Savchenko Solutions

Stefan Nicov, Aliaksandr Melnichenka et al.

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Here you can find the the solutions to the Savchenko problem book. My advice would be to try really hard to solve the problem before checking the solution.
Feedback and suggestions are welcomed at:
stefannicov@gmail.com
astrosander01@gmail.com

1 Kinematics

1.1 Constant Speed Motion

1.1.1 $v = 200\ \frac{m}{s}$.

1.1.2 $v = 0.7\ \frac{km}{s}$; southeast.

1.1.3 $v = 3\ \frac{m}{s}$; $1\ m$ from the ceiling and $2\ m$ from the side wall.

1.1.4 At a distance of $1.15\ m$ from counter A.

1.1.5 $AO = L\frac{3t_A - 2t_B - t_C}{2(t_A - t_B)}$, $t_O = t_B - \frac{1}{2}(t_A - t_B)$.

1.1.6 $l' = l\frac{(v-u)}{(v+u)}$

1.1.7 $v = c\frac{(\tau_0 - \tau)}{(\tau_0 + \tau)}$

1.1.8 $\nu' = \nu\frac{(w-u)}{(w-v)}$

1.1.9 a. At $t < \frac{l}{v}$ the boundary of the region is a cone with the apex at distance vt from the end of the rod, passing into a sphere of radius ut touching it. At $t > \frac{l}{v}$ - spheres with centers at the end of the rod and radius ut and $u(t - \frac{l}{v})$ with a tangent conic surface. b^*. $\cos\alpha = \frac{u}{v}$.

1.1.10 From the region bounded by the angle $\alpha = 2\arcsin(\frac{u}{v})$ with vertex at point A, whose bisector is the highway.

1.1.11 $v = \frac{cl}{\sqrt{l^2 - c^2\Delta t^2}}$.

1.1.12 $u = \frac{v}{\sin\alpha}$

1.1.13 See figure

1.1.14 The ordinate and abscissa of the intersection point of the graphs $x_1 = vt$ and $x_2 = a + \frac{v(t-t_1)}{2}$ give the time and coordinate of the point of particle collision: $t' = \frac{(2a - vt_1)}{v}$, $x' = 2a - vt_1$

1.1.15 See Fig.; b) $v_{av} = 0$ c) $v_{av} = 1\ \frac{m}{s}$

1.1.16 See Fig.

1.1.17 See Fig. (a) The return of the beam along the x-coordinate takes a very short time, so that few electrons fall per unit length of the luminescent screen surface.
See Fig. b) for $\tau_y \tau_x = \frac{m}{n}$, where m and n are any integers.

1.1.18 $x = 2lv\frac{v \sin\alpha + \sqrt{c^2 - v^2\cos^2\alpha}}{c^2 - v^2}$

1.1.19 $\beta = 2\alpha$. In the direction opposite to the initial one.

1.1.20 $tg\varphi = \frac{2ma}{(nb)}$ where m and n are any integers.

1.1.21 $(-c_x, c_y, c_z)$, $(-c_x, -c_y, -c_z)$.

1.1.22 $\frac{\Delta t}{t} = \sqrt{\frac{(r^2-h^2)}{(R^2-h^2)}}$

1.1.23 See Fig. Zero at the walls. Highest at any location at a distance from the walls, greater than $2R$, and equal to $\frac{2R}{(L-2R)}$ at $L > 4R$; anywhere at a distance from the walls greater than $L - 2R$, and equal to one at $4R > L > 2R$.

1.2 Variable speed movement

1.2.1 $v_{av} = \frac{2}{\pi}v\frac{R-r}{R+r}$ is directed along the interface.

1.2.2 $t = 12\ s$, $x = 24\ m$

1.2.3 $L = v_0 t + \frac{v_0(t-t_0)^2}{2t_0}$

1.2.4 Any graph with a coordinate change over the specified time of $20\ m$ and with the greatest "slope" of the tangent is $15\ \frac{m}{s}$.

1.2.5 $x > l(\frac{v_1}{v_2} - 1)$

1.2.6 $x = (\frac{\pi}{4})v_0 t_0$

1.2.7 The average speed is greater than the initial speed, and the final speed is zero.

1.2.8 $v = \sqrt{La}$

1.2.9 $v = \sqrt{\frac{N}{b}}$

1.2.10 $t = \frac{R}{q}$

1.2.11 a. $v = \frac{\pi v_0^3 t^2 tg^2\alpha}{s}$ b. $v = \frac{1}{2}\sqrt{\frac{q}{\pi h t}}$

1.2.12 $q = 126\ \frac{cm^3}{s}$

1.2.13 $a = 277\ \frac{m}{s^2}$; 28 times.

1.2.14 $v_1 = 43\ \frac{m}{s}$; $v_2 = 423\ \frac{m}{s}$

1.2.15 See Figure; $v = 600\ \frac{m}{s}$. From 6 to $6.9km$.
$x = 6.9km$. Check the equality of the areas in the graph of the acceleration above and below the t-axis

1.2.16 4 and 16

1.2.17 See Fig. The ratio of the acceleration modules is 2.

1.2.18 See Fig.

1.2.19 $v = 0.72\ \frac{cm}{s}$

1.2.20 $t = (2 + \sqrt{2})t_0$

1.2.21 $t = \frac{(2t_1t_2 - t_1^2 + t_2^2)}{[2(t_1 - t_2)]}$

1.3 Motion in the field of gravity. Curvilinear motion

1.3.1 $t = \frac{v}{g} - \frac{\Delta t}{2}$

1.3.2 a. $t = \sqrt{\frac{2D}{g}}$. b. On a circle of diameter $\frac{gt^2}{2}$ with upper point A.

1.3.3 At an angle of $\frac{\varphi}{2}$ to the vertical.

1.3.4 $v_B = \sqrt{v_A^2 + 2gh}$.

1.3.5 $t = \frac{v}{g}(sin\varphi - \cos\varphi tg\alpha)$

1.3.6 a) $v_x = v\cos\varphi, v_y = v\sin\varphi - gt$. b) $x = (v\cos\varphi)t$, $y = (v\sin\varphi)t - \frac{gt^2}{2}$. c)$y = xtg\varphi - \frac{gx^2}{2v^2cos^2\varphi} = xtg\varphi - \frac{gx^2}{2v^2}(tg^2\varphi + 1)$. d) $T = \frac{2v}{g}sin\varphi$,$H = \frac{v^2}{2g}sin^2\varphi$,$L = \frac{v^2}{g}siN_2\varphi$

1.3.7 $L = \sqrt{2}\frac{v^2}{g}$

1.3.8 $L = \frac{2v^2}{g}\frac{cos^2\beta}{cos\alpha}((tg\beta - tg\alpha)$

1.3.9 $v = \sqrt{L(a+g)}$.

1.3.10 $H = \frac{2u}{g}(v\cos\alpha - u)tg^2\alpha$.

1.3.11 $L = \frac{2v^2}{g(tg\beta + tg\alpha)}$.

1.3.12 $m = 7kg$.

1.3.13 a) $tg\varphi = \frac{v^2 \pm \sqrt{v^4 - 2gv^2y - g^2x^2}}{gx}$. b) $y = \frac{v^2}{2g} - \frac{gx^2}{2v^2}$.
c)$v_{min} = \sqrt{g(y + \sqrt{x^2+y^2})}$

1.3.14 $x_{rel} = \frac{(v\cos\varphi)}{Deltat}$; $y_{rel} = (v\sin\varphi)\Delta t - \frac{g\Delta t^2}{2} - g\Delta t \cdot t$, where t is time elapsed after the second body took off. Relative velocity is constant and vertically downward and is equal modulo to $g\Delta t$.

1.3.15 $v = \sqrt{\frac{2\pi Rgn}{sin2\alpha}}$, where n is any natural number; at $\alpha = 0$ the velocity can be any modulo.

1.3.16 $t = \frac{2v}{g} ctg\alpha$ at $v\cos\alpha < \sqrt{2gl\sin\alpha}$;

$t = \frac{v}{g} ctg\alpha(1 - \sqrt{1 - \frac{2gltg\alpha}{v^2 cos\alpha}})$ at $v\cos\alpha > \sqrt{2glsin\alpha}$

1.3.17 $v_1 = g\Delta t \sin\alpha$, $v_2 = g\Delta t \cos\alpha$.

1.3.18 $R = \frac{gT_1T_2}{(2\sqrt{2})}$.

1.3.19 $v = \sqrt{g[2(H-h)+L]}$.

1.3.20 $v\varepsilon = 1675\ \frac{km}{h}$, $a\varepsilon = 0,034\ \frac{m}{c^2}$

$v_L = 838\ \frac{km}{h}$, $a_L = 0,017\ \frac{m}{c^2}$.

1.3.21 $v = \sqrt{g_R = 8}\frac{km}{s}$.

1.3.22 $a < \frac{(4+\pi^2)v^2}{(2\pi l)}$.

1.3.23 See Fig.

1.3.24 $At(\frac{\sqrt{3}}{2}) - 102\ \frac{m}{s}; at 5 \cdot 10^{-5} rad$;

$\omega = 5 \cdot 10^{-3}\ s^{-1}$.

1.3.25 $a = \sqrt{k^2 + \frac{k^4t^4}{r^2}}$.

1.3.26 $v = \sqrt{gr}$.

1.3.27 $v = \sqrt{5gR}$.

1.3.28 27.5 and $42.4\ km$; 18.3 and $52\ km$;

0.2 and $73.4\ km$.

1.3.29 $.a = (\frac{v^2}{R}) cos^2\alpha$.

1.3.30 $t = (\frac{V}{g})\sqrt{9sin^2\alpha - 8}$ at $sin\alpha > \sqrt{\frac{8}{9}}$;

$t = 0$ at $sin\alpha < \sqrt{\frac{8}{9}}$.

1.4 Galileo's transformation

1.4.1 In the frame of reference of the second ship the first ship moves in a straight line along the vector $v_1 - v_2$. The perpendicular dropped on this straight line from the location of the second ship will be the shortest distance.

1.4.2 See Fig.

1.4.3 Exactly the same as the observer moving with particle A.

1.4.4 See Fig.

1.4.5 a. The bucket must be tilted in the direction of the movement of the platform at an angle φ to the vertical: $tg\varphi = \frac{u}{v}$. b.$u = 10\sqrt{3}\frac{m}{s}$.

1.4.6 $v_{max} = v\sqrt{3}$.

1.4.7 $t = \frac{2Lpv^2 - u^2 sin^2\alpha}{v^2 - u^2}$. Along the trace.

1.4.8 a) $\Delta v = -2(v+u)$. b) $\Delta v = -2(v-w)$. (The projection on the direction of the initial is considered positive).

1.4.9 a) $u = v$. b) $u = \sqrt{v^2 + 4vw\cos\alpha + 4w^2}$. c) $u = \sqrt{v^2 + 4vw\cos\alpha\cos\beta + 4w^2 cos^2\beta}$

1.4.10 $\nu = \sqrt{\frac{v^2+u^2}{2(R-r)}}$.

1.4.11 $t = 2\sqrt{\frac{u^2}{g^2} + \frac{2h}{g}}$.

1.4.12 The projection of velocity in the horizontal direction $v_x = v - 2u$; the projection of velocityin the vertical direction $v_y = \frac{(2n-1)Lg}{(v-u)}$.

1.4.13 $n = \frac{(v_1+v_2)}{(2R)}$.

1.4.14 $sin\alpha = \frac{u}{v}$.

1.4.15 $u = v\sqrt{3}$.

1.4.16 In the new frame of reference, the geometry of the beams and, hence, the area of their intersection are the same are the same as before. The velocity of the particles is not necessarily directed along the beam.

1.4.17 In $\sqrt{1 + \frac{v^2}{u^2}}$ times will change.

1.4.18 $\alpha = 60°$, $l = 200\sqrt{3} \approx 345\ m$.

1.5 Motion with links

1.5.1 $v_B = 2v_A$.

1.5.2 $v_k = \omega R$; $v_g = \omega(R - r)$.

1.5.3 $u = v\sqrt{3}$.

1.5.4 $a = gctg\alpha$

1.5.5 See Fig.

1.5.6 $(-2, 8; 3, 1)$

1.5.7 a. $u_{AB} = \frac{v}{\sqrt{2}}$. b.$u_1 = \sqrt{u^2 - v^2}$

1.5.8 See Fig. $a = (\frac{v^2}{R^2})r$; $r_B = \frac{(R+r)^2}{r}$, $r_H = \frac{(R-r)^2}{r}$.

1.5.9 $u = \frac{vR}{R\cos\alpha - r}$; $\omega = \frac{v}{R\cos\alpha - r}$;

to the right for $\cos\alpha > \frac{r}{R}$,

to the left for $\cos\alpha < \frac{r}{R}$.

1.5.10 The trajectory of the wheel rim point runs along the diameter of the cylinder.

1.5.11 a. One turn. b. For $4\ min$.

1.5.12 $a = 4\omega^2 R$.

1.5.13 $u = v\cos\alpha$.

1.5.14 At the center of the square in time $t = \frac{a}{v}$.

1.5.15 See Fig.; $v_B = \frac{2v_A^2 t}{\sqrt{L^2+v_A^2 t^2}}$

1.5.16 $u = \frac{v^2 t}{\sqrt{L^2-v^2t^2}}$.

1.5.17 $\omega = \frac{(vsin^2\alpha)}{H}$

1.5.18 $\omega' = \frac{\omega}{2sin^2(\frac{\alpha}{2})}$

1.5.19 $v = \frac{uR}{\sqrt{R^2-h^2}}$.

1.5.20 $d = \frac{\pi(R^2-r^2)}{(vt)}$.

2 DYNAMICS

2.1 Newton's Laws

2.1.1 Since the sum of the external forces is zero, the center of mass of the baron-horse system is at rest (according to the condition of the problem, the baron is stuck in a swamp).

The mechanical system, which includes the baron-horse, will be considered closed. There are no external forces inside a closed mechanical system, the interaction is caused only between the bodies of the baron-horse system, more precisely by the hand and the pigtail on the baron's head.

According to Newton's third law, forces arise in pairs, equal in modulus, directed along one straight line and opposite in direction. Thus, the geometric sum of the internal forces and the moments of these forces relative to the fixed center is zero. The Baron violated Newton's third law.

2.1.2 According to Newton's second law,

$m\vec{a} = m\vec{g} + \vec{N} + \vec{F_{mp}}$

In projection to the direction of motion

$ma = F_{mp}(1)$

To find the friction force, we need to determine the acceleration of the body. Let's propose one of the methods. The distance traveled by the body to a stop is

$l = v_{av}t = \frac{v_0+0}{2}t = \frac{v_0}{2}t$

Acceleration of the body

$0 = v_0 - at$, from where $v_0 = at$

Then

$l = \frac{at}{2}t = \frac{at^2}{2}$, from where $a = \frac{2l}{t^2}$

Making a substitution in (1)

$F_{mp} = \frac{2ml}{t^2}$

Calculations

$F_{mp} = \frac{2\cdot 0{,}1kg\cdot 20m}{(5s)^2} = 0.16N$

Interestingly, acceleration can be obtained even easier when using the "method from the opposite" in solving the problem, if from the stop point, accelerate the puck back to the stick, then

$l = \frac{at^2}{2}$, and $a = 2lt2$

Answer:

$\boxed{F = \frac{2ml}{t^2} = 0,16N.}$

2.1.3 The electric force informs the electron acceleration

$a = \frac{F_{el}}{m_e}$

where $a = \frac{v_y - v_{0y}}{t}$, because $v_{0y} = 0$ – the electron flies perpendicular to the screen, then

$\frac{v_y}{t} = \frac{F_el}{m_e}$, and $F_{el} = \frac{m_e v_y}{t}(1)$

Here v_y is the vertical component of the velocity that the electron will acquire when leaving the plates. To determine the electric force, it is necessary to find the vertical velocity and the time of movement of the electron before departure from the plates. The velocity is determined from the condition

$y = \frac{at^2}{2} = \frac{v_y t^2}{2t} = \frac{v_y t}{2}$

$v_y = \frac{2y}{t}$, where $t = \frac{l}{v}$, (2)

therefore, the vertical component of the velocity

$v_y = \frac{2yv}{l}(3)$

Substituting (2) and (3) in (1), we get

$F_{el} = \frac{2ym_e v^2}{l^2}(4)$

After leaving the plates, the electron then moves by inertia. The tangent of the angle at which the electron flies horizontally, after departure from the plates, is

$tg\alpha = \frac{v_y}{v} = \frac{2y}{vt} = \frac{2y}{l}$

On the other hand

$tg\alpha = \frac{Y-y}{L-\frac{l}{2}} = \frac{2(Y-y)}{2L-l}$

Then

$\frac{2y}{l} = \frac{2(Y-y)}{2L-l}$

After simple transformations , we find

$y = \frac{Yl}{2L}.(5)$

Let's make a substitution (5) in (4) and find the answer to the question

$F_{el} = \frac{2m_e v^2}{l^2}\frac{Yl}{2L} = \frac{m_e Y v^2}{lL}$

Answer:

$\boxed{F = \frac{m_e Y v^2}{lL}}$

2.1.4 Forces acting on the cargo Newton's second law:

$\vec{T_1} + \vec{T_2} + \vec{T_3} + \vec{T_4} + m\vec{g} = m\vec{a}$

Write down the system of equations in projections on

the Ox and Oy axes:

$\begin{cases} T_4 - T_3 = Mg \\ T_2 - T_1 = ma \end{cases}$

Hence

$a = g\frac{T_2 - T_1}{T - 4 - T_3}$

Answer:

$\boxed{a = g\frac{T_2 - T_1}{T_4 - T_3}}$

2.1.5 If we consider a rod with a mass of m as a single whole, then it will move with acceleration

$a = \frac{F}{M}$.

Because the rod is inextensible, then the acceleration of all its parts is the same and equal to a

Consider a small section of a rod of length dx and mass dm. Since the rod is homogeneous

$dm = m\frac{dx}{l}$,

we write down Newton's second law for this section.

$adm = F(x + dx) - F(x)(1)$

Where

$F(x + dx)$ and $F(x)$ are the force of interaction with neighbors

Let 's integrate expression (1) along the horizontal x coordinate:

$\int_x^l am\frac{dx}{l} = \int_x^l dF$

$ma\frac{l-x}{l}$

$F(x) = \boxed{F(1 - \frac{x}{l})}$

Answer:

$\boxed{T = F(1 - \frac{x}{l})}$

2.1.6 Let's write down Newton's second law for both bars along the horizontal axis $\begin{cases} m_2 a = F_2 - T \\ m_1 a = T - F_1 \end{cases}$

Reduce by acceleration a

$\frac{m_2}{m_1} = \frac{F_2 - T}{T - F_1}$

$\frac{m_2}{m_1} = \frac{2\alpha t - T}{T - \alpha t}$

Express T

$T = \alpha t\frac{2m_1 + m_2}{m_1 + m_2}$

From where, the time before the thread breaks T

$\boxed{t = \frac{T(m_1 + m_2)}{\alpha(2m_1 + m_2)}.}$

Answer:

$t = \frac{T(m_1 + m_2)}{\alpha(2m_1 + m_2)}$.

2.1.7 Let's write down Newton's second law with an astronaut

$kx_2 = (m_0 + m)a_2$

And with an empty chair

$kx_1 = m_0 a_1$

Let 's take into account the equidistant motion

$\begin{cases} k\frac{a_1 t_0^2}{2} = m_0 a_1 \\ k\frac{a_2 t^2}{2} = (m_0 + m)a_2 \end{cases}$

From the first equation

$k = \frac{2m_0}{t_0^2}$

We substitute it into the second

$\boxed{m = m_0(\frac{t^2}{t_0^2} - 1).}$

Answer:

$m = m_0[(\frac{t}{t_0})^2 - 1]$.

2.1.8 Let's write down Newton's second law for the horizontal axis Ox

$\begin{cases} F_2 - F = m_2 a \\ F - F_1 = m_1 a \end{cases}$

From here we express the mass ratio

$\boxed{\frac{m_2}{m_1} = \frac{F_2 - F}{F - F_1}}$

Answer:

$\frac{m_2}{m_1} = \frac{F_2 - F}{F - F_1}$

2.1.9 1. The movement of the container can be divided into three sections: on the acceleration section OA. on the piston side, a force $F = nmg$ acts, which accelerates the container to a speed of v_0; in the second section AB, the container moves like a body thrown vertically upwards, in the third section, after stopping, the container with the equipment will make a free fall to the bottom of the shaft.

2. We write down the equation of the basic law of dynamics for the acceleration section, which, in combination with the kinematic conditions of equidistant motion, allows us to determine the values y_1, t_1 and v_0

$nmg - mg = ma$

$a = g(n-1) = 1240\ \frac{m}{s^2}$

$V_0 = a\Delta t = g(n-1)\Delta t = 50\ \frac{m}{s}$

$y_1 = \frac{a\Delta t^2}{2} = \frac{g(n-1)\Delta t^2}{2} = 2\ m$

3. Determine the time of lifting the container from the point A and to point B and the value y_2

$t_2 = \frac{v_0}{g} = (n-1)\Delta t = 5\ s$

$y_2 = v_0 t_2 - \frac{gt_2^2}{2}$

$y_2 = g(n-1)^2\Delta t^2 - \frac{g}{2}(n-1)^2\Delta t^2$

4. Thus, the container will stop when it reaches a height:

$y_3 = y_2 + y_1 = \frac{g(n-1)\Delta t^2}{2} + \frac{g(n-1)^2\Delta t^2}{2}$

5. The time of the container falling from a height of y_3

$t3 = \sqrt{\frac{2y_3}{g}} = \Delta t\sqrt{n(n-1)} = 5\ s$

6. The residence time of the container with the equipment in the "airless" space

$t = \Delta t + t_2 + t_3$

$t = \Delta t[1 + (n-1) + \sqrt{n(n-1)}]$

$t = \Delta t[n + \sqrt{n(n-1)}] = 10\ s$

7. The weightlessness condition of the equipment in the container will be tested for a time

$t_H = 10\ s$

Answer:

$t = n\Delta t(1 + \sqrt{1 - \frac{1}{n}}); t_H \approx 10s$

2.1.10 When simulating weightlessness in a swimming pool, astronauts are affected by the resistance force of the medium, which will prevent movement by inertia. In addition, the internal organs of astronauts in the pool will not be in a state of weightlessness and will function differently than in zero gravity.

2.1.11 Let's write down Newton's second law for the vertical axis

$$\begin{cases} m_1 a = T - m_1 g \\ m_2 a = m_2 g - T \end{cases}$$

From here we express the acceleration, which will be the same for both loads due to the inextensibility of the thread

$$\boxed{a = g\frac{m_2 - m_1}{m_2 + m_1}}$$

We substitute into the system of equations and find the tension force of the thread T

$$\boxed{T = \frac{2m_1 m_2 g}{m_1 + m_2}}$$

The answer:

$a_1 = a_2 = g\frac{m_1 - m_2}{m_1 + m_2}$

$T_1 = \frac{2m_1 m_2}{m_1 + m_2}g, T_2 = 2T_1.$

The positive direction of acceleration corresponds to the lowering of the load m_1.

2.1.12 Let 's denote the mass of the painter by M_1 , and the mass of the chair by M_2. Let's write down the equations of motion of the painter and the chair:

$$\begin{cases} M_1 a = T - M_1 g + P \\ M_2 a = T - M_2 g - P \end{cases}$$

where P is the pressure force of the painter on the chair.

Subtracting the lower equation from the upper one, we find

$a = \frac{2P - (M_1 - M_2)g}{M_1 - M_2} = \frac{1}{3}g$

Then adding up the equations of motion, we find

$2T = (M_2 + M_2)(a + g) = \frac{4}{3}(M_1 + M_2)g = 1.1 \cdot 10^3$ N

This is the full load on the block:

$N = 2T = 1.1 \cdot 10^3\ N$

Answer:

$a = 3.5\frac{m}{c^2}$

$T \approx 1.1 \cdot 10^3\ N$

2.1.13 Let's write down the equilibrium condition for the two lower balls on the vertical and horizontal axes

$$\begin{cases} mg = F_1 \sin\alpha \\ F_x = F_1 \cos\alpha \end{cases}$$

where P is the pressure force of the painter on the chair.

And for the upper ball

$T = mg + 2F_1 \sin\alpha$

$T = 3mg$

Respectively, when the thread burns out, the force $T = 3mg$ will act down on the upper ball. From Newton's second law, its initial acceleration is found as

$a = \frac{T}{m} = 3g$

On the lower balls, a force F_x will act in the horizontal direction, which will be compensated by a force $F_1 \cos\alpha$, and gravity $mg - F_1 \sin\alpha$

Thus, the lower balls will be in zero gravity $a = 0$

Answer:

The acceleration of the upper ball is $3g$, the acceleration of the lower balls is zero.

2.1.14 When the vibrations have already stopped, the balls move with the same acceleration a.

Let's write down Newton's second law

$$\begin{cases} m_1a_1 = -k\Delta x \\ m_2a_2 = -F + k\Delta x \end{cases}$$

Reduce by a

$\frac{m_1}{m_2} = \frac{-k\Delta x}{-F+k\Delta x}$

Express Δx

$\boxed{\Delta x = \frac{F}{k}\frac{m_1}{m_1+m_2}}$

Immediately after after the termination of the force , the elastic force $k\Delta x$ will act on the bodies, which will cause acceleration

$$\begin{cases} m_1a_1 = -k\Delta x \\ m_2a_2 = k\Delta x \end{cases}$$

From where we express a_1

$\boxed{a_1 = -\frac{F}{m_2+m_1}}$

And similarly a_2

$\boxed{a_2 = \frac{Fm_1}{m_2(m_1+m_2)}}$

The answer:

$x = -\frac{Fm_1}{k(m_1+m_2)}$;

$a_1 = -\frac{F}{m_2+m_1}$,

$a_2 = \frac{Fm_1}{m_2(m_1+m_2)}$.

2.1.15 1. The springs in this problem are connected in parallel, their deformation is the same

$\Delta x_1 = \Delta x_2 = \Delta x$

2. The force acting on the mass from the springs. It is defined as the sum of

$F = F + F_1$

, or

$k\Delta x = k_1\Delta x + k_2\Delta x$

3. Write down the equation of mass motion under the action of an equivalent spring with a stiffness of co, which will determine the maximum displacement

$ma = (k_1 + k_2)\Delta x_{max}$

$\Delta x_{max} = \frac{ma}{(k_1+k_2)}$

4. The maximum values of the forces acting on the mass

$F_{1max} = k_1\Delta x_{max}$

$F_{2max} = k_2\Delta x_{max}$

Answer:

$x_{max} = \frac{ma}{(k_1+k_2)}$;

$F_{1max} = k_1x_{max}$,

$F_{2max} = k_2x_{max}$.

2.1.16 1. When the springs are connected in series, their deformation will be different with the same acting force, this circumstance allows us to determine the total stiffness of the springs as follows

$\Delta x_0 = \Delta x_1 + \Delta x_2 = \frac{F}{k_1} + \frac{F}{k_2} = \frac{F}{k_0}$

$\boxed{k_0 = \frac{k_1k_2}{k_1+k_2}}$

2. The combined effect on the mass of the springs at rest will be equal to the applied force F

$k_0\Delta x_0 = F$

$\boxed{\Delta x_0 = \frac{F(k_1+k_2)}{(k_1k_2)}}$

Answer:

$x = F\frac{(k_1+k_2)}{(k_1k_2)}$.

2.1.17 1. The body can be considered as free if the bonds are replaced by their reactions. The friction force in this case is caused by the action of a magnetic force, i.e.

$F_{fr} = \mu F_m$

2. The mass equilibrium condition in this case will take place when the modules of gravity and friction force are equal

$m_0g = \mu F_m \Rightarrow \boxed{F_m = \frac{gm_0}{\mu}}$

3. When the body begins to move at $m > m_0$, then the equation of Newton's second law will be valid, which in projection on the y axis will be written as follows

$mg - m_0g = ma$

$\boxed{a = g\frac{m-m_0}{m}}$

Answer: $F = \frac{m_0g}{\mu}$;

$a = \frac{g(m-m_0)}{m}$

2.1.18 1. If a horizontal force is applied to the body, and despite their efforts they do not move, then it is natural to assume that something prevents this. And this "something" is the friction force of rest, equal in magnitude to the applied force. The magnitude of the resting friction force may vary depending on the magnitude of the applied force. The greatest value of the friction force, at which sliding does not occur yet, is defined as:

$F_{fr(max)} = \mu N = \mu mg = 51\ N$

2. The friction force of rest, like any decent force, has a direction, it is directed towards a possible (virtual) movement, and with zero external force, the friction force will also be zero. Thus, the resting friction force varies linearly

from zero to the maximum value, remaining constant thereafter. An external force begins to inform the body of acceleration.

2.1.19 Let's conduct a visual experiment, put a pen on a piece of paper and slowly pull it aside

At some point, the pen will go in the opposite direction and slip out from under your hand

Using a special program, we will determine the angle of inclination of the handle at this moment

It turned out

$\alpha = 7°$

Let's display the forces acting on the handle

Since the friction force is the friction force at rest

$F_{fr} = \mu N$

We write down the equilibrium condition at the critical moment

$\begin{cases} \mu N = F \sin\alpha \\ N = F\cos\alpha \end{cases}$

From where

$\boxed{\mu = \tan\alpha}$

We substitute experimental data

$\mu = \tan 7° = 0.12$

2.1.20 1. The friction force modulo cannot exceed the value $F_{fr}(max) = \mu N$, where N is the sum of the projections of all forces in the direction perpendicular to the possible displacement. At equilibrium, the friction force is equal to the sum of the projections of forces on the direction of motion. Thus, at rest

$F_{fr} = mg\sin\alpha$

2. Otherwise, at $\mu \geq \tan\alpha$

$F_{fr} = \mu mg\cos\alpha$

3. The maximum value of the friction force will occur at angle α_0

$\boxed{\alpha_0 = \arctan\mu}$

Answer:

$F_{fr} = mg\sin\alpha$ at $\tan\alpha \leq \mu$

$F_{fr} = \mu mg\cos\alpha$ at $\tan\alpha \geq \mu$.

2.1.21 1. Discarding the bonds imposed on the box and replacing them with reactions, it can be considered as a free body capable of moving along the OX axis. The friction force in this case is directed towards acceleration, i.e. against the possible movement of the box.

2. The equation of Newton's second law allows us to determine the maximum acceleration value

$\mu mg\cos\alpha - mg\sin\alpha \leq ma$

$\boxed{a \leq g(\mu\cos\alpha - \sin\alpha)}$

Answer:

$a_{max} = g(\mu\cos\alpha - \sin\alpha)$

2.1.22 A body on an inclined plane is under the action of three forces: gravity $\vec{F_1} = m\vec{g}$, friction force $\vec{F_2} = -\mu mg$ and the normal coupling reaction $\vec{N}$, however, with further consideration of the motion, the normal reaction can not be considered, because its projection onto the horizontal axis along which it moves the body is equal to zero.

The equation of Newton's second law in projection on the OX axis when the body moves upwards in vector form is written as follows

$\vec{F_1} + \vec{F_2} = m\vec{a}$

Let's determine the projections of the acting forces on the Ox axis and write down the equation of Newton's second law

$F_{1x} = mg\sin\alpha, F_{2x} = \mu mg\cos\alpha$

$\mu mg\cos\alpha + mg\sin\alpha = ma$

Divide both parts of the last equation by the mass m and express the acceleration value

$a = g(\sin\alpha + \mu\cos\alpha)$

The time of the body's upward movement t_1 is determined from the condition that the velocity is equal to zero at the end of the ascent

$0 = v_0 - at \Rightarrow t_1 = v_0 a = v_0 g(\sin\alpha + \mu\cos\alpha)$

The downward movement of the body corresponds to the equation

$mg\sin\alpha - \mu mg\cos\alpha = ma$

$a = g(\sin\alpha - \mu\cos\alpha)$

The velocity will become equal to v_0 only at the end of the descent, because the conservation law no one canceled the energy, so

$\frac{dv}{dt} = g(\sin\alpha + \mu\cos\alpha)$

$\int_0^{v_0} dv = g(\sin\alpha + \mu\cos\alpha) = \int_0^{t_2} dt$

$t_2 = v_0 g(\sin\alpha + \mu\cos\alpha)$

The required time is determined as the sum of $t = t_1 + t_2$

$\boxed{t = \frac{2v\sin\alpha}{[g(sin^2\alpha - \mu^2 cos^2\alpha)]}}$

The answer: $t = \frac{2v\sin\alpha}{[g(sin^2\alpha - \mu^2 cos^2\alpha)]}$

2.1.23 1. The normal coupling reaction in this case will be determined by both gravity mg and the projection of the applied force on the OY axis:

$N = mg - F\sin\alpha$

The friction force is defined as:

$F_{fr} = (mg - F\sin\alpha)$

2. The basic law of dynamics, thus. it will be written as follows:

$F\cos\alpha = \mu(mg - F\sin\alpha)$

3. It is easy to determine the desired acceleration from the equation of Newton's second law

$a = \frac{1}{m}(F\cos\alpha - \mu mg + F\sin\alpha)$

$a = \frac{F}{m}(\cos\alpha - \mu\sin\alpha)$

Answer:

$a = (\frac{F}{m})(cos\alpha + \mu\sin\varphi) - \mu g$, if the expression is greater than zero, otherwise $a = 0$

2.1.24 Forces acting on the cylinder: gravity $\vec{F_t} = m\vec{g}$ two forces of normal reaction of the faces of the dihedral angle $\vec{N_1}$ and $\vec{N_2}$, two forces of friction of the cylinder on the face $\vec{f_{fr1}}$ and $\vec{f_{fr2}}$ (see figures). Since the cylinder has axial symmetry and the planes of the dihedral angle are symmetrical with respect to the vertical

$|\vec{N_1}| = |\vec{N_2}| = N\ \vec{f_{fr1}}| = |\vec{f_{fr2}}| = f_{fr}$

According to the Coulomb — Amonton law

$f_{fr} = \mu N.$

The basic law of dynamics for a cylinder has the form

$m\vec{a} = m\vec{g} + \vec{N_1} + \vec{N_2} + \vec{f_{fr1}} + \vec{f_{fr2}}$

Since the cylinder is stationary in the plane of section perpendicular to the edge of the dihedral angle. that is, projecting this equation onto the axis. perpendicular to the edge, we obtain (see Figure b)

$2N\sin\frac{\alpha}{2} = mg\cos\beta$

In projection onto the edge (axis OX), the dynamics equation for the cylinder is written as

$ma_x = mg\sin\beta - 2N\mu$

Substituting N here, we find the acceleration of the cylinder

$a_x = g(\sin\beta - \frac{\mu\cos\beta}{\sin\frac{\alpha}{2}})$

Answer:

$a = g(\sin\beta - \frac{\mu\cos\beta}{\sin\frac{\alpha}{2}})$ at $\mu \le \tan\beta\sin\frac{\alpha}{2}$

$a = 0$ at $\mu > \tan\beta\sin\frac{\alpha}{2}$

2.1.25 1. Due to the weightlessness and inextensibility of the thread, as well as the ideal properties of the block (no losses and low weight), the problem can be solved in the following approximation)

$a_1 = a_2 = a$

$T_1 = T_2 = T$

2. The equations of motion of goods in projection on the vertical axis in this case are written as follows: $\begin{cases} m_1 a = m_2 g - T \\ m_2 a = T - m_2 g - F_{fr} \end{cases}$

3. Solving the equations together, we get

$a = \frac{(m_1 - m_2)g - F_{fr}}{m_1 + m_2}$

4. Substituting the acceleration magnitude into the first equation of the system allows us to determine the tension threads

$\boxed{T = m_1\frac{2m_2 g + F_{fr}}{m_1 + m_2}}$

Answer:

$T = m_1\frac{2m_2 g + F_{fr}}{m_1 + m_2}$

2.1.26 Consider the forces acting on the box (Fig.). These are gravity $m\vec{g}$, rope tension force $\vec{F}$, impact reaction force $\vec{N}$ and friction force $\vec{F_{fr}}$, the value of which $\vec{F_{fr}} = \mu N$. We will project all forces in the direction along the convergence and perpendicular to them and write down the corresponding equations of motion.

Since the box does not move in the direction perpendicular to the convergence, the sum of the projections of forces in this direction should be zero, that is

$N + F\sin(\beta - \alpha) - mg\cos\alpha = 0, (1)$

Along the convergence, the box moves with acceleration a (in the special case, with uniform motion $a = 0$), so the sum of the force projections should be equal to ma:

$F\cos(\beta - \alpha) - mg\sin\alpha - \mu N = ma.(2)$

From equations (1) and (2) we obtain:

$F = \frac{ma + mg(\sin\alpha + \mu\cos\alpha)}{cos(\beta - \alpha) + \mu\sin(\beta - \alpha)}.(3)$

The resulting expression for the force F angle β includes only the denominator. Therefore, the magnitude of the force F will be minimal at such a value of the angle β at which the denominator in formula (3) is maximal, that is, the maximum value

$\cos(\beta - \alpha) + \mu\sin(\beta - \alpha).$

Let's do some transformations. Let's imagine the coefficient of friction μ as the tangent of some angle γ:

$\tan\gamma = \mu; \gamma = \arctan\mu;$

$\sin\gamma = \frac{\mu}{\sqrt{1+\mu^2}}; \cos\gamma = \frac{1}{\sqrt{1+\mu^2}}.$

Then you can write:

$\cos(\beta - \alpha) + \mu\sin(\beta - \alpha) = \sqrt{1+\mu^2}\cos(\beta - \alpha - \gamma).$

The last expression is maximal and equal to $\sqrt{1+\mu^2}$ at $\beta - \alpha - \gamma = 0$, that is , at $\beta = \alpha + \gamma = \alpha + \arctan\mu.(4)$

With such a value of the angle β and the minimum force F. Moreover, if the box moves uniformly ($a = 0$), then

$F_{min} = \frac{mg(\sin\alpha+\mu\cos\alpha)}{\sqrt{1+\mu^2}}$.

and when moving with acceleration a

$F_{min} = \frac{ma+mg(\sin\alpha+\mu\cos\alpha)}{\sqrt{1+\mu^2}}$.

However, this solution is not true for any acceleration. Since the direction of force $\vec{F}$ does not depend on a, and the absolute magnitude of the force $\vec{F}$ increases with increasing acceleration , then at a certain acceleration value $a = a_0$, the force $\vec{F}$ will become such that its component $F\sin(\beta-\alpha)$, perpendicular to the inclined plane, will be equal in absolute magnitude to the component of gravity $mg\cos\alpha$. In this case , both the force $\vec{N}$ and the force $\vec{F}$ will vanish. In the future (at $a > a_0$), in order for the box not to break away from the skids, the direction of force $\vec{F}$ must change with increasing acceleration so that the component of force $\vec{F}$ perpendicular to the inclined plane remained equal to the component of gravity, that is,

$F\sin(\beta-\alpha) = mg\cos\alpha$

For the components of these forces parallel to the inclined plane, we can write

$F\cos(\beta-\alpha) - mg\sin\alpha = ma$

From the last two equalities we find

$\tan(\beta-\alpha) = \frac{g\cos\alpha}{g\sin\alpha+a}$

from where

$\beta = \alpha + \arctan\frac{g\cos\alpha}{g\sin\alpha+a}$

The value of a_0 can be found from the considerations that for $a = a_0$ the value of the angle β from (4) and (5) coincide:

$\alpha + \arctan\frac{g\cos\alpha}{g\sin\alpha+a_0} = \alpha + \arctan\mu$

from where

$a_0 = g(\frac{\cos\alpha}{\mu} - \sin\alpha)$

So, for $a \leq a_0$ (hence, with uniform motion too)

$\beta = \alpha + \arctan\mu$

For $a > a_0$

$\beta = \alpha + \arctan\frac{g\cos\alpha}{g\sin\alpha} + a$

We have solved the problem. However, here is another solution. This is a beautiful geometric solution. With uniform movement along the slopes, the sum of all forces should be zero. Replace the forces $\vec{N}$ and $\vec{F_{fr}}$ with their resultant $\vec{Q} = \vec{N} + \vec{F_{fr}}$ (Fig.) and add the forces $\vec{Q}, \vec{F}$ and $m\vec{g}$. They should form a closed triangle. Let us replace that the direction of force $\vec{Q}$ is an angle δ with a perpendicular to the inclined plane such that

$\tan\delta = \frac{F_{fr}}{N} = \frac{\mu N}{N} = \mu$

Thus, when the magnitude and direction of force $\vec{F}$ change, the direction of force $\vec{Q}$ remains unchanged. Therefore, the absolute magnitude of the force $\vec{F}$ will be minimal if it is perpendicular to the vector $\vec{Q}$ (Fig.). (since the magnitude and direction of the vector $m\vec{g}$ are unchanged), the minimum force will be:

$F_{min} = mg\sin\alpha + mg\mu\cos\alpha = mg(\sin\alpha)$

Thus, the value of the minimum force required to lift the box with acceleration $a \leq a_0$ is

$F_{min} = mg(\sin\alpha + \mu\cos\alpha)$

If acceleration $a > a_0$, then for the minimum force $\vec{F}$ directed at an angle β horizontally, we have:

$\beta = \alpha + \arctan\frac{g\cos\alpha}{g\sin\alpha+a}$

Answer:

(a)

$\beta = \alpha + \arctan\mu$

2.1.27 $\boxed{1}$ (a) Let the man not slide

$\vec{R} = \vec{F_{fr}} + \vec{N}$

$\tan\varphi = \frac{\mu N}{N} \Rightarrow \varphi = \arctan\mu$

$T - min$ if $\vec{T} \perp \vec{R}$ (Because Mg and R fixed in the direction,see fig.)

Fig.

$\alpha = \varphi$

$\alpha = \arctan\mu$

(b) $T_{min}-?$

$\begin{cases} T\cos\alpha = \mu N \\ N + T\sin\alpha = Mg \end{cases}$

$T\cos\alpha = \mu(Mg - T\sin\alpha)$

$T(\cos\alpha + \mu\sin\alpha) = \mu Mg$

$\begin{cases} T = \frac{\mu Mg}{\cos\alpha+\mu\sin\alpha} \\ \tan\alpha = \mu \end{cases}$

$(1) 1 + \tan^2\alpha = \frac{\cos^2\alpha+\sin^2\alpha}{\cos^2\alpha} \Rightarrow \frac{1}{\sqrt{1+\mu^2}}$

$(2) \sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1-\frac{1}{1+\mu^2}} = \sqrt{\frac{1+\mu^2-1}{1+\mu^2}} = \frac{\mu}{\sqrt{1+\mu^2}}$

$(3) T = \frac{\mu Mg}{\frac{1+\mu^2}{\sqrt{1+\mu^2}}} = \boxed{\frac{\mu Mg}{\sqrt{1+\mu^2}}}$

(c)Let's find out at what mass ratio it is possible to choose the optimal angle $\alpha = \arctan\mu$ so that a person does not slide:

$T\cos\alpha \le \mu n$

$T\cos\alpha \le \mu(mg + T\sin\alpha)$

$T(\cos\alpha - \mu\sin\alpha) \le \mu mg \longleftarrow (1,2,3)$

$\frac{\cancel{\mu} M \cancel{g}}{\sqrt{1+\mu^2}}\left(\frac{1-\mu^2}{\sqrt{1+\mu^2}}\right) \le \cancel{\mu} m \cancel{g}$

$m \ge M\frac{1-\mu^2}{1+\mu^2}$

$\boxed{2}$ (a) Consider $m < M\frac{1-\mu^2}{1+\mu^2}$ in this case, the person will slide by himself and will not be able to move the box. Obviously, you need to increase α in this case, the person will slide by himself and will not be able to move the box. Obviously, you need to increase it until the F_{fr} for the box becomes the limit. Then the optimal angle will be when $\mu N = T\cos\alpha = \mu n$ so $N = n$:

$$\begin{cases} Mg + T\sin\alpha = mg - T\sin\alpha \\ T\cos\alpha = \mu N \end{cases}$$

$$(4)\ \begin{cases} (m-M)g = 2T\sin\alpha \\ T\cos\alpha = \mu Mg + \mu T\sin\alpha \end{cases}$$

$(5)\ T = \frac{\mu Mg}{\cos\alpha - \mu\sin\alpha} \longrightarrow (4)$

$\frac{(m-M)g}{2} = \frac{\mu Mg\sin\alpha}{\cos\alpha - \mu\sin\alpha} = \frac{\mu Mg}{\frac{1}{\tan\alpha} - \mu}$

$\frac{1}{\tan\alpha} - \mu = \frac{2\mu M + \mu m - \mu M}{m-M} = \frac{\mu(m+M)}{m-M}$

$(6)\ \tan\alpha = \frac{m-M}{\mu(m+M)}$

$$\boxed{\alpha = \arctan\frac{m-M}{\mu(m+M)}}$$

(7)

$\cos\alpha = \frac{1}{\sqrt{1+\tan^2\alpha(9)}} = \frac{1}{\sqrt{1+\frac{(m-M)^2}{\mu^2(m+M)^2}}}$

$(8)\ \sin\alpha = \sqrt{1-\cos\alpha} = \sqrt{1-\frac{1}{1+\frac{(m-M)^2}{\mu^2(m+M)^2}}} = \sqrt{\frac{1+\frac{(m-M)^2}{\mu^2(m+M)^2}-1}{1+\frac{(m-M)^2}{\mu^2(m+M)^2}}} = \frac{(m-M)}{\mu(m+M)}\cdot\frac{1}{\sqrt{1+\frac{(m-M)^2}{\mu^2(m+M)^2}}}$

$(7,8) \longrightarrow (5)$

$$T = \frac{\mu Mg}{\left(\frac{1-\frac{m-M}{m+M}}{\sqrt{1+\frac{(m-M)^2}{\mu^2(m+M)^2}}}\right)} = \frac{\mu Mg}{\left(\frac{2M}{m+M}\right)}\sqrt{1+\frac{(m-M)^2}{\mu^2(m+M)^2}} =$$

$$= \frac{\mu g}{2}(m+M)\sqrt{1+\frac{(m-M)^2}{\mu^2(m+M)^2}} =$$

$$= \frac{\mu g}{2}\sqrt{(m+M)^2 + \frac{(m-M)^2}{\mu^2}} =$$

$$\boxed{T = \frac{g}{2}\sqrt{\mu^2(m+M^2)+(m-M)^2}}$$

Answer:

At $m \ge M\frac{1-\mu^2}{1+\mu^2}$:

$\alpha = \arctan\mu$;

$T_{min} = \frac{\mu Mg}{\sqrt{1+\mu^2}}$

At $m < mM\frac{1-\mu^2}{1+\mu^2}$:

$\alpha = \arctan\frac{m-M}{\mu(m+M)}$;

$T = \frac{g}{2}\sqrt{\mu^2(m+M)^2+(m-M)^2}$

Note: In many places, this problem is formulated without the question about the angle, but with the clarification $m < M$. The answer in the solutions given turns out to be very often the same only with our case 2 (see $1,2,3$).

As we found out, in fact, this solution is suitable not for $m < M$, but for $m < M\frac{1-\mu^2}{1+\mu^2}$. By the way, the above links suggest a shorter way to find the force, since it does not involve finding the optimal angle.

Let's estimate whether the error is large in the range $M\frac{1-\mu^2}{1+\mu^2} < m < M$.

The red line in the graph below shows the answer specified in Savchenko's taskbook — this answer cannot be correct, since there should be no gap between the blue and red lines. The blue dotted line for clarity shows the behavior of the blue function outside the range of its applicability, i.e. at

$m < M\frac{1-\mu^2}{1+\mu^2}$.

A small gap in the green line shows the accepted value for $m = 70\ kg$. It can be seen that the difference between the green curve to the right of the gap and the blue solid one is not very large.

Answer:

At $m \ge M\frac{1-\mu^2}{1+\mu^2}$:

$\alpha = \arctan\mu$;

$T = \frac{\mu Mg}{\sqrt{1+\mu^2}}$

At $m < mM\frac{1-\mu^2}{1+\mu^2}$:

$\alpha = \arctan\frac{m-M}{\mu(m+M)}$;

$T = \frac{g}{2}\sqrt{\mu^2(m+M)^2+(m-M)^2}$

2.1.28 1. The external force when the car is moving is the friction force $F = \mu mg$, therefore, without taking into account the resistance from the air, the dynamic equation of motion has the form

$ma = \mu mga = \mu g$ 2. The kinematic equations of motion in this case are as follows

$$\begin{cases} v = v_0 - at \\ x = v_0 - \frac{at^2}{2} \end{cases}$$

$t = \frac{v_0}{a} = \frac{v_0}{\mu g}$

3. When substituting acceleration and time values into the second equation, we obtain the equation of the braking distance of the car

$x = \frac{v_0^2}{\mu g} - \frac{v_0^2}{2\mu g} = \frac{v_0^2}{2\mu g}$

$v_0 = \sqrt{2\mu gx}$

From here, the speed should be reduced by a factor of $\sqrt{10}$

Answer:

In $\sqrt{10}$ times.

2.1.29 We find the magnitude of the deceleration

$a = -\frac{\Delta v}{\Delta} t = -4\ \frac{m}{s}$

$\Delta t = \frac{\Delta v}{a}$

2. The kinematic equations of motion in this case are as follows

$\boxed{x = v_0 t - \frac{v \Delta t^2}{2} = \frac{v_0^2}{2a} = 50\ m}$

Let's find the coefficient of friction μ :

$ma = \mu mg$

$\boxed{\mu = \frac{a}{g} = 0.4}$

Answer:

$\mu \approx 0,4$ and $l \approx 50\ m$

2.1.30 $a)$ In order for the body to start sliding, the applied force must exceed the friction force. When determining the magnitude of the friction force, it must be taken into account that in accordance with Newton's third law, the body acts on the board, and the board acts on the body, therefore, a double friction force

$F_t = \mu g(m_1 + m_2)$

Condition for the beginning of movement:

$\boxed{F > \mu g(m_1 + m_2)}$

To determine Let's use the kinematic equations of sliding time:

$l = \frac{at^2}{2} \to t = \sqrt{\frac{2L}{a}}$

Acceleration a when acting along the board of a constant force F_0, we determine from the equation of Newton's second law in projection on the direction of motion:

$F_0 - F_t = m_1 a$

$a = \frac{F_0 - \mu g(m_1 + m_2)}{m_1}$

Sliding time:

$\boxed{t = \sqrt{\frac{2Lm_1}{F_0 - \mu g(m_1 + m_2)}}}$

$b)$ Two external forces F_0 and a friction force F_{fr} act on the body. Let's write down Newton's second law for a body

$m_1 a_1 = F_0 - F_{fr}$

Given that

$F_{fr} = \mu N = \mu m_1 g$

$m_1 a_1 = F_0 - \mu m_1 g$

From where we find a_1 as

$\boxed{a_1 = \frac{F_0 - \mu m_1 g}{m_1}}$

Meanwhile, only the friction force with the bar acts on the board from external forces

Then Newton's second law for the board

$m_2 a_2 = \mu m_1 g$

We expressa_2

$\boxed{a_2 = \mu g \frac{m_1}{m_2}}$

The answer:

a. $F > \mu(m_2 + m_1)g; t = \sqrt{\frac{2lm_2}{F_0 - \mu(m_2 + m_1)g}}$.

b. $a_1 = F_0 - \mu m_1 g m_1, a_2 = \mu g m_1 m_2$

2.1.31 Renumber the loads as shown in the figure, and the axis X is directed to the right.

It is clear that then none of the loads can have a negative acceleration.

Let's prove that the loads 3 and 4 are moving as one. To do this, let's assume the opposite: let the cargo 3 slides on the load 4. Then a friction force

$F_{fr} = \mu mg$

arises between them, and an elastic force

$T > \mu mg$

arises in the thread, while the acceleration of the load 2 would be directed to the left, which cannot be. Consequently, the acceleration of loads 2, 3 and 4 are the same.

Let's denote the acceleration of these loads by $\bar{a_1} = \bar{a_2}$, and the cargo is accelerated 1 through $\bar{a_1}$.

Now let's consider two cases.

Case 1. Let the loads 1 and 2 they are in relative rest and $\bar{a_1} = \bar{a_2}$.

Let's denote the modulus of the friction force at rest between them by F_1-the modulus of the friction force between the weights 3 and 4 through F_2 and the modulus of the elastic force of the thread through T.

Then: for cargo 1

$F - F_1 = Ma_1$

for cargo 2

$F_1 - T = ma_2$

for cargo 3

$T - F_2 = ma_2$

for cargo 4

$F_2 = Ma_2$

Solving this system of equations, we get:

$F_1 = \frac{2m+M}{2(M+m)}F, a_1 = a_2 = \frac{F}{2(M+m)}$

The same result can be obtained in another way. Since the friction between all surfaces is a rest friction, the cargo system moves as one body with mass $M = 2(M+m)$.

Therefore,

$\bar{F} = m\bar{a_1}, \bar{a_1} = \bar{a_2} = \frac{F}{2(M+m)}$

Case 2.

Let the cargo 2 slides the load 1. Then on the cargo 1 the friction force

$F'_{fr} = \mu mg$

acts and this load receives acceleration $a_1 = \frac{F-\mu mg}{m}$.

Cargo system 2,3 and 4 moves as a single body whose mass $M_0 = 2m + M$ with acceleration

$a_2 = \frac{\mu mg}{2m+M}$

The first case is realized if

$F \geq \frac{2\mu m(m+M)g}{2m+M}$

Answer:

At $F \leq \frac{2\mu m_1 g(m_1+m_2)}{m_2+2m_1} \equiv F_0$ we obtain $a_{1left} = a_{1right} = a_{2right} = \frac{F}{2(m_1+m_2)}$;

At $F \geq F_0$ we obtain $a_{2right} = \frac{F-\mu m_1 g}{m_2}, a_{1left} = a_{1right} = a_{2left} = \frac{\mu m_1 g}{m_2+2m_1}$.

2.1.32 The appearance of the action on the wedge from the chalk is due to the accelerated movement of the body along the wedge

Motion becomes possible under the condition

$mg\sin\alpha \geq \mu mg\cos\alpha$

otherwise the body will rest and acceleration will not occur.

Using the principles of liberability, let's imagine the body as a free material particle under the action of a system of forces $mg; F_fr$

The equation of Newton's second law in projection on the direction of motion of the body is represented as follows:

$ma_x = mg\sin\alpha - \mu mg\cos\alpha$

$a_x = g\sin\alpha - \mu g\cos\alpha$

The force acting on the wedge and the vertical wall

$F = mg\cos\alpha(\sin\alpha - \mu\cos\alpha)$

The answer:

$F = mg\cos\alpha(\sin\alpha - \mu\cos\alpha)$ at $\mu \leq \tan\alpha$

$F = 0$ at $\mu \geq \tan\alpha$

2.1.33 Two forces act on a falling drop: the constant force of gravity, which accelerates the movement of the drop, and the force of air resistance, which slows down its movement and increases with increasing drop velocity. The force of air resistance increases until it becomes equal to gravity. Then the speed change stops, and the drops fall at a constant speed.

As the size of the droplet increases, gravity increases in proportion to the volume, i.e. proportional to the third power of the radius, and the resistance force increases in proportion to the section of the droplet, i.e. proportional to the square of the radius. Therefore, as the radius of the drop increases, gravity increases faster than the force of air resistance, which means that the constant speed at which the drop falls to the ground increases as the size of the drop increases.

$\alpha \approx 0.7\ \frac{kg}{m}$

2.1.34 In this case, the movement occurs solely due to the friction force, which, in fact, is the driving force. If there were no friction force, then the bike, as well as the car, would not move from its place. Displacement with acceleration becomes possible when the projection of the friction force on the horizontal axis exceeds the modulus of the resistance force from the air

$\mu mg \geq f$

When moving without acceleration, at a constant speed, the latter inequality turns into equality

$\mu mg = f$

$\mu mg = \alpha v^2$

$\boxed{\alpha = \frac{\mu mg}{v^2} \approx 0.7\ \frac{kg}{m}}$

Answer:

$\alpha \approx 0.7\ \frac{kg}{m}$

2.1.35 Five forces act on the ball (see Fig.): gravity $\vec{F}_g = M\vec{g}$, buoyant force $\vec{F}$, air resistance force $\vec{F}_r$, Earth reaction force $\vec{N}$ and friction force from the Earth $\vec{F}_{fr}$.

Denote by $\vec{v}'$ the velocity of the ball relative to the Earth. Then

$\vec{F}_r = -\alpha(\vec{v}' - \vec{u})$

From the condition that the balloon moves uniformly in the horizontal direction, it follows

$|\vec{F}_r| - |\vec{F}_{fr}| = 0$

$|\vec{F}| + |\vec{N}| - M|\vec{g}| = 0$

In addition,

$|\vec{F}_{fr}| = \mu|\vec{N}|$

Taking into account that

$|\vec{F}_r| = -\alpha(|\vec{v}'| - |\vec{u}|)$

from the last three equations we obtain

$|\vec{v}'| = |\vec{u}| - \frac{\mu}{\alpha}(M|\vec{g}| - |\vec{F}|)$

Answer:

$v = u\sqrt{(\frac{\mu}{\alpha})(mg - F)}$ at $\alpha u^2 \geq \mu(mg - F)$

otherwise $v = 0$

2.1.36 The equation of Newton's second law for the direction of motion:

$ma = F_r$

$\frac{F_r}{m} = \frac{dv_x}{dt}$

The time derivative of velocity

$\frac{dv_x}{dt} = \frac{d}{dt}(v_0 - \beta x)$

$\frac{dv_x}{dt} = \frac{dv_0}{dt} - \beta\frac{dx}{dt} = -\beta t$

the minus sign indicates that the acceleration vector is directed in the direction opposite to the velocity vector. Combining the equations, we obtain the value of the resistance force as a function of velocity

$\boxed{F = \beta mv}$

Answer: $F = \beta mv$

2.1.37 Two forces act on a falling drop: the constant force of gravity, which accelerates the movement of the drop, and the force of air resistance, which slows down its movement and increases with increasing drop velocity. The force of air resistance increases until it becomes equal to gravity. Then the speed change stops, and the drops fall at a constant speed.

Let's write the equation after a long period of time:

$mg = Ap_0r^2v^2(1)$

Find m through volume V:

$m = pV = \frac{4}{3}\rho\pi r^3$

And substitute in (1):

$\frac{4}{3}\rho\pi r^3 g = Ap_0r^2v^2$

Hence:

$\boxed{v = \sqrt{\frac{4}{3}\rho\pi rg \cdot \frac{1}{Ap_0}} \approx 5.5\ \frac{m}{s}(2)}$

Of(2), the greater the r, the greater the V. This means that large drops fall to the ground at a higher speed

Answer:

Large; $v \approx 5.5\ \frac{m}{s}$

2.1.38 Two forces act on a falling drop: the constant force of gravity, which accelerates the movement of the drop, and the force of air resistance, which slows down its movement and increases with increasing drop velocity. The force of air resistance increases until it becomes equal to gravity. Then the speed change stops, and the drops fall at a constant speed.

Let's write down the equation after a long period of time:

$mg = \gamma rv(1)$

Find m through volume V:

$m = pV = \frac{4}{3}\rho\pi r^3$

And substitute in (1):

$43\rho\pi r^3 g = \gamma rv$

Hence:

$v = \frac{4}{3}\frac{\rho\pi g}{\gamma} \cdot r^2 = \alpha r^2(2)$

$\alpha = \frac{4}{3}\frac{\rho\pi g}{\gamma} = \frac{v}{r^2} = 10^8\ \frac{1}{m\cdot s}$

Substitute and find the answer

$v(\frac{r}{2}) = \alpha\frac{r^2}{4} = 0.25\ \frac{m}{s}$

$v(\frac{r}{10}) = \alpha\frac{r^2}{100} = 0.01\ \frac{m}{s}$

Answer:

$v_1 \approx 0.25\ \frac{m}{s}$; $v_2 \approx 0.01\ \frac{m}{s}$

2.1.39 This is due to the Reynolds number for a given situation.

If the Reynolds number is $Re < 2000$, then the drag force is proportional to the velocity

If $Re > 2000$, then the resistance force is proportional to the square of the velocity

2.1.40 1. The acceleration of the washer is determined by the equation of Newton's second law:

$\mu mg = ma$

$a = \mu g$

2. With the width of the conveyor belt d, the washer will travel an equidistant distance x due to the movement of the belt. From the similarity of right triangles obtained on the vectors of given velocities and geometric parameters of the movement of the washer, we find the ratio:

$\frac{d}{x} = \frac{v}{v_0}$

$\frac{d}{x} = \frac{v}{\sqrt{v^2+u^2}}$

$d = x\frac{v}{\sqrt{v^2+u^2}}$

3. The distance traveled by the washer we find x from kinematics:

$$\begin{cases} v = v_0 - at \\ x = v_0t - \frac{at^2}{2} \end{cases}$$

$v = 0 \rightarrow t = \frac{v_0}{a} = \frac{\sqrt{v^2+u^2}}{\mu g}$

$x = \frac{v^2+u^2}{\mu g} - \frac{v^2+u^2}{2\mu g} = \frac{v^2+u^2}{2\mu g}$

4. Next, substitute the value of x into the equation for d

$d = v\frac{\sqrt{v^2+u^2}}{2\mu g}$

Answer:

$d = \frac{v}{2\mu g}\sqrt{v^2+u^2}$.

2.1.41 1. A translationally moving washer has only the kinetic energy of translational motion

$K_1 = \frac{mv^2}{2}$

the plane motion of the washer, which is a superposition of translational and rotational movements, is characterized by two components of kinetic energy: translational and rotational:

$K_2 = \frac{mv^2}{2} + J_z\frac{\omega^2}{2}$

$K_2 = \frac{mv^2}{2} + \frac{mr^2}{2}\cdot\frac{\omega^2}{2}$

$K_2 = \frac{m}{2}(v^2 + \frac{r^2\omega^2}{2})$

$K_2 = \frac{3}{4}mv^2$

$K_2 > K_1$

2. The initial kinetic energy of the washers in both cases will be spent on work against the friction force

$A_{fr} = \mu mg\Delta x$

3. According to the kinetic energy change theorem:

$\frac{K_2}{K_1} = \frac{\Delta x_2}{\Delta x_1} = 1.5$

Answer:

Rotating

2.1.42 The sliding friction force acts along the line of motion in the opposite direction, its magnitude is determined only by the coefficient of friction of the surfaces (i.e., for a given nail, it is conditionally constant in magnitude and does not depend on the method of movement).

If you pull directly, you need to apply a lot of force along the axis of the nail. And if you rotate, then the vector of the friction force rotates from the axis of the nail, decomposing into two perpendicular forces: a small axial and a large radial. A large radial force is overcome using a lever (the length of the pliers handles is much larger than the diameter of the nail). There remains a small axial component, which is easy to overcome.

2.1.43 $v = \frac{\omega RF}{\sqrt{F_{fr}^2 - F^2}}$.

2.1.44 Since the speed changes quickly, the body does not have time to move in the horizontal direction and moves all the time in the direction $\vec{v}$

Since $v = const$, there is no acceleration in the direction along the velocity $\vec{v}$

$mg\sin\alpha = \mu mg$

$cos\alpha$

$cos\beta$

$cos\beta = \frac{\tan\alpha}{\mu}$

For geometric reasons, the modules of the vectors $\vec{v}$ and $\vec{u}$ are related by the ratio

$v = \frac{u}{\tan\beta}$

$v = u\frac{\frac{\tan\alpha}{\mu}}{\sqrt{1-\frac{\tan^2\alpha}{\mu^2}}}$

$\boxed{v = u\frac{\tan\alpha}{\sqrt{\mu^2-\tan^2\alpha}}}$

Answer:

$v = u\frac{\tan\alpha}{\sqrt{\mu^2-\tan^2\alpha}}$

2.1.45 Projecting $m\vec{g}$ onto OY:

From the figure, we find the reaction force of the support:

$N = mg\cos\alpha$

According to the Amonton — Coulomb Law:

$F_{fr} = \mu N = \mu mg\cos\alpha$

Because $\mu = \tan\alpha$ (by condition):

$F_{fr} = \mu mg\cos\alpha$

$F_{fr} = mg\cdot\tan\alpha\cdot\cos\alpha = mg\sin\alpha$

$F = mg\sin\alpha$

Redraw in the XY plane. At the initial moment($\varphi = 90°$):

At the final moment($\varphi = 0°$):

Consider an arbitrary moment:

Note that φ varies from $0°$ to $90°$:

Let's write down Newton's second law:

$$\begin{cases} \frac{du_x}{dt} = \frac{F\sin\varphi}{m} \\ \frac{du_y}{dt} = \frac{F(1-\cos\varphi)}{m} \end{cases}$$

$$\begin{cases} \frac{d}{dt}(u\sin\varphi) = \frac{F\sin\varphi}{m} \\ \frac{d}{dt}(u\cos\varphi) = \frac{F(1-\cos\varphi)}{m} \end{cases}$$

Solve the system of differential equations:

$$\begin{cases} \frac{d\varphi}{dt} = \frac{F\sin\varphi}{mu}(a) \\ \frac{du}{dt} = \frac{F(1-\cos\varphi)}{m}(b) \end{cases}$$

Divide (b) by (a):

$\frac{du}{d\varphi} = u\frac{1-\cos\varphi}{\sin\varphi}$

$\frac{du}{u} = \frac{1-\cos\varphi}{\sin\varphi} d\varphi$

$\frac{du}{u} = \tan(\frac{\varphi}{2}) d\varphi$

Integrate both parts of the equation:

$\int \frac{du}{u} = \int \tan(\frac{\varphi}{2}) d\varphi (c)$

$\int \frac{du}{u} = ln|u| (d)$

$\int \tan(\frac{\varphi}{2}) d\varphi = -2ln(\cos(\frac{x}{2}))(e)$

Substitute (e) and (d) in (c):

$ln(v) + C = ln(\sin(\varphi)) - (ln(\sin(\frac{\varphi}{2}) - \cos(\frac{\varphi}{2})))(e)$

$v_x = \frac{v}{2}$

Answer:

$\frac{v}{2}$

2.1.46 1. Since bodies of the same mass move down the plane with a stretched thread, the upper body will have a rough surface. Bodies with a stretched thread will have the same accelerations.

2. Write down the equations of Newton's second law in projection to the direction of motion for each body separately:
$\begin{cases} m_1 g \sin\alpha - T = m_1 a_1 \\ m_1 g \sin\alpha + T - Ft = m_2 a_2 \end{cases}$

$a_1 = a_2$

$m_1 = m_2$

$mg\sin\alpha + T - F_{fr} = mg\sin\alpha - T$

$F_{fr} = 2T$

Answer:

$F = 2T$

2.1.47 A body of mass m_3 moves with the acceleration of the center of mass of the system

Considering the whole system (without taking into account internal forces)

$\boxed{a_3 = \frac{F}{M+2m}}$

In this case, a force equivalent to the double thread tension force 2T

$2T = Ma_3$

$T = F\frac{M}{2(M+2m)}$

acts on the body m_3.

Two forces $\vec{T}$ and $\vec{F}$ act on the body m_1

Newton's second law for the first body is written as

$F - T = ma_1$

$\boxed{a_1 = \frac{F(M+4m)}{2m(M+2m)}}$

Similarly, only $\vec{T}$

$m_2 = T$

$\boxed{a_2 = \frac{FM}{2m(M+2m)}}$

applies to the second body m_2

The answer:

Accelerations of weights $1-3$:

$a_1 = \frac{F(M+4m)}{2m(M+2m)}$

$a_2 = \frac{FM}{2m(M+2m)}$

$a_3 = \frac{F}{M+2m}$

2.1.48 Let's consider a small displacement over time dt from the point of view of kinematics:

For geometric reasons:

$dy = dx \cdot \cot\beta$

we differentiate both parts of the expression twice:

$\frac{dy}{d^2t} = \frac{dx}{d^2t} \cdot \cot\beta$

$a_1 = a_2 \cdot \tan\beta (1)$

Because the author did not say anything about friction, example $F_{fr} = 0$ Next, we write down Newton's law 2 for a bar m_1 on the OY axis:

$m_1 a_1 = N_1 \cos\beta$

$N_1 = m_1 a_1 \cos\beta (a)$

Similarly for m_2:

$m_2 a_2 = m_2 g - 2N_1 \sin\beta (b)$

Substitute (a) in (b):

$m_2 a_2 = m_2 g - 2m_1 a_1 \cdot \tan\beta$

Substituting (1) into (c):

$m_2 a_2 = m_2 g - 2m_1 a_2 \cdot \tan^2\beta$

We express a_2:

$\boxed{a_2 = \frac{m_2 g}{m_2 + 2m_1 \tan^2\beta} (d)}$

Considering (1), multiply (d) by $\tan\beta$:

$\boxed{a_1 = \frac{m_2 g \tan\beta}{m_2 + 2m_1 \tan^2\beta}}$

Answer:

$a_1 = \frac{m_2 g \tan\frac{\alpha}{2}}{m_2 + 2m_1 \tan^2\frac{\alpha}{2}}$

$a_2 = \frac{m_2 g}{m_2 + 2m_1 \tan^2\frac{\alpha}{2}}$

2.1.49 Let's apply Newton's second law:

$OX : T - T\sin\alpha = m_0 a_{0x}$

$T\sin\alpha = ma_{1x}$

$OY : N - mg - T\cos\alpha = m_0 a_{0y} = 0$

$T\cos\alpha - mg = ma_{1y}$

$\tan\alpha = \frac{x_0 - x_1}{y_0 - y_1}$

Considering $\alpha = const$

$\frac{d}{dt}(\tan\alpha) = \frac{(\frac{dx_0}{dt}-\frac{dx_1}{dt})(y_0-y_1)-(\frac{dy0}{dt}-\frac{dy1}{dt})(x_0-x_1)}{(y_0-y_1)^2} = 0$

$\frac{d}{dt}((\frac{dx_0}{dt}-\frac{dx_1}{dt})(y_0-y_1)) = \frac{d}{dt}((\frac{dy0}{dt}-\frac{dy1}{dt})(x_0-x_1))$

$(\frac{d^2x_0}{dt^2}-\frac{d^2x_1}{dt^2})(y_0-y_1)+(\frac{dx_0}{dt}-\frac{dx_1}{dt})(\frac{dy_0}{dt}-\frac{dy_1}{dt}) = (\frac{d^2y_0}{dt^2}-\frac{d^2y_1}{dt^2})(x_0-x_1)+(\frac{dy_0}{dt}-\frac{dy_1}{dt})\cdot(\frac{dx_0}{dt}-\frac{dx_1}{dt})$

$(a_{0x}-a_{1x})(y_0-y_1) = (a_{0y}-a_{1y})(x_0-x_1)$

$\tan\alpha(a_{0y}-a_{1y}) = a_{0x}-a_{1x}$

Of (1)

$a_{0y} = 0$

$-\tan\alpha a_{1y} = a_{0x}-a_{1x}$

Rope length:

$L = xw - x_0 + \frac{y_0-y_1}{\cos\alpha}$

Because the thread is inextensible

$\frac{d^2L}{dt^2} = -\frac{d^2x_0}{dt^2}+\frac{1}{\cos\alpha}(\frac{d^2y_0}{dt^2}-\frac{d^2y_1}{dt^2}) = 0$

$(\frac{d^2x_0}{dt^2} = a_{0x}, \frac{d^2y_0}{dt^2} = a_{0y}, \frac{d^2y_1}{dt^2} = a_{1y})$

$a_{0x}\cdot\cos\alpha = a_{0y}-a_{1y}$

From $(1) : a_{0y} = 0$

$a_{0x}\cos\alpha = -a_{1y}$

Dividing the equations from (1):

$\frac{T\sin\alpha}{T\cos\alpha} = \frac{ma_{1x}}{m(g+a_{1y})} \longrightarrow \tan\alpha(g+a_{1y}) = a_{1x}$

$\tan\alpha a_{1y} = a_{1x} - g\tan\alpha$

and using (4)

$\boxed{a_{0x} = g\tan\alpha}$

Dividing the equations from (1):

$\frac{T(1-\sin\alpha)}{T\cos\alpha} = \frac{M}{m}\cdot\frac{a_{0x}}{g+a_{1y}}$

Note that:

$a_{1y} = -g\sin\alpha$

$\frac{1-\sin\alpha}{\cos\alpha}\frac{M}{m} = \frac{g\tan\alpha}{g-g\sin\alpha} \longrightarrow m = \frac{M\sin\alpha}{(1-\sin\alpha)^2}$

$\boxed{m = \frac{M\sin\alpha}{(1-\sin\alpha)^2}}$

Answer:

$a = g\tan\alpha$

$m = \frac{m_0\ sin\alpha}{(1-\sin\alpha)^2}$

2.1.50 Let's divide the acceleration of the bar $\vec{a}_2$ into normal $\vec{a}_n$ and tangential $\vec{a}_\tau$ components

From the figure:

$a_n = a_2\cdot\sin\beta$

$a_\tau = a_2\cdot\cos\beta$

$a_n = a_\tau\cdot\tan\beta(0)$

Consider a small change in the coordinate of the wedge and the bar on it in a small interval dt: The wedge moved by dx_1 Meanwhile, the bar in the frame of reference associated with the wedge, the bar moved along the OX axis by dx, and along the OY—by dy

Relative to the starting point, in the direction of the Observer, the bar has shifted by

$OX : dx = dx_0 + dx_1$

$OY : dy = dy_0$

Because the bar does not come off,

$dy = dx\cdot\tan\alpha(a)$

Let 's iterate twice both parts of the expression (a):

$\frac{dy}{d^2t} = \frac{dx}{d^2t}\cdot\tan\alpha$

$a_n = (a_\tau + a_1)\cdot\tan\alpha(1)!$

Let 's write down Newton 's second law for a bar on the axis:

$OX : ma_\tau = N\sin\alpha(2)$

$OY : ma_n = mg - N\cos\alpha(3)$

According to Newton 's third law , the pressure force $\vec{N}'$ exerted by the bar on the wedge is equal in modulus and opposite in direction to the force of the normal reaction of the support $\vec{N}$:

$\vec{N}' = -\vec{N}$

Newton's second law for a wedge on the OX axis:

$Ma_1 = N\sin\alpha(4)$

Let's compose and solve a system of equations $(0), (1), (2), (3)$ and (4) with respect to M:

$$\begin{cases} a_n = a_\tau\cdot\tan\beta \\ a_n = (a_\tau + a_1)\cdot\tan\beta \\ ma_\tau = N\sin\alpha \\ ma_n = mg - N\cos\alpha \\ Ma_1 = N\sin\alpha \end{cases}$$

$$\begin{cases} a_\tau\cdot\tan\beta = (a_\tau + a_1)\cdot\tan\alpha \\ ma_\tau = N\sin\alpha \\ ma_n = mg - N\cos\alpha \\ Ma_1 = N\sin\alpha \end{cases}$$

$M = ma_\tau\cdot\frac{1}{a_1}$

$a_\tau(\tan\beta - \tan\alpha) = a_1\tan\alpha$

$\frac{a_\tau}{a_1} = \frac{\tan\alpha}{\tan\beta-\tan\alpha}$

$M = m\frac{a_\tau}{a_1}$

$\boxed{M = m\frac{\tan\alpha}{(\tan\beta-\tan\alpha)}.}$

Answer:

$M = m\frac{\tan\alpha}{(\tan\beta-\tan\alpha)}.$

2.1.51 The movement of the plate is complex, consists of horizontal (due to the movement of the rollers) and at an angle to the horizon, and is progressive. Let's set up our forces. The vectors in blue are the forces acting on the rink (N_1 is the force from the plate to the rink, it is equal to and opposite to the force from the rink to the plate, F_{fr} is the friction force acting on the rink (under its action the rink rotates), and the reaction force of the support R at point P).

Similar forces act on a skating rink with a smaller radius. The red vectors indicate the forces acting on the plate (N is the total reaction force of the supports, mg - gravity, F_{fr} - friction force from the rollers on the plate).

Let's choose the Ox axis in the direction of the plate speed. The velocity itself is the instantaneous velocity of the point of contact between the plate and the roller, and the point P is the instantaneous center of velocities, and v is perpendicular to PC(instantaneous radius of rotation). Let's write down the equation of motion of the plate in projections on the Ox axis

$ma = mg\sin\frac{\alpha}{2} + N\sin\frac{\alpha}{2} - F_{fr}\cos\frac{\alpha}{2}(1)$

There are three unknowns in this equation. More equations need to be drawn up. Let's write down the equation of rotational motion for the roller. This equation has the form

$I\varepsilon = \Sigma M$

The product of the moment of inertia of a body by angular acceleration is equal to the sum of the moments of forces acting on the body. Let's find the shoulders of the forces acting on the roller PC

$2r\cos\frac{\alpha}{2}$

The shoulder of the friction force is

$2r\cos^2\frac{\alpha}{2}$ and the shoulder of force N is $2r\cos\frac{\alpha}{2}\cdot\sin\frac{\alpha}{2}$

The shoulder of force R is zero.

Let's write down the equation of rotational motion for the roller (since the mass of the roller is zero according to the condition of the problem, then $I = 0$):

$0 = -F_{fr}2r\cos^2\frac{\alpha}{2} + Nr\sin\alpha(2)$

From where

$F_{fr} = N\tan\frac{\alpha}{2}$

substituting this value into the equation of motion (1), we get

$ma = mg\sin\frac{\alpha}{2}$,

i.e. the desired acceleration is equal to

$\boxed{a = g\sin\frac{\alpha}{2}}$

Answer:

$a = g\sin\frac{\alpha}{2}$

2.1.52 Since the system is closed, there are no external forces

Accordingly, the center of mass of the system has no acceleration

$a_C = 0(1)$

Considering that $\vec{a}_1$ and $\vec{a}_2$ are directed in different directions, the acceleration of the center of mass of the system is described by the expression

$a_C = \frac{m_1a_1 - m_2a_2}{m_1+m_2}$

Considering the ratio (1)

$m_1a_1 = m_2a_2$

From where, the mass of the second star

$\boxed{m_2 = m_1\frac{a_1}{a_2}}$

Answer:

$m_2 = m_1\frac{a_1}{a_2}$

2.1.53 Let's make the drawing larger. Let's set up our forces. At the initial moment, the dumbbell does not move, so we write down Newton's second law, taking into account that the acceleration is zero.

We write down the equilibrium condition on the axis:

$0 = N_1 - mg - T\cos\alpha$

$0 = N_2 - mg\cos(\pi - 2\alpha) + T\cos\alpha.$

It is not difficult to guess that the design was carried out in the directions N_1 and N_2. From this system we find

$N_1 - N_2 \geq 2mg$

$N_1 + N_2 \geq mg - mg\cos(\pi - 2\alpha)$

because $\cos(\pi - 2\alpha) = \cos^2\alpha$, then

$N_1 + N_2 = mg - mg\cos^2\alpha$

By the condition of the problem $\alpha = 90°$

With this in mind, we finally get

$\boxed{N_1 = 3mg}$

$\boxed{N_2 = -\frac{mg}{2}}$

The sign "$-$" means that the force is directed in the opposite direction, as indicated in the figure.

Answer:

For the upper ball $N_1 = \frac{mg}{2}$,

for the lower ball $N_2 = \frac{3mg}{2}$

2.1.54 1. The stationary circular orbit of an electron, which is a negatively charged particle with a rest mass m_e, will take place if the Coulomb force of attraction and the inertia force

$F_k = F_i$

are equal in modulus

2. The inertia force is directly proportional to the square of the linear velocity of the particle and inversely proportional to the distance to the axis of rotation

$F_i = \frac{m_e v^2}{r}$

Thus, the Coulomb force in this case is inversely proportional to the distance between the electron and the charged filament.

3. As the charge of the filament increases,

$\boxed{F_k > \frac{m_e v^2}{r}}$

to restore equilibrium, the radius of the orbit should decrease, while decreasing the charge, on the contrary, the radius will increase.

Answer: $F = \frac{m_e v^2}{r}$.

Close to parabolas, touching circle from the inside;

from the outside.

2.1.55 Because the thread does not sag

$T_1 = T_2 = T$

As soon as the nail appears, both bodies continue to move at a speed of v along the new trajectory $r = \frac{l}{2}$

$T = ma_n$

$T = m\frac{v^2}{\frac{l}{2}}$

We obtain the tension force of the thread immediately after that

$\boxed{T = \frac{2mv^2}{l}}$

Answer:

$T = \frac{2mv^2}{l}$

2.1.56 1. In the case of a weightless and inextensible thread, its tension is defined as:

$T = F_i = \frac{mv^2}{L} = \frac{m\omega^2 L}{2L}$

$\boxed{T = m\omega^2 L}$

2. Select a given section of the rope and determine the mass of its part length $(L - x)$

$m_x = m\frac{L-x}{L}$

3. Determine the distance from the axis of rotation Oz to the center of mass of the rope segment

$r_x = L - \frac{L-x}{2}$

$r_x = \frac{L+x}{2}$

4. The tension of the rope in the section x will be due to the rotating mass M and the mass of the rope m_x

$\boxed{T = M\omega^2 L + \frac{m\omega^2(L^2-x^2)}{2L}}$

Answer:

$T = M\omega^2 l,$

$T_x = M\omega^2 l + \frac{m\omega^2(l^2-x^2)}{(2l)}$

2.1.57 Projecting the tension force of the thread T, we write down Newton's second law on the vertical and horizontal axes:

$T\sin\alpha = ma_c$

$mg - T\cos\alpha = 0$

Hence

$mg\tan\alpha = ma_c(1)$

We find the centripetal acceleration through the angular velocity of rotation ω

$a_c = \omega^2 r = \omega^2 R\sin\alpha$

We substitute in (1)

$g = \omega^2 R\cos\alpha$

From where we find α

$\cos\alpha = g\omega^2 R$

Given the area of definition of the cosine

$-1 \le \cos\alpha \le 1$

At $g > \omega^2 R$, the angular velocity will no longer be enough to lift the body to some angle and the body will take a stable position at the lowest point of its trajectory corresponding to

$\boxed{\alpha = 0}$

Answer:

$\cos\alpha = \frac{g}{(\omega^2 R)}$ at $\frac{g}{(\omega^2 R)} < 1$

$\alpha = 0$ at $\frac{g}{(\omega^2 R)} > 1$.

2.1.58 Projecting the tension force of the thread T for the lower ball, we write down Newton's second law on the vertical and horizontal axes:

$Tsin\beta = ma_c$

$mg - T\cos\beta = 0$

Hence

$mg\tan\alpha = ma_c(1)$

We find the centripetal acceleration through the angular velocity of rotation ω

$a_c = \omega^2 r = \omega^2(l_1 + l_2)$

$a_c = \omega^2 l(\sin\alpha + \sin\beta)$

We substitute in (1)

$g\tan\alpha = \omega^2 l(\sin\alpha + \sin\beta)$

From where we find ω

$\boxed{\omega = \sqrt{\frac{g\tan\beta}{l(\sin\beta+\sin\alpha)}}.}$

The answer:
$\omega = \sqrt{\frac{g\tan\beta}{l(\sin\beta+\sin\alpha)}}.$

2.1.59 We write down Newton's second law for the load, taking into account the elastic force of the spring F_{el}

$ma_c = F_{el} = k\Delta x$

We express Δx in terms of the length of the undeformed spring l

$ma_c = F_{el} = k(R-l)(1)$

We find the centripetal acceleration through the angular velocity of rotation ω

$a_c = \omega^2 R$

$a_c = \omega^2(l+\Delta x)$

Substitute in (1)

$\frac{k}{m}(R-l) = \omega^2 R$

From where we find l

$\boxed{l = R(1-\frac{m\omega^2}{k})}$

The answer:

$l = (1-\frac{m\omega^2}{k})R$

2.1.60 Consider a small piece of harness of length $dl = 2\alpha R$

Tension forces act on a piece of the harness

$T_1 = T_2 = 2k\alpha(R-R_0),$

taking into account $\sin\alpha \approx \alpha$ at small angles

$T_n = 2T\alpha$

Due to the uniformity of the harness, a piece of length $dl = 2\alpha R$ will have a mass

$dm = m\frac{\alpha}{\pi}$

At the same time it will be affected by centripetal acceleration

$a = \omega^2 R$

We write Newton's second law as

$dma = T_n$

$m\frac{1}{\pi}\omega^2 R = 4k(R-R_0)\alpha$

From where we find R

$\boxed{R = \frac{R_0}{1-\frac{m\omega^2}{4\pi^2 k}}}$

Analyzing the resulting expression,the resulting elastic force will act at $R < R_0\vec{T}_n$ aimed at stretching, which will not be compensated by centrifugal force. Thus, the tourniquet will stretch endlessly and eventually break.

Answer:

$R = \frac{R_0}{(1-\frac{m\omega^2}{4\pi^2 k})}$ at $\omega < 2\pi\sqrt{\frac{k}{m}}$

at $\omega > 2\pi\sqrt{\frac{k}{m}}$ the ring stretches indefinitely.

2.1.61 To begin with, I advise you to familiarize yourself with solution 2.1.60

Consider a small piece of harness of length $dl = 2\alpha R$

A piece of the harness is affected by the friction force $\vec{F_{fr}}$

$F_{fr} = \mu N$

Due to the uniformity of the harness, a piece of length $dl = 2\alpha R$ will have a mass

$dm = m\frac{\alpha}{\pi}$

While centripetal acceleration

$a = \omega^2 R$

will act on it

Newton's second law is written as

$dma = N$

$m\frac{\alpha}{\pi}\omega^2 R = N$

Where does the friction force

$F_{fr} = \mu m\frac{\alpha}{\pi}\omega^2 R$

come from Considering $sin\alpha \approx \alpha$ at small angles, we write the equilibrium condition on the vertical axis

$2T\alpha + F_{fr} = dmg$

$2T\alpha + \mu m\frac{\alpha}{\pi}\omega^2 R = mg\frac{\alpha}{\pi}$

From where we find μ

$\boxed{\mu = \frac{mg}{(2\pi T - m\omega^2 R)}}$

Answer:

$\mu = \frac{mg}{(2\pi T - m\omega^2 R)}$

2.1.62 $\alpha = \arctan\frac{v^2}{gR}$

2.1.63 Due to the fact that the angular velocity varies, in addition to the centripetal velocity a_n, there will also be a tangential acceleration a_τ

According to the Pythagorean theorem, we find the total acceleration $\vec{a}$

$a = \sqrt{a_n^2 + a^2\tau}$

$a = \sqrt{(\omega^2 R)^2 + (\varepsilon R)^2}$

$a = \varepsilon R\sqrt{1+\varepsilon^2 R^4}$

Newton's second law is written as

$ma = F_{fr}$

$ma = \mu mg$

$\varepsilon R\sqrt{1+\varepsilon^2 R^4} = \mu g$

This equality holds for

$t = \sqrt{\frac{\mu^2 g^2}{\beta^4 R^2} - \frac{1}{\beta^2}}$

From where we find ω

$$\boxed{\omega = \sqrt[4]{\frac{\mu^2 g^2}{R^2} - \varepsilon^2)}}$$

In this case, if $\varepsilon > \frac{\mu g}{R}$, then a_τ will be so large that the force of friction at rest will instantly turn into the force of sliding friction, even when $a_n = 0$

Answer:

$\omega_1 = 0$ at $\varepsilon > \frac{\mu g}{R}$

$\omega_1 = (\frac{\mu^2 g^2}{R^2 - \varepsilon^2})^{\frac{1}{4}}$ at $\varepsilon < \frac{\mu g}{R}$

2.1.64 Let's depict the forces acting on the motorcycle (see Figure above), and write down Newton's second law

$\vec{N} + m\vec{g} + \vec{F}_{fr} = m\vec{a}.$

In the projection on the axis:

$Ox : F_{fr} = ma;$

$Oy : N - mg = 0.$

Considering that when moving along a circle $a = \frac{v^2}{R}$,the friction force is equal to $F_{fr} = \mu N$, we get that the maximum speed of a motorcyclist is equal to

$v = \sqrt{\mu g R}$

When turning, the motorcyclist deviates to the center of the circle by a certain angle (see Figure below). Then according to Newton 's second law

$\vec{N} + m\vec{g} = m\vec{a}$

$Ox : N \sin\beta = ma;$

$Oy : N \cos\beta = mg.$

Where from

$\tan\beta = \frac{a}{g} = \frac{\mu g}{g}$

$\boxed{\beta = \arctan\mu}$

Consider the movement of a cyclist along a horizontal drift (Fig.).

The cyclist is affected by: gravity $m\vec{g}$ and the normal component of the reaction force of the support $\vec{N}$.

The centripetal acceleration of a cyclist can only be reported here by the frictional force of rest, directed along the radius of the circle to the center of O and arising when the cyclist leans towards the center of the circle.

The resultant of the forces $\vec{N}$ and $\vec{F}_{fr} \to F = \vec{F}_{fr} + \vec{N}$ passes through the center of gravity of the cyclist, and otherwise there would be a tipping moment of forces.

According to Newton's second law, for projection into the radial direction X $F_{fr} = ma_n = m\frac{v^2}{R}$, where v is the speed of the cyclist.

Since the resting friction force $F_{fr} \le \mu N = \mu mg$, we obtain the inequality

$m\frac{v^2}{R} \le \mu mg$

$v^2 \le \mu g R \to v \le \sqrt{\mu g R}$

the maximum value of the velocity on the horizontal track $v_1 = \sqrt{\mu g R}$.

Consider the movement of a cyclist on an inclined track. The forces acting on it are shown in Fig. ($\vec{F}$ is the resultant of the reaction forces of the support $\vec{N}$ and the friction forces of rest $\vec{F}_{fr}$). According to Newton's second law for projections on the X and Y axes:

on the Y-axis $N\cos\alpha - F_{fr}\sin\alpha - mg = 0(1)$

along the X-axis $N\sin\alpha + F_{fr}\cos\alpha = ma_n(2)$, where $a_n = \frac{v^2}{R}$

(v is the speed of movement on an inclined track). Let's rewrite the system in the form:

$$\begin{cases} N\cos\alpha - F_{fr}\sin\alpha = mg, (1') \\ N\sin\alpha + F_{fr}\cos\alpha = m\frac{v^2}{R}(2'). \end{cases}$$

Let us express from this system N and F_{fr}. To do this, multiply the equation $(1')$ by $\cos\alpha$, and the equation $(2')$ - on $\sin\alpha$:

$$\begin{cases} N\cos^2\alpha - F_{fr}\sin\alpha = mg\cos\alpha, \\ N\sin^2\alpha + F_{fr}\cos\alpha\sin\alpha = m\frac{v^2}{R}\sin\alpha. \end{cases}$$

After addition, we get

$N(\cos^2\alpha + \sin^2\alpha) = mg\cos\alpha + m\frac{v^2}{R}$

$N = m(g\cos\alpha + \frac{v^2}{R})$

Multiply $(1')$ by $\sin\alpha$, and $(2')$- on $\cos\alpha$, then

$$\begin{cases} N\cos^2\alpha - F_{fr}\sin\alpha = mg\cos\alpha, \\ N\sin^2\alpha + F_{fr}\cos\alpha\sin\alpha = m\frac{v^2}{R}\sin\alpha. \end{cases}$$

After calculations, we find

$F_{fr} = (\frac{v^2}{R} - g\sin\alpha)$

Since F_{fr} is the friction force at rest, then

$F_{fr} \le \mu N \to m(\frac{v^2}{R}\cos\alpha - g\sin\alpha) \le \mu m(g\cos\alpha + \frac{v^2}{R}\sin\alpha)$

$\frac{v^2}{R}(\cos\alpha - \mu\sin\alpha) \le g(\mu\cos\alpha + \sin\alpha)$

Divide both parts into $\cos\alpha$(from the condition $\cos\alpha > 0$).

$\frac{v^2}{R}(1 - \mu tg\alpha)(\mu + tg\alpha)$

If $(1 - \mu tg\alpha) > 0$, then $v^2 \le \frac{gR(\mu + tg\alpha)}{1 - \mu tg\alpha}$, or $v \le \sqrt{\frac{gR(\mu + tg\alpha)}{1 - \mu tg\alpha}}$

This means that the maximum speed when moving along an inclined track is

$v^2 = \sqrt{\frac{gR(\mu + tg\alpha)}{1 - \mu tg\alpha}}$

The ratio

$$\boxed{\frac{v_2}{v_1} = \sqrt{\frac{\mu+tg\alpha}{\mu(1-tg\alpha)}}}$$

Answer:

$v = \sqrt{\mu g R}$, $\beta = \arctan\mu$, $\frac{u}{v} = \sqrt{\frac{\mu+\tan\alpha}{\mu(1-\mu\tan\alpha)}}$.

2.1.65 The skater is informed by the centripetal acceleration of the friction force on the ice

$\vec{F}_{fr} = \mu\vec{N}$

where $\vec{N}$ is the force of the normal reaction of the ice (Fig. a).

Since the skater does not move in the vertical direction, the force $\vec{N}$ is equal in modulus to the force of gravity acting on the skater $M\vec{g}$

Therefore,

$F_{fr} = \mu mg$

$\frac{mv^2}{R} = \mu mg$

Hence

$v = \sqrt{\mu g R}$

When making a turn, the skater passes the distance

$S = \pi R$

during the time

$t = \frac{s}{v} = \pi\sqrt{\frac{R\mu}{g}}(1)$

The larger the radius of the circle along which the skater moves, the greater It's time.

Although the maximum speed of a skater increases with an increase in the turning radius, the distance traveled by him increases even more: while the speed is proportional to $\sqrt{R}$, the distance traveled is proportional to R. That is why the skater tries to pass the turn as close as possible to the inner edge.

2.1.66 $v_{min} = \sqrt{\frac{gR\sin\alpha(\tan\alpha+\mu)}{\mu\tan\alpha-1}}$

2.1.67 According to Newton's second law, we have

$m\omega^2 r = N + mg\sin\alpha$

where N is the force of normal pressure.

In order to avoid slippage, the condition

$mg\cos\alpha \leq k(m\omega^2 r - mg\sin\alpha)$

must be fulfilled

from where

$\omega^2 \geq \frac{g}{r}(\cos\alpha + \sin\alpha)$ at $k = l$.

Thus,

$$\boxed{\omega = \sqrt{\frac{g\sqrt{2}}{R}}}$$

Answer:

$\omega = \sqrt{\frac{g\sqrt{2}}{R}}$

2.2 Impulse. Center of mass

2.2.1 $u = \frac{5v}{2}$.

2.2.2 $F_{fr} = \frac{F}{3}$

2.2.3 $t = \frac{2p\sin(\frac{\alpha}{2})}{F}$

at an angle $\beta = \frac{(\pi+\alpha)}{2}$ to the initial velocity.

2.2.4 $m = \frac{F\Delta t^2}{(16L)}$. Using the experimental data, plot the dependence of the span time on the source voltage.

2.2.5 $t = \frac{mv(\sin\alpha-\mu\cos\alpha)}{[\mu(m+M)g]}$ at $\tan\alpha > \mu$

at $\tan\alpha \leq \mu$ the box will not move.

2.2.6

2.2.7 $\frac{m_1}{m_2} = \frac{(u_2-u_1)}{(v_1-v_2)}$. It is necessary to take into account the change in the velocity of the Earth.

2.2.8

2.2.9 $u_1 = \frac{F_0 t_0}{m_1}; u_2 = v - \frac{F_0 t_0}{m_2}$.

2.2.10 $m = \frac{m_0}{3}$.

2.2.11 $u_1 = u_2 = 0, 2v$.

2.2.12 At a distance of $4L$ horizontally from the gun

2.2.13 $\frac{S}{L} = \frac{35}{36}$

2.2.14 $w = \frac{m_1}{m_2}\sqrt{u^2+v^2}$.

2.2.15 $p = \sqrt{p_1^2 + 2p_1p_2\cos\alpha + p_2^2}$.

2.2.16 $V = \frac{\sqrt{m_1^2v_1^2+m_2^2v_2^2+m_3^3v_3^2}}{m_1+m_2+m_3}$

2.2.17 $l_1 = \frac{lm_2}{(m_1+m_2)}$, $l_2 = \frac{lm_1}{(m_1+m_2)}$

2.2.18 The trajectory of the particle is obtained by stretching with similarity coefficient 2 the trajectory of a particle whose mass is $2m$.

2.2.19 Circles whose center lies in the center of mass of the system station - astronaut. Radii of circles:

$R_1 = \frac{Rm_2}{(m_1+m_2)}, R_2 = \frac{Rm_1}{(m_1+m_2)}$.

2.2.20 On the bisector of the angle at a distance $l = \frac{L\sqrt{2}}{4}$ from the vertex, where L is the length of half of the rod; at the point of intersection of the medians; on the line connecting the centers of the disk and the hole, at a distance $l = \frac{dr^2}{(R^2-r^2)}$ from the center of the disk.

2.2.21 $u = \frac{\rho Svl}{m}$.

2.2.22 $v = \frac{uV(\rho_0-\rho)}{(\rho V+\rho_0 V_0)}$.

2.2.23 $T = 2\pi\sqrt{\frac{2R}{3g}}$

2.2.24 $F = \frac{m_1 m_2 v^2}{(m_1+m_2)l}$

2.2.25 $\omega = \sqrt{\frac{m_2T_1+m_1T_2}{Lm_1m_2}}; m = \frac{2m_1m_2(T_1-T_2)}{m_1T_2-m_2T}$

2.2.26 $T_{12} = \frac{m_1m_2}{m_1+m_2+m_3}l\omega^2$ for the thread connecting m_1 and m_2; the expressions for the other threads are similar.

2.2.27 $F = mg - \rho Va$

2.2.28 $F = S\rho La$

2.2.29 At a speed of $\frac{u}{4}$ upwards.

2.2.30 $n = \frac{(m_2gtg\alpha)}{(m_1v)}$.

2.2.31 $F = Nmg$. Increasing.

2.2.32 $H = h\frac{(M+Nm)^2}{Nm(Nm+2M)}$.

2.2.33 $F = \frac{mv^2}{R}; p = \frac{F}{S} = \frac{Nmv^2}{3}$

2.2.34 $\Delta v_N = (\frac{M}{M+m})^{2N}(v_2 - v_1)$.

2.2.35 $F = \rho Su^2$.

2.2.36 $\mu = \frac{Mg}{u}; \mu' = \frac{M(g+a)}{u}$

2.2.37 $F = \mu_2(u-v) + \mu_1 u$.

2.2.38 $v = \frac{\rho Su}{(\rho S+k)}$

2.2.39 $v = \sqrt{\frac{F(\rho-\rho_0)}{(\pi r^2\rho\rho_0)}}$.

2.2.40 See Fig.

2.2.41 $F = 3mg(1 - \frac{x}{l})$.

2.2.42 $F = \frac{m(v^2+gl)}{l}$.

2.2.43 $v = \sqrt{gh}$

2.2.44 $N = 2(F - \rho v^2)\cos\frac{\alpha}{2}$

at $v \geq \frac{F}{\rho}$

2.2.45 $K = k^n$.

2.2.46 $u = v\ln n$

2.2.47 $m \approx 5.5 \cdot 10^5\ kg$; 7.4 times less.

2.3 Kinetic energy. Work. Potential energy

2.3.1 At $m = \frac{2Fl}{v^2}$

2.3.2 $F = \frac{m(v_2^2-v_1^2)}{(2l)}$; if $F > 0$, the direction of the force coincides with the direction of the particles, and if $F < 0$, the direction of this force is opposite.

2.3.3 $F \approx 2.5 \cdot 10^6\ H$

2.3.4

2.3.5 $v = \sqrt{\frac{F_0(l_1+2l_2+l_3)}{m}}$.

2.3.6 $v > \sqrt{2\mu gL}$.

2.3.7 $A \approx 0.8J$

2.3.8 $x = v\sqrt{\frac{m}{k}}; x' = \sqrt{x_0^2 + \frac{mv^2}{k}}$

2.3.9 $E_k = \frac{F^2}{(8k)}$.

2.3.10 At the greatest force we can develop, the bow should stretch as far as the arm span allows. For a tighter bow, as for a less tight bow, the stored elastic energy will be less.

2.3.11 $K = mgl\cos\alpha$, $K' = mgl(\cos\alpha - \mu\sin\alpha)$

2.3.12 $h = \frac{v^2}{[2g(1-\mu\cot\alpha)}$.

2.3.13 $v = \sqrt{4gh - \frac{2A}{m}}$.

2.3.14 $A_{min} = mgl$.

2.3.15 $A_{min} = \frac{mgl}{2}$

2.3.16 $v = r\sqrt{\frac{g}{l}}$

2.3.17 $n = \frac{mv^2}{(4\pi FR\cos\alpha)}$.

2.3.18 $v = 2\sqrt{(l-h)Tm}$.

2.3.19 Moving through the pipe.

2.3.20 $sin\beta = \frac{v\sin\alpha}{\sqrt{v^2+2gh}}$.

2.3.21 $sin\beta = \frac{\sin\alpha}{\sqrt{1-2F\frac{l}{mv^2}}}$ at $Fl > \frac{mv^2}{2}\cos^2\alpha$

2.3.22 At the bottom. In the upper one. At angle $\alpha = arctg\frac{1}{\sqrt{3}}$ between the thread and the vertical

2.3.23 $x = l\frac{T-3mg}{T-mg}$.

2.3.24 $F = 5mg$ for the rod; $F = 6mg$ for the thread.

2.3.25 $L_{min} = \frac{R}{2(\tan\alpha-\mu)}$.

2.3.26 $h = \frac{2R}{3}$

2.3.27 $h = 2,5R.$

2.3.28 $F = \frac{2\pi Rmg}{4\pi^2R^2+h^2}\sqrt{4\pi^2R^2 + h^2 + 16\pi^2H^2}$

2.3.29 $F = mg(1 - \frac{1}{k^2}).$

2.3.30 $F = mg\cos\alpha(3\sin\alpha - 2)$ at $\sin\alpha \geq \frac{2}{3}$
$F = 0$ at $\sin\alpha \leq \frac{2}{3}$.

2.3.31 $v = \sqrt{2gl}.$

2.3.32 $A = 2\pi\mu mv^2$

2.3.33 $K = K_{0e}^{-2\mu\alpha}.$

2.3.34

2.3.35 $F = \frac{l}{(m\alpha^2)}$

2.3.36 $A = \frac{x^2}{(2m\alpha^2)} - \frac{px}{(\alpha m)}.$

2.3.37 $U = \frac{F_0x^2}{(2x_0)}$ at $|x| \leq x_0$
$U = F_0(|x| - \frac{x_0}{2})$ at $|x| > x_0$
Movement area: $|x| \leq \sqrt{\frac{2Kx_0}{F_0}}$ at $K \leq \frac{F_0x_0}{2}$
$|x| \leq (\frac{K}{F_0} + \frac{x_0}{2})$ at $K > \frac{F_0x_0}{2}$.

2.3.38 $K = \frac{kqQ}{r^2}$; at $qQ > 0$ - repulsion,
at $qQ < 0$ - attraction

2.3.39 No

2.3.40 At $E > 0$, the region of motion
$r \geq l\frac{V}{E}(-1 + \sqrt{1 + \frac{E}{V}})$
at $E < 0$, r is between $r_{1,2} = l\frac{V}{E}(-1 \pm \sqrt{1 + \frac{E}{V}})$.

2.3.41 $h = \frac{2mg}{k}$

2.3.42 $h = \frac{2mg}{k}; v = g\sqrt{\frac{m}{k}}$

2.3.43 $H_1 = \frac{3h}{2}; H_2 = \frac{4h}{3}.$

2.3.44 $k = \frac{mgx_0}{2(\sqrt{l^2+x_0^2}-l)^2}$

2.3.45 $F = mg(1 + \sqrt{1 + \frac{2k(h-l)}{(mg)}}).$

2.3.46 $F = (m_1 + m_2)g.$

2.3.47 $x = (\frac{m}{k})(g - a); x_{max} = (\frac{m}{k})(g + \sqrt{2ga - a^2}).$

2.3.48 $F = \mu g(m_1 + \frac{m_2}{2})$

2.3.49 $m = \frac{\mu u m_0}{2}.$

2.4 System energy. Energy transfer. Power

2.4.1 In a moving frame of reference, the force of tension performs work. No

2.4.2 $K = \frac{m_1v^2}{2} - Fl.$

2.4.3 $K_1 = \frac{k(x_1+x_2)x_1}{2}$
$K_2 = \frac{k(x_1+x_2)x_2}{2}.$

2.4.4 $A_1 = \frac{mu^2}{2} - \frac{mv^2}{2}; A_2 = -mu^2$

2.4.5 $A = 2Fr(2\sin\frac{\alpha}{2} - 1)$, $\alpha = 60°$.

2.4.6 The sum of the works of the mutual forces depends only on the change in the distance between the particles.

2.4.7 $x = v\sqrt{\frac{m}{(3k)}}.$

2.4.8 $v' = \sqrt{2}v.$

2.4.9 $v = \sqrt{gl}.$

2.4.10 $x_{max} = \frac{4l}{3}$

2.4.11 $v_m = \tan\alpha\sqrt{\frac{2Mgh}{M+m\tan^2\alpha}}$
$v_M = \sqrt{\frac{2Mgh}{M+m\tan^2\alpha} + 2g(H - h)}.$

2.4.12 $v = (\frac{4}{3})\sqrt{\frac{gR}{3}}.$

2.4.13 $F = \frac{7mg}{9}.$

2.4.14 $h \approx 0.25m$

2.4.15 $K' = 0.01K.$

2.4.16 $v_x = (l - l_0)\sqrt{\frac{k}{(2m)}}\cos\alpha$
$x = (l - l_0)\sin\alpha$

2.4.17 a. In translational motion. The acceleration of the center of mass and the total external force for the system is related in the same way as for an individual particle.

2.4.18 $\mu = \frac{v^2}{(2gl)}.$

2.4.19 $v = \sqrt{2h(g - \frac{T}{m})}; K = mgh, E_{rot} = Th.$

2.4.20 $x = \frac{Lm^2}{(M^2-m^2)}$

2.4.21 $l_{min} = l_0; l_{max} = l_0 + \frac{F}{k}$

2.4.22 $x = \frac{\mu mg\cos\alpha}{k}$ at $\mu \leq \tan\alpha$
$x = \frac{\mu mg\cos\alpha}{2k}[1 + \sqrt{1 - 2(1 - \frac{\tan\alpha}{\mu})^2}]$ at
$\tan\alpha \leq \mu \leq 3\tan\alpha$
$x = \frac{2mg\sin\alpha}{k}$ at $\mu \geq 3\tan\alpha$

2.4.23 The kinetic energy of the particle is $K = \frac{m(\vec{u}+\vec{V})^2}{2}$, where $\vec{u}$ is its velocity relative to the center of mass and $\vec{V}$ is the velocity of the center of mass. In sum over all particles of the system, the summands $m\vec{u}\vec{V}$ give zero.

2.4.24 $K_{max} = \frac{F^2}{(2k)}; U_{max} = \frac{2F^2}{k}; v_{rel} = F\sqrt{\frac{(m_1+m_2)}{(km_1m_2)}}$

2.4.25 At a velocity of the center of mass equal to zero

2.4.26 $\Delta W = Fl.$

2.4.27 $\Delta W = F(l - \frac{Ft^2}{2m})$

2.4.28 $\Delta W = \frac{F^2m_2^2}{[k(m_1+m_2)^2]}, U = \frac{\Delta W}{2}$

$K = Fl + \frac{F^2m_1m_2}{[k(m_1+m_2)^2]}$

2.4.29 $A = mu^2$. Half of the work goes to increase the internal energy.

2.4.30 $\frac{\Delta W}{A} = \frac{\mu}{(\tan\alpha+\mu)}.$

2.4.31 $W = W_1 + W_2 + \frac{m_1m_2}{2(m_1+m_2)}(V_1 - V_2)^2$, No.

2.4.32

2.4.33 $Q = m(\frac{v^2}{2} - gh)$

2.4.34 $Q = \frac{m_1gh(m_1-m_2)}{(m_1+m_2)}$

2.4.35 $Q = 2mgR(1 - \sqrt{1 - \frac{l^2}{(4R^2)}})\sqrt{1 - \frac{l^2}{(4R^2)}}.$

2.4.36 $E \approx 200\ MJ.$

2.4.37 $m \approx 3\ kg.$

2.4.38 8 times

2.4.39 $v = \mu gt$ at $t \le t_0 \equiv \frac{N}{m\mu^2g^2}$

$v = \sqrt{\frac{2N}{m}(t - \frac{N}{2m\mu^2g^2})}$ at $t > t_0$

2.4.40 $N = m_0g\omega(1 - \frac{\omega}{\omega_0})$, $m = \frac{m_0}{2}.$

2.4.41 $m = \frac{n_2m_0}{(2n_1)}$

2.4.42 $v \approx 20\ \frac{km}{h}$; $\alpha = \arcsin\frac{\sqrt{2}}{4}$

2.4.43 $N = \rho S(v - \omega R)^2\omega R.$

2.4.44 $\eta = \frac{2v}{(v+u)}.$

2.4.45 $N = \frac{mgu}{2}.$

2.5 Collisions

2.5.1 $\frac{m_1}{m_2} = 1$; yes.

2.5.2 $\alpha = \frac{\pi}{2}.$

2.5.3 $\frac{u_1}{u} = \frac{(k-1)}{(k+1)}; \frac{u_2}{u} = \frac{2k}{(k+1)}.$

2.5.4 The mass of the neutron is close to the mass of the deuteron ($m_n \approx \frac{m_d}{2}$), so the energy loss in elastic collisions with deuterons is much greater than collisions with heavy lead nuclei.

2.5.5 $m = \sqrt{m_1m_2}.$

2.5.6 $\cos\beta = \frac{v_1v_2\cos\alpha}{(u_1\sqrt{v_1^2+v_2^2-u_1^2})}$

2.5.7 $v_1' = 2v - v_1; v_2' = 2v - v_2$

2.5.8 After any odd number of collisions the velocities

$v_1' = \frac{(m_1-m_2)v_1+2m_2v_2}{m_1+m_2}$, $v_2' = \frac{(m_2-m_1)v_2+2m_1v_1}{m_1+m_2}$

After any even - are equal to the initial ones.

2.5.9 $v_1 = v\sqrt{\frac{m_2m_3}{m_1(m_1+m_3)}}; v_3 = v\sqrt{\frac{m_2m_1}{m_3(m_1+m_3)}}$

2.5.10 $\tan\beta = \tan\alpha\frac{m_1+m_2}{m_2-m_1}.$

2.5.11 $d = 2\sqrt{2}R.$

2.5.12

2.5.13 The two nearest balls obtain velocities $v_1 = v\cos\alpha$ and $v_2 = \sin\alpha$, directed on mutually perpendicular sides of the cell, and the originally moving ball will stop. These velocities are then transferred to the next balls in corresponding rows

2.5.14 $t = t_n - t_{n-1} = 2\Delta t.$

2.5.15 $t = \frac{(2R\cos\alpha)}{v}.$

2.5.16 $\frac{m_1}{m_2} = \frac{\sin^2(\alpha+\beta)-\sin^2\beta}{\sin^2\alpha}$; m_1 is the mass of an incoming particle, m_2 is the mass of a resting particle.

2.5.17 $\sin\alpha = \frac{m_2}{m_1}.$

2.5.18 $u = \frac{2m_1v\cos\alpha}{(m_1+m_2)}$

2.5.19 $m_2 = \frac{m_1(p^2+p_0^2-2pp_0\cos\alpha)}{(p_0^2-p^2)}$

2.5.20 $u = \sqrt{v^2 + (v + u_0)^2}$

swivel angle $\varphi = \frac{\pi}{2} + \arctan\frac{v}{u_0+v}$

2.5.21 $v = \sqrt{2gh(1 + \frac{m_2}{m_1})}.$

2.5.22 $v_1 = 0; v_2 = v$ at $v > v_0 \equiv \sqrt{2gh(1+\frac{m_2}{m_1})}$

$v_1 = v\frac{2m_2}{m_1+m_2}$, $v_2 = v\frac{m_2-m_1}{m_1+m2}$ at $v < v_0$

Here v_1 is the speed of the slide, v_2 is the velocity of the body.

2.5.23 $v_1 = \frac{m_2}{m_1}\sqrt{\frac{2gRm_1}{(m_1+m_2)}}$, $v_2 = \sqrt{\frac{2gRm_1}{(m_1+m_2)}}$

$N = m_2 g(3 + \frac{2m_2}{m_1})$

2.5.24 $u = x\sqrt{\frac{k(m_1+m_2)}{m_1 m_2}}$, $u_1 = \frac{m_1-m_2}{m_1+m_2}x\sqrt{\frac{k(m_1+m_2)}{m_1 m_2}}$

$u_2 = \frac{2m_1}{m_1+m_2}x\sqrt{k(m_1+m_2)}m_1 m_2$

then $u_1 = u$, $u_2 = 0$ and so on.

2.5.25 $h_{1max} = \frac{m_2 U}{m_1 g(m_1+m_2)}$, $h_{2max} = \frac{m_1 U}{m_2 g(m_1+m_2)}$

2.5.26 1.5 times.

2.5.27 $K = 35.7\ keV$

2.5.28 $E_{min} = E(1+\frac{m_e}{m})$.

2.5.29 $E_{min} \approx 27.2\ eV$

2.5.30 $v_1 = \sqrt{\frac{2Em_2}{m_1(m_1+m_2)}}$, $v_2 = \sqrt{\frac{2Em_1}{m_2(m_1+m_2)}}$

2.5.31 $E = \frac{p_1^2 m_2^2 + p_2^2 m_1^2 - 2p_1 p_2 m_1 m_2 \cos\Theta}{2m_1 m_2(m_1+m_2)}$

2.5.32 $E = K\sin\alpha_1 \sin\alpha_2$

2.5.33 $E = 4.1\ MeV$.

2.5.34 $\cos\alpha = \frac{(p_2-2mE)}{(p_2+2mE)}$ if $2mE < p_2$

$\alpha = \frac{\pi}{2}$ if $2mE > p_2$

2.5.35 $\frac{h}{h_0} = \left[\frac{(m_1-m_2)}{(m_1+m_2)}\right]^2$.

2.5.36 $\frac{Q}{K} = \frac{(3-\frac{m_1}{m_2})}{4}$

2.5.37 $Q_1 = 2\sqrt{Q_2 m}(v - 2\sqrt{\frac{Q_2}{m}})$.

2.5.38 $v_n = \sqrt{\frac{Fl}{m}(1+\frac{1}{n})}$, $u_n = \sqrt{\frac{Fl}{m(1+\frac{1}{n})}}$

$v_n \to \sqrt{\frac{F}{lm}}$ at $n \to \infty$

2.5.39 $\tan\beta = \tan\alpha - 2\mu$ with $\tan\alpha > 2\mu$

otherwise $\beta = 0$

2.6 The force of gravity. Kepler's laws

2.6.1

2.6.2 $a = \frac{K}{R^2}$ where R is the distance from the planet to the Sun, K is the constant.

2.6.3 $h \approx 700\ km$

2.6.4

2.6.5

2.6.6 According to the given data $\gamma = \frac{r^2 a}{(2M)} \approx 5\cdot 10^{-11}$ $H\cdot\frac{m^2}{kg^2}$, which is comparatively close to the results of exact measurements.

2.6.7 $M \approx 6\cdot 10^{24}\ kg$

2.6.8 $M \approx 6\cdot 10^{30}\ kg$

2.6.9

2.6.10 0.3 times.

2.6.11 6 times.

2.6.12 $R = (\frac{\gamma m_\odot}{\omega^2})^{\frac{1}{3}}$

2.6.13 $T_1 \approx 0.7$ years.

2.6.14 $T = 2\pi\sqrt{\frac{R^3}{\gamma(m_1+m_2)}}$.

2.6.15 $\frac{m}{m_\odot} = \mu = \frac{r^3}{T^2}$

2.6.16 $\omega^2 = \frac{3\gamma m}{l^3}$

2.6.17 $v_1 \approx 7.9\ \frac{km}{s}$, $v_2 \approx 1.7\ \frac{km}{s}$, $T_1 = 84\ min$, $T_2 \approx 105\ min$.

2.6.18 $p = 2m_0\sqrt{\frac{\gamma m}{R}}\sin\frac{\alpha}{2}$

2.6.19 $F = \frac{3mv^2}{(4R)}$

2.6.20 $\Delta N \approx 9\cdot 10^3\ H$

2.6.21 a. $R \approx 42\cdot 10^3\ km$. b. The "figure of eight", "touching" the 60th parallels with the point self-intersection at the equator.

2.6.22 $\Delta U = \gamma\frac{Mmh}{R(R+h)}$; $\frac{mgh-\Delta U}{\Delta U} = \frac{h}{R}$

2.6.23 $v = 4.6\ \frac{km}{s}$

2.6.24 $u = \sqrt{v^2 - v_0^2}$

2.6.25 10 times less

2.6.26 $v_1 \approx 11.2\ \frac{km}{s}$, $v_2 \approx 2.4\ \frac{km}{s}$

2.6.27 $\Delta v = (\sqrt{2}-1)v$

2.6.28 $h = 59\ km$.

2.6.29 $\rho_{max} = \frac{3H^2}{(8\pi\gamma)}$.

2.6.30 $v = \sqrt{\frac{3\gamma m}{R}}$.

2.6.31 $v \approx 42\ \frac{km}{s}$.

2.6.32 $v \approx 16.7\ \frac{km}{s}$.

2.6.33 $v_{min} = 29\ \frac{km}{s}$.

2.6.34 $U = -2K$

2.6.35

2.6.36 $S = (\frac{1}{2})vrt\sin\alpha$

2.6.37 $\frac{\omega_p}{\omega_a} \approx 45$

2.6.38 $\rho = R\sqrt{1 + \frac{2\gamma M}{(Rv^2)}}$.

2.6.39 $V = v(\frac{2\gamma M}{rv^2 - 1})$, $R = \frac{r}{\frac{2\gamma M}{rv^2} - 1}$,
M is the mass of the Earth.

2.6.40 $E = \frac{\gamma Mm}{(r_a + r_p)}$

2.6.41 $\frac{R_1}{R_2} = \frac{2u^2}{v^2}$

2.6.42 $dv = \frac{\gamma M d\varphi}{(v_p r_p)}$. The vector dv is directed to the center of the planet.

2.6.43 The velocity momentum (vector product of the velocity on the radius-vector drawn from the center of the orbit) of the probe is the same as the station's velocity momentum.
When the probe and station rotate by the same angle, the velocity vectors will also change in the same way. From the constancy of the probe's velocity momentum:
$up = (v - V\sin\alpha)r$ it follows that
$r = \frac{p}{(1-\varepsilon\sin\alpha)}$, where $\varepsilon = \frac{V}{u}$.
At $\varepsilon < 1$ the probe trajectory is an ellipse,
at $\varepsilon = 1$ it is a parabola,
at $\varepsilon > 1$ it is a hyperbola

2.6.44 When $V < u$, $r_p = \frac{pu}{u+V}, r_a = \frac{pu}{u-V}$
$\alpha_{pr} = \arcsin\frac{u}{V}$

2.6.45 This velocity is parallel to the major axis and perpendicular to vector V, so $V_0 = \sqrt{u^2 - V^2}$
Since $a = \frac{1}{2}(r_a + r_p) = \frac{pu^2}{u^2 - V^2}$,
then $a = \frac{\gamma M}{V_0^2}$ (From the equation $\frac{u^2}{p} = \frac{\gamma M}{p^2}$ for a circular orbit, it follows that $pu^2 = \gamma M$)
Finally, $V_0 = \sqrt{\frac{\gamma M}{a}}$

2.6.46 Speed of "sweeping" the area $\frac{dS}{dt} = \frac{1}{2}bV_0 = \frac{1}{2}b\sqrt{\frac{\gamma M}{a}}$. (See the solution of the problem 2.6.45.) The orbital period of the satellite $T = \frac{2\pi ab}{(bV_0)} = \frac{2\pi a^{\frac{3}{2}}}{\sqrt{\gamma M}}$.
One can solve this problem without referring to the solution of Problem 2.6.45. The radius of curvature of the orbit at the apex of the major axis of the ellipse $R = \frac{a}{k^2} = \frac{b^2}{a}$. Therefore $\frac{v^2}{R} = \frac{v^2 a}{b^2} = \gamma\frac{M}{r^2} \rightarrow vr = \sqrt{\gamma\frac{Mb^2}{a}}, \frac{dS}{dt} = \frac{1}{2}vr = \frac{1}{2}b\sqrt{\gamma\frac{M}{a}}$.
Satellite orbital period $T = 2\pi\frac{ab}{\frac{dS}{dt}} = \frac{2\pi a^{\frac{3}{2}}}{\sqrt{\gamma M}}$..

2.6.47 In 1910.

2.6.48 $t = \pi\sqrt{\frac{R}{g}}[\frac{(\frac{1+R_c}{R})}{2}]^{\frac{3}{2}}$.

2.6.49 $t \approx 65$ days.

2.6.50 $\Delta v \approx 70\ \frac{m}{s}$.

2.6.51 $F = \frac{\gamma Mm(R_2^3 - R_1^3)}{(R_1+R_2)R_1^2R_2^2}$

2.6.52 $N = \frac{\gamma m^2}{4r^2} - \frac{\gamma mM(3R^2r + r^3)}{R(R^2-r^2)^2}; R = {}^3\sqrt{12}R_0$.

2.6.53 $\sigma \approx 1,8 \cdot 10^{12}\ Pa$

2.7 Rotation of a solid

2.7.1 $\frac{K_2}{K_1} = 32$.

2.7.2 $K = \frac{mR^2\omega^2}{2}$. The disk has less energy.

2.7.3 $M = \frac{mR^2\omega}{t}; M = \frac{mR^2\omega^2}{(4\pi N)}$.

2.7.4 $t = \frac{\omega R}{(\mu g)}, n = \frac{\omega 2R}{(4\pi\mu g)}$.

2.7.5 $J = m_1r_1^2 + m_2r_2^2$.

2.7.6 $n = \frac{\omega 2R(1+\mu^2)}{[4\pi g\mu(1+\mu)]}$.

2.7.7 $n = \frac{\omega 2R(1+\mu^2)}{[8\pi g\mu(1+\mu)]}$.

2.7.8

2.7.9 $w = \frac{|m_1 - m_2|gR}{(J + m_1R^2 + m_2R^2)}$

2.7.10 $P_1 = \frac{mg}{2} - \frac{Jw}{l}; P_2 = \frac{mg}{2} + \frac{Jw}{l}$.

2.7.11 $a = \frac{F}{(m_1+m_2)}; w = \frac{F}{(m_2R)}$

2.7.12 $a = (\frac{1}{2})g\sin\alpha. F_{fr} = (\frac{1}{2})mg\sin\alpha$.

2.7.13 $T = (\frac{1}{7})mg\sin\alpha$

2.7.14 $v = \sqrt{gl(sin\alpha - 2\mu\cos\alpha)}$.

2.7.15 $a = \frac{2m_2g}{(2m_2+m_1)}$

2.7.16 $J = mr^2[\frac{gt^2}{(2h)} - 1]$

2.7.17 $a_1 = g\frac{(m_1R_1-m_2R_2)R_1}{J+mR_1^2+m_2R_2^2}$, $a_2 = -g\frac{(m_1R_1-m_2R_2)R_2}{J+m_1R_1^2+m_2R_2^2}$; $T_1 = m_1g\frac{J+m_2R_2(R_2+R_1)}{J+m_1R_1^2+m_2R_2^2}$ $T_2 = m_2g\frac{J+m_1R_1(R_2+R_1)}{J+m_1R_1^2+m_2R_2^2}$.

2.7.18 $a = \frac{g}{1+\frac{J}{mr^2}}; T = \frac{1}{2}\frac{mg}{1+\frac{mr^2}{J}}$.

2.7.19 $a = \frac{g}{2}$

2.7.20 $cos\alpha > \frac{r}{R}$.

2.7.21 See Fig. $t = \frac{\omega_0 R}{(2\mu g)}$. $\frac{Q}{E} = \frac{1}{2}$.

2.7.22 $t = v_0(3\mu g)$. $\frac{Q}{E} = \frac{1}{3}$.

2.7.23 $t = \frac{v}{(\mu g)}$.

2.7.24 $\omega > \frac{3v}{R}$.

2.7.25 $\omega_1 = \omega_3 = \frac{\omega}{3}; \omega_2 = -\frac{\omega}{3}$

2.7.26 $\alpha = 60°$. Less.

2.7.27 $N = \frac{4m_1m_2g}{(m_1+m_2)}$

2.7.28 $N = \frac{mgl^2}{(l^2+3a^2)}$.

2.7.29 $cos\alpha = \frac{2g(m_1-m_2)}{\omega^2 l(m_1+m_2)}$ at $|\frac{2g(m_1-m_2)}{\omega^2 l(m_1+m_2)}| < 1$; otherwise $\alpha = 0$ or π.

2.7.30 $\omega = \frac{J_1\omega_1+J_2\omega_2}{J_1+J_2}$. $Q = \frac{J_1J_2(\omega_2-\omega_1)^2}{2(J_1+J_2)}$

2.7.31 $\omega = \frac{v}{(2R)}$.

2.7.32 $\omega_0' = \frac{(3\omega_1-\omega_2)}{4}; \omega_2' = \frac{(3\omega_2-\omega_1)}{4}$.

2.7.33 $u \approx \frac{m_2v}{m_1}; \omega = \frac{2m_2vh}{(m_1R^2)}$

2.7.34 $\omega = \frac{2m_2vr}{(m_1R^2+2m_2r^2)}$.

2.7.35 $\Delta\omega = \frac{\omega mR^2}{J}$. It increases by a factor of $(1 + \frac{mR^2}{J})$.

2.7.36 $n = \frac{33}{8}h^{-1}$

2.7.37 West. Such a wind in the northern hemisphere is called a northeasterly trade wind

2.7.38 $m \approx 4 \cdot 10^{16}\ kg$.

2.7.39 a. "Humps" of tidal deformations of the Earth and tides in its oceans are delayed in relation to the passing of the zenith and the antizenith by the Moon or the Sun. b. The tide in the Earth's atmosphere produces a momentum of forces that accelerate the diurnal rotation.

2.7.40 $v = \sqrt{3gL}$

2.7.41 $Q = \frac{1}{10}mv^2$

2.7.42 $cos\alpha = 1 - \frac{3m_2^2v^2}{gl(4m_1+3m_2)(m_1+m_2)}$.

2.7.43 At a distance of $\frac{2l}{3}$ from the hand.

2.7.44 $F' = F(\frac{mRx}{J} - 1)$. At $x = \frac{J}{(mR)}F' = 0$.

2.7.45 After the first strike, the velocity of the dumbbell centers is $\frac{(v_1-v_2)}{2}$, and they rotate in opposite directions with angular velocity $\frac{(v_1+v_2)}{l}$. After a time $\frac{\pi l}{2(v_1+v_2)}$ the second stroke will occur; the rotation will stop and the dumbbells will fly with the same velocities as before the first impact.

2.7.46 $h = H(\frac{3m_2}{m_1+6m_2})^2$.

2.7.47 $M = \mu(u - \omega R)R$.

2.7.48 $N = \mu(u - \omega R)R\omega.\omega = \frac{u}{R} - \frac{M}{(\mu R^2)}$

2.8 Statics

2.8.1 $T = 98\ H$, $F = 138\ H$

2.8.2 $F = 0,98\ H$.

2.8.3 $h \approx 700\ m$

2.8.4 Neighboring strands form an angle of $120°$

2.8.5 $m_2 = \frac{m_1 \sin\alpha}{sin(\frac{l}{R}-\alpha)}$

2.8.6 $T \approx 2,6\ H$; $\alpha = arctg(3\sqrt{3})$.

2.8.7 $x = \frac{5F}{k}$

2.8.8 $l_0 = 2l_2 - l_1$

2.8.9 $T = \frac{mg}{(2tg\alpha)}; T' = \frac{mg}{(2\sin\alpha)}$.

2.8.10 $F_A = \frac{mg\sin\beta}{sin(\beta-\alpha)}; F_B = \frac{mg\sin\alpha}{sin(\beta-\alpha)}$

2.8.11 $F_A = mgtg\alpha; F_B = \frac{mg\cos 2\alpha}{cos\alpha}$

2.8.12 $\mu = tg(\frac{\alpha_{min}}{2})$.

2.8.13 $d_{max} = d_0 + 2R(1 - \frac{1}{\sqrt{1+\mu^2}})$.

2.8.14 $tg\alpha = \frac{(\mu_1-\mu_2)}{(1+\mu_1\mu_2)}$.

2.8.15 $\mu = \frac{1}{\sqrt{3}}$

2.8.16 $f_n = F(\frac{f}{F})^n$

2.8.17 $F = F_0e^{-\mu\theta}$.

2.8.18 $a.F_1 = F_2 = 98\ H; b.F_1 = 24,5\ H$, $F_2 = 171,5\ H$.

2.8.19 $m \leq 7,5\ g$.

2.8.20 $m = \sqrt{m_1 m_2}$

2.8.21 $\Delta m = (\frac{h}{L}) m_0 tg\alpha$

2.8.22 $\Delta m_{\pm} = \frac{\mu(M+2m)r}{(L\mp\mu r)}$, "excess" and "deficiency" are possible.

2.8.23 $\alpha = arctg(\frac{1}{3})$.

2.8.24 $T = \frac{mgL}{2h}; P = mg\sqrt{1+(\frac{L}{2h})^2}$

2.8.25 $T_n = \frac{(2n-1)mg}{sqrt3}$.

2.8.26 $P = (\frac{1}{4}) mgctg\alpha$

2.8.27 $\mu \geq \frac{1}{3}$

2.8.28 $l < L < l\sqrt{1+\mu^2}$

2.8.29 $\alpha \leq arctg 2\mu$.

2.8.30 $\alpha > \frac{\pi}{3}$.

2.8.31 $cos\varphi = \frac{ctg\alpha}{\sqrt{3}}$, consider $\mu > tg\alpha$.

2.8.32 $tg\alpha \leq \frac{1}{\mu}$

2.8.33 $F = \frac{mg}{2}, \alpha = 0$ at $\mu \geq \frac{1}{2}; F = \frac{mg}{2\mu}\sqrt{5\mu^2 - 4\mu + 1}, tg\alpha = \frac{1-2\mu}{\mu}$ at $\mu = \frac{1}{2}$.

2.8.34 $tg\alpha \geq \frac{(1-\mu_1\mu_2)}{(2\mu_1)}$.

2.8.35 $F' = \frac{F(l+\mu h)}{(l-\mu h)}$

2.8.36 $sin\alpha = \frac{\mu R}{(l+R)\sqrt{1+\mu^2}}$

2.8.37 $\omega = \frac{vh}{R^2}$.

2.8.38 Reasonable

2.8.39 $F_{ax} = mg; F_{pr} = \frac{mg}{4}$, one spring is compressed, the other is stretched.

2.8.40 $m = \frac{Mr}{(R-r)}$.

2.8.41 $T = 3mg$

2.8.42 $\Delta S = (\frac{Nl}{\mu})\alpha(t_2 - t_1) tg\varphi$

2.8.43

2.8.44 $F = \mu mg(\sqrt{2} - 1)$

3 OSCILLATIONS AND WAVES

3.1 Small deviations from equilibrium

3.1.1 $F' = -2\cos\alpha = -2F\frac{x}{\sqrt{l^2+x^2}} = [x \ll l] = \boxed{-2F\frac{x}{l}}$

$dU = 2F\cos\alpha dx = 2F\sqrt{\frac{x}{l^2+x^2}} = 2F\frac{x}{l}dx$

$\int_0^U dU = \int_0^x 2F\frac{x}{l}dx = \frac{2F}{l}\int_0^x dx = \boxed{\frac{Fx^2}{l}}$

$\frac{Fx_0^2}{l} = \frac{mv^2}{2} \Rightarrow v = \boxed{x_0\sqrt{\frac{2F}{ml}}}$

3.1.2 $F = -kx$ - spring force

$dU = Fdx = kxdx$

$\int_0^U dU = \int_0^x kxdx = k\int_0^x xdx = \boxed{\frac{kx^2}{2}}$

3.1.3 a) $v_0 = x_0\omega = x_0\sqrt{\frac{k}{m}} \Rightarrow v_0^2 = \frac{kx_0^2}{m} \Rightarrow k = \boxed{\frac{v_0^2 m}{x_0^2}}$

b) $F = -kx$ is the elastic force of the spring (obviously), since in any case in a normal spring pendulum the return force is equal to the elastic force of the spring. Then: $U = \boxed{\frac{kx^2}{2}}$

3.1.4

3.1.5 $OX: F = -mg\sin\varphi = \boxed{-mg\frac{x}{l}}$

$U = \int_0^U dU = \int_0^x mg\frac{x}{l}dx = \boxed{\frac{mgx^2}{2l}}$

3.1.6 $v_{max} = A\omega = \boxed{x_0\sqrt{\frac{g}{l}}}$

3.1.7 $E_1 = E_2$

$\frac{mgx_1^2}{2r} = \frac{mgx_2^2}{2R} \Rightarrow \frac{x_1^2}{x_2^2} = \frac{R}{r} \Rightarrow \frac{x_1}{x_2} = \sqrt{\frac{R}{r}}$

3.1.8 $ma_\tau = -qE\sin\varphi = [\varphi - \text{small}] \approx -qE\varphi = -qE\frac{2x}{l}$

$m\ddot{x}(t) + \frac{2qE}{l}x(t) = 0$

$\ddot{x}(t) + \frac{2qE}{ml}x(t) = 0$ - we got the harmonic motion equation.

$\omega = \sqrt{\frac{2qE}{ml}}$

$v_0 = x_0\omega = x_0\sqrt{\frac{2qE}{ml}} \Rightarrow m = \boxed{\frac{2qEx_0^2}{lv_0^2}}$

3.1.9 $dU = Fdx$

$F = mg\sin\varphi = mg\frac{x}{R-r}$

$U = \int_0^U dU = \int_0^x mg\frac{x}{R-r} = \frac{mg}{R-r}$

$\int_0^x xdx = \boxed{\frac{mg}{R-r}\cdot\frac{x^2}{2}}$

3.1.10 $U_1 = \frac{2kQq}{L}$ - initial energy of the system.

$U_2 = \frac{kQq}{L-x} + \frac{kQq}{L+x}$ - system energy after bead displacement

$\Delta U = U_2 - U_1 = kQq(\frac{1}{L-x} + \frac{1}{L+x} - \frac{2}{L}) \approx \boxed{\frac{Qqx^2}{2\pi\varepsilon_0 L^3}}$

$E_k = \Delta U$

$\frac{mv^2}{2} = \frac{Qq\Delta x^2}{2\pi\varepsilon_0 L^3} \Rightarrow \Delta x = \sqrt{\frac{mv^2\pi\varepsilon_0 L^3}{Qq}} = \boxed{v\sqrt{\frac{m\pi\varepsilon_0 L^3}{Qq}}}$

3.1.11 $F_y = k\Delta l = kx\cos\alpha$

$mg = 2F_y\cos\alpha = 2kxcos^2\alpha$

$m = \frac{2kx\cos^2\alpha}{g}$

3.1.12 $a.F = -\frac{2mg}{R}x.\ b.R' = R\sqrt{3}.F' = -\frac{6mg}{R'}x$

3.1.13 $v = \frac{\Delta m}{m}\sqrt{\frac{gR}{2}}$.

3.1.14 $\Omega = \varphi_0\frac{l}{L}\sqrt{\frac{g}{h}}$.

3.1.15 $\frac{mg(x_0+y_0)^2}{2l} = \frac{mv^2}{2} + \frac{MU^2}{2}$

$mv = MU$

Solve the upper equation with respect to v:

$v = \sqrt{\frac{Mg}{(M+m)l}(x_0+y_0)^2} = \sqrt{\frac{x_0 g}{(x_0+y_0)l}(x_0+y_0)^2} = x_0\sqrt{\frac{g}{l}(1+\frac{y_0}{x_0})}$

Solve the upper equation with respect to U:

$U = \sqrt{\frac{mg}{(M+m)l}(x_0+y_0)^2} = \sqrt{\frac{y_0 g}{(x_0+y_0)l}(x_0+y_0)^2} = y_0\sqrt{\frac{g}{l}(1+\frac{x_0}{y_0})}$

3.1.16 $T_{min} = mg\cos\varphi \approx mg(1-\frac{\varphi^2}{2}) = \boxed{mg(1-\frac{x_0^2}{2l^2})}$

$ma = T_{max} - mg \Rightarrow T_{max} = m(\frac{v^2}{l} + g)$

$mgl(1-\cos\varphi) = \frac{mv^2}{2}$ - Law of Conservation of Energy

$v^2 = 2gl(1-\cos\varphi) \approx gl\varphi^2 = \frac{gx_0^2}{l}$

Let us substitute this expression for the velocity into the expression for T_{max}:

$\boxed{T_{max} = mg(1+\frac{x_0^2}{l^2})}$

3.1.17 $N = m\cos\varphi \approx mg(1-\frac{\varphi^2}{2}) = mg(1-\frac{A^2}{2R^2})$ (1)

$N + \Delta = m(a+g) = m(\frac{v^2}{R} + g)$

$mgR(1-\cos\varphi) \approx mgR\frac{\varphi^2}{2} = \frac{mv^2}{2}$ - Law of Conservation of Energy

$N + \Delta = m(\frac{gR\varphi^2}{R} + g) = mg(1+\varphi^2) = mg(1+\frac{A^2}{R^2})$ (2)

Then we solve the system of equations (1) and (2) and obtain that:

$\boxed{A = R\sqrt{\frac{2\Delta}{3N+\Delta}}}$

3.2 Period and frequency of free oscillations

3.2.1 a) The equilibrium position is at the level of the centre of the wheel.

$F = -kx = [\Omega^2 = \frac{k}{m}] = m\Omega^2 x$

The values of the velocity and displacement of the load are repeated after time $t = T = \frac{2\pi}{\Omega}$

The velocity vector will only change its direction, and the displacement will change sign.

b) $\Omega = \sqrt{\frac{k}{m}}$

$R = x_0$, since point A is always at the same level as the weight.

3.2.2 $T = 2\pi\sqrt{\frac{m}{k}}$

$F = -k\Delta l = mg \Rightarrow k = \frac{mg}{\Delta l}$

$T = 2\pi\sqrt{\frac{m\Delta l}{mg}} = 2\pi\sqrt{\frac{\Delta l}{g}}$

3.2.3 $T_1 = 2\pi\sqrt{\frac{m}{k}}, T_2 = 2\pi\sqrt{\frac{m}{k'}}$

$k' = 2k + 2k$ - since the spring is split in two and the length of each half is $\frac{l}{2}2$

$T_2 = 2\pi\sqrt{\frac{m}{4k}} = \pi\sqrt{\frac{m}{k}}$

$\frac{T_1}{T_2} = 2$

3.2.4 a) $k' = k_1 + k_2 \Rightarrow T_1 = 2\pi\sqrt{\frac{m}{k_1+k_2}}$

b) $k' = \frac{k_1k_2}{k_1+k_2} \Rightarrow T_2 = 2\pi\sqrt{\frac{m(k_1+k_2)}{k_1k_2}}$

c) $k' = k_1 + k_2 \Rightarrow T_3 = 2\pi\sqrt{\frac{m}{k_1+k_2}}$ (does not depend on the distance between the walls)

3.2.5 $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow l = \frac{T^2 g}{4\pi^2} = 24.8$ cm

3.2.6 $T = 2\pi\sqrt{\frac{l}{g^*}} = [g^* = g\sin\alpha] = 2\pi\sqrt{\frac{l}{gsin\alpha}}$

3.2.7 a) $T_0 = 2\pi\sqrt{\frac{l}{g}}, T = \frac{2\pi}{w}$

$m\ddot{x} = -(mg+F)\sin\varphi = -(mg+F)\frac{x}{l}$

$w = \sqrt{\frac{mg+F}{ml}}$

$\frac{T^2}{T_0^2} = \frac{mg}{mg+F} \Rightarrow \boxed{F = \frac{mg(T_0^2-T^2)}{T^2}}$

b) $m\ddot{x} = -(\sqrt{(mg)^2+F^2})\sin\varphi$

$m\ddot{x} = -(\sqrt{(mg)^2+F^2})\frac{x}{l}$

$w = \sqrt{\frac{\sqrt{(mg)^2+F^2}}{ml}}$

$\frac{T_0^4}{T^4} = \frac{(mg)^2+F^2}{(mg)^2} \Rightarrow \boxed{F = mg\sqrt{\frac{T_0^4-T^4}{T^4}}}$

3.2.8 Using the law of gravity, let us write down the equations for determining the free-fall accelerations, taking into account that the acceleration above the field will be greater than far away from the field

$g_0 = \frac{4}{3}\pi RG\rho_0 \approx 4\pi RG\rho_0$

$g = g_0 + g_k \approx g_0 + 4\pi RG(\rho - \rho_0)$

Let's write down the ratio of the periods of oscillation of the pendulum given by the problem condition

$\alpha = \frac{T-T_0}{T_0} = 0,1; \xi = \frac{T}{T_0} = 1 + 10^{-3}$

Let's express the ratio of periods through the values of free-fall acceleration

$\xi^2 = \frac{g_0+g}{g_0} = 1 + \frac{r(\rho-\rho_0)}{R\rho_0}$

$r = \frac{(\xi^2-1)R\rho_0}{\rho-\rho_0} \approx 30$ km

3.2.9 $T_0 = 2\pi\sqrt{\frac{l}{g}}, T_1 = 2\pi\sqrt{\frac{l}{g^*}}$

$g = \frac{GM}{R^2}, g^* = \frac{GM}{(R+H)^2}$

$\frac{g}{g^*} = \frac{(R+H)^2}{R^2} \Rightarrow g^* = \frac{gR^2}{(R+H)^2}$

$T_1 = 2\pi\frac{R+H}{R}\sqrt{\frac{l}{g}}$

$\Delta T_1 = T_1 - T_0 = T_0(\frac{R+H}{R} - 1) = 2$ min

$\Delta T_2 = T_2 - T_0 = T_0(\frac{R+h}{R} - 1) = 6.75$ s

3.2.10 $F = 2Tsin\varphi$

$m\ddot{x}(t) + 2T\sin\varphi = 0$ $(\sin\varphi \approx \varphi = \frac{x}{l})$

$\ddot{x}(t) + \frac{2T}{ml}x(t) = 0$

$\omega^{*2} = \frac{2T}{ml} \Rightarrow T = \frac{ml\omega^{*2}}{2}$

3.2.11 $m\ddot{x}(t) - F = 0$

$F = kqQ(\frac{1}{(L-x)^2} - \frac{1}{(L+x)^2})$

$F = -\frac{4kqQLx}{(L^2-x^2)^2} \approx -\frac{4kqQx}{L^3}$

$\ddot{x}(t) + \frac{4kqQx}{mL^3}x(t)$

$\omega = \sqrt{\frac{4kqQ}{mL^3}} = \sqrt{\frac{qQ}{m\pi\varepsilon_0 L^3}}$

3.2.12 $F = mg\frac{x}{R}$

$m\ddot{x}(t) + \frac{mg}{R}x(t) = 0$

$\ddot{x}(t) + \frac{g}{R}x(t) = 0$

$\omega = \sqrt{\frac{g}{R}} \Rightarrow T = 2\pi\sqrt{\frac{R}{g}}$

$t = \frac{T}{2} = \pi\sqrt{\frac{R}{g}} = 42$ min

3.2.13 $m\ddot{x}(t) + F = 0$

$F = mgcos\varphi = mg\frac{x}{R} \Rightarrow \omega = \sqrt{\frac{g}{R}} \Rightarrow T = 2\pi\sqrt{\frac{R}{g}}$

$t = \frac{T}{2} = \pi\sqrt{\frac{R}{g}} \approx 42$ min

3.2.14 $ma = F_{fr1} - F_{fr2} + F - F = F_{fr1} - F_{fr2}$

$\begin{cases} F_{fr1} = \frac{\mu mg(l+x)}{l} \\ F_{fr2} = \frac{\mu mg(l-x)}{l} \end{cases}$

$ma = \frac{\mu mg(l+x)}{l} - \frac{\mu mg(l-x)}{l} = \frac{\mu mg(l+x-l+x)}{l} = \frac{2\mu mgx}{l}$

$\ddot{x}(t) + \frac{2\mu g}{l}x(t) = 0$

$\omega = \sqrt{\frac{2\mu g}{l}}$

3.2.15 $t = 22s$

3.2.16 $T = 2\pi\sqrt{\frac{l}{g^*}}$

$(g^*)^2 = a^2 + g^2 - 2ag\cos\alpha$

$g^* = \sqrt{a^2 + g^2 - 2ag\cos\alpha}$ - Cosine Theorem

$T = 2\pi\sqrt{\frac{l}{\sqrt{a^2+g^2-2ag\cos\alpha}}}$

3.2.17 The weight of a pendulum in space is zero.

$m\ddot{x}(t) + m\Omega^2(R+l)sin\varphi = 0$

$\ddot{x}(t) + \Omega^2(R+l)\varphi = 0$

$\ddot{x}(t) + \frac{\Omega^2(R+l)}{l}x(t) = 0$

$T = \frac{2\pi}{\Omega}\sqrt{\frac{l}{R+l}}$

3.2.18 $m\ddot{x}(t) + kx - m\Omega^2 x = 0$

$\ddot{x}(t) + (\frac{k}{m} - \Omega^2)x(t) = 0$

$\omega = \sqrt{\frac{k}{m} - \Omega^2}$

3.2.19 $I\varepsilon = \sum M = mgxsin\varphi - Mglsin\varphi \approx \varphi(mgx - Mgl)$

$I = Ml^2 + mx^2$

$(Ml^2 + mx^2)\ddot{\varphi} + \varphi(Mgl - mgx) = 0$

$\ddot{\varphi} + \frac{Mgl-mgx}{Ml^2+mx^2}\varphi = 0$

$\omega = \sqrt{\frac{g(Ml-mx)}{Ml^2+mx^2}}$

3.2.20 $\omega = \sqrt{\frac{\alpha}{\beta}}$

$E_k = \frac{mv^2}{2} \Rightarrow \beta = \frac{m}{2}$

$E_p = mgl(1 - cos\varphi) + \frac{k(x/2)^2}{2} = \frac{mgl\varphi^2}{2} + \frac{kx^2}{8} = \frac{mgl}{2}\frac{x^2}{l^2} + \frac{kx^2}{8} = x^2(\frac{mg}{2l} + \frac{k}{8}) \Rightarrow \alpha = \frac{mg}{2l} + \frac{k}{8}$

$\omega^2 = \frac{g}{l} + \frac{k}{4m}$

3.2.21 $I\varepsilon = \sum M$

$I = MR^2 + mR^2 = R^2(M+m)$

$R^2(M+m)\ddot{\varphi} + mgRsin\varphi = 0$

$\ddot{\varphi} + \frac{mg}{R(M+m)}\varphi = 0$

$\omega^2 = \frac{mg}{R(M+m)} \Rightarrow M = m(\frac{g}{\omega^2 R} - 1)$

3.2.22 a) $2m\ddot{x}(t) + 2mgsin\varphi = 0$

$\ddot{x}(t) + \frac{g}{\sqrt{R^2-l^2}}x(t) = 0$

$\omega^2 = \boxed{\frac{g}{\sqrt{R^2-l^2}}}$

b) $\omega^2 = \frac{\alpha}{\beta}$

$E_p = 2mgR'(1-cos\varphi) = 2mg\sqrt{R^2-l^2}(1-\cos\varphi)$

$E_p \approx \frac{2mg\sqrt{R^2-l^2}}{2}\varphi^2$

$\alpha = \frac{2mg\sqrt{R^2-l^2}}{2}$

$E_k = \frac{2mv^2}{2} = \frac{2m\omega^2 R^2}{2} = \frac{2mR^2}{2}\dot{\varphi}^2$

$\beta = \frac{2mR^2}{2}$

$\omega^2 = \boxed{\frac{g\sqrt{R^2-l^2}}{R^2}}$

3.2.23 $\omega = \sqrt{\frac{\alpha}{\beta}}$

$E_p = \frac{kx^2}{2} \Rightarrow \alpha = \frac{k}{2}$

$E_k = \frac{mv^2}{2} + \frac{I\omega^2}{2} = \frac{2mv^2}{2} \Rightarrow \beta = \frac{2m}{2}$

$\omega = \boxed{\sqrt{\frac{k}{2m}}}$

3.2.24 $I = mR^2 + md^2$, where d is the distance from the past axis to the present axis:

$I = mR^2 + mR^2 = 2mR^2$

$L = \frac{2mR^2}{mR} = 2R$

$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{2R}{g}}$

$\omega = \sqrt{\frac{g}{2R}}$

3.2.25 $m_1x_c = m_2(l-x_c) \Rightarrow x_c = \frac{m_2l}{m_1+m_2}$

$k_1x_c = k_2(l-x_c)$

$\frac{k_1m_2l}{m_1+m_2} = k_2(l - \frac{m_2l}{m_1+m_2}) = \frac{k_2m_1l}{m_1+m_2}$

$\frac{k_1}{k_2} = \frac{m_1}{m_2}$

$k = \frac{k_1k_2}{k_1+k_2} = \frac{k_1\frac{m_2}{m_1}}{k_1+k_1\frac{m_2}{m_1}} = \frac{k_1m_2}{m_1+m_2}$

$\omega = \sqrt{\frac{k_1}{m_1}} = \sqrt{\frac{k_2}{m_2}} = \sqrt{\frac{k(m_1+m_2)}{m_1m_2}}$

3.2.26 $\frac{\omega_{HD}}{\omega_{H_2}} = \frac{\sqrt{3}}{2}$

3.2.27 Let us denote the stiffness of the springs of the molecule model k, the mass of the balls - oxygen atoms M and the mass of the ball - carbon atom m ($m/M = 12/16$).

Making oscillations of type a), both oxygen atoms oscillate synchronously relative to the fixed carbon atom. This is due to the fact that by virtue of the symmetry of oscillations of oxygen atoms on the carbon atom at any moment act on both sides equal in absolute value and oppositely directed forces that "balance" each other. Therefore, in case (a) oxygen atoms make free oscillations, the period of which is equal to

$T_a = 2\pi\sqrt{\frac{M}{k}}$

At oscillations of type b), equal in absolute value forces act on the carbon atom, and they are directed in the same direction. If the ball - carbon atom is divided into two equal parts, it is clear that they will oscillate as one whole: equal forces always act on them and, consequently, the ball-halves and any moment will have the same acceleration, velocity and coordinates. The frequency of oscillation of the molecule CO_2, is equal to the frequency of oscillation of a system consisting of an oxygen atom and half a carbon atom. Thus, the problem is reduced to the determination of the period of oscillation of balls of masses M and $m/2$ connected by a spring. Such balls oscillate about the stationary centre of mass of the system. If the length of the spring in the unstretched state is l, then the centre of mass of the system is at a distance $l\frac{m}{m+2M}$ from the ball of mass M. Therefore, we can consider that the ball of mass M (oxygen atom) oscillates relative to the centre of mass on a spring of length

$l_1 = l\frac{m}{m+2M}$.

The stiffness of a part of the spring is greater than the stiffness of the whole spring. Since the stiffness is inversely proportional to the ratio of the length of this part to the length of the whole spring, the stiffness of the part of the spring is

$k_1 = k\frac{l}{l_1} = k\frac{m+2M}{m}$

The period of oscillation of a ball of mass M on a spring of stiffness k_1 is equal to

$T_b = 2\pi\sqrt{\frac{M}{k_1}} = 2\pi\sqrt{\frac{mM}{k(m_2M)}}$, $\frac{T_b}{T_a} = \sqrt{\frac{m+2M}{m}} = \sqrt{1+\frac{2M}{m}}$.

Since $\frac{M}{m} = \frac{16}{12} = \frac{4}{3}$, then:

$\frac{T_b}{T_a} = \sqrt{1+\frac{8}{3}} = \sqrt{\frac{11}{3}}$.

Hence, the ratio of frequencies:

$\frac{\nu_b}{\nu_a} = \sqrt{\frac{3}{11}}$.

3.2.28 $T = 2\pi\sqrt{\frac{\beta}{\alpha}}$

$E_k = \frac{mv^2}{2} + \frac{MU^2}{2}$

$mv = MU \Rightarrow U = \frac{mv}{M}$

$E_k = \frac{mv^2}{2} + \frac{m^2v^2}{2M} = \frac{m}{2}(1+\frac{m}{M})v^2$

$\beta = \boxed{\frac{m}{2}(1+\frac{m}{M})}$

$E_p = mgl(1-cos\varphi) \approx mgl\frac{\varphi^2}{2} = \frac{mg}{2l}x^2$

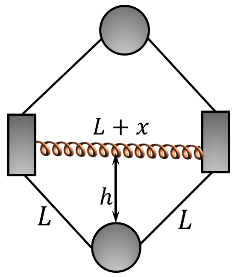

Figure 1: For the 3.2.31

$\alpha = \boxed{\frac{mg}{2l}}$

$T = 2\pi\sqrt{\frac{l(m+M)}{Mg}}$

3.2.29 This system can be considered as oscillations of four pendulums on springs of stiffness $2k$ each. The stiffness of the springs increases as the balls oscillate relative to the centres of mass of each system consisting of two balls and a spring (the centre of mass of this system is the centre of the spring).

$T = 2\pi\sqrt{\frac{m}{k^*}} = 2\pi\sqrt{\frac{m}{2k}}$

a) $t_1 = \frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{m}{2k}}$

b) $t_2 = \frac{3}{4}T = \frac{3}{2}\pi\sqrt{\frac{m}{2k}}$

3.2.30 When the thread is twisted, the force is proportional to the angle of twist

$F = -k\varphi$

$I_0\ddot{\varphi} = -k\varphi d$

$T_0 = 2\pi\sqrt{\frac{T_0}{kd}}$

Likewise

$(I_0 + I_1)\ddot{\varphi} = -k\varphi d$

$\frac{T^2}{T_0^2} = \frac{I+I_0}{I_0}$

$I = I_0(\frac{T^2}{T_0^2} - 1)$

3.2.31 Let spring is stretched to the x.

$h = \sqrt{L^2 - \frac{(L+x)^2}{4}} = L\sqrt{\frac{3}{4} + \frac{1}{2}\frac{x}{L}}$

$h =\approx L\frac{\sqrt{3}}{2} - \frac{1}{2\sqrt{3}}x$

$E_p = \frac{kx^2}{2}$, $\ddot{E}_p = k$

$h' = -\frac{1}{2\sqrt{3}}\dot{x}$

$E_k = \frac{2mh^2}{2} = \frac{1}{2}\dot{x}^2(2m\frac{1}{12})$

$\alpha_0 = \frac{m}{6}$

$\omega_0 = \sqrt{\frac{6k}{m}}$

3.2.32 $\omega_1 = \sqrt{\frac{Mg}{(ml)}}, \omega_2 = 2\omega_1 = 2\sqrt{\frac{Mg}{(ml)}}$.

3.2.33 $T = 2\pi\sqrt{\frac{\beta}{\alpha}}$

$E_k = \frac{mv^2}{2} \Rightarrow \beta = \frac{m}{2}$

$E_p = m'gx = m\frac{x}{l}gx = \frac{mg}{l}x^2 \Rightarrow \alpha = \frac{mg}{l}$

$T = 2\pi\sqrt{\frac{l}{2g}}$

3.2.34 $T = 2\pi\sqrt{\frac{\beta}{\alpha}}$

$E_k = \frac{m(\dot{x})^2}{2} + \frac{M(\dot{x}')^2}{2}$

$S_1x = S_2x' \Rightarrow x' = \frac{S_1x}{S_2}$

$E_k = \frac{\rho HS_1(S_1+S_2)}{2S_2}(\dot{x}^2)$

$\beta = \boxed{\frac{\rho HS_1(S_1+S_2)}{2S_2}}$

$E_p = mg\frac{x}{2} + MG\frac{x'}{2} = \frac{\rho gS_1x^2}{2} + \frac{S_1}{S_2}\rho gS_2\frac{x^2}{2} = \frac{\rho gS_1(S_1+S_2)}{2S_2}x^2$

$\alpha = \boxed{\frac{\rho gS_1(S_1+S_2)}{2S_2}}$

$T = 2\pi\sqrt{\frac{H}{g}}$

3.2.35 $\omega = \sqrt{\frac{g}{H}}$

3.2.36 $\omega = \sqrt{\frac{k}{[m+\frac{\pi\rho lR^4}{r^2}]}}$.

3.2.37 $T_0 = 2\pi\sqrt{\frac{H_1}{g}}$

$T = 2\pi\sqrt{\frac{H_2}{g}} = 2\pi\sqrt{\frac{H_1+\Delta H}{g}}$

$M = \rho_w H_1 S \Rightarrow H_1 = \frac{M}{\rho_w S}$

$M + m = \rho_w H_2 S \Rightarrow H_2 = \frac{M+m}{\rho_w S}$

$M = \frac{T_0^2\rho_w gS}{4\pi^2}$

$M + m = \frac{T^2\rho_w gS}{4\pi^2}$

$m = \frac{T^2\rho_w gS}{4\pi^2} - \frac{T_0^2\rho_w gS}{4\pi^2} = \frac{\rho_w gS(T^2-T_0^2)}{4\pi^2} \approx 900$ T

3.3 Harmonic motion

3.3.1 The coordinate depends on time according to the law

$x(t) = A\cos\omega t$

Differentiating by time, we obtain the time dependence of velocity

$\frac{dx}{dt} = A\frac{d}{dt}\cos\omega t$

$\boxed{v = -A\omega\sin\omega t}$

Similarly, differentiate the velocity to get the acceleration

$\frac{dv}{dt} = -A\omega\frac{d}{dt}\sin\omega t$

$\boxed{a = -A\omega^2 \cos \omega t}$

According to Newton's second law, the force acting on a body is defined as

$F = ma$

$\boxed{F = -mA\omega^2 \cos \omega t = -m\omega^2 x}$

By Hooke's law

$F = -kx$

Where is the stiffness of the spring

$\boxed{k = m\omega^2}$

3.3.2 $a)x = 5sin(3,13t)$. $b)x = 5cos(3,13t)$. Displacement is measured in millimeters, time in seconds

3.3.3 $T = 0,06\ s$

3.3.4 From 3.3.1, we find that velocity depends on time according to the law

$v(t) = v_0 \sin \omega t$

Given that kinetic energy is defined by the expression

$E_k = \frac{mv^2}{2}$

The dependence of kinetic energy on time will have the following form

$E_k = E_0 \sin^2 \omega t$

The moment of time when E_k is $E_k = \frac{E_0}{2}$ is described by $Eq.$

$\sin^2 \omega t = \frac{1}{2}$

Whence

$\sin \omega t = \frac{\sqrt{2}}{2}$

This will occur at the nearest moment equal to

$t = \frac{\pi}{4\omega}$

3.3.5 The first part of the path "up to the nail", the pendulum will pass in time

$T_1 = \frac{1}{2}2\pi\sqrt{\frac{l}{g}} = \pi\sqrt{\frac{l}{g}}$ (1).

After hitting the nail, half of its length remains in place and the other half continues to move at the same speed

Thus, the length of the pendulum is halved.

$l \to \frac{l}{2}$

Making the substitution in (1)

$T_2 = \pi\sqrt{\frac{l}{2g}}$

Thus the total period of oscillation

$T = T_1 + T_2$

$$\boxed{T = (1 + 1/\sqrt{2})\pi\sqrt{\frac{l}{g}}}$$

3.3.6 Similarly to 3.3.5, the oscillation will consist of two half-oscillations When a body slides along a trough of radius R, its motion, from the point of view of kinematics, is no different from the motion of a mathematical pendulum with a long thread R Thus, the period of oscillation is

$$T_0 = 2\pi\sqrt{\frac{R}{g}}$$

I.e. half of the chute, it will pass in time

$$T_1 = \pi\sqrt{\frac{R}{g}}$$

Similarly, for a gutter of radius r

$$T_2 = \pi\sqrt{\frac{r}{g}}$$

Thus the total period of oscillation

$$T = T_1 + T_2$$

$$\boxed{T = (\sqrt{\frac{R}{g}} + \sqrt{\frac{r}{g}})}$$

3.3.7 $t = \frac{\pi}{2\sqrt{\frac{l}{g}}}$; will not change

3.3.8 $t = \pi\sqrt{\frac{m}{(2\pi R\Delta p)}}$.

3.3.9 Focused at distances $l = \pi(n + \frac{1}{2})v_0\sqrt{\frac{m}{k}}$, where n is an integer

3.3.10 The number of intersections is equal to an integer part of the value $\frac{l}{\pi v_0}\sqrt{\frac{g}{R}}$

3.3.11 $T = (\frac{4}{3})\pi\sqrt{l}g$

3.3.12 $l = Acos[\pi(1 - \frac{T}{T_0})]$.

3.3.13 $t = [\pi + 2arctg\sqrt{\frac{mg}{2k(H-h)}}]\sqrt{m}k$

3.3.14 $t = \frac{\pi}{2}\sqrt{\frac{l}{(\mu g)}}$ at $v =\leq \sqrt{\mu g l}, t = \frac{v'}{\mu g} + \sqrt{\frac{l}{(\mu g)}}arccos\frac{v'}{v}$ at $v > \sqrt{\mu g l}$, where $v' = \sqrt{v^2 - \mu g l}$.

3.3.15 $w = \frac{2R}{(\pi A)}$ at $A \gg R, w = \frac{1}{3}$ at $A = 2R$. Increase

3.3.16 $u = \frac{d}{2\pi n}\sqrt{\frac{k}{m}}$, where n is an integer.

3.3.17 $t = \frac{T}{4} + \frac{\tau}{2}$

3.3.18 $a)x = \frac{mg}{k}(cos\omega t - 1).b)x = \frac{mg}{k} + l(cos\omega t - 1)$. The x-axis is directed vertically upwards, the origin is at the initial position.

3.3.19 $v = \frac{mv}{m+M}cos\sqrt{\frac{k}{m+M}}t, x = \frac{mv}{\sqrt{k(M+m)}}\sin\sqrt{\frac{k}{m+M}}t.$

3.3.20 From the moment the ball hits the wall for the first half-period there is compression and return of the spring to an undeformed state. Then a second strike at the moment when the spring is not deformed, after which the balls start moving with constant velocity v. The period $T = 2\pi\sqrt{\frac{m}{(2k)}}$.

3.3.21 $v_1 = \frac{m_1}{m_1+m_2}v(1 + \frac{m_2}{m_1}\cos\omega t), v_2 = \frac{m_1}{m_1+m_2}v(1 - \cos\omega t)$.

3.3.22 $F_{max} = 2F; \tau = \frac{T}{2}$

3.3.23

3.3.24 $A = \sqrt{A_0^2 + \frac{F^2}{k^2} - \frac{2A_0F}{k}\cos\omega t_0}$. When $t_0 = \pi(2n+1)\omega$, where n is an integer, the amplitude is the largest; at $t = \frac{2\pi n}{\omega}$ it is the smallest.

3.3.25 $x_0 = u\sqrt{m}k$.

3.3.26 If $u \geq \mu g\sqrt{\frac{m}{k}}$ a harmonic oscillation with amplitude $A = \frac{\mu mg}{k}$, at lower u an oscillation with amplitude $A = usqrt\frac{m}{k}$.

3.3.27 $\mu = \frac{kl}{(4Mgn)}$.

3.3.28 $BC = \frac{g(M+m)}{(M\omega^2)}$.

3.3.29 $F = -m\omega^2 x = -m\omega^2 Acos(\omega t + \varphi)$, the force $m\omega^2 A$ is reached at time $t = \frac{(\pi n - \varphi)}{\omega}$, where n is an integer.

3.3.30 When $\omega^2 A > g$, the load bounces and its detachment from the surface of the diaphragm occursabove its middle position.

3.3.31 $A = \frac{F}{(m\omega^2)}$.

3.3.32 $h = A + \frac{g}{(2\omega^2)} + \frac{\omega^2 A^2}{(2g)}$ at $\omega^2 A > g$.

3.3.33 $A = (\frac{g}{\omega^2})\sqrt{\pi^2 n^2 + 1}$, where n — integer.

3.3.34 At an amplitude $A \gg 10^{-11}$ cm, the acceleration of the plate face is much greater than the acceleration $g = 0.8$ $\frac{m}{s}$ that friction can provide, so the load practically stays in place with almost no effect on frequency. At amplitude $A < 10^{-11}$ cm, the weight moves with the end and affects the frequency in a noticeable way. $v_{max} = \frac{\pi g}{(2\omega)} \approx 1.57 \cdot 10^{-6}$ $\frac{m}{s}$.

3.3.35

3.3.36 $u_{av} = \frac{\pi v_0 tg\alpha}{(2\mu)}$

3.4 Overlapping oscillations

3.4.1 There will be a superposition of horizontal and vertical harmonic oscillations with frequencies $\omega_1 = \sqrt{\frac{2k_1}{m}}$ and $\omega_2 = \sqrt{\frac{2k_2}{m}}$. At $k_1 \neq k_2$ rectilinear motion is possible only vertically and horizontally.

3.4.2 A body deflected from its equilibrium position by a distance r needs to be given a velocity $v = \omega r$, where $\omega = \sqrt{\frac{k}{m}}.T = \frac{2\pi}{\omega}$.

3.4.3 a. The trajectory is an ellipse with semi-axes A and $\frac{v}{\omega}$. The limits of the variation of the distance from $\frac{v}{\omega}$ to A.

b. The trajectory is an ellipse with semi-axes

$$\sqrt{\frac{1}{2}(A^2 + \frac{v^2}{\omega^2} \pm \sqrt{(A^2 + \frac{v^2}{\omega^2})^2 - 4(\frac{xv}{\omega})^2})}$$

3.4.4 $2\varphi = \frac{\pi}{6}$.

3.4.5 When $2\varphi = \pi n$, where n is an integer, the screen shows a segment; when $2\varphi = \pm\frac{\pi}{2} + 2\pi n-$ circle. The length of the semiaxes of the ellipse is $A\sqrt{2}\cos\varphi$ and $A\sqrt{2}\sin\varphi$.

3.4.6 Ellipse with axes vertically and horizontally.

3.4.7 The segment along the diagonal of the screen will become an ellipse extended along the diagonal. ellipse, whose semi-axes will gradually become equal in length. Then a circle will appear, which will begin to turn into an ellipse stretched along the other diagonal of the screen, and so on. After a time of $\frac{2\pi}{\Omega}$ the whole cycle will repeat.

3.4.8 $T_x : T_y = 1 : 2$, except in the case d, when $T_x : T_y = 2 : 1$.

3.4.9 If $T_x : T_y = p : q$, where p and q are integers, then in time $pT_y = qT_x$ the point will return to its initial position. If $T_y = T_x$, the trajectory of the point is an ellipse.

3.4.10 $\omega_y : \omega_x = p : q = 3 : 4$.

3.4.11 $\mu_{min} = \frac{2F}{(M+m_1+m_2)}$, except for the case $\sqrt{\frac{m_1}{m_2}} = \frac{p}{q}$, where p and q are odd integers.

3.4.12 $F = k[A_2cos(\omega t + \varphi_2) - A_1cos(\omega t + \varphi_1)].E_{max} = \frac{k}{2}[A_1^2 + A_2^2 - 2A_1A_2cos(\varphi_2 - \varphi_1)].E_{av} = \frac{k}{4}[A_1^2 + A_2^2 - 2A_1A_2cos(\varphi_2 - \varphi_1)]$. When $\varphi_2 - \varphi_1 = \pi$ the average energy takes the highest value, when $\varphi_2 - \varphi_1 = 0$ - the lowest.

3.4.13 $F = 2kA\sin(\frac{\omega_2 - \omega_1}{2}t)\sin(\frac{\omega_2 + \omega_1}{2}t).E_{av} = \frac{k}{4}(A_1^2 + A_2^2)$

3.4.14 $N = (\frac{1}{2})\omega F_0 A\sin\varphi$

3.4.15 $a.\omega_1 = \sqrt{\frac{3k}{m}}, \omega_2 = \sqrt{\frac{k}{m}}$.

$b.v_1 = \frac{v(cos\omega_2 t + \cos\omega_1 t)}{2}, v_2 = \frac{v(cos\omega_2 t - \cos\omega_1 t)}{2}; x_1 = x_2 = \frac{v(\frac{1}{\omega_1}+\frac{1}{\omega_2})}{2}; \Delta x = \frac{v}{\omega_1}$.

$c.v_1 = v(2cos\omega_2 t + \cos\omega_1 t), v_2 = v(2cos\omega_2 t - \cos\omega_1 t); x_1 = x_2 = \frac{v(\frac{2}{\omega_1}+\frac{1}{\omega_2})}{2}; \Delta x = \frac{2v}{\omega_1}$..

3.4.16 The motion of the atoms will be the sum of the following motions: a) all atoms move progressively with velocity v_0; b) the carbon atom is stationary, and the velocities of the oxygen atoms are equal in modulo and oppositely directed: $v_0^{(1)} = \pm v_1 \cos\omega_1 t, \omega_1 = \sqrt{\frac{k}{M}}$; c) oxygen atoms move with the same velocity $v_2 \cos\omega_2 t$ towards the carbon atom whose velocity which is equal to

$-v_2 \frac{2M}{m} cos\omega_2 t, \omega_2 = \sqrt{k(\frac{1}{M}+\frac{2}{m})}$

Shift of the oxygen atom toward the carbon atom $\Delta x = \frac{|v_1|}{\omega_1} + (1+2\frac{M}{m})\frac{|v_2|}{\omega_2} = \frac{1}{2}v(\frac{1}{\omega_1}+\frac{1}{\omega_2})$.

3.4.17 $x_{max} = \frac{v(\omega_1+\omega_2)[l(\omega_1^2-\omega_1\omega_2+\omega_2^2)-g]}{\omega_1\omega_2[l(\omega_1^2+\omega_2^2)-2g]}, L = \frac{g^2}{l(\omega_1\omega_2)^2}$

3.4.18 $k = \frac{m(\omega^2-\omega_0^2)}{2}$.

3.4.19 $A_{1,2} = \frac{(A\pm B)}{2}; \omega_{1,2} = \frac{2\pi}{\tau} \pm \frac{\pi}{T}$

3.5 Forced and damped oscillations

3.5.1 See Fig.

3.5.2 See Fig.

3.5.3 See Fig. If the shocks follow each other at intervals of time T_0, then amplitude

$A_n = \sqrt{[\frac{v_0}{\omega} + \frac{np}{(m\omega)]^2} + x_0^2}$.

If at intervals $\frac{T_0}{2}$, the amplitude

$A_n = \sqrt{[\frac{v_0}{\omega} + \frac{np}{(m\omega)]^2} + x_0^2}$. for odd n

$A_n = \sqrt{\frac{v_0^2}{\omega^2} + x_0^2}$ for even $n, \omega = \frac{2\pi}{T_0}$

3.5.4

3.5.5 About $63\ cm$.

3.5.6 Potholes on the road on the entry side are less frequent than on the exit side

3.5.7 Before the course and speed of the boat was changed, there was a resonant swaying.

3.5.8 As the amplitude increases, the loss per period increases. When they are equal to energy gain due to the shock, further rocking will stop.

3.5.9 $N = bv^2$

3.5.10 $\frac{d}{dt}(\frac{kx^2}{2} + \frac{mv^2}{2}) = -bv^2$, hence $m\frac{dv}{dt} = -kx - bv$.

3.5.11 See Fig. a: after a single shock there is a gradual damping of oscillations;

Fig. b: with periodic shocks, initially the oscillations swing, and then,when the energy gain of the order of pv compares with the loss per period having the order bv^2T, the oscillations are established.

3.5.12

3.5.13

3.5.14 At $\gamma\omega_0 \approx 1$.

3.5.15 The speed of the oscillator is less in n^2, n^3 times its initial velocity

3.5.16 In τ_2 the energy will decrease fourfold. In time $\frac{\tau_2}{2}$ the energy will be halved.

3.5.17 See Fig.

3.5.18

3.5.19 $\gamma = 10^2\ s^{-1}, \omega = \pi \cdot 10^3\ s^{-1}$. The error in replacing ω by ω_0 is quadratic on the small value of $\frac{\gamma}{\omega_0}$.

3.5.20 $a.\gamma \approx 10^{-2}\ s^{-1}$.

$b.\gamma' = \frac{\gamma}{4}$.

3.5.21 $a.Q = \frac{\omega_0}{(2\gamma)}, n = \frac{Q}{(2\pi)}$.

b. About 50 times for $Q = 10^8$ and only 1.5 times at $Q = 10^9$

3.5.22 $v_{max} = \frac{p}{m}\frac{2}{1-exp(\frac{-2\pi\gamma}{\omega})}$.

$v_{max} \approx \frac{2p}{m}$ if $\frac{2\pi\gamma}{\omega} \gg 1$;

$v_{max} \approx \frac{2\omega p}{(2\pi\gamma m)}$ if $\frac{2\pi\gamma}{\omega} \ll 1$

3.5.23

3.5.24 $A = \frac{F_0}{(m\omega^2)}$.

3.5.25

3.5.26 $a.A = \frac{F_0}{[m(\omega^2-\omega_0^2)]}, \omega_0 = \sqrt{\frac{k}{m}}$.

$b.A = \frac{F_0}{[m(\omega_0^2-\omega^2)]}, \omega_0 = \sqrt{\frac{k}{m}}$.

3.5.27 $A = \frac{F_0}{[m(\omega^2-\omega_0^2)]}$. The quantities B and φ are chosen so that at time $t = 0$ the initial conditions $x(0) = x_0, v(0) = v_0$.

3.5.28 $x_0 = F0/[m(\omega_0^2 - \omega)], v0 = 0$, then $B = 0$.

3.5.29 The additional acceleration associated with free oscillations multiplied by the by the mass of the oscillator is equal to the additional internal force.

3.5.30 Let us consider an example of vibrations of a body attached to a spring. Forced vibrations of this body with a frequency less than the natural frequency can be imagined as free vibrations on the same spring of the body with additional mass. The force with of this mass can be considered as a forcing force. It is directed against the elastic force and therefore in the direction of displacement. The forced oscillations with a frequency greater than the of the same body with an additional spring attached to it. The elastic force of this spring can be can be seen as a forcing force. It is directed against the displacement.

3.5.31

3.5.32 See Fig.$x(t) = \frac{2F_0}{m(\omega_0^2 - \omega^2)} sin(\frac{\omega - \omega_0}{2}t)\sin(\frac{\omega+\omega_0}{2}t)$.

3.5.33 $x(t) \approx \frac{F_0 t}{m(\omega+\omega_0)}\sin(\frac{\omega+\omega_0}{2}t)$.

3.5.34 $x(t) \approx \frac{F_0 t}{2m\omega_0}\sin\omega_0 t$.

3.5.35 At $|\omega - \omega_0|\gamma$, the initially occurring beats gradually transform into forced oscillations due to a decrease according to the law $e^{-\gamma t}$ of the term changing with the frequency ω_0. At $\omega = \omega_0$ the initial swing of oscillations with linearly increasing amplitude smoothly decreases and forced oscillations are established. The characteristic establishment time is equal to the time of damping of free oscillations $\tau = \frac{1}{\gamma}$, when their amplitude decreases by e. times.

3.5.36 $a.F = -2A\gamma m\omega_0 \sin(\omega_0 t - \varphi)$. $b.A = -F_0(2\gamma m\omega_0)$; in $\frac{\omega_0}{(2\gamma)}$ times.

3.5.37 $\gamma = \frac{F_0}{(2x_0\omega m)}$.

3.5.38 $\omega_0 = 550\ s^{-1}, \gamma = 50\ s^{-1}, Q = 5,5$.

3.5.39 About $10^5\ s$.

3.5.40 $v = \frac{\omega_0\lambda}{(2\pi)}$.

3.5.41 The velocity of the particles after the time of flight $v = \frac{F_0}{m\omega}(1-\cos\omega t)$; their average velocity $v_{av} = \frac{F_0}{(m\omega)}$; the highest velocity $V_{max} = \frac{2F_0}{(m\omega)}$ is attained by these particles at a distance of $\frac{F}{m\omega^2}\pi(2n+1)$ from the source, where n is an integer.

The velocity of the particles emitted at time $t = \frac{\pi}{\omega}, v = \frac{F_0}{m\omega}(cos\omega t - 1)$; their average velocity $v_{av} = \frac{F_0}{(m\omega)}$; the highest velocity $v_{max} = \frac{2F_0}{(m\omega)}$ is reached by these particles on the other side of the source at the same distance.

The velocity of the particles emitted at moment $t = \frac{\pi}{(2\omega)}, v = \frac{F_0}{m\omega}\sin\omega t$; their mean velocity $v_{av} = 0$; the highest velocity of these particles, $v_{max} = \frac{F_0}{(m\omega)}$, is attained at distance $\frac{F_0}{(m\omega^2)}$ from the source.

3.5.42 Cycloid; the average velocity $v_{av} = \frac{F_0}{(m\omega)}$ is directed along the x-axis. If at $t = 0$ $v_x = \frac{-F_0}{(m\omega)}$ and $v_y = 0$, the particle will move on a circle of radius $r = \frac{F_0}{(m\omega^2)}$.

3.6 Deformations and stresses. Wave velocity.

3.6.1 $\frac{F}{k}; \frac{(N-1)F}{k}$

3.6.2 Increase by $10^{-14}\ m$

3.6.3 $k = \frac{ES}{L}, F = ES(\frac{\Delta L}{L})$.

3.6.4 $k = Ea$

3.6.5 See Fig. $l = 3\ mm$

3.6.6 10^8 to -0.5 to $10^8\ Pa$

3.6.7 $F = 5 \cdot 10^4\ H$

3.6.8 At $1.2 - 10^{-4}m$

3.6.9 $\Delta l = \frac{mal}{(2ES)}$.

3.6.10 $w = \frac{E\varepsilon^2}{2} = \frac{\sigma^2}{(2E)}$

3.6.11 $A_{min} = \frac{\pi^2}{6}\frac{Ea^4}{l}$

3.6.12 $\nu = \frac{k}{(k+2k_0)}$

3.6.13 $\nu = \frac{k}{(k+2k_0)}$

3.6.14 Increasing. $\nu = 0,5$.

3.6.15 $\chi = \frac{3(1-2\nu)}{E}$

3.6.16 Increases by about $30\ m$. The density of water is greater by $50\ \frac{kg}{m^3}$. The energy in unit volume is $2.5 \cdot 10^6\ \frac{J}{m^3}$.

3.6.17 The horizontal component of the tension force of the thread is equal to F; by the slope of the nonhorizontal part of the thread, the vertical components of the tension force are found, and by them the required forces.

3.6.18 See Fig. The forces applied to the bending points $1,2,3: F_1 = \frac{-F0b}{L}, F_2 = F_0(\frac{b}{L} + \frac{b}{l}), F_3 = \frac{-F_0 b}{l}$.

3.6.19 $u = -c\varepsilon$.

3.6.20 $a.\frac{dp}{dt} = -\rho^c 2\varepsilon$. $b.F = F_0\varepsilon; c = \sqrt{\frac{F_0}{\rho}}$

3.6.21 $a.\varepsilon = \frac{-b}{L}, w = \frac{Eb^2}{(2L^2)}; u = -c\varepsilon = \frac{cb}{L}$. $b.c = \sqrt{\frac{E}{\rho}}$.

3.6.22 $a.\frac{dp}{dt} = \rho cuS = -\rho c^2\varepsilon S$. $b.\sigma = -E\varepsilon, c = \frac{pE}{\rho}$.

3.6.23 5 $\frac{km}{s}$. Let's make a thin rod in a sheet of steel. Its transverse displacement is "interfered" by neighboring parts of the sheet. The stiffness of such a rod is greater than that of a rod with a free side surface.

3.6.24 $550, 1400$ and 340 $\frac{m}{s}$

3.6.25 $c^2 = \frac{\rho(P-P_0)}{[\rho_0(\rho-\rho_0)]}$.

3.6.26 With compression smoothly decreasing toward the wave front, the speed of sound is greater at the more distant sections, the disturbances of the medium catch up with each other. In the case of rarefaction, the distant sound velocity is lower, they lag behind, and the disturbance blurs.

3.6.27 See Fig. Particle velocity and the height of water level rise in a running wave are related by the relation $\frac{u}{c} = \frac{\Delta h}{h}$. We equate the rate of change in momentum with the difference in pressure forces; $\rho hcu = \rho gh\Delta h$. Hence $c = \sqrt{gh}$

3.6.28 $c = \frac{\omega l}{2arcsin(\frac{\omega}{2\omega_0})}$. At $\omega \ll \omega_0$ $c = \omega_0 l, \omega_0 \approx 0.5 \cdot 10^{14}$ Hz.

3.7 Wave propagation

3.7.1 $p = \rho cbS$.

3.7.2 $a.q_p = \Delta\rho c^2$. $b.v = \frac{\Delta\rho c}{\rho}\frac{l}{L}; x = \frac{\Delta\rho}{\rho}l$.

3.7.3 $P(t_0 - \frac{r}{c})$, where r is the distance to the sensor.

3.7.4 The momentum flux density $q_p = \rho cu(x_0 - ct)$.

3.7.5 $F = 1400$ N

3.7.6 $u = \frac{F}{(S\sqrt{E\rho})}, \varepsilon = \frac{-F}{(SE)}; \rho' = \rho[1 + \frac{F}{(SE)}]$. The momentum $p = 0.5F\tau$ $p' = F\tau$; energy $W = \frac{0.5F^2\tau}{(S\sqrt{E\rho})}, W' = \frac{F^2\tau}{(S\sqrt{E\rho})}$.

3.7.7 $A = 12.5 \cdot 10^3$ J, $\frac{K}{A} = 0.25$.

3.7.8 See Fig.;$u = \frac{c_1c_2}{c_1+c_2}\frac{F\perp}{F\|}$, $c_1 = \sqrt{\frac{F\|}{\rho_1'}}, c_2 = \sqrt{\frac{F\|}{\rho_2'}}$

3.7.9 The vertical forces $F_{1,3} = \frac{(\rho v^2-F)b}{L}$ and $F_2 = \frac{2(F-\rho v^2)b}{L}$. When $v \to \sqrt{\frac{F}{\rho}}$the forces, acting on the string, tend to zero - the string "does not resist" bending. If the forces on the are fixed in one way or another, then if $v \to \sqrt{\frac{F}{\rho}}$the deformations of the string increase infinitely. deformations of the string.

3.7.10 The velocities of the "bend" waves and the disturbance will coincide, which will lead to a sharp increase in the amplitude of the waves in the tire. This in turn can cause the tire to rupture.

3.7.11 The speed of the boat and the speed of the wave that the boat excites in the river coincided.

3.7.12

3.7.13 Flat front. The direction of propagation forms an angle α with the normal to the of the interface $(sin\alpha = \frac{c}{v})$.

3.7.14 $\alpha_1 = \alpha, \sin\alpha_2 = (\frac{c_2}{c_1})\sin\alpha$.

3.7.15 Engine noise propagates slower than the shockwave front generated by of a supersonic aircraft.

3.7.16 $sin\alpha_0 = \frac{c_1}{c_2}$

3.7.17 Only the direction of the refracted wave will change:

$sin\alpha_2 = \frac{c_2\sin\alpha_1}{c_1+v\sin\alpha_1}$

where c_1 and c_2 are sound velocities in still air and water, v is air flow velocity, α_1 is angle of incidence.

3.7.18 a. The parts of the wave front farther from the shore move at a greater speed than those less distant. Therefore, the angle between the wave front and the shore near the shore decreases. b. See Fig.

3.7.19 A complete internal reflection is possible at the depth interface.

3.7.20 See figure showing "sound rays" that are orthogonal to the wave surfaces; in the wind direction the sound goes almost along the Earth's surface, and in the opposite direction it goes away from it.

3.7.21 $\nu = \frac{\nu_0}{(1-\frac{v}{c})}$.

3.7.22 $\nu_{1,2} = \nu_0(1 \pm \frac{v}{c})$; $\nu 3 = \nu_0[1 - (\frac{v}{c})\cos\alpha]$.

3.8 Wave superposition and reflection .

3.8.1 In the first case (see Fig. a to Problem 3.8.1) the kinetic energy is zero, and potential energy $U = 2E$. In the second case (see Fig. b to Problem 3.8.1) the kinetic energy $K = 2E$, and potential energy is equal to zero.

3.8.2 Spreading strain waves with $\varepsilon = -0.5 \cdot 10^{-3}$.

3.8.3 See Fig.

3.8.4 See Fig. $P = 2\rho c\omega A \cos\omega t$. Wavelength $\lambda = \frac{2\pi c}{\omega}$. There is a velocity node and a pressure beam near the wall. The first pressure node is away from the wall at a distance of $\frac{\lambda}{4}$

3.8.5 See the figure in the problem condition. In a "non-inverted" displacement wave, the sign of the deformation is opposite to the sign of deformation of the incident wave.

3.8.6 $A = \frac{v_0}{2\omega}$. At the end of the rod there is a velocity beam and a pressure node. The first node of velocity is at a distance $\frac{\lambda}{4}$ from the end of the rod (see Fig.).

3.8.7 When a wave reflects from the inner surface of the glass, it creates a region of of high tension (stretching).

3.8.8 $u = \frac{2P}{(\rho c)} = 250\ \frac{m}{s}\ ; l = \frac{c\tau}{2} = 1cm.$

3.8.9 $l = \frac{1}{2}(L - \frac{c}{\omega} arcsin\frac{\sigma}{\sigma_0}) = \frac{L}{2}(1 - \frac{1}{\pi} arcsin\frac{\sigma}{\sigma_0}).l = \frac{L}{2}$ at $\sigma_0 \gg \sigma, l = \frac{L}{4}$ at $\sigma_0 \approx \sigma$.

3.8.10 $P = \rho cu = 3.9 \cdot 10^4\ atm$. The force applied to the end of the rod from the side of the wall generates a compression wave in it. Reaching the free end, it is reflected from it. The reflected wave is a tensile wave. When the reflected wave and the force wave from the wall overlap, the deformation disappears and the velocity of the rod sections changes sign. When the front of the reflected wave reaches the wall, the entire rod is undeformed and its contact with the wall ceases. The contact time $\tau = \frac{2l}{c} = 4 \cdot 10^{-4}\ s$.

3.8.11 $v_l = v, v_L = v|1 - \frac{2l}{L}|$.

3.8.12

3.8.13 $v_1 = 0, v_2 = \frac{vl_1}{l_2}$.

3.8.14 $\frac{u_{refl}}{u_{fal}} = \frac{\sqrt{\rho_1 E_1} - \sqrt{\rho_2 E_2}}{\sqrt{\rho_1 E_1} + \sqrt{\rho_2 E_2}}, \frac{u_{refr}}{u_{fal}} = \frac{2\sqrt{E_1 \rho_1}}{\sqrt{E_1 \rho_1} + \sqrt{E_2 \rho_2}}$

3.8.15 $D \approx \frac{4\rho_1 c_1}{\rho_2 c_2} \approx 1,1 \cdot 10^{-3}$.

3.8.16 In the presence of the spacer, the coefficient of passage of the wave received by the sensor increases from 0.25 to 0.41. Secondary signals ("echo-signals") appear, following each other at $\frac{2l}{s}$ intervals, the power of which decreases geometrically. At high frequency of signals repetition "echo-signals" overlap each other, then By selecting the thickness of the spacer it is possible to achieve almost complete passing or reflection of the signal.

3.8.17 $n = (\frac{\rho_1 c_1 - \rho_2 c_2}{\rho_1 c_1 + \rho_2 c_2})^2, L = \frac{2lc_1}{c_2}$.

3.8.18 $L = \frac{2lc_1}{c_2}.n = 1$. No

3.8.19 $l_1 = 1.25\ mm, l_2 = 2.5\ mm$

3.9 Sound. Acoustic resonators

3.9.1 $\lambda = \frac{c}{\nu} = 6,6\ m$

3.9.2 $l = \frac{c}{4\nu} = 82.5\ cm.$

3.9.3 $c = \frac{2l}{\nu}$.

3.9.4 $v_1 = 6.8\ \frac{cm}{s}$, $v_2 = 6.8 \cdot 10^{-8}\ \frac{m}{s}$, $x_1 = 0.11\ mm$, $x_2 = 1.1 \cdot 10^{-11}\ m$, $P_1 = 3 \cdot 10^{-4}\ atm$, $P_2 = 3 \cdot 10^{-12}\ atm$

3.9.5 $I > 3\ \frac{kW}{m^2}$

3.9.6 $F = 2L^2 \rho cv$. At $\omega \ll \frac{c}{L}$ there is almost complete pressure equalization in the of the air jet, so the emission of sound is weak

3.9.7 $E = 2\pi R^2 \omega^2 A^2 \rho c$. The pressure amplitude in the wave is inversely proportional to the distance to the center of the ball.

3.9.8 a. Two divergent waves: velocities

$u = \frac{F_0}{2S\rho c} \cos\omega(t \mp \frac{x}{c})$

(the coordinate x starts at the cross section where the source of the force F is located) and deformation $\varepsilon \mp \frac{u}{c}$.

b. A standing wave occurs between the force sources:

$u = \frac{F_0}{S\rho c} \cos\omega(t - \frac{l}{2c}) \cos\frac{\omega x}{c}$;

outside the sources are two scattering waves:

$u = \frac{F_0}{S\rho c} \cos\omega l2c \cos\omega(t - \frac{x}{c})$

(x-coordinate counting starts at the point located in the middle between the sources of force F). If at the distance l there is an even number of half-waves, the power of the resultant the power of the resulting wave is maximal, if there is an odd number of half-waves - the power of the resulting wave is zero. is equal to zero.

3.9.9 When $l = (\frac{1}{4} + n)\lambda$; when $l = (\frac{3}{4} + n)\lambda, \lambda = \frac{2\pi c}{\omega}$.

3.9.10 $L = 2\lambda, c = \frac{L\omega}{4\pi}$.

3.9.11 a. The stress nodes are at distances from the free end divisible by $\frac{\lambda}{2}.F_0 = \frac{\sigma_0 S}{sin(\frac{2\pi L}{\lambda})}$

b. See Fig.; $\omega = \frac{2\pi nc}{(2L)}$, where $n$is an integer, $c = \frac{\omega\lambda}{(2\pi)}$is the speed of sound. We can.

3.9.12 $\nu_n = n \cdot 2500\ Hz$. At a distance of $25\ cm$ from its ends.

3.9.13 It will be halved.

3.9.14 $A = \frac{A_0}{|sin(\frac{\omega L}{c})|}.\tau = \frac{2\pi}{\omega|sin(\frac{\omega L}{c})|}$.

3.9.15 $\nu = \frac{c}{(2L)} = 8.25\ Hz$.

3.9.16 As the height of the air column in the vessel changes, its resonance frequencies change. The sound is amplified when the difference between the frequency of the tuning fork and one of the resonant frequencies of the air column.

3.9.17 $50, 250, 450\ m$, etc.

3.9.18 $\nu_0^{(1)} = 300\ Hz;\nu_0^{(2)} = 150\ Hz$.

3.9.19 To make the instrument's natural frequency set as rich as possible. The tone decreases as the size increases.

3.9.20 The sound of the voice is influenced by the air's own vibrations. The corresponding wavelengths in a helium-oxygen medium will be unchanged, and the frequencies will increase as the of sound velocity. The overall tone of the voice will increase. The frequency of vibration of the tuning fork will not change, the same frequency as the sound.

3.9.21 $F = 4l^2\nu^2\mu = 144\ N$.

3.9.22 Near the displacement bundles at a distance of $\frac{l}{6}$ or $\frac{l}{3}$ from the end of the string.

3.9.23 Because of the friction between the hand and the rod, there will be a large loss of energy. They are lowest for the middle of the rod, where there is a velocity bundle, the highest for its ends, where there is a velocity bundle.

3.9.24 The main energy losses are associated with the transition of the wave from one medium (sapphire) to another (air). The transmission coefficient is. $D = \frac{4\rho_{air}c_{air}}{\rho_{sapf}c_{sapf}} = 0.7 \cdot 10^{-4}$(see Problem 3.8.15). The losses will increase about 10^4 times.

3.9.25 The power of a passing wave is the same fraction of the incident wave power whether the sound travels from air to water or from water to air, and this fraction is very small. Pressure is another matter. When a sound wave is reflected in the air at the boundary with water, a pressure beam is formed, so the pressure in the wave passing into the water is almost twice as much as the pressure in the incident sound wave. (We consider only the normal falling wave at the boundary of the two media; in other cases the picture is qualitatively the same). When, however. sound wave falls on the interface out of water, a pressure node is formed at that interface, and the pressure in the passing wave in the air is nearly zero. This approximation is based on the fact that ρc for the wave and air differ many times (about 330times). It is possible to calculate the pressure change accurately. The pressure in a passing wave in the first medium

$P_{refr1} = \frac{2\rho_1 c_1}{\rho_1 c_1 + \rho_2 c_2} P_{fal2}$

where P_{fal2} is the pressure of the incident wave in the second medium. When passing from water to air, the pressure decreases by a factor of about 150.

3.9.26 $M = \frac{\sqrt{mk}}{\omega} ctg\omega\sqrt{\frac{m}{k}}$.

4 FLUID MECHANICS

4.1 Fluid pressure

4.1.1

4.1.2 $F_1 = 2000\sqrt{2}\ N$. $F_2 = 0$.

4.1.3 $P = \frac{4}{\sqrt{3}}\frac{F}{a^2}$

4.1.4 Yes.

4.1.5 $F = 2\pi r^2 P$.

4.1.6 $F = \pi(R^2 - r^2)P$.

4.1.7 $\sigma = \frac{(R-\Delta)^2}{R^2-(R-\Delta)^2} P$

4.1.8 The force F_1 acting per unit length of the circumference of the sausage cross section is less than the force F_2 acting per unit length of the perimeter of its longitudinal section.

4.1.9 $At distance l = \frac{d_1^2 - d_3^2}{d_1^2 + d_2^2 + d_3^2} a$ to the left of the center of the stick.

4.1.10 $h = 727\ cm$.

4.1.11 $F_N = 4392\ N$; $F_c = 4314\ N$; $F_b = 4353\ N$; $F = 78\ N$.

4.1.12 $F = (\frac{1}{12})\rho g a^2(3\sqrt{3}h - \sqrt{2}a) + (\frac{1}{4})Pa^2\sqrt{3}$.

4.1.13 Constituent forces parallel and perpendicular to the bottom of the vessel: $F_{\parallel} = a^3(\rho - \rho_0)g\sin\alpha, F_{\perp} = a^3\rho_0 g(\frac{\rho}{\rho_0}cos\alpha + \frac{1}{2}\sin\alpha + \frac{h}{a}) + Pa^2$.

4.1.14 $x = H - (\frac{R^2}{r^2})(1+\frac{a}{r})(\frac{\rho}{\rho_0 - 1})h$.

4.1.15 $h = 85\ cm$.

4.1.16 $h = 10,1\ m$.

4.1.17 $m = \frac{\pi R^3 \rho}{3}$.

4.1.18

4.1.19 $A = \pi r^2(h + \frac{1}{2}l\frac{r^2}{R^2})\rho g l$.

4.1.20 The pressure P_r can be found from the equilibrium condition of the highlighted thin cylindrical volume in the figure: the force of attraction of this volume to the center of the planet, equal to the product of the mass of the volume by the acceleration of the gravity field in the center of the volume, is balanced by the pressure acting on the lower section,

$P_r = \frac{2}{3}\pi\gamma\rho^2(R^2 - r^2), P_0 = \frac{2}{3}\pi\gamma\rho^2R^2$.

4.1.21 In the direction of acceleration of the vessel.

4.1.22 $\beta = \alpha - arctg\mu$

4.1.23 The pressure $P(x)$ can be found from the condition that the pressure force on the inner base of the thin cylindrical volume highlighted in the figure is equal to $m\omega^2 y$, where y is the distance from the center of the cylinder to the axis of rotation, m is the mass of the selected volume: $P(x) = \frac{\rho\omega^2[(R-x)^2 - \frac{R^2}{4}]}{2}$.

4.1.24 $y = \frac{1}{2}\frac{\omega^2}{g}x^2$.

4.2 Swimming. Archimedes' Law

4.2.1 $P = \frac{mg}{S} + P_0$

4.2.2 $h = \frac{H(\rho - \rho_1)}{(\rho_2 - \rho_1)}$

4.2.3 $H = \frac{(m - \rho_1 hS)}{[S(\rho_2 - \rho_1)]}$.

4.2.4 . If at a small rotation of the parallelepiped around the axis passing through the point O, the momentum of the forces acting on the parallelepiped is directed in the opposite direction to the direction of rotation. to the direction of rotation, its position is stable. This condition is satisfied at

$\frac{a}{b} > \sqrt{6\frac{\rho}{\rho_0}(1 - \frac{\rho}{\rho_0})}$.

4.2.5 $A = 34$

4.2.6 $V = 147\ cm^3$

4.2.7 $\rho = 1.5\ \frac{g}{cm^3}$.

4.2.8 $\frac{\rho_1'}{\rho_2'} = \frac{\rho_1}{\rho_2}$.

4.2.9 $x = \frac{4m}{[\pi(d_1^2 + d_2^2)]}$.

4.2.10 $F = 0,8 \cdot 10^{-3}\ N$

4.2.11 $F = (\frac{2}{3})\pi r^3\rho g(1 + \frac{2r}{l})$

4.2.12 $F = 1,2 \cdot 10^{-2}\ N$.

4.2.13 $\rho = \frac{2}{3}\ \frac{g}{cm^3}$

4.2.14 $F = \frac{mg}{\sqrt{3}}$

4.2.15 $.a.F = \rho g R(H + \frac{L}{2})^2$. $b.F = \frac{\rho g L(H+R)^2}{2}$.

4.2.16

4.2.17 $m = 520\ g$

4.2.18 $m = (\frac{4}{3})\pi(R^2 + r^2)^{\frac{3}{2}}\rho$

4.2.19 $m_1 = \frac{\rho a^3(6 + 5tg\alpha + tg^3\alpha)}{24}$; $m_2 = \frac{\rho a^3(6 - 5tg\alpha - tg^3\alpha)}{24}$

4.2.20 $T = \frac{\sqrt{3}mg}{72}$.

4.2.21 $a.\ Q = 1\ kJ$ $.b.\ Q = \pi r^2\rho g h H[1 + \frac{1}{2}\frac{h}{H}\frac{\rho}{\rho_0}(1 - \frac{r^2}{R^2})]$.

4.2.22 $Q = (\frac{4}{3})\pi R^3\rho g H = 410\ J$, ρ is the density of water.

4.2.23 $A = 2.5 \cdot 10^6\ J$

4.2.24 $a.$ Can. $\rho[\frac{g}{cm^3}] = (1 + \frac{\Delta}{2R - 2H - 2\Delta + l})(1 - \frac{\Delta}{l})$.

4.2.25 $F = (\frac{4}{3})\pi r^3(R - r)\rho\omega^2$

4.2.26 $\omega = \sqrt{\frac{(gtg\alpha)}{[R - (l + r)\sin\alpha]}}$.

4.2.27 $F \approx \frac{(m_1 - m_2)\omega^2 R}{2}$.

4.3 The motion of an ideal fluid

4.3.1 $28.5; 27.0; 25.6\ \frac{m}{s}$. To the twelfth floor.

4.3.2 $\Delta T = 2gh\rho S.$

4.3.3 $N = \rho V[gh + \frac{V^2}{(2S^2)}].$

4.3.4 a. Because of the pressure difference in sections 1and 2, the fluid between these sections in the direction of its movement, a resultant pressure force greater than of the force acting on the side of section A. $b. = \frac{\rho v^2 S_1(1-\frac{S_2}{S_1})^2}{2}$.

4.3.5 Pressure in the vessel $P_c = P_0 + \rho gx$, pressure in the tube $P_t = P_0 + \rho g(x - H)$.

4.3.6 $F = \sqrt{2}(P + pv^2)S$

4.3.7 $v = \sqrt{\frac{2FS}{[\rho(S^2-s^2)]}}$.

4.3.8 $x = 5l$

4.3.9 $h = \frac{1}{2g}[v^2 - (\frac{mg}{\rho vSN})^2]$

4.3.10 $a = \frac{\rho-\rho_0}{\frac{\rho+\rho_0 r^2}{(R^2-r^2)}}g, \Delta P = \frac{\rho R^2}{\rho(R^2-r^2)+\rho_0 r^2}\rho_0 gh.$

4.3.11 The size of the longitudinal section of the jet will increase by a factor of 2. The velocity of similar sections in the jet will increase by $\sqrt{2}$ times. Therefore, the discharge will increase by $2\sqrt{2}$ times

4.3.12 The jets will be similar. All dimensions of the jet will decrease by $\frac{H}{h}$ times as the water level decreases, the velocity of similar sections in the jet will decrease by $\sqrt{\frac{H}{h}}$ times. Therefore, the velocity of level decrease will decrease in $(\frac{H}{h})^2\sqrt{\frac{H}{h}} = (\frac{H}{h})^{\frac{5}{2}}$times.

4.3.13 From the law of conservation of energy it follows that the velocity of sections $2, 3$of the jet on the plane will be equal to the velocity of section $1v$, and from the law of conservation of momentum it follows that
$h_1 = \frac{h(1+\cos\alpha)}{2}, h_2 = \frac{h(1-\cos\alpha)}{2}$.

4.3.14 We need to move to a frame of reference in which the plates move along their planes. In this system, the plates will move as two counter jets shown in in figure a. Their motion above and below the plane OO' repeats the motion of the jet considered in in problem 4.3.13. Then it is necessary to return to the previous frame of reference (b). $v_1 = vtg\frac{\alpha}{2}, v_2 = vctg\frac{\alpha}{2}$.

4.3.15 Cone; $cos\alpha = \frac{(R^2-r^2)}{(R^2+r^2)}$.

4.3.16 The problem is reduced to Problem 4.3.15if we move to a frame of reference in which the counter velocities of the armor and the metal jet are equal in modulo. $v = 1\ km/s$.

4.3.17 $h = lcos(t\sqrt{\frac{g}{l}}).P = \frac{x\rho g}{2}$in the vertical part of the tube. $P = \frac{y\rho g}{2}$in the horizontal part of the tube.

4.3.18 $a = g(\frac{s}{S})^2$.

4.3.19 $E = PV.$

4.3.20 $v = \sqrt{\frac{2}{3}\frac{P}{\rho}(\frac{R^3}{r^3} - 1)}, \rho$ is the density of water.

4.3.21 If atmospheric pressure is not able to give the water velocity equal to the propeller edge velocity v, a cavity may appear behind the edge;$v > 14\ \frac{m}{s}$.

4.4 Viscous fluid flow

4.4.1 The force with which fluid layers act on each other across a unit area surface area $AA', F = \eta\frac{dv}{dx}$. In a stationary flow, the resultant force acting on the fluid layer between any interface surfaces AA' and $BB'$is zero. Therefore, the velocity gradient is the same everywhere and equals $\frac{v_0}{h}$, and the velocity at distance $x$from the stationary plane is $\frac{v_0 x}{h}, 0 < x < h, F = \frac{\eta v_0}{h}$.

4.4.2 $v = \frac{P}{2\eta}x(h - x), 0 < x < h; Q = \frac{P}{12\eta}h^3$.

4.4.3 $a.\ Q = \frac{h^3\rho g}{3\eta}\sin\alpha.\ b.\ \alpha \approx 8 \cdot 10^{-8}\ rad$

4.4.4 $v = \frac{2mg\Delta^2}{(\pi r^2 h\eta)}$.

4.4.5 a) The resultant pressure force on the ends of the separated cylindrical volume $P \cdot \pi x^2$is balanced by the force of viscous friction $2\pi xl\eta\frac{dv}{dx}$. Therefore $\frac{dv}{dx} = -\frac{xP}{2l\eta}, 0 < x < R.\ b).v = \frac{P}{4\eta l}(R^2 - x^2)$. Volume of fluid flowing per unit time, $Q = \frac{\pi R^4 P^2}{(8\eta l)}$.

4.4.6 $t = T$

4.4.7 $t = \frac{32\eta l}{(\rho g d^2 \sin\alpha)}$.

4.4.8 a) The momentum of forces acting on the cylindrical interface between the layers is independent of the cylinder radius x, because only in this case the resultant moment of forces acting on the liquid between two cylindrical surfaces is is zero and the fluid moves stationary. Therefore $M_x = -x \cdot 2\pi x \cdot \eta x\frac{d\omega_l}{d_x} = M, \frac{d\omega_l}{d_x} = -\frac{M}{2\pi\eta x^3}, r < x < R$.
$b.\omega_l\frac{M}{4\pi\eta}(\frac{1}{x^2} - \frac{1}{R^2}), \omega\frac{M}{4\pi\eta}(\frac{1}{r^2} - \frac{1}{R^2})$.

4.4.9 $F = P_2S_2 - P_1S_1 - \rho v_1^2 S_1(1 - \frac{S_1}{S_2})$.

4.5 Liquid surface tension

4.5.1

4.5.2

4.5.3 $r \approx 0.5\ cm$

4.5.4 $F = 2(\sigma_1 - \sigma_2)l.$

4.5.5 $\sigma = \frac{k(2\pi R-l)}{(2R)}.$

4.5.6 $a.A \approx \frac{2V\sigma}{\Delta}.\ b.n \approx 4.$

4.5.7 $a = 2.1\ cm$

4.5.8 $\sigma = \frac{r\rho gh}{2}.$

4.5.9 Less than $0.2\ \frac{cm}{s^2}.$

4.5.10 The figure shows forces acting on a section of the plate of unit length (double arrows), and forces acting on sections of the lateral surface of the liquid of unit length (bold arrows): Fx is the desired force, mg is the force of gravity acting on the plate, $F_0 = \rho gxl$ and $F_\| = \frac{\rho gx^2}{2}$ - forces caused by negative fluid pressure, σ- surface tension. It follows from the condition of equilibrium of the lateral surface of the liquid,that
$F_\| = \frac{\rho gx^2}{2} = \sigma - \sigma\cos\theta, \cos\theta = 1 - \frac{\rho gx^2}{(2\sigma)}.$
From the equilibrium condition of the plate we have
$F_x = F_0 + mg + 2\sigma\sin\theta = mg + \rho gx(l + 2\sqrt{\frac{\sigma}{\rho g} - \frac{x^2}{4}}).$

4.5.11 $m = 0.55\ \frac{g}{cm^2}.$

4.5.12 $a.h = \sqrt{\frac{2\sigma(1-\sin\theta)}{(\rho g)}}.\ b.h = 3,9\ mm$

4.5.13 $a.x = 2sin\frac{\theta}{2}\sqrt{\frac{\sigma}{(\rho g)}}.\ b.x = 5,4\ mm$

4.5.14 $a.x = \sqrt{\frac{2\rho_l(\sigma_m+\sigma_{l.m.}-\sigma_l)}{\rho_m(\rho_l-\rho_m)g}}$ if $\sigma_l \le \sigma_m + \sigma_{l.m.}; x = 0$ if $\sigma_l \ge \sigma_m + \sigma_{l.m.}$ About $2.5\ km^2$

4.5.15 The vertical component of the surface tension force is equal to the perimeter of the wand cross section multiplied by $\sigma\cos\vartheta$. Therefore, the volume of liquid lifted by surface tension does not depend on the shape of the wand cross section, but depends on its perimeter.

4.5.16

4.5.17 $P_{max} = \frac{2\sigma}{R} + \rho g(h+R), P_{min} = \frac{2\sigma}{R} + \rho g(h-R).$

4.5.18 $R = \frac{\sigma}{(\rho gx)}.$

4.5.19 $P = P_0 + 2\sigma[\frac{1}{R} + \frac{1}{(R-h)}]$

4.5.20 About 3 liters.

4.5.21

4.5.22 $.h = 2r\sqrt{\frac{\sigma_1+\sigma_2-\sigma}{\sigma_1+\sigma_2+\sigma}\frac{\sigma^2}{\sigma^2-(\sigma_1-\sigma_2)^2}}.$

4.5.23 $R = \frac{rR_0}{(R_0-r)}.\alpha = 120°$

4.5.24 $m = \pi r^2(\rho h + \frac{2\sigma}{Rg})$

4.5.25 $h = 0,14\ mm.$

4.5.26 $A = 1.4 \cdot 10^{-5}\ J$

4.5.27 In a thin jet, the sum of $\frac{\rho v^2}{2} + \rho gh + \frac{\sigma}{r}$ (here $\rho, \sigma$and v are density, surface tension and velocity of the jet, and r and h are the radius of the jet and the distance to the tap) does not change. $h \approx 2\ cm.$

4.6 Capillary phenomena

4.6.1 b. From the inside

4.6.2 $h = \frac{2\sigma}{(\rho gR)}; A = \frac{4\pi\sigma^2}{(\rho g)}; U = \frac{2\pi\sigma^2}{(\rho g)}.$ Part of the energy is converted into heat.

4.6.3 $r = 1.5\ \mu m.$

4.6.4 $a.V = \frac{\pi r^3\sigma}{(4\eta h)}.\ b.V = 1,1\ \frac{cm^3}{s}.$

4.6.5 $\Delta = 0,4\%.$

4.6.6 $r_2 = -1,5\ mm$, $r_4 = 1,5\ mm.$

4.6.7 $\Delta x = \frac{2\sigma}{(\rho gr)}$ if $0 < x < h - \frac{2\sigma}{(\rho gr)}; \Delta x = h - x$ if $h - \frac{2\sigma}{(\rho gr)} < x < h + \frac{2\sigma}{(\rho gr)}; x_0 = h + \frac{2\sigma}{(\rho gr)}.$

4.6.8 $r_x = \frac{2r}{cos\theta}.$

4.6.9 $\omega = \frac{2}{l}\sqrt{\frac{\sigma}{(r\rho)}}.$

4.6.10 $x = 2h$ if $l > h; x = l + h$ if $l < h$

4.6.11 $t = 17°\ C.$

4.6.12 The wetting liquid will move towards the narrow part of the capillary, the non-wetting liquid will move towards its wide part.

4.6.13 $x = \frac{1}{2}H(1 - \sqrt{1 - \frac{16\sigma}{\rho g\alpha H^2}}), \alpha > \frac{16\sigma}{\rho gH^2}; x = H, \alpha < \frac{16\sigma}{\rho dH^2}.$

4.6.14 $x = \frac{\sigma}{\rho g\Delta}(cos\theta_1 + \cos\theta_2).$

4.6.15 $F = a\sigma(1 - \cos\theta).$

4.6.16 $F = \frac{2a\sigma^2}{(\rho g\Delta^2)}$

4.6.17 $h = \frac{\sigma l}{(S\rho g)}, T = 2\pi\sqrt{\frac{h}{g}}.$

5 MOLECULAR PHYSICS

5.1 Thermal motion of particles

5.1.1 $K \approx 5.8 \cdot 10^{-21}\ J$, $\sqrt{\langle v^2 \rangle} \approx 1.5 \cdot 10^{-4}\ \frac{m}{s}$

5.1.2 Twice as much.

5.1.3 $m \geq 0.01mg$.

5.1.4 $\sqrt{\langle x^2 \rangle} \approx 6.4 \cdot 10^{-8}\ m$.

5.1.5 $\Delta r \approx 7\ cm.At\ T = 100\ K\ d \approx 4\ cm$.

5.1.6 The lighter, i.e. more mobile particles pass through the baffle faster. Therefore, at first the number of light particles per unit volume is equalized, and the pressure in the section where the heavy particles were, increases

5.1.7 $\frac{P_{He}}{P_{H_2}} = \frac{1}{\sqrt{2}}$.

5.1.8 $\frac{N_1}{N_2} = (\frac{\mu_2}{\mu_1})^{\frac{(n-1)}{2}}$.

5.1.9 $\tau' = n\tau$

5.1.10 In $\frac{L^2}{R^2}$ times

5.1.11 $N_1 = N\frac{\sqrt{T_2}}{(\sqrt{T_2}+\sqrt{T_1})}, N_2 = N\frac{\sqrt{T_1}}{(\sqrt{T_2}+\sqrt{T_1})}$. Toward the first volume.

5.2 Distribution of gas molecules by velocities

5.2.1 $a.\ N = 1.3 \cdot 10^{15}\ l^{-1}$. $b.\ N_1 = 1.3 \cdot 10^{17}\ m^{-3}$; $N_2 = 1.3 \cdot 10^{19}\ m^{-3}$.

5.2.2 $N_1 = 6.2 \cdot 10^4\ cm^{-3}$, $N_2 = 1.2 \cdot 10^4\ cm^{-3}$

5.2.3 $a)\ N_1 \approx 10^{16}\ cm^{-3}$, $b)\ N_2 \approx 2 \cdot 10^{22}\ m^{-3}$

5.2.4 $T = 21\ K$

5.2.5 $n = 0, 13$.

5.2.6 $n = 6 \cdot 10^{-3}$.

5.2.7 The temperature will decrease.

5.2.8 $L = l\frac{\tau}{t_0}(1 + \frac{2t}{t_0+2\tau})$

5.2.9 $v = 300\ \frac{m}{s}$.

5.2.10 $v = \frac{h\omega}{(2\pi)}$.

5.2.11 $a.\ f' = \frac{1}{l}f(\frac{v}{l})$. $b.\ f' = \frac{1}{kl}f(\frac{v}{kl})$.

5.2.12 $f(v) = \frac{1}{v_0}$ at $v_0 \leq v \leq 2v_0, f(v) = 0$in the rest region of v values. The distribution function $f(v)$will shift by $\Delta v = \frac{F\tau}{m}$to the region of higher velocities.

5.2.13 $a.v_{min} = v + \frac{Fr}{m}, v_{max} = v + \frac{Fr}{m} + \Delta v, n' = n$. $b.\ v_{min} = v\sqrt{1 + \frac{2Fl}{mv^2}}, v_{max} = (v + \Delta v)\sqrt{1 + \frac{2Fl}{m(v+\Delta v)^2}} \approx v\sqrt{1 + \frac{Fl}{mv^2}} + \Delta v.\sqrt{1 + \frac{2Fl}{mv^2}}, n' = n.\sqrt{1 + \frac{2Fl}{mv^2}}$.

5.2.14 a. It will decrease by a factor of $exp(\frac{2Fl\alpha}{m})$. $b.\rho = \rho_0 exp(\frac{-mgh}{kT})$; it will not change.

5.2.15 $m = 10^{-24}\ kg$, $r = 10^{-9}\ m$.

5.2.16 $h_1 \approx 111\ km$, $h_2 \approx 123\ km$.

5.2.17 $a.\ n \approx n_0 exp(\frac{-q}{kT})$. $b.\ n = 10^{15}\ cm^{-3}$

5.3 Collisions of molecules. Transport processes

5.3.1 $d \approx 0.3\ nm$.

5.3.2 $l \approx 60\ nm$

5.3.3 $\nu_1 \approx 6 \cdot 10^{28}\ s^{-1} \cdot cm^{-3}$; $\nu_2 \approx 3 \cdot 10^{28}\ s^{-1} \cdot cm^{-3}$

5.3.4 Increased by a factor of 1.5.

5.3.5 $l_1 \sim \pi^{-1}[4R_1^2 n_1 + (R_1 + R_2)^2 n_2]^{-1}; l_2 \sim \pi^{-1}[4R_2^2 n_2 + (R_1 + R_2)^2 n_1]^{-1}$

5.3.6 $t \approx 30\ ps$.

5.3.7 $n = \frac{2\sqrt{2}r_{AB}}{(r_A+r_B)}$.

5.3.8 a. A horizontal unit area ABlocated at height h is crossed from top to bottom by a flux of radioactive atoms whose density is estimated by the formula $W_1 \approx \frac{\bar{v}_z n_{h+\lambda}}{2}$, where $\bar{v}_z$is the velocity close to the RMS $\sqrt{(\bar{v}_z^2)} = \sqrt{\frac{kT}{m}}$, and $n_{h+\lambda} = \alpha(h+\lambda)$is the number of atoms per unit volume at height $h + \lambda$. The density of the flux of atoms coming from below, $W_2 = \frac{\bar{v}_z n_{h-\lambda}}{2} \approx \frac{\sqrt{\frac{kT}{m}}\alpha(h-\lambda)}{2}$. The resulting flux density of radioactive atoms on the Earth $W = W_1 - W_2 \approx \alpha\lambda\sqrt{\frac{kT}{m}}$. $b.\ D \approx 12\ \frac{\mu m}{s}$.

5.3.9 $D = \frac{nD_1D_2}{(n_1D_2+n_2D_1)}$.

5.3.10 $t \approx \frac{L}{D}; m = \frac{DS\rho}{L}$.

5.3.11 a. The solution is similar to the solution of Problem $5.3.8a : W \approx n\alpha\lambda k\sqrt{\frac{kT_0}{m}}$. No change. b. By a factor of 6.2.

5.3.12 $W \approx 12\ Wt, t \approx 2\ h$. Due to air convection.

5.3.13 $\chi = \frac{\chi_1}{1+\frac{1}{4\alpha}[1+(\frac{\chi_1}{\chi_2}\sqrt{\frac{\mu_1}{\mu_2}})^{\frac{1}{2}}]^2} + \frac{\chi_2}{1+\frac{\alpha}{4}[1+(\frac{\chi_2}{\chi_1}\sqrt{\frac{\mu_2}{\mu_1}})^{\frac{1}{2}}]^2}$

5.3.14 $t' = nt$

5.4 Particulate gases. Interaction of molecules with the surface solid

5.4.1 $\nu \approx 10^{24}\ s^{-1} \cdot sm^{-2}$, $\frac{\Delta p}{\Delta t} \approx 10\ N$.

5.4.2 Will decrease by a factor of 1to $\frac{k}{2}$.

5.4.3 $F \approx \pi r^2 nmv^2$

5.4.4

5.4.5 $F \approx 4\pi r^2 Pv\sqrt{\frac{\mu}{(RT)}}$, where R is the gas constant.

5.4.6 $F = \frac{Ps}{2}$.

5.4.7 $F \approx PSv\sqrt{\frac{\mu}{(RT)}}$.

5.4.8 As long as the free path length of gas molecules is greater than the distance between the disks, the momentum of the viscous friction force depends on the pressure. $\varphi = (\frac{\varphi_1}{P_1})P$.

5.4.9 $\omega' = \omega(\frac{r_1}{r_2})^2$.

5.4.10 When the plates are illuminated, the temperature of the blackened surface becomes higher than the mirror surface. Therefore, in the rarefied gas the pressure on it is somewhat higher. The rotor will rotate in the direction of the mirror surface.

5.4.11 $F \approx 10^{-2}\ N$.

5.4.12 $v \approx 1\ \frac{m}{s}$

5.4.13 $P \approx \frac{FT_1}{[S(T_2-T_1)]}$.

5.4.14 $P_0 = P\sqrt{\frac{T_0}{T}}$

5.4.15 $P' = P(1+\sqrt{2}) \cdot 2^{\frac{-5}{4}}, T' = T\sqrt{2}$.

5.4.16 $w = 1,5kn\Delta T\sqrt{\frac{3kT}{\mu}}$.

5.4.17 The division value of the temperature scale should be reduced by $\sqrt{14}$times.

5.4.18 $m \approx 0.1\ kg$.

5.4.19 $r \approx (\frac{W_1}{W_2 \pi n \delta})^{\frac{1}{2}}$.

5.4.20 In case a, the thermal conductivity does not change: in case b, it decreases by a factor of N.

5.5 Equation of state of an ideal gas

5.5.1 Three times.

5.5.2 $V = \frac{(P_2V_2 - P_1V_1)}{(P_2-P_1)}$.

5.5.3 $\Delta m = \frac{m(k-1)n}{(n-1)}$.

5.5.4 $P = P_0 + \frac{mgh}{(2\pi r^2 L)}$.

5.5.5 $V = 885\ l$

5.5.6 $x = \frac{L(1+\frac{\rho g L}{2P})}{2}$.

5.5.7 $P = 1,166\ MPa$

5.5.8 $\delta t = 140°C$.

5.5.9 In operation, when the gas in the cylinder is heated, its pressure must not exceed atmospheric pressure.

5.5.10 $T = \frac{9T_0}{8}$.

5.5.11 $n = \frac{(P-P_0)V_0}{P_0 V}$.

5.5.12 $n = \frac{ln(\frac{P_0}{P})}{ln(1+\frac{V}{V_0})}$.

5.5.13 It doesn't depend.

5.5.14 The burner smokes due to a lack of oxygen. A vertical glass tube causes the flow of oxygen to the burner flame.

5.5.15 $\Delta P = 137\ Pa$

5.5.16 $T = T_0\frac{2V0+S(l+2x)}{2V0+S(l-2x)}$.

5.5.17 $P = 1146\ hPa$.

5.5.18 $\frac{V_B}{V_H} = 1,9$.

5.5.19 $x = \frac{1}{2}\{l + H + \frac{P_0}{\rho g} - \sqrt{(l+H+\frac{P_0}{\rho g})^2 - 4lH}\}$.

5.5.20 $x = (H_0 - H)(1 - \frac{\rho g h_0}{P_0} + \rho g H)$

5.5.21 $a.\ H = \frac{h}{2}(1 + \frac{2\rho g h}{2P_0 + \rho g h})$. $b.H = \frac{h}{2} + \frac{P_0}{\rho g}$

5.5.22 $x = \frac{\sqrt{33}-5}{2}a$

5.5.23 $P = P_0 + \rho g H$.

5.5.24 $P_1 = 0.17\ MPa$, $P_2 = 0.18\ MPa$.

5.5.25 N_2O_3.

5.5.26 $m = 210\ \frac{g}{m^3}$

5.5.27 $\frac{F_{He}}{F_{H_2}} = \frac{25}{27}$.

5.5.28 $M = 13,5m$

5.5.29 $r = 15cm$.

5.5.30 $m = \frac{\mu P_0 V(T-T_0)}{(RTT_0)}$.

5.5.31 $V = 15\ m^3$

5.5.32 $N = 0,28$

5.5.33 $T = T_0 \frac{\mu(P_0 r + 4\sigma)}{\mu P_0 r - 3\delta R T_0 \rho}$,
where R is the gas constant.

5.5.34 $\sigma = \frac{P}{4} \frac{r_1^3 + r_2^3 - r^3}{r^2 - r_1^2 - r_2^2}$

5.5.35 $T = 2\pi \sqrt{\frac{ml}{(2P_0 S)}}$

5.5.36 $V_3 = \frac{V_2^2}{V_1}$.

5.6 The first beginning of thermodynamics. Heat capacity

5.6.1 $\bar{\varepsilon}_{H_2} = \bar{\varepsilon}_{N_2} = (\frac{5}{2})kT, \bar{\varepsilon}_{H_2O} = \varepsilon_{CH_4} = 3kT$.

5.6.2 $U_1 = 0.25\ J, U_2 = 0.2\ MJ$.

5.6.3 It hasn't changed.

5.6.4 $P = \frac{P_1V_1 + P_2V_2}{V_1 + V_2}, T = T_1T_2 \frac{P_1V_1 + P_2V_2}{P_1V_1T_2 + P_2V_2T_1}$

5.6.5 Twice.

5.6.6 $v_{1max} \approx \sqrt{\frac{3P_0V_0m_2}{m_1(m_1+m_2)}} v_{2max} \approx \sqrt{\frac{3P_0V_0m_1}{m_2(m_1+m_2)}}$

5.6.7 $T_{max} = T_0 + \frac{2mv^2}{(3R)}$, where R is the gas constant

5.6.8 $v \approx 10\ \frac{m}{s}$

5.6.9 When expanding without heat, the gas performs work and cools down.

5.6.10 In isobaric expansion.

5.6.11 $a)\ A = PV; b)\ A = \frac{3PV}{2}$.

5.6.12 $A = 460\ J$.

5.6.13 $Q = (\frac{c}{R})(P_2V_2 - P_1V_1) + P_2(V_2 - V_1)$, where R is the gas constant.

5.6.14 $A = 2.6\ kJ$

5.6.15 $A = 240\ J$

5.6.16 $Q \approx 7.94\ kJ.\ A \approx 2.27\ kJ$.

5.6.17 $A \approx R(\sqrt{T_3} - \sqrt{T_1})^2$.

5.6.18 $T = T_0(1 + \frac{Mu^2}{3P_0V_0}), V = V_0(\frac{3P_0V_0}{3P_0V_0 + Mu^2})^{\frac{3}{2}}$.

5.6.19 $A = \frac{7\nu R(T_1 - T_2)}{2}$.

5.6.20 $\Delta t \approx 10°C$

5.6.21 $u_{max} = \sqrt{2gHh[1 - \frac{PS}{Mg} + \frac{PS}{Mg} ln \frac{PS}{Mg}]}$;

$u_{max} = \sqrt{2gHh[1 - \frac{5}{2}(\frac{PS}{Mg})^{\frac{3}{5}} + \frac{3}{2}\frac{PS}{Mg}}$

5.6.22 $V_1 = \sqrt{V_0V_2}, A_{min} = 5P_0V_0[(\frac{V_0}{V_2})^{\frac{1}{5}} - 1]$. Each compressor performs the work $\frac{A_{min}}{2}$.

5.6.23 $Q = 450\ kJ.\ \Delta U = 321\ kJ$

5.6.24 Oxygen

5.6.25 $T = T_0 + \frac{Q}{c}$ when $Q \leq Q_1 = \frac{cT_0F}{P_0S}$;

$T = \frac{Q + cT_0 + RT_0(1 + \frac{F}{2P_0S})}{c(1 + \frac{F}{P_0S}) + R(1 + \frac{F}{2P_0S})}(1 + \frac{F}{P_0S})$ when $Q \geq Q_1$

5.6.26 $Q = 10\rho g S h^2$

5.6.27 $c = (\frac{1}{(1-n)} + \frac{3}{2})R, n = \frac{5}{3}; n = 1$

5.6.28 Cooled

5.6.29 $c = \frac{2P_0V_0}{T_0}$

5.6.30 $x = \frac{3H(1 - \frac{PS}{M}g)}{5}$.

5.6.31 $x = \frac{3H(1 - \frac{PS}{M}g)}{5}$.

5.7 Gas leakage

5.7.1 $v = \sqrt{\frac{2cPT}{\mu}}$.

5.7.2 $v = \sqrt{\frac{7(k+1)RT}{(k\mu_1 + \mu_2)}}$.

5.7.3 $a.\ T \approx 3150\ K.\ b.\ v \approx 3\ \frac{km}{s}$.

5.7.4 $a)\ v \approx 5.2\ \frac{km}{s}; b)\ v \approx 5.7\ \frac{km}{s}; v \approx 7\ \frac{km}{s}$

5.7.5 $m = \frac{Mg}{\sqrt{\frac{2cPT}{\mu}}} \approx 3.8\ \frac{t}{s}$.

5.7.6 $v = \{\frac{2\gamma RT_1}{\mu(\gamma - 1)}[1 - (\frac{P_2}{P_1})^{\frac{(\gamma-1)}{\gamma}}]\}^{\frac{1}{2}}$.

5.7.7 $T \approx 120\ K, v \approx 1370\ \frac{m}{s}$

5.7.8 $T \approx 193\ K\ .\ P \approx 0.33\ MPa$

5.7.9 $v' = v\frac{1}{1+\gamma}[1 - \frac{P}{\rho v^2} + \sqrt{(\gamma + \frac{P}{\rho v^2})^2 - \frac{2(\gamma^2 - 1)q}{\rho S v^3}}]$

$F = \rho S v(v' - v)$, where $\rho = \frac{P\mu}{(RT)}$.

5.8 Probability of thermodynamic state

5.8.1 a . $t=\frac{\tau}{4}$. b. $t=\frac{\tau}{8}$. c. $t=\frac{\tau}{2^N}$

5.8.2 a. $p_1=\frac{1}{4}, p_2=\frac{1}{2}$. b. $p=\frac{1}{2}$. c. $p_2=\frac{3}{8}, p_0=\frac{1}{8}$.

5.8.3 a. $p=(1-\frac{V}{V_0})^N$ b. $V=V_0(1-10^{\frac{-2}{N}})$.

5.8.4 $p\sim 10^{-10^{15}}, V\sim 10^{-17}-10^{-18}\ cm^3$.

5.8.5 a. In the figure, the motion along the trajectory is expanded by mirror images into motion between two parallel lines. The corresponding points of the trajectories are marked with the same letters. It follows from this figure:

$v'\approx \frac{x}{2A'B'}v\approx v\Delta\sqrt{2}; \Delta\approx\frac{1}{2}[tg(\frac{\pi}{4}+\Delta)-1]=\frac{k}{2n}$,

where k and $n$are integers without a common divisor

$tg(\frac{\pi}{4}+\Delta)-1=\frac{k}{n}; h_1\approx\frac{2a\Delta}{k}, h_2=0$.

b. It is unlikely that $tg(\frac{\pi}{4}+\Delta)-1$ is exactly equal to a simple fraction such as 0.03, because there can be any number in the vicinity of this number, e.g., numbers like $0,03+\frac{\sqrt{2}}{n}$, n being an integer, which are as small as 0.03. These numbers are called irrational, and mathematics proves that the set of these numbers is more is more powerful than the set of prime fractions. If a number is irrational, then the trajectory is not closed. $p=\frac{S}{a^2}$

c. $p=\frac{V}{a^3}$.

5.8.6 a. $v'\approx v\Delta\sqrt{1-\frac{1}{m^2}}; tg\alpha=m, tg(\alpha+\Delta)-1=\frac{k}{n}, h_1=\frac{2\alpha\Delta}{k}, h_2=0$. b. $p=\frac{S}{a^2}$. c. $p=\frac{V}{a^3}$

5.8.7 $\tau\approx\frac{R}{v\Delta}; \tau'\sim\frac{\tau H}{R}$ at $H\gg R, \tau'\sim\frac{\tau R}{H}$ at $H\ll R$ and $\tau'\sim\tau$ at $H\sim R$

5.8.8

5.8.9 $p=(\frac{V}{V_0})^N$

5.8.10 $A=200\ kJ$.

5.8.11

5.8.12 $(1-\frac{V^2}{V_0^2})^N$ times.

5.8.13 $10^{4,8\cdot 10^{22}}$ times.

5.8.14 a. The probability of states that differ only in potential energy, are the same. Figures a and c show two states of an ideal gas half-full of the same volume and have the same probability. Moving from state a to state c at constant temperature, using two pistons as shown in the figures. The change in the logarithm of the state probability at this transition $\Delta S=\frac{NU}{T}+Nklnc$, where N is the number of gas molecules, c is the ratio of gas pressure values above and below the dashed line, separating regions of different potentials. But ΔS is zero. Therefore, $c=exp(\frac{-U}{kT})$.

5.8.15 Unreal.

5.8.16 Unreal. Real

5.9 The second beginning of thermodynamics

5.9.1

5.9.2 $\Delta S=1.2\ \frac{kJ}{K}$

5.9.3 $\Delta S=7\ \frac{kJ}{K}$

5.9.4 a,b. $\Delta S=(\frac{m}{\mu})Rln2$.

5.9.5 $a-c$. $\Delta S=\frac{m}{\mu}Rln\{\frac{V_2}{V_1}(\frac{T_2}{T_1})^{\frac{3}{2}}\}$

5.9.6 $\Delta S\approx 20\ \frac{J}{K}$

5.9.7 $\Delta S\approx 60\ \frac{J}{K}$

5.9.8 $\Delta S=(\frac{PV}{T})ln2$

5.9.9 $a.\Delta S=-\frac{Q_1}{T_1}+\frac{Q_2}{T_2}=\frac{5}{2}R\frac{(T_1-T_2)^2}{T_1T_2}$ where $-Q_1$ and Q_2 are the amount of heat transferred to the heater and the cooler during one cycle.

b. $\Delta S=\frac{R}{2}[\frac{3P_2V_2}{P_1V_1}+\frac{3P_1V_1}{P_2V_2}+\frac{2V_2}{V_1}+\frac{2V_1}{V_2}-10]$.

5.9.10 a. $\eta=1-(\frac{V_1}{V_2})^{\frac{2}{3}}$. b. $\eta=\frac{2(T_2-T_1)ln(\frac{P_2}{P_1})}{5(T_2-T_1)+2T_2ln(\frac{P_2}{P_1})}$.

5.9.11 It doesn't exist.

5.9.12 We can.

5.9.13 $\eta\approx 10,8\%, \eta=30\%$.

5.9.14 For any thermal cyclic process

$\frac{-Q_h}{T_h}+\frac{Q_r}{T_r}>0, Q_h-Q_r=A, \eta=\frac{A}{Q_h}$,

where T_h and T_r are the temperatures of the heater and refrigerators, respectively, $-Q_h$ and Q_r are the amount of heat transferred to the heater and refrigerator during one cycle, A is the work per cycle. From the above relations it follows, that the efficiency factor $\eta\leq\frac{(T_h-T_r)}{T_r}$, wherein the sign of equality takes place in case $\frac{Q_h}{T_h}-\frac{Q_r}{T_r}=0$, i.e. when entropy does not change.

5.9.15 During detonation, the entropy of the system increases.

5.9.16 $A\approx 33\ kJ$.

5.9.17 $A \approx 3 \cdot 10^{16}\ J.\ t \approx 60\ days$

5.9.18 $A = C[T - T_0 - T_0 ln(\frac{T}{T_0})]$.

5.9.19 $A = C_1T_1 + C_2T_2 - (C_1 + C_2)T_1^{\frac{C_1}{(C_1+C_2})} T_2^{\frac{C_2}{(C_1+C_2)}} \approx 32\ kJ$

5.9.20 Increase

5.9.21 $Q_{max} = \frac{A(1-\eta)}{\eta}$.

5.9.22 $N = 0,29\ MW$, $N' = 0,11\ W$.

5.9.23 $m = 5\ kg$.

5.9.24 $N = 138\ W$

5.9.25 $A = 46\ kJ$

5.9.26 No. The process lasts until there is saturation of the environment with water vapor.

5.10 Phase transitions

5.10.1 $t \approx 1\ h$.

5.10.2 No

5.10.3 In a vessel without a lid, water evaporates from the surface, which requires additional heat.

5.10.4 $\Delta p \approx 10^{-8}\ Pa$

5.10.5 13% water

5.10.6 A mixture of $100.5\ g$ water and $30.5\ g$ ice at $0\ °C$.

5.10.7 $m = 98\ \frac{g}{s}$.

5.10.8 $x \approx 0,11\ m$.

5.10.9 a. As long as there is water in the pot, the bottom temperature is about $100°C$. b. You can.

5.10.10

5.10.11 A layer of steam forms between the surface of the hot plate and the drop, which makes it difficult to bring heat to the water.

5.10.12 The low air temperature in the Dewar vial is maintained by boiling air, and the low temperature of solid carbon dioxide by its strong evaporation from the surface.

5.10.13 There is evaporation of ice in the dry air.

5.10.14 $v \approx 8\ \frac{m}{s}$.

5.10.15 Carbon tetrachloride boils off 25 times faster.

5.10.16 To prevent condensation of steam.

5.10.17 You can't.

5.10.18 At the critical temperature, liquid and vapor are indistinguishable.

5.10.19 Faster.

5.10.20 $m = 11,7\ g$

5.10.21 $P = 0.2\ MPa$, $A = 35\ kJ$.

5.10.22 $P = 0,37P_0$.

5.10.23 $\Delta v = \frac{m\lambda RT}{[P_0(\mu q+RT)]}.A = \frac{m\lambda RT}{(\mu q+RT)}$.

5.10.24 $h \approx 580\ m$.

5.10.25 5% water.

5.10.26 6% ice.

5.10.27 a . $n = exp(\frac{mgh}{RT}) = exp(\frac{2m\sigma}{\rho r})$. b. $\Delta h = 15\ cm$

5.10.28 $\Delta t = \frac{2\varphi\lambda\mu_{H_2O}P_H}{(7RP)} = 23°C$

5.10.29 It will be halved.

5.10.30 $P = P_0(\frac{R}{r})^2$

5.10.31 $P = 2P_0(\frac{R}{L})^2$

5.10.32 a. $m\sqrt{n}$ times. b. $P = 200P_0$.

5.10.33 $a = 1,0\ \frac{m}{s^2}$

5.10.34 $m_1 = 1,7\ \frac{kg}{s}, m_2 = 170\ frackgs$.

5.10.35 $T \approx 1720\ K$

5.11 Thermal radiation

5.11.1 a. $\Phi \approx 0.2\ kW$. b. $\varphi = 89\ \frac{MW}{m^2}$.

5.11.2 $T_1 \approx 600\ °C$, $T_2 \approx 2000\ °C$.

5.11.3 $w = 7.56 \cdot 10^{-16} T^4\ \frac{J}{m^3}$

5.11.4

5.11.5 a. Quartz, unlike steel, almost does not absorb visible light, so when heated, it emits much weaker radiation in the visible region.

b. Unlike black coal, which almost completely absorbs visible light, white chalk reflects that light. Therefore, when heated, chalk emits much less light and looks darker on the background of strongly emitting coal.

5.11.6 a. $T = \frac{T_0}{\sqrt[4]{2}}$. b. $T = \sqrt[4]{\frac{(T_1^4+T_2^4)}{2}}$.

5.11.7 $a.T = T_0\sqrt{\varepsilon(\frac{R}{2L})^2}$. b . $\varphi = 1.7\ \frac{kW}{m^2}$

5.11.8 $a.\ T = 200, 70, -35°C$. $b.\ \Phi \approx 4\cdot10^{26}\ W$. $c. T = 140\ °C$

5.11.9 $T = 2,4\ K$

5.11.10 $T = 20°C$

5.11.11 $\Phi = \frac{\varepsilon_1\varepsilon_2}{\varepsilon_1+\varepsilon_2-\varepsilon_1\varepsilon_2}\sigma S(T_1^4 - T_2^4)$

5.11.12 $a.\ T' = \frac{T}{\sqrt[4]{2}}$. $b.\ n = 32$

5.11.13 $T = \frac{T_0}{\sqrt[4]{6,5+\frac{4R}{r}}}$

5.11.14 $T = \sqrt[4]{T_1^4 - T_2^4 + [T_1 + \frac{\varepsilon}{2-\varepsilon}(T_1^4 - T_2^4)\frac{h}{\chi}]^4}$.

5.11.15

5.11.16 $a.\ a = \frac{S\Phi}{2\pi R^2 mc}$. $b.\ v = (\frac{1}{R_1} - \frac{1}{R_2})\sqrt{S\Phi}\pi mc$

5.11.17 a. The tail of the comet is affected by the pressure of the sun's rays. b . $r \approx 1\ \mu m$.

6 ELECTROSTATICS

6.1 Coulomb's law. Electric field strength

6.1.1 $a. F = 1.8\cdot10^4\ N$. $b.\ F = 2.3\cdot10^{-8}\ N$. $4.17\cdot10^{42}$ times.

6.1.2 $q \approx 1.05\cdot10^{-5}\ Cl \approx 3.16\cdot10^4\ CGS$.

6.1.3 $a.\ E = 1\ \frac{V}{m} = 3.3\cdot10^{-5}\ CGS$. $b.\ E = 3\cdot10^5\ \frac{V}{m} = 10\ CGS$.

6.1.4 At a distance of $1\ m$ $E_1 = 9\cdot10^{10}\ \frac{V}{m} = 3\cdot10^6\ CGS$; at a distance of $20\ m$ $E_2 = 2.25\cdot10^8\ \frac{V}{m} = 7.5\cdot10^3\ CGS$.

For a charge of $0.001\ Cl$, $F_1 = 9\cdot10^7\ N$, $F_2 = 2.25\cdot10^5\ N$;

per $1,000\ CGS\ charge F_1 = 3\cdot10^9\ dyn$, $F_2 = 7.5\cdot10^6\ dyn$

6.1.5 $F = 2.56\cdot10^9 N$

6.1.6 $q = 3.5\cdot10^3\ Cl$

6.1.7 $T_{12} = \frac{q_1(4q_2+q_3)}{16\pi\varepsilon_0 l^2}, T_{23} = \frac{q_3(4q_2+q_1)}{16\pi\varepsilon_0 l^2}$

6.1.8 At a distance $x = \frac{l\sqrt{q_1}}{(\sqrt{q_1}+\sqrt{q_2})}$ from the charge $q1$. Yes. No.

6.1.9 $q = l\sqrt{8\pi\varepsilon_0 mg}$

6.1.10 $T = \frac{1}{4\pi\varepsilon_0 l^2}(Q^2 - \frac{q^2}{3\sqrt{3}})$.

6.1.11 $\beta = 2arctg(\frac{q}{Q})^{\frac{2}{3}}, \alpha = \pi - \beta$.

6.1.12 $r = 1,4\cdot10^{-8}\ cm$

6.1.13 $\omega = q\sqrt{\frac{(3\sqrt{2}-4)}{(8\pi\varepsilon_0 ml^3)}}$.

6.1.14 $q_{min} = \frac{32\pi\varepsilon_0 mgR^2}{Q}$

6.1.15 $k = \frac{q^2\sqrt{a^2+l^2}}{32\pi\varepsilon_0 a^3(\sqrt{a^2+l^2}-l)}$

6.1.16 $T = \frac{q^2}{8\pi\varepsilon_0 l^2}(\frac{9}{4} + \frac{\sqrt{3}}{3})$

6.1.17 $E_1 = 0, E_2 = \frac{Qh}{[(4\pi\varepsilon_0(R^2+h^2)^{\frac{3}{2}}]}$.

6.1.18 $E = \frac{\rho l}{[4\pi\varepsilon_0 x(l+x)]}$.

6.1.19 $a)\ E = \frac{\sigma}{(6\varepsilon_0)}$; $b)\ E = \frac{(\sigma_1-\sigma_2)}{(4\varepsilon_0)}$; $c)\ E = \frac{\sigma}{(2\varepsilon_0)}$; $d)\ E = \frac{\sqrt{\sigma_1^2+\sigma_2^2+\sigma_3^2-\sigma_1\sigma_2-\sigma_2\sigma_3-\sigma_1\sigma_3}}{(3\varepsilon_0)}$; $e)\ E = \frac{\rho h(1-\cos\alpha)}{(2\varepsilon_0)}$; $f) E = \frac{\sqrt{3}l\rho}{(12\varepsilon_0)}$.

6.1.20 b. Yes

6.1.21 $a.\ q = \sqrt{10}Q$. $b.\ q = 9Q$.

6.2 The flux of electric field strength. Gauss' theorem

6.2.1 $a.\ \Phi = \frac{El^2}{2}$. $b.\ \Phi = -Eh^2, \Phi = Eh^2$.

6.2.2 $\Phi = E\cos\alpha\cdot\pi(R^2 - r^2)$.

6.2.3

6.2.4 $F = \sigma\Phi$

6.2.5 $a. F_1 = F_2 = \frac{q\sigma}{(2\varepsilon_0)}, E = \frac{\sigma}{(2\varepsilon_0)}$. $b.\ F = \frac{\sigma q}{(4\varepsilon_0)}$.

6.2.6 $a)\ E = 0$ at $r < R, E = \frac{Q}{(4\varepsilon_0 r^2)}$ at $r > R$; $b)\ E = \frac{\rho}{(2\pi\varepsilon_0 r)}$; $c)\ E = \frac{\sigma}{(2\varepsilon_0)}$; $d)\ E = \frac{\rho r}{(3\varepsilon_0)}$ at $r \le R; E = \frac{\rho R^3}{(3\varepsilon_0 r^2)}$ at $r \ge R$; $e)\ E = \frac{\rho r}{(2\varepsilon_0)}$ at $r \le R; E = R; E = \frac{\rho R^2}{(2\varepsilon_0 r)}$ at $r \ge R$; $f)\ E = \frac{\rho x}{\varepsilon_0}$ at $x \le \frac{h}{2}$ (x is the distance from the central plane of the plate); $E = \frac{\rho h}{(2\varepsilon_0)}$ at $x \ge \frac{h}{2}$.

6.2.7 $a)\ \rho = \frac{2E_0\varepsilon_0}{r}$; $b)\ \rho = \frac{E_0\varepsilon_0}{r}$.

6.2.8 The force acting on the selected face of the cube, $F = \sigma\int E_n ds$, where $\int Ends$ is the flux through this face the electric field strength created by other five faces. As a closed surface, let us construct a cube slightly larger than the given one. Then all six charged faces give a flux of electric field strength through all six sides of the constructed surface $\Phi = \frac{q}{\varepsilon_0} = \frac{6\sigma l^2}{\varepsilon_0}$ and through one face $\Phi_0 = \frac{\sigma l^2}{\varepsilon_0}$. But

$\Phi' = \int E_n ds + \frac{\sigma l^2}{2\varepsilon_0}$, therefore,

$\int Ends = \frac{\sigma l^2}{\varepsilon_0} - \frac{\sigma l^2}{2\varepsilon_0} = \frac{\sigma l^2}{2\varepsilon_0}$.

So, the force $F = \frac{\sigma^2 l^2}{(2\varepsilon_0)}$. Similarly, for the tetrahedron we obtain

$F = \sqrt{3}\frac{\sigma^2 l^2}{(8\varepsilon_0)}$.

6.2.9 Between planes $E_1 = \frac{\sigma}{\varepsilon_0}, E_2 = 0$. Outside the planes $E_1' = 0, E_2' = \frac{\sigma}{\varepsilon_0}$.

6.2.10 $E_1 = (\frac{\sigma}{\varepsilon_0})sin(\frac{\alpha}{2}), E_2 = (\frac{\sigma}{\varepsilon_0})cos(\frac{\alpha}{2})$.

6.2.11 $E_{max} = \frac{\rho h}{\varepsilon_0}$

6.2.12 $E_A = \frac{\rho h}{(6\varepsilon_0)}, E_B = \frac{\rho_h}{(3\varepsilon_0)}, E_{(r)} = \frac{\rho r}{(3\varepsilon_0)}$.

6.2.13 At any point inside the cavity the field strength is directed along the line,connecting the centers of the ball and the cavity, and $E = \frac{\rho l}{(3\varepsilon_0)}$.

Outside the cavity $E = \frac{\rho}{3\varepsilon_0}[x + \frac{r^3}{(l-x)^2}]$ at $0 < x < l - r; E = \frac{\rho^3}{\varepsilon_0}[x - \frac{r^3}{(x-l)^2}]$ at $l + r < x < R; E = \frac{\rho^3}{\varepsilon_0}[R^2x^2 - \frac{r^2}{(x-l)^2}]$ at $x > R$.

6.2.14 $a.\ E = \frac{\rho l}{(3\varepsilon_0)}$. $b.\ \sigma = 3\varepsilon_0 E\cos\alpha$, where α is the angle between the field direction and the radius drawn to a point on the sphere. $\sigma_{max} = 3\varepsilon_0 E$.

6.3 Electric field potential. Conductors in a constant electric field

6.3.1 $a.v = 10^7\ \frac{m}{s}$. $b.v = 1,25 \cdot 10^6\ \frac{m}{s}$

6.3.2 $a.\Delta\varphi = 850\ V.\ v = \sqrt{3} \cdot 10^7\ \frac{m}{s}.\ b.v = 8.8 \cdot 10^6\ \frac{m}{s}$

6.3.3 $\varphi = 2,7 \cdot 10^8\ B.$

6.3.4 $\varphi = \frac{\sqrt{2}q}{(\pi\varepsilon_0 l)}$

6.3.5 $\Delta\varphi \approx -11,9\ B.$

6.3.6 $\varphi = 13,5\ kV = 45\ CGS.$

6.3.7 $\varphi = \frac{Q}{(4\pi\varepsilon_0 R)}$. No. Yes.

6.3.8

6.3.9

6.3.10 $b.E_{max} = \frac{nQ}{(4\pi\varepsilon_0 R^2)}, E_{min} = \frac{Q}{(4\pi\varepsilon_0 R^2)}.\ c.E = \rho(2\pi b).$

6.3.11 $\sigma_1' = \frac{(\sigma_1+\sigma_2)}{2}, \sigma_1'' = \frac{(\sigma_1-\sigma_2)}{2}, \sigma_2' = -\frac{(\sigma_1-\sigma_2)}{2}, \sigma_2'' = \frac{(\sigma_1+\sigma_2)}{2}$.

6.3.12 $a.\Delta\varphi = 37.7\ GHS = 11.3\ kV.\ b.\ \varphi = 18.8\ GHS = 5.65\ kV.$

6.3.13 $\varphi_3 - \varphi_1 = \frac{[(\sigma_3-\sigma_1)(h_1+h_2)+\sigma_2(h_1-h_2)]}{(2\varepsilon_0)}$.

6.3.14 $E_{12} = \frac{\varphi}{a}; E_{23} = \frac{\varphi}{b}$.

6.3.15 $a.$ The field strength near the upper plate $E_u = \frac{\sigma b}{[\varepsilon_0(a+b)]}$, near theof the lower plate $E_n = \frac{\sigma a}{[\varepsilon_0(a+b)]}$. Accordingly, the surface density $\sigma_v = -\frac{\sigma b}{(a+b)}, \sigma_n = -\frac{\sigma a}{(a+b)}$.

$b.\ q_a = -\frac{qb}{(a+b)}; q_b = -\frac{qa}{(a+b)}$.

6.3.16 $Q' = -Q, \sigma = \frac{Q}{(4\pi R^2)}, E = \frac{(Q+q)}{(4\pi\varepsilon_0 L^2)}$. No. No.

6.3.17 The surface of the cavity has a charge $-q$, and the surface of the conductor has a charge q,which (except for the area near the ends of the conductor) is uniformly distributed over thesurface of the conductor. Therefore, $E = 0$ at $0 < x < r, E \approx \frac{q}{(2\pi\varepsilon_0 xL)}$ at $r < x < R, E = 0$ at $x > R; x$ is the distance from the axis.

6.3.18 The surface charge density at the corresponding parts of the conductor surface will remain the same.

6.3.19 See Fig.

6.3.20 $\varphi_1 = \frac{q}{(4\pi\varepsilon_0 r)}, \varphi_2 = \frac{q}{(8\pi\varepsilon_0 r)}, \varphi_3 = 0.$

6.3.21 $q_r = -8\pi\varepsilon_0 r\varphi, q_{2r} = 16\pi\varepsilon_0 r\varphi$

6.3.22 $\varphi_1 = \varphi\frac{(R_2-R_1)}{R_2}, \varphi_2 = \frac{\varphi R_1}{R_2}$.

6.3.23 $E = 0, \varphi = 0$ for $r > R_2; E = \frac{q}{4\pi\varepsilon_0 r^2}, \varphi = \frac{q}{4\pi\varepsilon_0}(\frac{1}{r} - \frac{1}{R_2})$ at $R_1 < r < R_2; E = 0, \varphi = \frac{1}{4\pi\varepsilon_0}(\frac{1}{R_1} - \frac{1}{R_2}$ at $r < R_1$.

6.3.24 $E = \frac{q}{4\pi\varepsilon_0 r^2}(1 - \frac{R_1}{R_2}), \varphi = \frac{q}{4\pi\varepsilon_0}(1 - \frac{R_1}{R_2})$ at $r > R_2; = -\frac{qR_1}{4\pi R_2 r^2}, \varphi = \frac{q}{4\pi\varepsilon_0 R_2}(1 - \frac{R_1}{r}$ at $R_1 < r < R_2; E = 0, \varphi = 0$ at $r < R_1$.

6.3.25 $E = \frac{\rho r}{3\varepsilon_0}, \varphi = \frac{\rho}{2\varepsilon_0}(R^2 - \frac{r^2}{3}$ at $0 < r < R; E = \frac{\rho R^3}{3\varepsilon_0 r^2}, \varphi = \frac{\rho R^3}{3\varepsilon_0 r}$ at $r > R$.

6.3.26 $\Delta\varphi = \frac{\rho R^2}{(6\varepsilon_0)}, \Delta\varphi = \frac{\rho R^2}{(4\varepsilon_0)}, \Delta\varphi = \frac{\rho h^2}{(8\varepsilon_0)}$.

6.3.27 $\varphi = \frac{\rho}{2\varepsilon_0}(r^2 ln\frac{R}{r} + \frac{r^2}{2} - \frac{x^2}{2}$ at $0 < x < R; \varphi = \frac{\rho r^2}{2\varepsilon_0}ln\frac{R}{x}$ at $r < x < R$.

6.3.28 $F = \frac{Q^2}{(16\pi\varepsilon_0 h^2)}$.

6.3.29 $F = \frac{3Q^2}{(32\pi\varepsilon_0 h^2)}$.

6.3.30 No$F = \frac{q^2(2\sqrt{2}-1)}{(32\pi\varepsilon_0 l^2)}$.

6.3.31 $\varphi = \frac{q}{(4\pi\varepsilon_0 L)}$.

6.3.32 $Q = -\frac{qR}{L}$.

6.3.33 Will increase by $F = \frac{Qq}{(4\pi\varepsilon_0 L^2)}$ at $L > R$; will not change at $L < R$

6.3.34

6.3.35 $h = \frac{3q^2R^2}{(16\pi^2\varepsilon_0\rho g r^6)}$, where h is counted from the center of the ball.

6.3.36 $'0 = \frac{qQ}{(Q-q)}$.

6.3.37

6.3.38

6.3.39 It will increase threefold.

6.3.40 $C = 4\pi\varepsilon_0 R$

6.4 Capacitors

6.4.1

6.4.2 a. Increased by a factor of four. b. Decreased by a factor of two; decreased by a factor of n.

6.4.3 $a.\ C = \frac{\varepsilon_0 S}{d}.\ b.\ C = 5.3\ cm = 5.9pF.$

6.4.4 a. Increase one and a half times. b. Increase $1 + \frac{S'}{(2S)}$ times. c. It will not change.

6.4.5 $C = 4\pi\varepsilon_0 \frac{R_1R_2}{R_1-R_2}$.

6.4.6 $C = 4\pi\varepsilon_0[\frac{1}{R_1} - \frac{1}{R_2} - \frac{d}{R_0(R_0-d)}]^{-1}$.

6.4.7 $C = \frac{2\pi\varepsilon_0 l}{ln(\frac{R_2}{R_1})}$.

6.4.8 $C = \frac{\varepsilon_0 al}{d}(1 + \frac{ld}{2\pi R^2})$.

6.4.9 $a.C = \frac{C_1C_2}{(C_1+C_2)}, C = C_1 + C_2.\ b.\ C_0 = \frac{4C}{3}.\ c.\ C_0 = \frac{7C}{5}.\ d.\ C_0 = \frac{(\sqrt{5}-1)C}{2}.\ e.\ C_0 = \frac{6C}{5}.$

6.4.10 $q = \pm\varepsilon_0 SE$.

6.4.11 $\Delta V = \frac{a}{d+a}(V_1 + V_2)$.

6.4.12 a. It will increase one and a half times. b. It will increase two times.

6.4.13 $\Delta q = \frac{qx}{d}$.

6.4.14 $\frac{V}{V_0} = (\frac{Cd+2\varepsilon_0 S}{Cd+\varepsilon_0 S})^{4n}$.

6.4.15 $F = 4.4 \cdot 10^{-2}\ N$. No.

6.4.16 Will increase by a factor of k. Will increase by a factor of n^2times.

6.4.17 $a)W = 4.4\ mJ; b)W = \frac{2\pi\varepsilon_0 r_1 r_2 V^2}{(r_2-r_1)}; c)\ W = \frac{\pi\varepsilon_0 lV^2}{[ln(\frac{r_2}{r_1})}]$.

6.4.18 $a.\ A = \frac{Q^2d}{(2\varepsilon_0 S)}.b.\ A = \frac{Q^2dx}{[2\varepsilon_0 a^2(a-x)]}.c.A_a = \frac{Q^2d}{(4\varepsilon_0 A)}; A_b = \frac{Q^2dx}{(2\varepsilon_0 a^3)}$.

6.5 Electric pressure. Electric field energy

6.5.1 $a.F = \frac{\sigma^2 S}{(2\varepsilon_0)}.P = \frac{\sigma^2}{(2\varepsilon_0)}.\ b.\ \sigma = \varepsilon_0 E.P = \frac{\varepsilon_0 E^2}{2}(inSI), P = \frac{E^2}{(8\pi)}\ (inSGS)\ .c.\ P = 4.325\ Pa$, $\sigma = 8.85\ \frac{\mu Cl}{m^2}$.

6.5.2 Will decrease by $1 + \frac{Q^2}{(2P_0\varepsilon_0 S^2)}$ times.

6.5.3 By Gauss theorem, we determine the surface charge density at the interface: $\sigma = \varepsilon_0 E$. Using the superposition principle,

$E' - \frac{\sigma}{(2\varepsilon_0)} = E$, $E' + \frac{\sigma}{(2\varepsilon_0)} = 2E$,

we find the external field strength: $E' = \frac{3E}{2}$. The force that acts on the charge falling on the unit of the surface area of the interface of the fields, i.e. the pressure from the external $P = E'\sigma = \frac{3\varepsilon_0 E^2}{2}$. For fields E and $-2E$, reasoning similarly, we obtain $\sigma = -3\varepsilon_0 E$ and $E' = \frac{-E}{2}$. Thus, in the second case the surface charge density is three times is greater, but the external field strength is three times less. Therefore the electric pressure will be the same: $P = E'\sigma = \frac{3\varepsilon_0 E^2}{2}$.

6.5.4 $P = \frac{\rho^2 h^2}{(2\varepsilon_0)}$.

6.5.5 $P = \frac{Q^2}{(32\pi^2\varepsilon_0 R^4)}$ (see solution of Problem 6.5.3).

6.5.6 $P = \frac{\varepsilon_0 R^2 V^2}{[2r^2(R-r)^2]}$.

6.5.7 $\rho = 2\pi R\sqrt{2\varepsilon_0 P}$.

6.5.8 $a.\ F_1 = \frac{q\sigma}{(4\varepsilon_0)}, F_2 = \frac{\sqrt{2}q\sigma}{(8\varepsilon_0)}, F_3 = \frac{\sqrt{3}q\sigma}{(16\varepsilon_0)}; E_1 = \frac{\sigma}{(4\varepsilon_0)}, E_2 = \frac{2\sigma}{(8\varepsilon_0)}, E_3 = \frac{\sqrt{3}\sigma}{(16\varepsilon_0)}.\ b.\ E = \frac{R\rho}{(4\varepsilon_0)}$.

6.5.9 $F = \frac{Q^2(R^2-h^2)}{(32\pi\varepsilon_0 R^4)}; q = \frac{-Q}{2}$.

6.5.10

6.5.11 $A = 2E\sigma dS$.

6.5.12 $a.\ \sigma = \varepsilon_0 E, P = \frac{\varepsilon_0 E^2}{2}.\ b.\ A = \frac{\varepsilon_0 E^2 hS}{2}$.

6.5.13 $A = \frac{\sigma^2 Sh}{(2\varepsilon_0)}$.

6.5.14 $A = \varepsilon_0 ShE_0(E_0 - E)$.

6.5.15 $W = \frac{Q^2}{(8\pi\varepsilon_0 R)}\ (inSI)\ ; W = \frac{Q^2}{(2R)}\ (inGHS)$.

6.5.16 $r = 1,4 \cdot 10^{-15}\ m$

6.5.17 $1,400$ times.

6.5.18 $W = \frac{3Q^2}{(20\pi\varepsilon_0 R)}$.

6.5.19 $A = \frac{Q^2}{(8\pi\varepsilon_0 R)}$.

6.5.20 $A = \frac{Q^2}{8\pi\varepsilon_0 R}(1 - N^{\frac{-2}{3}})$.

6.5.21 At n times

6.5.22 $A_3 = 3A; A_n = \frac{n(n-1)}{2}A$.

6.5.23 $A' = 6A$.

6.5.24 $A' = \sqrt{2}A$.

6.5.25 $\Delta W = \frac{Q^2}{(4\pi\varepsilon_0 l)}$

6.5.26 $A = \frac{(Q_1\Delta\varphi_1 + Q_2\Delta\varphi_2)}{2}$.

6.5.27 $a)F = \frac{2Q^2 dc(d-c)}{[\varepsilon_0 a^3(2d-c)^2]}$;

$b)F = \frac{\varepsilon_0 acV^2}{[2d(d-c)]}$.

6.5.28 $A = \frac{Q^2 ab}{[2S\varepsilon_0(a+b)]}$.

6.5.29 $F = \frac{q^2 Sd}{(8\pi^2\varepsilon_0 r^5)}$.

6.5.30 The field strength of the charges distributed over a spherical shell, the hole in which is closed with a cork, in the center of the sphere is zero and can be represented as

$E(0) = E_{oftheplug} + E_{ofthespherewithouttheplug} = 0$.

At $\Delta \ll r \ll R$ the plug field is a dipole field, the field strength of which at point O is $E_{ofplug}(0) = \frac{q'\Delta}{(2\pi\varepsilon_0 R^3)}$. After removal of the plug, redistribution of charges on the remaining part of the spherical shell at Δr, will be negligible, and for estimation it can be assumed that

$E(0) = E_{spherewithoutcork} \approx -E_{cork} = \frac{-q'\Delta}{(2\pi\varepsilon_0 R^3)}$.

By Gauss theorem $q' = \frac{-qr^2}{(4R^2)}$. Taking this into account we have

$E(0) = \frac{q^2 r^2 \Delta}{(8\pi_0\varepsilon R^5)}$.

6.6 Electric field in the presence of a dielectric

6.6.1

6.6.2 $p = 7.4 \cdot 10^{-37}\ Cl \cdot m$.

6.6.3 $p_{av} = 1 \cdot 10^{-34}\ Cl \cdot m$.

6.6.4 $\sigma_{pr} = \pm\frac{\sigma(\varepsilon-1)}{\varepsilon}$. Field strength:

$E = \frac{\sigma}{(\varepsilon_0\varepsilon)}$ - in dielectric,

$E = \frac{\sigma}{\varepsilon_0}$ - in the gap.

The potential difference between the plates $V = (\frac{\sigma}{\varepsilon_0})(d - h + \frac{h}{\varepsilon})$.

6.6.5 $E = E_0\sqrt{sin^2 + \frac{(cos^2\alpha)}{\varepsilon^2}}$.

6.6.6 It will increase ε times.

6.6.7 $\varepsilon = 2$.

6.6.8 $q = (\varepsilon - 1)CV$

6.6.9 $\Delta V = \frac{\varepsilon-1}{\varepsilon+1}V.\Delta V = \frac{\varepsilon-1}{\varepsilon(n-1)+1}V$

6.6.10 $\Delta V = \frac{k(\varepsilon-1)V}{n}$

6.6.11 $C = \frac{\varepsilon_0(\varepsilon_1+\varepsilon_2)S}{(2d)}$.

6.6.12 $C = \frac{\varepsilon_0\varepsilon_1\varepsilon_2 S}{\varepsilon_2 d_1 + \varepsilon_1 d_2}; q_{pol} = \frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1\varepsilon_2}q$.

6.6.13 $C = \frac{\varepsilon_0 d_2(\varepsilon-1)(S_2-S_1)+\varepsilon_0\varepsilon d_1 S_1}{d_1(d_1-d_2)+d_1 d_2}$

6.6.14 $\rho = \frac{-q}{(\varepsilon_1 Sd)}$.

6.6.15 a. To the thread. b. $F_2 = \frac{\varepsilon_1(\varepsilon_2-1)}{\varepsilon_2(\varepsilon_1-1)}F_1$. c. $F \sim V_2, F \sim \frac{1}{r^3}\ d$. In $(\frac{R}{r})^3$ times.

6.6.16 $F = \frac{(\varepsilon-1)SQ^2}{8\pi^2\varepsilon_0\varepsilon R^5}\delta$

6.6.17 $M = \frac{\varepsilon_0(\varepsilon-1)SdE^2 siN_2\alpha}{2\varepsilon}; A = -\frac{\varepsilon_0(\varepsilon-1)SdE^2 sin^2\alpha}{2\varepsilon}$.

6.6.18 $\sigma_{internal} = \frac{(\varepsilon-1)Q}{4\pi\varepsilon r^2}, \sigma_{external} = \frac{(\varepsilon-1)Q}{4\pi\varepsilon R^2}$. See Fig.

6.6.19 $.P = \frac{(\varepsilon-1)Q^2}{32\pi_2\varepsilon_0\varepsilon}(\frac{1}{r^4} - \frac{1}{R^4})$.

6.6.20 $F = \frac{Q^2 d}{2\varepsilon_0 b}\frac{\varepsilon-1}{[a+x(\varepsilon-1)]^2}$

6.6.21 $h = \frac{\varepsilon_0(\varepsilon-1)V^2}{(2\rho g d^2)}$.

6.6.22 $h = \frac{(\varepsilon-1)Q^2}{(2\varepsilon_0\varepsilon\rho g S^2)}$

6.6.23 $W = \frac{q^2}{2C}\frac{\varepsilon-1}{\varepsilon}$.

6.6.24 $W = \frac{V^2 C}{2}(\varepsilon - 1)$.

6.6.25 $V = [\frac{2W}{(\varepsilon_1-\varepsilon_2)C}^{\frac{1}{2}}].E \sim 10^9\ \frac{V}{m}$.

6.6.26 The dipole moments in the dielectric are oriented in the electric field with a lag; $\varepsilon = 2$.

6.6.27 a. $V' = \frac{\varepsilon-1}{\varepsilon}V$. b. $\Delta T \sim 10^{-5}\ K$.

6.6.28 $r = 0.12\ nm$.

6.6.29 $p = 4\pi\varepsilon_0 r^3 E$.

6.6.30 $\varepsilon = 1 + 4\pi r^3 n$.

7 MOTION OF CHARGED PARTICLES IN AN ELECTRIC FIELD

7.1 Motion in a constant electric field

7.1.1 When the initial velocity of a particle is directed along a straight line of force.

7.1.2 $t = 0.56\ \mu s$; $x = 2.8\ m$.

7.1.3 $E = \frac{2(d-vt)m}{(qt^2)}$.

7.1.4 $u = v\sqrt{1 + \frac{2qEd}{(mv^2)}}, tg\beta = tg\alpha\sqrt{1 + \frac{2qEd}{(mv^2 sin^2\alpha)}}$.

7.1.5 Twice as much.

7.1.6 $K = \frac{qEl}{[2cos^2\alpha(tg\alpha+tg\beta)]}$.

7.1.7 $N = \frac{neUbl^2}{(2m_e vd)}$.

7.1.8 $q = 10^{-17}\ Cl$.

7.1.9 $\beta = \alpha - arctg\frac{\sqrt{1+sin^2\alpha}}{cos\alpha}; tg\beta = \frac{cos\alpha\sqrt{1+cos^2\alpha}-cos^2\alpha}{cos\alpha\sqrt{1+cos^2\alpha}+sin^2\alpha}$

7.1.10 $tg\beta = tg\alpha\sqrt{\frac{1+2e(\varphi_2-\varphi_1)}{(m_e v^2 sin^2\alpha)}}$

7.1.11 $V = 19\ kV$.

7.1.12 $V = \frac{(\frac{V_0}{2})}{ln(\frac{R_2}{R_1})}$.

7.1.13 The velocity will not change, but the time of the positron's flight will be longer. The positron can not reach point B at all if its initial kinetic energy K_0 is less than $e\varphi_0$.

7.1.14 $t' = t\sqrt{3}$.

7.1.15 $\sqrt{n}$

7.1.16 $K \to 0$ when $l \to 2R; K = \frac{eQ}{4\pi\varepsilon_0}(\frac{1}{R} + \frac{1}{l-R} - \frac{4}{l})$ at $l > 2R$. The condition of minimality will be the arrival of the electron at the midpoint of the interval connecting the centers of the spheres with zero velocity.

7.1.17 $t = \frac{2R}{v}[1 - \frac{qQ}{(m+M)2\pi\varepsilon_0 RmMv^2}]^{\frac{-1}{2}}$.

7.1.18 $v = \sqrt{\frac{qEl}{m}}$.

7.1.19 $T = 2\pi\sqrt{\frac{ml}{(2qE)}}$.

7.1.20 $T = 2\pi\sqrt{\frac{ml}{(mg+qE)}}$ when $mg + qE > 0; T = 2\pi\sqrt{\frac{ml}{\sqrt{(mg)^2+(qE)^2}}}$.

7.1.21 $\omega = \sqrt{\frac{qQ}{(2\pi\varepsilon_0 ml^2)}}$.

7.1.22 $\omega = \sqrt{\frac{g}{l} - \frac{qQ(h+l)}{4\pi\varepsilon_0 mh^3 l}}$ at $\frac{qQ(h+l)}{4\pi\varepsilon_0 h^3} < mg$.

7.1.23 $T = 2\pi(\frac{g}{R} + \frac{qQ}{32\pi\varepsilon_0 mR^3})^{\frac{-1}{2}}$ at $\frac{qQ}{32\pi\varepsilon_0 R^2} > -mg$.

7.1.24 $\Delta t = \frac{\pi\varepsilon_0 R(m_e v^2)^2}{(2e^2C)}$.

7.1.25 $\alpha = \frac{\pi}{4}; \delta \approx \frac{4K(\Delta\alpha)^2}{(eE)}$.

7.1.26 $k = \frac{l}{(2d)}$.

7.1.27 $b.\ p_1 = \frac{q_1 q_2}{(2\pi\varepsilon_0 vr)}.\ c.\ l = 3,4 \cdot 10^{-13}\ m$.

7.1.28 $k = \frac{1}{(2\varepsilon_0)}$.

7.2 Focusing of charged particles

7.2.1 Increase by half.

7.2.2 $x = v\frac{\pi}{2}\sqrt{\frac{m_e\varepsilon_0}{(e\rho)}}$ at $x \leq l; x = l + v\sqrt{\frac{2m_e\varepsilon_0}{e\rho}}ctg(\frac{1}{v}\sqrt{\frac{e\rho}{(2m_e\varepsilon_0)}}R)$ at $x \geq l$.

7.2.3 a. By a factor of k will increase. b. By a factor of k will decrease.

7.2.4

7.2.5 $y = \frac{y_0 f}{(x_0-f)}$.

7.2.6 a. Does not depend. b. If the electron moves along a trajectory close to the straight line AA',then the transverse momentum that the electron will receive in the region of the hole is close to $p \perp= \frac{e\Phi}{(2\pi rv)}$, where $\Phi = \pi r^2 E$ is the flux of electric field strength across the surface of a cylinder of radius r in the region of the hole, v is the speed of the electron in that region. The focal distance $f = \frac{-r \cdot m_e v}{p\perp} = -\frac{2m_e v^2}{(eE)} = -4d$. (The minus sign at f means that electron scattering occurs. scattering of electrons.)

7.2.7 $f = \frac{4}{3}d[(1 + \frac{V_0}{V})(2\frac{V_0}{V} + 2\sqrt{\frac{V_0}{V}(\frac{V_0}{V+1})} - 1)]$ at $V < 8V_0$. At a distance of $\frac{8dV_0}{V}$ from the first cover at $V > 8V_0$.

7.2.8 $f = d(\frac{4V_0}{V})^2$.

7.2.9 A particle of mass m having charge q and flying with velocity v through a charged ball will receive from the ball field a transverse momentum $p \perp= \frac{q\Delta q}{(2\pi\varepsilon_0 vx)}$, where $x \ll R$ is the minimum distance between the particle and the center of the ball, $\Delta q \approx \pi x^2\rho 2R$ is the charge of the area of the sphere cut by a cylinder of radius $x; f = \frac{mv}{p\perp}\frac{1}{x} = \frac{R}{2}\frac{V_0}{V}$.

7.2.10 At a distance $f = 2R(\frac{V_0}{V})^2$ from the center of the spheres.

7.2.11 $x \approx \frac{1}{(\frac{1}{f} - \frac{1}{L})}$, where $f = 2R(\frac{V_0}{V})^2$.

7.2.12 $\Delta E \perp = \frac{(a-b)^2}{(16d^2)}$

7.2.13 $V = V_0\sqrt{\frac{2d}{l}}$.

7.2.14 $f = \frac{4V(2E_2-E_1)}{(E_2-E_1)^2}$

7.3 Motion in an alternating electric field

7.3.1 $t = \frac{2m_e l}{(eE\tau)}$

7.3.2 $a)\sqrt{\frac{2eV}{m_e}} - \frac{e\Delta V\tau}{m_e l} \leq v \leq \sqrt{\frac{2eV}{m_e}} + \frac{e\Delta V\tau}{m_e l}$;

$b) v_1 = \sqrt{\frac{2e(V+\Delta V)}{m_e}}; v_2 = \sqrt{\frac{2e(V-\Delta V)}{m_e}}$ at $\Delta V < V$.

7.3.3 $\nu_{max} \approx 10^9\ Hz$

7.3.4 $a.\ S = \frac{Ll}{(2Vd)}$. $b.\ S = 0.09\ \frac{mm}{V}$.

7.3.5 The circle radius is 5 cm.

7.3.6 $\nu > l\sqrt{\frac{2eV}{m_e}}$.

7.3.7 $V = \frac{\pi\delta\nu\tau}{(2S)}$.

7.3.8 $\frac{e}{m_e} = \frac{l^2 f^2}{2V(n+\frac{1}{2})^2}$ where n is an integer

7.3.9 $\Delta\alpha = \pm arctg\{\frac{V_0}{d\omega}\sqrt{\frac{2e}{m_e V}}[1 - cos(\omega l\sqrt{\frac{m_e}{2eV}})]\}$.

7.3.10 $a. v = \frac{\omega l}{(2\pi n)}$. $b.\ \Delta b = \frac{4\pi e V_0 n}{(m_e\omega^2 d)}$, where n is an integer.

7.3.11 $|u_{max}| = \frac{2eE_0}{m_e\omega}|cos\varphi|, v_{av} = \frac{eE_0}{m_e\omega}\cos\varphi$.

7.3.12 $K = 0.4\ keV$.

7.3.13 Because of the departure from the plasma of the electrons accelerated by the high-frequency electric plasma's potential will increase until even the fastest electrons even the fastest electrons can no longer escape. $V = \frac{2eE_0^2}{m_e\omega_0^2}(\frac{E_0}{\omega})^2$.

7.3.14 $A = \frac{eE_0}{[m_e\sqrt{(\omega^2-\omega_0^2)^2+4\gamma^2\omega^2}]}$.

7.3.15 $\varepsilon = 1 + \frac{4\pi n_e e^2}{[m_e\sqrt{(\omega^2-\omega_0^2)^2+4\gamma^2\omega^2}]}$.

7.4 Interaction of charged particles

7.4.1 $v = \frac{e}{\sqrt{4\pi\varepsilon_0 m_e r}}\sqrt{\frac{\gamma-1}{\gamma}}$.

7.4.2 $v = \sqrt{\frac{e^2(4+\sqrt{2})}{(8\pi\varepsilon_0 m_e a)}}$.

7.4.3 $\frac{v_p}{v_e} = \sqrt{(\frac{m_e}{m_p})(4\sqrt{2}+1)} \approx 0,01$. For the estimation, we can assume that the light positrons will have time to travel far before the protons move.

7.4.4 $r_{min} = \frac{e^2}{(4\pi\varepsilon_0 m_e v^2)}$.

7.4.5 $r_{min} = \frac{e^2}{[4\pi\varepsilon_0 m_e(v_1+v_2)^2]}$

7.4.6 $v = \sqrt{\frac{q_1 q_2(m_1+m_2)}{[2\pi\varepsilon_0 m_1 m_2(R_1+R_2)]}}$.

7.4.7 $r_{min} = \frac{de^2}{(e^2+4\pi\varepsilon_0 m_e v^2 d\cos\alpha)}$.

7.4.8 $\alpha = \frac{\pi}{2}$.

7.4.9 $v = \sqrt{\frac{q^2}{(8\pi\varepsilon_0 md)}}$.

7.4.10 $v = v_0\sqrt{1 - \frac{q^2(2\sqrt{2}-1)}{8\pi\varepsilon_0 m v_0^2 d}}$ at $\frac{mv_0^2}{2} \geq \frac{q^2(2\sqrt{2}-1)}{16\pi\varepsilon_0}$. If instead of the dihedral angle we place charge $+q$ at point A, the electric field in the region outside the conductor, and hence the forces do not change. This allows us to consider the motion of the system of four charges shown in the figure.

7.4.11 $v = \sqrt{\frac{4e^2r^2}{[\pi\varepsilon_0 m_e(4r^2+R^2)^{\frac{3}{2}}]}}$

7.4.12 $K_{min} = \frac{Ze^2}{(8\pi\varepsilon_0 r)}$

7.4.13 $K_{min} = \frac{e^2(2-\sqrt{2})}{(4\pi\varepsilon_0 r)}$.

7.4.14 $n_{min} = \frac{(\sqrt{2}-1)m}{M} + \sqrt{2}$.

7.4.15 $v_{min} = 2v$

7.4.16 Impossible

7.4.17 $r_{min} = \frac{e^2}{2\pi\varepsilon_0 m_p v^2} + \sqrt{\rho^2 + (\frac{e^2}{2\pi\varepsilon_0 m_p v^2})^2}$.

7.4.18 $m = \frac{4q^2(l-r)}{rl[u^2+v^2+2uvcos(\alpha+\beta)-\frac{l^2}{r^2}(u\sin\alpha - v\sin\beta)^2]}$.

7.4.19 $t = 2\sqrt{2}t_0$.

7.4.20 $v \geq \sqrt{\frac{qQ(m+M)}{(2\pi\varepsilon_0 RmM)}}$ when $qQ > 0$; any when $qQ < 0$.

7.4.21 $v = \frac{mv_0}{m+M} + \sqrt{(\frac{MV_0}{m+M})^2 - \frac{QqM}{2\pi\varepsilon_0 Rm(m+M)}}$

7.4.22 $v = \sqrt{\frac{3qQ(m+M)}{(4\pi\varepsilon_0 mMR)}}$ at $qQ > 0; v = 0$ at $qQ \leq 0$.

7.4.23 $v_c = \sqrt{\frac{q^2}{(6\pi\varepsilon_0 ml)}}; v_{kr} = \sqrt{\frac{q^2}{(24\pi\varepsilon_0 ml)}}$.

7.4.24 $v=\sqrt{\frac{q^2m(2R-l)}{[2\pi\varepsilon_0 RlM(M+2m)]}}$.

7.4.25 $x=\frac{R}{2}(\frac{Q^2}{4\pi\varepsilon_0\mu mgR^2}-1), v_{max}=\sqrt{\mu gR}(\frac{Q}{\sqrt{4\pi\varepsilon_0\mu mgR^2}}-1)$.

7.4.26 $h=\frac{h_0 cos^2\alpha}{mg}[\frac{q^2}{8\pi\varepsilon_0(H-h_0)H\sin\alpha}(1-\mu ctg\alpha)-mg(1-\mu tg\alpha)]$

7.4.27 $W=\frac{3q^2}{(32\pi\varepsilon_0 l)}$.

7.4.28 $k=[\frac{q^2}{(2\pi\varepsilon_0 l_1 l_2)}](l_1+l_2+2l_0)$.

7.4.29 $v_{max}=v\sqrt{\frac{1+q^2}{(4\pi\varepsilon_0 Rmv^2)}}$

7.4.30 $a)W=\frac{4}{3}\pi R^3\rho v^2+\frac{Q^2}{4\pi\varepsilon_0 R}+4\pi R^2\sigma(2-2^{\frac{2}{3}})$

$b)W=\frac{4}{3}\pi R^3\rho v^2+\frac{Q^2+q^3}{8\pi\varepsilon_0 R}-\frac{(Q-q)^2}{8\pi\varepsilon_0\sqrt[3]{2R}}+4\pi R^2\sigma(2-2^{\frac{2}{3}})$

7.4.31 The charge will oscillate along the axis of the cylindrical hole. Its velocity is maximal at the point O

7.4.32 $v=\sqrt{2gh[1-\frac{S\sigma^2}{(4\varepsilon_0 mg)}]}$ at $mg>\frac{S\sigma^2}{(2\varepsilon_0)}$;

$v=\sqrt{\frac{2\varepsilon_0 mg^2h}{(\sigma^2 S)}}$ at $mg<\frac{S\sigma^2}{(2\varepsilon_0)}$.

7.4.33 $v=\sqrt{\frac{q^2}{4\pi\varepsilon_0 m}(\frac{1}{r}-\frac{1}{R})}$.

7.4.34 $v=v_0\sqrt{1-\frac{\rho^2 l}{2\pi\varepsilon_0 mv^2}ln\frac{R_1}{R_2}}$.

7.4.35 $T=2\pi\sqrt{\frac{4\pi\varepsilon_0 ml^3}{(\sqrt{2}q^2)}}$.

7.4.36 $a)$ Electrons and ions separate completely. The electric field of the ions $E_i=\frac{neh}{(2\varepsilon_0)}$ will stop the electrons after a time

$t\approx\frac{2\varepsilon_0 m_e v}{(e^2hn)};\nu\approx\frac{e^2hn}{(8\varepsilon_0 m_e v)}$.

$b)$ Part of the ions and electrons form charged areas at the layer boundaries (see figure) whose electric field causes the harmonic motion of the electrons with period $T=2\pi\sqrt{\frac{e^2n}{(\varepsilon_0 m_e)}}$. Therefore, the electrons will stop after time $t=\frac{T}{4}=(\frac{\pi}{2})\sqrt{\frac{e^2n}{(\varepsilon_0 m_e)}},\nu=\frac{1}{4t}$.

7.4.37 $n=8sin^2(\frac{\alpha}{2})$.

7.4.38 $x=\sqrt{l^2+l_0^2}-l+l_0$, where $l_0=\frac{q^2}{(8\pi\varepsilon_0\mu Mg)}$.

8 ELECTRIC CURRENT

8.1 Current. Current density. Current in vacuum

8.1.1 $a.\ I\approx\frac{nec}{l}=0,02A$. $b.\ I=\sqrt{\frac{e^4}{[16\varepsilon_0 m_e(\pi r)^3]}}=0,0012A$.

8.1.2 $v=\frac{Il}{q}$

8.1.3 $I=2\varepsilon_0 Eav=1,3\cdot 10^{-4}\ A$.

8.1.4

8.1.5 $v=0,4\ \frac{cm}{s}$.

8.1.6 $j=e\nu$.

8.1.7 $j=-en_e u$.

8.1.8 $I=sj\sin\alpha=10\ A$

8.1.9 $t=8\cdot 10^{-6}\ s$

8.1.10 $\rho=\frac{j}{v}$

8.1.11 $E\approx\frac{I}{(2\pi\varepsilon_0 vr)}=6\cdot 10^5\ \frac{V}{m};L\approx[\frac{8m_e rv^2}{(3eE)}]^{\frac{1}{2}}\approx 0.1\ m$.

8.1.12 $a)\rho=\frac{\rho_0 v_0}{\sqrt{v_0^2-\frac{2eEx}{m_e}}}$ where x is the distance to the front grid. $b)$ $\rho_2=2\rho_1$ at $x<x_0=\frac{m_e v_0^2}{(2eE)};\rho=0$ at $x>x_0$. From the dependence of ρ_2 on x the greatest charge field strength between the grids: $E_1=\frac{1}{2\varepsilon_0}\int_0^{x_0}\rho_2 dx=\frac{\rho_0 m_e v_0^2}{\varepsilon_0 eE}$.
The charge field of the beam can be neglected if $E_1\ll E$. When E_1 is comparable to E, i.e. $\frac{\rho_0 m_e v_0^2}{(\varepsilon_0 eE)}\approx E$, it must be taken into account. Hence the estimate $\rho\approx\frac{e\varepsilon_0 E^2}{(m_e v_0^2)}$.

8.1.13

8.1.14 The curve T_1 corresponds to the iso-temperature cathode and the curve T_3 corresponds to the high-temperature cathode.

8.1.15 If the field were not close to zero, then all of the electrons from this boundary would either toward the anode or toward the cathode, depending on the sign of the field.

8.1.16 $\rho=\frac{I}{S}\sqrt{\frac{m_e d}{2eV}}\frac{1}{\sqrt{x}}=1,75\cdot 10^{-6}\frac{1}{\sqrt{x}}\frac{Cl}{m^3}$.
When $x\to 0$ the charge density $\rho\to$ inf, nevertheless the charge per unit area ($\sigma=\int_0^d\rho dx$), is limited: $\sigma=3.5\cdot 10^{-6}\sqrt{d}$. Therefore the largest value of the of the spatial charge field strength is limited: $E'=\frac{\sigma}{(2\varepsilon_0)}$. In this case $E'\ll\frac{V}{d}$ and the action of the spatial charge can be neglected.

8.1.17 $n = \frac{4}{3}; j = \frac{4}{9}\varepsilon_0\sqrt{\frac{2e}{m_e}}\frac{V^{\frac{3}{2}}}{d^2}, I = jS$

8.1.18 The charge density increases by a factor of n, and the current increases by a factor of $n^{\frac{3}{2}}$ times.

8.1.19 $j = \frac{i}{(2\pi r)}$.

8.1.20 $a. j_1 = \frac{2I}{4\pi r^2}\sqrt{1 - \frac{l^2}{r^2}}; j_2 = \frac{2I}{4\pi r^2}\frac{l}{r}$ where l is the distance from the middle of the segment AB to point in which j is determined; r is the distance from A or B to this point. In the first case the current is perpendicular to the symmetry plane, in the second case it lies in it. The total currents through the plane are I and 0, respectively.

$b.\ j = \frac{2I}{4\pi r^2}\sqrt{1 - \frac{h^2}{r^2}}$ where r is the distance from the source to the point at which j

8.1.21 $j = \frac{qvl}{(2\pi r^3)}$

8.2 Conductivity. Resistance. Sources of EMF .

8.2.1 $a.\ \lambda = \frac{e^2 n_e \tau}{m_e}$. $b.\ \tau = 2,4 \cdot 10^{-15}\ s$.

8.2.2 $\frac{\Delta N}{N} = 1,5 \cdot 10^{-10}$.

8.2.3 $f = \frac{-n_e^2 v}{\lambda}$.

8.2.4 $I = \frac{m_e \omega r \lambda s}{(e\tau)} = 1,7\ mA$.

8.2.5 The change in the field occurs at the speed of light.

8.2.6 The ratio $\frac{\chi}{\lambda}$ is almost the same for these metals. Theoretical estimate: $\frac{\chi}{\lambda} = \frac{\pi^2 k^2 T}{(3e^2)}$, where k is Boltzmann constant, T is temperature, e is carrier charge

8.2.7 $E = \frac{j}{\lambda}; V_1 = (\frac{jl}{\lambda})\cos\alpha; V_2 = \frac{\pi j l}{(2\lambda)}$

8.2.8 $\sigma = \varepsilon_0 j(\frac{1}{\lambda_1} - \frac{1}{\lambda_2})$.

8.2.9 $tg\alpha_2 = \frac{\lambda_2}{\lambda_1}tg\alpha_1; \sigma = \varepsilon_0 j\cos\alpha_1(\frac{\pi j l}{(2\lambda)})$

8.2.10 $\rho = \frac{\varepsilon_0 j}{(\lambda a)}$.

8.2.11 $a\ .I = \frac{Q_0}{(\varepsilon_0\rho)}$. $b.\ Q = Q_0 exp[\frac{-t}{(\varepsilon_0\rho)}]$.

8.2.12

8.2.13 $I = \frac{\lambda S V}{l}; R = \frac{l}{(\lambda S)}$.

8.2.14 $R_I = \frac{l}{\pi r^2}(\frac{1}{\lambda_1} + \frac{1}{\lambda_2}), R_{II} = \frac{1}{\pi}(\frac{l_1}{r_1^2\lambda_1} + \frac{l_2}{r_2^2\lambda_2}); I_I = \frac{V}{R_1}, I_{II} = \frac{V}{R_{II}}$ at $|r_2 - r_1| \ll l_1, l_2$.

8.2.15 $R = 0,0566\ Om$

8.2.16 $R = \frac{R_0}{cos^2\alpha}$.

8.2.17 $I = 4\pi r\lambda V; R = \frac{1}{(4\pi r\lambda)}$.

8.2.18 $R = 0,14\ Om$

8.2.19

8.2.20 $R = \frac{1}{4\pi\lambda}(\frac{1}{r_1} - \frac{1}{r_2}); I = \frac{\lambda q}{\varepsilon\varepsilon_0}$

8.2.21 $C = \frac{\varepsilon\varepsilon_0}{(\lambda R)}$; no.

8.2.22 The electrodes should touch the center of the plate from different sides.

8.2.23 $K = \frac{1}{2}m_e(\frac{I}{en_e S})^2 = 2 \cdot 10^{-15}\ EV$

8.2.24 $I = \frac{Fl}{(qR)}; v = \frac{Fl^2}{(q^2 R)}$

8.2.25 $a.\ \varphi = \frac{qvR}{l}$. $b.\ \varphi = \frac{Fl}{q}$.

8.2.26 $V = \frac{W}{e}; I_{max} = e\nu$. If $R < \frac{W}{e^2\nu}$ the current does not change with the load

8.2.27 $I = I_0(1 - \sqrt{\frac{V}{V_0}})$.

8.2.28 See Fig. $W = E_c l$

8.2.29 $\varepsilon = 1,13\ V$.

8.2.30 $\varepsilon = 1.07\ V$. There is an inflow of heat from the environment.

8.2.31 $\nu = 1.4 \cdot 10^{-2}\ mol$

8.2.32 The capacitor will not discharge completely due to the appearance of a chemical counter-EMF, increasing as the number of baths increases.

8.2.33

8.2.34 $k = \frac{V}{(2\varepsilon)}$.

8.3 Electrical circuits

8.3.1 $r = 1.5$ and $50\ kOhm$.

8.3.2 $r = 20\ Ohm$

8.3.3 $V = 1\ kV$

8.3.4 In circuits a and e the instruments will show a decrease in current, in circuit d an increase in current, in circuit b and f the current does not change. In circuit c the upper ammeter will show increasing current, the lower ammeter will show a decrease in current.

8.3.5 $a.\ \frac{\Delta V}{V} = \frac{R}{(R+r)}$. $b.\ \frac{\Delta I}{I} = \frac{r}{(R+r)}$.

8.3.6 $\frac{I_V}{I_6} = \frac{10}{64}, V \approx 40\ V$

8.3.7 $100\ Ohm$.

8.3.8 More

8.3.9 $V = 48\ V$; $I = 15\ A$

8.3.10 $r_x = \frac{rR_2}{R_1}$;saved.

8.3.11 $R_B = \frac{V_1}{I_1}; R = \frac{V_2V_1}{(I_2V_1 - I_1V_2)}$;
$R_A = \frac{(V_1V_3I_2 - V_3V_2I_1 - V_2V_1I_3)}{I_3(V_1I_2 - V_2I_1)}$.

8.3.12 Here is the part of the circuit that includes the required resistance. To the nodes A and O we connect a battery, and a voltmeter to nodes C and O, and ammeters to nodes C and A, C and B, and nodes A and B are connected by a wire. The current through resistance R is $I_{CA} + I_{CB}$. Then $R = \frac{V}{(I_{CA}+I_{CB})}$, where V is the reading of the voltmeter.

8.3.13 $R = 7\ Ohm$

8.3.14 $a. r = \sqrt{3}R$. $b.\ r = (\sqrt{3} - 1)R$. $c.\ I_n = I(2 - \sqrt{3})^{n-1}$ through a resistance of $2R$; $I'_n = I(2 - \sqrt{3})^{n-1}(\sqrt{3} - 1)$ through resistance R, n is the cell number, $R_0 = (\sqrt{3} + 1)R$.

8.3.15 $R_1 = 9r; R_2 = \frac{10r}{9}$.

8.3.16 In section a:$V = \varepsilon - I(r+R)$; $b : V = -\varepsilon - I(r+R)$; $c : V + \varepsilon_1 + \varepsilon_2 - I(r_1 + r_2 + R)$; $d : V = \varepsilon_1 - \varepsilon_2 - I(r_1 + r_2 + R)$.

8.3.17 $\varepsilon = 34.3\ V$; $r = 1.43\ Ohm$

8.3.18 Battery with EMF $E = 10\ V$ and internal resistance $r = 14\ Ohm$

8.3.19 See Fig.

8.3.20 $I = 10\ A$, $r = 20\ Ohm$; $\varepsilon = 200\ V$, $r = 20\ Ohm$.

8.3.21 $I = 80\ A$

8.3.22 $I_2 = \frac{I_3R_3}{R_2}; I_1 = \frac{I_3(R_2+R_3)}{R_2}; V = \frac{I_3(R_1R_2+R_1R_3+R_2R_3)}{R_2}$.

8.3.23 See Fig.

8.3.24

8.3.25 $a.V = 5ir; R = \frac{5r}{6}; I = 6i$. b. See Fig. $I = \frac{7i}{2}; R = \frac{12r}{7}$; $c.\ R_{AB} = \frac{13r}{7}; R_{CD} = \frac{5r}{7}$.

8.3.26 $I = 8\ A$.

8.3.27 $a.\ I = \frac{i}{2}; R = \frac{r}{2}$. $b.\ R = \frac{r}{3}$. $c.\ R_{AB} = \frac{2r}{3}; R_{AC} = r$

8.3.28 $\varepsilon = \frac{(\varepsilon_1r_2+\varepsilon_2r_1)}{(r_1+r_2)} = 21\ V$, $r = \frac{r_1r_2}{(r_1+r_2)} = 3,75\ Ohm$.

8.3.29 See Fig.

8.3.30 It will decrease by a factor of three.

8.3.31 $V = 0; I = 0,75\ A$.

8.3.32 $V = 0,75\ V$.

8.3.33 In $12, 54$ and $27\ min$.

8.3.34 $N = I^2R$.

8.3.35 $N' = \frac{N_0(N-N_0)}{N}$.

8.3.36 $R = 9(n - 1)r$

8.3.37 $r = \sqrt{R_1R_2}$.

8.3.38 2 and $100\ V$; 20 and $0.1\ W$. The current is almost unchanged. will not change, but the power will almost double

8.3.39 $S = 42\ mm^2$; approximately 10 times.

8.3.40 $N = (E - Ir)I; R = r$.

8.3.41 $N_1 = 125\ W$; $N_2 = 80\ W$; $N_3 = 45\ W$.

8.3.42 At $r = R$

8.3.43 $N_p = \frac{(V-\varepsilon)\varepsilon}{r}; N_t = \frac{(V-\varepsilon)^2}{r}$.
If $\varepsilon > \frac{V}{2}$, then the useful power is greater than the thermal power.

8.3.44 $N = 4\ W$

8.3.45 $N = \frac{\lambda CV^2}{\varepsilon_0}$

8.3.46 $N = I(\frac{m_ev^2}{2e} - IR)$.

8.3.47 $q = 4\pi^2\varepsilon_0a^3en_eRv, v \gg \frac{a^2e^2n_eR}{m_e}$.

8.3.48 $T = T_0 + \frac{R_0I^2}{(\chi - I^2R_0\alpha)}, \chi > I^2R_0\alpha$. When $\chi < I^2R_0\alpha$ the temperature T increases indefinitely.

8.4 Capacitors and nonlinear elements in electrical circuits

8.4.1 $a.\ q = 8 \cdot 10^{-4}\ Cl$. $b.\ V = 60\ s$. $30, 30, 60\ V$

8.4.2 $V = \frac{V_0x}{(2x-l)}$; swap sources

8.4.3 $\varphi_A = \varphi_B + 2(l - \frac{x}{2})\sqrt{\frac{kx}{\varepsilon_0S}}$.

8.4.4 .$\varphi_A - \varphi_B = \varepsilon(\frac{R_1}{R_1+R_2} - (\frac{C_2}{C_1+C_2}$. It should be measured with an electrostatic voltmeter, $q_1 = \frac{C_1R_1\varepsilon}{(R_1+R_2)}; q_2 = \frac{C_2R_2\varepsilon}{(R_1+R_2)}$. In this case, the influence of of these voltmeters on the electrical circuit.

8.4.5 $W_1 = \frac{CV^2}{4}\frac{R_1}{R_1+R_2}; W_2 = \frac{CV^2}{4}\frac{R_2}{R_1+R_2}$

8.4.6 $W = A - \frac{q^2}{C}$.

8.4.7 $q = C\varepsilon; W = \frac{C\varepsilon^2}{4}$.

8.4.8 $W = \frac{C(\varepsilon - V_0)^2}{2}, \varepsilon > V_0; W = 0, \varepsilon < V_0$.

8.4.9 $W = C(V - \varepsilon)\varepsilon; W = \frac{C(V-\varepsilon)^2}{2}$.

8.4.10 First the capacitor must be charged from one cell, then from two cells connected in series, and so on. Then the energy loss will be $\frac{1}{n}$ fraction of the stored energy

8.4.11 $N_g = \frac{Iq}{C} > N_k = \frac{Iq}{(2C)}$. These quantities differ from each other because of the work done by changing the capacitance of the capacitor.

8.4.12 In $\tau \approx 10^{-3}RC$.

8.4.13 $q = C\frac{\varepsilon_1 R_2 + \varepsilon_2 R_1}{R_1 + R_2} q = C\frac{\varepsilon_1 R_2 + k\varepsilon_2 R_1}{kR_1 + R_2}$

8.4.14 $V = \frac{V_0 R\tau}{(rT + R\tau)}$.

8.4.15 $\frac{dV}{dt} = -\frac{V}{RC}; V = V_0 exp(\frac{-\tau}{RC})$.
$I = \frac{V_0}{R} exp(\frac{-\tau}{RC})$

8.4.16 $R < 40\ kOhm$

8.4.17 $\nu = (RCln\frac{V-V_0}{V-V_1})^{-1}$.

8.4.18 $a.\ I = \frac{qv}{d}.\ b.\ No$

8.4.19 $I = \frac{\varepsilon_0(\varepsilon-1)\varepsilon av}{d}$.

8.4.20 $I = \frac{1}{2\alpha R^2} + \frac{\varepsilon}{R} - [(\frac{1}{2\alpha R^2} + \frac{\varepsilon}{R})^2 - \frac{\varepsilon^2}{R^2}]^{\frac{1}{2}}$

8.4.21 On the volt-ampere characteristic draw the line $I = \frac{(\varepsilon - V)}{R}$; the point of their intersection gives gives a current of $2\ mA$. Drawing the corresponding straight lines through the ends of the rectilinear of the characteristic line, we find that at $R > 0.3\ kOhm$ and $R > 3\ kOhm$ the diode stops working at the straight line part of the volt-ampere characteristic.

9 PERMANENT MAGNETIC FIELD

9.1 Induction of a magnetic field. The effect of a magnetic field on a current

9.1.1 $B = 100\ Tl$

9.1.2 $B = 20\ Tl$

9.1.3 $a)\ F_1 = F\frac{I_1}{I}\sqrt{1 + \frac{L^2}{l^2} - 2\frac{L}{l}cos\varphi}.\ b) F_2 = 2F\frac{RI_2}{lI}$

9.1.4 $\Delta h = \frac{a\lambda VB}{(b\rho g)}$.

9.1.5 $\alpha = 45°$

9.1.6 $I = \frac{mg}{2aB}ctg\alpha$

9.1.7

9.1.8 $\omega = \sqrt{\frac{6BI}{m}}$.

9.1.9 $tg\alpha = \frac{IB}{(4\rho g)}$

9.1.10 The current frame will be divided into trapezoidal microcircuits with current I as is shown in the figure. The moment of forces acting on all microcircuits when $\Delta h \to 0$ coincides with the momentum of forces acting on the frame with current:
$\vec{N}_{\Delta\vec{h}\to 0} \to \Sigma_i[\Delta M_i \times B] = [(\Sigma_i \Delta M_i \times B)]_{\Delta\vec{h}\to 0} \to [\vec{M} \times \vec{B}]$

9.1.11 $a.\ tg\alpha = \frac{IB}{2\rho g}.\ b.\ tg\alpha = \frac{\pi(4+\pi)IB}{4(1+\pi)(2+\pi)\rho g}$.

9.1.12 $N = \frac{\pi R^2 IB(sin\alpha + \cos\alpha)}{2}$.

9.1.13 $B = \frac{P}{(\pi RIn)}$

9.1.14 $a = \frac{2\pi RIB\sin\alpha}{m}$.

9.1.15 $B = \frac{F}{(RI)}$.

9.2 The magnetic field of a moving charge. The induction of the magnetic magnetic field of a linear current

9.2.1 $B = \frac{\mu_0 \rho v}{(2\pi r)}$, where r is the distance to the thread

9.2.2 $B = \frac{\mu_0 I}{(2\pi r)}$, where r is the distance to the wire.

9.2.3 $\mu = 1,25$.

9.2.4 $B = 1,88 \cdot 10^{-5}\ Tl$

9.2.5 $B = \frac{\mu_0 I}{2\pi}(\frac{1}{x} + \frac{1}{y})$.

9.2.6 $B = \frac{\mu_0 I}{2\pi l}\sin\frac{\alpha}{2}$ where l is the distance to the intersection point of the wires.

9.2.7 $a.\ B = \frac{\mu_0 qv}{4\pi r^2}sin\alpha.\ b.\ B = \frac{\mu_0 Il}{4\pi r^2}sin\alpha$.

9.2.8

9.2.9 $B = \frac{\mu_0 I}{(2R)}; B_h = \frac{\mu_0 IR^2}{[2(R^2+h^2)^{\frac{3}{2}}]}$.

9.2.10 $n = \sin(\frac{\alpha}{2})$.

9.2.11 $B = \frac{\mu_0 I}{2\pi R}(1 + \frac{\pi}{2})$

9.2.12 $B = \frac{\mu_0 I}{(4R)}$.

9.2.13 $B_0 = \frac{\mu_0 I(\pi+1)}{2\pi R}$

9.2.14 $B_h = \frac{\mu_0 I}{2}[\frac{1}{\pi^2(R^2+h^2)} + \frac{R^4}{(R^2+h^2)^3} + \frac{2R^3}{\pi(R^2+h^2)^{\frac{5}{2}}}]^{\frac{1}{2}}$.

9.2.15 *a.* $I = I_0\sqrt{10}$. *b.* $I = 2I_0\sqrt{10}$.

9.2.16 $B = \frac{\mu_0 M}{(2\pi h^3)}$.

9.2.17 $B = \frac{\mu_0 M\sqrt{1+3sin^2\alpha}}{(4\pi r^3)}, M = Ia^2$

9.2.18 Two flat circuits with current I, having different shapes but the same area, break them into square microcircuits with current as shown in the figure. The induction of the magnetic field created by these microcircuits when $\Delta h \to 0$ coincides with the induction of the contours inside which the microcircuits are located. The magnetic field of the circuits in question is close to the field of a single micro-loop at a large distance multiplied by the number of microcontours inside each loop. But this product when $\Delta h \to 0$ for each loop tends to the same value since the number of microcontours depends only on the area of the contour.

9.2.19 *a.* In the figure each microcircuit with momentum M_0 is surrounded by a circuit with current $I = \frac{M_0}{a^2}$. At distances much greater than the distance between neighboring microcircuits, the field of microcircuits tends to the field of surrounding current I, which coincides with the field of the current I, flowing along a large circuit. The magnetic moment of such a loop $M = Ib^2 = \frac{M_0 b^2}{a^2} = nM_0$.

b. The magnetic field of a thin plate is close to the magnetic field of the contour current $I = hM$, where M is the magnetic moment of a unit volume of the plate. But the magnetic field induction B is related to I by the relation $B = \frac{\mu_0 I\sqrt{8}}{(\pi a)}$. Therefore, $M = \frac{B\pi a}{(\mu_0 h\sqrt{8})}$.

9.2.20 $B = \frac{\mu_0 MR^2 h}{[2(R^2+l^2)^{\frac{3}{2}}]}$

9.2.21 $B = 4,9 \cdot 10^{-2}\ Tl$.

9.2.22 The vector B_0 must be parallel to the surface of the disk. $N = \frac{2\pi BB_0R^3}{\mu_0}$.

9.2.23 $M = \sqrt{\frac{\pi HF}{(2\mu_0 ah^2)}}$.

9.3 The magnetic field of a current distributed over a surface or space

9.3.1 $B = \frac{\mu_0\sigma v}{2}$.

9.3.2 $B = 10^{-10}\ Tl$.

9.3.3 $\frac{\mu_0 i}{2}$.

9.3.4 Between planes $B = \frac{\mu_0(i_1-i_2)}{2}$, outside planes $B = \frac{\pm\mu_0(i_1+i_2)}{2}$.

9.3.5 $F = \frac{\mu_0 I^2}{(2b)}$.

9.3.6 *a.* $\Delta = \frac{\mu_0 aI^2}{(8Eb^2)}$. *b.* $B_1 \approx 10\ Tl$. $B_2 \approx 35\ Tl$

9.3.7 $B_\parallel = \mu_0 e_0 E_\perp v = \frac{\mu_0 i\Omega}{(4\pi)}$, where $E_\perp = \frac{\sigma\Omega}{(4\pi\varepsilon_0)}$ is the component of of electric field of current carriers, perpendicular to the surface, σ - their surface density, v - velocity

9.3.8 *a.* $B = \frac{\mu_0 i}{4}$. *b.* $B = \mu_0 i$; independent. *c.* $B = \frac{\mu_0 aj}{(4\sqrt{3})}$.

9.3.9 $T = \frac{\mu_0 nRI^2}{2}$

9.3.10 *a.* $B_\parallel = \frac{\mu_0 i\Omega}{(4\pi)}$, where Ω is the solid angle at which the surface of the cylinder (see problem 9.3.7). In the section AA' the solid angle $\Omega = 2\pi$, so $B_\parallel = \frac{\mu_0 i}{2}$.

b. $B = \frac{1}{2}\mu_0 i(1 - \frac{1}{\sqrt{1+(\frac{R}{x_1})^2}}), B =_{x_1 \vec{\to} \inf} \frac{1}{4}\mu_0 i(\frac{R}{x_1})^2$

$B = \frac{1}{2}\mu_0 i(1 + \frac{1}{\sqrt{1+(\frac{R}{x_2})^2}}), B =_{x_2 \vec{\to} \inf} \mu_0 i$

9.3.11 *a.* The magnetic field of a cylinder is composed of the magnetic fields of thin discs of thickness Δ into which this cylinder can be divided. The magnetic field of each disk coincides with the magnetic field of the current flowing with linear density M (M is the magnetic moment unit volume of iron); on the outer surface of the disk (see the solution of problem 9.2.19).

b. The direction of magnetic field induction in the center of the cube coincides with the direction of of magnetization. The modulus of this vector will be as many times smaller than the modulus of induction of the magnetic field inside the rod as many times $\frac{8\pi}{3}$ (the solid angle at which the side faces of faces of cube $1-4$) is less than 4π, i.e. $n = 1.5$ times.

c. $B = \frac{\mu_0 M}{sqrt1+4(\frac{r}{l})^2}; B_{(\frac{r}{l})\vec{\to}0} \to \mu_0 M, B_{(\frac{r}{l})\vec{\to}\inf} \to \frac{\mu_0 Ml}{2r}$.

d. $B = \mu_0 M(1 - \frac{1}{\sqrt{1+4(\frac{r}{l})^2}}); B_{(\frac{r}{l})\vec{\to}0} \to \frac{2\mu_0 Mr^2}{l^2}, B_{(\frac{r}{l})\vec{\to}\inf} \to \mu_0 M$.

9.3.12 The induction of the magnetic field inside a rectangular column will be as many times greater than B, how many times greater 4π is the solid angle at which the side faces of the plate can be seen from its center. $B = \frac{\pi aB_0}{(2\sqrt{2h})}$.

9.3.13 $B_{\parallel} = 6,28 \cdot 10^{-4}\ Tl$, $B_{\perp} = 0,377\ Tl$

9.3.14 $\Delta B = \frac{B_0 \chi h}{(2R)}$

9.3.15 $a. B = \frac{\mu_0 I x}{(2\pi r^2)}, 0 < x < r; B = \frac{\mu_0 I}{(2\pi x)}, x > r.$

$b.\ B = \mu_0 x j, x = \frac{a}{2}; B = \frac{\mu_0 a j}{2}, x < \frac{a}{2}.$

9.3.16 $B_{max} = \frac{\mu_0 NI}{(2\pi r)}, B_{min} = \frac{\mu_0 NI}{(2\pi R)}.$

9.3.17 $a.$ Above the plane $B = \frac{\mu_0 I}{(2\pi x)}$, the magnetic field induction lines coincide with with the field induction lines of an infinite straight wire; below the plane $B = 0$.

$b.$ Above plane $B = \frac{\mu_0 I}{(2\pi x)}$, below plane $B = \frac{\mu_0 (I - I_0)}{(2\pi x)}$.

$c.$ Inside the cable $B = \frac{\mu_0 I}{(2\pi x)}$, outside the cable $B = 0$.

9.3.18 $B = \frac{\mu_0 I}{2\pi r} tg \frac{\beta}{2} v$

9.3.19 See Fig. $B_{max} = \frac{\mu_0 h j}{2}$.

9.3.20 $B = \frac{\mu_0}{2} j x, 0 < x < \frac{h}{2}; B = \frac{\mu_0}{2} h j (1 - \frac{h}{4x}), x > \frac{h}{2}$ where x is the distance to point O.

9.3.21 $B = \frac{\mu_0 j d}{2}$.

9.3.22 $a.\ B = \frac{\mu_0 j a}{2}$. $b.\ i = 2B_0 sin \frac{\varphi}{\mu_0}, i_{max} = \frac{2B_0}{\mu_0}$. See Fig.

9.3.23 The component induction of the magnetic field along the solenoid axis $B_{\parallel} = \mu_0 n I$, and component of the magnetic field induction perpendicular to the solenoid axis, $B_{\perp} = \mu_0 n I tg\alpha$.

9.3.24 To determine the equivalent surface currents (see the solution of the problem $9.3.11a$) the cylinder should be divided into thin layers, one of which is shown in the figure. The planes of the layers should be perpendicular to the direction of magnetization. $B = \frac{\mu_0 M}{2}$ when $x < r$; $B = (\frac{\mu_0 M}{2})(\frac{r}{x})^2$ at $x > r$.

9.4 Magnetic flux

9.4.1 $a.\ \Phi = \sqrt{3} \frac{B a^2}{2}$ $b.\ \Phi = BS \sin\alpha.$

9.4.2 $\Phi = B \cdot \pi R^2 (sin^2\alpha - sin^2\beta)$

9.4.3

9.4.4

9.4.5

9.4.6

9.4.7 $n = \frac{sin\alpha}{sin\beta}, i = (\frac{B}{\mu_0}) \cos\alpha (1 - tg\alpha ctg\beta).$

9.4.8 $B_2 = B_4 = B_1 \frac{a_1}{a_2} = \sqrt{\frac{B_1^2 + B_3^2 + 2B_1 B_3 \cos\alpha}{2cos(\frac{\alpha}{2})}}$

9.4.9 $a.\ B_r = \frac{1}{2} B_0 \frac{r}{x}, tg\alpha = \frac{1}{2} \frac{r}{x}$; See Fig. $b.\ B_r = \frac{1}{2} n B_0 \frac{r}{x_0} (\frac{x}{x_0})^{n-1}, B_r = \frac{1}{2} r B_0 \frac{\delta f}{\delta x}$

9.4.10 Since the magnetic flux of the radial component of the field induction outside the cylinder is conserved, the magnetic field induction will decrease as $\frac{\alpha R}{r}$, where r is the distance to the axis 340 of the cylinder, $\alpha = \frac{B_0 R}{(2x_0)}$ is the radial component of the magnetic field induction near of the cylinder surface.

9.4.11 $a.$ At a sufficiently large distance from the end of the cylinder the magnetic field induction $B_0 = \mu_0 i$, and the magnetic flux in section πR^2 is equal to $\pi R^2 B_0$. Part of this flux (Φ_1) comes out of the cylinder through section AA', part (Φ_2) - through the side surface: $\pi R^2 B_0 = \Phi_1 + \Phi_2$. Hence $\Phi_2 = \pi R^2 B_0 - \Phi_1$. As in section $AA' B_{\parallel} = \frac{B_0}{2}$ (see solution of problem $9.3.10a$), then $\Phi_1 = \pi R^2 B_{\parallel} = \frac{\pi R^2 B_0}{2}$ and $\Phi_2 = \frac{\pi R^2 B_0}{2} = \frac{\mu_0 \pi i R^2}{2}$.

$b.$ The force acting on a dedicated section of one half of the solenoid in the axial direction, $\Delta F k = B_{\perp} \Delta S - nI = nI = \Delta\Phi$, where $\Delta\Phi$ is the magnetic flux from the other half of the solenoid through this section. Therefore, the total axial force $F_{\parallel} = nI - \Phi$, where the total magnetic flux from the second half of the solenoid through the surface of the first half $\Phi = \frac{\mu_0 \pi n I R^2}{2}$. So $F_{\parallel} = \frac{\mu_0 \pi (nIR)^2}{2}$.

9.4.12 $B = \sqrt{2\mu_0 F} (\pi R^2)$.

9.4.13 $F = nI(\Phi_1 - \Phi_2)$.

9.4.14 $a.\ L = \frac{\mu_0 \pi (rR)^2}{l^3}$. $b.\ L = \mu_0 n \pi r^2$

10 MOTION OF CHARGED PARTICLES IN COMPLEX FIELDS

10.1 Motion in a homogeneous magnetic field

10.1.1 $R = 0,2\ m$

10.1.2 $R = 0,68\ m$

10.1.3 $a.\ \omega = \frac{qB}{m}$. $b.\ \omega = 1,75 \cdot 10^{11}\ s^{-1}$

10.1.4 $\frac{R_1}{R_2} = \sqrt{\frac{K_1}{K_2}}$

10.1.5 $t = \frac{2\pi m}{(qB)}$.

10.1.6 $K = \frac{3(eBR)^2}{(4m_p)}$

10.1.7 $sin\alpha = \frac{eBl}{(m_e v)}$ at $\frac{eB}{m_e} \leq \frac{v}{l}; \alpha = \pi$ at $\frac{eB}{m_e} > \frac{v}{l}$.

10.1.8 $x_1 = 0.29\ m$, $x_2 = 0.41\ m$, $x_3 = 0.5\ m$, $x_4 = 0.58\ m$, $\Delta l = 3.7\ mm$.

10.1.9 $\frac{\Delta V}{V_0} < 0,025$

10.1.10 $l = \frac{2mv}{(qB)}, \Delta z = \frac{mv(\delta\alpha)^2}{(4qB)}$

10.1.11 $R = \frac{mv \sin\alpha}{(qB)}, h = \frac{2\pi mv \cos\alpha}{(qB)}$.

10.1.12 $x = \frac{2\pi m_e v}{(eB)}, \Delta y = \frac{\pi m_e v(\delta\alpha)^3}{(4eB)}$

10.1.13 See fig. $a.\ B > B_0 = \frac{2\sqrt{2m_e k}}{(eR)}$. $b.\ P_2 > P_1$.

10.1.14 $B = \frac{m_e v}{(eR)} + \frac{e}{(16\pi\varepsilon_0 v R^2)}$.

10.1.15 $\omega = \omega_0 - \frac{eB}{(2m_e)}$.

10.1.16 $V' = \frac{2Vh}{R} - Bh\sqrt{\frac{2eV}{m_e}}$.

10.1.17 $a. y = \frac{m_e E}{eB^2 lL} z^2$.
$b.\ y[m] = 1,1 \cdot 10^{-4}\ m^{-1} \cdot z^2$
$c.\ y = \frac{m_e E}{eB^2 lL} z\sqrt{z^2 + (\frac{eBlL}{m_e c})^2}$.

10.1.18 $t = \frac{\pi m_p}{e^2 BV}(\frac{e^2 B^2 R^2}{2m_p} - K)$

10.1.19 $V = \frac{eB^2 d^2}{2\pi^2 m_e} \cdot \frac{1}{k^2}$ where $k = 1, 2, ...$ The size of the spot is determined by the initial velocity of electrons.

10.1.20 $v = \frac{mg}{qB\mu}(sin\alpha - \mu\cos\alpha)$ at $\mu \leq tg\alpha; v = 0$ at $\mu > tg\alpha$

10.1.21 $M = 2\pi R^2 \rho v BR$.

10.1.22 $v = \frac{Q(B_2 - B_1)R}{(2m)}$

10.1.23

10.1.24

10.1.25 $M = \frac{QR^2(B_1 - B_2)}{2}$. Preserved

10.1.26 The time of motion of the electron through the highlighted area $t = \frac{\Delta l}{v}$, where v is the projection of velocity on the plane passing through it and the axis. The change of momentum in the direction perpendicular to this plane is $\Delta p_\perp = -\frac{eB_\perp v \Delta l}{v} = -eB_\perp \Delta l = -\frac{e\Delta\Phi}{(2\pi R)}$, where $\Delta\Phi$ is the magnetic flux through the plot. Change of momentum $\Delta M = R\Delta p_\perp = -(\frac{e}{2\pi})\Delta\Phi$. Therefore, $M_2 - M_1 = (\frac{e}{2\pi})(\Phi_1 - \Phi_2)$.

10.1.27 $n = \frac{(1 - \frac{pB_1}{B_2})}{2}$.

10.1.28 $r = R\sqrt{\frac{B_2}{B_1}}$.

10.2 Drift motion of particles

10.2.1 $v_{dr} = \frac{2v(B_1 - B_2)}{[\pi(B_1 + B_2)]}$

10.2.2 $v_{dr} \approx \frac{\alpha m_e v^2}{(eB_0)}$.

10.2.3 See Fig.$R = \frac{1}{B}\sqrt{\frac{2mEl}{q}}$
$v_{dr} = \frac{2\sqrt{ql}E}{2\sqrt{ql}B + \pi\sqrt{mE}}$

10.2.4 $v = \frac{E}{B}$.

10.2.5 $v_{dr} = \frac{E}{B}$

10.2.6 $v_{dr} = (\frac{E}{B})sin\alpha$

10.2.7

10.2.8 $v \leq \frac{eBh}{(4m_e)}$ or $v = \frac{V}{(hB)}$

10.2.9 $V = \frac{eB^2 d^2}{(2m_e)}; V = 3,5 \cdot 10^5\ V$

10.2.10 In a coordinate system moving with drift velocity $\frac{E}{B}$, the electron moves on a circle of radiusIn a coordinate system moving with drift velocity E/B, the electron moves on a circle of radius $\frac{m_e v'}{eB}$, where $v' = (v^2 + 2\frac{E}{B}\cos\alpha + \frac{E^2}{B^2})^{\frac{1}{2}}$

10.2.11 $v_{dr} = \frac{F}{(qB)}$.

10.2.12 $v_e \approx 8 \cdot 10^{-7}\ \frac{m}{s}$, $v_p \approx 1.5 \cdot 10^{-3}\ \frac{m}{s}$

11 ELECTROMAGNETIC INDUCTION

11.1 Motion of conductors in a constant magnetic field. Electric motors

11.1.1 Between the ends of the wings

11.1.2 $V = 0.03\ V$.

11.1.3 $V = vbB; \sigma = \varepsilon_0 vB$

11.1.4 $v < \frac{Ze}{(4\pi\varepsilon_0 Br^2)}$.

11.1.5 $V < 7\ MV$.

11.1.6 $E = vB$.

11.1.7 $B = \frac{V}{(a^2\omega)}$.

11.1.8 $a.$ See Fig. $b.\ M = (\frac{a^2 b^2 B^2 \omega}{R})sin^2\omega t$

11.1.9 $W = \frac{B^2 vab}{(2\rho)}, a < b; W = \frac{B^2 vb^2}{(2\rho)}, a > b.$

11.1.10 $W = \frac{B^2 l^2 vtg\alpha}{(2\rho)}$.

11.1.11 $N = \frac{(vB)^2 SL}{(4\rho)} = 1W$.

11.1.12 $I = \lambda BvS = 10\ kA$, $V = vBh = 200\ V$

11.1.13 $V = \frac{IB}{(\rho h)}$.

11.1.14 $a.v = \sqrt{\frac{2BIlL}{m}}$. $b.\ v \approx 1,1 \cdot 10^7\ \frac{m}{s}$

11.1.15 $v = \sqrt{IB}(\rho b)$

11.1.16 $I_t = \frac{2\pi r_0^2 Bv}{[R_0(r_0+vt)]}$.

11.1.17 $Q = \frac{SB}{R}$

11.1.18 $B = 1.1 \cdot 10^{-2}\ Tl$

11.1.19 $v = \frac{gmR}{(Bl)^2}$. In heat.

11.1.20 $v(t) = g\frac{mR}{B^2l^2}[1 - exp(-\frac{B^2l^2}{mR}t)]; v(t) = \frac{gtm}{(m+CB^2l^2)}$.

11.1.21 $k = I$

11.1.22 $v = \frac{mgR}{(B_0\pi a^2\alpha)^2}$.

11.1.23 $I = (\frac{mg}{BL})\cos\omega t$.

11.1.24 $a.\omega_{inst} = \frac{2\varepsilon}{BL^2}(1-\frac{2FR}{B\varepsilon L}), I = \frac{2F}{BL}$. $b.\ \omega(t) = \frac{2\varepsilon}{BL^2}[1-exp(-\frac{3B^2L^2}{4mR}t)]$

11.1.25 $I = \frac{\omega Br^2}{(2R)} = 0,4\ A$.

11.1.26 $\omega = \omega_0 - \frac{4M\rho}{(a^3B^2)}$.

11.1.27 When the rotor stops, the circuit will have maximum current flow because there will be no induction EMF

11.1.28 $\varepsilon = 40\ V$

11.1.29 $f = f_0(\frac{\varepsilon}{\varepsilon_0} - \frac{2\pi MRf_0}{E_0^2})$.

11.1.30 $\varepsilon = 120\ V$. $N = 240\ W$

11.1.31 $M = \frac{2\varepsilon I_0\omega}{\omega_0^2}$.

11.1.32 $l = \frac{2V(I_1-I_2)+R(4I_1^2-I_2^2)}{2\rho(I_2^2-I_1^2)}, v = \frac{I_2}{2F}[2V - I_2(2\rho l + R)]$.

11.2 Vortex electric field

11.2.1 $\Phi = 1\ Vb$, $100\ Vb$, $300\ Vb$

11.2.2 $E = \frac{\alpha r^2}{(2l)} = 2.5 \cdot 10^{-5}\ \frac{V}{m}$

11.2.3 In position C, because of the axial symmetry of the magnetic field, the induction flux through the ring does not change. Therefore, there is no EMF in the ring

11.2.4 $E_1 = 6.4 \cdot 10^{-6}\ \frac{V}{m}$, $E_2 = 2.56 \cdot 10^{-5}\ \frac{V}{m}$

11.2.5 $E = \mu_0\alpha x$, where x is the distance from the center line

11.2.6 $E = (\frac{\mu_0\pi\nu n_0 I_0}{l_0})xcos(2\pi\nu t)$, where x is the distance from the coil axis; $E = 0.12\ V$.

11.2.7 $a.\ q = C\varphi$. $b.\ q_1 = q_2 = \frac{C_1C_2}{C_1+C_2}\varphi$.

11.2.8 $a.\ q_1 = C_1\frac{\varphi}{2}, q_2 = C_2\frac{\varphi}{2}$. $b.\ q_3 = \frac{C_3(C_2-C_1)}{C_1+C_2+C_3}\frac{\varphi}{2}$

11.2.9 $a.\ I = 1.44\ mA$. $b.\ I = 2.5\ mA$, the current through the jumper is zero. $c.\ I_1 = 2.79\ mA$, $I_2 = 1.77\ mA$, $I_3 = 0.96\ mA$.

11.2.10 $b.\ \Delta I = \frac{IkT}{(RC)}$

11.2.11 $\Phi_{max} = VRC = 5 \cdot 10^{-7}\ Vb$.

11.2.12 $a.\ V_1 = t\sqrt{\frac{2\mu_0 ma^3}{(hd)}}, V_2 = t^3\sqrt{\frac{32\mu_0 mb^2}{(9hd)}}$

$b.\ V_1 = (8,7 \cdot 10^8\ \frac{V}{s})t, V_2 = (1,2 \cdot 10^{14}\ \frac{V}{s^3})t^3$

11.2.13 $\varepsilon = (\frac{\pi r^2}{3})nB_0\omega\sin\omega t$.

11.2.14 $\omega = \frac{qBl^2}{(2mr^2)}$. There will be no change in

11.2.15 $B(t) = \alpha t(1 + \frac{r^2}{r_0^2})$.

11.2.16 Decreasing. As the magnetic field induction increases, the Lorentz force and the electron velocity increase. But the latter is not fast enough for the electron to stay on a circle of the same radius

11.2.17 $l = \frac{3r_0}{4}$. By a factor of 100. If the initial radius $r < l$, the electron will move along the convergent to the center spiral, at $r > l$ - along the divergent spiral.

11.2.18 $\omega = \frac{2\sigma B}{[r(\rho+2\mu_0\sigma^2)]}$.

11.2.19 $a.\ 2.6 \cdot 10^{12}$ times. $b.\ nSr \approx 7 \cdot 10^{-14}\ m^2$ where n is the number of turns per unit length of the solenoid, r is the radius of the solenoid, S is the wire cross-section.

11.2.20 $m_{e.m.} = \varepsilon_0\mu_0 CV^2 = \frac{CV^2}{c^2}$ where c is the speed of light

11.2.21 $m_{e.m.} \approx 10^{-27}\ kg$

11.3 Mutual inductance. Conductor inductance. Transformers

11.3.1 $\Phi = \mu_0 ISn\sin\alpha, L_{12} = \mu_0 Sn\sin\alpha$.

11.3.2 $L_{12} = (\frac{\mu_0\pi r^2 n}{2})(cos\alpha + \sin\alpha)$.

11.3.3 $L_{12} = \mu_0 \pi r^2 n N$

11.3.4 $V = \mu_0 \pi r^2 n N \omega I_0 \cos \omega t$

11.3.5 $L = \mu_0 \pi r^2 n^2 l$. b. Equation of motion of the electron in the solenoid

$e(E - \frac{L}{l}\frac{dI}{dt}) = m_e \frac{dv}{dt}, l = 2\pi r N.$

But $e n_e S v = I$. Therefore, the first equation can be rewritten as

$El = V = (L + \frac{m_e l}{e^2 n_e S})\frac{dI}{dt}.$

So $L_1 = L + \frac{m_e l}{e^2 n_e S}$. Can.

11.3.6 $L = \frac{\mu_0 \pi (r_1^2 + r_1 r_2 + r_2^2) n^2}{3} = 2.3\ \frac{Gn}{m}$

11.3.7 $t = \frac{B\sqrt{v}}{(V\sqrt{\mu_0})} = 8.9 \cdot 10^{-2}\ s.$

11.3.8 At $h \ll d L = \frac{\mu_0 h}{d} = 6.3 \cdot 10^{-8}\ \frac{Gn}{m}$.

11.3.9 $L = \frac{\mu \mu_0}{2\pi} ln \frac{r_1}{r_2}$.

11.3.10 $L = \frac{\mu_0}{4\pi}(\mu_1 + 2\mu_2 ln\frac{r_1}{r_2})$

11.3.11 $L = \frac{\mu_0}{\pi} ln \frac{h}{r}$.

11.3.12 Increased by a factor of k

11.3.13 $L1 = \mu_0 \pi (n_1^2 r_1^2 l_1 + n_2^2 r_2^2 l_2 + 2 n_1 n_2 r_1^2 l_2); L_2 = \mu_0 \pi (n_1^2 r_1^2 l_1 + n_2^2 r_2^2 l_2 - 2 n_1 n_2 r_1^2 l_2).$

11.3.14 $L = L_1 + L_2 + 2L_{12}$

11.3.15 $L_{12} = \sqrt{L_1 L_2}$.

11.3.16 $E_2 = (\frac{\mu \mu_0 N_1 N_2 S}{l}) I_0 \omega \cos \omega t. V_1 = (\frac{\mu \mu_0 N_1^2 S}{l}) I_0 \omega \cos \omega t.$

11.3.17 $V_2 = const$

11.3.18

11.3.19

11.3.20

11.3.21 $\nu = 100\ Hz$

11.3.22 To reduce Foucault currents

11.3.23

11.3.24 $V = 10\ V$

11.3.25 $V = 60\ V$

11.4 AC electrical circuits

11.4.1 $I(t) = \frac{\varepsilon t}{L}, A = \frac{\varepsilon^2 \tau^2}{(2L)}$. In the energy of the magnetic field

11.4.2 $a)\ V = \alpha(Rt + L)$. $b)\ V = I_0(R \sin \omega t + L\omega \cos \omega t)$

11.4.3 $W_{max} = \frac{(LI)^2}{(RT)}$

11.4.4 $I(t) = (\frac{\varepsilon_0}{\omega L})(1 - \cos \omega t).$

11.4.5 See Fig.

11.4.6 $C(t) = C_0[1 - \frac{t^2}{(2LC_0)}]$

11.4.7 $V_{max} = V_0 R \sqrt{\frac{C}{L}}$

11.4.8 a. When open. b. $C = \frac{1}{[(2\pi \nu N)^2 L]} \approx 1\ \mu F$

11.4.9 $I_{max} = \varepsilon \sqrt{\frac{C}{L}}, q_{max} = 2\varepsilon C.$

11.4.10 $I_{1max} = V \sqrt{\frac{C L_2}{L_1(L_1 + L_2)}}, I_{2max} = V \sqrt{\frac{C L_1}{L_2(L_1 + L_2)}}$

11.4.11 $a. I = V_0 \sqrt{\frac{C}{L}} \sin \omega_0 t$, where $\omega_0 = \frac{1}{\sqrt{LC}}$

$b.\ I = \frac{V_0}{L(\omega_0^2 - \omega^2)}(\omega_0 \sin \omega_0 t - \omega \sin \omega t); I_{max} = \frac{V_0}{L|\omega - \omega_0|} \approx 4,8\ kA$

11.4.12 a. See Fig $VR = RI_0, VL = \omega L I_0, VC = \frac{I_0}{(\omega C)}$.

$b.\ V_0 = I_0 \sqrt{R^2 + [\omega L - \frac{1}{(\omega C)}]^2}, \varphi = arctg \frac{\omega L - \frac{1}{(\omega C)}}{R}.$

11.4.13 $\varepsilon_0 = 208\ V.$

11.4.14 $I(t) = \frac{\varepsilon_0(\omega^2 LC - 1)}{\omega L(2 - \omega^2 LC)} cos \omega t$

11.4.15 $L = 2.8\ Gn$

11.4.16 $V = V_0 sin(\omega t - \varphi)$, where $\varphi = arctg \frac{2\omega C_0 R_0}{(\omega CR)^2 - 1}$.

11.4.17 $a.\ I_L = 0, I_R = (\frac{\varepsilon_0}{R}) \sin \omega t, N = 200\ W$

$b.\ I_R = (\frac{\varepsilon_0}{R}) \sin \omega t, I_C = -\varepsilon_0 \omega C(sin \omega t + \cos \omega t), N = 200\ W$

11.4.18 $L = 0.16\ Gn$

11.4.19 See Fig.

11.4.20 If V_{C_0} and V_C are the potential differences respectively on the capacitor C_0 and C, and I is the current in the circuit, then $V_{C_0} - V_C = \frac{LdI}{dt} = V_0 \cos \omega t, \omega = \sqrt{\frac{LCC_0}{(C + C_0)}}$. But $(V_0 - V_{C_0})C_0 = V_C C$. From these equations we find

$V_C = (1 + \frac{C}{C_0})^{-1} V_0 (1 - \cos \omega t)$

Therefore, at $V < 2V_0(1 + \frac{C}{C_0})^{-1}$ the breakdown occurs after time

$\tau = \frac{1}{\omega} arccos[1 - (1 + \frac{C}{C_0})\frac{V}{V_0}]$

and at $V > 2V_0(1 + \frac{C}{C_0})^{-1}$ the capacitor of capacity C does not break through

11.4.21 *b*. If I_1 and I_2 are currents through the inductance coils L_1 and L_2, and $\omega = \frac{1}{\sqrt{(L_1+L_2)C}}$ and $I_0 = \frac{V_0}{(\omega L_1)}$, then $L_1I_1 + L_2I_2 = LI_0, I_1 - I_2 = I_0 \cos\omega t$. From these equations we find

$I_2 = \frac{L_1}{L_1+L_2}(1+\cos\omega t)I_0, I_{max} = 2V_0\sqrt{\frac{C}{L_1+L_2}}$

11.4.22 *a*. $L_1I_1 + L_2I_2 = L_1I = (L_1 + L_2)I_0$, where I_0 is the steady-state current through the coils of inductors L_1 and L_2.

$W = \frac{1}{2}L_1I^2 - \frac{1}{2}(L_1+L_2)I_0^2 = \frac{L_1L_2}{2(L_1+L_2)}I^2$.

b. I_1 to $I_1 - \frac{2(I_1-I_2)}{(1+\frac{L_1}{L_2})}$; I_2 to $I_2 + \frac{2(I_1-I_2)}{(1+\frac{L_2}{L_1})}$.

11.4.23 $R = 1,4 \cdot 10^{-3}\ Ohm$

11.4.24 $W = \frac{(L+CR^2)(I_1^2-I_2^2)}{2}$.

11.4.25 $\varphi = 2arcsin(\omega\sqrt{\frac{LC}{2}}).v = \frac{\omega l}{\varphi}$ at $\omega < \frac{2}{\sqrt{LC}}; v = \frac{l}{\sqrt{LC}}$ at $\omega \ll \frac{1}{\sqrt{LC}}$.

11.5 Conservation of magnetic flux. Superconductors in a magnetic field

11.5.1

11.5.2 $B = B_0(\frac{r_0}{r})^2$

11.5.3 It will be halved.

11.5.4 One and a half times

11.5.5 Reduced by a factor of three

11.5.6 Only the axial component of the magnetic field induction changes. In the region of the external field it is equal to $(\frac{1}{2})B_0\cos\alpha$, and outside this field it is equal to $-(\frac{1}{2})B_0\cos\alpha$.

11.5.7 $I = I_0 - (\frac{\pi r^2}{L})B_0\cos\alpha$.

11.5.8 $I_0 = \frac{\pi D^2 B}{(4L)}$.

11.5.9 Outside the steel cylinder the induction will decrease by $\frac{B_0}{2}$, inside it will increase by $\frac{B_0}{2}$.

11.5.10 See Fig. x - coordinate of the front end of the rod, counted from the beginning of the coil.

a. $I_{max} = \frac{I_0}{1-\frac{\sigma}{S}}$. *b*. $I_{max} = \frac{I_0}{1-\frac{\sigma l}{(Sh)}}$.

11.5.11 $L = \mu_0\pi r^2(1 - \frac{r^2}{R^2})\frac{N^2}{l}$

11.5.12 $I = a\sqrt{\frac{2\rho C_u gh}{\mu_0}} = 380\ A$, ρ_{C_u} is the density of copper.

11.5.13 The magnetic field above the superconducting plane AA' coincides with the magnetic field, which is the result of superposing the magnetic fields of a straight wire with current I and a wire with current $(-I)$, symmetrically located under the AA' plane. There is no magnetic field above the AA' plane. Therefore, $P = \frac{\mu_0 I^2}{[2(\pi h)^2]}$. The interaction with the superconducting plane of a long wire with current I is equivalent to the interaction of two wires that are $2h$ apart, with currents flowing in opposite directions. Therefore $f = \frac{\mu_0 I^2}{(4\pi h)}$.

11.5.14 $v = \frac{V}{(\pi r^2 nB)} = 2\ \frac{km}{s}$

11.5.15

11.5.16 From the laws of conservation of energy and magnetic flux in the solenoid follows

$\frac{1}{2\mu_0}B_0^2(W-w) + \frac{1}{2}mv_0^2 = \frac{1}{2\mu_0}B^2W + \frac{1}{2}mv^2, B_0(W-w) = BW$

where $B_0 = \frac{\mu_0 NI}{L}$ and B are the maximum magnetic field induction in the solenoid before and after the projectile, $W = \pi R^2 L$ and $w = \pi r^2 l$ are the volumes of solenoid and projectile. From the given equations we obtain

$\Delta v = \sqrt{v_0^2 + \pi\mu_0(\frac{NI}{L})^2 r^2 l[1 - \frac{r^2 l}{(R^2 L)}]} - v_0$

11.5.17 $v = NIr\sqrt{\frac{\pi\mu_0}{(12lm)}}$.

11.5.18 When entering a magnetic field in a superconducting rod, a current arises that creates a field inside the rod, the induction of which is equal in modulo to the induction of the external field and is directed in the opposite direction to it. The work to create this current $A = \frac{B^2 Sl}{(2\mu_0)}$ is equal to the change in kinetic energy of the rod. Hence, $v_{min} = B\sqrt{\frac{Sl}{(\mu_0 m)}}$.

11.5.19 The magnetic flux in any cross section of the tube during the projectile's flight does not change:

$\pi r_1^2 B = \pi(r_1^2 - r_0^2)B_1, \pi r_2^2 B = \pi(r_2^2 - r_0^2)B_2$.

Using these equations and the law of conservation of energy gives

$\Delta K = (\frac{lB^2}{2\mu_0})[\frac{r_1^4}{(r_1^2-r_0^2)} - \frac{r_2^4}{(r_2^2-r_0^2)}]$.

11.5.20 $v_1 = v, v_2 = 3v$ if $mv^2 < \frac{B^2 lSs^2}{[4\mu_0(2S-s)(S-s)]}; v_1 = 3v, v_2 = v$ if $mv^2 > \frac{B^2 lSs^2}{[4\mu_0(2S-s)(S-s)]}$].

11.5.21 $v_1' = v_1, v_2' = v_2$ if $\frac{(v_2-v_1)^2}{\frac{1}{m_1}+\frac{1}{m_2}} > \frac{B^2 lSs^2}{2\mu_0(2S-s)(S-s)}$

$v_1' = \frac{2m_2v_2 + (m_1-m_2)v_1}{m_1+m_2}, v_2' = \frac{2m_1v_1 + (m_2-m_1)v_2}{m_1+m_2}$,

if $\frac{(v_2-v_1)^2}{\frac{1}{m_1}+\frac{1}{m_2}} < \frac{B^2 Ss^2}{2\mu_0(2S-s)(S-s)}$

11.5.22

11.5.23 $T' = \frac{T}{\sqrt{1+\frac{B^2r^4T^2}{(4LJ)}}}$

11.5.24 $\omega = 2i\sqrt{\frac{\mu_0 ah}{[m(l-d)]}}$

11.5.25 $v = v_0(1+\sqrt{1+\frac{LxI_0^2}{(mv_0^2)}})$

11.5.26 $B = B_0 + \frac{2\mu_0\rho v^2\Delta}{(Br_0)} \approx 500\ Tl$, $P = B^2(2\mu_0) \approx 10^{11}$ Pa

11.5.27 Equation of motion of the electron in the tube

$m_e\frac{dv}{dt} = eE = e\frac{r}{2}\frac{d(B-B')}{dt}$,

where B and B' are the inductions of the external magnetic field and the field created by the moving electrons. Therefore, $m_e v = \frac{er(B-B')}{2}$. On the other hand, $en_e vh = j, B' = \mu_0 j$, where j is the linear current density. From the last equations we get $j = \frac{e^2hB}{(2m_e+e^2r\mu_0 n_e h)}$ and then $B - B' = \frac{m_e B}{\frac{m_e+e^2r\mu_0 n_e h}{2}} = 5.7\cdot 10^{-5}\ Tl$.

11.5.28 $B = \frac{2m_e\omega}{e}$

11.6 Relation of the alternating electric field to the magnetic field

11.6.1 $C_B = \frac{1}{c}\frac{dN}{dt}$(in GHS); $C_B = \mu_0\varepsilon_0\frac{dN}{dt}$ (in SI). C_B is the circulating magnetic field induction, N is the electric displacement flux, c is the speed of light, ε_0 and μ_0 are the electric and magnetic constants.

11.6.2 a. $\frac{dN}{dt} = vlE, C_B = \mu_0\varepsilon_0 vlE, C_B = \mu_0\varepsilon_0\frac{dN}{dt}$ (in SI), $C_B = \frac{1}{c}\frac{dN}{dt}$ (in GHS)

11.6.3 $N = 9\cdot 10^5\ V\cdot m$

11.6.4 According to Gauss's law the electric displacement flux inside the capacitor $N = \frac{Q}{\varepsilon_0}$, where Q is the charge of the capacitor and the rate of change of flux $\frac{dN}{dt} = \frac{1}{\varepsilon_0}\frac{dQ}{dt} = \frac{1}{\varepsilon_0}I$, where I is the current in the circuit. Therefore, the circulation of magnetic field induction $C_B = \mu_0\varepsilon_0\frac{DN}{dt} = \mu_0 I$ coincides with the circulation of magnetic field induction that would be created by the current I.

11.6.5 $B = 2,5\cdot 10^{-6}\ Tl$

11.6.6 $n = \frac{2\pi Nr}{L}$

11.6.7 $B = \mu_0\varepsilon_0 Ev\cos\alpha$

11.6.8 $\sigma = \frac{B}{(\mu_0 v})$

11.6.9 a. $B = \frac{\mu_0\varepsilon_0 vV}{h}$ inside conductor, $B = \frac{-\mu_0\varepsilon_0 vV}{h}$ between conductor and capacitor's coils.

b. Will decrease by a factor of $\frac{(\varepsilon+1)}{(\varepsilon-1)}$.

11.6.10 See Fig. In the first case, because of the polarization current flowing through the circuit $abb'a'$, the circulation of the magnetic field induction vector through this contour will be ε times greater than in the second case. Therefore, the motion of the medium together with the contour reduces the induction of the magnetic field by a factor ε

11.6.11 a. The induction of the magnetic field caused by an alternating electric field, $B_1 = \frac{\pi r^2\alpha\mu_0\varepsilon_0}{(2\pi r)} = \frac{\mu_0\varepsilon_0\alpha r}{2}$.The induction of the magnetic field caused by the polarization current of the dielectric is $\varepsilon - 1$ times greater:$B_2 = (\varepsilon-1)B_1$. Therefore $B = B_1 + B_2 = \varepsilon B_1 = \frac{\mu_0\varepsilon_0\varepsilon\alpha r}{2}$.

b. $B_1 = \frac{\mu_0\varepsilon\varepsilon_0\alpha Vr}{2h}, B_2 = \frac{\mu_0\varepsilon_0\alpha V}{2hr_0}[r^2(\varepsilon-1)+r_0^2]$.

11.6.12 See Fig. $B_0 = \frac{\mu_0 Ir}{(2\pi r_0^2)}$. When $x < r$ the value of $B = \frac{\mu_0 Ix}{2\pi r_0^2}$, at $r_0 > r > r$ the value $B = \frac{\mu_0 Ir^2(r_0^2-x^2)}{2\pi x^2 r_0(r_0-r^2)}$, for $x > r_0$ the value of $B = 0$.

12 ELECTROMAGNETIC WAVES

12.1 Properties, radiation and reflection of electromagnetic waves

12.1.1 In the direction of the z-axis

12.1.2 $a), b)$ Will be reversed

12.1.3 $E = E_0\sin[\frac{2\pi}{\lambda}(z-ct)]$

12.1.4 $E_0 = \sqrt{E_1^2+E_2^2+2E_1E_2cos(\varphi_1-\varphi_2)}$

$\varphi = \omega(t-\frac{z}{c}) + arctg\frac{E_1\sin\varphi_1+E_2\sin\varphi_2}{E_1\cos\varphi_1+E_2\cos\varphi_2}$.

12.1.5 $E = 2E_0, w = \frac{1}{2\pi}E_0^2cos^2[(t-\frac{z}{c})\Delta+\varphi]$

12.1.6 $B = \frac{E}{c}(inSI), B = E(inGHS)$

12.1.7 $B = E\frac{\sqrt{\varepsilon}}{c}(inSI), B = E\sqrt{\varepsilon}(inGHS)$.

12.1.8

12.1.9 $B = E\frac{\sqrt{\varepsilon\mu}}{c}(inSI), B = E\sqrt{\varepsilon\mu}(inGHS)$.

12.1.10 See fig. $\frac{1}{2}, \frac{1}{2}; 1.0; \frac{1}{2}, \frac{1}{2}$.

12.1.11 a. Two plane waves running in opposite directions. Wave length d, electric field strength in the wave $\frac{E}{2}$. b. Two plane waves propagating perpendicularly to the planes AB and $A'B'$ in opposite directions. Induction of the electric field in the wave $\frac{cB}{2}$

12.1.12 a. $E_{rad} = \frac{1}{2}\frac{v}{c}E$

b. When the sphere stops, all the energy of the magnetic field will be transferred to the radiation energy. At any point, the magnetic field induction of a moving charge is equal in GHS to the electric field strength of the electric field multiplied by $(\frac{v}{c})\sin\theta$. Therefore, the energy transferred to radiation, would be equal to the electric field energy $\frac{Q^2}{(2r)}$ multiplied by $(\frac{v}{c})^2$ if there were no multiplier $sin\theta$. Because of this multiplier, the energy of the magnetic field decreases by another factor and a half. Thus,

$W = \frac{Q^2}{3r}(\frac{v}{c})^2 (inGHS), W = \frac{Q^2}{12\pi\varepsilon_0 r}(\frac{v}{c})^2 (inSI)$.

c. The intensity of the "extra" fields will increase by a factor of two. Radiated energy is proportional to the square of the intensity. Therefore, the power of radiation will increase by a factor of four.

12.1.13 Interference of radiation from different plates.

$\nu' k = \frac{c}{d}k, \nu'' k \approx \frac{c}{d}(k + \frac{1}{2})$,

k is an integer

12.1.14

12.1.15 a. See Fig. At time t at point A, the electric field strength radiation $E_{rad} = E_1 + E_2$, where E_1 and E_2 are the field strengths in the wave emitted by the upper and bottom plates:

$E_1 = \frac{1}{2c}Ev_{t-\frac{x}{c}} = \frac{1}{2c}Ea(t - \frac{x}{c}), E_2 = -\frac{1}{2c}Ev_{t-\frac{(x+d)}{c}} = -\frac{1}{2c}Ea(t - \frac{x+d}{c})$.

So, $E_{rad} = E_1 + E_2 = \frac{adE}{(2c^2)}$.

b. $E_{rad} = \frac{\mu_0 c i_0}{2} = \frac{i_0}{(2c\varepsilon_0)}(inSI); E_{rad} = \frac{2\pi i_0}{c}(inGHS)$.

c. In the electric field of the wave $E_0 \sin\omega t(\omega = 2\pi\lambda)$ the speed of the electrons $v = \frac{eE_0}{m_e\omega}cos\omega t$.The amplitude of the electric field strength in the wave emitted by these electrons, $E_{rad} = \frac{eE_0}{m_e\omega}\frac{n_e e}{2c\varepsilon_0}$. Reflection coefficient $k = (\frac{E_{rad}}{E_0})^2 = [\frac{n_e e^2 x}{(4\pi m_e \nu\varepsilon_0 c)}]^2$.

You can also find the reflection coefficient by determining by how much the wave is attenuated after passing through the film. In this case secondary emission of electrons caused by their interaction with the wave already emitted by the same electrons during interaction with the incident wave. Due to superposition of secondary radiation coming in the antiphase on the wave passing through the film the intensity of the wave decreases, and due to superposition of primary radiation coming in the shifted phase on it the intensity of the wave decreases. radiation coming with a phase shift of $\frac{\pi}{2}$, the intensity of the wave increases. The first effect is two times stronger than the second one. Therefore, the intensity of the wave after passing through the film will decrease by an amount equal to the intensity of the reflected wave

12.1.16 $\lambda = 4 \cdot 10^{-5}\ cm$

12.1.17 As the film thickness increases, more and more electrons become involved in the radiation reflection. more electrons and the amplitude of the reflected wave increases linearly (region $x < x_1$). The linear dependence of the amplitude on the film thickness is broken when the fraction of reflected radiation is large. This is the case when $x > x_2$.

12.1.18 $\Delta \approx \frac{4\pi m_e \nu\varepsilon_0 c}{(n_e e^2)} \approx 10^{-5}\ cm.$

12.1.19 $E = 0, B = \frac{2E}{c}$

12.1.20 $\lambda = 4 \cdot 10^{-5}\ cm$, $x = 2 \cdot 10^{-5}\ cm.$

12.1.21 $j = 2\varepsilon_0 cE, P = 2\varepsilon_0 E^2 (inSI); j = \frac{cE_0}{(2\pi)}, P = \frac{E^2}{(2\pi)})(inGHS)$

12.1.22 $P = c_0 E_0^2$

12.1.23 $P = 2\ mPa$, $P = 0.5\ mPa$

12.1.24 $r \approx 1\mu m$

12.1.25 See Fig. $a)\ E' = -E, B' = B.\ b)\ E' = E$

12.1.26 $E = \sqrt{P\varepsilon_0 cos^2\alpha}$

12.1.27 The force acting on an electron moving along a metal surface is $F = e(E - \frac{v}{c}B) = 0$. Therefore $\frac{E}{B} = \frac{v}{c}$.

12.1.28 $P = 2\varepsilon_0 E_0^2 \frac{c+v}{c-v}(inSI); P = \frac{1}{8\pi}E_0^2\frac{c+v}{c-v}(inGHS)$.

12.1.29 $v = \frac{c\Delta}{(2\nu_0 + \Delta)}$

12.1.30 $v = c\frac{k-1}{k+1}$.

12.1.31 a. The charges induced on the flat boundary create an electric field perpendicular to the flat boundary. Therefore, only the perpendicular component of electric field strength of the wave decreases by a factor of ε

b. Induced surface currents create a magnetic field whose induction is parallel to the surface.

Therefore, only the parallel component of the wave's magnetic field induction increases by a factor μ.

12.1.32 On different sides of the interface the electric field strength and the magnetic field induction are the same: $E - E_0 = E_n, B + B_0 = B_n$, and $B = E\frac{\sqrt{\varepsilon_1}}{c}, B_0 = E_0\frac{\sqrt{\varepsilon_1}}{c}, B_n = E_n\frac{\sqrt{\varepsilon}}{c}$ (see Problem 12.1.7). From these equations it follows that $\frac{E_0}{E} = \frac{(\sqrt{\varepsilon_2}-\sqrt{\varepsilon_1})}{(\sqrt{\varepsilon_2}+\sqrt{\varepsilon_1})}$. At $\sqrt{\varepsilon_1} < \sqrt{\varepsilon_2}$ the signs of E_0 and E are the same, and for $\sqrt{\varepsilon_1} > \sqrt{\varepsilon_2}$ are opposite. This means that in the first case the phase of the reflected wave does not change, and in the second case it changes by π.

12.1.33

12.1.34 See Fig. In the GHS $W_1 = \frac{7E_0^2r_0^3}{3}, W_2 = 2E_0^2r_0^3$.

12.2 Propagation of electromagnetic waves

12.2.1

12.2.2 The wavelength and speed decrease by a factor of n, the frequency does not change.

12.2.3

12.2.4 $sin\alpha_k = \frac{k\lambda}{b}$,where k is an integer.

12.2.5 In k^2 times

12.2.6 $\Delta\alpha = 13,5'$

12.2.7 $l = \frac{2r^2}{\lambda}, l_k = \frac{2r^2}{[\lambda(k+1)k]}$.

12.2.8 The intensity of radiation at other points will increase.

12.2.9 Four times

12.2.10 See Fig.

12.2.11 Increased by a factor of 100 (a) and 324 (b).

12.2.12 $c = \frac{i}{\lambda}, a = \frac{A\Delta Si}{(r\lambda)}$ (multiplying by i means the phase shift of the secondary wave by $\frac{\pi}{2}$).

12.2.13 $a.\ R \approx 1\ km.\ b.\ R \approx 1.5\ m.$

12.2.14 $l \approx 1\ m$, $0.5\ km$, $150\ km$

12.2.15 $a.$ The blue part of the filament spectrum is scattered on the matte surface stronger.

$c.$ Because of fluctuations in atmospheric air density, the blue part of the spectrum is scattered stronger.

13 GEOMETRIC OPTICS. PHOTOMETRY. QUANTUM NATURE OF LIGHT

13.1 Straight-line propagation and reflection of light

13.1.1 See Fig.

13.1.2 See Fig

13.1.3 See Fig

13.1.4 An image of the Sun appears on the wall. In the case where the size of the hole will be is larger than the image of the Sun on the wall.

13.1.5 The mirror does not "flip" the image. But an opaque object appears to us upside down from right to left, because usually we only see the side reflected by the mirror if the object is rotated $180°$

13.1.6 $H = \frac{h}{2}$

13.1.7 Doesn't change

13.1.8 The double reflection results in an inverted image. From anywhere in the room

13.1.9 Note: Look in the kaleidoscope.

13.1.10 $\alpha = 120°$

13.1.11 The course of the rays is shown in the figure.

13.1.12 $x = \frac{h}{2}$.

13.1.13 $f = \frac{R}{2}$.

13.1.14

13.1.15 $f = 36\ cm$

13.1.16 $l = 20\ cm.$

13.1.17 $f = 48\ cm.$

13.1.18 See Fig

13.1.19 See Fig

13.1.20 A paraboloid of rotation if its axis is parallel to the rays

13.2 Refraction of light. Lens formula

13.2.1 $\alpha = 48°$

13.2.2 $a.\ h = 4m.\ b.\ h = 4\ km.$

13.2.3 Decrease in $\frac{L+l}{L+\frac{l}{n}}$ times

13.2.4 $n = 1,13.$

13.2.5 $a.\ \alpha_a = 24,6°$, $\alpha_c = 49°, \alpha_a, c = 33,7°$

$b.$ Because of the total internal reflection of the rays from the bubbles.

13.2.6 No

13.2.7 $R = \frac{ln}{(n-1)}.$

13.2.8 At $\alpha > arccos(\frac{2}{3})$ the light already at the first reflection will partially leave the cone. At $\alpha < arccos(\frac{2}{3})$ the light will first be completely reflected from the side surface. After each reflection the angle between the ray and the normal to the surface of the cone will diminish by 2α, and after several reflections the light will leave the cone through its surface again.

13.2.9 $r = \frac{R}{2}$

13.2.10 $r' = \frac{r}{n}$

13.2.11 $sin\alpha = \frac{n}{k^{N-1}}$ if $\frac{n}{k^{N-1}} < 1$; always passes if $\frac{n}{k^{N-1}} > 1$

13.2.12 $H = \frac{1}{2}(\frac{n_0}{\alpha - R}).$

13.2.13

13.2.14 $c.\ \frac{1}{F} = (n-1)(\frac{1}{R_1} + \frac{1}{R_2})$

13.2.15 $a.\ F = 0.25\ m$, $D = \frac{1}{F} = 4\ dpt.\ b.\ R = 0.6\ m$

13.2.16 $R = 0.26\ m.$

13.2.17 $a) \frac{1}{x} = \frac{1}{nf} - \frac{n-1}{nr}$; $b)\ \frac{1}{x} = \frac{1}{f} - \frac{n-1}{r}.$

13.2.18 $f = -\frac{1}{(n-1)}\frac{R^2}{\delta}$

13.2.19 $\Delta = \alpha(n_1 - n_2)f.$

13.2.20 From the part of the lateral surface of the half-cylinder bounded by the angle $\alpha = 2arcsin(\frac{1}{n})$

13.2.21 $n = \frac{4}{3}.$

13.2.22 $n = \frac{3}{2}$

13.2.23 $y = \frac{x}{n}$

13.3 Optical systems

13.3.1 See Figure: $a)\ k = \frac{1}{2}$; $b)\ k = \frac{3}{2}$; $c)\ k = \frac{1}{4}$; $d)\ k = \frac{3}{4}$

13.3.2 See Fig.

13.3.3 $f = 20\ cm$

13.3.4 $f = 2f.$

13.3.5 $v = 2\omega f$

13.3.6 $k = \frac{f^2}{(a-f)^2 - \frac{l^2}{4}}$

13.3.7 $f = \frac{3}{7}\ m$

13.3.8 $t = 5\ ms$

13.3.9 $\frac{df}{dt} = \frac{vk}{1+k}, \frac{db}{dt} = vk.$

13.3.10 To the lens at distance $l = \frac{r_1 - r_2}{\frac{D}{2} + r_2} f$

13.3.11 $2.3\ m < l_1 < 3.2\ m$; $1.6\ m < l_2 < 8\ m.$

13.3.12 $D_1 = -5\ Dpts$, $D_2 = 2\ Dpts$

13.3.13 Severely short-sighted.

13.3.14 The aperture limits the working area of the lens and allows viewing objects less than $25\ cm$ away from the eye. The magnification will be $k = \frac{25}{x}$, where x distance.

13.3.15 $f = 2.5\ cm$

13.3.16 $tg\alpha' = (1 - \frac{\alpha}{f})tg\alpha$

13.3.17 $k = 2.$

13.3.18 $a.\ l_1 = \frac{f(a-2f)}{(a-f)}, l_2 = a - f.\ b.\ f' = \frac{f}{2}$

13.3.19 $f' = \frac{f_1 f_2}{f_1 + f_2}; \frac{1}{f'} = \frac{1}{f_1} + \frac{1}{f_2}$

13.3.20 $f' \approx \frac{f}{2} + \frac{3l}{4}$ from the first lens

13.3.21 At a distance greater than $10\ cm$ from the nearest lens

13.3.22 $f > 0.6\ m$

13.3.23 $x_1 = \frac{(d-R)f}{d-R-f}, x_2 = \frac{df}{d-f}$ when $d > R + f; x = \frac{fd}{d-F}$ at $f < d < R + f$; at $d < F$ there is no solution.

13.3.24 $n = \frac{(R - \frac{l}{2})}{(R-l)}$

13.3.25 $f_1 = \frac{n}{2}f; f_2 = \frac{n(n_0 - 1)}{2(n_0 - n)}f$

13.3.26 $\frac{h_2}{h_1} = (l - f)f$

13.3.27 Will increase by $\frac{a}{(a-f)}$ times

13.3.28 At $\frac{25l}{f_1 f_2}$; will decrease by a factor of k.

13.4 Photometry

13.4.1 $h = 1\ m$

13.4.2 $E_1 = 130\ lx$, $E_2 = 71\ lx$, $E_2 = 25\ lx$

13.4.3 $E = 41\ lux.$

13.4.4 $E = \frac{I[h^2+(h+2x)^2]}{h^2(h+2x)^2}$

13.4.5 See Fig.

13.4.6 At $80,000$ times.

13.4.7 $I' = \frac{I(1-k)}{(1+k)}$.

13.4.8 $x \sim 5$ light years.

13.4.9 $\frac{N'}{N} \sim (\frac{R}{r})^2$ where R is the radius of Venus, and r is the distance from Earth to Venus.

13.4.10 $x \sim \frac{R^2}{r}$, where r is the characteristic size of the car

13.4.11 The illumination of the image will decrease: the upper part of the arrow-object - slightly more than twice as much, the lower part slightly less than twice as much

13.4.12 It will increase twice.

13.4.13 $\frac{E_{left}}{E_{right}} = (\frac{R}{2l})^2$

13.4.14 $E = E_0 \frac{a^2 f^2}{[xf-(a-x)(x-f)]^2}$. If $x = \frac{a}{2}$

13.4.15 $L = \frac{L_0 D^2}{D_0^2}$

13.4.16 No. $E_{max} = \frac{B\pi D^2}{R^2}$.

13.4.17 $D = 1.85\ m$

13.4.18 $N \approx \frac{4x^2}{d^2}(\frac{T}{T_C})^4 \approx 770$, where $T_C \approx 6 \cdot 10^3\ K$ is the surface temperature of the Sun

13.4.19 It won't change.

13.4.20 The luminous flux from the star to the eye increases dramatically.

13.4.21 $d_2 = \frac{5d_1}{6}$

13.4.22 $t_1 = t_2(\frac{k_1+1}{k_2+1})^2$.

13.4.23 $n \approx (10\pi r^2 L)^{-1}$

13.4.24 $\rho = 0.2\ \frac{g}{m^3}$

13.4.25 Eight times

13.5 The quantum nature of light

13.5.1 $E_1 \approx 10^{-6}\ \frac{W}{m^2}$, $E_2 \approx 4 \cdot 10^{-6}\ \frac{W}{m^2}$, $E_3 \approx 4 \cdot 10^{-5}\ \frac{W}{m^2}$

13.5.2 $W = h\nu - eV_0$

13.5.3 The velocities of the electron and positron must be equal in modulo and oppositely directions. $\nu = 1.24 \cdot 10^{20}\ Hz$

13.5.4 $a.\ v = c\frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2}$. $b. v = c\frac{sin(\theta_1+\theta_2)}{sin\theta_1 + \sin\theta_2}$

13.5.5 $a.\ m = (1-\cos\theta)\frac{h\nu^2}{c^2\delta\nu}$. $b.\ \Delta\nu = (1-\cos\theta)\frac{h\nu'}{m_e c^2}$

13.5.6 a) When a photon is emitted in the direction of the atom's motion

$\frac{mv^2}{2} = \frac{m(v-\Delta v)^2}{2} + hv + \varepsilon, mv = m(v - \Delta v) + \frac{hv}{c}$

When a photon is emitted in the direction opposite to that of the atom,

$\frac{mv^2}{2} = \frac{m(v+\Delta' v)^2}{2} + h\nu + \varepsilon, mv = m(v + \Delta' v) - \frac{h\nu'}{c}$

In (1) and (2) m is the mass of the atom, Δv and $\Delta' v$ are the change in velocity, ε is the change in the internal energy of the atom, ν' is the unknown photon frequency. For $\Delta v, \Delta' v \ll v$ it follows from (1) and (2) that $\nu' = \nu\frac{1-\frac{v}{c}}{1+\frac{v}{c}}$. $b)$ $\nu' = \nu(1 - \frac{v}{c})$.

13.5.7 The attraction of photons to the star.

13.5.8 $\Delta\nu = \frac{\nu\gamma M}{(Rc^2)}, \gamma$ is the gravitational constant. $\Delta\nu_C \approx 10^9\ Hz$. Thermal atoms on the surface of the Sun affects the frequency of photons emitted by it to a greater than the gravitational field.

13.5.9 $f \sim \frac{R^2c^2}{(6\gamma M)} \sim 10^9\ pc$

14 SPECIAL THEORY OF RELATIVITY

14.1 Constancy of the speed of light. Addition of velocities

14.1.1 $l = 15\ km.$

14.1.2 $v = 6 \cdot 10^7\ \frac{m}{s}$.

14.1.3 $tg\alpha = \frac{v}{c}$.

14.1.4 $tg\frac{1}{2}\Delta = \frac{sin\alpha.}{(\frac{c}{v}+\cos\alpha)} \simeq \frac{v}{c}\sin\alpha = 10^{-4}\sin\alpha$, where $v = 30\ \frac{km}{h}$ is the velocity of the Earth relative to the Sun.

14.1.5 For station observers, the travel time of the light signal, which three times traveled distance l between stations is $\frac{3l}{c}$, and the probe travel time is $\frac{l}{v}$, v being the speed of the probe. The travel times of the probe and the signal coincide: $\frac{3l}{c} = \frac{l}{v}$. Therefore, $v = \frac{c}{3}$. The probe's apparatus records a light signal that moves away from the probe at speed c. Toward of the probe, a second station moves towards it with speed v. Therefore, the time of movement of the light signal from the first station to the second one measured by the probe equipment is equal to $\frac{l'}{(c+u)}$, l' is the distance between the stations measured by the probe equipment. The time of movement of the light signal from of the second station to the first one is equal to $\frac{l'}{(c-u)}$, and the total time of movement of the light signal is equal to $\frac{l'}{(c+u)} + \frac{l'}{(c-u)} + \frac{l'}{(c+u)}$, and it is equal to the time of movement of the second station towards the probe $\frac{l}{u}$. From the equation $\frac{l'}{(c+u)} + \frac{l'}{(c-u)} + \frac{l'}{(c+u)} = \frac{l'}{u}$ we find that $u = \frac{c}{3}$. Thus, the observers of the station and the instrumentation of the probe will record the same speed of approach of the probe with the second station, equal to $\frac{c}{3}$.

14.1.6 For an observer at the first station, the travel time of the light signal to the second station will be $\frac{L}{(c+u)}$, L is the distance between the stations at the time of emission of the signal and the probe. The signal will return to the first station for the same time. Therefore, at the moment of signal reflection from the first station the second station will move to the distance $2\frac{l}{c+u} \cdot u$ $(Fig.a)$ and the distance between stations will be equal to $l = L\frac{c-u}{c+u}$. Therefore, the third time the signal will be in transit time $\frac{l}{c+u} = L\frac{c-u}{(c+u)^2}$, and the total signal travel time will be equal to $L\frac{3c+u}{(c+u)^2}$. Exactly the same time is in the way of the probe, the time of movement of which is determined through the desired velocity of the probe v_1 by the formula $\frac{L}{v_1+u}$. Equating these times, we obtain the equation $L\frac{3c+u}{(c+u)^2} = \frac{L}{v_1+u}$, from which we determine $v_1 = c\frac{c-u}{3c+u}$.

For the observer of the second station, the signal travel time from the first station to the second will be $\frac{L'}{c}$. During this time the first station will move by distance $\frac{L'}{c}u$ $(Fig.b)$, and therefore the signal will return to the first station after time $\frac{L' - \frac{L'u}{c}}{c+u} = \frac{L'}{c}\frac{c-u}{c+u}$. After the reflection from the first station the signal will return to the second station after the same time. Thus, the total time of motion of the signal will be equal to $\frac{L'}{c} + 2\frac{L'}{c}\frac{c-u}{c+u} = \frac{L'}{c}\frac{3c-u}{c+u}$ and is equal to the probe travel time, and the probe velocity is equal to the distance between the stations at the moment of the probe launch, divided by this time: $v_1 \simeq \frac{L'}{\frac{3c-u}{c+u} \cdot \frac{L'}{c}} = c\frac{c+u}{3c-u}$.

Exactly the same velocities will be recorded by the probe hardware: the first station will move away from the probe with velocity $c\frac{c-u}{3c+u}$ and the second station is approaching with speed $c\frac{c+u}{3c-u}$.

14.1.7 The velocities of both signals as observed from the ship are the same. Therefore, for the spacecraft observer at the time of reflection the stations were at the same distance, and the signal from them was reflected simultaneously, as in this case simultaneously sent signals and will return after the reflection at the same time. And what is observed from the stations? The signals relative to the ship are no longer equal to the speed of light, but equal to either $c+v$, or $c-v$. Therefore, the signal cannot be reflected simultaneously from the stations at the moment when the ship was at the same distance from them. In this case, the signal would travel faster to the ship from the station to which the ship is moving. Moreover, signals cannot be reflected at all at the same time. Indeed, for the simultaneously reflected signals to also arrive at the ship simultaneously, the ship must be at a distance $\frac{(c+v)}{(c-v)}$ greater from the station it is approaching than the distance to the station it is moving away from. But then it would have to send signals to these stations at different times, since only then would they arrive at these stations at the same time. Therefore, the stations necessarily observe the arrival of the signals at different times, and at the moment of reflection the ship is necessarily is observed at different distances from the stations. To determine the difference in the timing of the reflections of the signals from the stations, we need to find the distance of the ship from stations x and $l-x$ at the time of the signals from the ship. These distances are found from the condition of equality of signal travel times:

$$\frac{x}{c} + \frac{x}{c-v}(1+\frac{v}{c}) = \frac{l-x}{c} + \frac{x}{c+v}(1-\frac{v}{c})$$

From this equation it follows that $x = \frac{1}{2}(1-\frac{v}{c})l, l-x = \frac{1}{2}(1+\frac{v}{c})l$. Therefore, the signal travel times from the ship to the stations are defined by the formulas

$$\tau_1 = \frac{x}{c} = \frac{1}{2}(1-\frac{v}{c})\frac{l}{c}, \tau_2 = \frac{l-x}{c} = \frac{1}{2}(1+\frac{v}{c})\frac{l}{c}$$

and the difference in signal reflection times by the formula

$$\tau_1 - \tau_2 = \frac{v}{c^2}l.$$

The distance to the stations at the moments of reflections is easily found through τ_1, τ_2, x and $l-x: x_1 = x_2 = \frac{1}{2}(1-\frac{v^2}{c^2})l$.

14.1.8 $v_1 = (1-\frac{1}{k})c, v_2 = \frac{(k-1)(1-\beta^2)}{(k-1)(1+\beta)+1}c$.

14.1.9 $Fig.a$ shows schematically two consecutive reflections of a radar pulse from an object. If τ_1 and τ_2 are the return times of the pulse, then $\frac{(\tau_1+\tau_2)}{2}$ time interval between the first and the second reflection from the object, and $\frac{c(\tau_1-\tau_2)}{2}$ is the path traveled by the object during this time. So, the velocity of the object is determined through the time of momentum return by the formula

$$v = \frac{\frac{c(\tau_1-\tau_2)}{2}}{\frac{c(\tau_1+\tau_2)}{2}} = c\frac{k-1}{k+1}$$

where k is the ratio of return times $\frac{\tau_1}{\tau_2}$. And what speed of the object will be obtained if we listen to the general? $Fig.b$ shows the speed of the radar pulse and the flight times of the pulse from reflection to reflection. In this case, the velocity of the object approaching the station is determined through the values shown in $Fig.b$ by the formula

$$v' = \frac{(c-u)\tau_1^+ + (c-u)\tau_2^-}{2(\tau_1^- + \tau_2^+)}$$

In this formula we have to $\tau_{1,2}^+$ and $\tau_{1,2}^-$ determine through the observed values τ_1 and τ_2. To do this it is necessary to use the following obvious relations

$$\tau_{1,2}^+ + \tau_{1,2}^- = \tau_{1,2}, \frac{\tau_{1,2}^+}{\tau_{1,2}^-} = \frac{(c-u)}{(c+u)}$$

from which it follows that $\tau_{1,2}^{\pm} = \frac{(1\mp\frac{u}{c})\tau_{1,2}}{2}$, and the velocity

$$v' = \frac{(1-\frac{u^2}{c^2})(k-1)c}{[k+1-\frac{u}{c}(k-1)]} = \frac{(1-\frac{u^2}{c^2})v}{(1-\frac{vu}{c^2})}.$$

This velocity v' is different from velocity v and is determined, as the general assumed, not only by the ratio of times k, but also by the velocity of the laboratory u relative to the Earth. But does the The velocity v' with the approaching velocity observed from Earth? After all, τ_1 and τ_2, the times of momentum return in the laboratory system, do not coincide with the times of return τ_1' and τ_2' observed from the Earth, only their ratios are the same: $\tau_1 : \tau_2 = \tau_1' : \tau_2'$. But the equality of these ratios is already sufficient for v' coincides with the approaching velocity, observed from the Earth. This result means that the difference in the approaching velocities recorded by the laboratory and Earth observers is due to the fact that these groups record different speeds of the momentum of light relative to the lab. The first group observes this velocity equal to the speed of light, while the latter, depending on whether the pulse is flying away from the lab or towards it, is less or more than the speed of light by the value u.

The speed of the laboratory u is found from the equation

$$v' - v = \frac{v(1-\frac{u^2}{c^2})}{1-\frac{uv}{c^2}} - v = \frac{vu(v-u)}{c^2-uv} = \alpha v,$$

where $\alpha = 10^{-4}$. At this small α the velocity $u \simeq \frac{\alpha c^2}{v} = 90\ \frac{km}{s}$. The velocity of the object relative to the Earth is equal to the difference between the velocity of the object approaching the laboratory and the velocity of the laboratory, observed from the Earth:

$$v_0 = v' = u = v \cdot \frac{c^2-u^2}{c^2-uv} - u = \frac{v-u}{1-\frac{vu}{c^2}} \simeq 100,000\ \frac{km}{s}$$
$$-90\ \frac{km}{s} = 99,910\ \frac{km}{s}.$$

14.1.10

14.1.11 $v = 2,9 \cdot 108\ \frac{km}{s}$.

14.1.12 $u = \frac{(v+\frac{c}{n})}{(1+\frac{v}{nc})}$.

14.1.13 $T = \frac{2nl}{c(1-\frac{v^2}{c^2})}$.

14.1.14 $v = \frac{L\tau c^2}{l(l+2L)}(\sqrt{1+\frac{l^3(l+2L)}{(L\tau c)^2}}-1)$; at $\frac{l}{\tau}, \frac{L}{\tau} \ll c$ we get $v = \frac{l^2}{(2L\tau)}$.

14.1.15 $v_0 = \frac{(c^2-vu-\sqrt{(c^2-v^2)(c^2-u^2)})}{(v-u)}$.

14.1.16 $N = \frac{(1+2\frac{u}{v}+\frac{u^2}{c^2})}{(1+\frac{vu}{c^2}+\frac{u^2}{c^2})}$.

14.1.17 The figure shows the trajectories of the light signal as observed from Earth and from the rocket. The minimum distance between the rocket and the Earth is the same for both observations and is equal to l. Therefore, the return time of the signal is equal to $\frac{2l}{c}$ for observations from the Earth, and the return time is equal to $\frac{2l}{c}$ for observations from the rocket. observations from the rocket, the return time is $(\frac{2l}{c}) \cdot \frac{1}{cos\alpha} = \frac{2l}{c\sqrt{1-\beta^2}}$, where $\beta = \frac{v}{c} = \sin\alpha$. Thus, the time interval between the departure and arrival of the light signal on the Earth increases when observing from a rocket by $\frac{1}{\sqrt{1-\beta^2}}$ times.

14.1.18 Suppose the following happens. Several observers move near the Earth with different velocities. A radar pulse reflected from one observer has returned to the Earth. While this pulse traveled, the hands of the clock at the starting place made three complete revolutions, during the second trip of the pulse the hands made two more revolutions. Both the observer from whom the impulse was reflected and all other observers will record the events: three turns of the hands of the earth clock during the first trip of the impulse and two turns of the hands during the second trip. Each revolution for any observer lasts the same amount of time. Therefore, for all observers, the ratio of the duration of the first and second journeys of the pulse is equal to the ratio of the number of turns of the hands of the clock $3:2$. The above example illustrates the independence of the ratio of the ratio of times characterizing the events from the speed of the observers.

14.1.19

14.1.20 The period of oscillations of light walkers irrespective of their orientation according to observations from the station will increase by $\frac{1}{\sqrt{1-\beta^2}}$ times, and therefore the walkers will "walk" in $\frac{1}{\sqrt{1-\beta^2}}$ times slower. To determine the distance between the mirrors l' which is observed from the station at the longitudinal walkers, we define the period of oscillations of the walkers through l':

$\tau_1 = \frac{l'}{c(1+\beta)} + \frac{l'}{c(1-\beta)} = \frac{2l'}{c(1-\beta^2)}$

This period of $\frac{1}{\sqrt{1-\beta^2}}$ times the period of the walker oscillations $\frac{2l}{c}$ measured in the rocket. So,

$\tau_1 = \frac{2l'}{c(1-\beta^2)} = \frac{2l}{c\sqrt{1-\beta^2}}$

It follows from the last equation that $l' = l\sqrt{1-\beta^2}$. This means that the walkers and the rocket, and the people in it, will "flatten" by $\frac{1}{\sqrt{1-\beta^2}}$ times in the direction of velocity βc according to observations from the Earth. Similarly, everything will "flatten" at the station according to observations from the rocket. The relative motion of the station introduces many changes in the observed motion picture. The former simultaneity of events is broken, and the clock on the station runs slower by $\frac{1}{\sqrt{1-\beta^2}}$ times, and everything shrinks by a factor of $\frac{1}{\sqrt{1-\beta^2}}$ in the direction of travel. But the "flattened" people at the station with their "flattened" instruments, using "slowed down" time and incorrectly determining simultaneity of events, they get, measuring the relative speed of light flying away from them, not the speed $c - \beta c$, but the speed c. The light, which is flying towards them, approaches to them not with speed $c + \beta c$, but, according to their distorted measurements, at c. So that would explain explain the difference in measurements of the relative speed of light by the observers on the rocket. But in the same way. the observers at the station could also explain it, thinking that they were fine and that the distortions are observed by the rocket guys.

14.1.21 In $\sqrt{1 - \frac{u^2}{c^2} + \frac{u^2}{v^2}}$ times

14.1.22 The speeds of the hares and Mazai are equal to the previous speed of the fourth hare.

14.1.23 See Fig. $\lambda_+ = \frac{\lambda}{2}, \lambda_- = 2\lambda, \lambda_\perp = \frac{5\lambda}{4}$

14.1.24 $N = \frac{(1+\beta)}{2}$

14.1.25 $\delta \simeq \sqrt{\frac{\Delta}{c}}$

14.1.26 $sin\alpha_1 = \frac{sin\alpha + 2\beta + \beta^2 sin\alpha}{1 + 2\beta\sin\alpha + \beta^2}$

14.1.27 In a frame of reference that moves with velocity $u\sin\alpha$ in the direction opposite to the ship motion, the missile velocity v_p is perpendicular to the ship motion direction $v_k; v_p$ and v_k are defined by the formulas

$v_p = u\sin\alpha\sqrt{1 - (\frac{u}{c})^2\cos\alpha}, v_k = \frac{(v - u\cos\alpha).}{(1 - \frac{vu\cos\alpha}{c^2})}$

In the reference frame in which the ship velocity is zero, the components of the missile velocity $v_\perp$ and $v_\parallel$, perpendicular and parallel to the former ship velocity v_k, are defined by the formulas

$v1 = \sqrt{v_\perp^2 + v_\parallel^2} = \frac{\sqrt{u^2 + v^2 - 2vu\cos\alpha - (\frac{vu}{c})^2 sin^2\alpha.}}{(1 - \frac{vu\cos\alpha}{c^2})}.$

14.1.28 $tg\nu = \gamma tg(\frac{\alpha}{2}), \gamma = \frac{1}{\sqrt{1-\beta^2}}.$

14.2 Time dilation, shrinking bodies in moving systems. systems. Lorentz transformation .

14.2.1 2.5 times

14.2.2 $v > \frac{c}{\sqrt{1 + (\frac{\tau c}{l})^2}}$

14.2.3 $\Delta v = 6 \cdot 10^4 \ \frac{km}{s}$

14.2.4 $\Delta\nu = 10^7 \ Hz$

14.2.5 At a point moving at the speed of the wall, the frequencies of the electromagnetic oscillations of the incident and reflected waves coincide. Therefore, the frequency of the incident wave ν is related to the frequency of the reflected wave ν. the frequency of the reflected wave ν' by the equality

$\frac{\nu}{(1+\beta)} = \frac{\nu}{(1-\beta)}, \nu' = \frac{\nu(1-\beta)}{(1+\beta)}.$

14.2.6 At points traveling at wall velocity, the frequency of electromagnetic oscillation of the wave in the dielectric and outside the dielectric is the same. Therefore, the frequency of the wave outside the dielectric ν is related to the frequency of the wave inside the dielectric ν' by the following equality

$\frac{\nu}{/}(1+\beta) = \frac{\nu'}{(1+n\beta)}, \nu - \nu' = \frac{(n-1)\beta}{(1+n\beta)}$.

14.2.7 $\tau = \frac{l(1-\frac{vu}{c^2})}{v\sqrt{1-\frac{u^2}{c^2}}}$

14.2.8 In $5 \cdot 10^4$ years.

14.2.9

14.2.10 A pencil case moves towards the pencil with velocity βc. The length of the pencil case is $\frac{l}{\gamma}(\gamma = 1\sqrt{1-\beta^2})$ is γ^2 times less than the length of the pencil γl. At the moment when the bottom of the pencil case reaches the front end of the pencil, the bottom will stop. However, the open end of the pencil case will move with velocity $c\beta$ until the wave of "stopping" sections of the pencil case, coming from its bottom with velocity $\frac{c}{\beta}$, reaches the open end. At this point, the length of the pencil case is equal to the length of the pencil. and the pencil case slams shut.

14.2.11

14.2.12 $tg\alpha = \frac{\beta\beta_1}{\sqrt{1-\beta_1^2}}$.

14.2.13

14.2.14 $\Delta v = \frac{c\nu_0^2(\nu_1^2-\nu_2^2)}{(\nu_1^2-\nu_0^2)(\nu_2^2-\nu_0^2)}$

14.2.15

14.2.16 $cos\theta = \frac{(cos\alpha+\beta)}{(1+\beta\cos\alpha)}, \nu' = \frac{(1+\beta\cos\alpha)}{\sqrt{1-\beta^2}}$.

14.2.17 $a)\ \tau = \frac{L}{(v+u)}, \tau_2 = \frac{\tau_1(1+vu)}{\sqrt{1-(\frac{u}{c})^2}}$

$b)\ \tau_1 = \frac{v}{a}(\sqrt{1+\frac{2al}{v^2}}-1), \tau_2 = \frac{\tau_1(1+\frac{va\tau_1}{2c^2})}{\sqrt{1-\frac{v^2}{c^2}}}$

14.2.18 The center of oscillation moves with velocity βc. The coordinates of the body relative to the center are related to time t' by the following relations: $a)\ z' = \frac{A}{\gamma}\sin\frac{\omega t'}{\gamma}(1+\frac{\beta z'}{\omega c})$; $b)\ y' = A\sin\frac{\omega t'}{\gamma}, \gamma = 1\sqrt{1-\beta^2}$.

14.3 Transformation of electric and magnetic fields

14.3.1 The distance between the charges in the plates will decrease by a factor of $\gamma = \frac{1}{\sqrt{1-\beta^2}}$, which will lead to an increase in the surface charge density of each plate by a factor of γ.Therefore, the electrical voltage will increase by a factor of γ:

$E' = \gamma E, B = \beta E' = \gamma\beta E$

14.3.2 $E_\perp = \gamma \cdot E\cos\alpha, E_\parallel = E\sin\alpha, B = \gamma\beta E\cos\alpha = \beta E_\perp, \gamma = 1\sqrt{1-\beta^2}$.

14.3.3 $E_r = \frac{2\gamma\rho}{r}, B_r = \frac{2\gamma\beta\rho}{r}$, where $\gamma = \frac{1}{\sqrt{1-\beta^2}}$, r is the distance to the thread.

14.3.4 $a.\ \rho_e = \frac{-\rho}{\gamma}, \rho_i = \gamma\rho, \gamma = \frac{1}{\sqrt{1-\beta^2}}$

$b.$ Will increase by a factor of γ

$c.$ Different changes in the charge density of electrons and ions during the movement of the conductor leads to the appearance of an uncompensated bulk charge density $\rho' = \frac{\gamma\rho-\rho}{\gamma} = \beta^2\gamma\rho$.The electric field of this charge is $E = \frac{\beta^2\gamma\rho s}{r}$, and the magnetic induction of a moving conductor is $B = \frac{\beta\gamma\rho s}{r}$, where s is the cross section of the conductor, and r is the distance to its axis. Therefore, $E = \beta B$.

14.3.5 $a.\ \rho_i = \gamma_1\rho$, where $\gamma_1 = \frac{1}{\sqrt{1-\beta_1^2}}$. To determine the electron density, we will go into a state of motion with a velocity of $\beta_1 c$ through an intermediate state of motion with a velocity of βc, in which the electrons are stationary and their density is equal to $\rho_e' = \frac{-\rho}{\gamma}, \gamma = 1\sqrt{1-\beta^2}$. Then, informing the intermediate state of the velocity $\beta^2 c = \frac{c(\beta_1-\beta)}{(1-\beta_1\beta)}$, we move to the desired state in which the electron density is determined by the formula $\rho_e = \frac{\rho_e'}{\sqrt{1-\beta_2^2}} = -\gamma_1(1-\beta\beta_1)\rho$.

$b.$ Will increase by a factor of γ_1.

$c.\ E_1 = \beta_1 B_1$.

14.3.6 $a.\ \vec{E} = -[\vec{\beta}\times\vec{B}]$.

$b.$ In the moving state, the electric field E is defined by the formula $E = -[\vec{\beta}\times\vec{B}]$, where $\vec{B}'$ is the magnetic field induction in the moving state. At small $\beta\vec{B}'$ is close to $\vec{B}$. Therefore, $\vec{E} \simeq -[\vec{\beta}\times\vec{B}']$.

$c.$ Both explanations are valid. This means that the absolute motion of the magnet cannot be determined.

14.3.7 $a.$ As a test body we choose a straight conductor, which is stationary in the initial state and in which conduction electrons move with velocity βc. The density of electrons per unit length of the conductor is $-\rho$, and the density of ions of the crystal lattice of the conductor is $+\rho$. Therefore, the conductor is not charged and the electric field in the initial state does not act on it. In the $-\beta c$ moving state, the conduction electrons are stationary and the ions are moving at a velocity of $-\beta c$. The density of electrons in the conductor will decrease by a factor of γ, and the density of ions will increase by a factor of γ. Therefore, the conductor will be charged after the transformation with density $\gamma\rho - \frac{\rho}{\gamma} = \beta^2\gamma\rho$, and the force $\beta^2\gamma\rho E$ will act on the unit length of the conductor in the transverse direction from the side of the electric field E. But the conductor is moving without acceleration. This means that the force on the electric field side E is compensated by the force acting on the magnetic field side: $\frac{IB}{c} + \beta^2\gamma\rho E = 0, I = -\gamma\rho\beta c$ is the current in the conductor after conversion, B is the magnetic field perpendicular to both the conductor and the electric field strength. It follows from the last formula that the magnetic field B appears in the transformed system, related to the electric field by the relation $\vec{B} = [\vec{\beta} \times \vec{E}]$.

b. In the transformed system (see problem a), the magnetic field is defined by the formula $\vec{B} = [\vec{\beta} \times \vec{E}']$, where $\vec{E}'$ is the electric field in the transformed system. At low drift velocities, $\vec{E}'$ is close to $\vec{E}$. Therefore, $\vec{B} \simeq [\vec{\beta} \times \vec{E}]$.

14.3.8

14.3.9 a) Increase by $\frac{1}{\sqrt{1-\beta^2}}$ times; b) Decrease in $\frac{1}{\sqrt{1-\beta^2}}$ times.

14.3.10 Increase by $\frac{1}{\sqrt{1-\beta^2}}$ times

14.3.11 $E_{max} = \frac{Q}{(R^2\sqrt{1-\beta^2})}, \sigma_{max} = \frac{Q}{(4\pi R^2\sqrt{1-\beta^2})}, \sigma_{min} = \frac{Q}{(4\pi R^2)}$

14.3.12 The figure shows a sphere around a stationary charge and an ellipsoid arising from this sphere when it is drifting together with the charge with velocity βc. The minor axis of the ellipsoid is in $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ times smaller than the sphere. On the surface of this ellipsoid there is an electric field, which used to be on the surface of the sphere. The transverse component of this field $E_\perp$ increases by γ times, while the longitudinal component $E_\parallel$ does not change. Therefore, the tangent of the angle between the new field strength and the drift direction will increase by a factor of γ. The tangent angle between the new field strength and the drift direction times the tangent of the angle of the radius vector. So the electric field will still be directed along the radius vector. However, the strength of the new field will depend not only on the the distance to the charge r, but also on the angle α between the direction of velocity βc and the radius vector r. For example, if we compare this strength with the strength of a stationary charge, it will will increase in the transverse direction by a factor of γ^2 times, and in the longitudinal direction it will decrease by γ^3 times. For the other directions, the intensity will be determined by the formula

$\vec{E} = \frac{q}{r^3} \cdot \frac{1-\beta^2}{(1-\beta^2 sin^2\alpha)^{\frac{3}{2}}} \cdot \vec{r}$.

There was no magnetic field in the initial state. Therefore, the magnetic field induction is determined by the formula $\vec{B} = [\vec{\beta} \times \vec{E}]$.

14.3.13 When the system moves at a velocity of $-\beta c$, the dielectric plate will stop and the capacitor shells will move at a velocity of $-\beta c$. The densities of surface charges on the linings will increase by $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ times and will be equal to $\pm\gamma\sigma$, where $\pm\sigma$ - densities of the surface charge densities of the fixed capacitor shells. In addition, there will be a current with linear density $\pm\gamma\sigma\beta c$. These surface charges and currents will create inside the stationary dielectric electric voltage $E' = \frac{4\pi\gamma\sigma}{\varepsilon}$ and magnetic induction $B' = +4\pi\gamma\beta\sigma$. The motion of the new system with velocity βc returns it to its original state. The electric and magnetic fields inside the dielectric are determined by the field conversion formulas, given in the condition of problem 14.3.8. a:

$E = 4\pi\sigma\gamma^2(\frac{1}{\varepsilon} - \beta^2), B = 4\pi\sigma\gamma^2\beta(1 - \frac{1}{\varepsilon})$.

14.3.14 The motion of the state with velocity $-\beta c$ leads to a state in which the stationary dielectric is in a magnetic field of induction $\gamma B, \gamma = \frac{1}{\sqrt{1-\beta^2}}$ and in an electric field of strength $\gamma\beta B$. The magnetic field has no effect on the dielectric, but the electric field, which is perpendicular to the plate, is weakened by a factor of $\varepsilon : E' = \frac{\gamma\beta B}{\varepsilon}$. The motion of the new state with velocity βc returns the old state, whose electric field is found by the electric field transformation formula given in the condition of Problem 14.3.8. a:$E = \gamma^2\beta(1 - \frac{1}{\varepsilon})B$. The potential induced by this field is $U = Eh = \gamma^2\beta hB(1 - \frac{1}{\varepsilon})$.

14.3.15 will increase in $\sqrt{\frac{(1+\beta)}{(1-\beta)}}$ times.

14.3.16 Increase in $\frac{(1+\frac{\beta}{n})}{\sqrt{(1-\beta^2)}}$ times

14.3.17 Will increase by a factor of $\frac{(1+\beta)}{(1-\beta)}$.

14.3.18 Will increase by a factor of $\frac{(1+\beta\sin\alpha)}{(1-\beta\sin\alpha)}$.

14.3.19 Will increase in $\frac{(1+\beta\beta_1)}{\sqrt{(1-\beta_1^2)}}$ times.

14.3.20 Increase in $\frac{1}{\sqrt{1-\beta^2}}$ times; $\rho = \frac{\beta\gamma j}{c}$.

14.3.21 No

14.3.22 $E = 4\pi\gamma[\sigma - j(t' - \frac{x'\beta}{c})] = 4\pi[\gamma\sigma - jt' + \frac{l'\beta\gamma^2}{c}]$.

14.3.23 The longitudinal field does not change during motion. Only the place and time of its of its appearance. Electric field in a stationary capacitor $E = 4\pi(\sigma - jt)$. The electric field in a capacitor moving with velocity βc,

$E0 = 4\pi[\sigma - j(t' - \frac{x'\beta}{c})] = 4\pi(\sigma - \frac{j}{\gamma}t' + \frac{l'\beta\gamma}{c})$,

where l is the distance from the front plate, $\gamma = \frac{1}{\sqrt{1-\beta^2}}$.

14.3.24 $P = vM$.

14.3.25 $P = vM$.

14.3.26 $F_\pm = \frac{2\mu ev}{R_\pm^3}, R_\pm = \frac{e^2}{(2\gamma m_e v^2)}$

14.3.27

14.3.28 No. In a moving capacitor, the components of the force F acting on the first plate along and across the velocity are equal to

$F_\parallel = QE\cos\alpha, F_\perp = QE\sin\alpha(1-\beta^2)$,

and the components of acceleration are

$a_\parallel = k\cos\alpha, a_\perp = k\sin\alpha, k = \frac{QE\sqrt{1-\beta}}{M}$,

where Q, M, E are respectively the charge, rest mass and electric field inside the capacitor. This acceleration is perpendicular to the plate, equal in magnitude to the acceleration of the second plate and is opposite to the acceleration of the second plate. Therefore, the capacitor will not rotate.

14.4 The motion of relativistic particles in electric and magnetic fields magnetic and electric fields

14.4.1 $a)$ In a system moving with velocity βc, the time interval between the two events - crossing of the field boundary by the electron - will be in $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ longer: $T = \gamma\tau$.

$b)$ In the first case, during the time τ the momentum of the electron has changed by the value $2\gamma m_e c\beta$, so $\tau = \frac{2\gamma m_e c\beta}{(eE)}$, where E is the electric voltage. In the second case, during the time of motion T, the momentum of the electron has changed by the value $\frac{\gamma_1 m_e c\beta}{(eE)}$, where $\beta_1 c = 2\beta c(1+\beta^2)$ - is the velocity of the electron after the field has acted on it. Therefore, $T = \gamma\tau$.

14.4.2 In a frame of reference in which the field is stationary,

$\tau_1 = \frac{2m_e v_1}{eE\sqrt{1-\frac{v_1^2}{c^2}}} = \tau\sqrt{1-\frac{u^2}{c^2}}$,

and the velocity of the electron $v_1 = \frac{(v+u)}{(1+\frac{vu}{c^2})}$.

Therefore

$E = \frac{2m_e(v+u)}{[e\tau(1-\frac{u^2}{c^2})\sqrt{1-\frac{v^2}{c^2}}]}$.

14.4.3 $E = \frac{m_e v}{(e\tau\sqrt{1-\frac{v^2}{c^2}})}$.

14.4.4 $a)$ Will increase by a factor of $1\sqrt{1-\frac{u^2}{c^2}}$.

$v' = \sqrt{v^2 + u^2 - \frac{v^2u^2}{c^2}}$.

$b)$ Will increase by a factor of $\frac{1}{\sqrt{1-\frac{u^2}{c^2}}}[1 + \frac{u}{v}(1 - \sqrt{1-\frac{u^2}{c^2}})$.

$v' = \frac{(v+u)}{(1+\frac{vu}{c^2})}$

14.4.5 $\tau = \frac{m_e}{e}\frac{v}{E}(\frac{1}{\sqrt{1-\frac{4v^2}{c^2}}} - \frac{1}{\sqrt{1-\frac{v^2}{c^2}}})$

14.4.6 $x = \frac{m_e c^2}{eE}$.

14.4.7 $p' = p$. In $\frac{1}{\sqrt{1-\beta^2}}$ times

14.4.8 $v = \frac{c}{\sqrt{1+(\frac{m_e c^2 R}{e^2 z})^2}}$.

14.4.9 In $\frac{1}{(1-\beta^2)}$ times. In $\sqrt{sin^2\alpha + \frac{cos^2\alpha}{(1-\beta^2)^2}}$ times

14.4.10 $v = \frac{c}{\sqrt{1+(\frac{mc\omega}{2qE})^2}}$

14.4.11 $a)$ In a system moving with velocity βc, distances are reduced by $\frac{1}{\sqrt{1-\beta^2}}$ times. $l' = l\sqrt{1-\beta^2}$. $b)$ In the first case

$c^2\Delta m = (\frac{m_e}{\sqrt{1-\beta^2}} - m_e)c^2 = eEl, l = \frac{m_e}{eE}(\frac{1}{\sqrt{1-\beta^2}} - 1)$.

In the second case, the initially stationary electron, gaining velocity βc, passes the distance

$l_1 = \frac{m_e c^2}{eE}(\frac{1}{\sqrt{1-\beta^2}} - 1)$,

moving in the direction of the field. During this time the field moves a distance $\Delta l = c\beta\tau$, where

$\tau = \frac{m_e c\beta}{(eE\sqrt{1-\beta^2})}$ is the time for the electron to gain the velocity βc. Therefore

$l' = l_1 + \Delta l = \frac{m_e}{eE}(1 - \sqrt{1-\beta^2}) = l\sqrt{1-\beta^2}$.

14.4.12 $E = \frac{m_e c^2}{\sqrt{1-\frac{v^2}{c^2}}}(\frac{1+\frac{uv}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} - \sqrt{1-\frac{u^2}{c^2}})$

14.4.13 $\tau = \sqrt{\frac{(2-\frac{lEl}{m_e c^2})m_e l}{eE}}$

14.4.14 $l = \frac{\varepsilon}{eE} = 1\ km.\ \tau = \frac{1}{eE}\sqrt{m_{\pi 0}\varepsilon(2 + \frac{\varepsilon}{m_{\pi 0}c^2})} = 0.34\ ms$

14.4.15 $2N^2 = 1 = 2 \cdot 10^6$ times more than $m_e c^2$. In $k = 2N - (\frac{1}{N}) \simeq 2000$ times more the energy of electrons in collisions.

14.4.16 $tg\alpha_e = \frac{m_e c^2+E}{2m_e c^2+E} \cdot \frac{2m_p c^2+E}{m_p c^2+E} tg\alpha_p$ at $\alpha_p \ll 1, \alpha_e \simeq \frac{m_e c^2+E}{2m_e c^2+E} \cdot \frac{2m_p c^2+E}{m_p c^2+E}\alpha_p = 0.075\ rad.$

14.4.17 $v_1 = \frac{v}{sin^2\alpha} + \sqrt{\frac{v^2}{cos^2\alpha}sin^4\alpha + (\frac{m_e c^2}{l})^2} - \frac{m_e c^2}{e}$.

14.4.18 $\varepsilon = \sqrt{(m_p c^2)^2 + (eBR)^2} - m_p c^2 = 4,3\ MeV.\ \varepsilon_e = 80,5\ MeV$

14.4.19 $B = \frac{sin\alpha}{el}\varepsilon(\varepsilon - m_e c^2) = 0,04\ Tl$, $N \simeq 4\%$

14.4.20 $R_p = \frac{1}{eB}\sqrt{\varepsilon^2 - (m_p c^2)^2} = 13\ km;\ R_e = \frac{1}{3}\ km.$

14.4.21 $\omega = \frac{eB}{m_e c(1+\frac{eU}{m_e c^2})}$

14.4.22 $B = \frac{m_e c^2}{(eR\sqrt{N^2-1}} = 0,28\ Tl$

14.4.23 $T = \frac{\pi m_e c^2}{(eB\sqrt{1-\beta^2}}$.

14.4.24 $T = \frac{\pi m_e c^2(1+\beta\beta_1)}{eB(1+\beta_1^2)\sqrt{1-\beta^2}}$

14.4.25 $\varepsilon = \sqrt{(m_e c^2)^2 + (eBh)^2} - m_e c^2 = 8.5\ MeV.$

14.4.26 $l = \frac{m_e vc}{eB\sqrt{1-\frac{v^2}{c^2}}}$

14.4.27 $\varepsilon = \sqrt{(m_e c^2)^2 + (eBR)^2[1 + (\frac{h}{2\pi R})^2]} - m_e c^2$.

14.4.28 $v_{dr} = c\frac{E}{B}$

14.4.29 If a constant homogeneous magnetic field with induction $B = \sqrt{1-k^2}, k = \frac{E}{B}$, in which the electron rotates, moves with drift velocity kc, we obtain a crossed field with magnetic induction B and electric intensity E, in which the electron makes drift motion. The maximum and minimum velocities of the electron are defined by the formulas

$v_{max} = c\frac{\beta_1+k}{1+\beta_1 k}, v_{min} = \beta c = c\frac{\beta_1-k}{1-\beta_1 k}$

where $\beta_1 c$ is the velocity of the electron in the initial state. From the above equations we find $v_{max} = c\frac{[2k+(1+k^2)\beta]}{(1+k^2+2k\beta)}$

14.4.30 $ev = \sqrt{(m_e c^2)^2 + (hH)^2} - m_e c^2$

14.4.31 The electron velocity βc at the moment the field is turned on is perpendicular to E and is the sum of the rotational velocity $\beta_1 c$ and the drift velocity $kc.k = \frac{E}{B}$ (see the solution of Problem 14.4.12). Therefore, $\beta_1 = \sqrt{\beta^2(1-k^2)+k^2}$, and $v_{max} = c\frac{(\beta_1+k)}{(1+\beta_1 k)}$.

14.5 Law of conservation of mass and momentum

14.5.1 $m = \frac{M}{2}$.

14.5.2 $m = \frac{W}{c^2} = 4,4\ \frac{\tau}{c}$

14.5.3 $m_1 = \frac{m}{2}, m_0 = \frac{m\sqrt{1-\beta^2}}{2}, E = \frac{mc^2(1-\sqrt{1-\beta^2})}{2}$.

14.5.4 $m = (k+1)m_p, v = c\sqrt{1-\frac{1}{k^2}}$

14.5.5 $\varepsilon_1 = c^2(m_p - m_e) = 938\ MeV, \varepsilon_2 = c^2\frac{(m\pi_0 - m_e)}{2} = 67\ MeV$

14.5.6 $M_1 = M + m, p = mc$

14.5.7 $v = \frac{cm}{(M-m)}$

14.5.8 $v = \frac{c\dot{m}t}{(M-\dot{m}t)}, m_0 = \sqrt{\dot{m}t(M-2\dot{m}t)}, t < \frac{M}{(2m)}$.

14.5.9 $M = m_1 + m_2, v = \frac{\sqrt{(m_1 v_1)^2+(m_2 v_2)^2+2m_1 m_2 v_1 v_2\cos\alpha}}{(m_1+m_2)}$.

14.5.10 $m_e = 0.51\ MeV$, $m_p = 939\ MeV$, $m\pi_0 = 135\ MeV$, $m_\psi = 2820\ MeV$

14.5.11 $\varepsilon_K = \frac{(\varepsilon - \frac{m\pi_0 c^2}{2})}{E} = 152\ MeV.$

14.5.12 $\varepsilon_K < 2m_e c^2(\frac{m\pi_0}{4m_e} - 1)^2$.

14.5.13 $\varepsilon_K = Mc^2 - \sqrt{(Mc^2)^2 - \varepsilon_e(\varepsilon_e + 2m_e c^2) - E_\nu^2}$.

14.5.14

14.5.15 $v = c \cdot \cos\alpha^2$.

14.5.16 $\varepsilon_1 = \frac{c^2 m_{\pi 0}(m_{\pi_0}+4m_p)}{(2m_p)}, \varepsilon_2 = \frac{c^2 m_\psi(m_\psi+4m_p)}{(2m_p\varepsilon_3)} = 6m_p c^2$.

14.5.17 $\varepsilon = \frac{2c^2(m_p^2-m_e^2)}{m_e}, N = 2(\frac{m_p}{m_e} - 1) = 3,7 \cdot 10^3$

14.5.18 $\varepsilon = m_p c^2\sqrt{1 + (1 - \frac{m_e^2}{m_p^2})ctg^2\alpha^2}$

14.5.19 $a.\ v = \frac{m}{M}c, \Delta m = m.\ b.\ u > \frac{m}{M}c$

14.5.20 $\varepsilon_{max} = \varepsilon[1 - \frac{(m\mu+m_e)^2}{m_{\pi_0}^2}][1 + \sqrt{1 - (\frac{m_{\pi 0}c^2}{E})^2}] = 4,4\ GeV;\ E_{min} = 0$

14.5.21 The range of neutrino energies is from zero to $\frac{1}{2}(m_\mu - \frac{2m_e^2}{m_\mu})c^2$, the range of kinetic electron energies from zero to $\frac{(m_\mu - m_e)^2 c^2}{(2m_\mu)}$.

14.5.22 $\varepsilon_{max} = \varepsilon \frac{\varepsilon + \sqrt{\varepsilon_e^2 + (m_e c^2)^2}}{2\varepsilon + \varepsilon_e - \sqrt{\varepsilon_e^2 - (m_e c^2)^2}}$

14.5.23 $m_\gamma = \frac{m}{1 + m m_e (1 - \cos\alpha)}, m_e' = m_e + m - m_\gamma$.