Recovering 3D Magnetic Turbulence from a Synchrotron Polarization Map

APS Division of Plasma Physics

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Long Beach, CA.

Aliaksandr Melnichenka

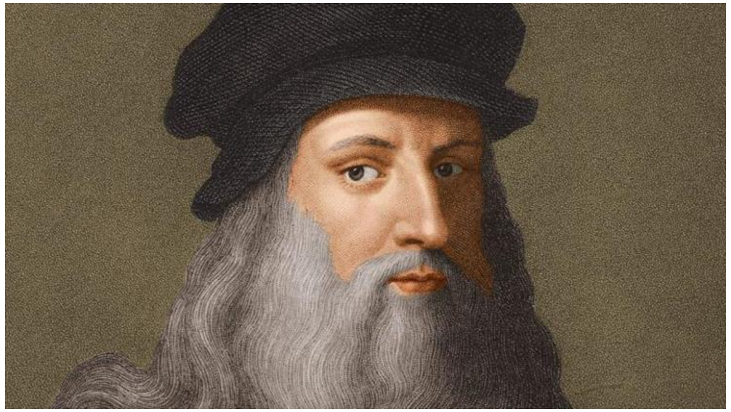
Berea College

Alex Lazarian (UW Madison)

Dmitry Pogosyan (University of Alberta)

Turbulence is a chaotic order

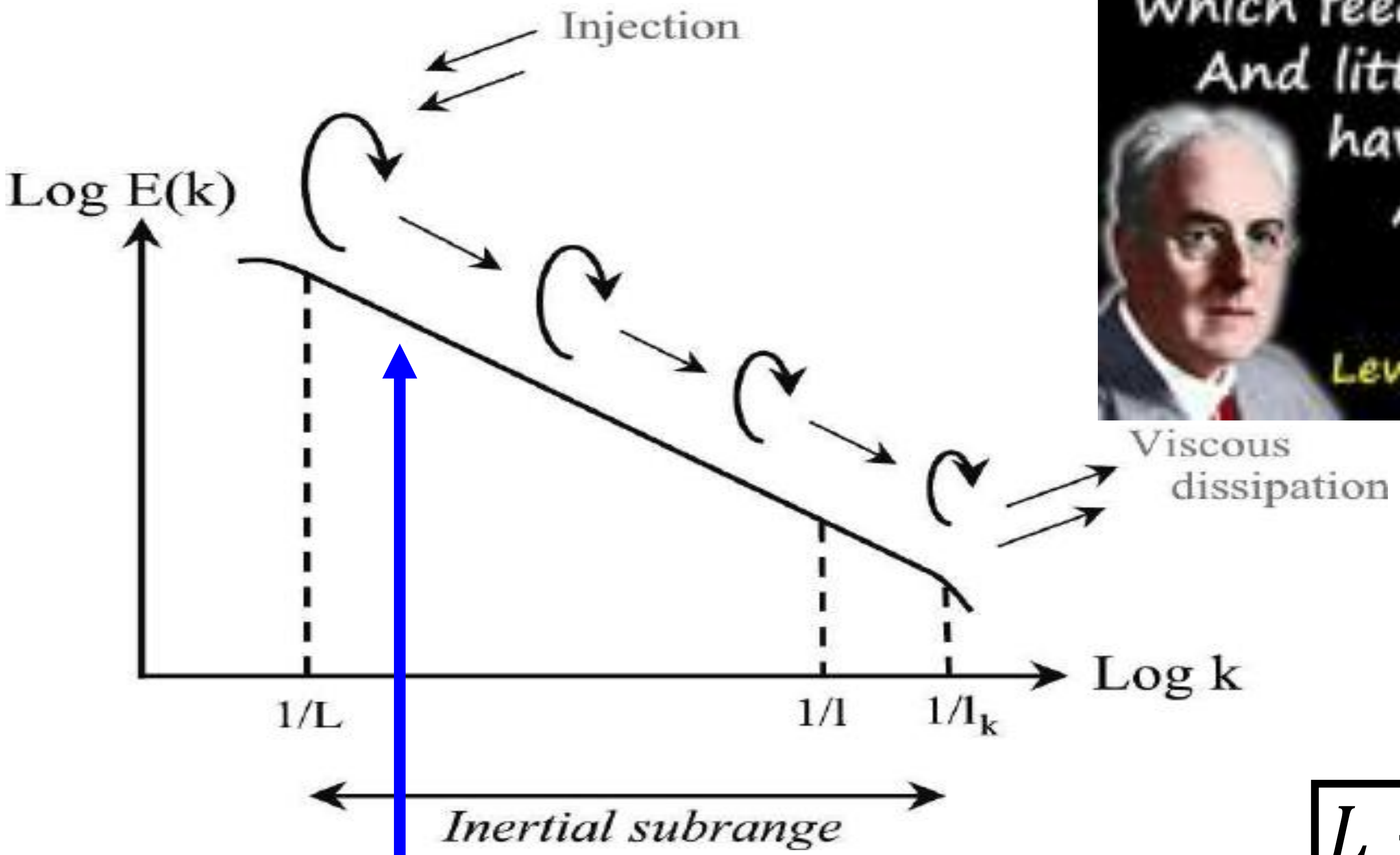
c.1500

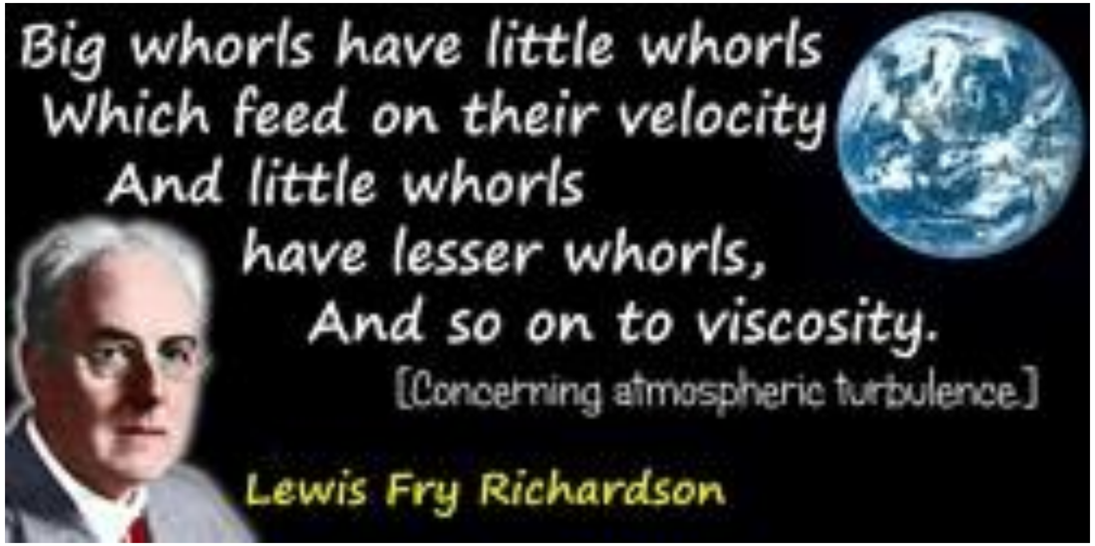


L. Da Vinci

It is important to know the laws of this order and use them

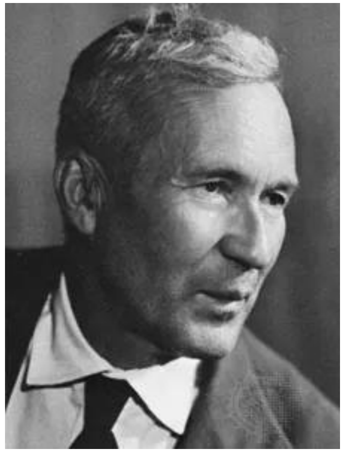
Turbulent energy cascade





Constant turbulent energy transfer rate

$$\boxed{L \to \ell \to \eta}$$



https://commons.wikimedia.org/w/index.php?curid=64829708

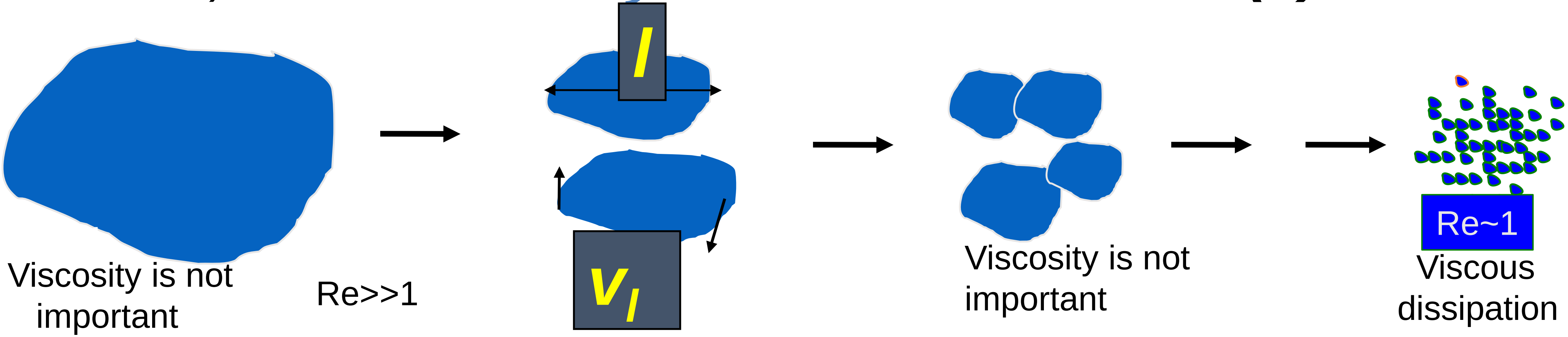
Kolmogorov theory reveals order in chaos for incompressible hydro turbulence

$$\frac{V_l^2}{t_{cas,l}} = const$$

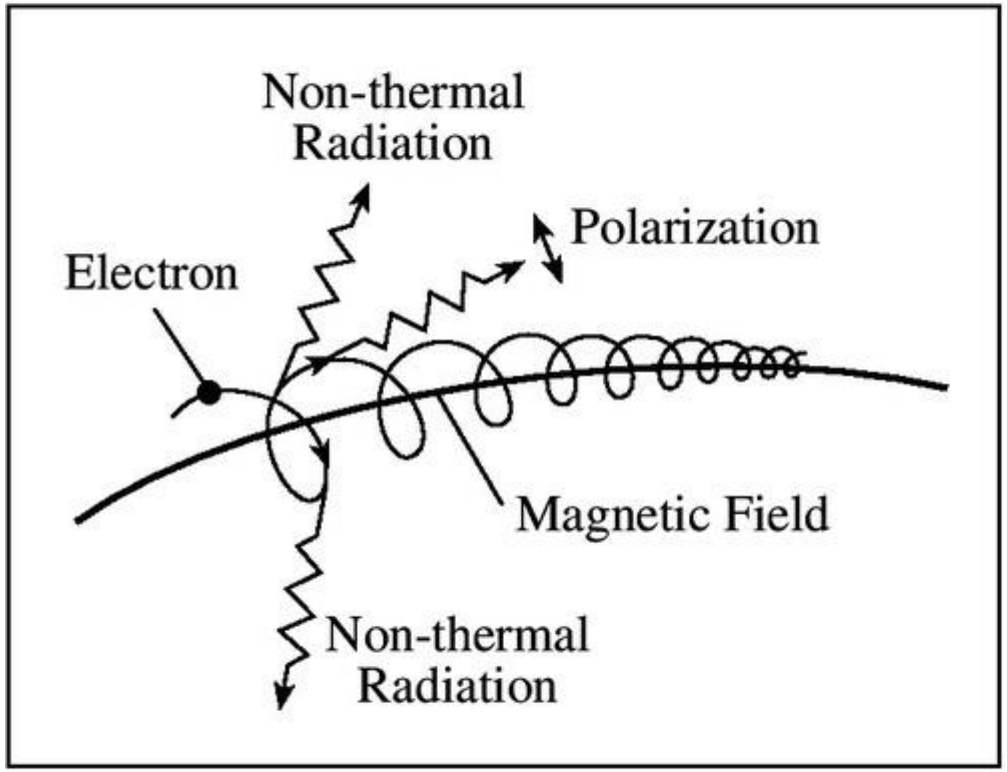
$$t_{cas,l} = l/V_l$$

$$\frac{V_l^3}{l} = const, \; V_l \sim l^{1/3}$$

Or, $E(k) \sim k^{-5/3}$

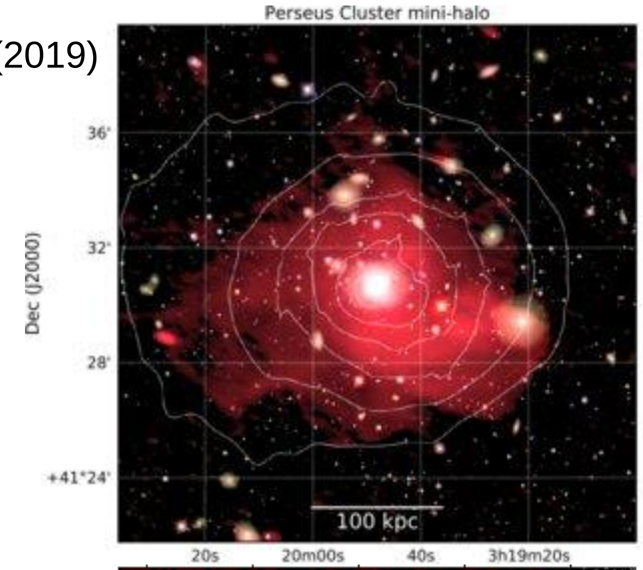


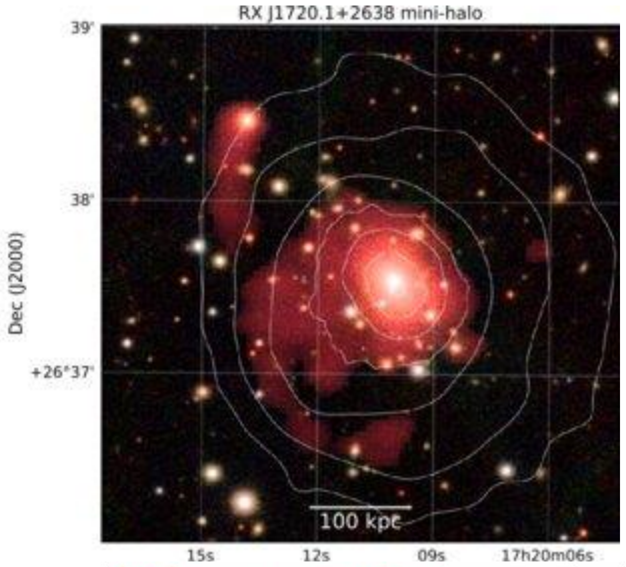
Typical magnetic field tracing method: synchrotron polarization

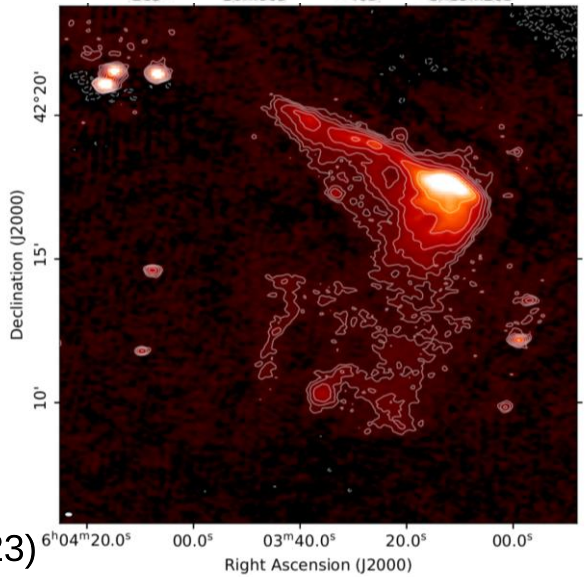


Synchrotron emission

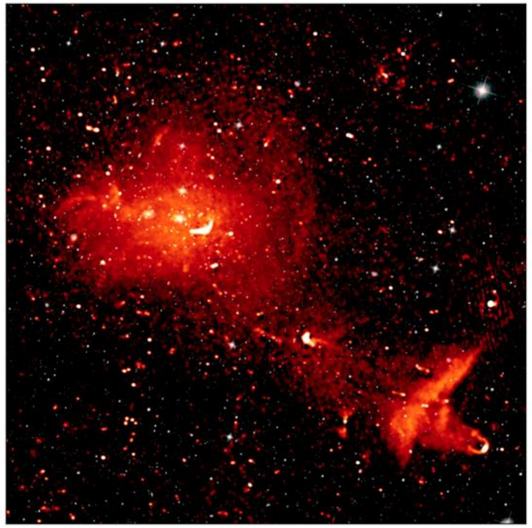
van Weeren et al. (2019)







Wittor Denis (2023)

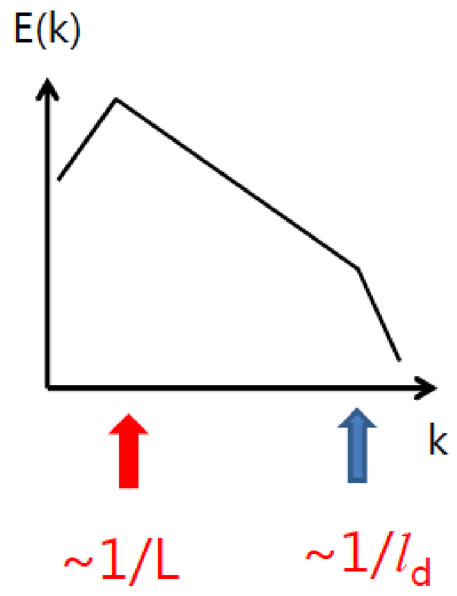


Bonafede et al. (2022)

For turbulence the cascade is self-similar from the injection to the dissipation scales

-Outer scale L
(=energy injection scale ~integral scale)

$$L_{int} = 2\pi \frac{\int E_b(k)/k \; dk}{\int E_b(k) \; dk} \sim \text{outer scale}$$



-Kolmogorov scale l_d (=dissipation scale)
← Reynolds number $(l_d v_d/\nu) = 1$

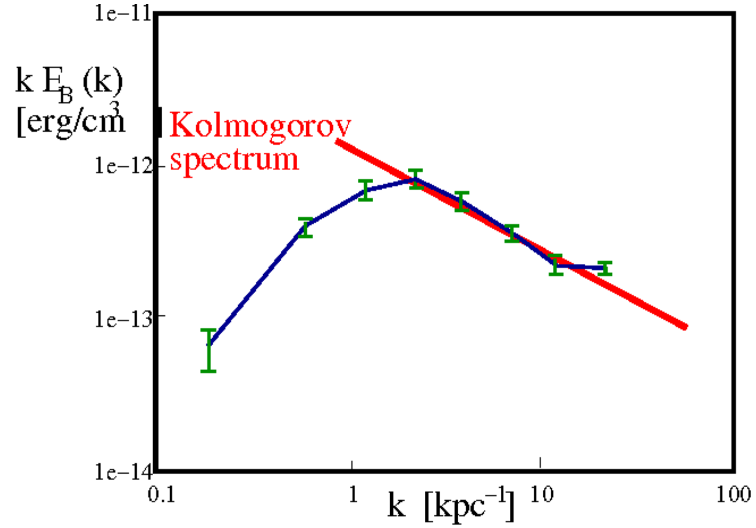
Since $v_d = v_L (l_d/L)^{1/3}$, we have $v_L (l_d)^{4/3} L^{-1/3} /\nu = 1$

→ $l_d = L\,(\mathrm{Re})^{3/4}$

Kolmogorov spectrum is common in Astrophysics

Clusters of Galaxies

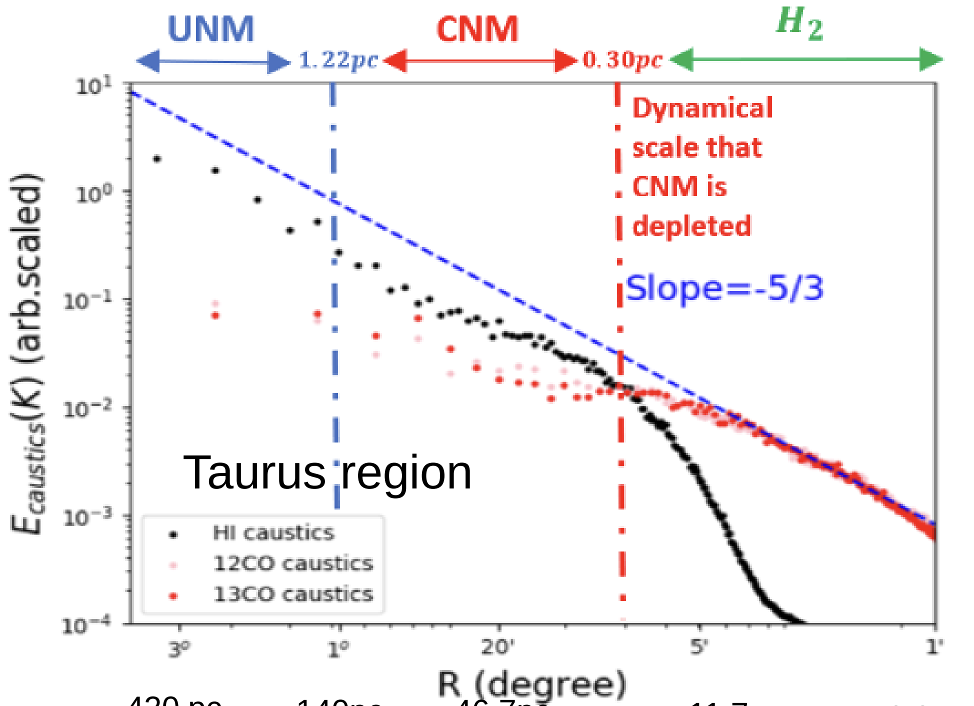
Magnetic fluctuations



Vogt & EnBlin 2005

Interstellar Medium

Velocity fluctuations



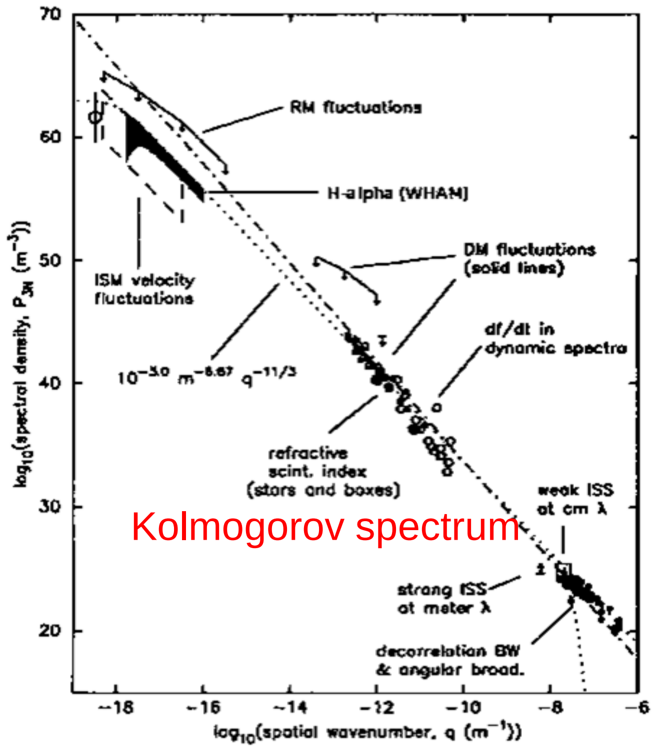
420 pc 140pc 46.7pc 11.7pc 2.3pc

Distance (degrees,pc)

Yuen, Ho & AL 2021

Interstellar Medium

Density fluctuations

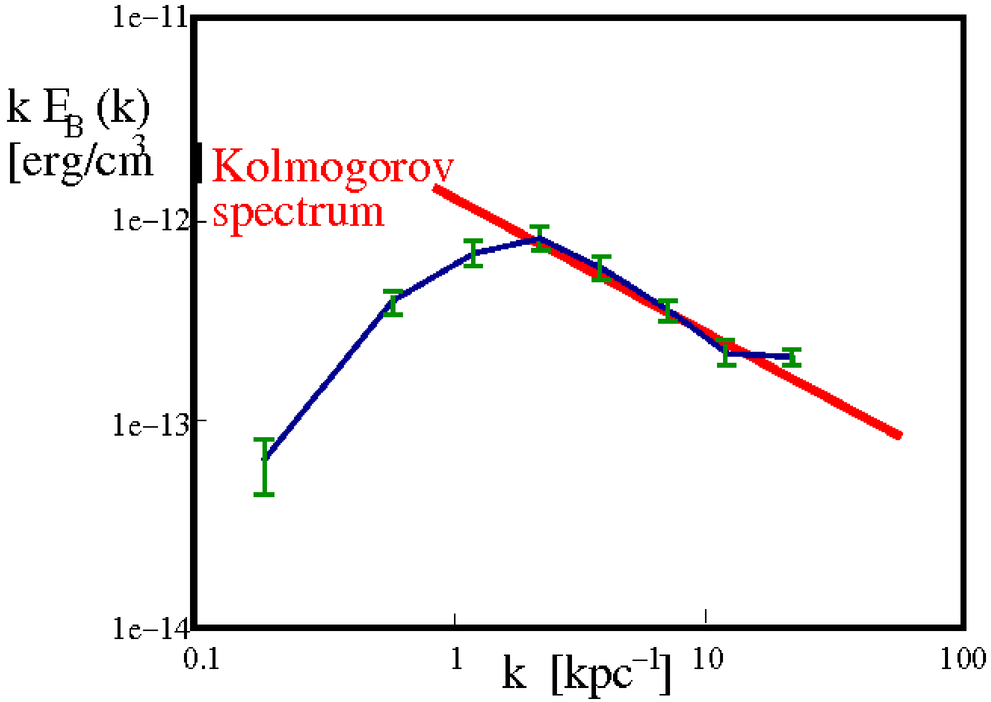


Armstrong+ 95; Chepurnov & AL 09

Clusters of Galaxies

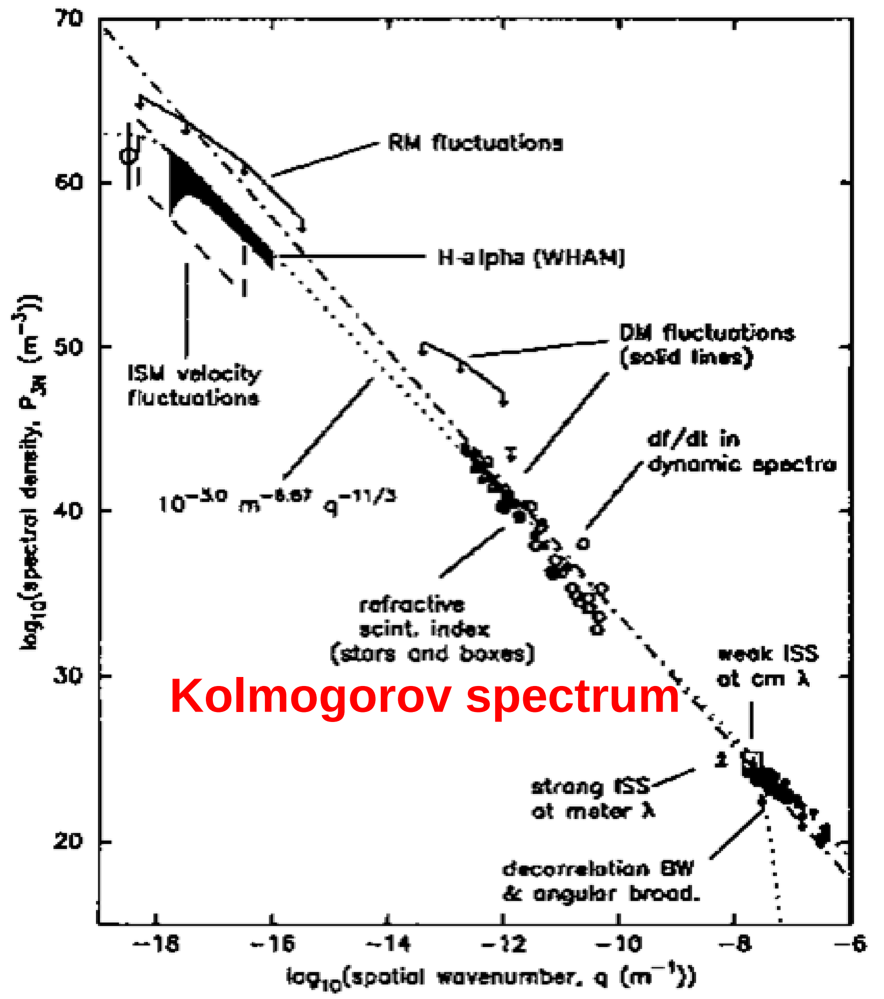
Interstellar Medium

Magnetic fluctuations



Vogt & EnBlin 2005

Density fluctuations

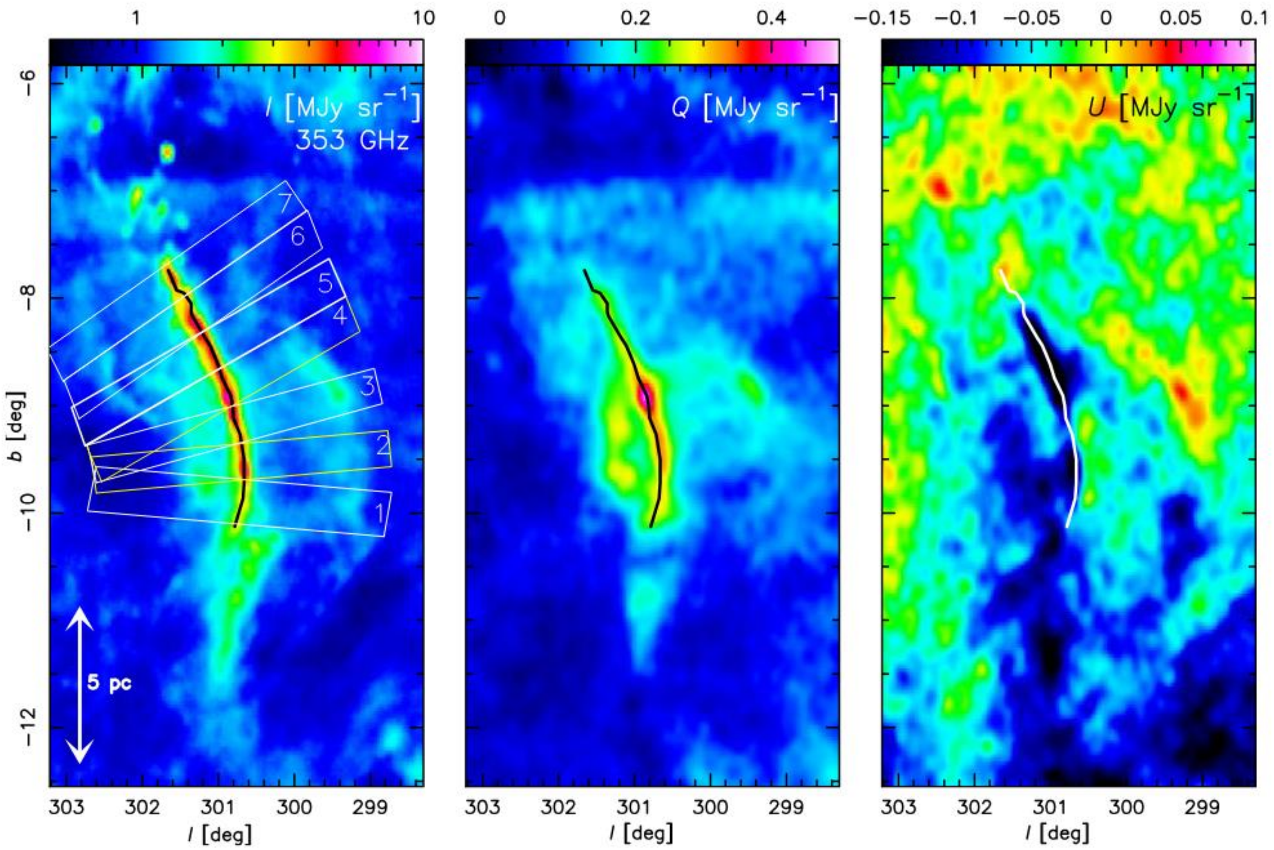


Armstrong+ 95; Chepurnov & AL 09

SKA to be commissioned in 2030: New data for SIGs

LOFAR is a gigantic European telescope measuring at 2 to 15 m radio wavelength

Why polarization maps? What current surveys give us



Planck PR3 (2018) $353-\mathrm{GHz}, I, Q, U$ full-sky maps (HEALPix)

- Stokes Q, U encode synchrotron emissivity and magnetic geometry.
- Faraday rotation imprints wavelength-dependent phase, mixing scales.
- Finally, to disentangle emissivity and Faraday effects to recover turbulence spectra.
  - Faraday depolarization flattens spatial spectra.

We analyze spatial structure from wavelength to separate physical effects

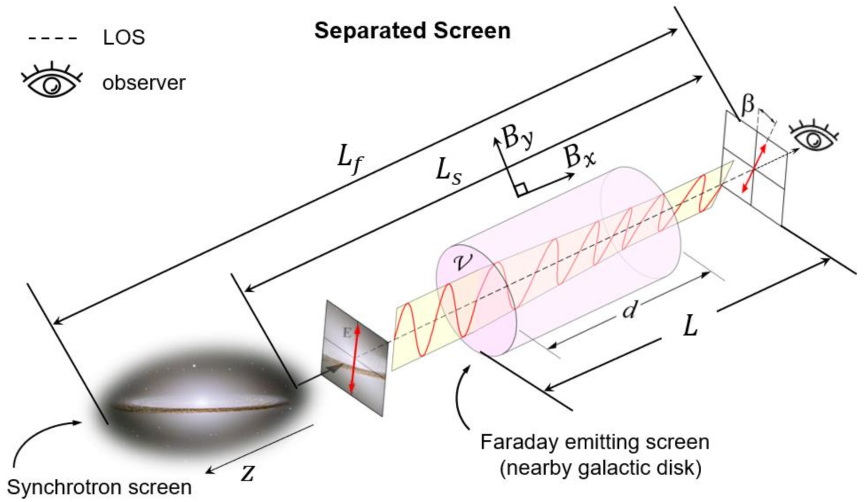
Propagation geometries: separated screen and mixed volume

With emissivity ($\gamma = 2$):

$$\Pi(\mathbf{x}, z) = {B_\perp}^{\gamma-2}\left(B_x + iB_y\right)^2$$

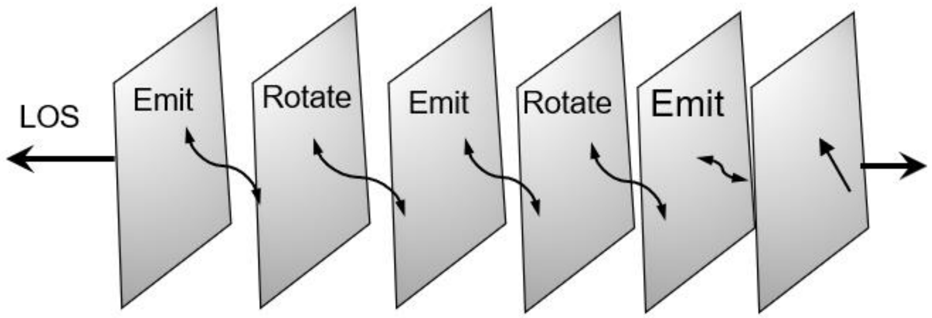
Separated screen
(external RM screen in front of emitter)

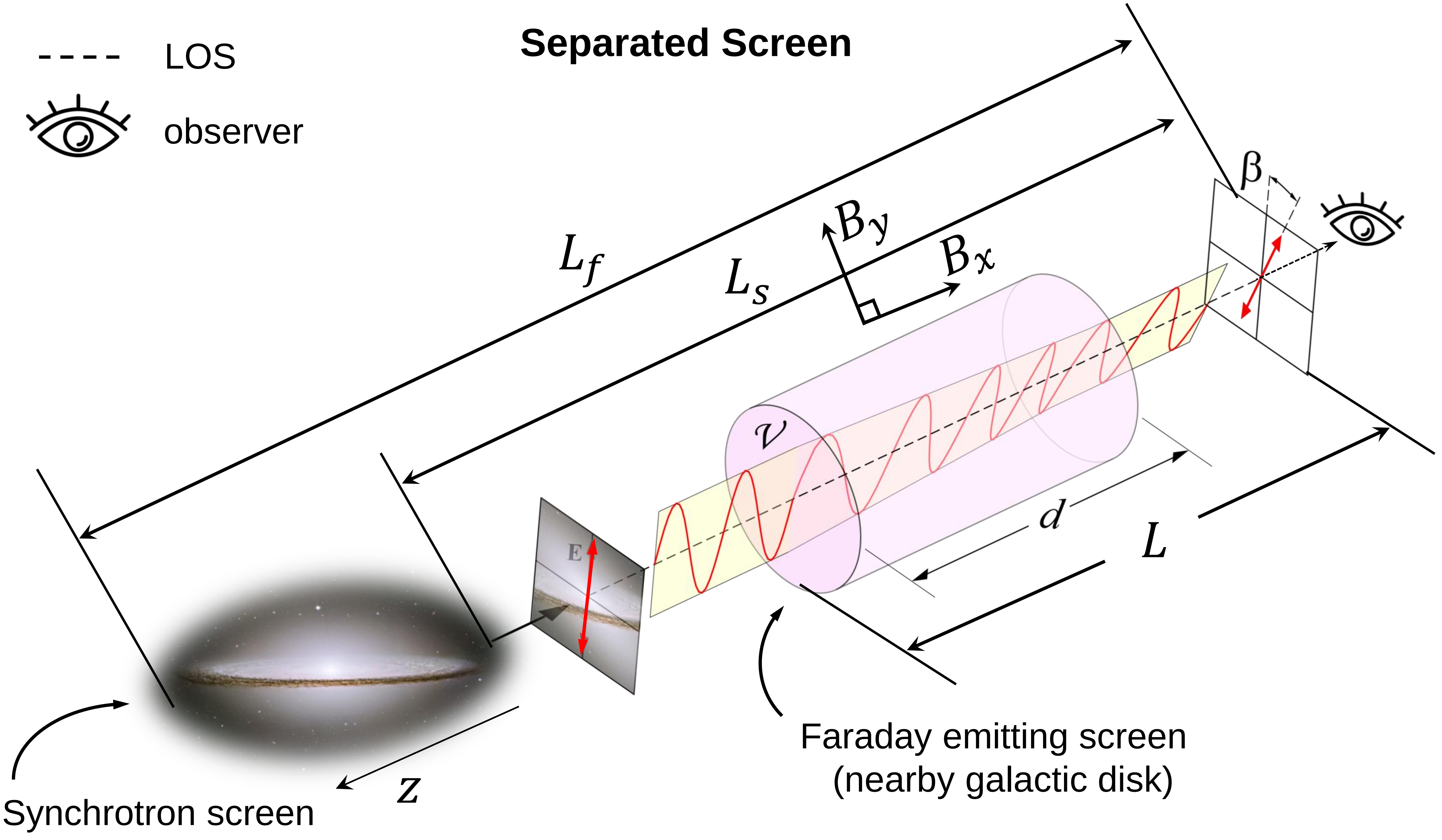
$$P(x,\lambda) = P_{\text{emit}}(\mathbf{x})\, e^{2i\,\Phi(\mathbf{x},z)} \qquad P_{\text{emit}}(\mathbf{x}) = \int \Pi(\mathbf{x},z)\, dz$$



Mixed volume
(emission and rotation in the same region)

$$P(x,\lambda) = \int \Pi(\mathbf{x},z)\, e^{2i\,\Phi(\mathbf{x},z)} dz$$

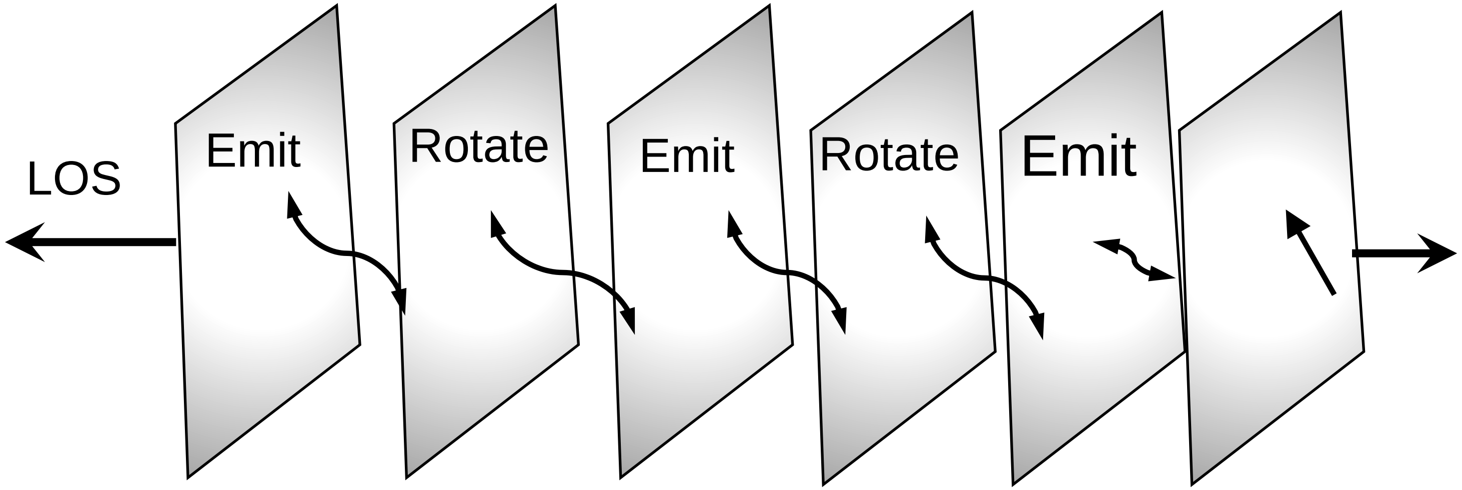




Separated Screen
LOS
observer
L_f
L_s
B_y
B_x
β
$\mathcal{V}$
E
d
L
z
Synchrotron screen
Faraday emitting screen
(nearby galactic disk)

Mixed Volume

$$P(x,\lambda) = \int \Pi(\mathbf{x}, z)\, e^{2i\, \Phi(\mathbf{x},z)} dz$$



Polarization Angle Directional Correlation (PADC)

Compute from one map :

$$S(R) = \langle \sin^2[2(\chi(x) - \chi(x+R))]\rangle$$

New measure: directional spectrum (2-D Fourier power)

$$P_{dir}(k) = |\mathrm{FFT}(\cos(2\chi)|^2 + |\mathrm{FFT}(\sin(2\chi)|^2$$

Why it works:
no angle unwrapping
robust to filtering
directly maps to power spectrum

Emission vs. Faraday Screen: who dominates?

Polarization model (external screen):

$$P(x,\lambda) = e^{2i\,[\,\psi_{emit}(x) + \lambda^2 RM(x)]}$$

Transition criterion (at scale k):

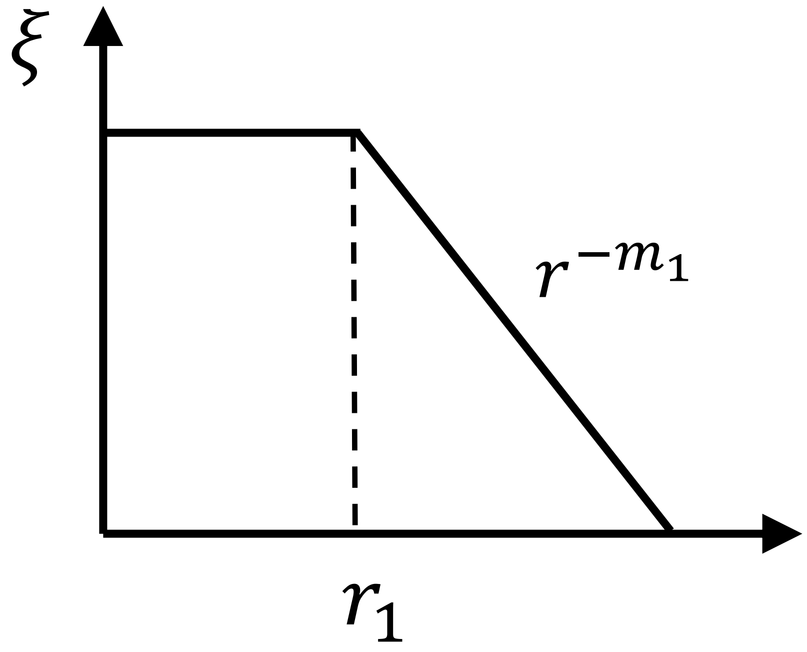
$$\lambda^2 \cdot \sigma_{RM}(k) \approx 1$$

defines crossover $k_\times\,(\lambda)$

Standard Deviation of Rotation Measure

Short λ : synchrotron emission angles dominate.
Long λ : Faraday screen ($n_e B_\parallel$) dominates.

$$\langle P_1 P_2 \rangle = \langle P_{i1} P_{i2} \rangle \left\langle e^{\Phi_1} e^{\Phi_2} \right\rangle$$

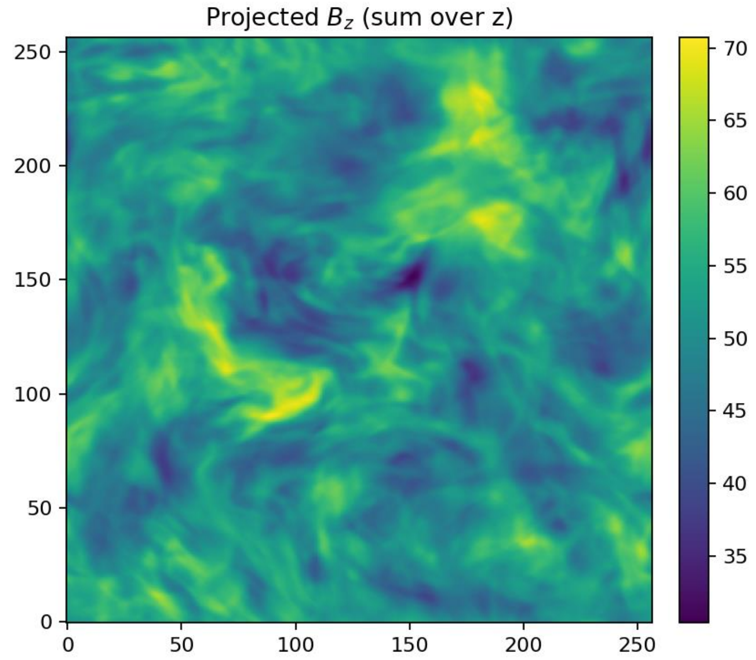


$$\begin{array}{cc} \xi_1(r) & \xi_\Phi(r) \\ \downarrow & \downarrow \\ r_1 & r_\phi \end{array}$$

$$\xi = \frac{\sigma^2}{1 + \left(\frac{r}{r_L}\right)^{m_1}}$$

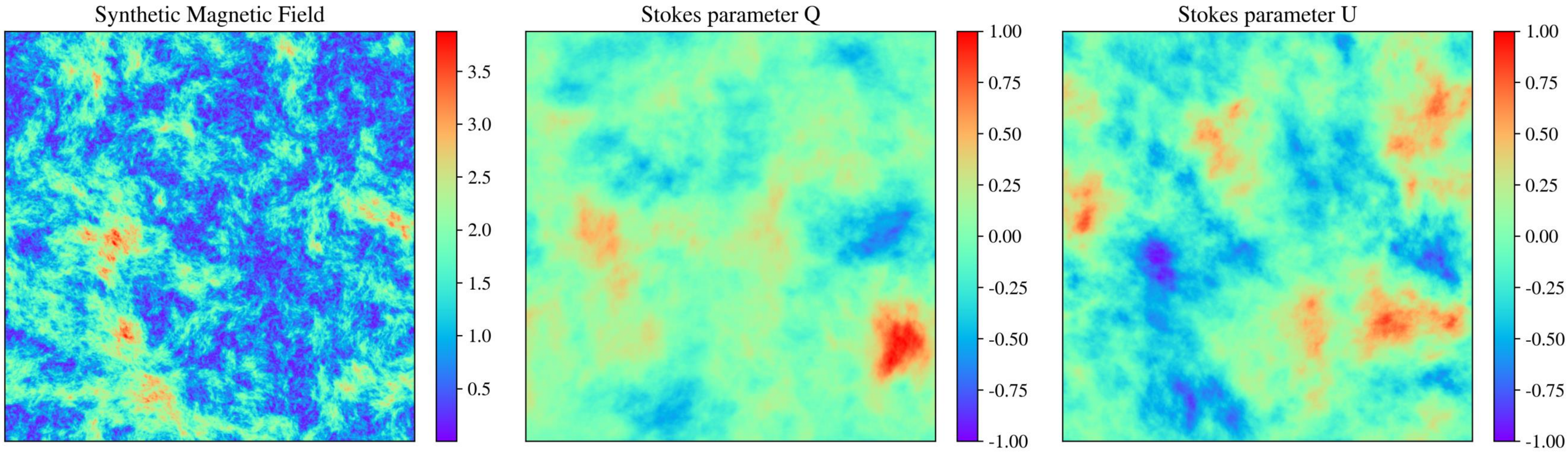
Simulation design

- Synthetic fields with controlled slopes (density & magnetic)
- ATHENA MHD snapshots (sub- and Alfvénic)
- Geometries: External screen; Mixed emission and screen; Two-screen tests
- Outputs: $S(R), P_{dir}(k)$, crossover $k_{\times}\ (\lambda)$

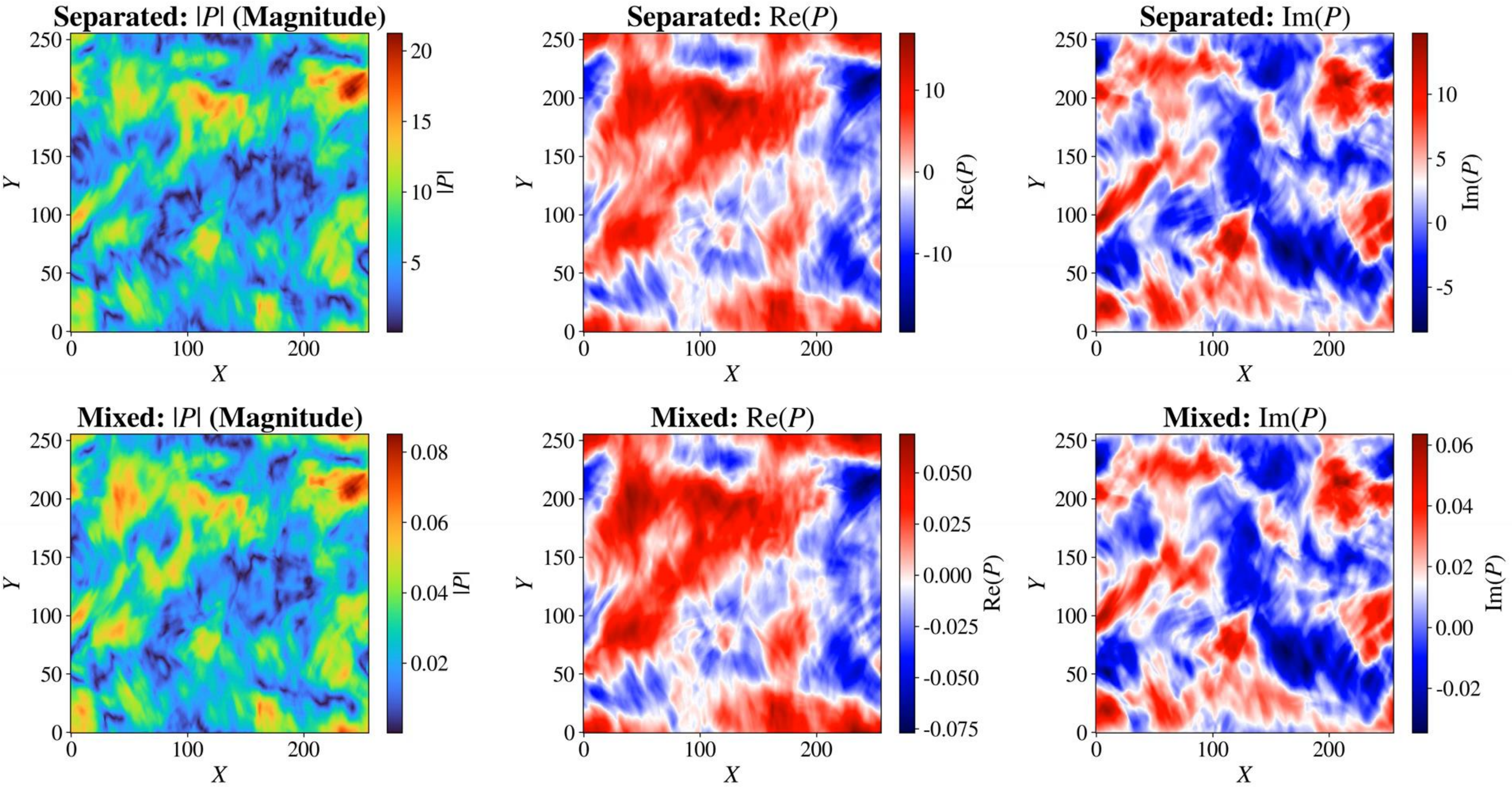


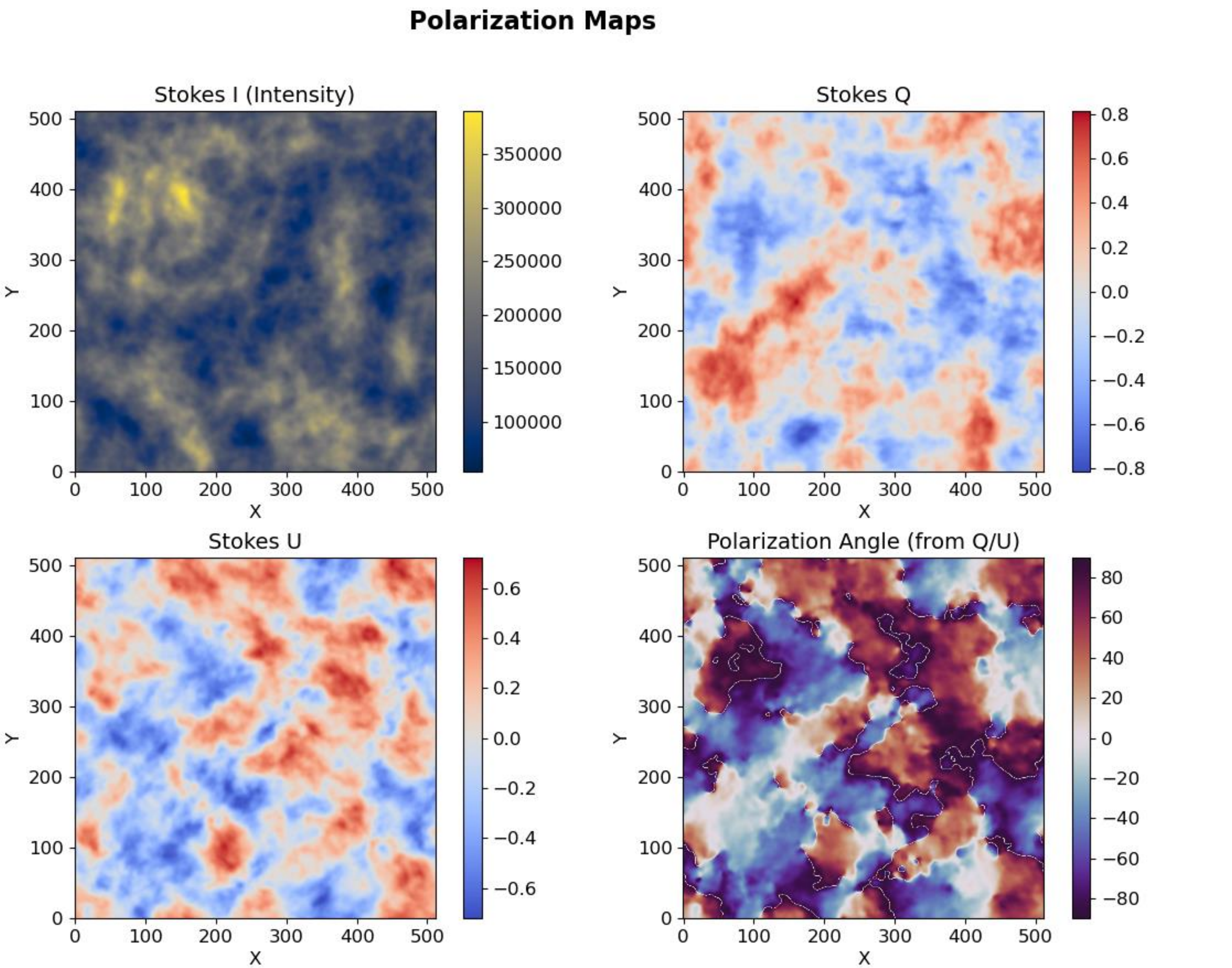
$M_A = 0.8, M_S = 1.0$

Stokes Parameters of the Synthetic Magnetic Field



Polarization Maps Comparison ($\chi = 0.003$, $\lambda = 0.033$) - Synchrotron-dominated

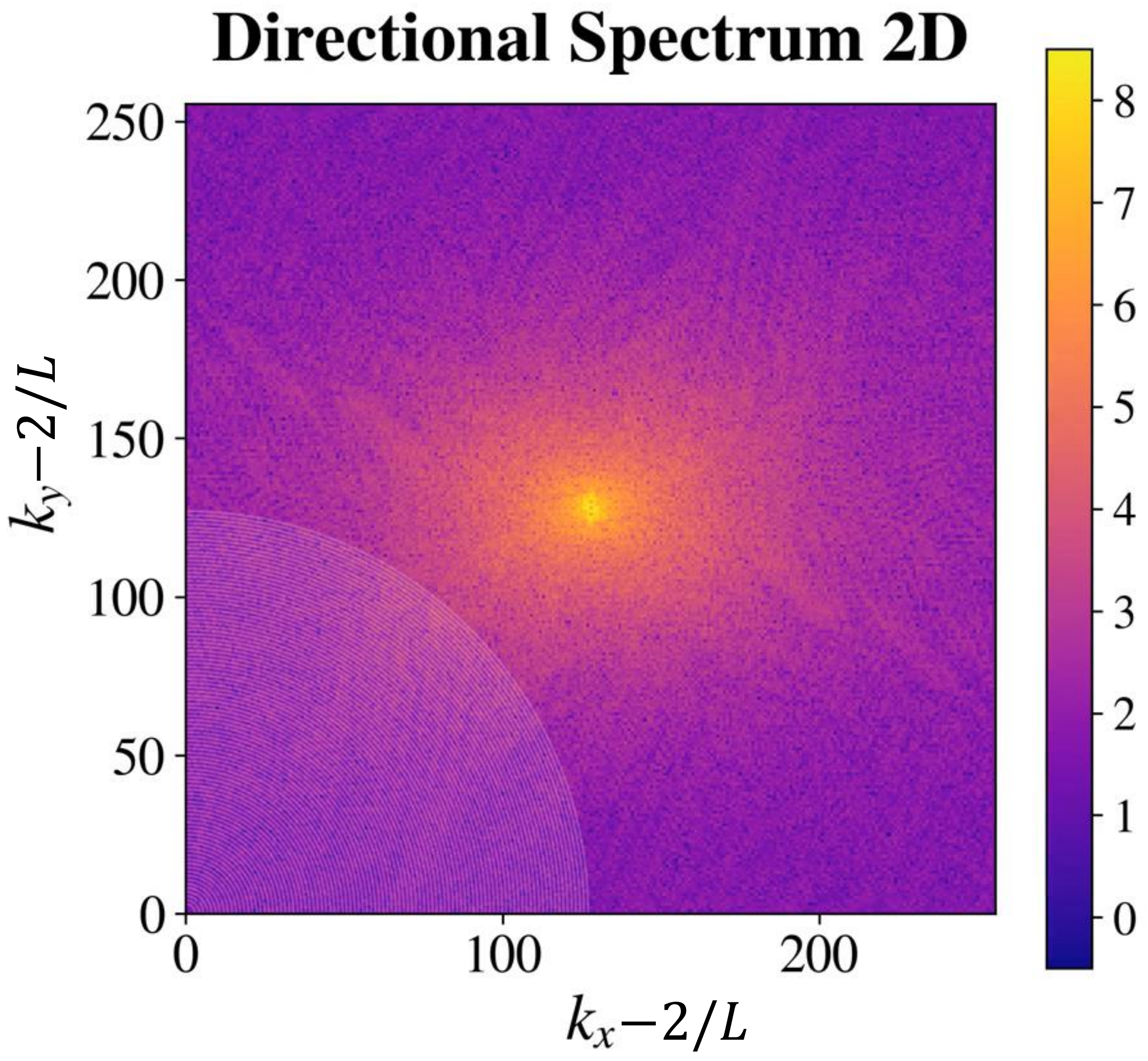


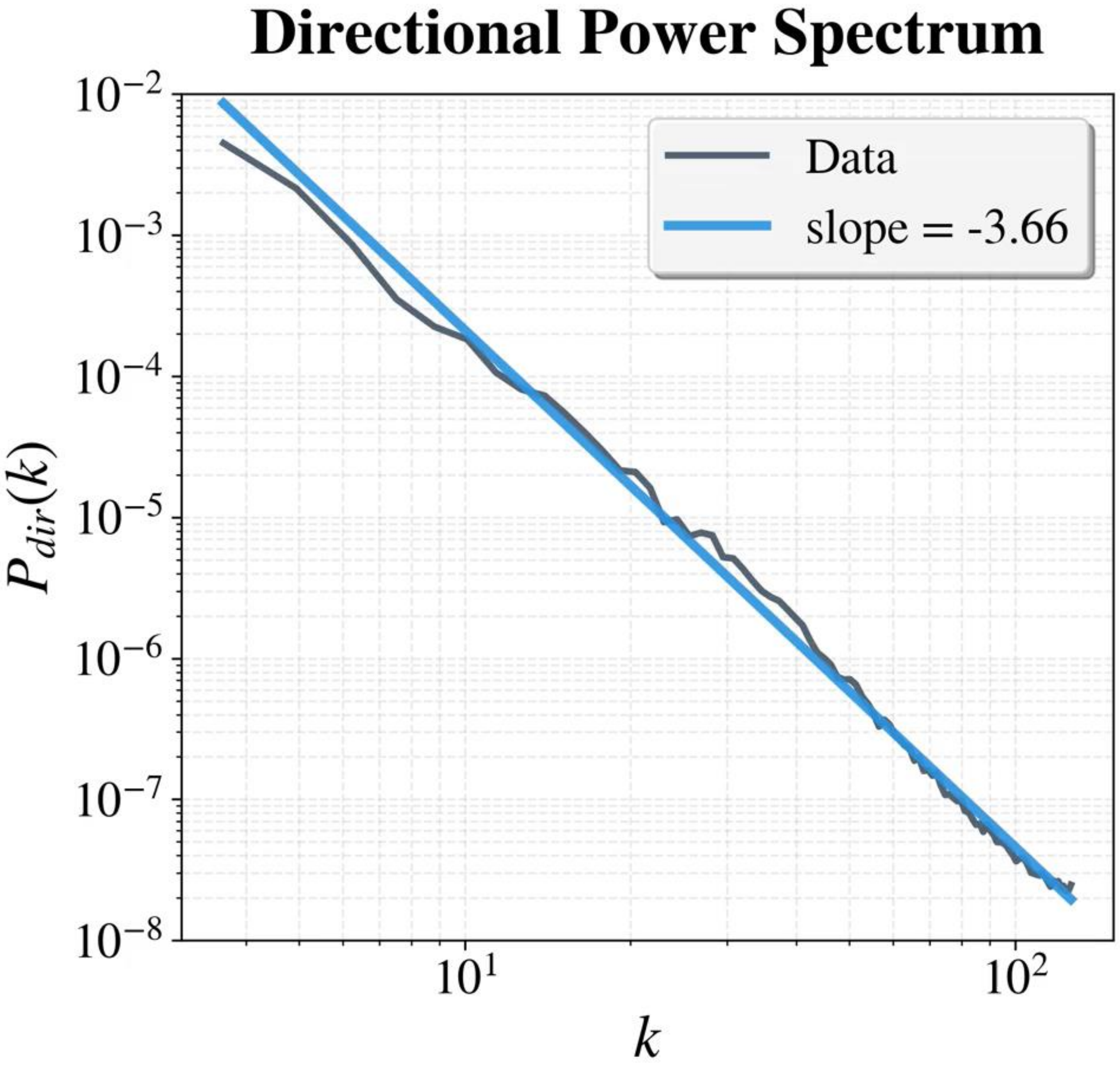


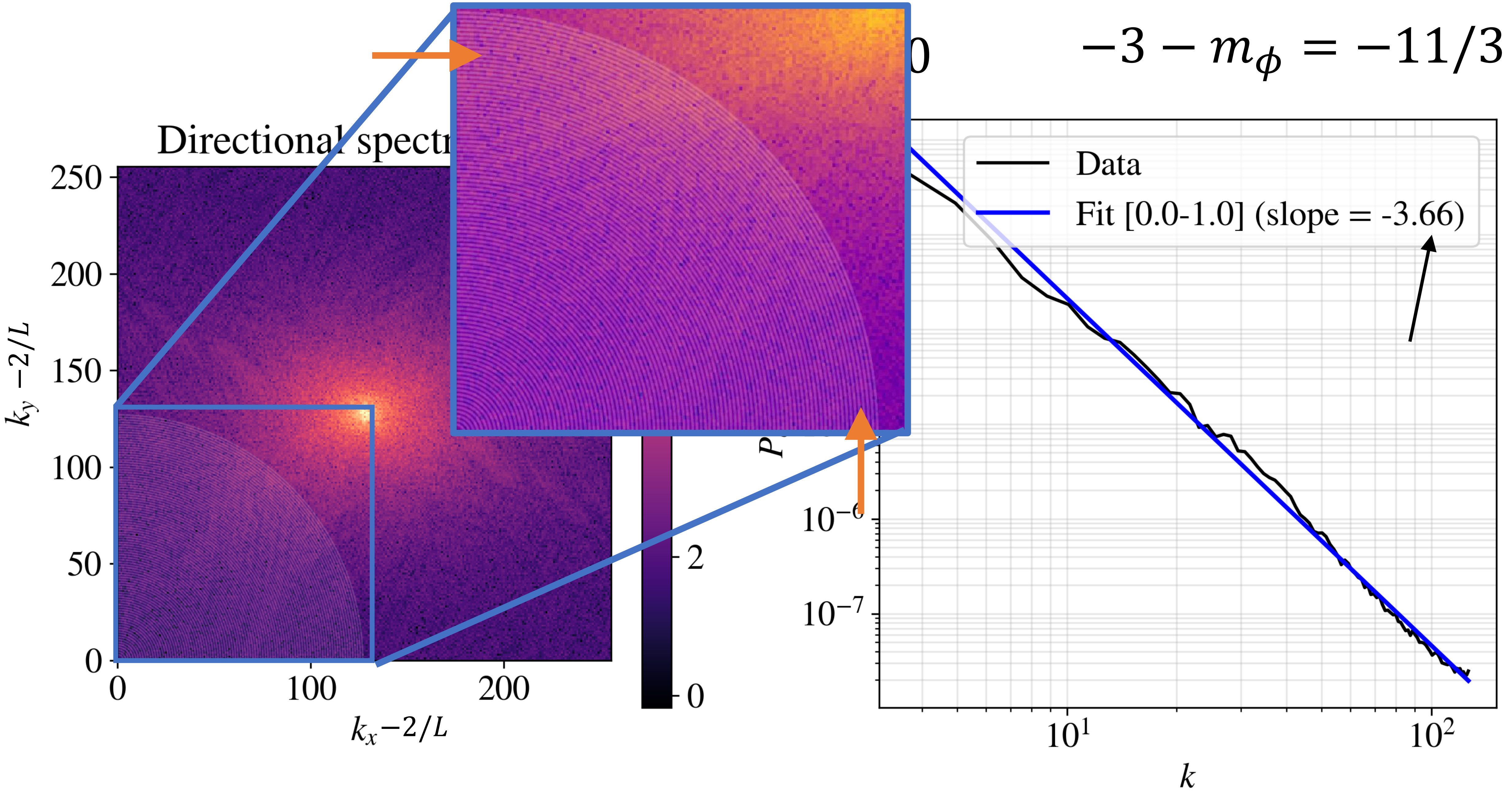
Polarization Maps
Stokes I (Intensity)
Stokes Q
Stokes U
Polarization Angle (from Q/U)
X
Y

Synchrotron-dominated $\chi = 0.00$

$m_\phi = -2/3$



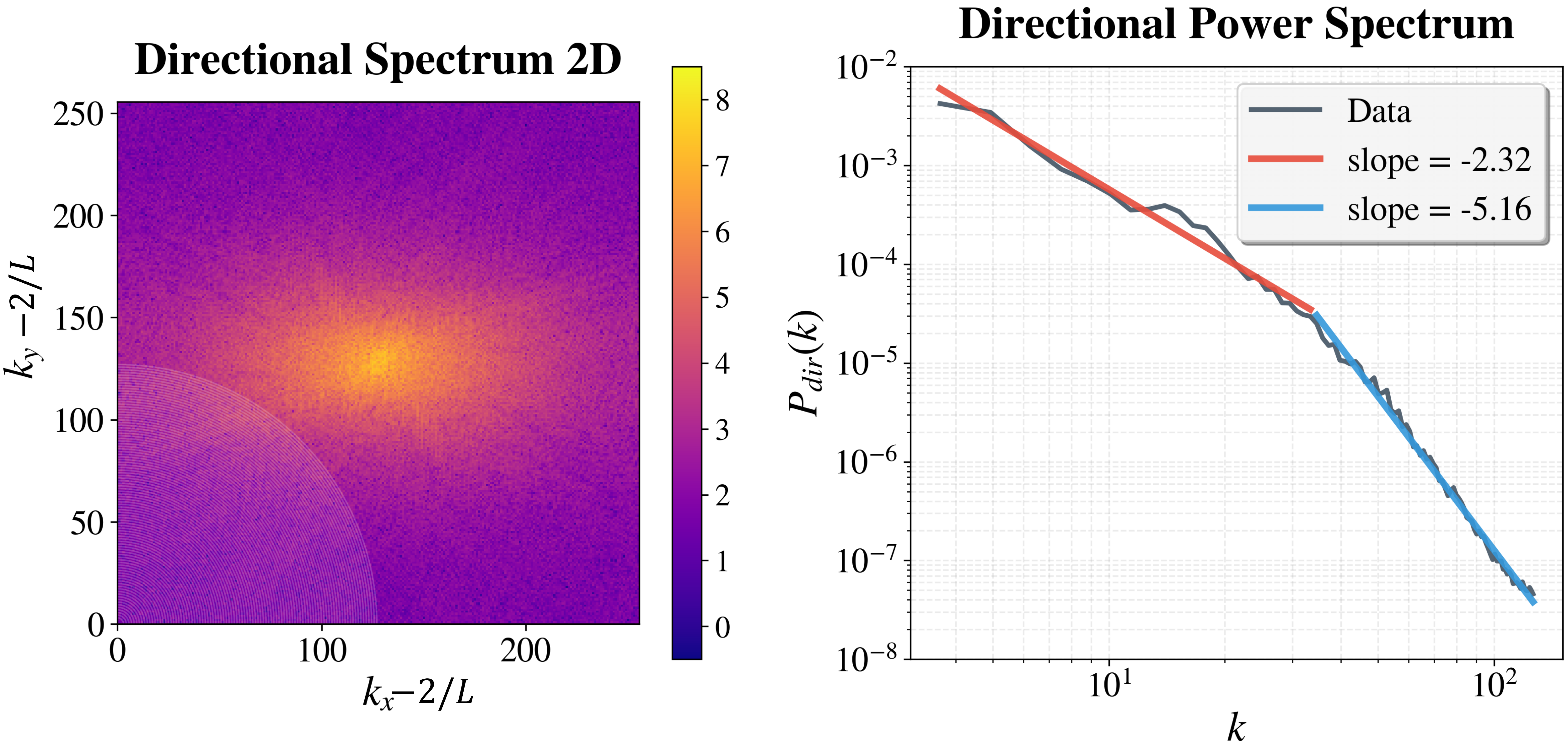


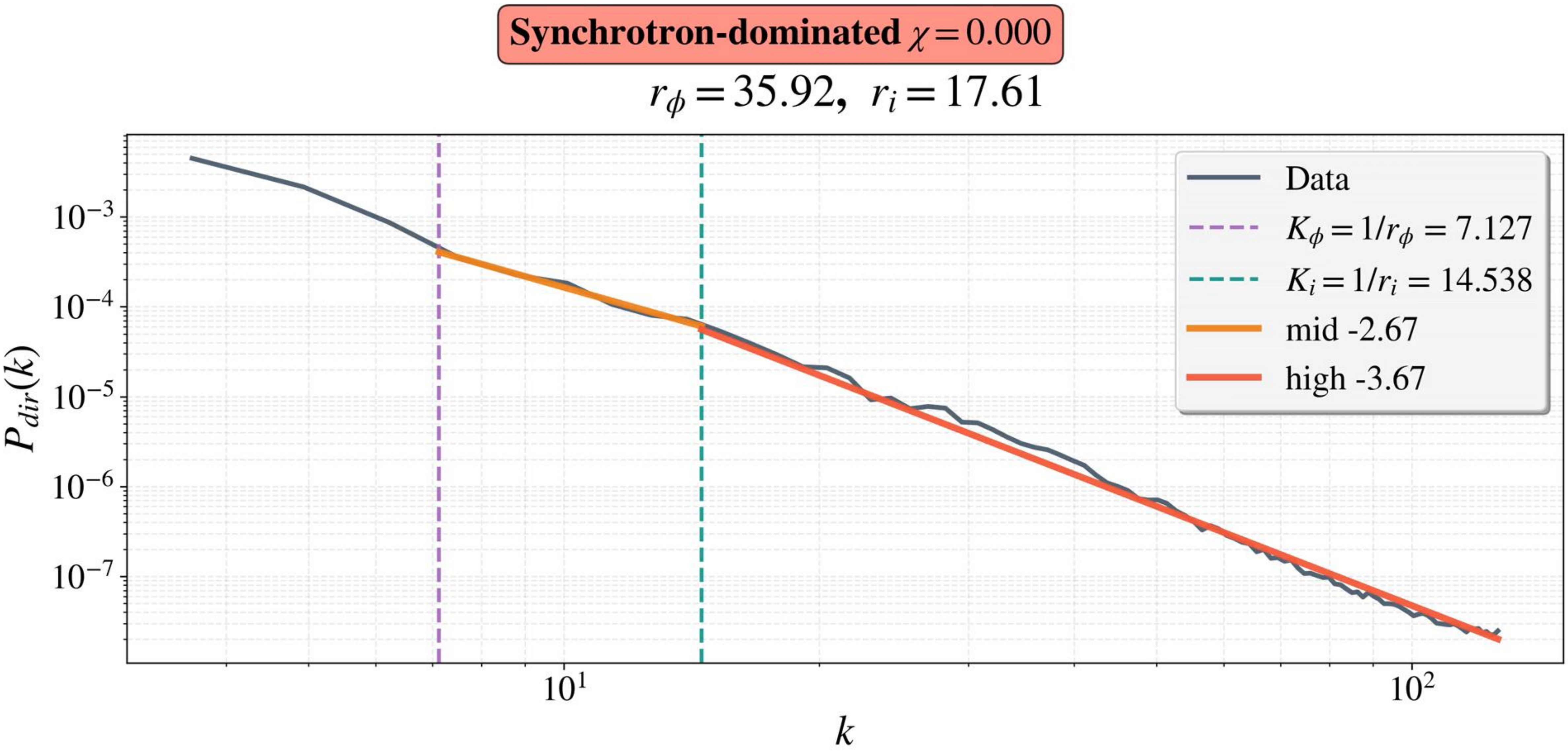


$-3 - m_\phi = -11/3$
Directional spectr
$k_y - 2/L$
$k_x - 2/L$
250
200
150
100
50
0
0
100
200
2
0
Data
Fit [0.0-1.0] (slope = -3.66)
10^{-6}
10^{-7}
10^1
10^2
k

$$-3 + m_\phi = -7/3$$

Faraday-dominated $\chi = 3.50$





Synchrotron-dominated $\chi = 0.000$
$r_\phi = 35.92,\ r_i = 17.61$
$P_{dir}(k)$
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{1}
10^{2}
k
Data
$K_\phi = 1/r_\phi = 7.127$
$K_i = 1/r_i = 14.538$
mid -2.67
high -3.67

Four complementary measures

LP16

- Polarization Frequency Analysis (PFA): $\langle|P(\lambda)|^2\rangle$
- Polarization Spatial Analysis (PSA): $P_{2D}(k;\lambda)$
- Derivative: PSA of $\frac{dP}{d\lambda^2}$
- **Directional spectrum:** $P_{dir}(k) = |\mathrm{FFT}(\cos(2\chi)|^2 + |\mathrm{FFT}(\sin(2\chi)|^2$

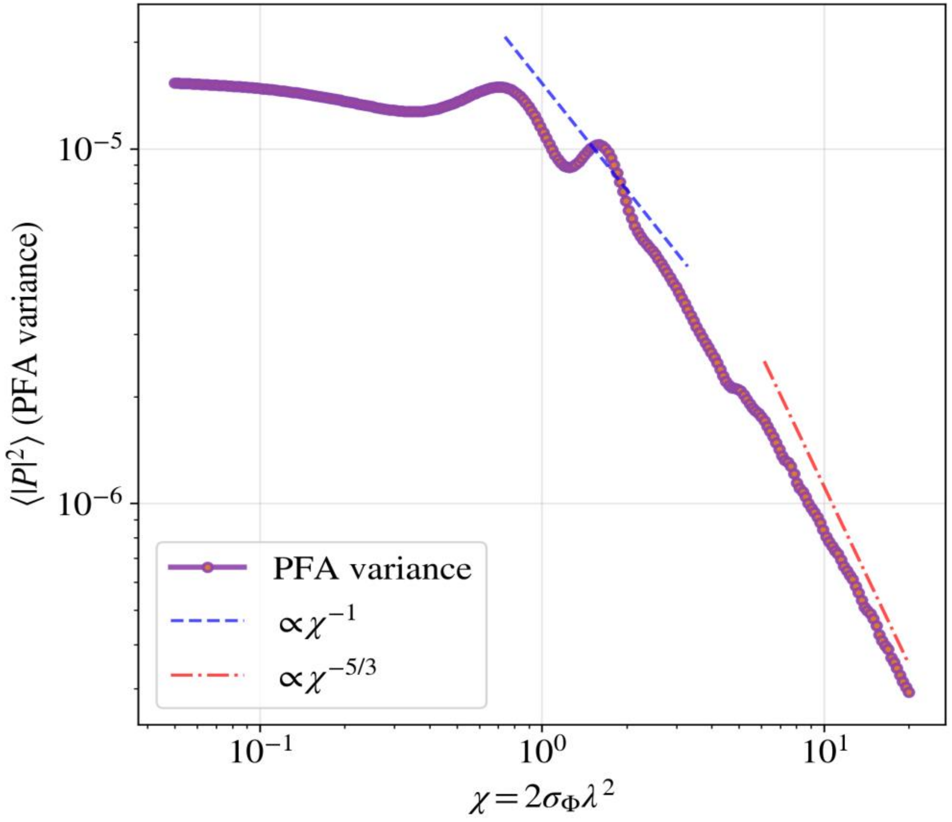
Polarization Frequency Analysis (PFA)

$$\boxed{\mathrm{PFA}(\lambda^2) = \langle |P(\mathbf{x};\lambda)|^2 \rangle_{\mathbf{x}}}$$

External Faraday screen:

$$P(\mathbf{X},\lambda) = P_{\mathrm{emit}}(\mathbf{X}) e^{2i\lambda^2 \Phi_{\mathrm{screen}}(\mathbf{X})}$$

Mixed

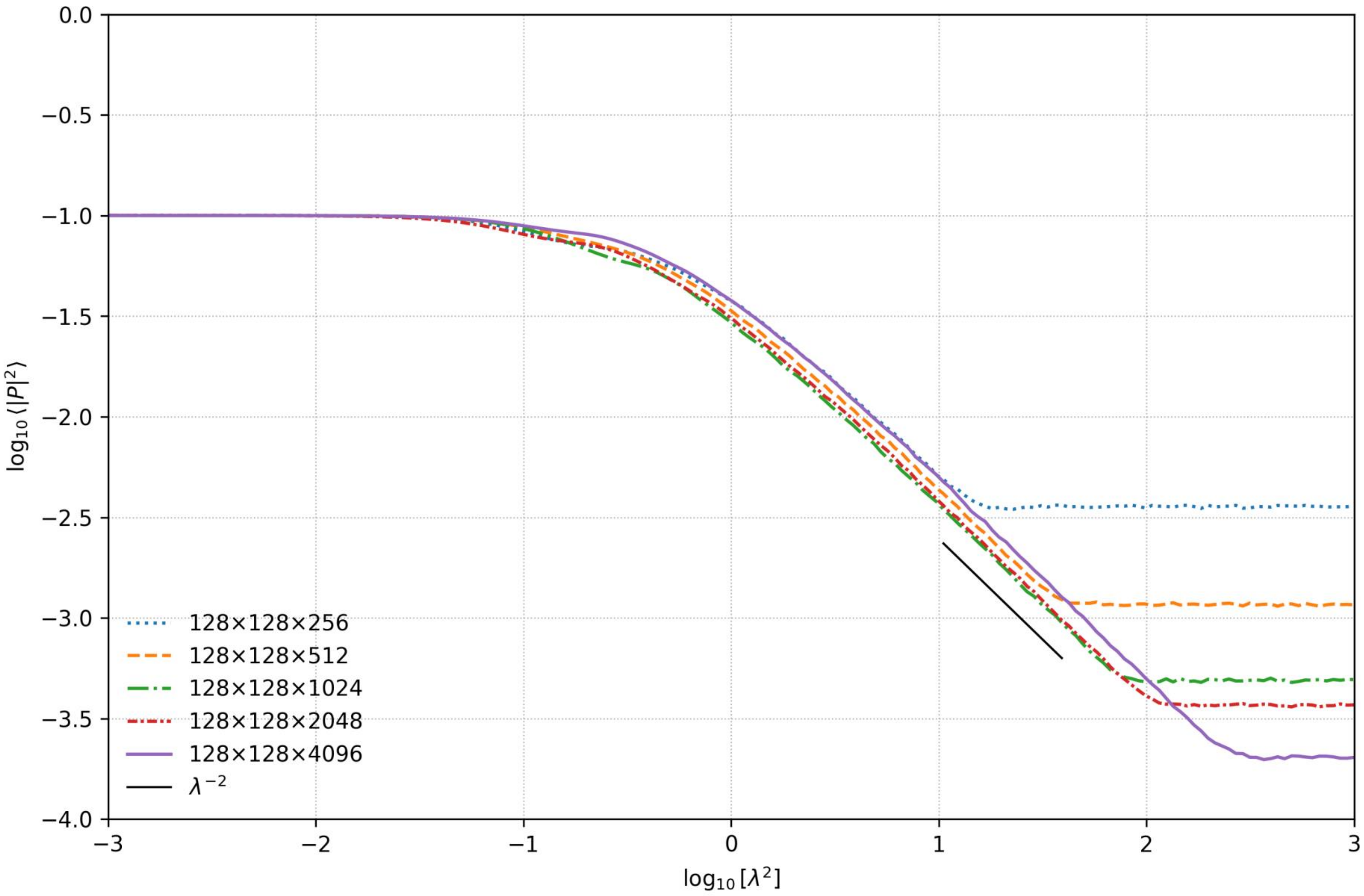


LP16 prediction:

Plateau for $\chi < 1$

$\propto \chi^{-1}$ ($\propto \lambda^{-2}$) for $1 < \chi < 10$

Polarization variance as a function of the square of wavelength λ^2 in the 3D case. The initial settings are $\beta = 11/3$ and $\eta = \infty$ (Zhang et al. 2016).



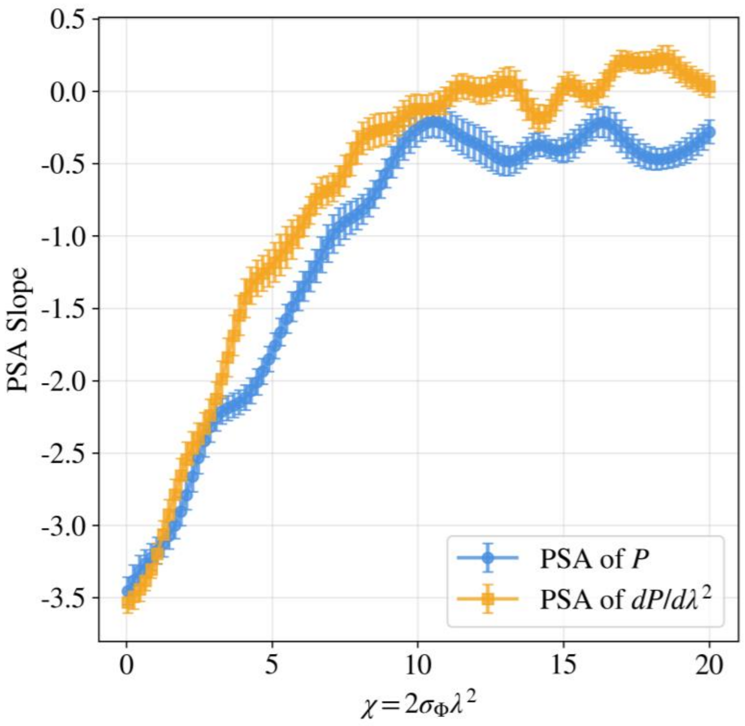
Polarization Spatial Analysis (PSA)

$$\mathrm{P}_{2D}(k;\lambda) = \left\langle |\, \tilde{P}(\mathbf{k};\lambda)|^2 \right\rangle_{|\mathbf{k}|=k}$$

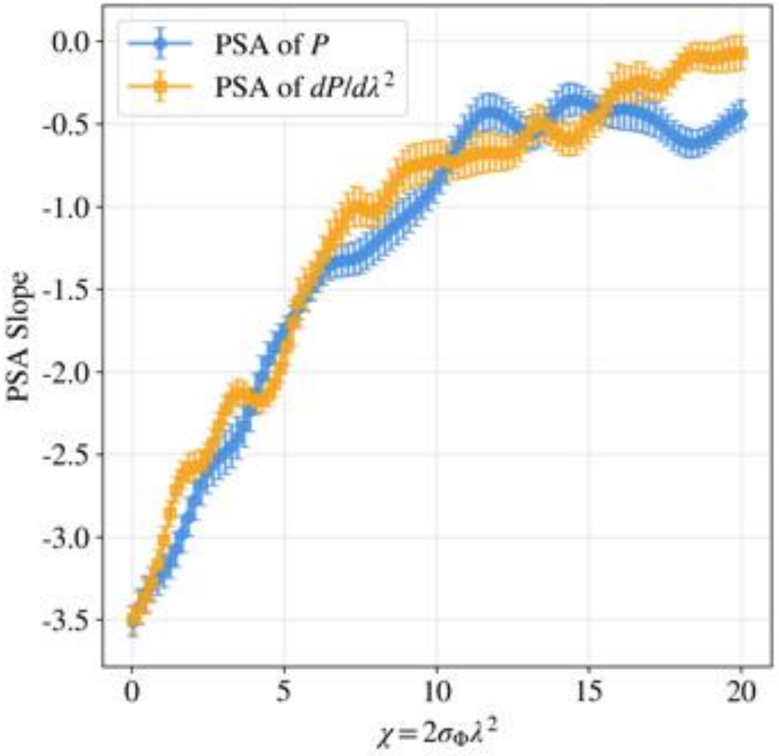
With energy-like spectrum

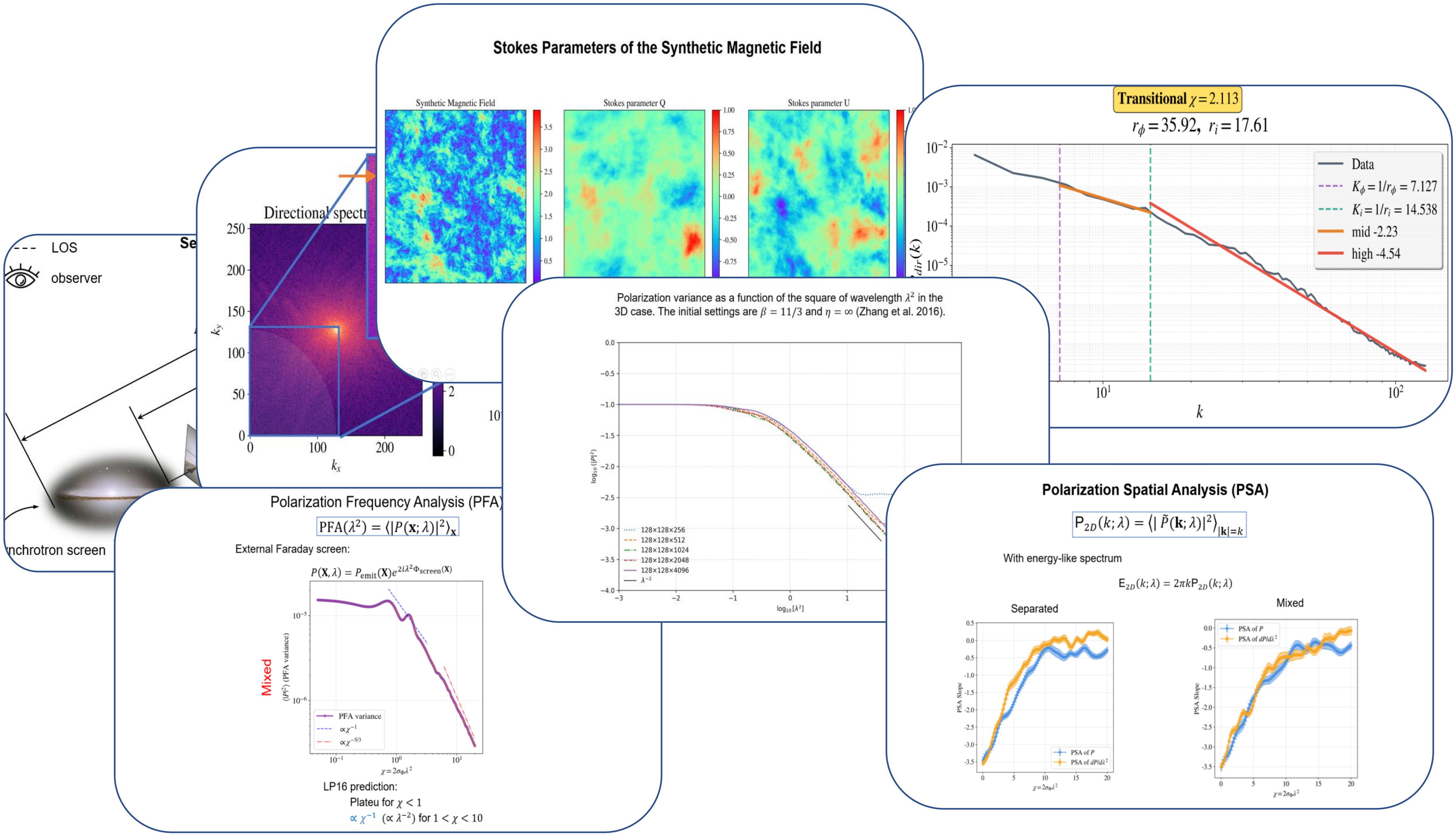
$$\mathrm{E}_{2D}(k;\lambda) = 2\pi k \mathrm{P}_{2D}(k;\lambda)$$

Separated



Mixed





Stokes Parameters of the Synthetic Magnetic Field
Synthetic Magnetic Field
Stokes parameter Q
Stokes parameter U
Transitional $\chi = 2.113$
$r_\phi = 35.92, \ r_i = 17.61$
Data
$K_\phi = 1/r_\phi = 7.127$
$K_i = 1/r_i = 14.538$
mid -2.23
high -4.54
k
LOS
observer
Directional spectr
k_x
k_y
nchrotron screen
Polarization variance as a function of the square of wavelength λ^2 in the 3D case. The initial settings are $\beta = 11/3$ and $\eta = \infty$ (Zhang et al. 2016).
128×128×256
128×128×512
128×128×1024
128×128×2048
128×128×4096
λ^{-2}
$\log_{10}[\lambda^2]$
$\log_{10}(|P|^2)$
Polarization Frequency Analysis (PFA)
$\mathrm{PFA}(\lambda^2) = \langle |P(\mathbf{x};\lambda)|^2 \rangle_{\mathbf{x}}$
External Faraday screen:
$P(\mathbf{X},\lambda) = P_{\mathrm{emit}}(\mathbf{X}) e^{2i\lambda^2 \Phi_{\mathrm{screen}}(\mathbf{X})}$
Mixed
PFA variance
$\propto \chi^{-1}$
$\propto \chi^{-5/3}$
$\chi = 2\sigma_\Phi \lambda^2$
LP16 prediction:
Plateu for $\chi < 1$
$\propto \chi^{-1}$ ($\propto \lambda^{-2}$) for $1 < \chi < 10$
Polarization Spatial Analysis (PSA)
$\mathrm{P}_{2D}(k;\lambda) = \langle |\tilde{P}(\mathbf{k};\lambda)|^2 \rangle_{|\mathbf{k}|=k}$
With energy-like spectrum
$\mathrm{E}_{2D}(k;\lambda) = 2\pi k \mathrm{P}_{2D}(k;\lambda)$
Separated
Mixed
PSA Slope
PSA of P
PSA of $dP/d\lambda^2$
$\chi = 2\sigma_\Phi \lambda^2$

Take-home and outlook

- Single-frequency polarization already encodes turbulence: $S(R)$ and $P_{dir}(k)$ recover slopes.
- Crossover $k_{\times}\ (\lambda)$ operationalizes separation of emission from screen.
- Future work with LOFAR, MeerKAT, VLA archives; sets the stage for SKA when spectral coverage is sparse.

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