

## Basic Astrophysics

1. What would be the radius of the Sun when it becomes a black hole (although the probability is almost zero)?

- a. 1 km
- b. 2 km
- c. **3 km (Answer)**
- d. 4 km
- e. 5 km
- a.  $8.94 \times 10^{47} \text{ J}$

Answer explanation:  $R_{\text{sch}}$  is the radius of the black hole. In the black hole, escape velocity is the speed of light.

$$\sqrt{2GM/R_{\text{sch}}} = c$$

$$R_{\text{sch}} = 2GM/c^2 = 2954\text{m} = 3\text{km}$$

2. A comet called "SMukherjee2017" which has physical values of  $e = 0.12$ ,  $a = 4\text{AU}$  was visible from the Earth in 2017. In which year is this comet visible again?

- a. 2021
- b. 2022
- c. 2023
- d. 2024
- e. **2025 (Answer)**

Answer explanation:

$$P^2 = 4\pi^2/G(M_1+M_2)*a^3$$

Inside the solar system this can be abridged to  $P^2 = a^3$ . Thus  $P=64^{0.5} = 8 \text{ years}$

SMukherjee was visible in 2017. Thus it will be visible again in  $2017+8 = 2025$ .

3. What is the perihelion of comet "SMukherjee2017"?

- a. 3.0 AU
- b. **3.5 AU (Answer)**
- c. 4.0 AU
- d. 4.5 AU

- e. 5.0 AU

Answer explanation:  $a_p = a(1-e) = 4 (1-0.12) = 4*0.88 = 3.52\text{AU}$

4. A comet called “LQj2017” which has physical values of  $e = 1.2$ ,  $a = 19\text{AU}$  was visible from the Earth in 2017. In which year is this comet visible again?

- a. 2080
- b. 2100
- c. 2109
- d. 2130
- e. **The comet is not visible again. (Answer)**

Answer explanation: For conic sections:

$e = 0$ : circle

$0 < e < 1$ : ellipse

$e = 1$ : parabola

$e > 1$ : hyperbola

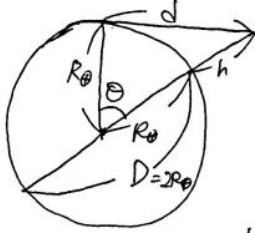
In the case of comet LQj2017,  $e = 1.2 > 1$ , so the orbit is a hyperbola. Thus, it will not be visible again.

5. How far away is the horizon from you, when your height is about 5ft 11in (1.8m)? You are standing on a plain that has no mountains near you and the elevation is 0ft. Ignore the effect of atmospheric refraction and the oblateness of the Earth.

- a. 2.4 km
- b. **4.8 km (Answer)**
- c. 24 km
- d. 48 km
- e. 240 km

Answer explanation:

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Method 1.

$$\cos \theta = \frac{R_{\oplus}}{R_{\oplus} + h}$$

$$\theta = \cos^{-1}\left(\frac{R_{\oplus}}{R_{\oplus} + h}\right)$$

$$\tan \theta = \frac{d}{R_{\oplus}}$$

$$d = R_{\oplus} \tan \theta$$

$$= R_{\oplus} \tan\left(\cos^{-1}\left(\frac{R_{\oplus}}{R_{\oplus} + h}\right)\right)$$

$$= (6.371 \times 10^6) \tan\left(\cos^{-1}\left(\frac{6.371 \times 10^6}{6.371 \times 10^6 + 1.8}\right)\right)$$

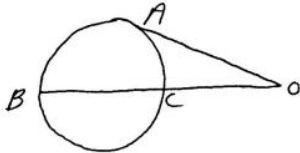
$$= 4789 \text{ m}$$

$$\approx 4800 \text{ m}$$

$$= \underline{4.8 \text{ km}}$$

Method 2.

Consider the secant-tangent theorem:



$$OA^2 = OB \cdot OC$$

$$\text{Then } d^2 = h(D+h)$$

$$d = \sqrt{h(D+h)}$$

$$= \sqrt{h(2R_{\oplus} + h)}$$

$$= 4789 \text{ m}$$

$$\approx 4800 \text{ m}$$

$$= \underline{4.8 \text{ km}}$$

6. Consider that the absolute magnitude of a star is  $m_0$ . Imagine that the first star gets split into  $N$  smaller identical stars with the same temperature and average densities as the initial star, and that the sum of the masses of all  $N$  smaller stars is equal to the initial star's mass (i.e., total mass is conserved). What is the total combined absolute magnitude ( $m$ ) of all the  $N$  stars assuming that none of the stars obstruct each other's light (i.e. their luminosities add linearly)?

- $m = m_0 - \log(N)$
- $m = m_0 - 2.5 \log(N)$
- $m = m_0 - \frac{2.5}{3} \log(N)$  (Answer)
- $m = m_0 - \frac{2.5}{N}$
- $m = m_0 - 2.5 N$

**Answer:** The initial star had a luminosity

$$L_0 = 4 \pi R^2 \sigma T^4$$

Let  $R$  be the initial star's radius and  $r$  be the radius of the smaller final stars. Conservation of mass implies:

$$M = N m$$

or

$$4 \pi R^3 / 3 = 4 \pi r^3 N / 3$$

(since we assume the stars have the same average densities).

Therefore:

$$r = R N^{-1/3}$$

The total final luminosity will be

$$L = N(4 \pi r^2 \sigma T^4) = N^{1/3} L_0$$

The final combined absolute magnitude will be:

$$M = M_0 - 2.5 \log(L/L_0) = m_0 - \frac{2.5}{3} \log N$$

7. Taking the radius of the black hole to be the Schwarzschild radius (the radius at which the escape velocity of an object would be equal to the speed of light), what is the surface area of a black hole of mass  $M$ ?

- a.  $A = \frac{16\pi}{c^4} G^2 M^2$  (Answer)
- b.  $A = \frac{4\pi}{c^4} G^2 M^2$
- c.  $A = \frac{4\pi}{3c^4} G^2 M^2$
- d.  $A = 16 \pi G^2 M^2$
- e.  $A = \frac{16\pi}{c^2} G^2 M^2$

**Answer:**

$$R_s = \frac{2GM}{c^2}, A = 4 \pi R_s^2 = 16 \pi G^2 M^2 / c^4$$

8. In 1974, Stephen Hawking proved that black holes emit blackbody radiation according to the Stefan-Boltzmann law (due to quantum effects near the event horizon). This radiation is called Hawking radiation and through this process, black holes slowly evaporate their mass away in the absence of new material to accrete. Assume that the Hawking temperature of a black hole is inversely proportional to its mass (i.e.  $T_H = \text{const.} / M$ ) and that our initial black hole of mass  $M$  gets split into  $N$  smaller black holes, each with a mass  $M / N$ . Using the results found in problems 6 and 7, what is the relation between the final combined luminosity of the smaller black holes ( $L$ ) and the luminosity of the initial black hole ( $L_0$ )?

- a.  $L = L_0$
- b.  $L = L_0 / N$
- c.  $L = N L_0$
- d.  $L = N^2 L_0$
- e.  $L = N^3 L_0$  (Answer)

**Answer:** From problem 9,  $A_0 = \frac{16\pi}{c^4} G^2 M^2$ , so the surface area of each small black hole will be

$A = \frac{16\pi}{c^4} G^2 \frac{M^2}{N^2} = \frac{A_0}{N^2}$ . Moreover, the temperature of a small black hole will be

$T = \text{const.}$   $N / M = T_0 M / M = N T_0$ . Finally, the combined luminosity of the smaller black holes will be:

$$L = N (A \sigma T_{H,0}^4) = N \frac{A_0}{N^2} \sigma N^4 T_{H,0}^4 = N^3 L_0$$

9. We observe that a quasar's brightness varies within less than a day. What is the best upper bound on the quasar's size that you can derive from this information?

- a. 8 kpc
- b. 170 AU
- c. 3 AU
- d. 3 Sun Radii
- e. 1 pc

**Answer explanation:** We conclude that whatever must be producing the light coming from the quasar must be less than the time of variation multiplied by the speed of light, since the "news" of the change takes different amount of time to reach us from different parts of the quasar. Speed of light  $c = 3 \times 10^8$  m/s. 1 day = 86400 s. Thus, quasar's  $2R < c \times 1 \text{ day} = 2.5 \times 10^{13} \text{ m} = 167 \text{ AU} = 8.3 \times 10^{-4} \text{ pc}$ .

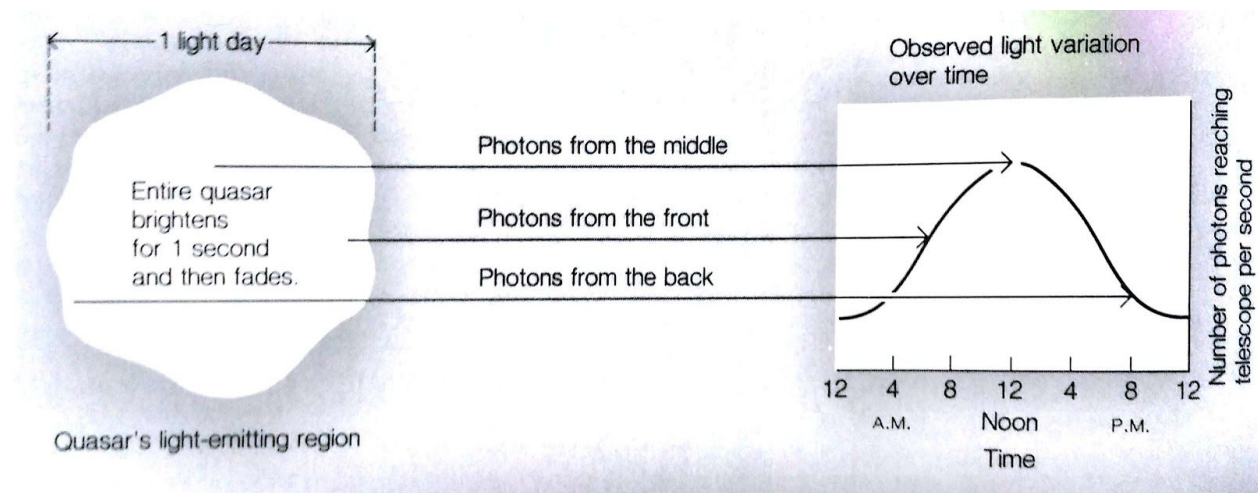


Photo credit: Donald Goldsmith - "The Evolving Universe", pg 182

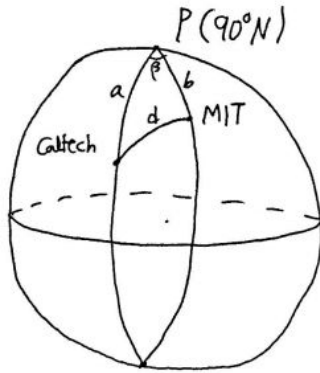
## Coordinates and Times

10. A student group from California Institute of Technology is planning a massive prank for this year. This year, they are planning on launching a rocket to Massachusetts Institute of Technology and making it explode above the so called 'Great Dome' of MIT. The rocket will contain a giant parachute printed with a logo of Caltech, so that the Great Dome will be covered with Caltech logo. How much distance does this rocket need to fly? The longitude and latitude information of two locations are given in the table below. (Assume that the Earth is a perfect sphere with radius of 6371 km.)

	Latitude	Longitude
The Great Dome of MIT	42.3601° N	71.0942° W
Caltech	34.1377° N	118.1253° W

- a. 3890 km
- b. **4160 km (Answer)**
- c. 4780 km
- d. 4910 km
- e. 5290 km

Answer explanation:



Spherical law of cosine:

$$\cos x = \cos y \cos z + \sin y \sin z \cos X$$

$$a = 90^\circ - 34.1377^\circ = 55.8623^\circ$$

$$b = 90^\circ - 42.3601^\circ = 47.6399^\circ$$

$$\beta = 116.1253^\circ - 71.0942^\circ = 47.0311^\circ$$

$$\cos d = \cos a \cos b + \sin a \sin b \cos \beta$$

$$= \cos(55.8623^\circ) \cos(47.6399^\circ) + \sin(55.8623^\circ) \sin(47.6399^\circ) \cos(47.0311^\circ)$$

$$= .794987$$

$$d = \cos^{-1}(.794987) = 37.346^\circ$$

$$2\pi R_E \times \frac{37.346^\circ}{360^\circ} \approx \underline{4160 \text{ km}}$$

This problem works best before the previous problem (10)

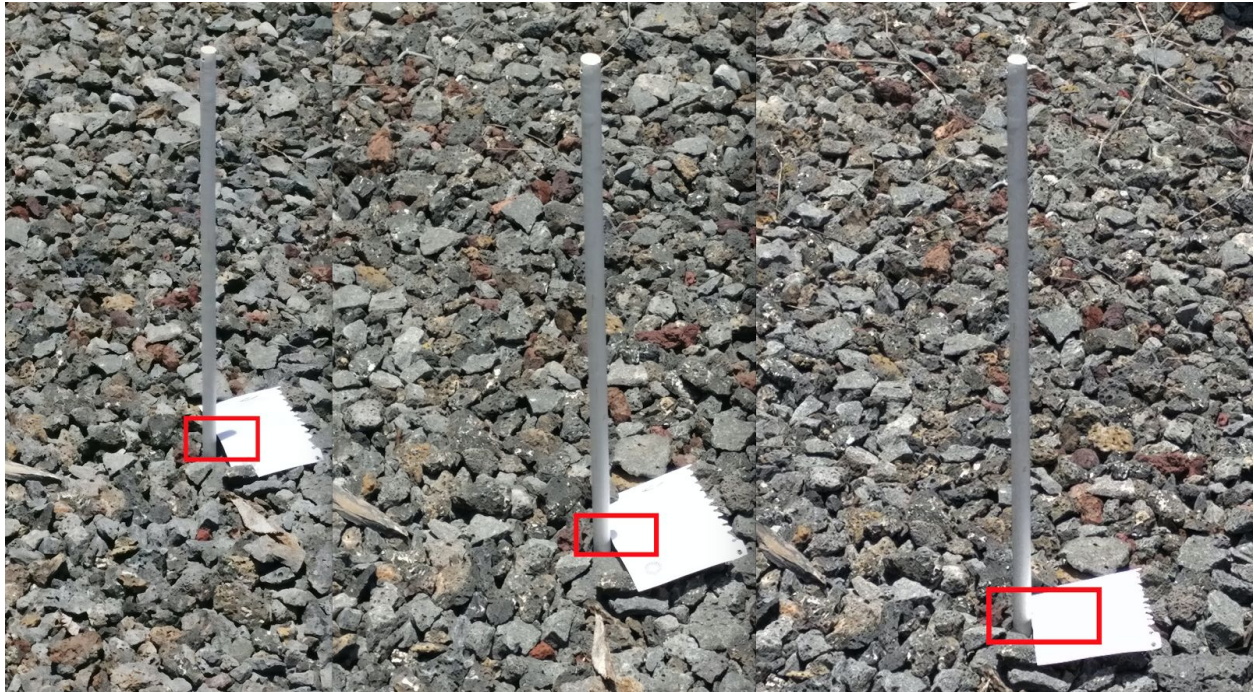
11. The MIT students have found out about the plans of the Caltech students of sending a rocket to fly a parachute above the "Great Dome" of MIT. They know the rocket will follow an elliptical orbit of semiaxis  $a$  with the center of the Earth at one of its foci. To counter this prank, they will aim another rocket at the original rocket, which upon collision will transfer enough energy to the Caltech rocket to make it reach the escape velocity of the Earth at an early point in its trajectory and make it unable to reach back to the Earth's surface again. What is the energy that needs to be transferred to the Caltech rocket in order for the MIT students to reach their goal? Assume that the Earth has mass  $M$ , and the Caltech rocket has mass  $m$ .

- $GMm/(3a)$
- $GMm/(3a^2)$
- $GMm/(2a)$**
- $GMm/a$
- $GMm/a^2$

**Answer explanation:** Total energy of the Caltech rocket in its original orbit is  $-GMm/(2a)$ . To put in an orbit where it escapes the gravitational field of the Earth, it must be able to reach infinity. To get the minimum energy, we can assume it has 0 speed at infinity. Thus, its total energy is 0 at infinity. Due to

*conservation of energy, it means the total energy must be zero right after the collision and that the Caltech rocket is now on a parabolic orbit. Thus, the transferred energy, equal to the difference between final (0) and initial  $(-GMm/(2a))$ , is  $GMm/(2a)$*

12. The following series of photos were taken on March 17 around noon time (each photo taken about 3 minutes apart from the previous one). In each photo you can see a white vertical pole (about 1 foot high), and a piece of paper at the base of the pole. Notice how the shadow cast by the pole on the piece of paper is slowly disappearing. Based on the date and time, and the length of the pole's shadow, where were the photos taken?



(photo for problem 12. Copyright: Ana-Roxana Pop. Ask for author's permission before using.)

- a. Seattle, USA ( $47.61^\circ$  N,  $122.33^\circ$  W)
- c. Los Angeles, USA ( $34.05^\circ$  N,  $118.24^\circ$  W)
- b. Cancun, Mexico ( $21.16^\circ$  N,  $86.85^\circ$  W)
- d. Galapagos Islands, Ecuador ( $0.74^\circ$  S,  $90.32^\circ$  W)**
- e. Sydney, Australia ( $33.87^\circ$  S,  $151.21^\circ$  E)

**Answer explanation:** *on the Vernal Equinox (March 21), the plane of Earth's equator passes through the center of the Sun. On this day, at a point on the equatorial line, the Sun is exactly overhead. Therefore, since the photos were taken close to March 21st, they should have been taken somewhere near the Equator, i.e. in the Galapagos Islands.*

13. Imagine you are at the South Pole on December 21st. Which of these statements best describes your shadow on this day?

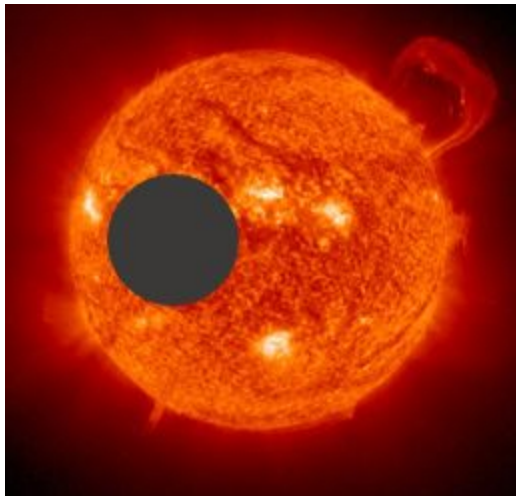


- a. on December 21st at the South Pole it is night all day so you have no shadow at all
- b. you have no shadow at noon when the Sun is over your head, but you can see your shadow the rest of the day
- c. the Sun will be up all day but it will be straight over your head, so you will have no shadow all day
- d. the Sun will be up all day and your shadow will trace out a 360° circle during one day**
- e. your shadow will point towards the North Pole

**Answer explanation:** on the winter solstice (December 21st), the Sun never sets at the South Pole. If you want to cast no shadow at noon, you have to be at a latitude between 23.5 N and 23.5 S. Anywhere outside this region, the Sun will never be directly overhead. Moreover, in the Southern Hemisphere your shadow will point towards the South Pole, except that if you are exactly at the South Pole, your shadow will have a base (at your feet) exactly at the Pole, while the tip of your shadow (i.e. the shadow cast by your head) will draw a 360 degree circle in 24 hours (roughly).

## Solar System

14. The USAAO team successfully launched a satellite to be in the orbit within the boundary of the Solar System. The satellite took a picture of the Moon passing in front of the Sun on April 24, 2017. Estimate the distance between the Moon and the satellite. (Angular diameter of the Sun is 32')



(The copyright of this picture is reserved to NASA.)

- a.  $5.56 \times 10^5$  km
- b.  **$1.12 \times 10^6$  km (Answer)**
- c.  $2.24 \times 10^6$  km
- d.  $1.12 \times 10^7$  km
- e.  $2.24 \times 10^7$  km

Answer explanation:

One needs to actually measure the ratio of radii. (By ruler, ideally.)

$$R_o : R_m = 3 : 1$$

Angular diameter of the Sun is  $32'$ .

Therefore angular diameter of the Moon is  $\frac{32'}{3} = 10.667'$

Let 'd' be the distance between the satellite and the Moon.

$$d \times \left( \frac{10.667'}{60'} \times \frac{\pi}{180} \right) = 2R_m$$

$$d = \frac{2R_m}{\left( \frac{10.667'}{60'} \times \frac{\pi}{180} \right)}$$
$$= \frac{2(1.737 \times 10^6 \text{ m})}{\left( \frac{10.667'}{60'} \times \frac{\pi}{180} \right)}$$

$$= 1.1196 \times 10^9 \text{ m}$$

$$\approx 1.12 \times 10^9 \text{ m}$$

$$\approx \underline{1.12 \times 10^6 \text{ km}}$$

15. At which of the following phases of the Moon's orbit is the tidal bulge of Earth largest?

- a. **Full**
- b. First Quarter
- c. Waxing Gibbous
- d. Waning Gibbous
- e. Waxing Crescent

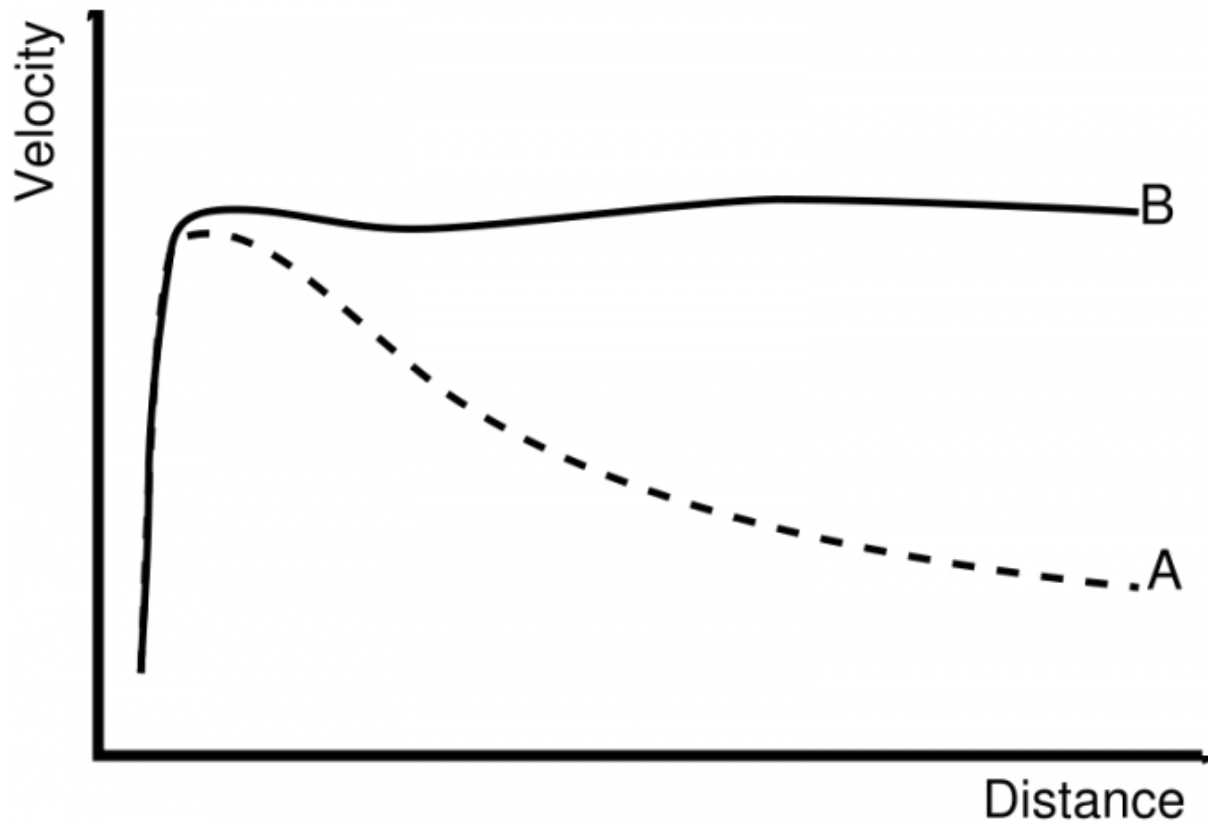
Answer explanation:

<http://earthsky.org/earth/tides-and-the-pull-of-the-moon-and-sun>

Or

<http://home.hiwaay.net/~krcool/Astro/moon/moontides/>

## Stars



16. A predicted (A) and observed (B) rotation curve of a typical spiral galaxy is shown above. What component of the galaxy causes this discrepancy?

- a. Baryons
- b. Neutrinos
- c. Gamma Rays
- d. **Dark Matter**
- e. Globular Clusters

Answer explanation:

See [https://www.wikiwand.com/en/Galaxy\\_rotation\\_curve](https://www.wikiwand.com/en/Galaxy_rotation_curve)

17. What is a star burning in its core when it is on the horizontal branch?

- a. Atomic Hydrogen
- b. Molecular Hydrogen
- c. **Helium**
- d. Carbon
- e. Oxygen

Answer Explanation: See [https://www.e-education.psu.edu/astro801/content/l6\\_p3.html](https://www.e-education.psu.edu/astro801/content/l6_p3.html)

And [https://www.wikiwand.com/en/Horizontal\\_branch](https://www.wikiwand.com/en/Horizontal_branch)

18. What is the final stage in the evolution of a G1 star with mass of  $1.1M_{\text{Sun}}$ ?

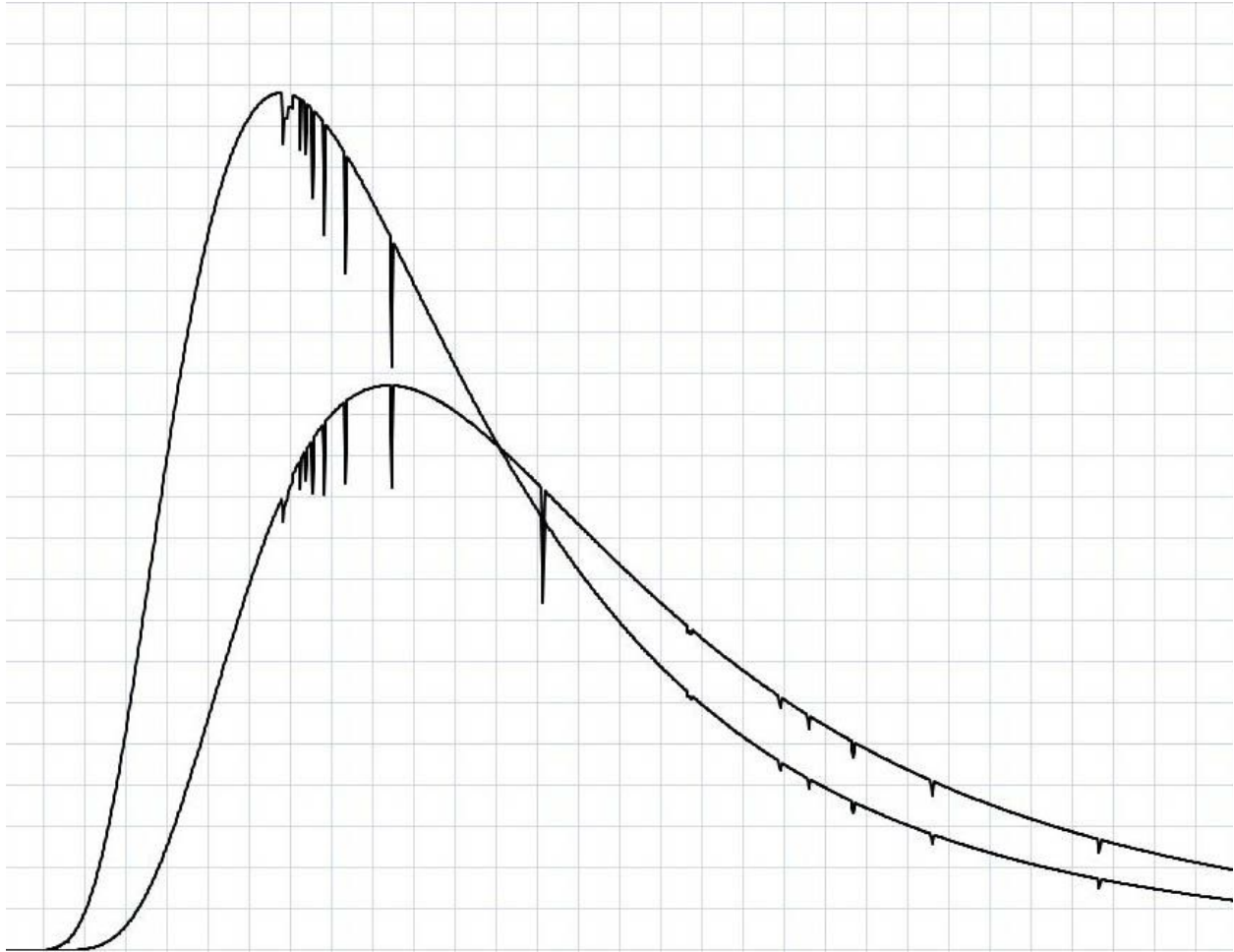
- a. Red Giant
- b. **White Dwarf**
- c. Black Hole
- d. Neutron Star
- e. Brown Dwarf

Answer explanation: See [https://www.e-education.psu.edu/astro801/content/l6\\_p3.html](https://www.e-education.psu.edu/astro801/content/l6_p3.html)

And <http://www.telescope.org/pparc/res8.html>

## Stellar Systems

19. The following graph shows spectral flux density as a function of wavelength for the 2 stars in a binary system. Assume that the stars are young and are comprised only of Hydrogen and a small amount of Helium. Moreover, assume that the stars can be treated as black bodies. Answer the following questions based on the graph. Take Wien's Displacement Constant to be  $2.8978 \text{ mm K}$  (The origin of the graph corresponds to zeroes for both wavelength and spectral flux density)



What are the surface temperatures of the 2 stars?

- a. 4000K, 8000K
- b. 6000K, 8000K**
- c. 3000K, 4000K
- d. 9000K, 12000K
- e. 5000K, 12000K

Solution: Looking at the Hydrogen absorption lines, we get the the lines to the left are part of the Balmer series. Therefore, the long line at the right is the H-alpha line. Using this, we can get the scale for the graph. After, that we simply use Wien's Displacement Law to get the temperatures.

20. What is the ratio of the radius of the hotter star to that of the colder star?

- a. 1.7
- b. 1.3
- c. 0.8
- d. 0.7
- e. 0.6**

Solution: The heights of the peaks are proportional to  $R^2T^5$ . Therefore, since we know the ratio of the temperatures, we can get the ratio of the radii.

21. Given that the apparent magnitude of the system is 8.2, what is the apparent magnitude of the brighter star?

- a. 8.4
- b. 8.7
- c. 8.9**
- d. 9.0
- e. 9.2

Solution: We can find the ratio of the luminosities and then use the definition for apparent magnitude.

22. Assuming that the radius of the smaller star is 2 solar radii, what is the distance to the system?

- a. 75 parsecs
- b. 85 parsecs**
- c. 100 parsecs
- d. 115 parsecs
- e. 150 parsecs

Solution: We can find the luminosity of the system and use the Sun as a reference point. (Note: Either the apparent or absolute magnitude of the sun should be included in the constant sheet)

Note: I am not sure whether the spectrum is supposed to look like this. I would appreciate if someone can make a better graph. Also, it might be a good idea to check the calculations once. (I will try to make a better graph when I have time)

## Cosmology

23. Astronomers have observed the Cosmic Microwave Background (CMB) radiation. Which of these is a correct description of the CMB radiation we observe today?

- a. Bright, uniform x-ray glow
- b. Faint, uniform radio signal**
- c. Faint, uniform x-ray glow
- d. Weak and patchy radio signal
- e. Weak background of cosmic neutrinos

Answer explanation:

[https://www.wikiwand.com/en/Cosmic\\_microwave\\_background](https://www.wikiwand.com/en/Cosmic_microwave_background)

24. Which is the correct chronological order of events?

- a. Inflation, Recombination, Reionization**

- b. Recombination, Reionization, Inflation
- c. Inflation, Reionization, Recombination
- d. Reionization, Recombination, Inflation
- e. Recombination, Inflation, Reionization

Answer explanation:

[https://www.wikiwand.com/en/Chronology\\_of\\_the\\_universe](https://www.wikiwand.com/en/Chronology_of_the_universe)

25. Edwin Hubble published in 1929 the paper "A Relation Between Distance and Extragalactic Nebulae" explaining his find that there exists a linear relationship between the radial velocities and distances for extragalactic objects. In the following graph you can see his data used in the paper. The units for the velocity are in km/s. (notice the "/" s" is missing from his original graph). What is the value for the Hubble constant that he derived from this data? Use the linear fit to the data marked with the continuous line to derive your estimation.

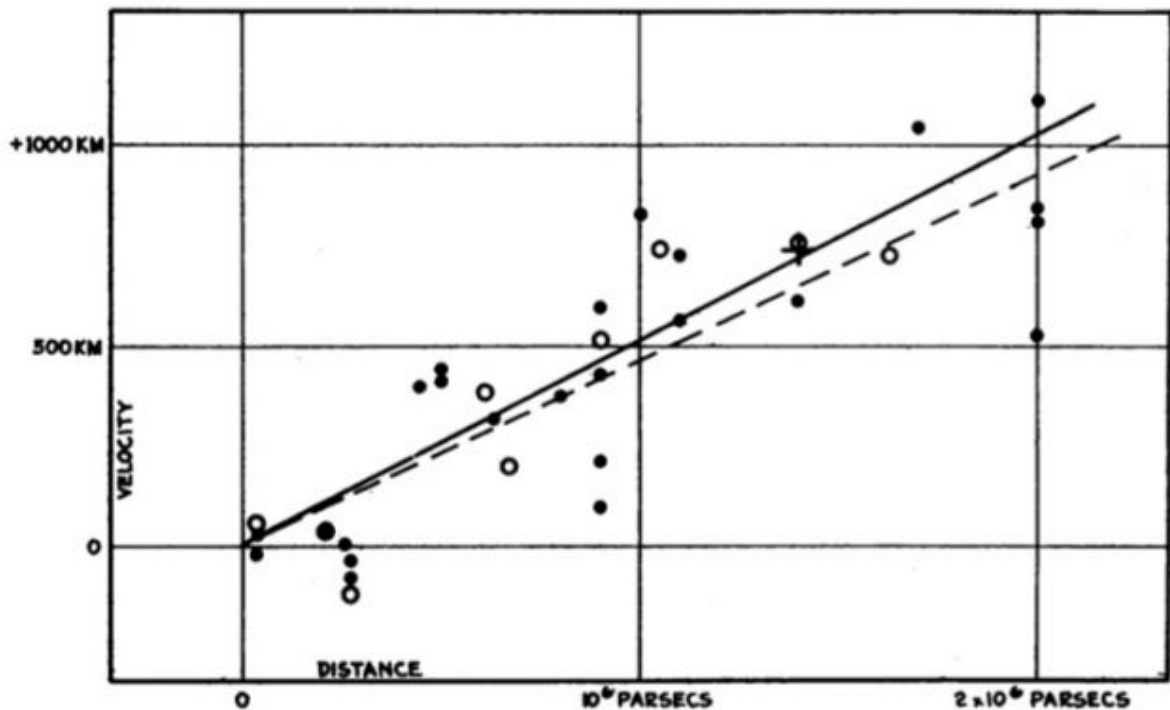


FIGURE 1

Velocity-Distance Relation among Extra-Galactic Nebulae.

- a. 500 km/s/pc
- b. 72 km/s/Mpc
- c. **500 km/s/Mpc**
- d. 50 km/s/Mpc
- e. 67km/s/Mpc

**Answer explanation:**  $v = H \cdot D$ . To determine  $H$ , pick two points along the line and take the ratio between their velocity and distance coordinates  $H = (v_1 - v_2) / (D_1 - D_2) = 500 \text{ km/s/Mpc}$ . Edwin Hubble's original 1929 paper, which includes the above graph, can be found at <http://www.pnas.org/content/15/3/168>. An interesting question comes in explaining the difference between this result and the currently established result of  $72 \text{ km/s/Mpc}$ .

## Instrumentation and Space Technologies

26. An astronomical interferometer is an array of separate radio telescope antennas that work together as a single telescope. Which of the following upgrades to your telescope setup will improve its resolving power **the most**?

- a. **Increasing the baseline (or distance between the telescopes) (Answer)**
- b. Increasing the number of telescopes per unit area
- c. Increasing the diameter of each telescope
- d. Increasing the electrical power supplied to each telescope
- e. Decreasing the baseline (or distance between the telescopes)

**Answer:** Very roughly, the interferometer achieves the effect of a telescope the size of the distance between the apertures, i.e. the size of your baseline (in terms of angular resolution). Note: this is not true in terms of the amount of light gathered. The angular resolution for a radio interferometer will be inversely proportional to the size of the baseline.

27. What is the diffraction limit of a 4.5-meter telescope if we are measuring at a wavelength of  $1.2 \text{ } \mu\text{m}$ ?

- a.  $1.5 \times 10^{-5} \text{ arcsec}$
- b.  $1.9 \times 10^{-5} \text{ arcsec}$  **(Answer)**
- c.  $2.2 \times 10^{-5} \text{ arcsec}$
- d.  $1.5 \times 10^{-4} \text{ arcsec}$
- e.  $1.9 \times 10^{-6} \text{ arcsec}$

**Note: The correct answer to this problem is**

**Answer:**  $\theta = 1.22 \frac{\lambda}{D} = 3.25 \times 10^{-7} \text{ rad} = 1.86 \times 10^{-5} \text{ degrees} = 0.07 \text{ arcsec}$

**As the units for the answer options a-e were wrong, this problem was excluded from the grading of the exam.**



## Practical

28. Choose the values (x, y, z) that would best complete the descriptions below for the 3 different types of twilights.

i) civil twilight, when the Sun is  $x^\circ$  below the horizon. We can start to see the brightest stars and the sea horizon can be clearly seen. At this point it becomes hard to read outdoors without artificial light.

ii) nautical twilight, when the Sun is  $y^\circ$  below the horizon. It is too dark to see the sea horizon and you can no longer make altitude measurements for navigation using the horizon as a reference.

iii) astronomical twilight, when the Sun is  $z^\circ$  below the horizon. Scattered sunlight becomes less than the average starlight and it is about the same brightness as the aurora or zodiacal light.

- a. Civil twilight -  $5^\circ$ , nautical twilight -  $10^\circ$ , astronomical twilight -  $15^\circ$
- b. **Civil twilight -  $6^\circ$ , nautical twilight -  $12^\circ$ , astronomical twilight -  $18^\circ$  (Answer)**
- c. Civil twilight -  $3^\circ$ , nautical twilight -  $6^\circ$ , astronomical twilight -  $9^\circ$
- d. Civil twilight -  $12^\circ$ , nautical twilight -  $6^\circ$ , astronomical twilight -  $18^\circ$
- e. Civil twilight -  $10^\circ$ , nautical twilight -  $20^\circ$ , astronomical twilight -  $30^\circ$

Answer explanation:

See <https://www.timeanddate.com/astronomy/different-types-twilight.html>

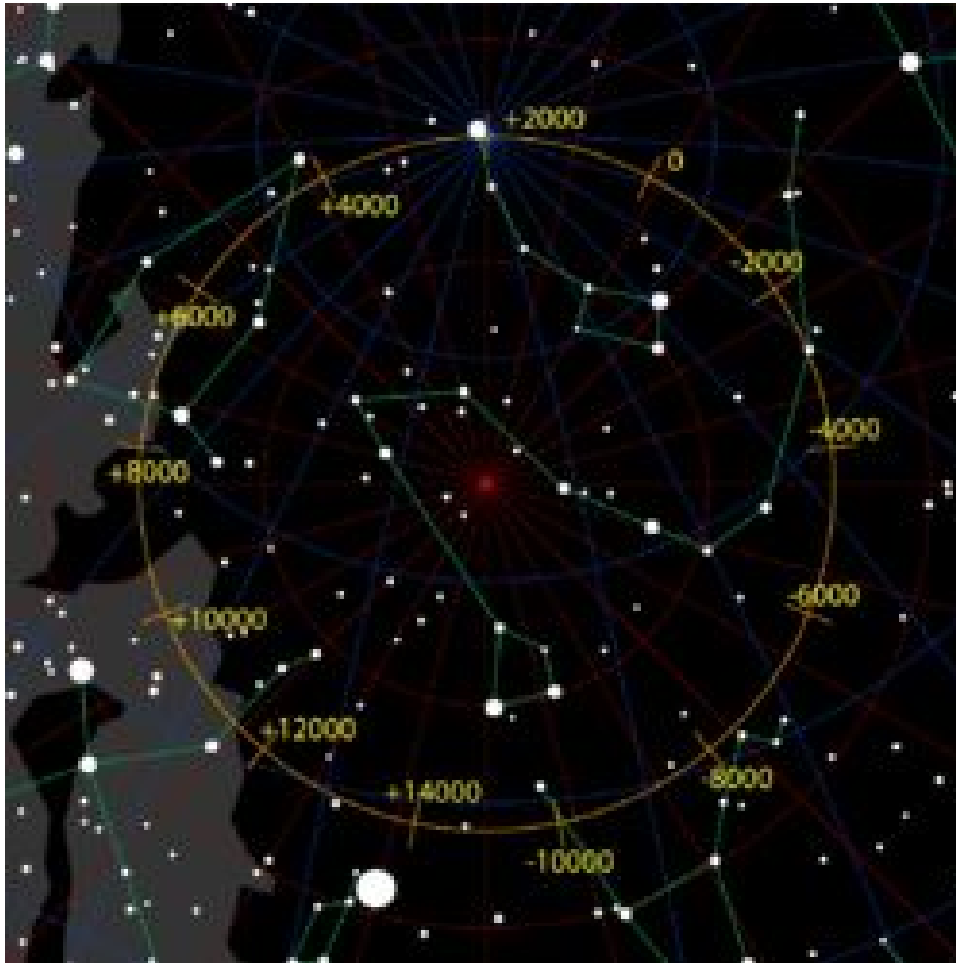
29. The Small Magellanic Cloud is located in the southeast corner of which of the following constellations:

- a. Orion
- b. Ophiucus
- c. Crux
- d. Draco
- e. **Tucana (Answer)**

30. Due to the precession of the equinoxes, the north and south celestial poles trace out circles on the celestial sphere with a period of about 25700 years. The circle traced by the North Celestial Pole is shown in the picture below. At the moment, Polaris is within one degree of the North Celestial Pole and this is why it is also called the North Star.

Based on the picture below, which star of magnitude  $\sim 1$  will be closest to the North Celestial Pole in the year 10000 CE?

(see next page)



(Image for P30. Image credit: Tau 'olunga, [https://commons.wikimedia.org/wiki/File:Precession\\_N.gif](https://commons.wikimedia.org/wiki/File:Precession_N.gif))

- a. Vega
- b. **Deneb (Answer)**
- c. Thuban
- d. Eltanin
- e. Alderamin

31. What is the 13'th constellation that crosses the ecliptic, which is not included in the 12 zodiacal constellations?

- a. Corona Borealis
- b. **Ophiuchus**
- c. Pegasus
- d. Aquila

e. Caelum