Set $x \in [0, L]$ from top down. $y(x,t)$ is vertical position.

Continuum model

$$\mu\, y_{tt} = K\, y_{xx} + \mu g$$

Let $w = y - \frac{1}{2}gt^2$ so

$$w_{tt} = c^2 w_{xx}, \qquad c = \sqrt{K/\mu}$$

Free ends after release: $w_x(0,t) = 0$, $w_x(L,t) = 0$. At $t = 0$: $w_t(x,0) = 0$.

Stall time

$$\text{signals move on } x \pm ct = \text{const}, \quad t_{\text{stall}} = \frac{L}{c}$$

Center of mass

$$\mu \int_0^L y_{tt}\, dx = K[y_x]_0^L + \mu g L = \mu g L$$

$$M = \mu L, \quad Y_{\text{CM}} = \frac{1}{L}\int_0^L y\, dx \Rightarrow \ddot{Y}_{\text{CM}} = g$$

Discrete cross-check

$$m\, \ddot{y}_j = k(y_{j+1} - 2y_j + y_{j-1}) + mg$$

$$w_j = y_j - \tfrac{1}{2}gt^2 \Rightarrow \ddot{w}_j = \frac{k}{m}(w_{j+1} - 2w_j + w_{j-1})$$

Continuum limit gives $c = \sqrt{k\,\Delta x^2/m}$ and $t_{\text{stall}} \approx L/c$.

Measure c

$$c \approx \frac{2L}{\Delta t_{\text{rt}}} \quad \text{(round trip top} \to \text{bottom} \to \text{top)}$$

Prediction

$$\text{slope of } t_{\text{stall}} \text{ vs } L = \frac{1}{c}$$