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On The Ontological Commitment of Potentials and Fields

In July 2017, the MMS spacecraft approached the Earth's dawn side in its regular orbit. As it came close to the reconnection region (a region where the magnetic field changes directions), sensors aboard Juno measured the electric field of the Earth with the purpose of mapping it (to eventually compare models of the reconnection process). Likewise, by measuring the impact of charge particles on a detector, the cluster spacecraft can measure the energy of ions and electrons in the region. In the paper modeling this flyby, we read statements such as: "the measured reconnection electric field (E_y in this article) is sufficient to energize ions to 10 times the initial thermal energy." (Sega & Ergun., 2024). This short essay will answer the question of how are we to understand this and other statements about electric fields (and potentials). The goal is to analyze the **E**-field in terms of the potential φ (or viceversa) on the grounds of one or the other being the more fundamental quantity. Hopefully, after doing this, we can get a better insight into how the above statement is to be understood.

One may ask if these measurements of the electric field around Earth are to be understood, ultimately, as describing the *potential difference* close to Earth, being this difference the real physical object. We can also consider that nothing more is to be understood by these electric field measurements other than just describing the electric field around Jupiter, which is a real physical object. The last question we'll be addressing is if it makes a difference? Is speaking in terms of "V" or "**E**" similar to the case of saying "car" or "automobile"? Can we really reduce one term to the other, and what do we gain by doing so?

We will also ask the same questions for the magnetic potential **A** and the magnetic field **B**. All the arguments for the electric case are equally applicable to the magnetic case and vice versa, so both situations will be analyzed in parallel. The magnetic potential **A** does appear with more protagonism in Quantum Mechanics, so the effect this has on the argument of potential vs. field will also be commented on shortly. At the end of the essay, we'll see that under Quine's notion of ontological commitment, our talk of field commits us to both **A** and **B,** and that no extra precision is gained by replacing one concept with the other.

Metaphysical Overture

I'll clarify the way in which I'll be using the terms "property" and "object". I'll start with the linguistic and somewhat vague subject-predicate distinction with the hope of capturing some common-sense ideas (also, for the present of the paper I don't need to be more precise than this). Note that we normally ascribe properties to objects through predication, while objects are referred to in the subject of sentences. If I say that "George is tall", then I'm ascribing to George the property of being tall. We may also speak of properties as subjects in our sentences, as in "red is a color". Is red, then, a property or an object? Our subject-predicate analogy breaks down here, and this dilemma usually leads to *universals and particulars*. Red, as seen in *particular* objects, is thought of as a property, but if we were to talk about red as an object in itself (let's call this object redness), we would be talking about a *universal*. Roughly speaking, universals are things that many particulars have in common;

luckily, we don't have to deal with this complicated universal/particular distinction since magnetic fields are usually mentioned as the subject of a sentence (as in the opening remark). [1]

What do I mean by "objects" then? I mean that we can talk about them in the same way that we talk about chairs and tables. This is not to say that chairs and tables are anything like electromagnetic fields, it is just to say that they appear in our sentences in a similar way that chairs and tables appear. One may also speak of magnetic fields and vector potentials as properties of a point **p**, or a region R, in spacetime as well, but in this essay I will speak about magnetic fields and vector potentials as extended objects in space. This choice is debatable, but the essay will be centered on the question of which one (**A** or **B**) [2] is more "real", and hopefully this is independent of this choice.

This brings us to my next point about *reality*, yet another debated term in metaphysics. I will start by saying that to be real is to exist, and what we mean by "to exist" is associated roughly with the existential quantifier ∃. In this, I'm following Quine on "On What There Is" (Quine W. V., 1961). In his famous essay, he roughly says that to exist is to be quantified over in a true sentence; we need some things to exist in order for some of the propositions we utter (in science) to come out as true, and this is explicit in the analysis of propositions through quantificational logic.[3]

Finally, the kind of analysis done here will have a flavor of revisionary metaphysics of Quine's type (the one criticized by Strawser in *Individuals*) (Strawson, 1959), following his idea of regimented theory. That is, the analysis is not just to describe what may be meant by statements containing **A** and **B** but rather I want to study the possibility of reformulating (at least in cases in which one is asked to be pedantically precise) statements about **A** (or a loop integral of **A**) into statements about **B** or vice versa. Quine writes:

> To some degree…the scientist can enhance objectivity and diminish the interference of language, by his very choice of language. And we [meaning we philosophers, we scientists at the abstract and philosophical end of the spectrum], concerned to distill the essence of scientific discourse, can profitably purify the language of science beyond what might reasonably be urged upon the practicing scientist. (Quine W. V., 1966)

This mission of regimentation seems quite strange at first. What's the point of distilling the language in such a way if the practicing scientist does not need the clarification in the first place? Also, how do we know what the practicing scientist wants in terms of clarity and language? To what scientific practice is Quine referring? My take on this claim is that Quine wants the language of science to be run through his notions of quantification and ontological commitment[4], and to be understood in those terms. While this will bring better conceptual understanding to the practicing scientist, it will not affect the way the measurements of Jupiter's magnetosphere are made, or the conclusions made from these data. Nevertheless, that doesn't mean that the improved conceptual understanding may not be useful to some practicing scientist in the field.

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[1] Ideally, I would like for this vocabulary to be similar to picking a basis in a manifold to elaborate a chart independent proof: it seems like I'm making hard choices unjustifiably but eventually my result won't depend on this choices. Sadly, I doubt that such a thing is possible in philosophy. Hopefully, though, some philosopher some something useful here and maybe adapts it to his/her own scheme.

[2] **A** is the magnetic potential and **B** the magnetic field. Likewise, in this essay, φ in the electric potential and **E** the electric field. All the statements made about A and B apply to the φ and **E** debate.

[3] This weak version of existence may allow (we'll see) for the existence of both **B** and **A**

[4] He indeed argued that "the framework of regimented theory is first-order logic with identity, that the variables of this theory range over physical objects and sets, and that the predicates of the theory, the only non-logical vocabulary, are physicalistic, in his somewhat complicated sense."

In summary, properties are instantiated by objects; we refer to objects as the subjects in our sentences, which in turn transform, under analysis, into an existential quantifier. To exist is to be picked out by the existential quantifier in a true proposition. And not to forget, these philosophical concepts are being brought forward because we are "regimenting" the language used to talk about magnetic fields and vector potentials. Next, we'll develop the mathematical formalism we use to define the concepts of **A** and **B.**

B versus A

Until the development of Quantum Mechanics, the magnetic potential **A** was thought of as a mere mathematical tool, without any existence of its own. Maxwell's equations historically only involved the fields **B** and **E,** and the potentials were later introduced to ease the calculations of some of these vector fields. The most important reason for giving potentials this low ontological status, is the fact of their multiplicity: infinitely many different potentials can represent the same physical phenomena and correspond to the same magnetic and electric fields. This feature of a field is known as gauge invariance. In the case of the potentials, we have:

$$\varphi \rightarrow \varphi' = \varphi - \frac{\partial \mathrm{f}}{\partial t}$$

$$\boldsymbol{A} \rightarrow \boldsymbol{A}' = \boldsymbol{A} + \nabla \mathrm{f}$$

The relation between fields and potentials being:

$$\boldsymbol{E} = -\nabla\varphi - \frac{\partial \boldsymbol{A}}{\partial t}$$

$$\boldsymbol{B} = \nabla \times \mathbf{A}$$ [5]

Given these equations, could we discuss the **A** field in the same way we speak about the **B** field? We can't. We can't because we would not be able to say: "the potential **A** at point **p** near Earth = (1,2,3)" in the same way we can do for **B,** for there's no way to distinguish the truth of this statement from "the potential **A** at point **p** near Earth = $(1 + \partial_x f(\boldsymbol{p}), 2 + \partial_y f(\boldsymbol{p}), 3 + \partial_z f(\boldsymbol{p}))$." Both statements can be true at the same time even if $\boldsymbol{A} \neq \boldsymbol{A}'$**,** and statement like "∃ an **A** such that **A** = (1,2,3)" needs such an **A** to exist. We could only then speak of there being one **A** of many at point **p,** and we'll further have to provide the function f to communicate with other scientists without confusion. The statements involving **B** would then be easily replaceable by those involving **A** and this arbitrary function f (in the electric potential case this will amount to specifying where we are defining the potential to be zero: where is our ground).

While we cannot replace **A** with **B** in the sentences in which we use **B**, we can consider the possibility that all the sentences pertaining to a potential **A**, that are compatible with **B** at a point are, in fact, true. That is, if one is committed to the existence of a particular **A**, one can still allow for any other types of **A** to exist at the same point **p**, together with **B**. Therefore, we would have at least two objects **A**, and **B,** at a point **p** or a region R, and we may have infinitely more **A** fields at that point as well (compatible with **B**). This seems to be redundant, but logically consistent. Such a plurality of fields has not been the traditional view in Physics, however. Physicists (before QM) had deemed the **A** as a mathematical tool because they use a stricter notion of existence than just the existential quantifier. This notion can be captured in these two conditions

[5] One obtains the magnetic field from the magnetic potential by taking "the curl" of **A**, which is a type of derivative. Similarly, one derived takes the gradient of the electric potential to obtain the electric field.

E1) To be real is to act locally[6]

E2) To be real is to have physical effects on the dynamics of a particle

The first one comes directly from Feynman's discussion of the problem in the lectures:

> A "real" field is then a set of numbers we specify in such a way that what happens at a point depends only on the numbers at that point. We do not need to know any more about what's going on at other places. It is in this sense that we will discuss whether the vector potential is a "real" field. (Feynman, 2017)

The second one is also alluded to in the same section, but it is somewhat less controversial. There's a more general notion of existence for physical objects that requires them to be able to interact with others for us to know they exist at all. An object that enters in an explanation of a physical effect is granted ontological status for free. Objects associated with dynamics in electromagnetism ought to have some dynamical effects in order for them to gain "real" ontological status. Feynman writes:

> In any region where $\boldsymbol{B}=0$ even if $\boldsymbol{A}$ is not zero, such as outside a solenoid, there is no discernible effect of $\boldsymbol{A}$. Therefore, for a long time it was believed that $\boldsymbol{A}$ was not a "real" field. (Feynman, 2017)[7]

If we consider **B** to exist, taking this last condition into consideration, our many **A**-fields fall under "inexistent." Simplicity and economy by themselves become reasons to deny the existence of **A.** It seems to have (so far) no explanatory role (if **B** is present), no effects on physical phenomena, but at the same time requires us to be committed to the existence of infinity many objects at every point in spacetime.

If we replace **A** for **B** (and talk about the curl of **A** as the true causal agent in the dynamics) we'll be referring to an **A** that we don't directly measure. The meaning of the sentence "the measured curl of A is sufficient to energize particles near the Earth" could not be easily analysed in terms of a vector field since the values of **A** are not known for the Earth. Most of the sentences involving vector potentials will be indeterminate (which **A** of the infinitely many occupying that point?). Nevertheless, note that if we choose a particular **A,** we are describing the situation as accurately as if we have used **B.** In terms of descriptive power, any **A** will do, but we would have to pay an ontological price and include f when communicating with others. By replacing **A** for **B**, we would have attempted against reductionism and gain nothing from it (but just loose clarity). For these reasons, **A** was primarily thought of as a computational device to get the field, but in no sense descriptive, by itself, of the physical system in hand.

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[6] This will come into play in the quantum mechanical case, but not now.

[7] In classical mechanics, **B**=0 can be associated with a non-zero potential ***A*** which produces no dynamical effect. Likewise, the electric field **E** can be zero everywhere, and to it we can associate a non-zero uniform potential everywhere. This non-zero potentials *will have no effect on the dynamics on any particle insofar their corresponding **E** and **B** are nil.* In quantum mechanics, however, particles in regions where **B** in zero (but non-zero somewhere else) and **A** is non-zero *do* have their dynamics measurably perturbed. This is called the Alharonov-Bohm effect. One can either take the non-zero **B,** which does not exist in the region in question, to be acting at a distance, or one can consider **A** to be acting in a way to involves a loop path integral in the region (because the dynamical effect does not depend in **A** per-se but on its loop integral $\oint \boldsymbol{A} \cdot d\boldsymbol{s}$)

Feynman, however, argues that, given that Quantum Mechanics (see footnote 7) leans the balances towards favouring the potentials (***A*** and φ) to be the real quantities. This is because, unlike the case of Maxwell Equations, they do appear directly in the equations of quantum mechanics:

> In quantum mechanics what matters is the interference between nearby paths; it always turns out that the effects depend only on how much the field $\boldsymbol{A}$ *changes* from point to point, and therefore only on the derivatives of $\boldsymbol{A}$ and not on the value itself. Nevertheless, the vector potential $\boldsymbol{A}$ (together with the scalar potential φ that goes with it) appears to give the most direct description of the physics. This becomes more and more apparent the more deeply we go into the quantum theory. In the general theory of quantum electrodynamics, one takes the vector and scalar potentials as the fundamental quantities in a set of equations that replace the Maxwell equations: $\boldsymbol{E}$ and $\boldsymbol{B}$ are slowly disappearing from the modern expression of physical laws; they are being replaced by $\boldsymbol{A}$ and ϕ. (Feynman, 2017)[8]

The **E** and **B** fields were originally introduced in mechanics as mediators; we wanted to *avoid* having to talk about action at a distance between two charges. Saying that the magnetic field now acts at a distance seems ironic. Nevertheless, taking this position has the advantage of having a separable description of the phenomena: a little bit of flux (say, consider an arbitrary small region in the solenoid with some flux σ) will cause a little bit of (discernible) phase change; to contrast, the vector potential has to come all in one inseparable looped package because its effect depends on the different potential paths of the particles, hence on a loop path integral. An arbitrary section of this loop (by itself) will not contribute any discernible change in the dynamics, meanwhile an arbitrarily small **B** field flux will have an arbitrarily small effect (albeit the effect will be non-local: the field acts at a distance). Again, we must relax either condition E1 or E2 for existence or, what amounts to the same, relax non-locality or non-separability.

Note in the quote above that Feynman advocates for **A** to be the “most direct description of the physics” and speaks of a replacement of the fields by their potentials, at least in modern equations. This appears to be revisionary, but at the same time Feynman never implicitly lowers the ontological status of the **B** field, which seems paradoxical. If one can describe the same situation with **B** and **A**, these notions are just different ways of saying the same thing. Shouldn’t we choose the one which “more directly describes the situation” to be precise in our (mathematical) language? Is language “stepping in the way” when we talk about **B**? It is of no doubt that working in terms of the magnetic field has its advantages and Feynman expresses the importance of this in the lectures, but if we want to be precise, we can still urge scientists to understand **B** as the curl of **A,** at least when asked to be pedantically precise. This is a reasonable demand, and it follows from saying that is a better description of the physics, and that we want to be as accurate as with our descriptions of physical systems.

Nevertheless, Feynman’s notion of revision is not what we set up to do, but Quine’s regimentation. Even if **A** is a better description, our notion of existence allows for quantifications of **B’s** and statements like the one above use magnetic fields and come out true. Under Quine’s regimentation, the notion of a **B** field is precise enough to be part of the (most precise) scientific language. This is

[8] A similar conclusion was reached by R. Hailey in “Non-Locality and The Aharonov-Bohm Effect”, where he argues that the double-slit experiment without the solenoid already features this non-separability (Healey, 1997).

good because of the mentioned impracticality of having to replace B by A in Quantum Mechanical equations.

Traditionally, it is stated (Healey, 1997) that we should choose between relaxing the conditions of separability or local action on our fields. I have argued that this amounts to the same as relaxing either E1 or E2 in our notions of existence. If we take a description in terms of **A**, point by point, to be a local description of these objects in space (and we would have to allow for gauge freedom: any single **A** would do), then we would be insisting on the existence of (many) fields that in most occasions have no physical effects, but that would be describable point-by-point. Note that while the Aharonov-Bohm effect itself needs a region ∂R, it could be a consequence of any separable **A**. it is the loop itself that, if consider as an object in itself, is non-separable. On the other hand, if we allow for action at a distance and relax E1, we can talk about the **B** on the solenoid acting on the particle. But, if we relax both and only strictly speak of Quine's ontology, we can have both![9]

Conclusion

After applying the notions of ontological commitment to the case in hand, we are left with the existence of many fields. Infinitely many **As**, and **Bs**. This comes about by analyzing sentences involving these fields with the existential quantifier, and noting that they come out true, as far as we verify these claims by looking at physical systems.

A causal notion of existence (E2) doesn't by itself secure the argument that the **A** potential is the existing quantity; E2 actually denies **A** and favors instead the loop integral of **A**, or a **B**-field that acts at a distance. Also, even if we are to consider **A** as an object (not just a mathematical abstraction), it doesn't follow from this that **B** is a mathematical abstraction not less real than **A**.

If I say "there exists a **B** such that B = (1T, 2T, 3T) at point **p**", analyzing such statements in terms of a loop integral of **A** brings no new insight into the statement. Likewise, if I say, "there's an **A** in region **R** such that $\oint \vec{A}\,.\,d\vec{s} = 3$", analyzing this statement in terms of **B** seems to be equally unfruitful. And this unfruitfulness is independent of how hard it is to go from one formulation to the other. The only problem we found with the magnetic field is the nature of this action at a distance of **B** on a charged particle. Is there any conceptual problems or disadvantages of taking this position? Maybe, but not under Quine's regimentation.

I agree that speaking of action at a distance of from a field is strange, particularly when fields were introduced to deal with this problem. Couldn't we introduce **A** as a field created by **B** to mediate this interaction with the wave-packets? I don't see why not; this will allow us to say that **B** is as causally responsible for the effect as **A** is, in the same way that charges are responsible for their fields *and* the Coulomb force. Nevertheless, I agree that the best description of the Aharonov-Bohm involves **A**. This doesn't, however, make all sentences involving **B** false, and free us of ontological commitment. Astronomers can rest assure that, under Quine's regimentation, they are being very precise in their description of the physical environment around Jupiter in terms of **B.**

[9] In the face of being too relaxed I'd just say that Quine is already more precise than we need.

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