

Solution to practice quiz 5

$$x = Ae^{i\omega t} + Be^{-i\omega t}$$

$$\left. \begin{aligned} x(0) &= A + B = 10 \\ \dot{x}(0) &= i\omega A - i\omega B = 2 \\ A - B &= \frac{2}{i\omega} \end{aligned} \right\}$$

$$2A = 10 + \frac{2}{i\omega}$$

$$\boxed{\begin{aligned} A &= \frac{10 - i}{2\omega} \\ B &= A^* = \frac{10 + i}{2\omega} \end{aligned}}$$

$$\omega = \sqrt{\frac{k}{m}} = 2$$

$$\text{Amplitude} = 2|A| = 2\sqrt{\frac{10^2 + 1}{4}} \approx 10.05 \text{ m} \checkmark$$

You could also do it with trig identities

$$\begin{aligned} x &= A \cos(\omega t + \phi) \Rightarrow x(0) = A \cos(\phi) = 10 \\ \dot{x} &= -A\omega \sin(\omega t + \phi) \Rightarrow \dot{x}(0) = -A\omega \sin(\phi) = 2 \end{aligned}$$

$$\phi = \cos^{-1}\left(\frac{10}{A}\right) ?$$

Harder to solve.

Problem 2.

$$\text{Energy} = \frac{1}{2} k x^2 = \frac{1}{2} k A e^{-\frac{\gamma}{2m} t}$$

$$E(0) = \frac{1}{2} k A$$

$$\frac{E(t)}{E(0)} = e^{-\frac{\gamma}{2m} t} = 0.5 \quad \rightarrow \text{key eqn.}$$

$$\ln(0.5) = -\frac{\gamma}{2m} t$$

$$\frac{2m}{\gamma} \ln(2) = t \quad \rightarrow t = 4 \ln(2)$$

Problem 3. This one is trivial. You do it.

Problem 4. The CM of the cylinder will advance as well

$$E = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} I \omega^2 = \frac{1}{2} k x^2 \quad ; \quad r\omega = v_{cm}$$

$$\frac{1}{2} M v_{cm}^2 + \frac{1}{2} I \frac{v_{cm}^2}{r^2} = \frac{1}{2} k x^2 \quad ; \quad I = \frac{1}{2} M r^2$$

$$\frac{1}{2} M v_{cm}^2 + \frac{1}{4} M v_{cm}^2 = \frac{1}{2} k x^2$$

$$v_{cm} = \sqrt{\frac{2}{3} \frac{k}{M}} x \quad \rightarrow \text{then with this you set the two energies}$$