

Solution to Practice Quiz 8.

Problem 1

Wave eqn.

$$\frac{\partial^2 \Psi}{\partial x^2} = \frac{1}{v_{\text{wave}}^2} \frac{\partial^2 \Psi}{\partial t^2} \quad \xrightarrow{\text{per string}} \quad \frac{\partial^2 \Psi}{\partial x^2} = \frac{\mu}{T} \frac{\partial^2 \Psi}{\partial t^2}$$

tension.

$$v_{\text{wave}}^2 = \frac{T}{\mu}$$

$$\begin{array}{c} \uparrow T \\ \downarrow mg \end{array} \quad \begin{array}{l} \Sigma F_y = 0 \\ T = mg \end{array}$$

$$v_{\text{wave}_0}^2 = \frac{mg}{\mu}$$

Then, a mass M is added

$$v_{\text{wave}_2}^2 = \frac{(m+M)g}{\mu} = \frac{\left(v_{\text{wave}_0}^2 \frac{\mu}{g} + M \right) g}{\mu}$$

$$M = \left(v_{\text{wave}_2}^2 - v_{\text{wave}_0}^2 \right) \frac{\mu}{g} = (45^2 - 20^2) \frac{5.6}{9.81}$$

Problem 2.

The pressure or density of the air oscillates along a sound wave. It oscillates in the direction of the wave propagation.

Problem 3. At the surface $C = P_{\text{atm}} + \frac{1}{2} \rho v_{\text{surface}}^2$
At the jet level, $C = P_{\text{atm}} + \frac{1}{2} \rho v_{\text{jet}}^2 + \rho gh$

$$v_{\text{surface}} A_f = v_{\text{jet}} A_j \quad \text{free fall} \quad \frac{1}{2} \rho (v_s^2 - v_{\text{jet}}^2) = \rho gh$$

$$X = v_{\text{jet}} t_{\text{flight}} \quad t_{\text{flight}} = \sqrt{\frac{2H}{g}} \quad \frac{1}{2} \rho v_{\text{jet}}^2 \left(\frac{A_j}{A_f} - 1 \right) = \rho gh \Rightarrow v_{\text{jet}} = \sqrt{\frac{2\rho gh}{\left(\frac{A_j}{A_f} - 1 \right)}}$$

$$X = \sqrt{\frac{2\rho gh}{\left(\frac{A_j}{A_f} - 1 \right)}} \sqrt{\frac{2H}{g}} \Rightarrow \text{Maximize } X' = 2\rho g(h_0 - H)H$$

condense first $\frac{\partial X}{\partial H} = 2\rho g h_0 - 4\rho g H = 0 \Rightarrow H = \frac{h_0}{2}$