i}\,k-\lambda}\right\}\right\}$$

In[∘]:=
```
Ssingledelta // MatrixForm
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} \frac{e^{2\,\mathrm{i}\,a\,k}\,\lambda}{2\,\mathrm{i}\,k-\lambda} & \frac{2\,k}{2\,k+\mathrm{i}\,\lambda} \\ \frac{2\,k}{2\,k+\mathrm{i}\,\lambda} & \frac{e^{-2\,\mathrm{i}\,a\,k}\,\lambda}{2\,\mathrm{i}\,k-\lambda} \end{pmatrix}$$

<3> Unitary Condition of S Matrix **S (k; λ, a).S (k; -λ, -a) = $1_{2\,x\,2}$**

In[∘]:=
$$\text{ssingledelta} = \left\{\left\{\frac{-e^{-2\,\mathrm{i}\,a\,k}\,\lambda}{2\,\mathrm{i}\,k+\lambda},\,\frac{2\,k}{2\,k-\mathrm{i}\,\lambda}\right\},\,\left\{\frac{2\,k}{2\,k-\mathrm{i}\,\lambda},\,\frac{-e^{2\,\mathrm{i}\,a\,k}\,\lambda}{2\,\mathrm{i}\,k+\lambda}\right\}\right\};$$
```
Ssingledelta.ssingledelta // Simplify // MatrixForm
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

<4> When Potential will have **Parity Symmetry** :

In[∘]:=
```
PSingledelta = {{Ssingledelta[[1, 1]] + Ssingledelta[[1, 2]], 0},
  {0, Ssingledelta[[1, 2]] - Ssingledelta[[2, 2]]}} // Simplify
```
Out[∘]=

$$\left\{\left\{\frac{2\,k-\mathrm{i}\,e^{2\,\mathrm{i}\,a\,k}\,\lambda}{2\,k+\mathrm{i}\,\lambda},\,0\right\},\,\left\{0,\,\frac{2\,k+\mathrm{i}\,e^{-2\,\mathrm{i}\,a\,k}\,\lambda}{2\,k+\mathrm{i}\,\lambda}\right\}\right\}$$

In[∘]:=
```
PSingledelta // MatrixForm
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} \frac{2\,k-\mathrm{i}\,e^{2\,\mathrm{i}\,a\,k}\,\lambda}{2\,k+\mathrm{i}\,\lambda} & 0 \\ 0 & \frac{2\,k+\mathrm{i}\,e^{-2\,\mathrm{i}\,a\,k}\,\lambda}{2\,k+\mathrm{i}\,\lambda} \end{pmatrix}$$

<5> At a = 0 **Phase Shifts :**

In[∘]:=
```
a = 0;
PSingledelta // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} \frac{2k - i\lambda}{2k + i\lambda} & 0 \\ 0 & 1 \end{pmatrix}$$

1. $e^{i\,\delta_+(k)} = T + R = \frac{2k - i\lambda}{2k + i\lambda}$

2. $e^{i\,\delta_-(k)} = T - R = 1$

Double Delta Function.

(a) Transfer Matrix.

In[∘]:=

$$M1 = \frac{1}{2k} \{\{2k - i\lambda 2, -i\lambda 2\, e^{-2ika2}\}, \{i\lambda 2\, e^{2ika2}, 2k + i\lambda 2\}\};$$

$$M2 = \frac{1}{2k} \{\{2k - i\lambda 1, -i\lambda 1\, e^{-2ika1}\}, \{i\lambda 1\, e^{2ika1}, 2k + i\lambda 1\}\};$$

```
M = M1.M2 // Simplify
```

Out[∘]=

$$\left\{\left\{1 - \frac{i\lambda}{k}, -\frac{i\lambda}{k}\right\}, \left\{\frac{i\lambda}{k}, 1 + \frac{i\lambda}{k}\right\}\right\}$$

In[∘]:=
```
M // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 1 - \frac{i\lambda}{k} & -\frac{i\lambda}{k} \\ \frac{i\lambda}{k} & 1 + \frac{i\lambda}{k} \end{pmatrix}$$

(b) S - Matrix and its Unitary Condition.

In[∘]:=

$$S = \frac{1}{M[[2, 2]]} \{\{-M[[2, 1]], 1\}, \{Det[M], M[[1, 2]]\}\} \text{ // Simplify}$$

Out[∘]=

$$\left\{\left\{-\frac{i\lambda}{k + i\lambda}, \frac{k}{k + i\lambda}\right\}, \left\{\frac{k}{k + i\lambda}, -\frac{i\lambda}{k + i\lambda}\right\}\right\}$$

To Check Unitary of S matrix - S(k,λ,a) S(k,-λ,-a) = 1*1M

In[∘]:=
```
M3 = 1/(2 k) {{2 k + ⅈ λ2, ⅈ λ2 ⅇ^(2 ⅈ k a2)}, {-ⅈ λ2 ⅇ^(-2 ⅈ k a2), 2 k - ⅈ λ2}}
M4 = 1/(2 k) {{2 k + ⅈ λ1, ⅈ λ1 ⅇ^(2 ⅈ k a1)}, {-ⅈ λ1 ⅇ^(-2 ⅈ k a1), 2 k - ⅈ λ1}}
```

Out[∘]=

$$\left\{\left\{\frac{2\,k + i\,\lambda}{2\,k}, \frac{i\,\lambda}{2\,k}\right\}, \left\{-\frac{i\,\lambda}{2\,k}, \frac{2\,k - i\,\lambda}{2\,k}\right\}\right\}$$

Out[∘]=

$$\left\{\left\{\frac{2\,k + i\,\lambda}{2\,k}, \frac{i\,\lambda}{2\,k}\right\}, \left\{-\frac{i\,\lambda}{2\,k}, \frac{2\,k - i\,\lambda}{2\,k}\right\}\right\}$$

Defining “s” as S(k,-λ,-a)

In[∘]:=
```
m = M3.M4 // Simplify
```

Out[∘]=

$$\left\{\left\{1 + \frac{i\,\lambda}{k}, \frac{i\,\lambda}{k}\right\}, \left\{-\frac{i\,\lambda}{k}, 1 - \frac{i\,\lambda}{k}\right\}\right\}$$

In[∘]:=
```
s = 1/m[[2, 2]] {{-m[[2, 1]], 1}, {Det[m], m[[1, 2]]}} // Simplify
```

Out[∘]=

$$\left\{\left\{\frac{i\,\lambda}{k - i\,\lambda}, \frac{k}{k - i\,\lambda}\right\}, \left\{\frac{k}{k - i\,\lambda}, \frac{i\,\lambda}{k - i\,\lambda}\right\}\right\}$$

In[∘]:=
```
S // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} -\frac{i\,\lambda}{k+i\,\lambda} & \frac{k}{k+i\,\lambda} \\ \frac{k}{k+i\,\lambda} & -\frac{i\,\lambda}{k+i\,\lambda} \end{pmatrix}$$

In[∘]:=
```
s // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} \frac{i\,\lambda}{k-i\,\lambda} & \frac{k}{k-i\,\lambda} \\ \frac{k}{k-i\,\lambda} & \frac{i\,\lambda}{k-i\,\lambda} \end{pmatrix}$$

In[∘]:=
```
s.S // Simplify
```

Out[∘]=

{{1, 0}, {0, 1}}

In[∘]:=
```
S.s // Simplify
```

Out[∘]=

{{1, 0}, {0, 1}}

In[∘]:=
```
S.s // MatrixForm // Simplify
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

In[∘]:=
```
s.S // Simplify // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

(c) Setting $\lambda 1$ / $\lambda 2$ = 0 and we need to get Single Delta Function :

Case 1 :

In[ ]:=
```
λ2 = 0
```
Out[ ]=
```
0
```

In[ ]:=
```
M // Simplify // MatrixForm
```
Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 - \frac{i\lambda}{k} & -\frac{i\lambda}{k} \\ \frac{i\lambda}{k} & 1 + \frac{i\lambda}{k} \end{pmatrix}$$

In[ ]:=
```
S // Simplify // MatrixForm
```
Out[ ]//MatrixForm=

$$\begin{pmatrix} -\frac{i\lambda}{k+i\lambda} & \frac{k}{k+i\lambda} \\ \frac{k}{k+i\lambda} & -\frac{i\lambda}{k+i\lambda} \end{pmatrix}$$

Case 2 :

In[ ]:=
```
Clear[λ2]
```

In[ ]:=
```
λ1 = 0
```
Out[ ]=
```
0
```

In[ ]:=
```
M // Simplify // MatrixForm
```
Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 - \frac{i\lambda}{k} & -\frac{i\lambda}{k} \\ \frac{i\lambda}{k} & 1 + \frac{i\lambda}{k} \end{pmatrix}$$

In[ ]:=
```
S // Simplify // MatrixForm
```
Out[ ]//MatrixForm=

$$\begin{pmatrix} -\frac{i\lambda}{k+i\lambda} & \frac{k}{k+i\lambda} \\ \frac{k}{k+i\lambda} & -\frac{i\lambda}{k+i\lambda} \end{pmatrix}$$

In[ ]:=
```
Clear[λ1]
```

Here we can clearly see that the Double Delta function changes to Single Delta function when $\lambda 1$ /$\lambda 2$ = 0

(d) a2 - a1 ---> 0

We need to consider a2 - a1 = 0 ; i.e a1=a2=a

In[ ]:=
```
a1 = a2 = a
```
Out[ ]=
```
0
```

In[∘]:=
```
M // Simplify // MatrixForm
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} 1 - \frac{i\lambda}{k} & -\frac{i\lambda}{k} \\ \frac{i\lambda}{k} & 1 + \frac{i\lambda}{k} \end{pmatrix}$$

In[∘]:=
```
S // Simplify // MatrixForm
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} -\frac{i\lambda}{k+i\lambda} & \frac{k}{k+i\lambda} \\ \frac{k}{k+i\lambda} & -\frac{i\lambda}{k+i\lambda} \end{pmatrix}$$

Here in this condition when a1=a2=a; simply the λ_i get added which shows that the potential strength getting increased.

In[∘]:=
```
Clear[a1]
```

In[∘]:=
```
Clear[a2]
```

(e) When Potential will have Parity Symmetry :

The Parity Symmetry is Simply V(-x) = V(x) ; in out Hamiltonian the Kinetic part wont get effected by the Parity Symmetry, So if Potential has Parity Symmetry the Hamiltonian of the System also have the parity Symmetry H(-x) = H(x).
The wave equations also changes because we replace **x ----> -x**
So here to have parity Symmetry the conditions are $\lambda 1 = \lambda 2$ and a1+a2 = 0.

(f) S - Matrix for Parity Symmetry Potential and check condition of Scattering Matrix for Parity Symmetries.

In[∘]:=
```
λ1 = λ2 = λ
```
Out[∘]=

λ

In[∘]:=
```
a1 = -a
```
Out[∘]=

0

In[∘]:=
```
a2 = a
```
Out[∘]=

0

In[∘]:=
```
M // Simplify // MatrixForm
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} 1 - \frac{i\lambda}{k} & -\frac{i\lambda}{k} \\ \frac{i\lambda}{k} & 1 + \frac{i\lambda}{k} \end{pmatrix}$$

In[ ]:=
```
S // Simplify // MatrixForm
```
Out[ ]//MatrixForm=

$$\begin{pmatrix} -\frac{i\lambda}{k+i\lambda} & \frac{k}{k+i\lambda} \\ \frac{k}{k+i\lambda} & -\frac{i\lambda}{k+i\lambda} \end{pmatrix}$$

Condition to check S matrix in Parity is S[[1,1]] = S[[2,2]] ; S[[1,2]] = S[[2,1]]

In[ ]:=
```
S[[1, 2]] - S[[2, 1]] // Simplify
```
Out[ ]=
```
0
```

In[ ]:=
```
S[[1, 1]] - S[[2, 2]]
```
Out[ ]=
```
0
```

(g) Phase Shifts in Parity Symmetry Basis.

In[ ]:=
```
PS = {{S[[1, 1]] + S[[1, 2]], 0}, {0, S[[1, 2]] - S[[2, 2]]}} // Simplify
```
Out[ ]=

$$\left\{\left\{\frac{k - i\lambda}{k + i\lambda}, 0\right\}, \{0, 1\}\right\}$$

In[ ]:=
```
PS // MatrixForm
```
Out[ ]//MatrixForm=

$$\begin{pmatrix} \frac{k-i\lambda}{k+i\lambda} & 0 \\ 0 & 1 \end{pmatrix}$$

In[ ]:=
```
a = 0
```
Out[ ]=
```
0
```

In[ ]:=
```
PS // MatrixForm // Simplify
```
Out[ ]//MatrixForm=

$$\begin{pmatrix} \frac{k-i\lambda}{k+i\lambda} & 0 \\ 0 & 1 \end{pmatrix}$$

In[ ]:=
```
Clear[a]
```
When we put a = 0 , the separation is 0 ; that is the same we got for the Single Delta Function.

We can check that pole of S matrix gives the **Bound States** , for that replace k----> $i\,\kappa$

1. Parity even Sector, S blows up if $e^{-2\,ka} = -\frac{2k+\lambda}{\lambda}$.
2. Parity odd Sector , S blows up if $e^{-2\,ka} = \frac{2k+\lambda}{\lambda}$.

In[ ]:=
```
Clear[a]
```
In[ ]:=
```
Clear[m1]
```
In[ ]:=
```
Clear[m2]
```
In[ ]:=
```
Clear[S]
```

In[∘]:=
```
Clear[PS]
```

★ Right Step Potential :

Lets Define the wave equations for the right step potentials

$\# \; \psi_1 \; (x) = A_1 \, e^{i \, k1 \, x} + B_1 \, e^{-i \, k1 \, x}$

$\# \; \psi_2 \; (x) = A_2 \, e^{i \, k2 \, x} + B_2 \, e^{-i \, k2 \, x}$

continuty and diffrentiability at a give the Conditions at x = a :

<1> Calculating the Transfer Matrix :

In[∘]:=
$$t1 = \{\{e^{i\,k2\,a}, e^{-i\,k2\,a}\}, \{k2\,e^{i\,k2\,a}, -k2\,e^{-i\,k2\,a}\}\};$$

$$t1inverse = \frac{1}{Det[t1]} \{\{t1[[2,2]], -t1[[1,2]]\}, \{-t1[[2,1]], t1[[1,1]]\}\}; \; // \; Simplify$$

In[∘]:=
```
t1inverse // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2} e^{-i\,a\,k2} & \frac{e^{-i\,a\,k2}}{2\,k2} \\ \frac{1}{2} e^{i\,a\,k2} & -\frac{e^{i\,a\,k2}}{2\,k2} \end{pmatrix}$$

In[∘]:=
$$t2 = \{\{e^{i\,k1\,a}, e^{-i\,k1\,a}\}, \{k1\,e^{i\,k1\,a}, -k1\,e^{-i\,k1\,a}\}\};$$

```
Trightstep = t1inverse.t2 // Simplify
```

Out[∘]=

$$\left\{\left\{\frac{e^{i\,a\,(k1-k2)}\,(k1+k2)}{2\,k2}, \frac{e^{-i\,a\,(k1+k2)}\,(-k1+k2)}{2\,k2}\right\}, \left\{\frac{e^{i\,a\,(k1+k2)}\,(-k1+k2)}{2\,k2}, \frac{e^{-i\,a\,(k1-k2)}\,(k1+k2)}{2\,k2}\right\}\right\}$$

In[∘]:=
```
Trightstep // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} \frac{e^{i\,a\,(k1-k2)}\,(k1+k2)}{2\,k2} & \frac{e^{-i\,a\,(k1+k2)}\,(-k1+k2)}{2\,k2} \\ \frac{e^{i\,a\,(k1+k2)}\,(-k1+k2)}{2\,k2} & \frac{e^{-i\,a\,(k1-k2)}\,(k1+k2)}{2\,k2} \end{pmatrix}$$

Calculating Inverse of Right Step Potential Transfer Matrix

In[∘]:=
$$Trightstepinvrse = \frac{1}{Det[Trightstep]} \{\{Trightstep[[2,2]], -Trightstep[[1,2]]\}, \{-Trightstep[[2,1]], Trightstep[[1,1]]\}\}; \; // \; Simplify$$

In[∘]:=
```
Trightstepinvrse // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} \frac{e^{-i\,a\,(k1-k2)}\,(k1+k2)}{2\,k1} & -\frac{e^{-i\,a\,(k1+k2)}\,(-k1+k2)}{2\,k1} \\ -\frac{e^{i\,a\,(k1+k2)}\,(-k1+k2)}{2\,k1} & \frac{e^{i\,a\,(k1-k2)}\,(k1+k2)}{2\,k1} \end{pmatrix}$$

Calculating S Matrix for Right Step Potential

In[∘]:=
```
Srightstep = 1/Trightstep[[2, 2]]
   {{-Trightstep[[2, 1]], 1}, {Det[Trightstep], Trightstep[[1, 2]]}} // Simplify
```
Out[∘]=
$$\left\{\left\{\frac{e^{2 i a k1}(k1-k2)}{k1+k2}, \frac{2 e^{i a (k1-k2)} k2}{k1+k2}\right\}, \left\{\frac{2 e^{i a (k1-k2)} k1}{k1+k2}, \frac{e^{-2 i a k2}(-k1+k2)}{k1+k2}\right\}\right\}$$

In[∘]:=
```
Srightstep // MatrixForm
```
Out[∘]//MatrixForm=
$$\begin{pmatrix} \frac{e^{2 i a k1}(k1-k2)}{k1+k2} & \frac{2 e^{i a (k1-k2)} k2}{k1+k2} \\ \frac{2 e^{i a (k1-k2)} k1}{k1+k2} & \frac{e^{-2 i a k2}(-k1+k2)}{k1+k2} \end{pmatrix}$$

At a = 0 ;

In[∘]:=
```
a = 0;
Srightstep // MatrixForm
```
Out[∘]//MatrixForm=
$$\begin{pmatrix} \frac{k1-k2}{k1+k2} & \frac{2 k2}{k1+k2} \\ \frac{2 k1}{k1+k2} & \frac{-k1+k2}{k1+k2} \end{pmatrix}$$

In[∘]:=
```
PSrightstep = {{Srightstep[[1, 1]] + Srightstep[[1, 2]], 0},
   {0, Srightstep[[1, 2]] - Srightstep[[2, 2]]}} // Simplify
```
Out[∘]=
```
{{1, 0}, {0, 1}}
```

In[∘]:=
```
PSrightstep // MatrixForm
```
Out[∘]//MatrixForm=
$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

In[∘]:=
```
Clear[a]
```

★ Left Step Potential :

In[∘]:=
$$l1 = \{\{e^{i k2 a}, e^{-i k2 a}\}, \{k2\, e^{i k2 a}, -k2\, e^{-i k2 a}\}\};$$
```
l1inverse =
   l1inverse = 1/Det[l1] {{l1[[2, 2]], -l1[[1, 2]]}, {-l1[[2, 1]], l1[[1, 1]]}}; // Simplify
```

In[∘]:=
```
l1inverse // MatrixForm
```
Out[∘]//MatrixForm=
$$\begin{pmatrix} \frac{1}{2} e^{-i a k2} & \frac{e^{-i a k2}}{2 k2} \\ \frac{1}{2} e^{i a k2} & -\frac{e^{i a k2}}{2 k2} \end{pmatrix}$$

In[ ]:=
```
l2 = {{ⅇ^(ⅈ k1 a), ⅇ^(-ⅈ k1 a)}, {k1 ⅇ^(ⅈ k1 a), -k1 ⅇ^(-ⅈ k1 a)}};
Tleftstep = l1inverse.l2 // Simplify
```

Out[ ]=

$$\left\{\left\{\frac{e^{i a (k1-k2)} (k1+k2)}{2\,k2}, \frac{e^{-i a (k1+k2)} (-k1+k2)}{2\,k2}\right\}, \left\{\frac{e^{i a (k1+k2)} (-k1+k2)}{2\,k2}, \frac{e^{-i a (k1-k2)} (k1+k2)}{2\,k2}\right\}\right\}$$

In[ ]:=
```
Tleftstep // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} \frac{e^{i a (k1-k2)} (k1+k2)}{2\,k2} & \frac{e^{-i a (k1+k2)} (-k1+k2)}{2\,k2} \\ \frac{e^{i a (k1+k2)} (-k1+k2)}{2\,k2} & \frac{e^{-i a (k1-k2)} (k1+k2)}{2\,k2} \end{pmatrix}$$

In[ ]:=
```
Trightstepinvrse // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} \frac{e^{-i a (k1-k2)} (k1+k2)}{2\,k1} & -\frac{e^{-i a (k1+k2)} (-k1+k2)}{2\,k1} \\ -\frac{e^{i a (k1+k2)} (-k1+k2)}{2\,k1} & \frac{e^{i a (k1-k2)} (k1+k2)}{2\,k1} \end{pmatrix}$$

In[ ]:=
```
Trightstep // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} \frac{e^{i a (k1-k2)} (k1+k2)}{2\,k2} & \frac{e^{-i a (k1+k2)} (-k1+k2)}{2\,k2} \\ \frac{e^{i a (k1+k2)} (-k1+k2)}{2\,k2} & \frac{e^{-i a (k1-k2)} (k1+k2)}{2\,k2} \end{pmatrix}$$

In[ ]:=
```
Tleftstep.Trightstep // Simplify // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} \frac{(k1-k2)^2+e^{2 i a (k1-k2)} (k1+k2)^2}{4\,k2^2} & \frac{e^{-2 i a k1} \left(1+e^{2 i a (k1-k2)}\right) (-k1+k2) (k1+k2)}{4\,k2^2} \\ \frac{\left(e^{2 i a k1}+e^{2 i a k2}\right) (-k1+k2) (k1+k2)}{4\,k2^2} & \frac{(k1-k2)^2+e^{-2 i a (k1-k2)} (k1+k2)^2}{4\,k2^2} \end{pmatrix}$$

In[ ]:=
```
a = -l / 2;
r = Trightstep
```

Out[ ]=

$$\left\{\left\{\frac{e^{-\frac{1}{2} i (k1-k2) l} (k1+k2)}{2\,k2}, \frac{e^{\frac{1}{2} i (k1+k2) l} (-k1+k2)}{2\,k2}\right\}, \left\{\frac{e^{-\frac{1}{2} i (k1+k2) l} (-k1+k2)}{2\,k2}, \frac{e^{\frac{1}{2} i (k1-k2) l} (k1+k2)}{2\,k2}\right\}\right\}$$

In[ ]:=
```
r // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} \frac{e^{-\frac{1}{2} i (k1-k2) l} (k1+k2)}{2\,k2} & \frac{e^{\frac{1}{2} i (k1+k2) l} (-k1+k2)}{2\,k2} \\ \frac{e^{-\frac{1}{2} i (k1+k2) l} (-k1+k2)}{2\,k2} & \frac{e^{\frac{1}{2} i (k1-k2) l} (k1+k2)}{2\,k2} \end{pmatrix}$$

In[ ]:=
```
Clear[a]
```

In[∘]:=
```
a = l / 2;
ll = Trightstepinvrse
```

Out[∘]=

$$\left\{\left\{\frac{e^{-\frac{1}{2} i (k1-k2) l} (k1+k2)}{2\,k1}, -\frac{e^{-\frac{1}{2} i (k1+k2) l} (-k1+k2)}{2\,k1}\right\}, \left\{-\frac{e^{\frac{1}{2} i (k1+k2) l} (-k1+k2)}{2\,k1}, \frac{e^{\frac{1}{2} i (k1-k2) l} (k1+k2)}{2\,k1}\right\}\right\}$$

In[∘]:=
```
Clear[a]
```

In[∘]:=
```
Tb = ll.r // Simplify
```

Out[∘]=

$$\left\{\left\{\frac{-e^{-i (k1+k2) l} (k1-k2)^2 + e^{-i (k1-k2) l} (k1+k2)^2}{4\,k1\,k2}, \frac{e^{-i\,k2\,l} \left(-1+e^{2\,i\,k2\,l}\right) (-k1+k2) (k1+k2)}{4\,k1\,k2}\right\}, \left\{\frac{e^{-i\,k2\,l} \left(-1+e^{2\,i\,k2\,l}\right) (k1-k2) (k1+k2)}{4\,k1\,k2}, \frac{-e^{i (k1+k2) l} (k1-k2)^2 + e^{i (k1-k2) l} (k1+k2)^2}{4\,k1\,k2}\right\}\right\}$$

In[∘]:=
```
Tb // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} \frac{-e^{-i (k1+k2) l} (k1-k2)^2+e^{-i (k1-k2) l} (k1+k2)^2}{4\,k1\,k2} & \frac{e^{-i\,k2\,l} \left(-1+e^{2\,i\,k2\,l}\right) (-k1+k2) (k1+k2)}{4\,k1\,k2} \\ \frac{e^{-i\,k2\,l} \left(-1+e^{2\,i\,k2\,l}\right) (k1-k2) (k1+k2)}{4\,k1\,k2} & \frac{-e^{i (k1+k2) l} (k1-k2)^2+e^{i (k1-k2) l} (k1+k2)^2}{4\,k1\,k2} \end{pmatrix}$$

In[∘]:=

$$\text{Sb} = \frac{1}{\text{Tb}[[2, 2]]} \{\{-\text{Tb}[[2, 1]], 1\}, \{\text{Det}[\text{Tb}], \text{Tb}[[1, 2]]\}\} \text{ // Simplify}$$

Out[∘]=

$$\left\{\left\{-\frac{e^{-i\,k2\,l} \left(-1+e^{2\,i\,k2\,l}\right) (k1-k2) (k1+k2)}{-e^{i (k1+k2) l} (k1-k2)^2 + e^{i (k1-k2) l} (k1+k2)^2}, \frac{4\,k1\,k2}{-e^{i (k1+k2) l} (k1-k2)^2 + e^{i (k1-k2) l} (k1+k2)^2}\right\}, \left\{-\frac{4\,e^{-i (k1-k2) l}\,k1\,k2}{\left(-1+e^{2\,i\,k2\,l}\right) k1^2 - 2 \left(1+e^{2\,i\,k2\,l}\right) k1\,k2 + \left(-1+e^{2\,i\,k2\,l}\right) k2^2}, \frac{e^{-i\,k2\,l} \left(-1+e^{2\,i\,k2\,l}\right) (-k1+k2) (k1+k2)}{-e^{i (k1+k2) l} (k1-k2)^2 + e^{i (k1-k2) l} (k1+k2)^2}\right\}\right\}$$

In[∘]:=
```
Sb // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} -\frac{e^{-i\,k2\,l} \left(-1+e^{2\,i\,k2\,l}\right) (k1-k2) (k1+k2)}{-e^{i (k1+k2) l} (k1-k2)^2+e^{i (k1-k2) l} (k1+k2)^2} & \frac{4\,k1\,k2}{-e^{i (k1+k2) l} (k1-k2)^2+e^{i (k1-k2) l} (k1+k2)^2} \\ -\frac{4\,e^{-i (k1-k2) l}\,k1\,k2}{\left(-1+e^{2\,i\,k2\,l}\right) k1^2-2 \left(1+e^{2\,i\,k2\,l}\right) k1\,k2+\left(-1+e^{2\,i\,k2\,l}\right) k2^2} & \frac{e^{-i\,k2\,l} \left(-1+e^{2\,i\,k2\,l}\right) (-k1+k2) (k1+k2)}{-e^{i (k1+k2) l} (k1-k2)^2+e^{i (k1-k2) l} (k1+k2)^2} \end{pmatrix}$$

In[•]:=
```
a = 0;
PSB = {{Sb[[1, 1]] + Sb[[1, 2]], 0}, {0, Sb[[1, 2]] - Sb[[2, 2]]}} // Simplify
```

Out[•]=

$$\left\{\left\{\frac{e^{-i\,k1\,l}\left(\left(1+e^{i\,k2\,l}\right)k1+\left(-1+e^{i\,k2\,l}\right)k2\right)}{k1+e^{i\,k2\,l}\,k1+k2-e^{i\,k2\,l}\,k2},\,0\right\},\left\{0,\,-\frac{e^{-i\,k1\,l}\left(k1-e^{i\,k2\,l}\,k1-\left(1+e^{i\,k2\,l}\right)k2\right)}{k1-e^{i\,k2\,l}\,k1+k2+e^{i\,k2\,l}\,k2}\right\}\right\}$$

In[•]:=
```
PSB // MatrixForm // Simplify
```

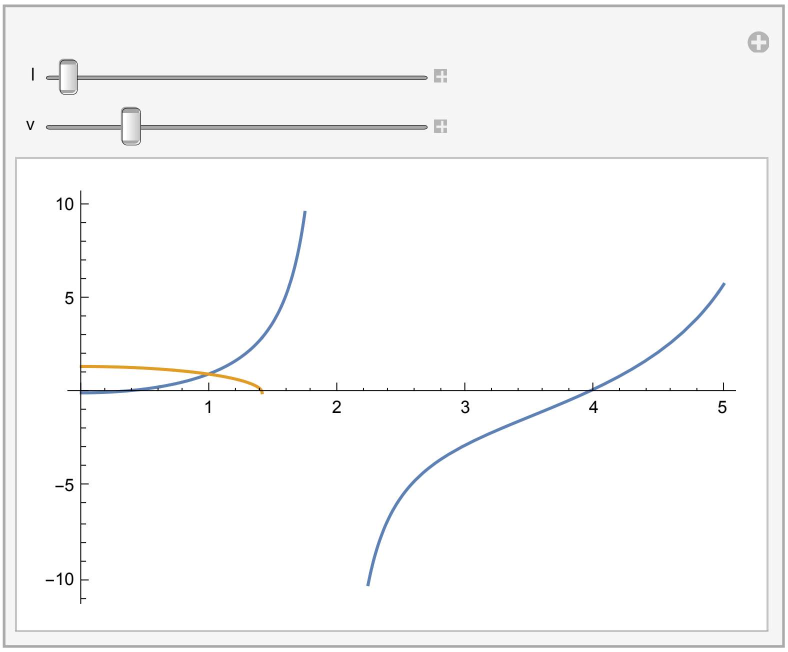
Out[•]//MatrixForm=

$$\begin{pmatrix}\frac{e^{-i\,k1\,l}\left(\left(1+e^{i\,k2\,l}\right)k1+\left(-1+e^{i\,k2\,l}\right)k2\right)}{k1+e^{i\,k2\,l}\,k1+k2-e^{i\,k2\,l}\,k2} & 0\\ 0 & -\frac{e^{-i\,k1\,l}\left(k1-e^{i\,k2\,l}\,k1-\left(1+e^{i\,k2\,l}\right)k2\right)}{k1-e^{i\,k2\,l}\,k1+k2+e^{i\,k2\,l}\,k2}\end{pmatrix}$$

Given in Notes $\mathbf{(}\boldsymbol{T_R}\mathbf{[k1, k2,}\ \boldsymbol{a}\mathbf{])^{-1} =}\ \boldsymbol{T_R}\mathbf{[k2, k1,}\ \boldsymbol{a}\mathbf{]}$ and $\mathbf{T_L[k1, k2, a] = (T_R[k2, k1, a])^{-1}}$

In[•]:=
```
Manipulate[Plot[{k Tan[k l/2], Sqrt[v - k^2]}, {k, 0, 5}], {l, 0, 100}, {v, 0, 10}]
```

Out[•]=



In[ ]:=
```
Manipulate[Plot[{-k Cot[k l/2], Sqrt[v - k^2]}, {k, 0, 5}], {l, 0, 100}, {v, 0, 10}]
```

$$\text{Manipulate}\left[\text{Plot}\left[\left\{-k\,\text{Cot}\left[k\,\frac{l}{2}\right],\ \sqrt{v-k^2}\right\},\ \{k, 0, 5\}\right],\ \{l, 0, 100\},\ \{v, 0, 10\}\right]$$

Out[ ]=