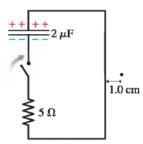
## PHY2049C, Homework 6

# A- Submit a handwritten version of the solutions (clearly readable) at the beginning of class.

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### **Problem 1**

The capacitor in the figure is charged to 50 V. The switch closes at t = 0 s. Draw a graph showing the magnetic field strength as a function of time at the position of the dot. On your graph indicate the maximum field strength, and provide an appropriate numerical scale on the horizontal axis.



### **Problem 2**

When a rigid charge distribution with charge Q and mass M rotates about an axis, its magnetic moment  $\mu$  is linearly proportional to its angular momentum L, with  $\mu = \alpha L$ . The constant of proportionality  $\alpha$  is called the gyromagnetic ratio of the object. We can write  $\alpha = g(Q/2M)$ , where g is a dimensionless number called the g-factor of the object. Consider a spherical shell with mass M and uniformly distributed charge Q centered on the origin Q and rotating about the z-axis with angular speed  $\omega$ .

- (a) A thin slice with latitude  $\theta$  measured with respect to the positive z-axis describes a current loop with width  $Rd\theta$  and radius  $r = R \sin \theta$  as shown in Figure 1. What is the differential current dI carried by this loop in terms of Q,  $\omega$ , R,  $\theta$  and d $\theta$ ,
- (b) The differential magnetic moment contributed by that slice is  $d\mu = A \, dI$ , where  $A = \pi \, r^2$  is the area enclosed by the loop. Express the differential magnetic moment in terms of Q,  $\omega$ , R,  $\theta$  and  $d\theta$ , as shown in Fig. P28.77. Figure P28.77 What is the differential current dI
- (c) Integrate over  $\theta$  to determine the magnetic moment  ${\pmb \mu}$
- (d) what is the magnitude of the angular momentum  $\mathbf{L}$ ?
- (e) Determine the gyromagnetic ratio  $\boldsymbol{\alpha}.$
- (f) What is the g-factor for a spherical shell?

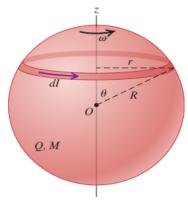


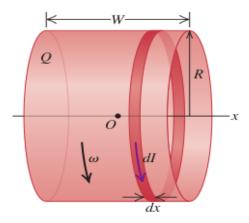
Figure 1

A cylindrical shell with radius R and length W carries a uniform charge Q and rotates about its axis with angular speed  $\omega$ . The center of the cylinder lies at the origin O and its axis is coincident with the x-axis, as shown the Figure below

- (a) What is the charge density  $\sigma$ ?
- (b) What is the differential current dI on a circular strip of the cylinder centered at x and with width dx?
- (c) Use Eq. (12.16) (see below) to write an expression for the differential magnetic field dB at the origin due to this strip. Note that on the figure, the coordinate x corresponds to the coordinate y in the equation.

$$ec{f B} = rac{\mu_0 \mu {f \hat j}}{2\pi (y^2 + R^2)^{3/2}}.$$

(d) Integrate to determine the magnetic field at the origin.



#### **Problem 4**

A rectangular loop of length *l* and width w is located a distance *a* from a long, straight wire, as shown in Figure 2. What is the mutual inductance of this arrangement?

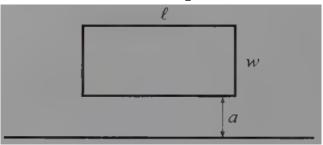


Figure 2

## **Problem 5** (Wolfson and Passachof)

A wire of length *l* and resistance R is formed into a closed rectangular loop twice as long as it is wide. It is mounted on a nonconducting horizontal axle parallel to its longer dimension, as shown in Figure 3. A uniform magnetic field **B** points into the page, as shown. A long string of negligible mass is wrapped many times around a drum of radius *a* attached to the axle, and a mass m is attached to the string. When the mass is released it falls and eventually reaches a speed that, averaged over one cycle of the loop's rotation, is constant from one rotation to the next. Find an expression for that average terminal speed.

