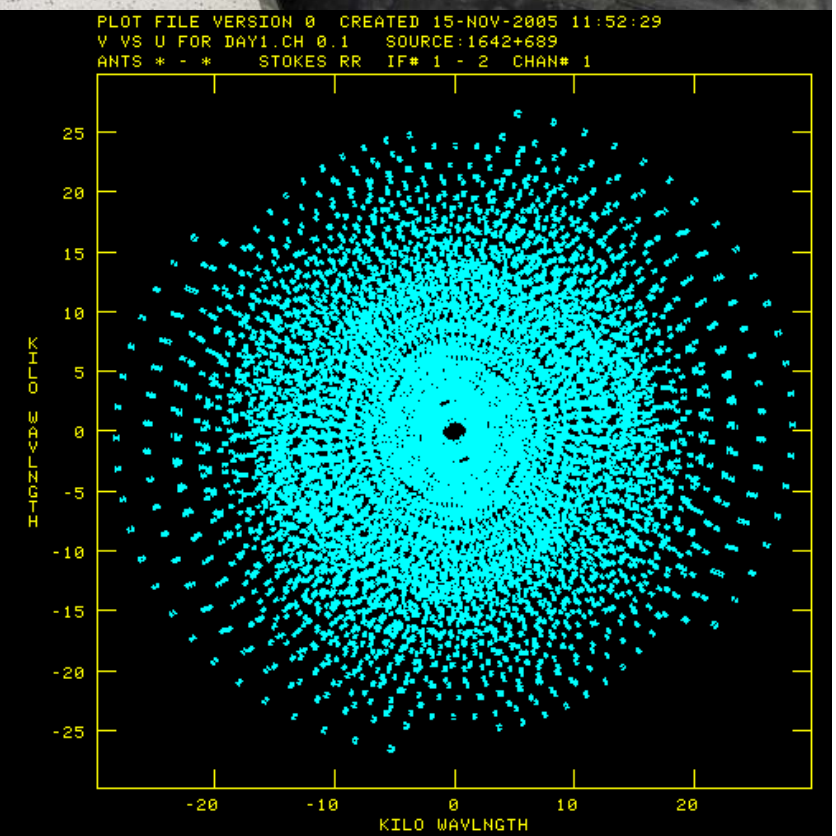

Cross correlators

for radio astronomy

Adam Deller

May 16, 2018

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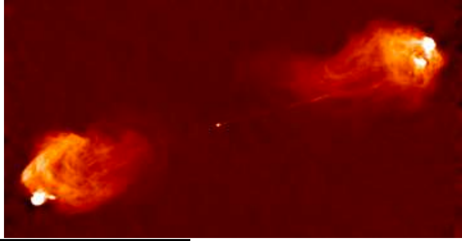
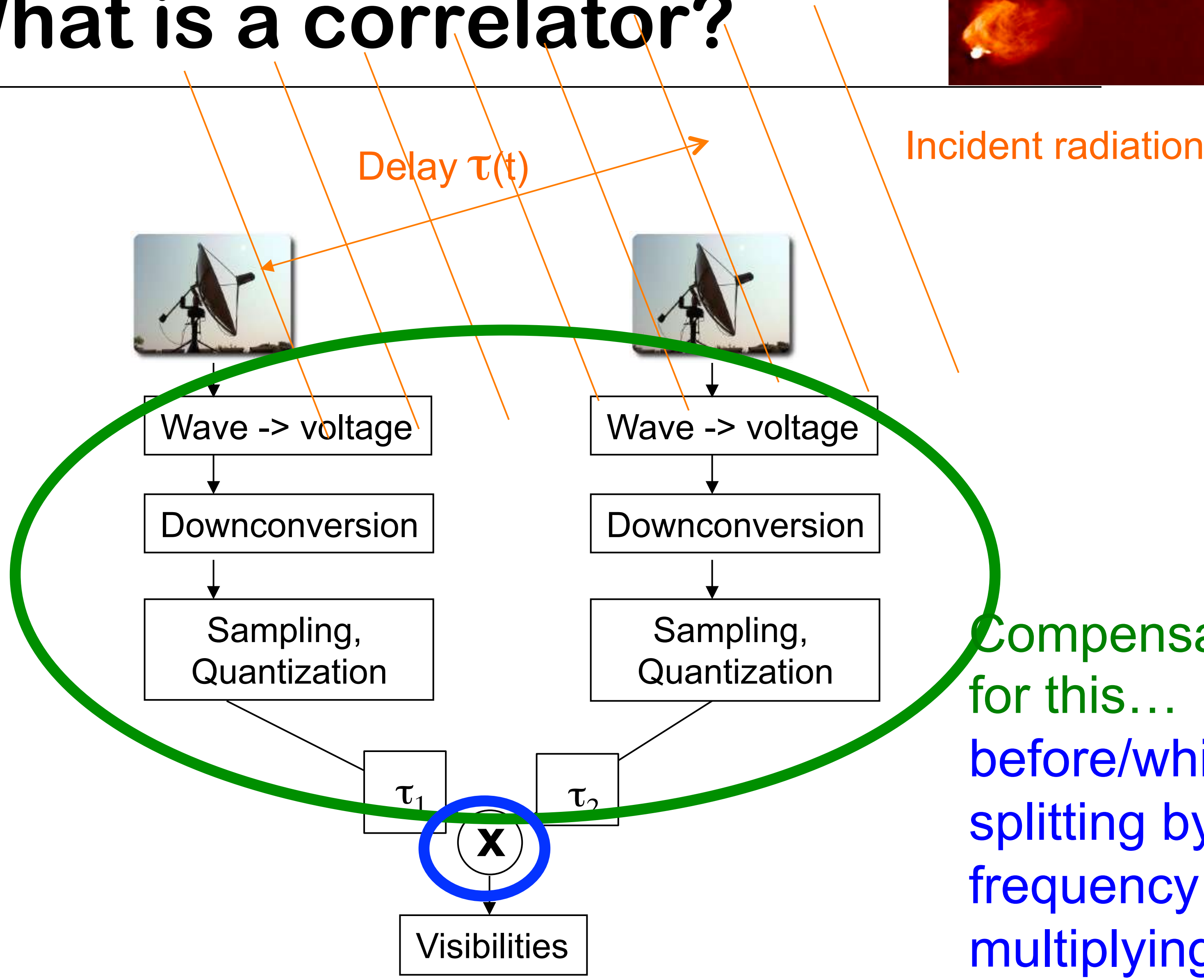
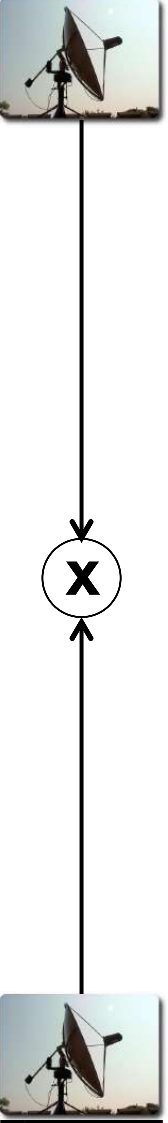
What is a correlator?

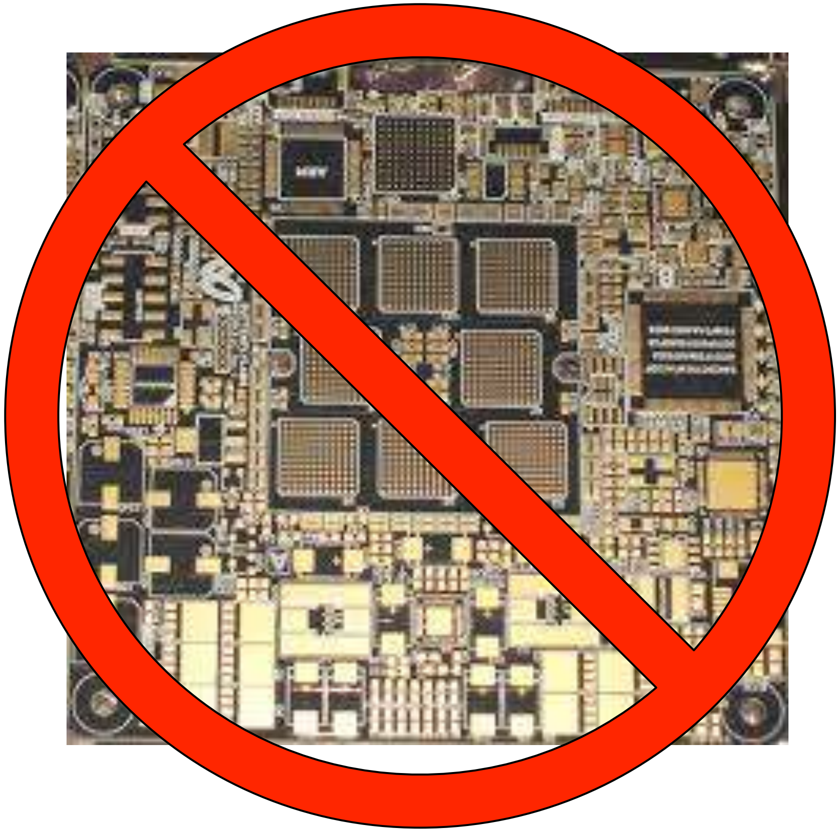


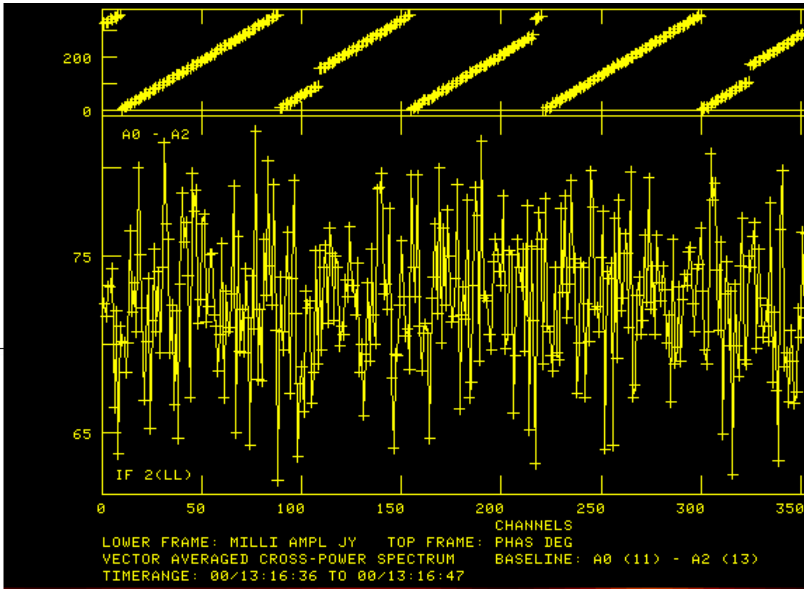


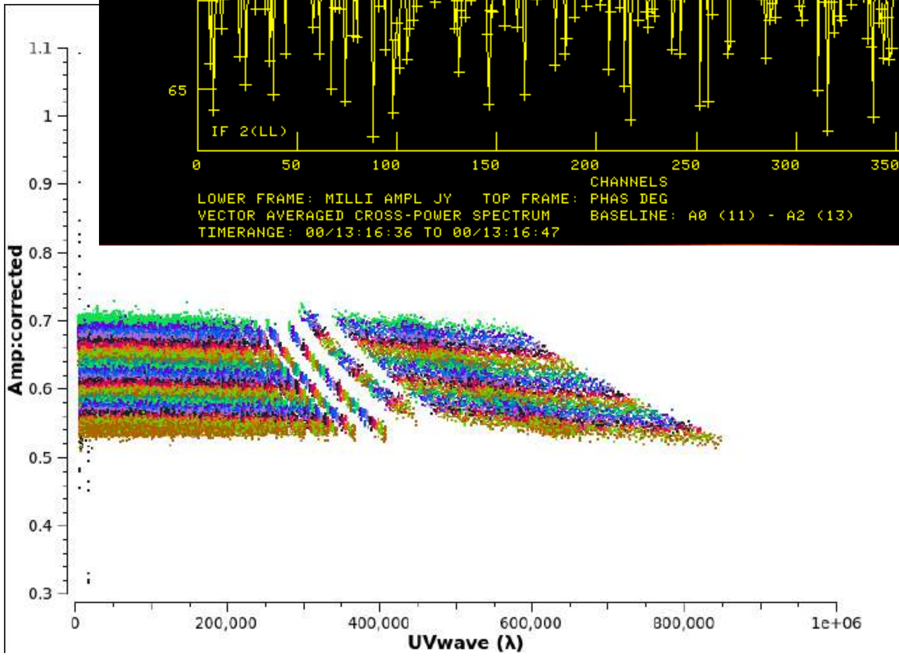
Compensate for this…
before/while splitting by frequency and multiplying



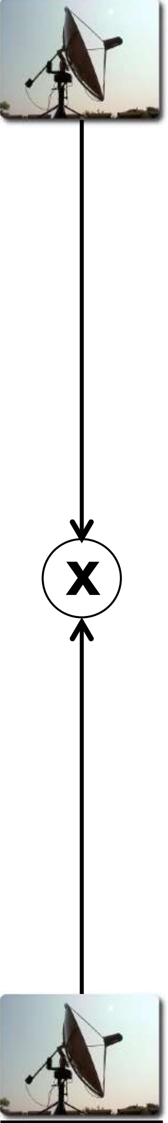
Why correlators matter to YOU



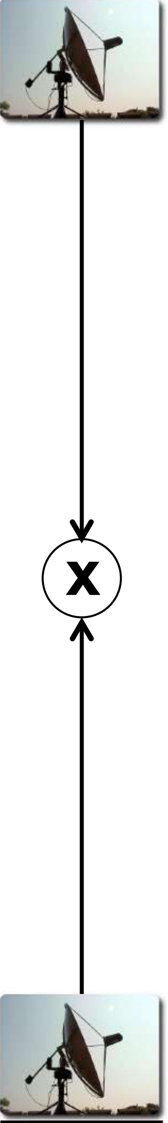


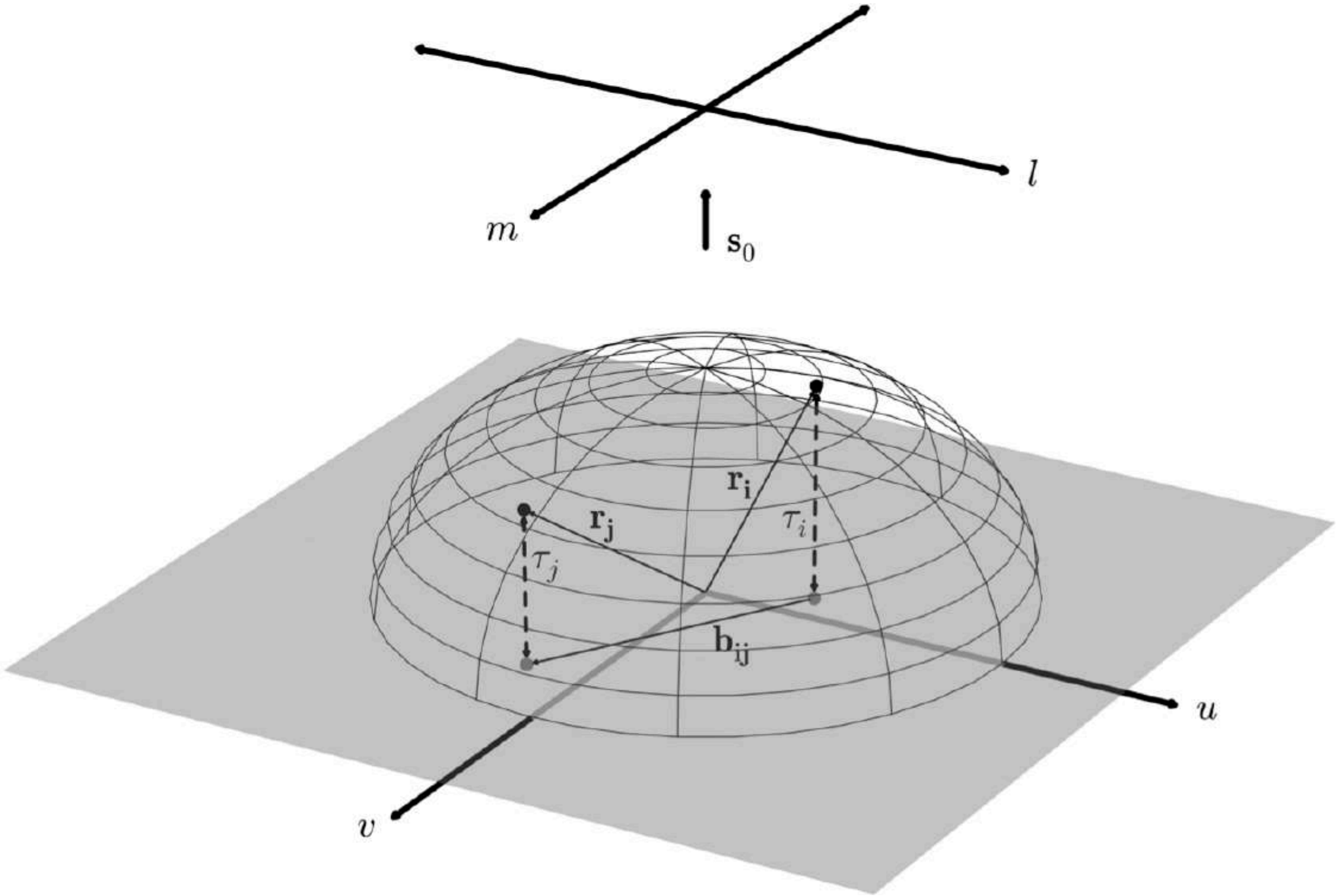


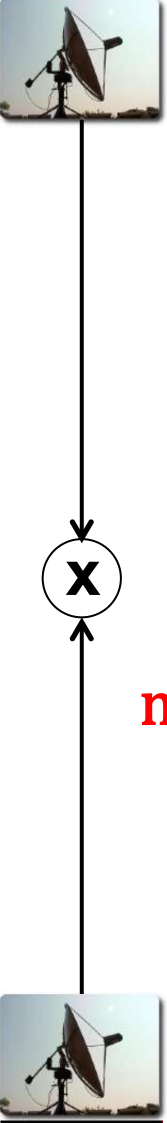
Correlators and interferometry



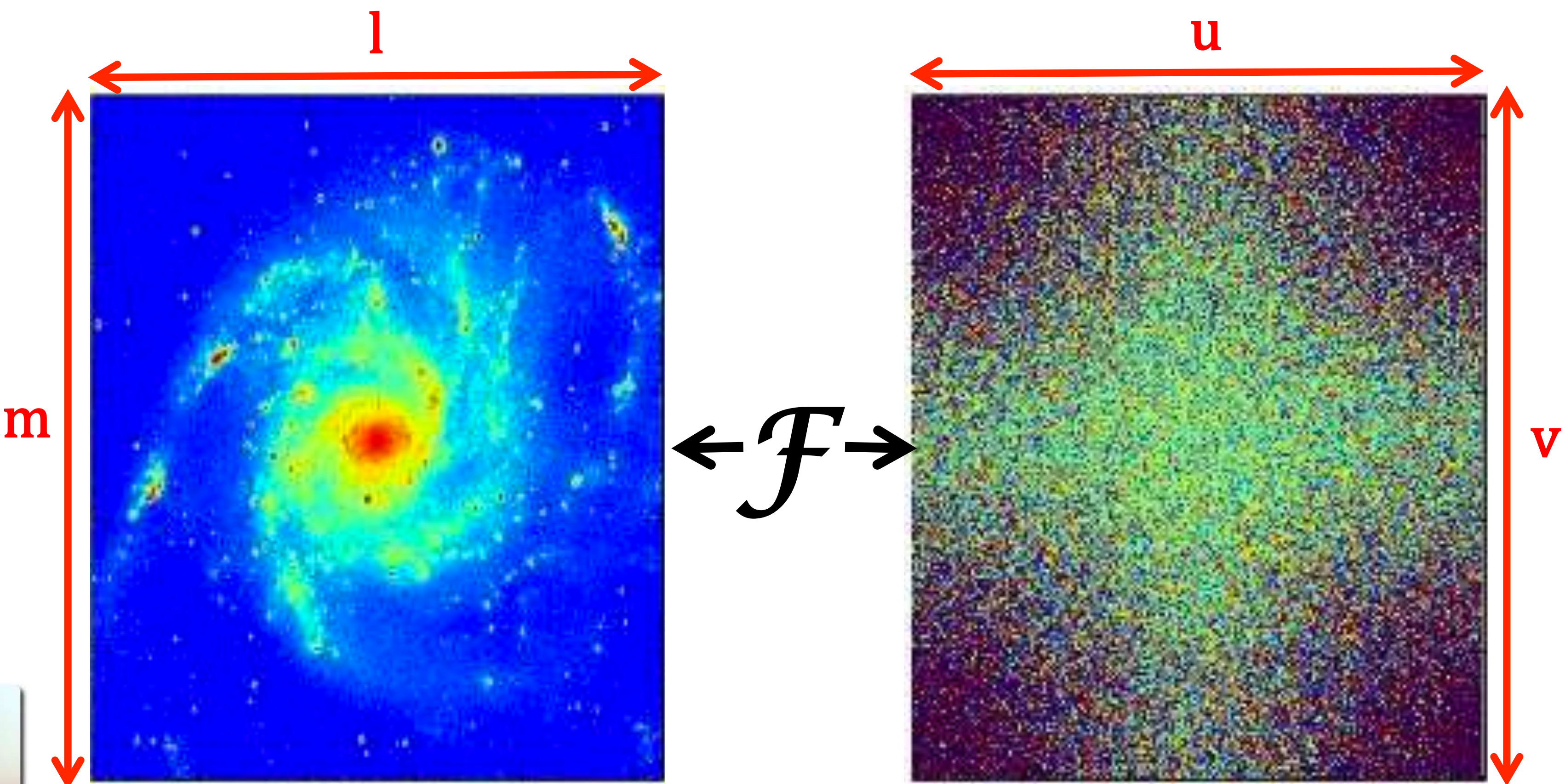
Correlators and Interferometry





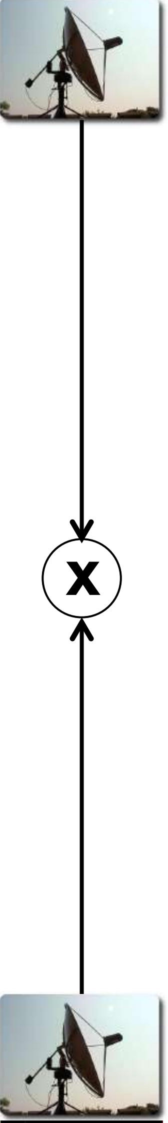


Correlators and Interferometry



Sky brightness at frequency ν_0

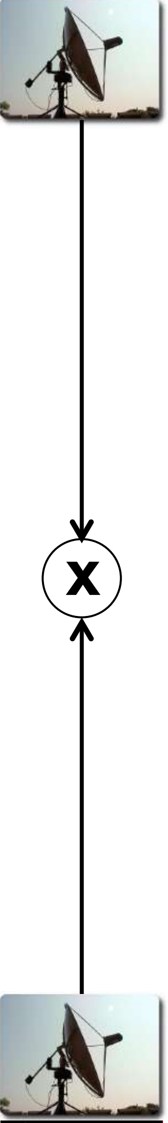
Visibilities (real component shown, unit is $\lambda_0 = c / \nu_0$)

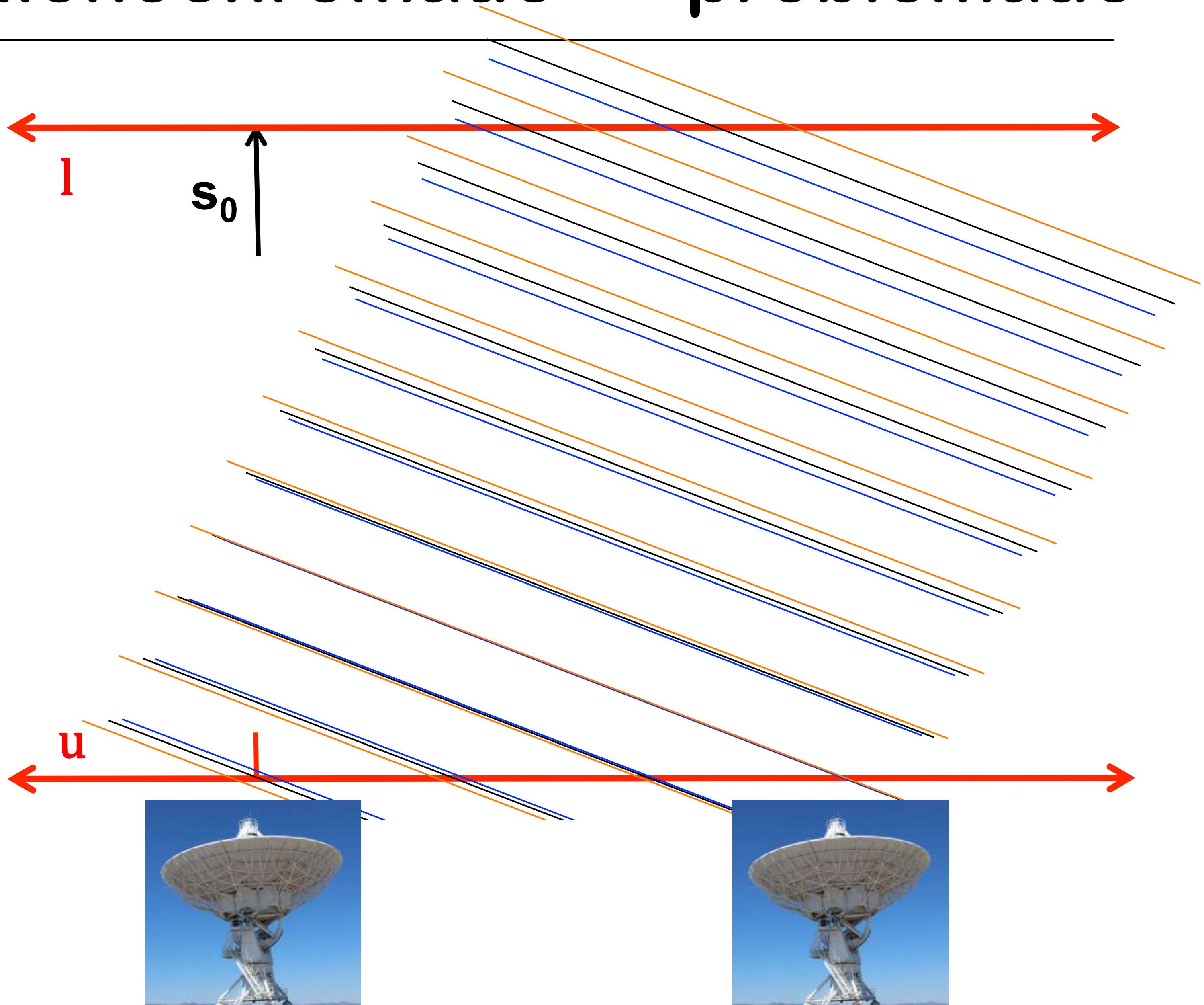


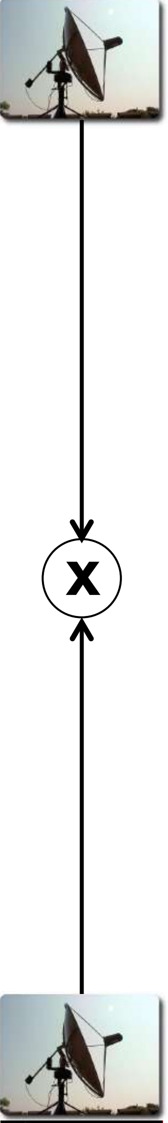
Monochromatic == problematic

- Mathematically: $\mathrm{u} \times \mathrm{l} + \mathrm{v} \times \mathrm{m}$ is supposed be constant, but both u and v are $\propto \nu$
- No truly monochromatic radiation!
- Fortunately, “fairly narrow” band of $\Delta\nu$ (*quasi-monochromatic*) can suffice:
  - Real world viewpoint: different frequency components stay “in phase” as wavefront propagates from one antenna to the next

Monochromatic == problematic







Monochromatic == problematic

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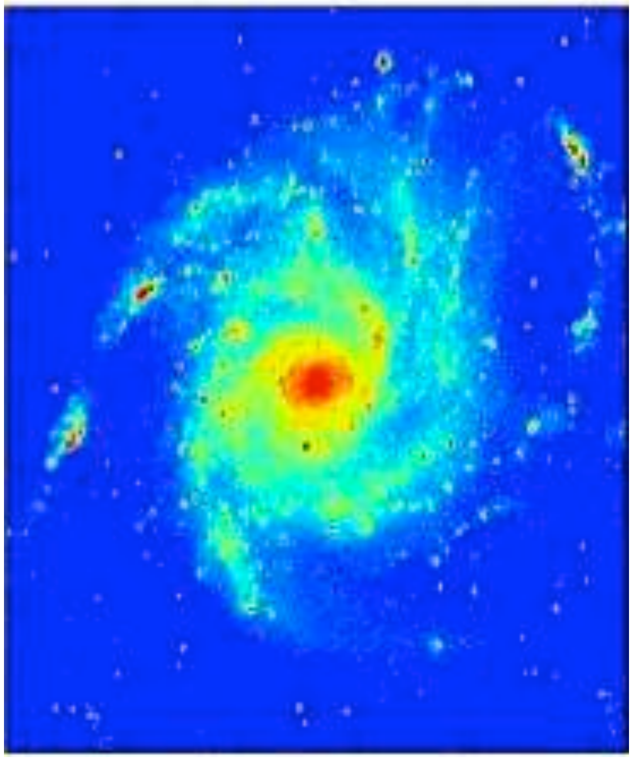
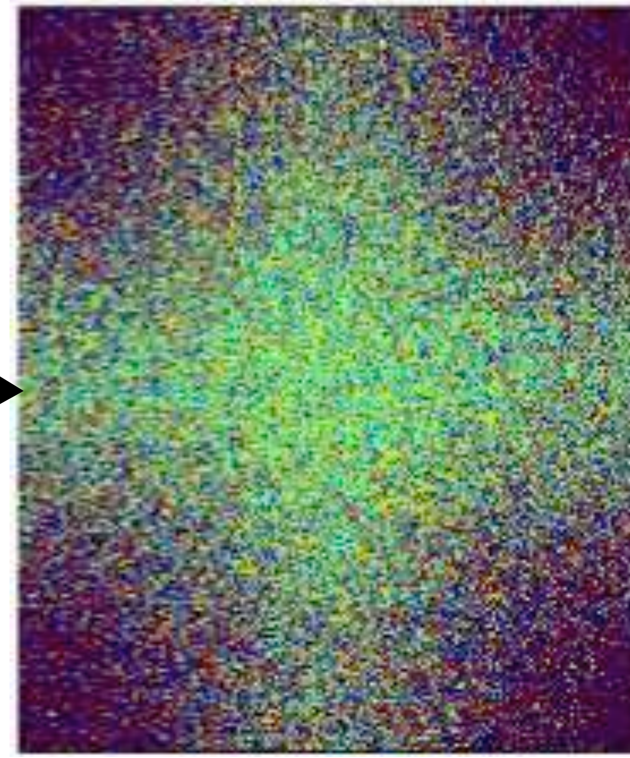
- No truly monochromatic radiation!
- Fortunately, “fairly narrow” band of $\Delta\nu$ (*quasi-monochromatic*) can suffice:
  - if $\Delta \mathrm{u} \times \mathrm{l} \ll 1$ and $\Delta \mathrm{v} \times \mathrm{m} \ll 1$ then the different frequency components stay in phase and we’re ok
  - Correlator needs to slice at least this finely

Correlators and Interferometry

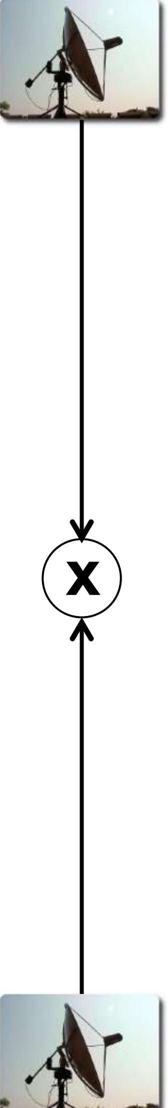


$\leftarrow \mathcal{F} \rightarrow$



Sky brightness at frequency ν_0

Visibilities (real component shown, unit is $\lambda_0 = c / \nu_0$)



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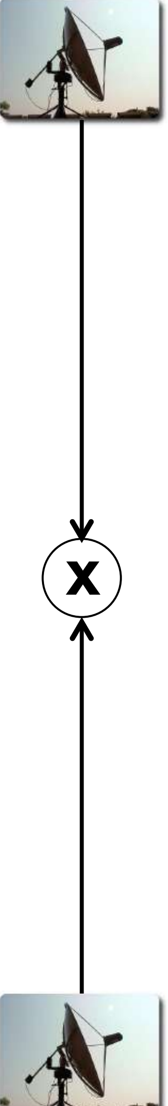
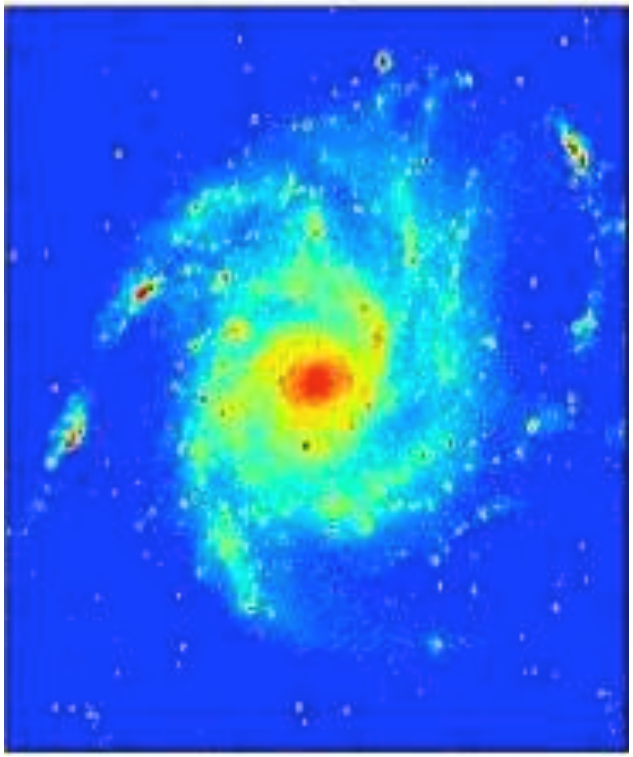


Correlators and Interferometry



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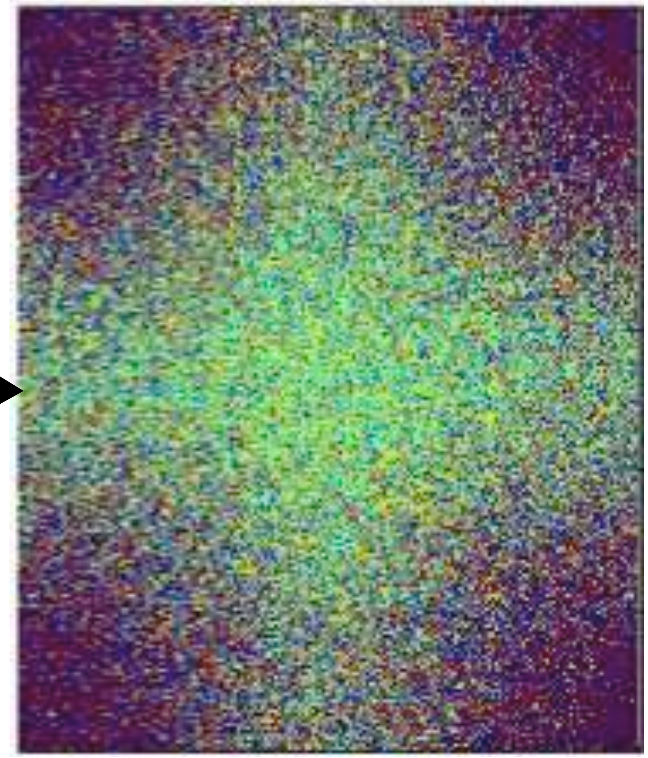
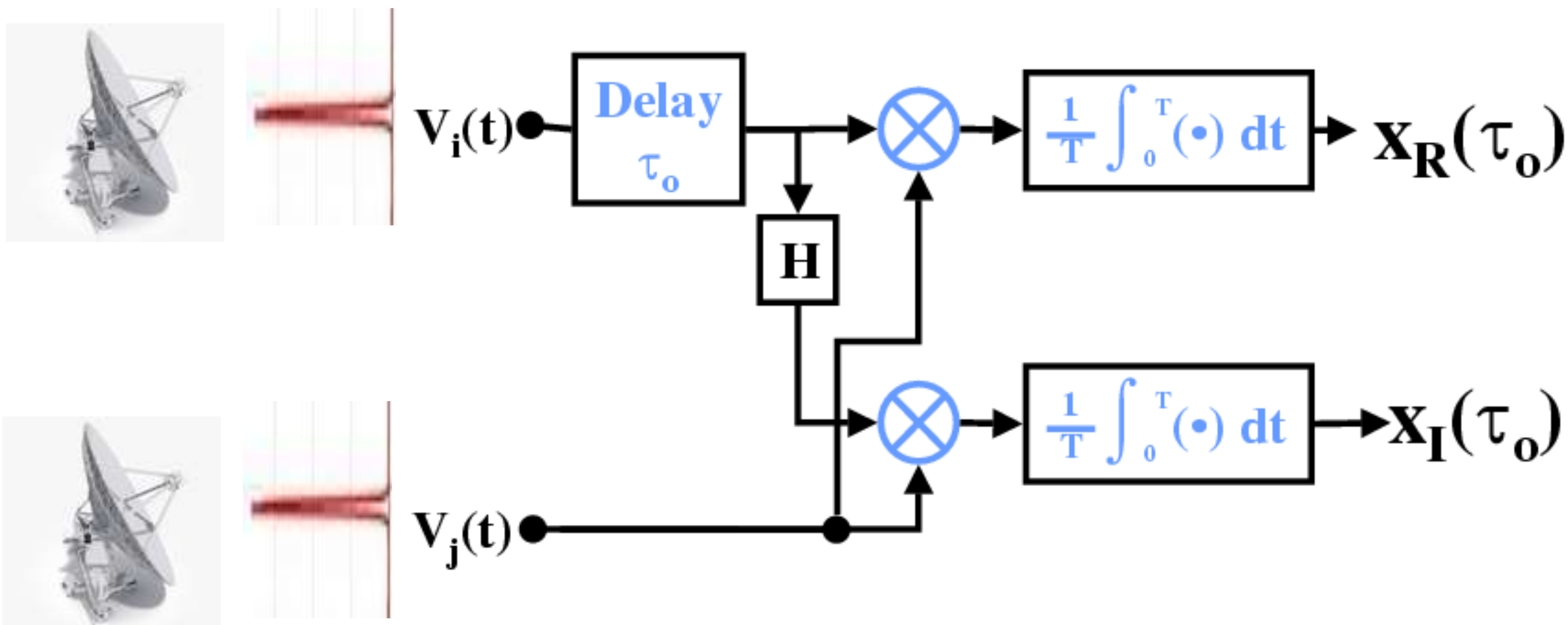


Sky brightness at frequency $\nu' = \nu_0 + \delta\nu$

Visibilities (real component shown, unit is $\lambda' = c / \nu'$)

A “dumb” correlator

- Use many analog filters to make many narrow channels; correlate each one separately with a standard complex correlator:



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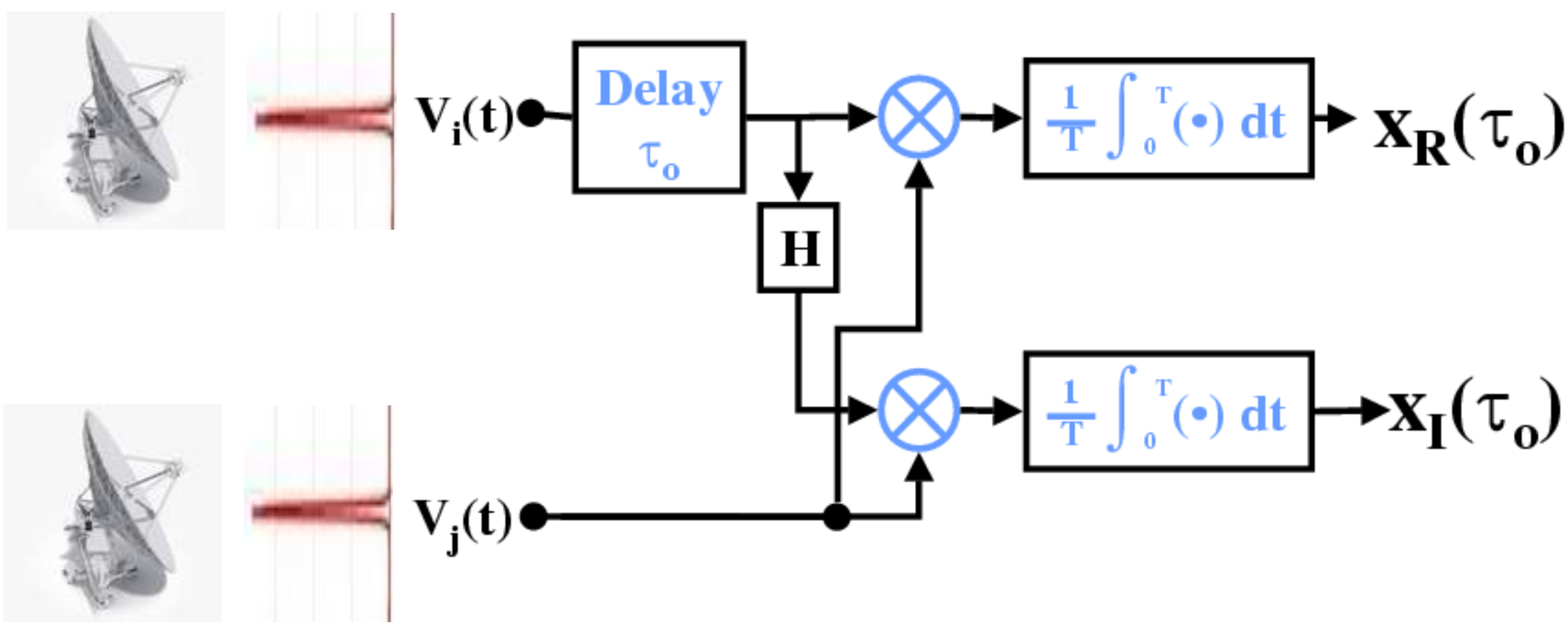
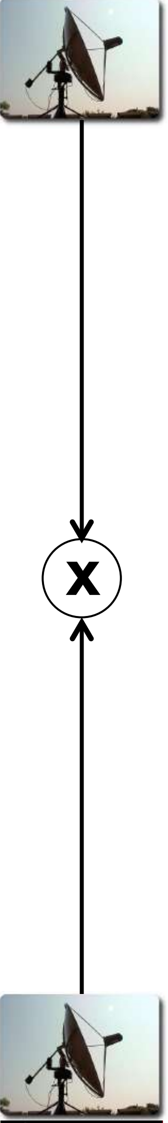
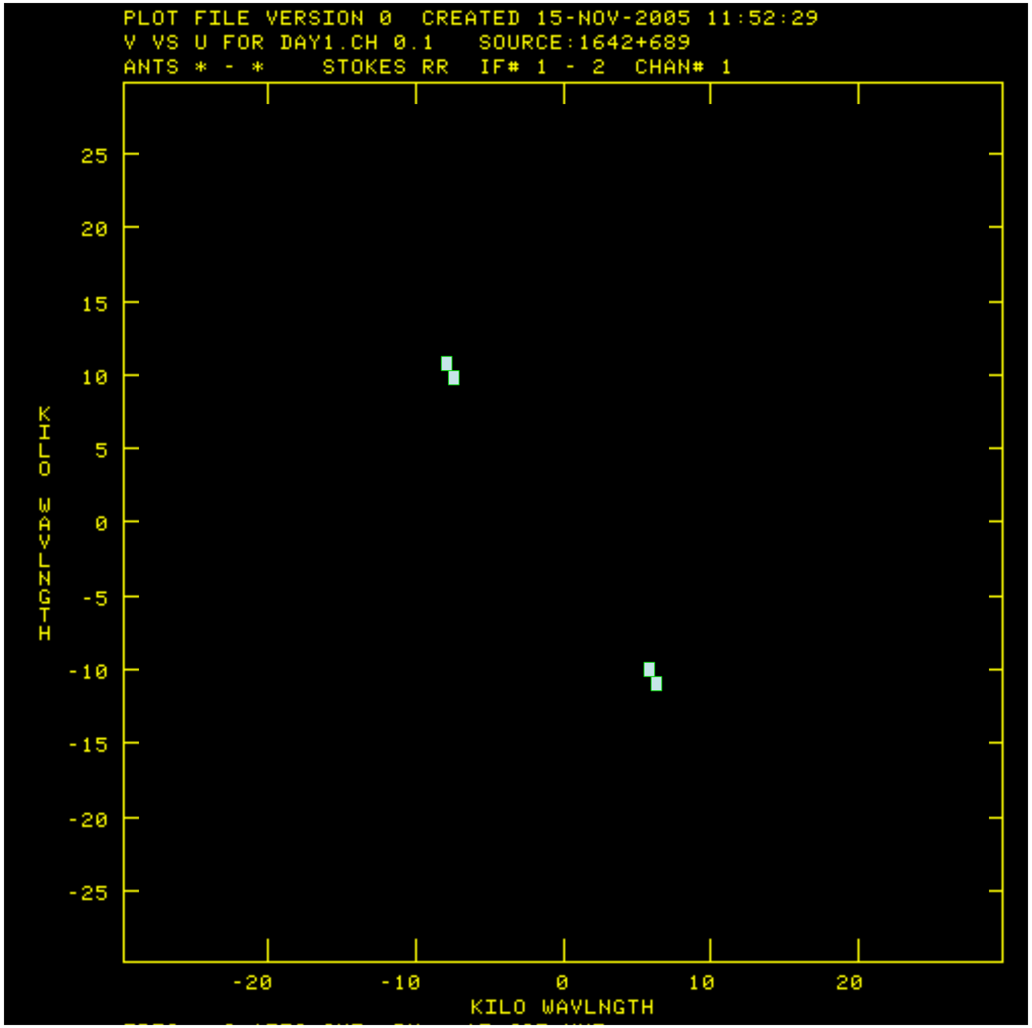
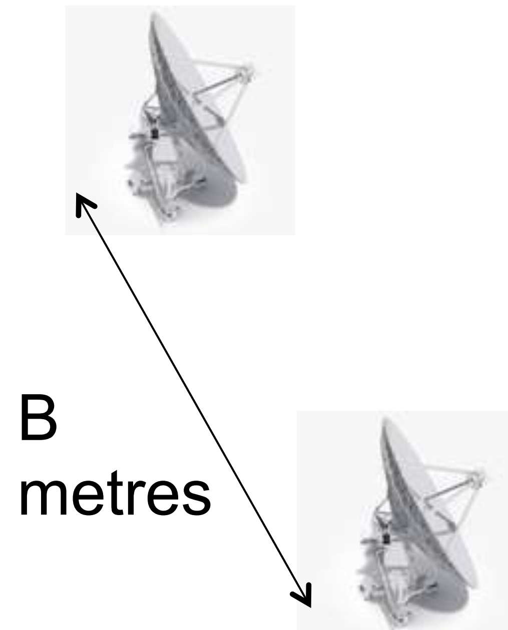
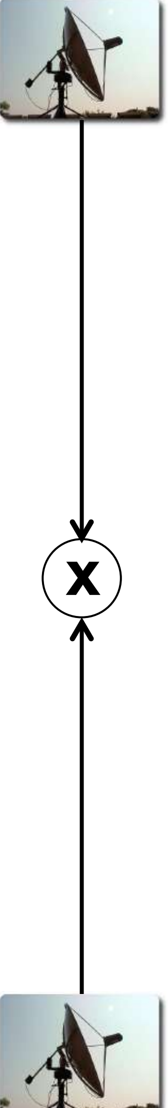


The output

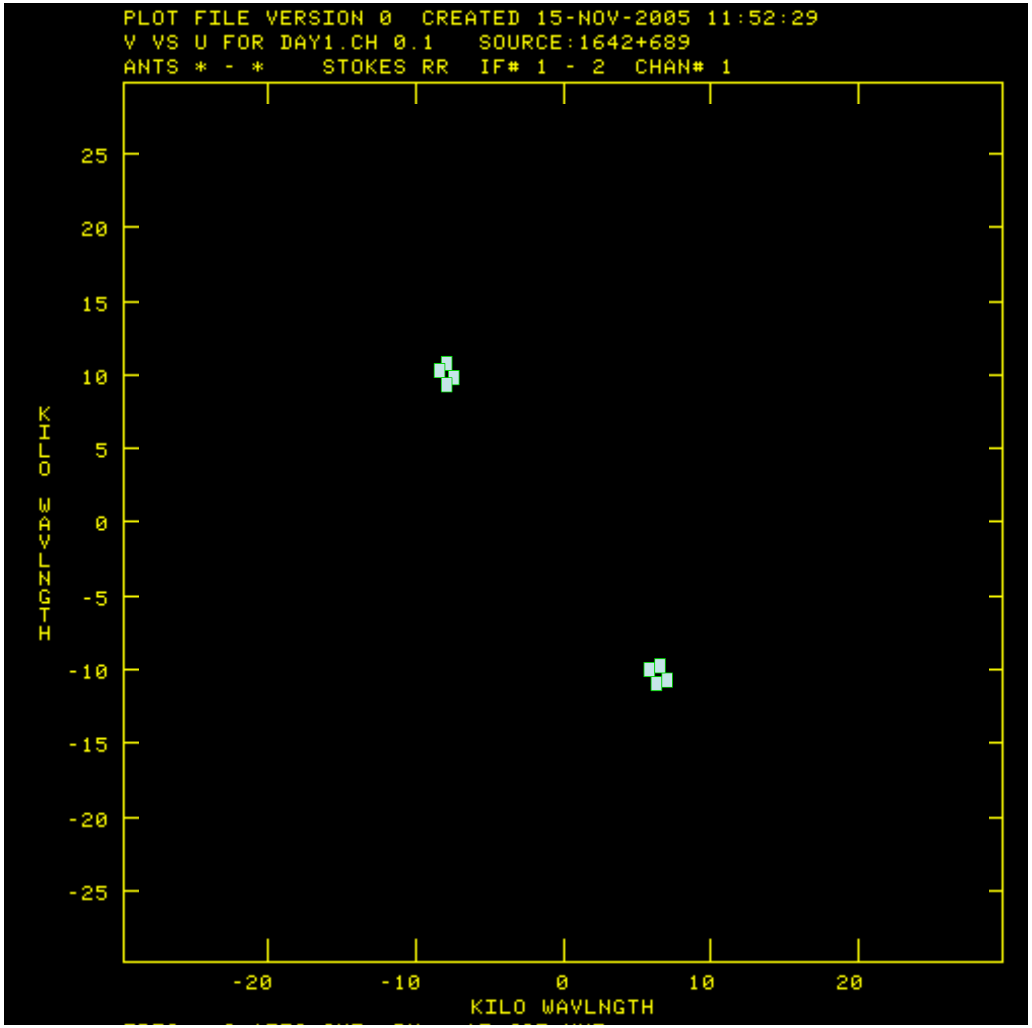

X

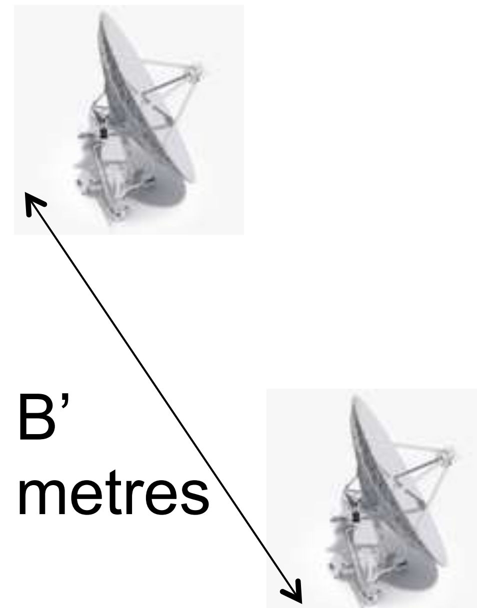

PLOT FILE VERSION 0 CREATED 15-NOV-2005 11:52:29
V VS U FOR DAY1.CH 0.1 SOURCE:1642+689
ANTS * - * STOKES RR IF# 1 - 2 CHAN# 1
KILO WAVLNGTH
25
20
15
10
5
0
-5
-10
-15
-20
-25
-20
-10
0
10
20
KILO WAVLNGTH


B
metres



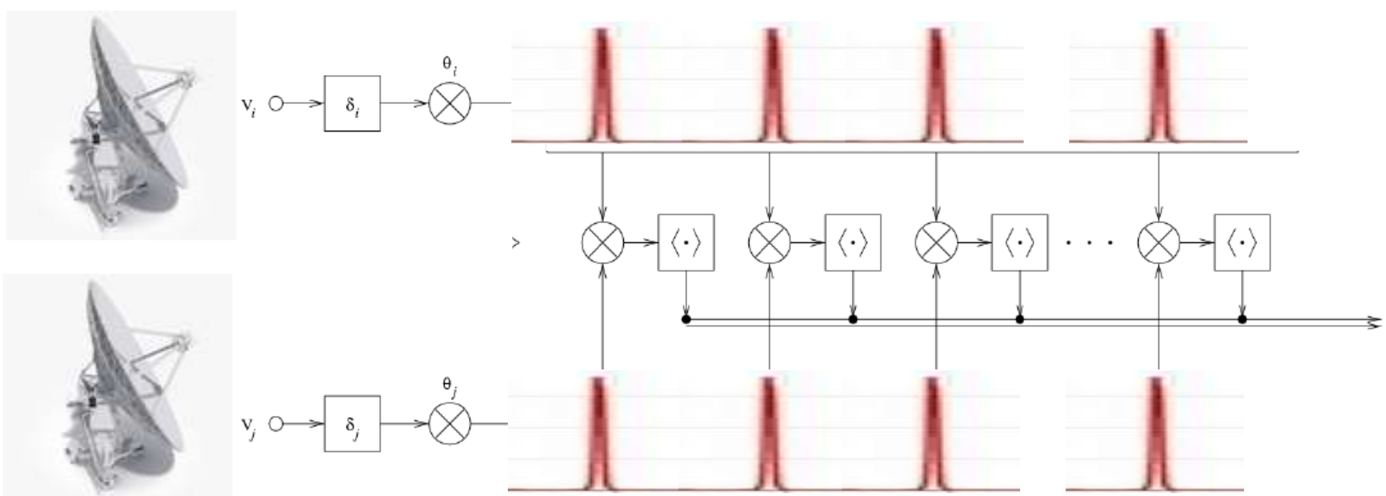
The output


PLOT FILE VERSION 0 CREATED 15-NOV-2005 11:52:29
V VS U FOR DAY1.CH 0.1 SOURCE:1642+689
ANTS * - * STOKES RR IF# 1 - 2 CHAN# 1
KILO WAVLNGTH
-20 -10 0 10 20
25 20 15 10 5 0 -5 -10 -15 -20 -25
KILO WAVLNGTH


B'
metres

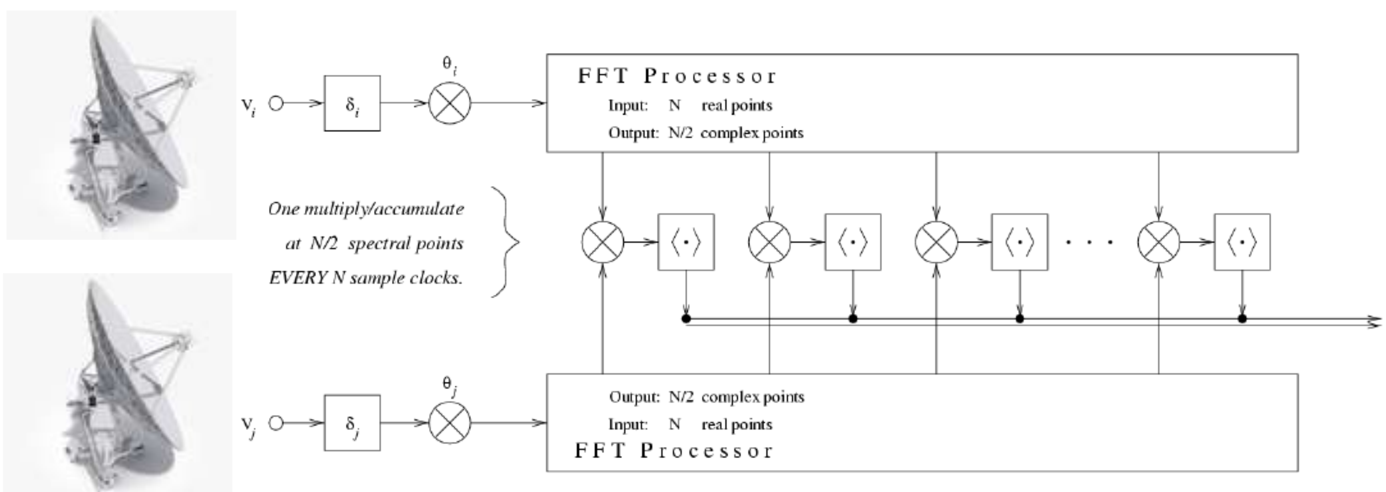
Making it feasible

- Analog filters are costly & finnicky; this would be expensive and temperamental



Making it feasible

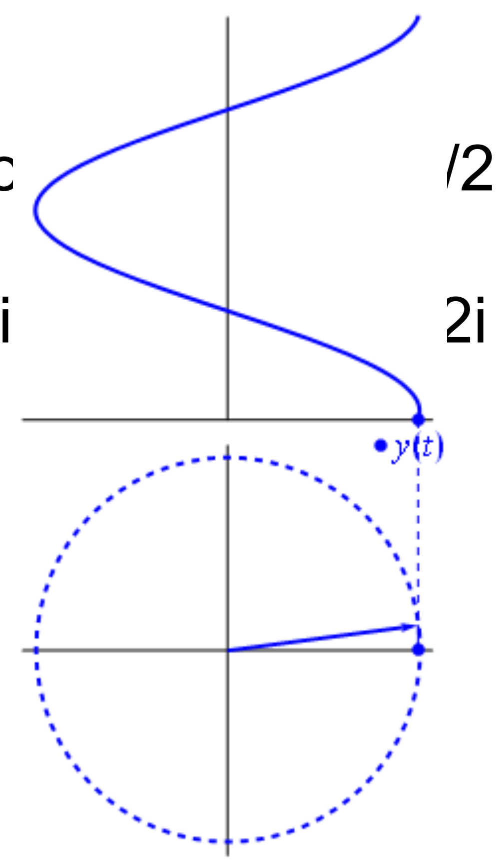
- Analog filters are costly & finnicky; this would be expensive and temperamental
- Fortunately, we can (and do) digitize the signal – meaning we can use a digital substitute: **digital filterbank**



The advantage of going digital

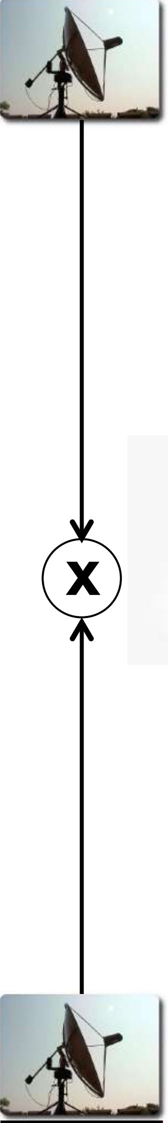
- Stable, cheap filters
- Produces complex output: when cross-multiplying, use 1 complex multiplier rather than 2 real multipliers and a phase shift

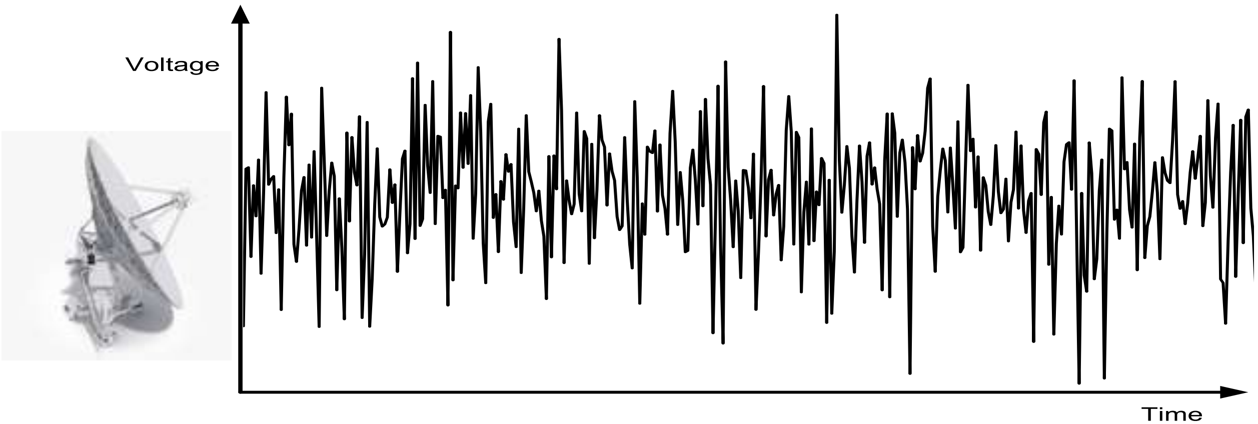
$e^{i\phi} = \cos\phi + i \sin\phi$



Animation from http://en.wikipedia.org/wiki/File:Unfasor.gif

The “FX” correlator





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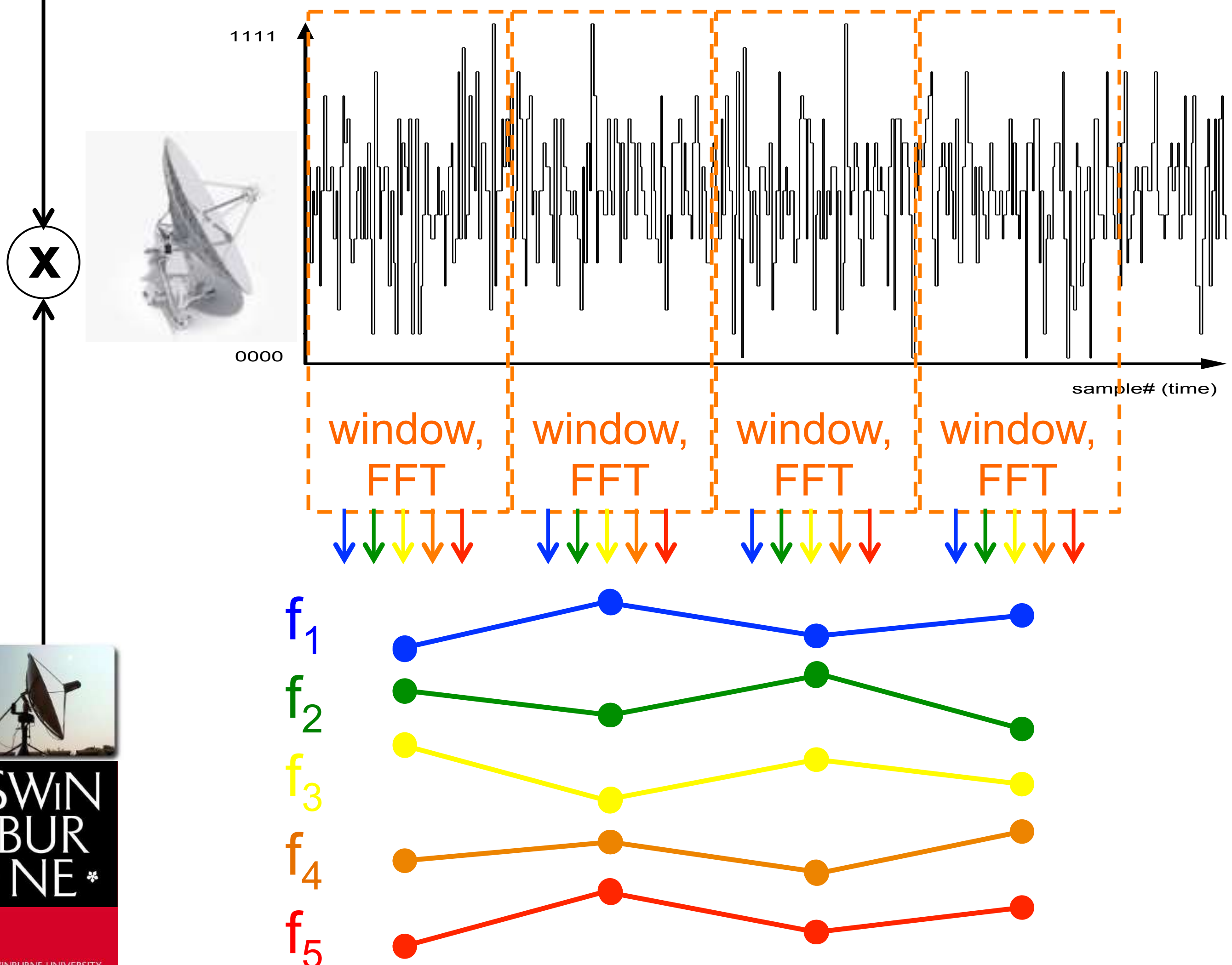

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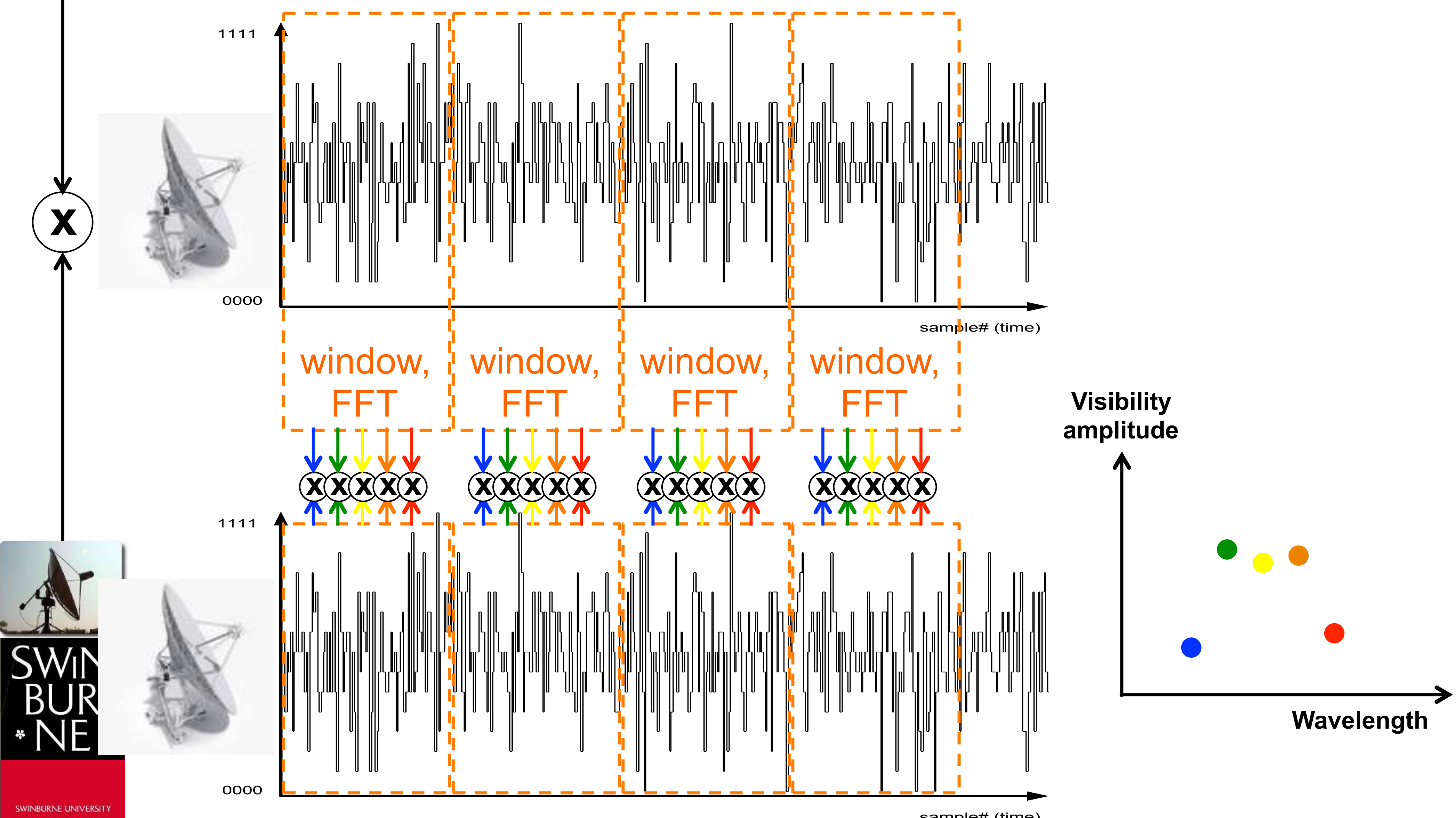

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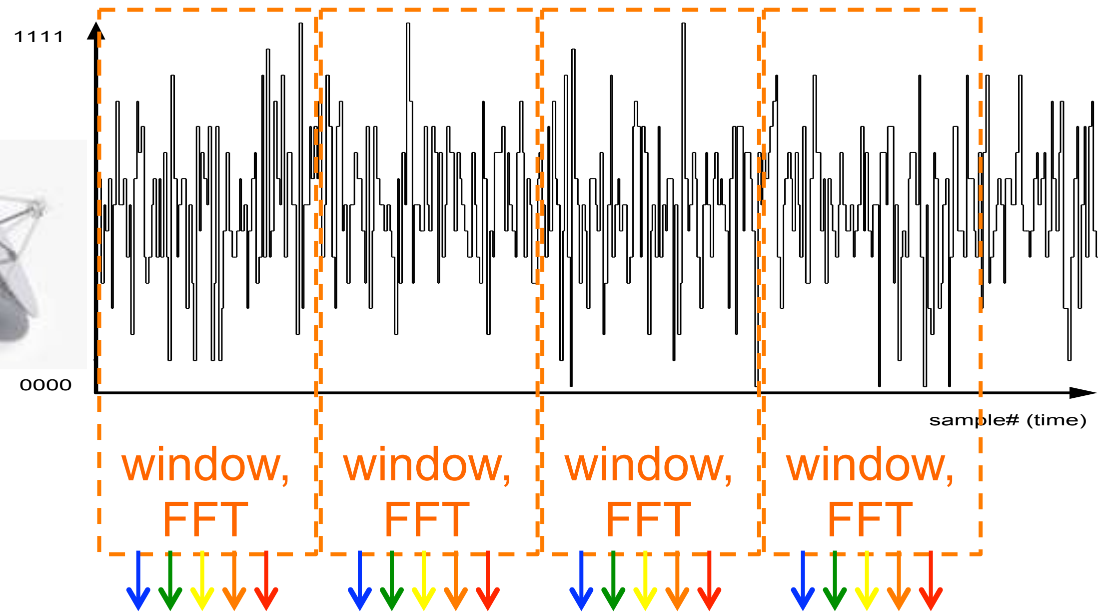

- Since this architecture consists of a Fourier transform (F) followed by cross-multiplication (X), we dub this the "FX" correlator

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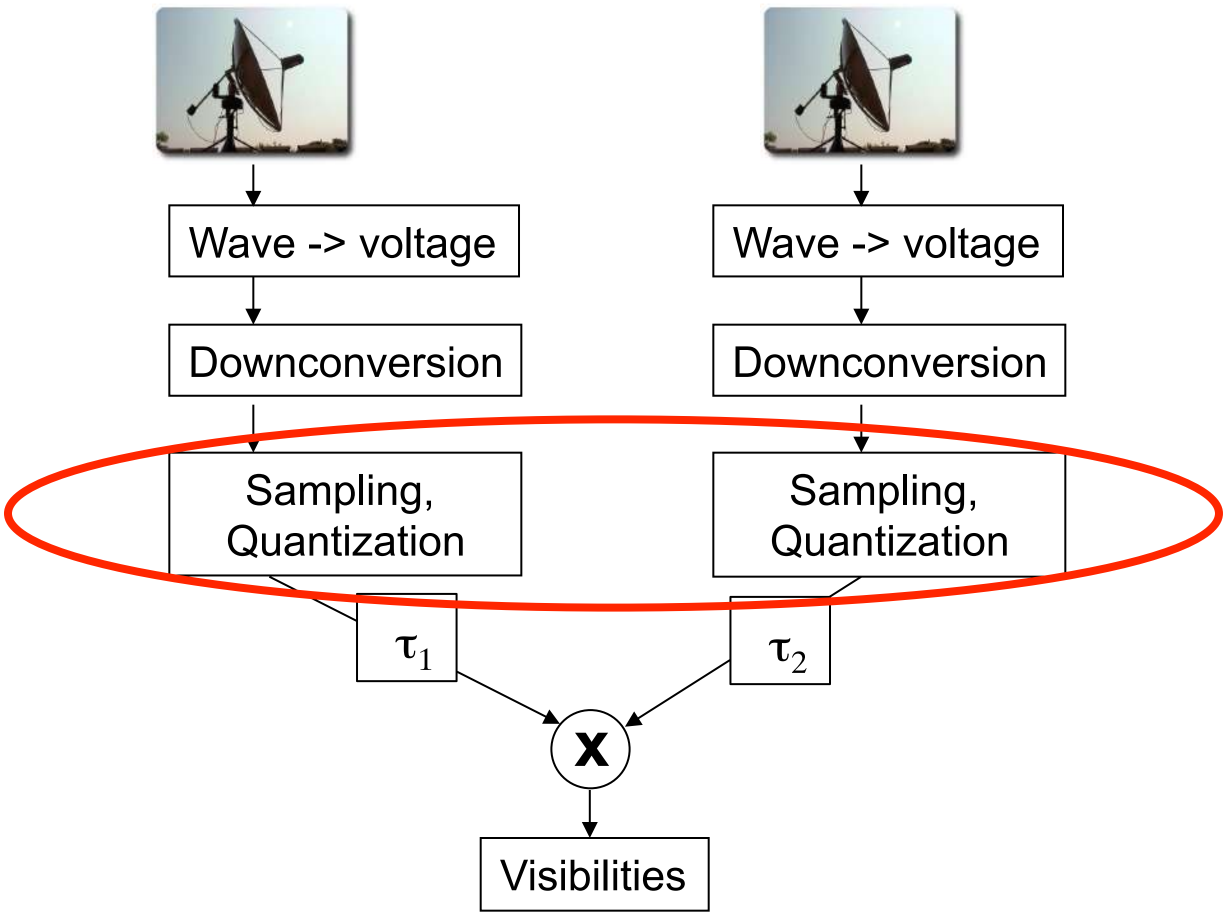
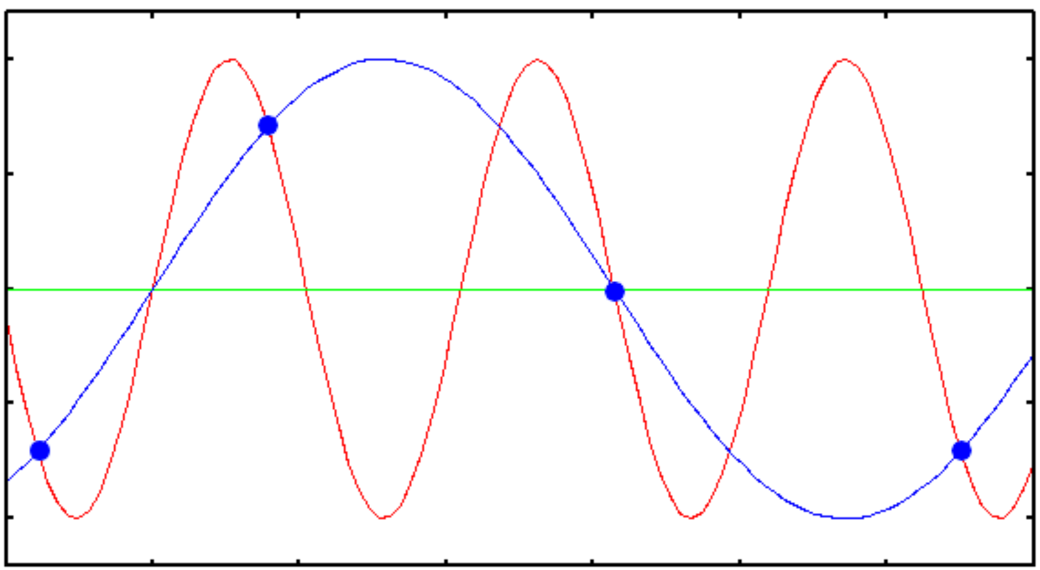
Righting the wrongs



Sampling

- Nyquist-Shannon sampling theorem:
  - real-valued signal is sampled every Δt sec
  - Original signal can be reconstructed perfectly so long as contains no power at frequencies $\geq 1 / (2\ \Delta t)$ Hz (*band-limited*)



Adequately sampled

Undersampled, cannot be reconstructed

Sampling

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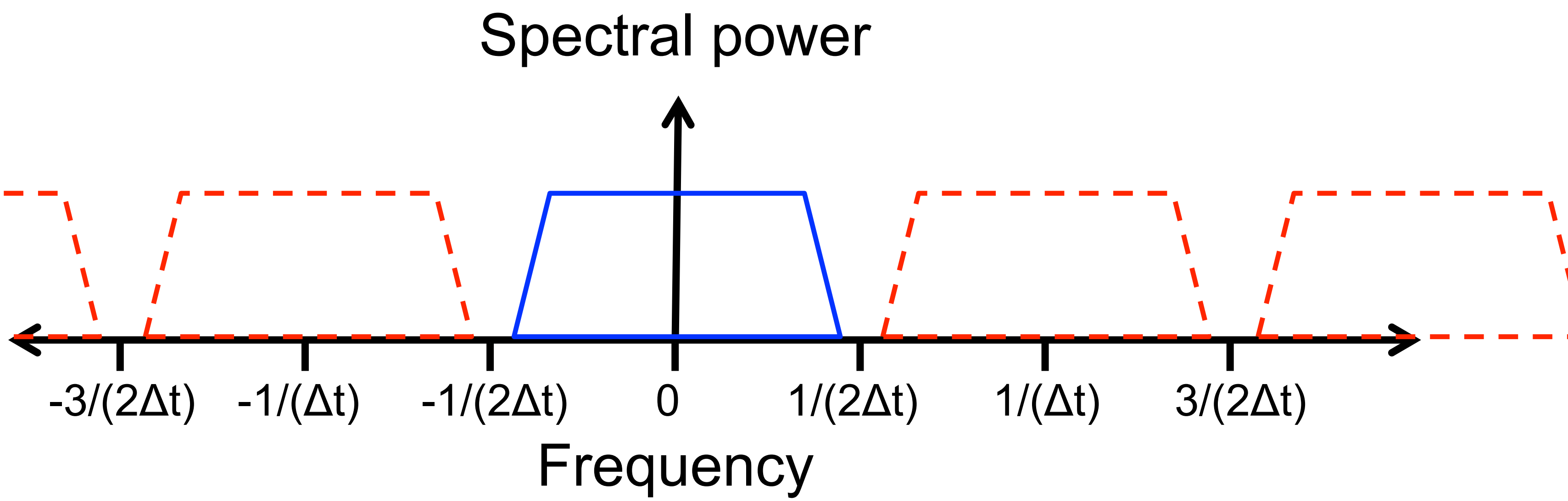

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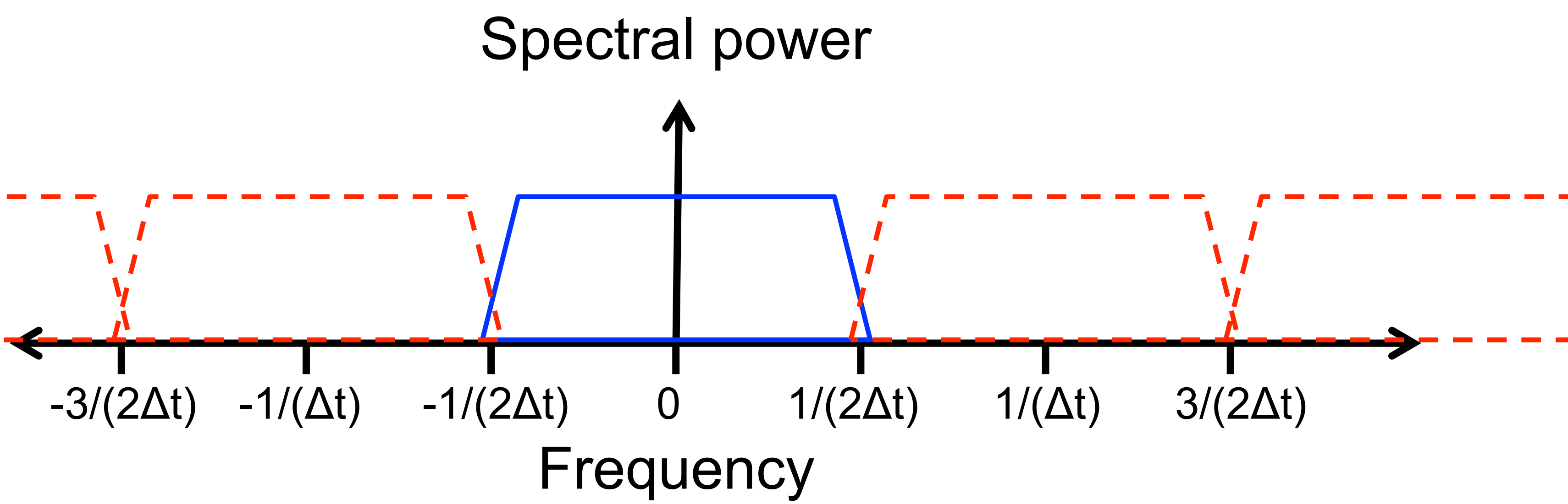
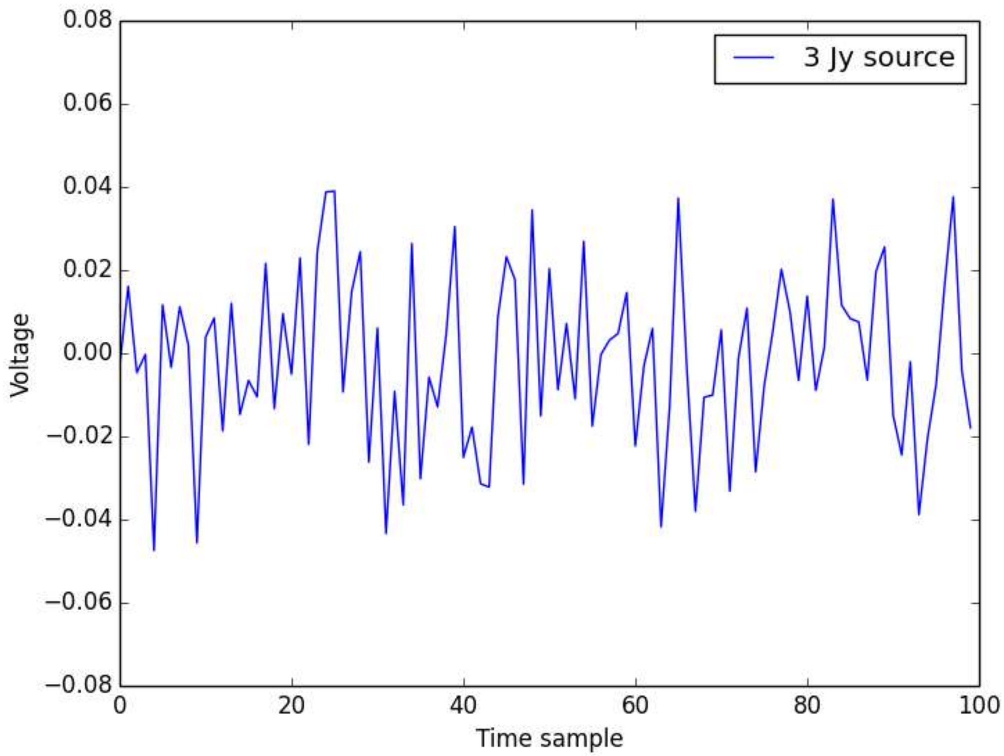


Quantization

- When correlation is low (almost always) even very coarse quantization is ok!



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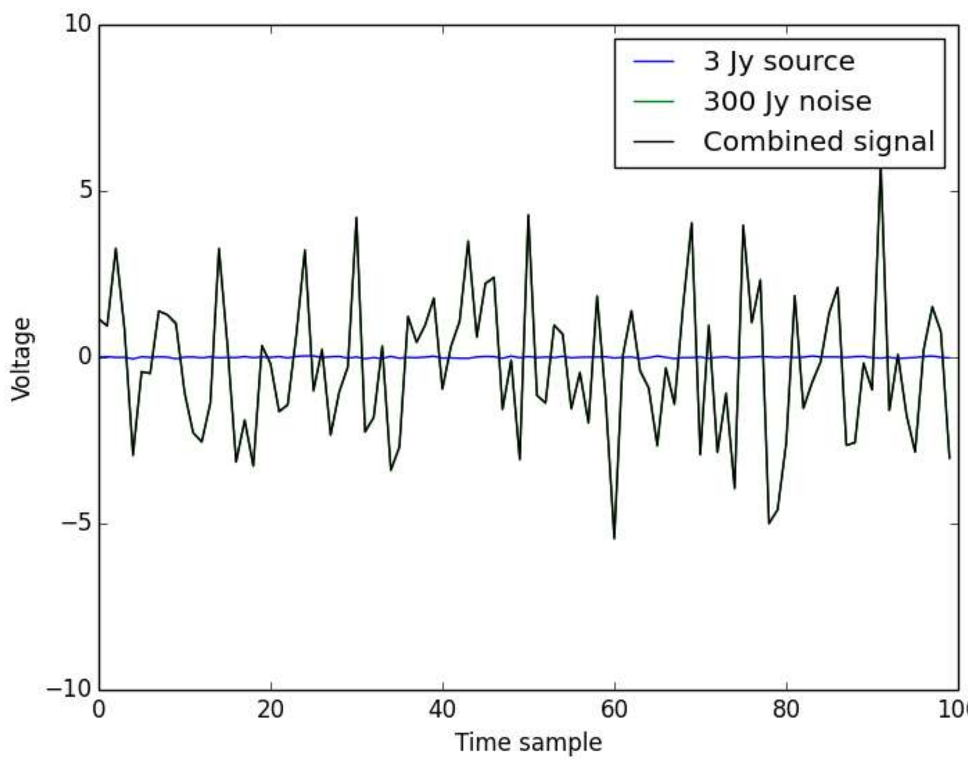

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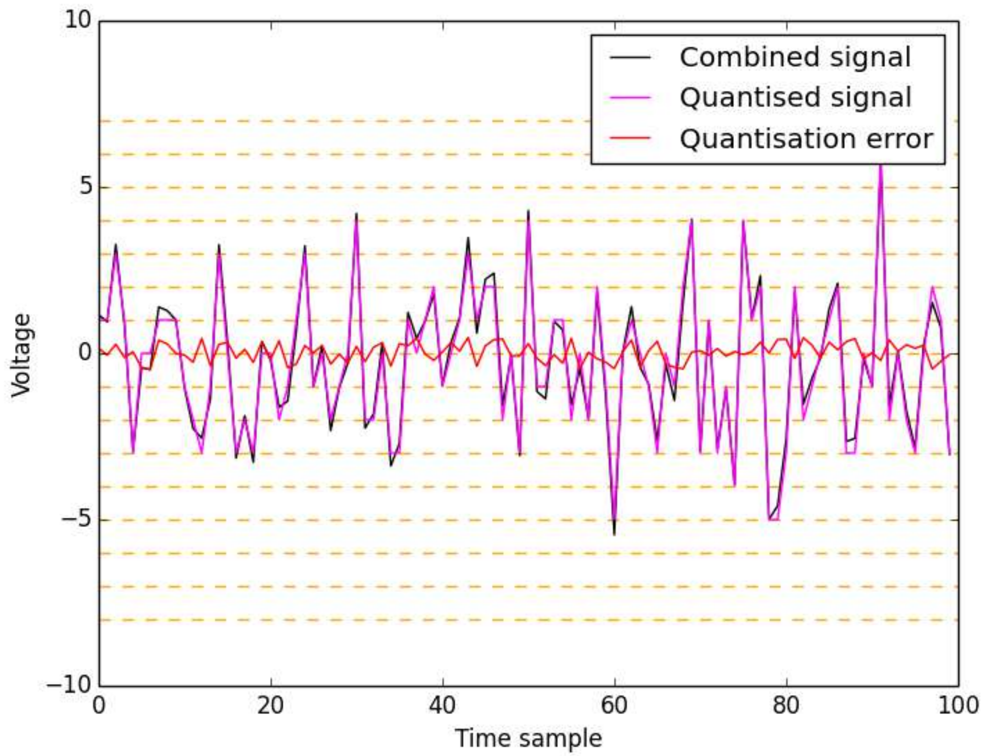

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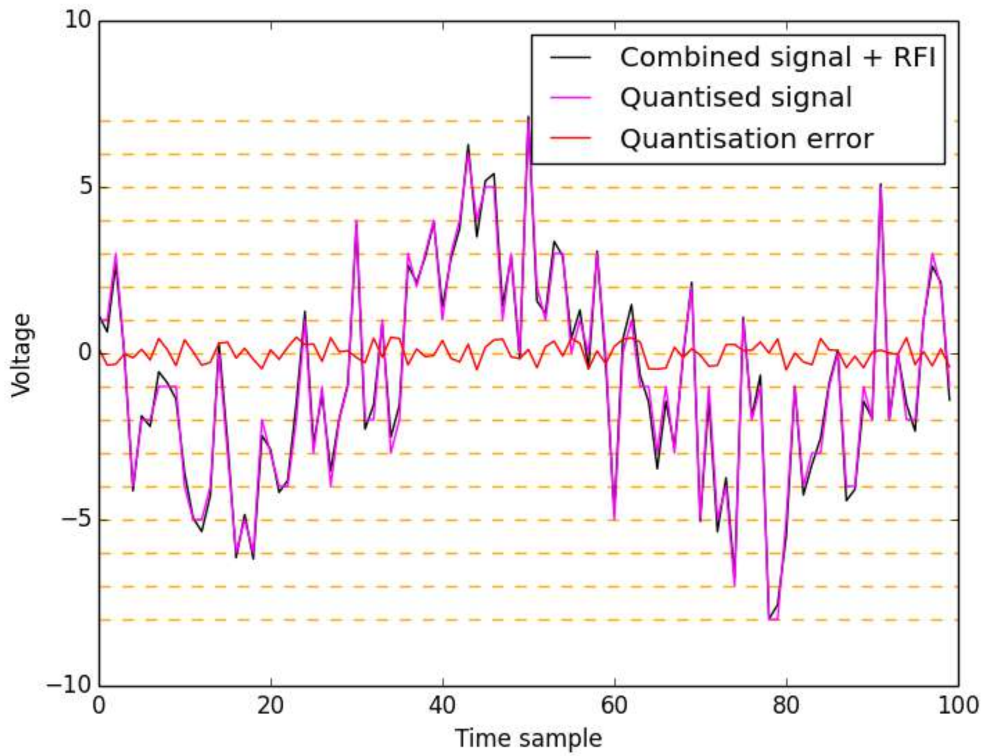

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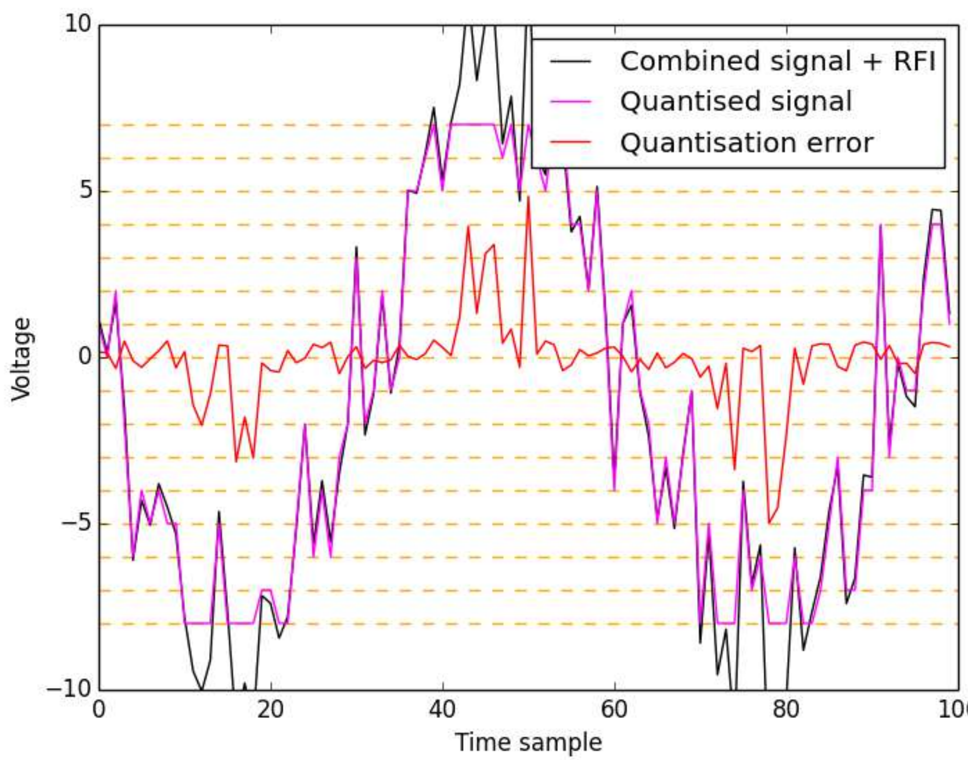
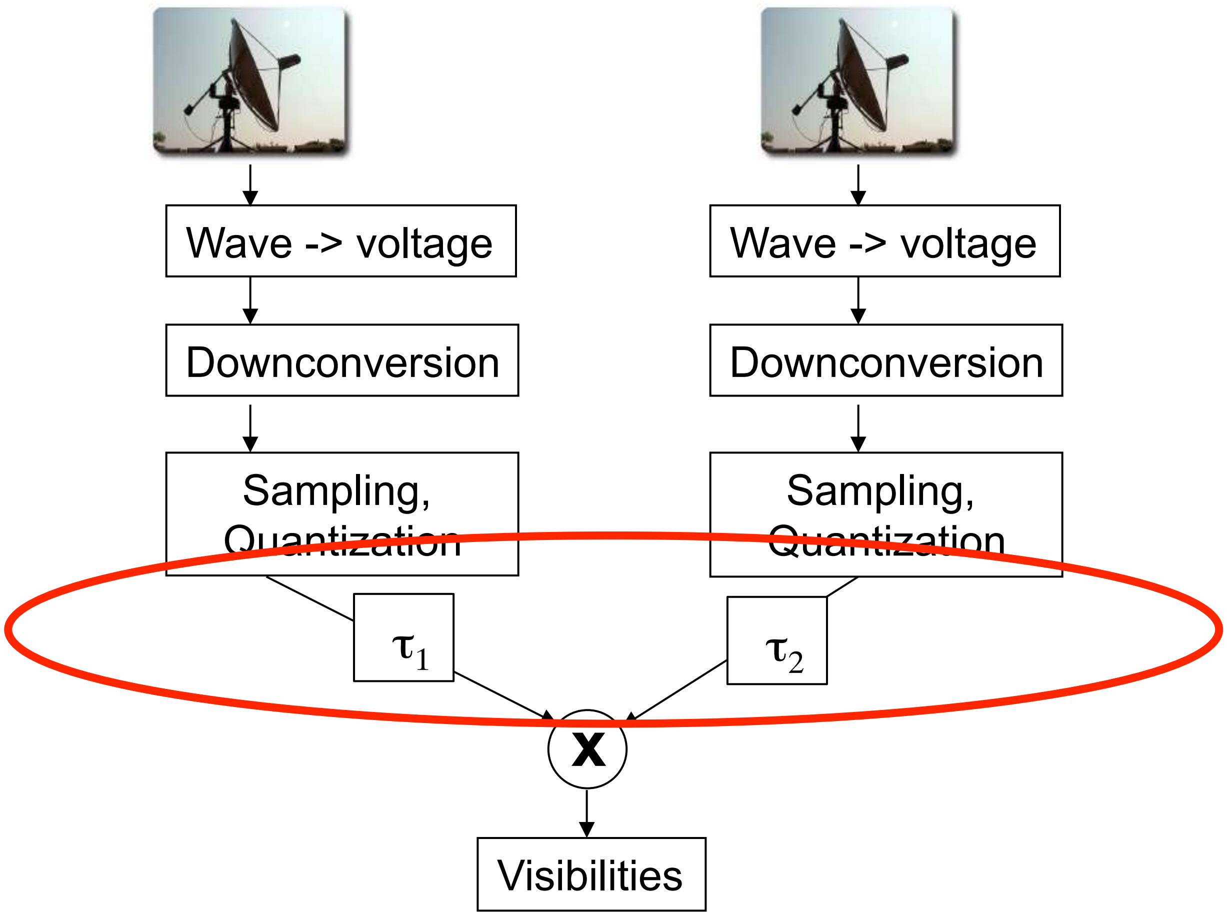


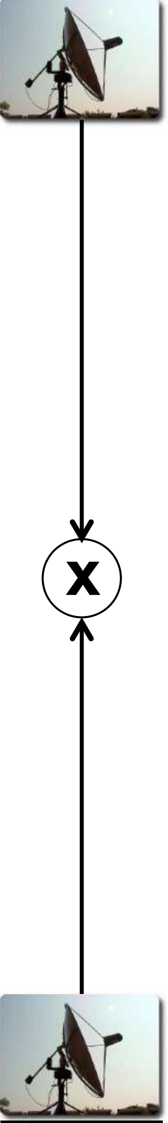
until the headroom runs out…

Quantization

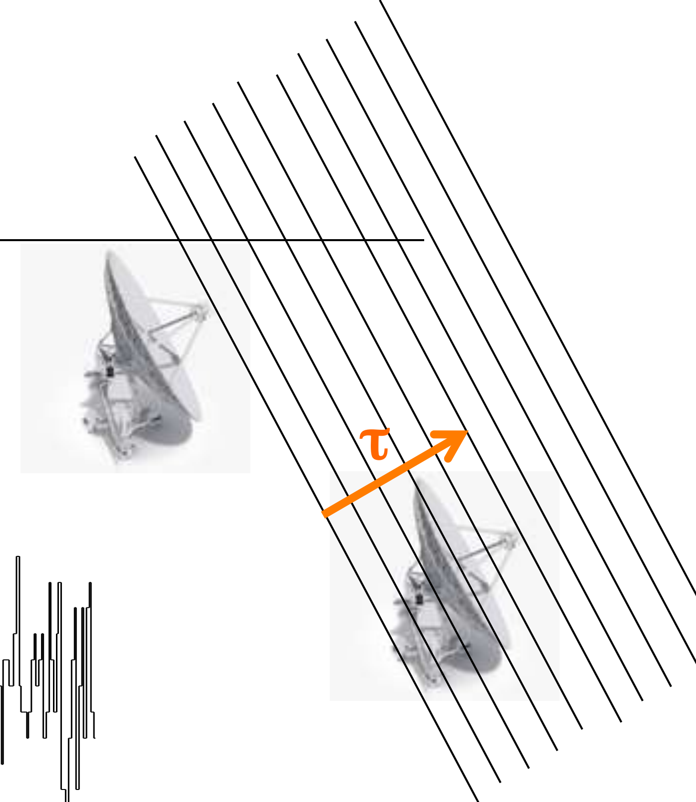
- When correlation is low (almost always) even very coarse quantization is ok!
- Sensitivity loss due to quantisation:
  - 8 bit: 0.1%
  - 4 bit: 1.3%
  - 2 bit: 12%
  - 1 bit: 36%
- Correct visibility amplitudes for this sensitivity loss

Righting the wrongs

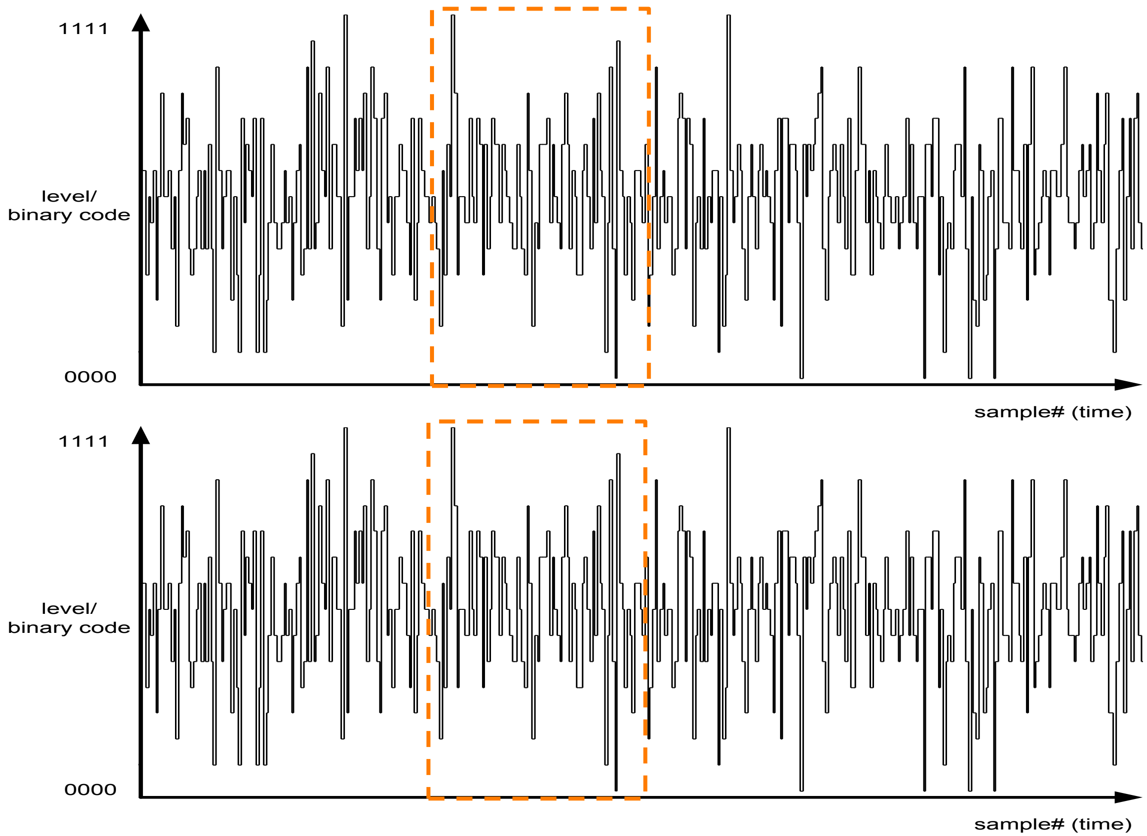




Delay compensation



- Delay to the nearest sample is easy:



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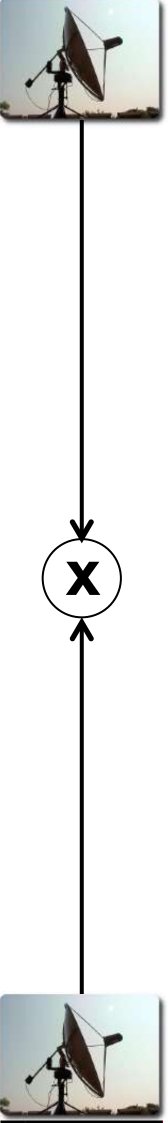

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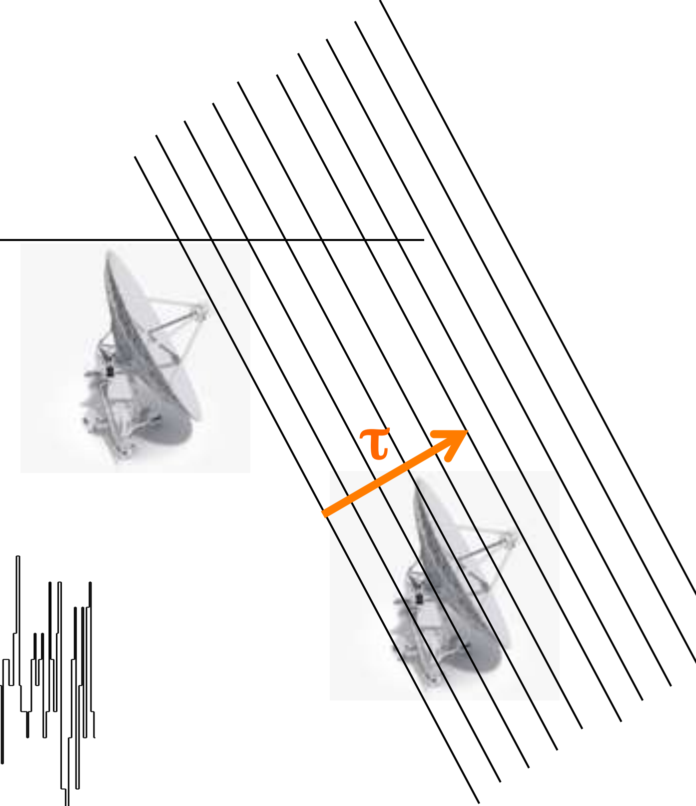
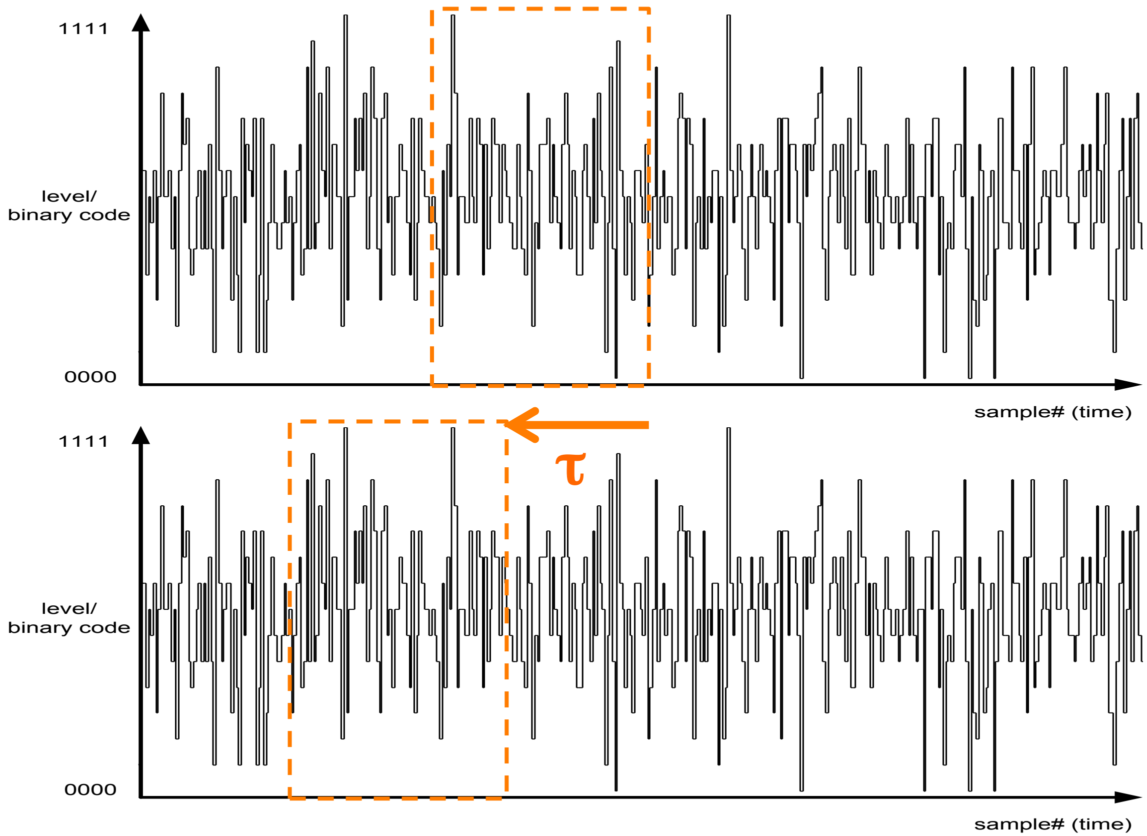
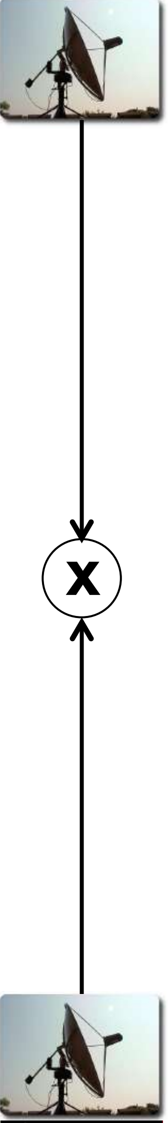
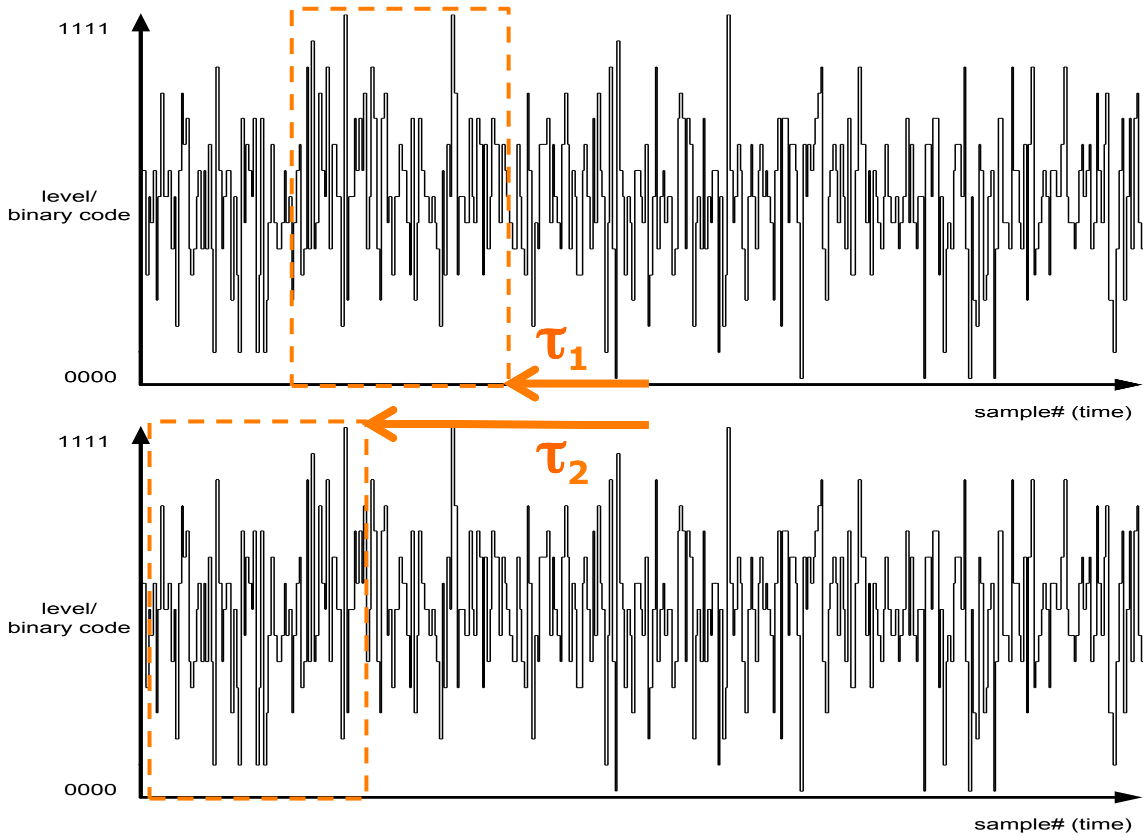


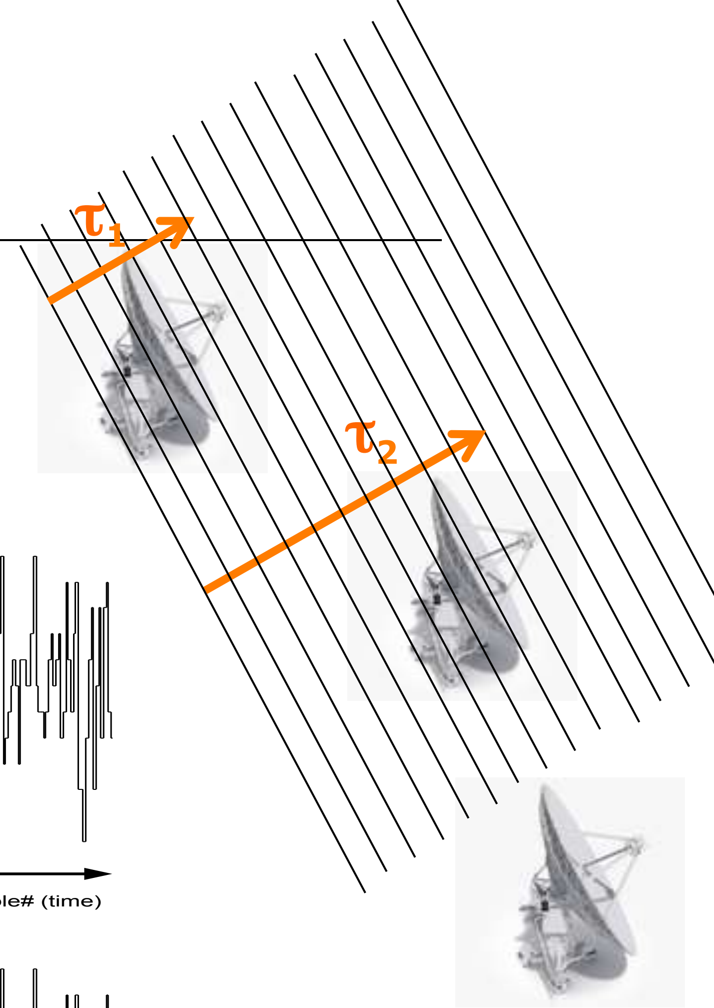


Delay compensation

- In practise, delay all to common reference

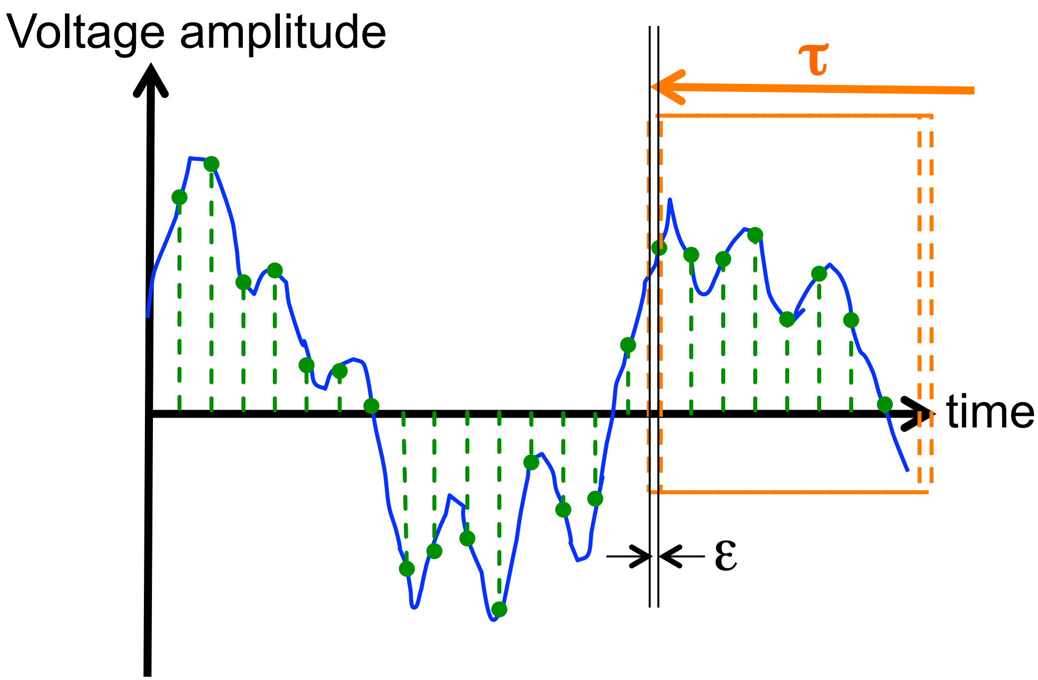


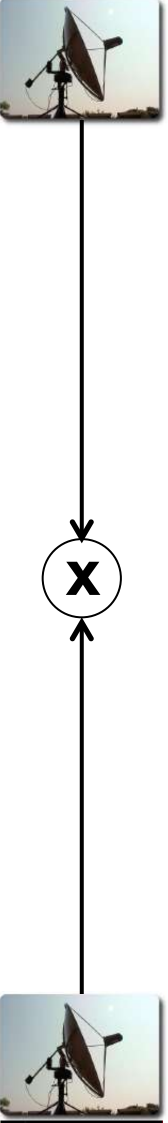




Fractional-sample correction

- Sampling prevents perfect alignment of datastreams; always a small error





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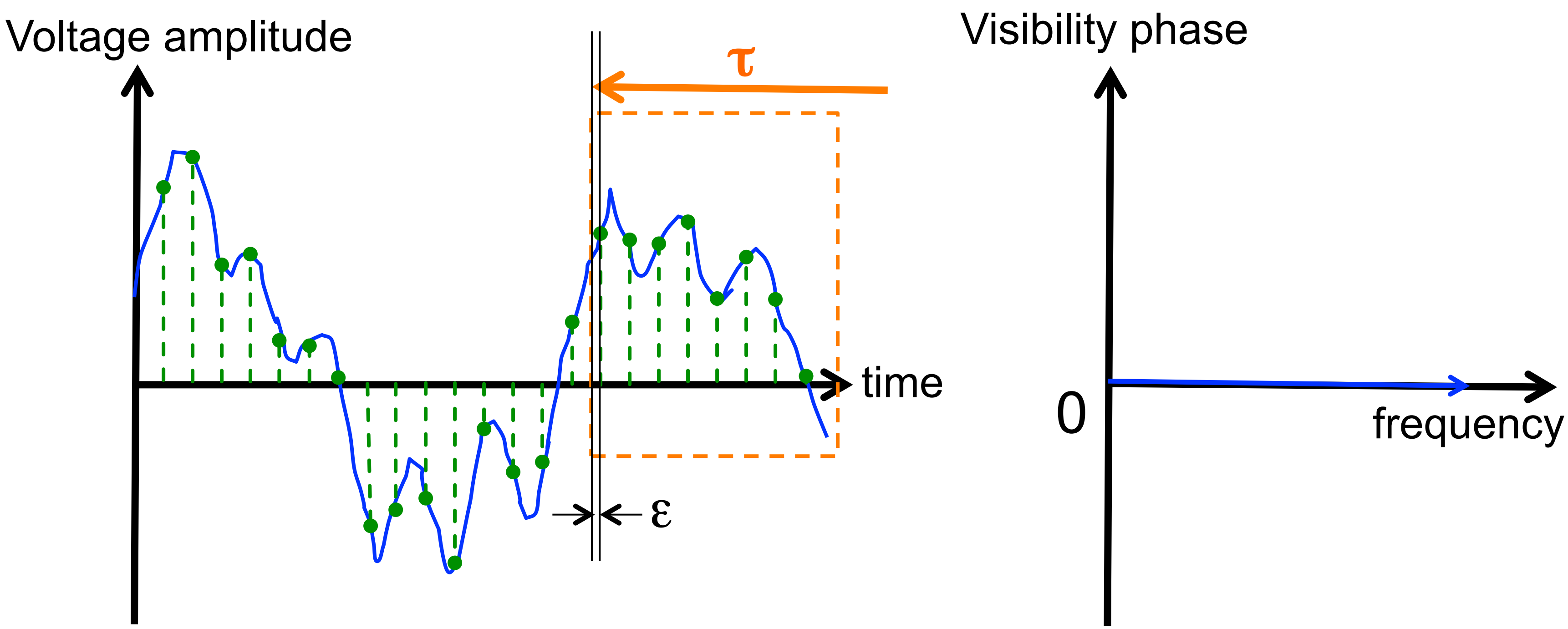

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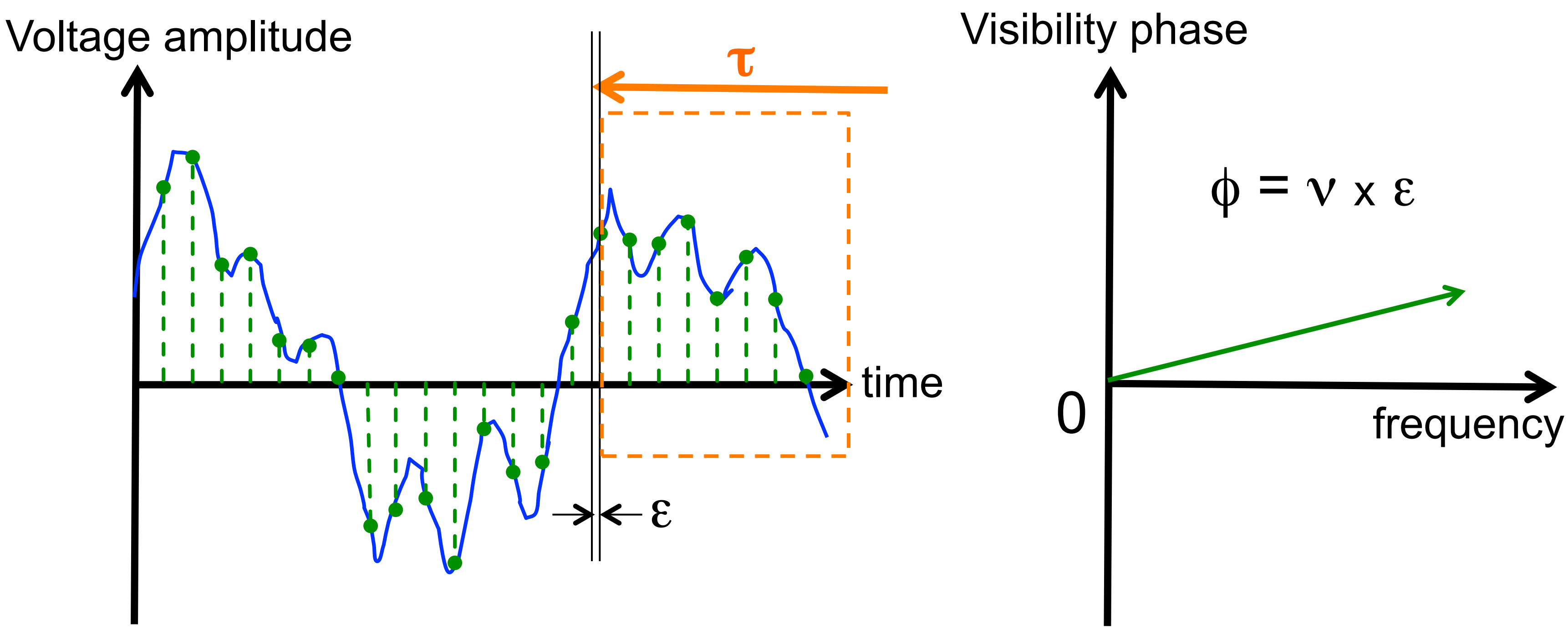

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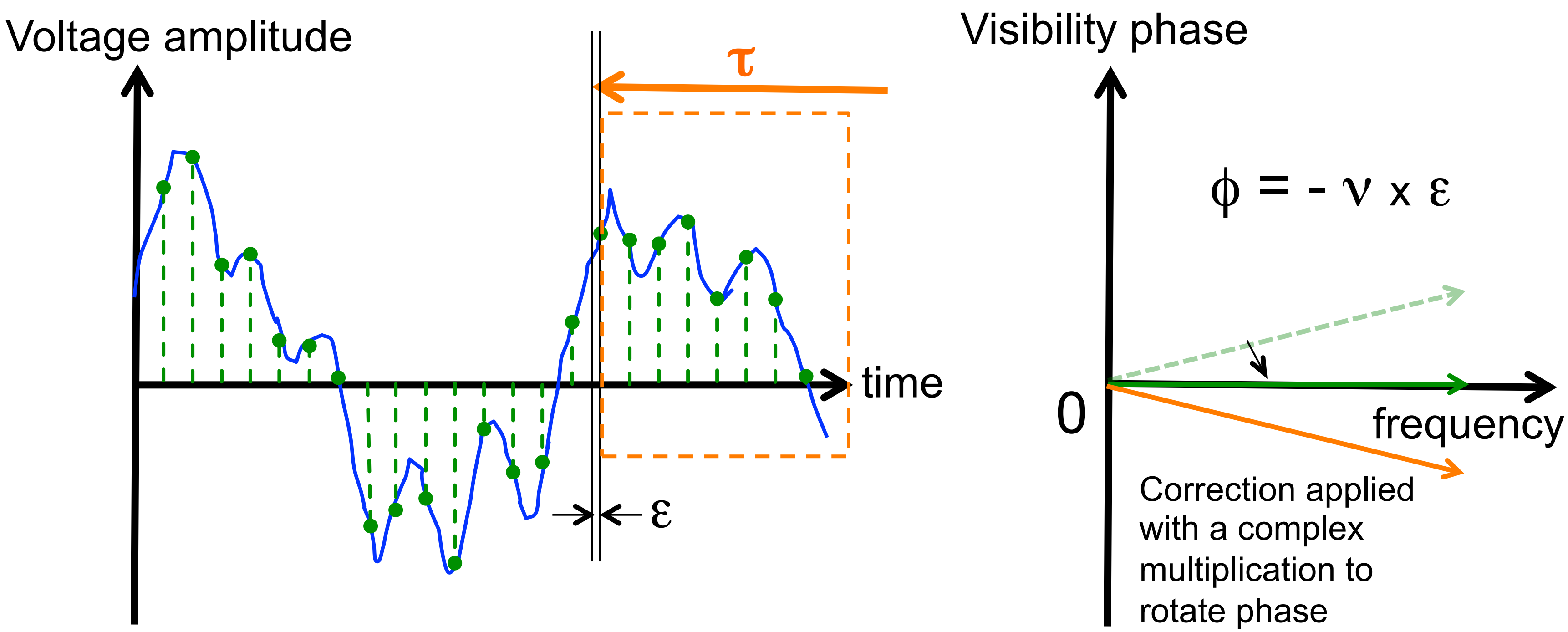
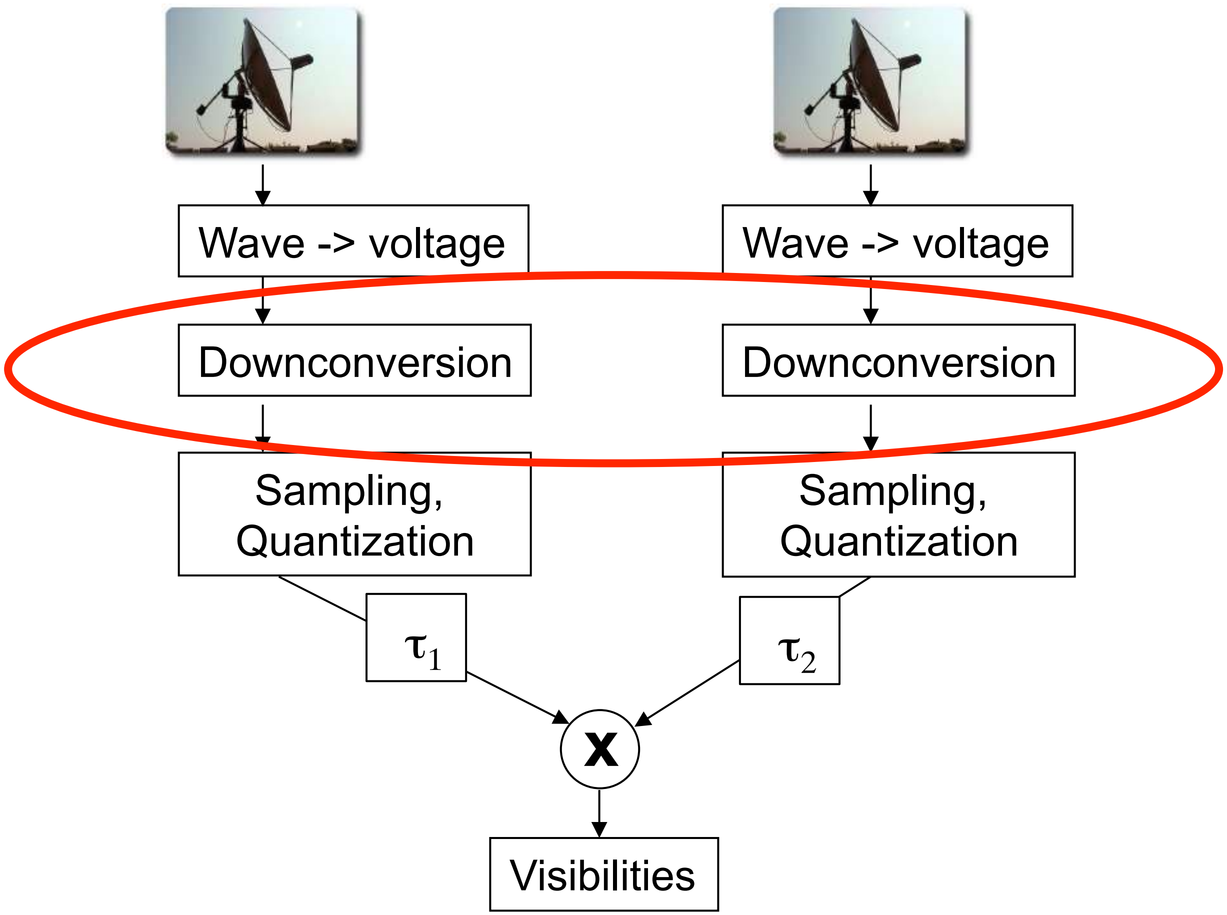
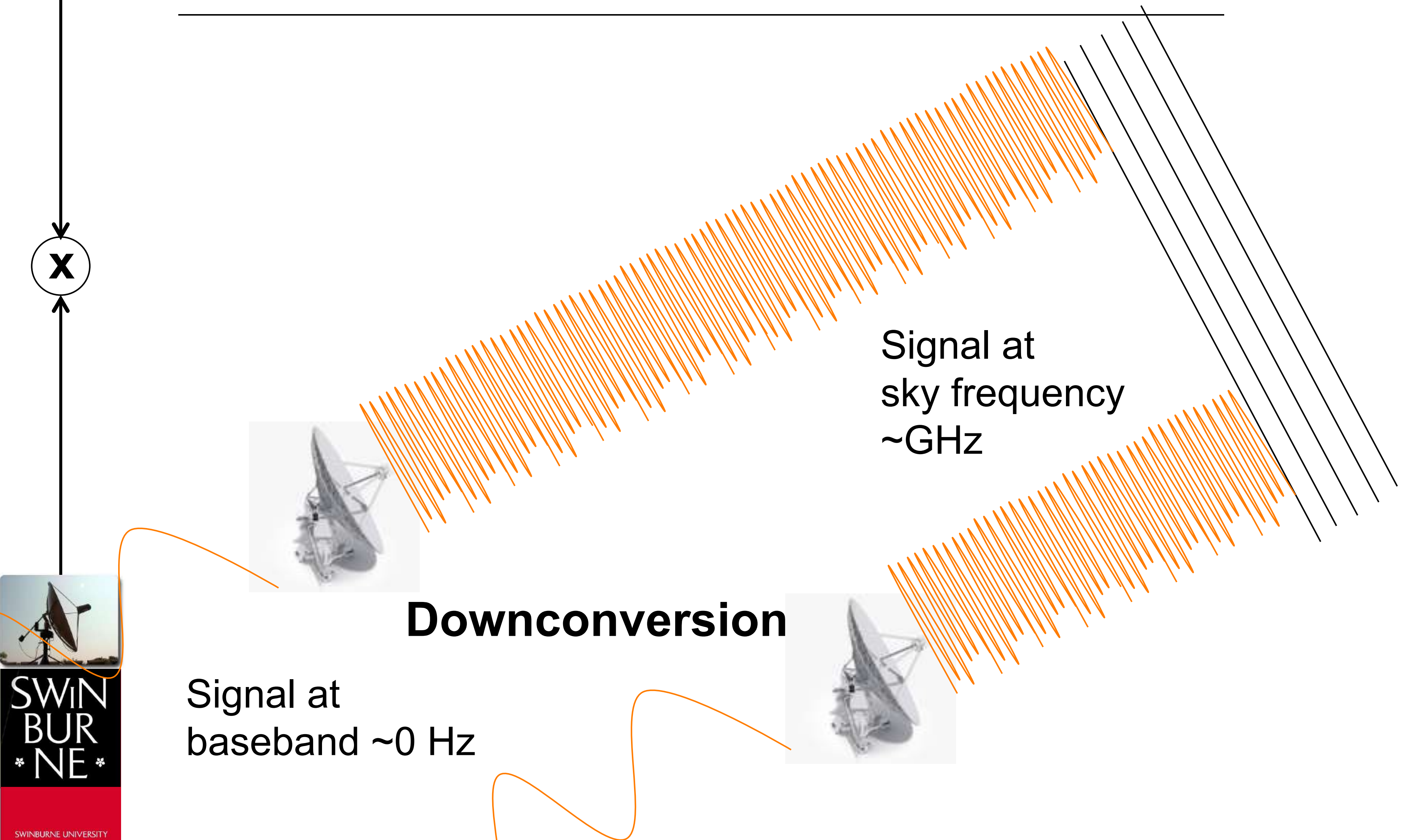
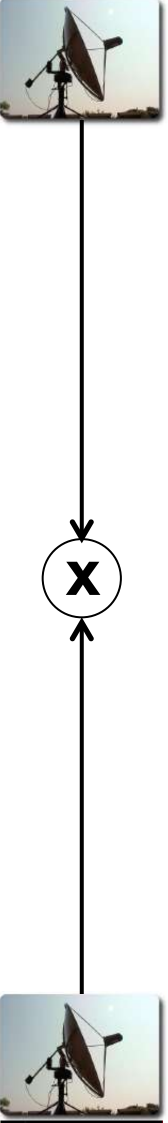


Righting the wrongs



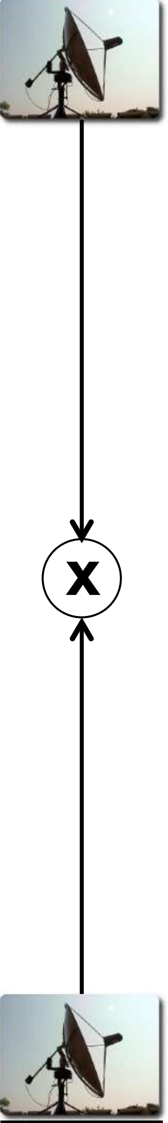
Fringe rotation





Fringe rotation

- Implementation: rotate phase using complex multiplier
- $\Delta\phi = 2\pi\, \nu_{lo}\, \tau_g$ ν_{lo} = local oscillator frequency, τ_g = applied delay
- Update rate of $\Delta\phi$ depends on how fast τ_g changes:
  - If τ_g is changing fast, update every sample in the time domain
  - For shorter baseline / low frequency instruments, can do post-channelisation or even post-accumulation



Alternate implementation

- We have shown how to build a practical FX correlator, which first Fourier transforms and then multiplies
- Convolution theorem: **Multiplication** in the frequency domain is equivalent to **convolution** in the time domain
- It is mathematically equivalent to convolve the two signals in the time domain and then Fourier transform

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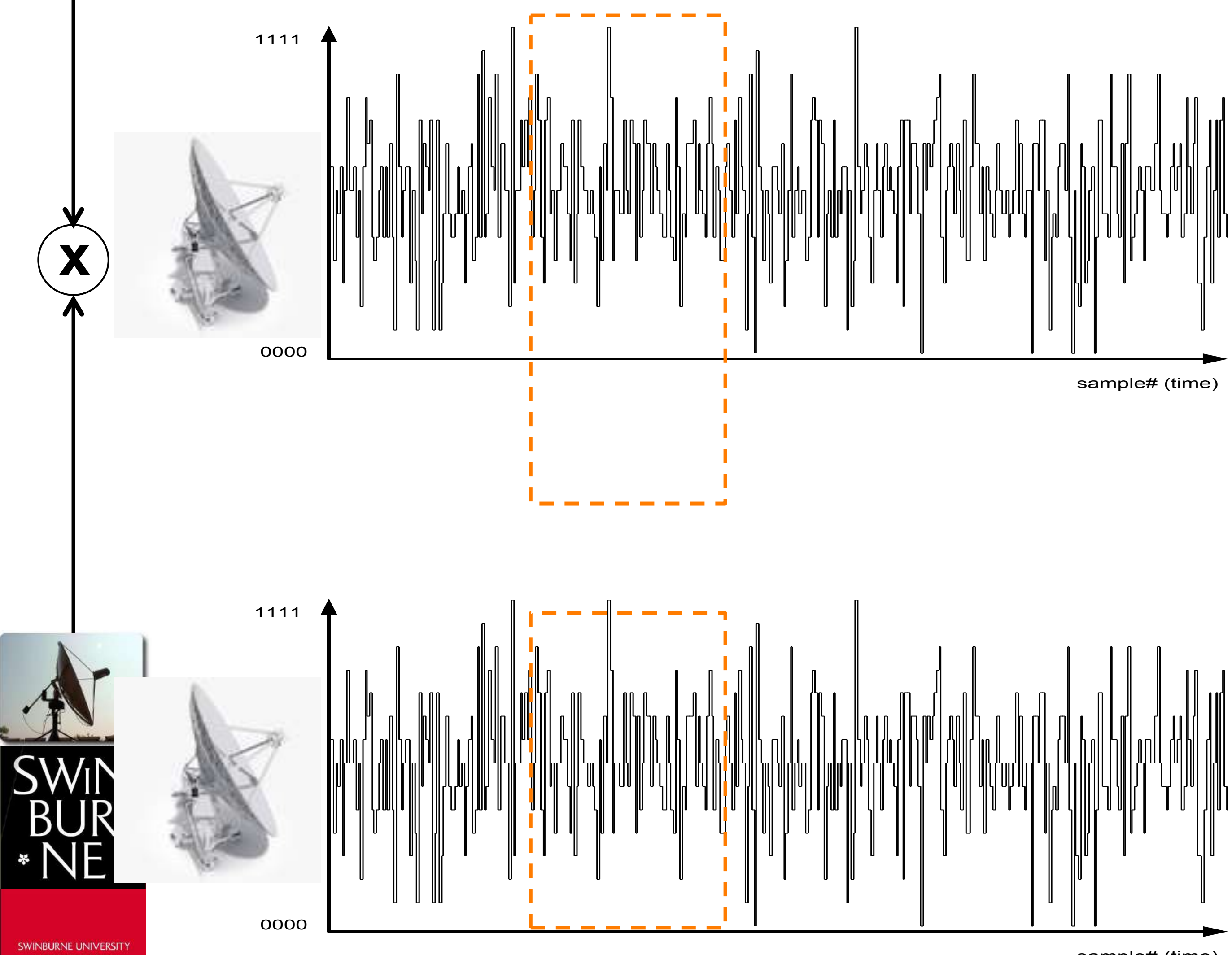
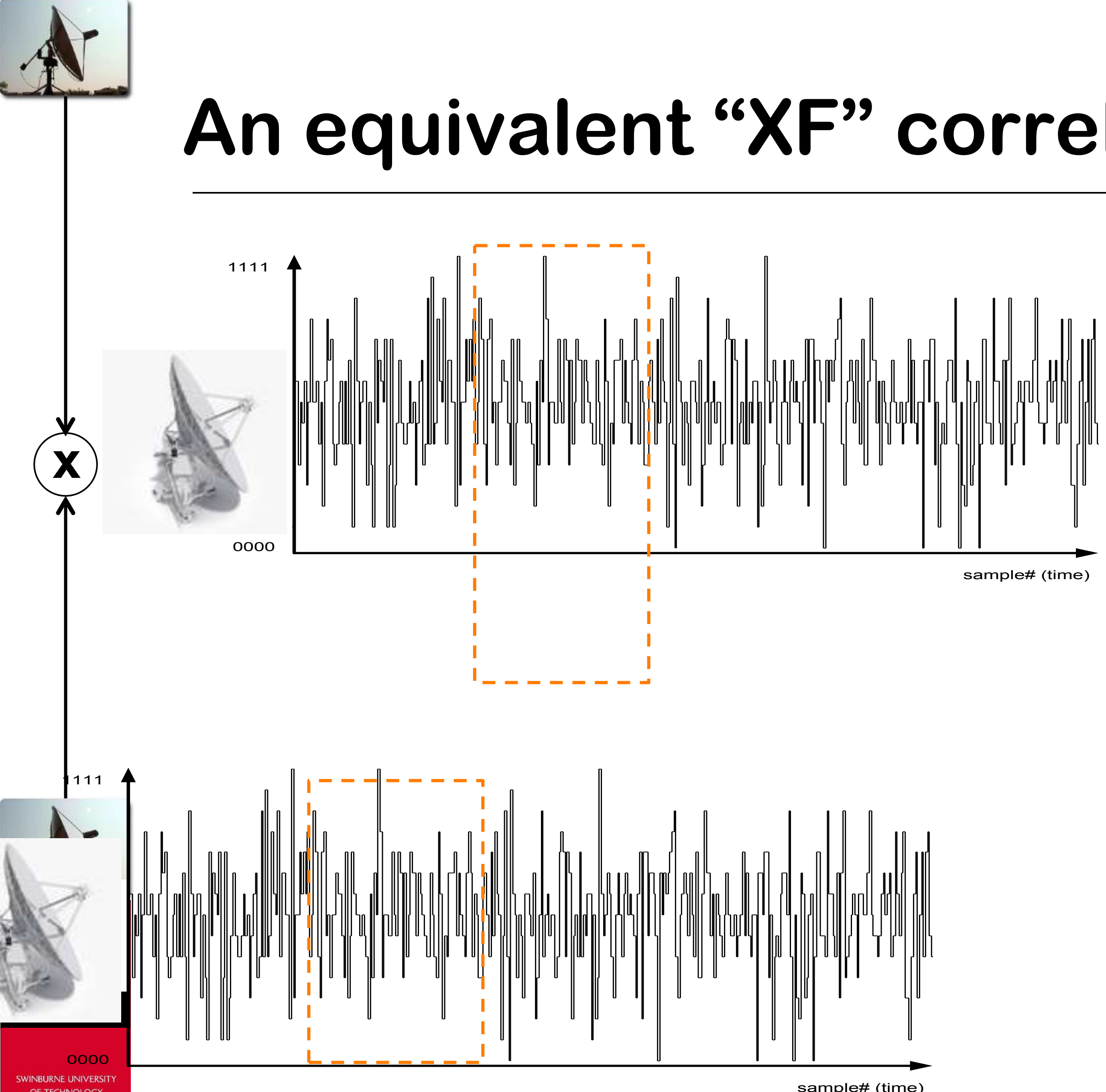
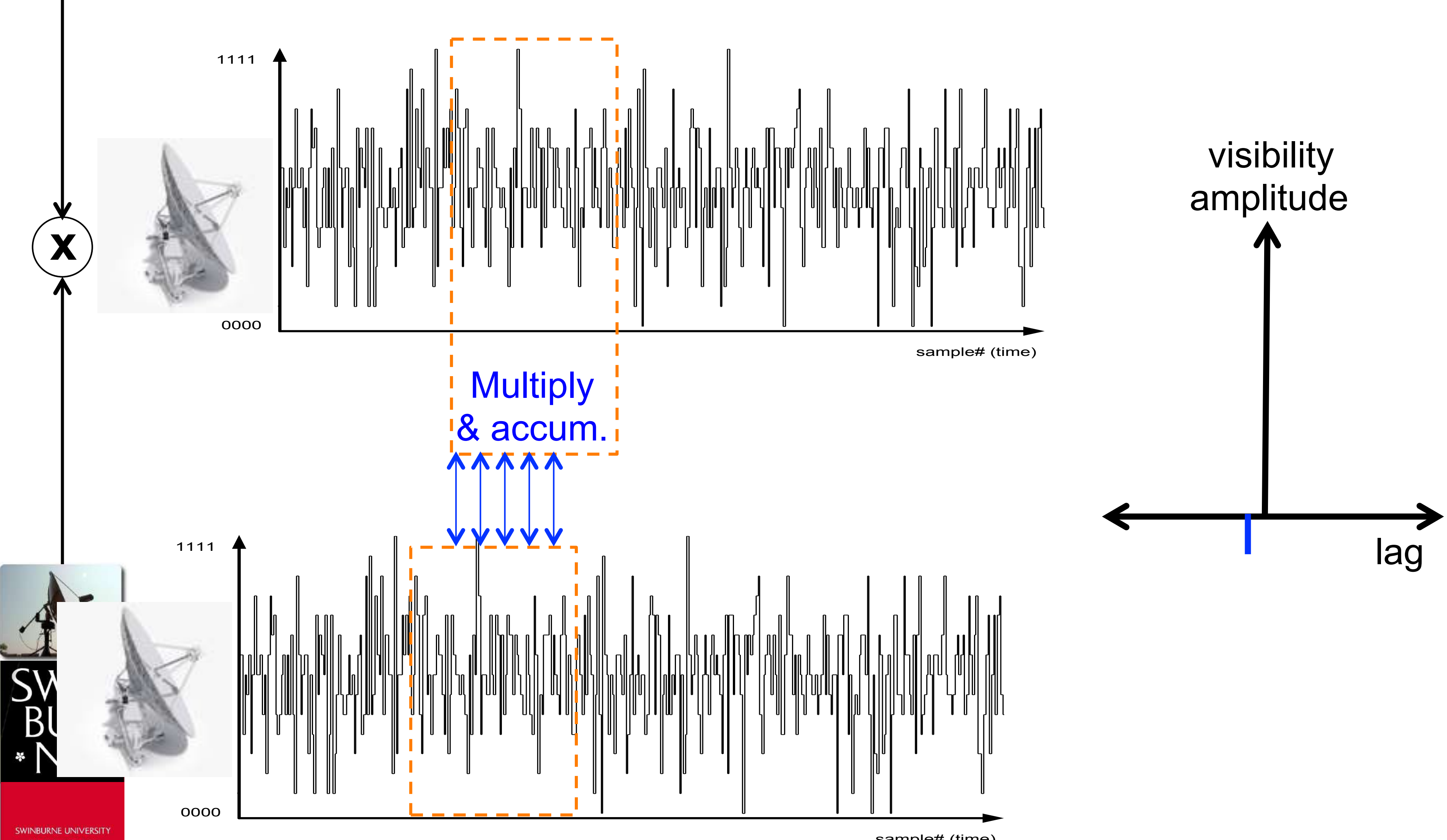
An equivalent “XF” correlator



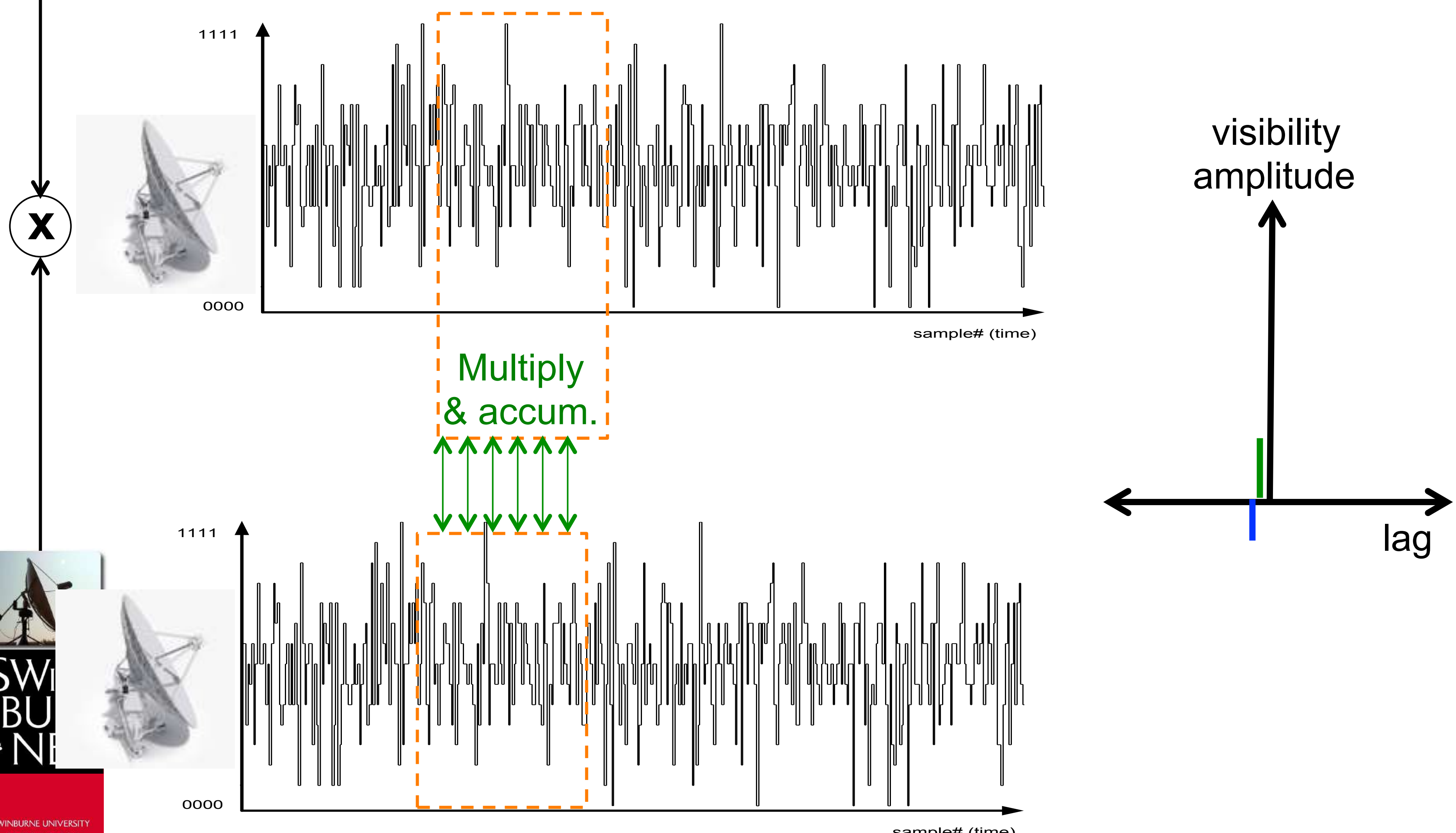
An equivalent “XF” correlator



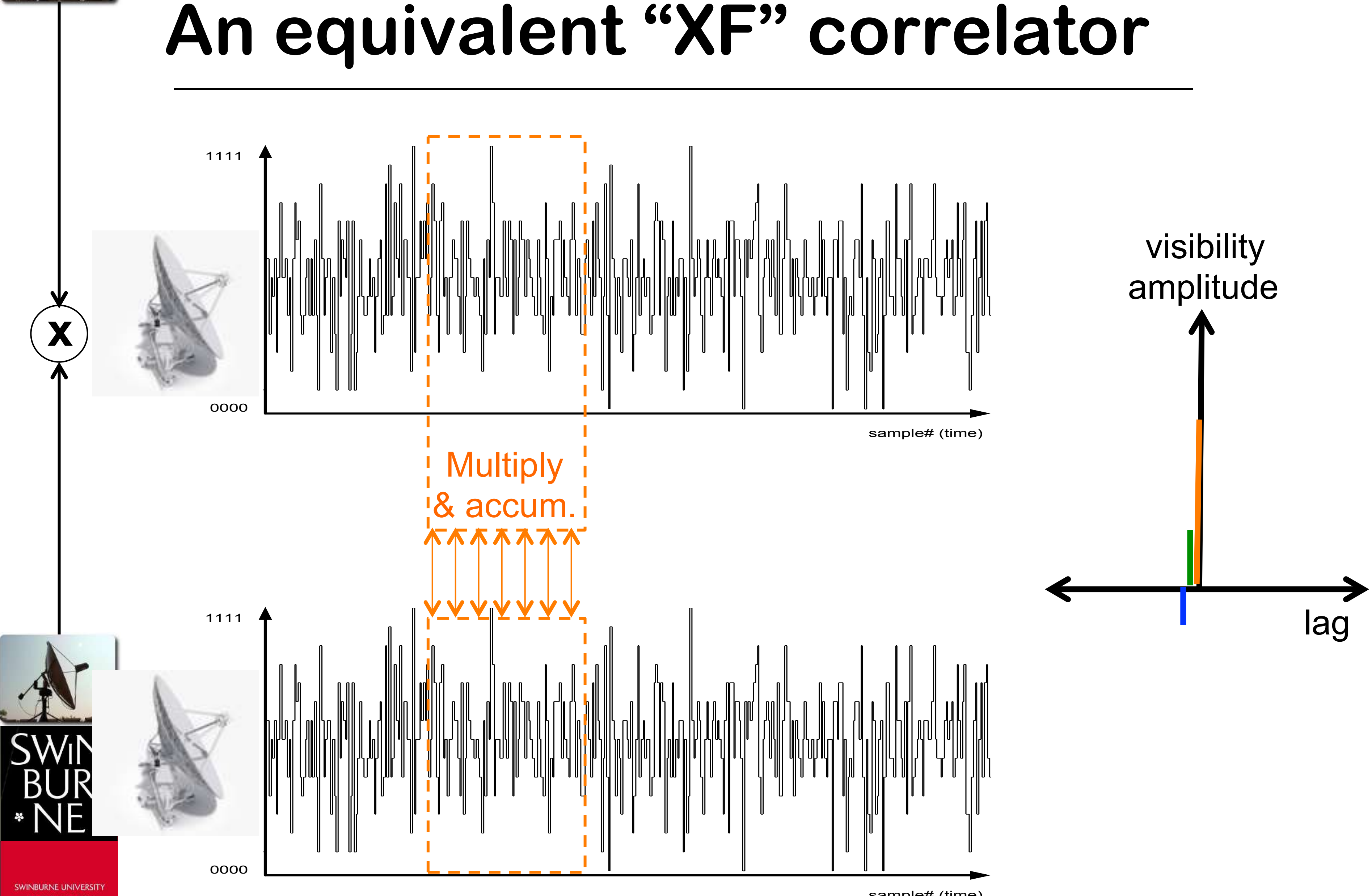
An equivalent “XF” correlator



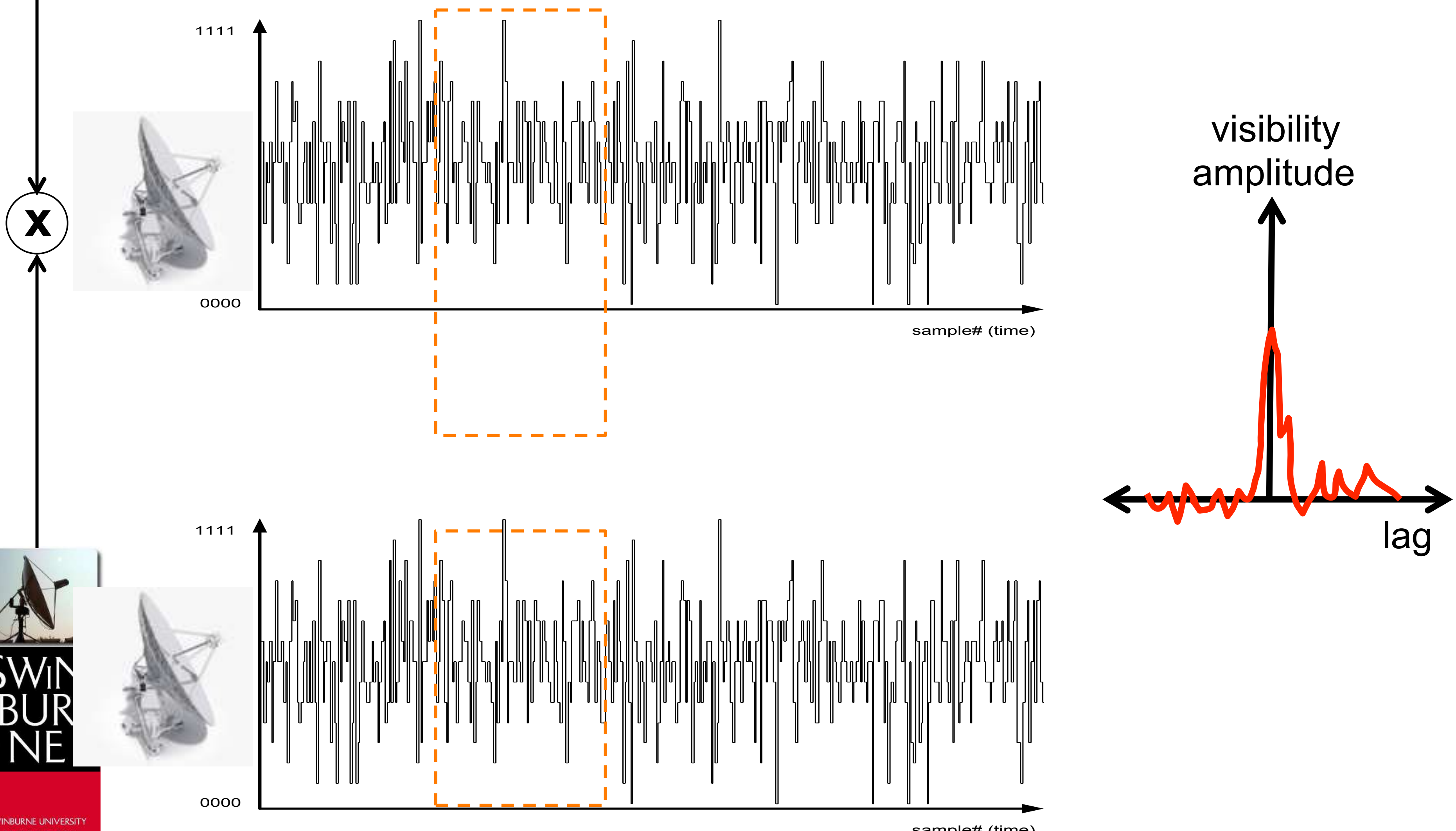
An equivalent “XF” correlator



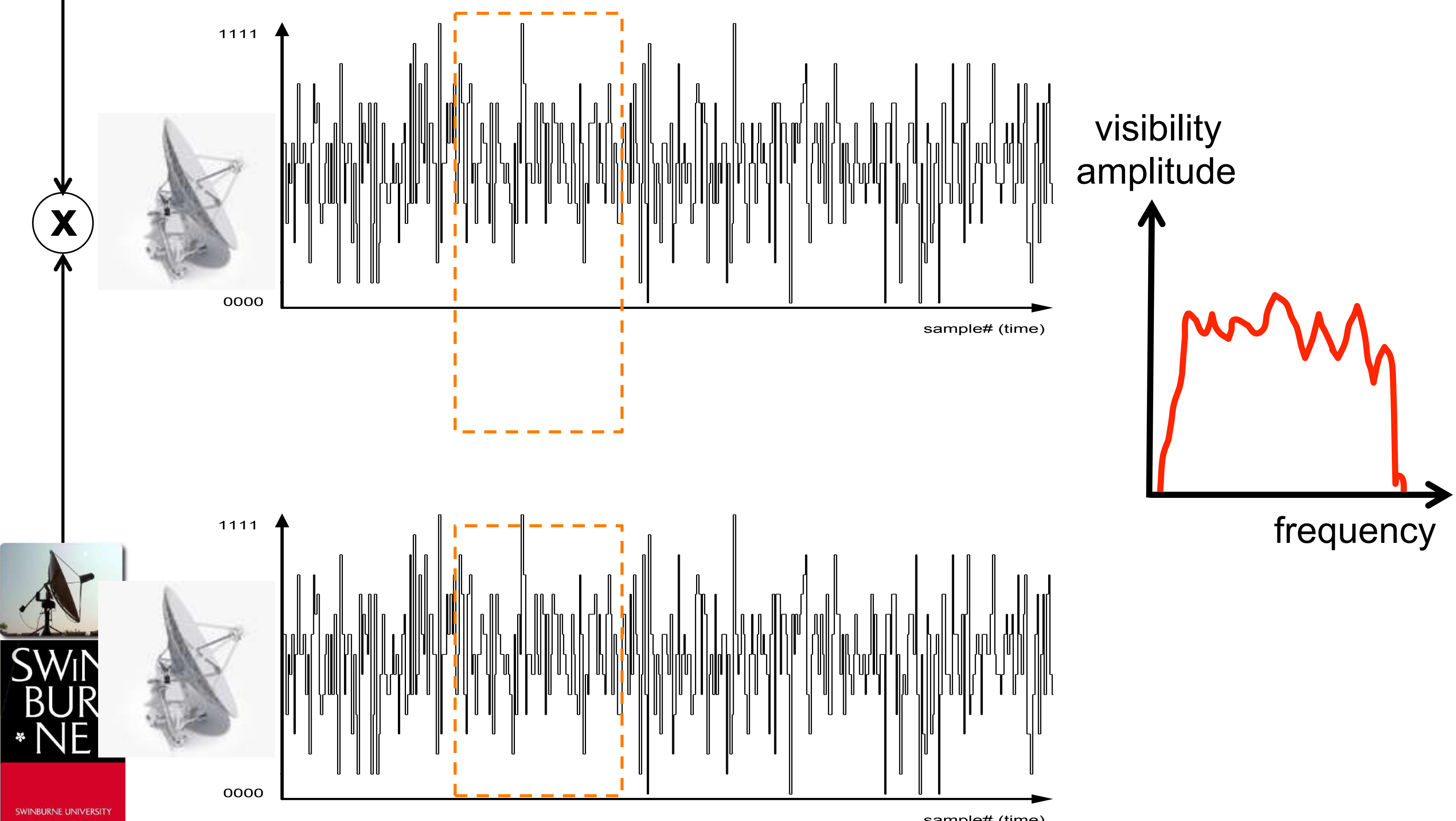
An equivalent "XF" correlator



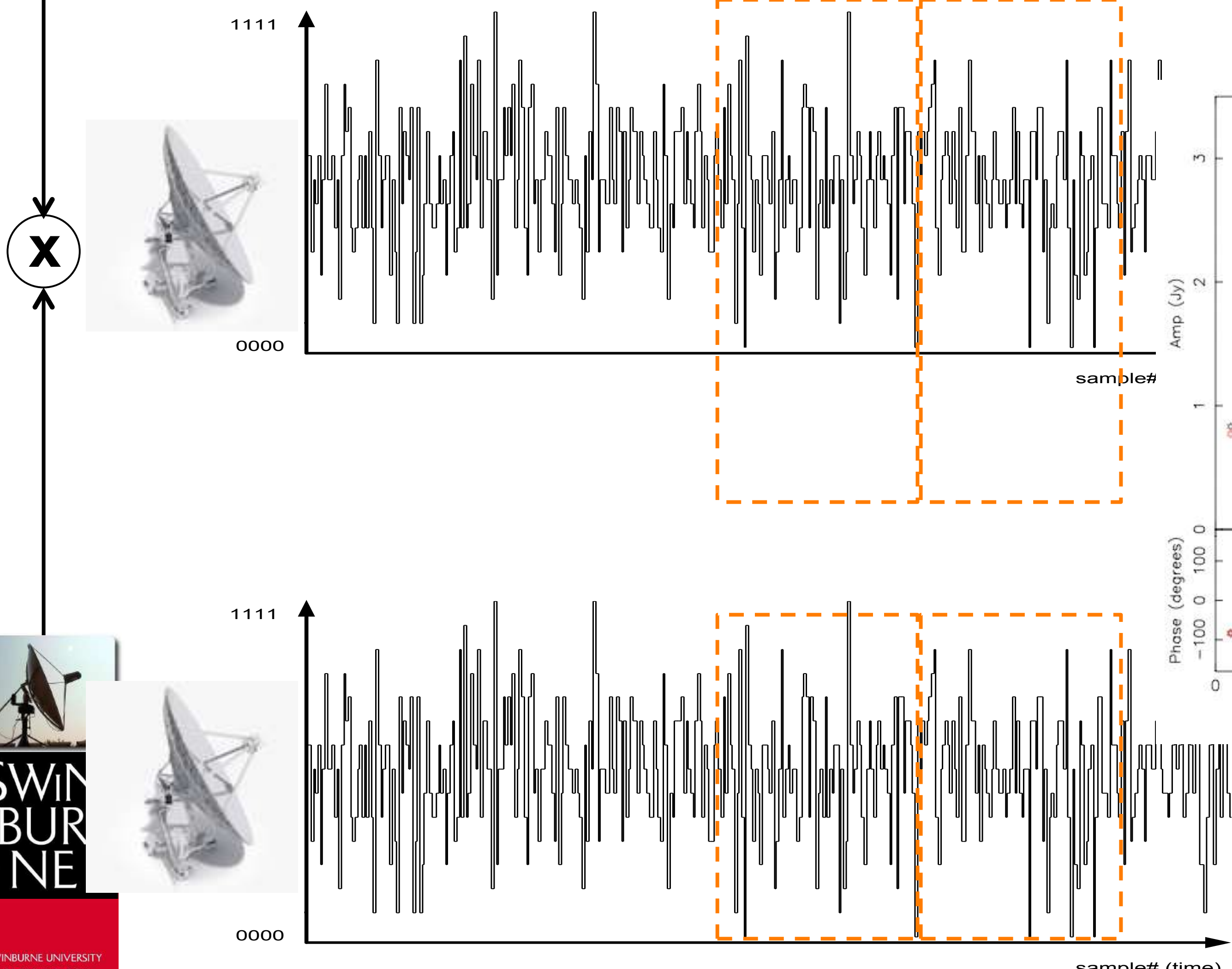
An equivalent “XF” correlator

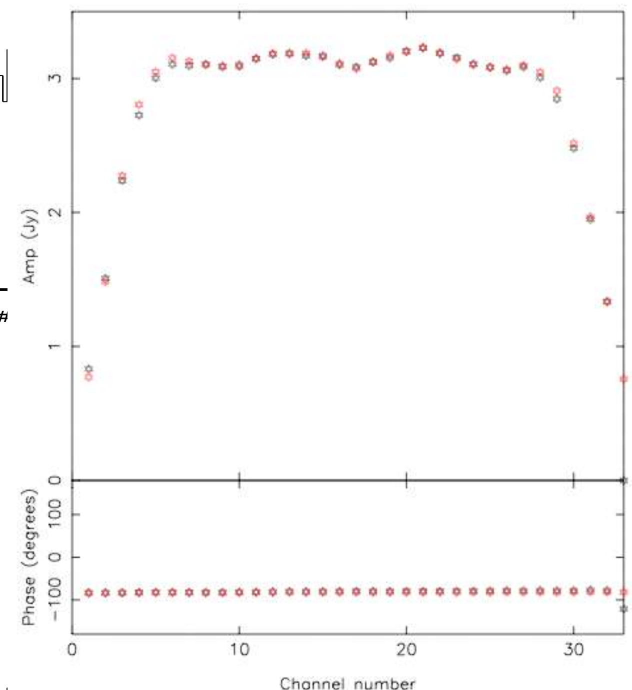


An equivalent “XF” correlator

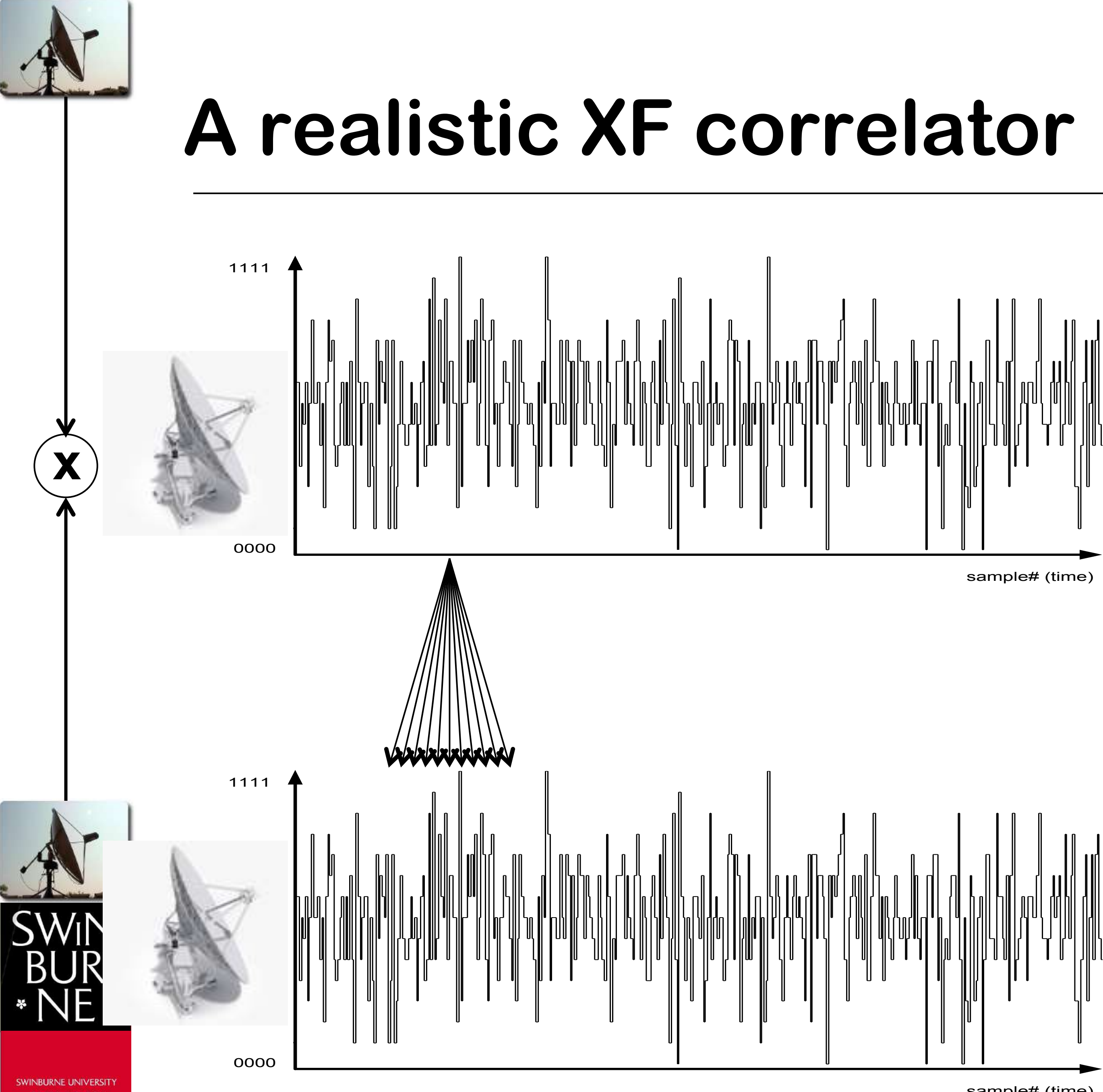


An equivalent “XF” correlator

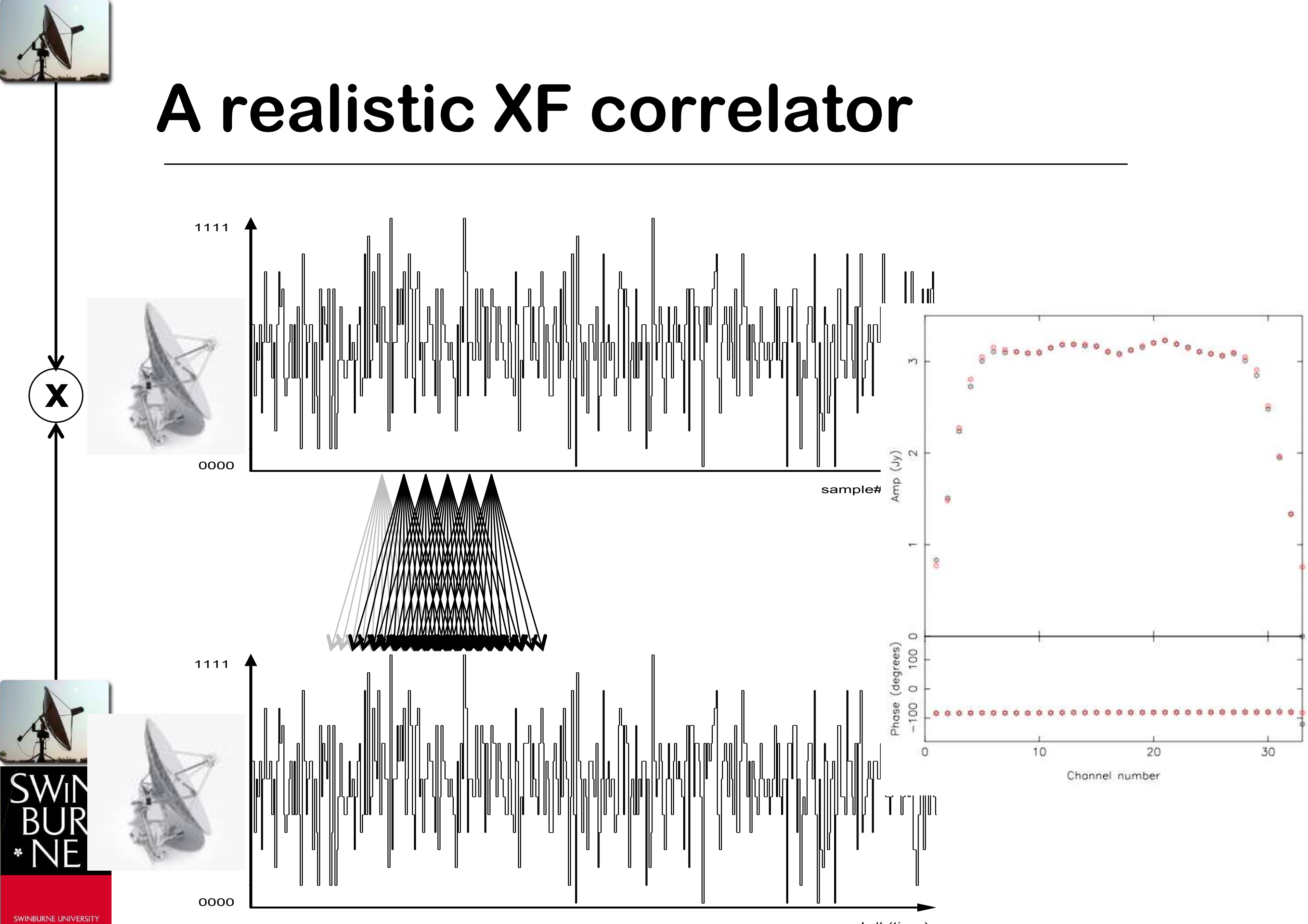




A realistic XF correlator

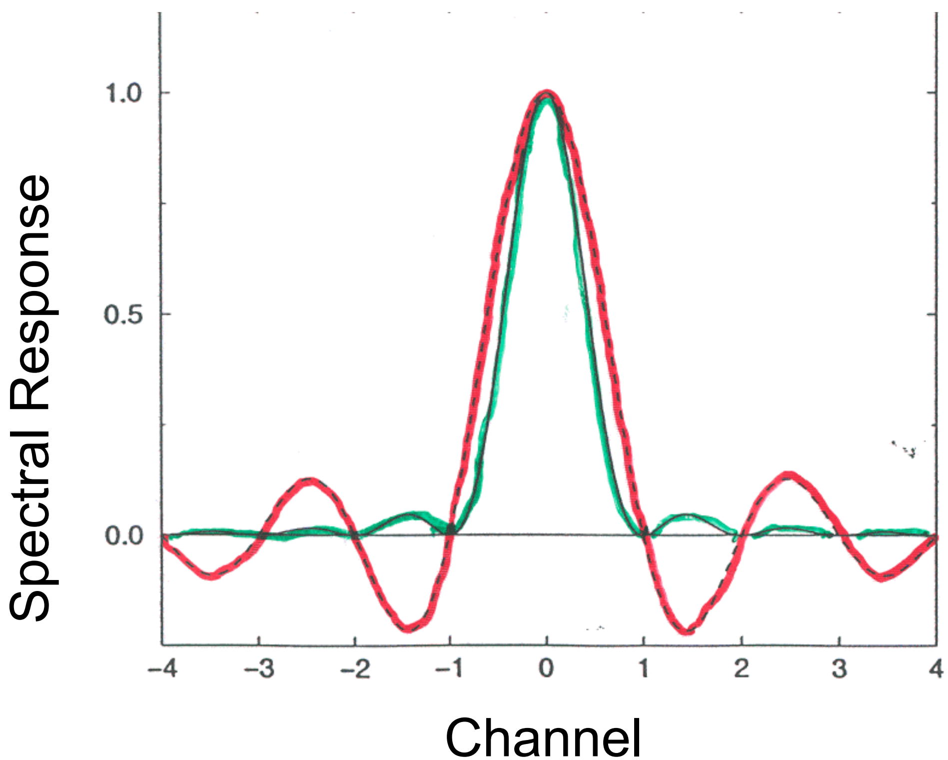


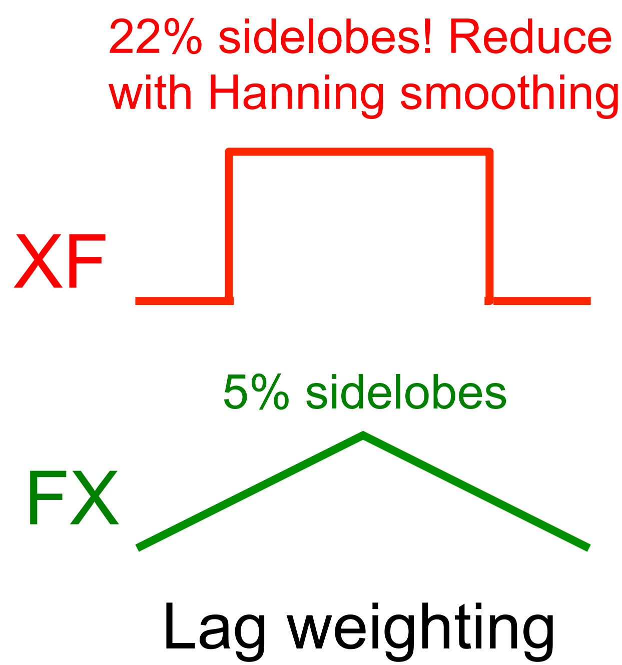
A realistic XF correlator

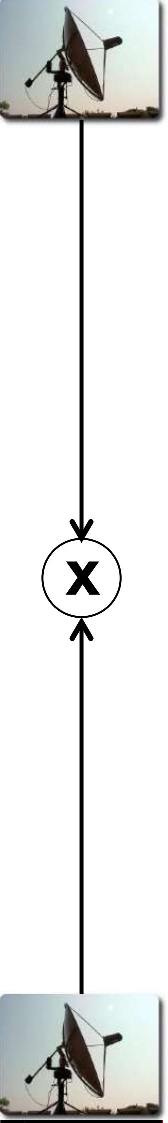


XF vs FX

- Different windowing in time domain gives different spectral response







XF vs FX: which is better?

- Advantages and disadvantages to both
  - FX many fewer operations overall
  - XF can make use of very efficient low-precision integer multipliers up-front (but need special-purpose hardware)
  - FX: access to frequency domain at short timescale allows neat tricks and higher precision correction of delay effects
  - Modern correlators mostly FX-style, but use digital filterbank to channelise rather than FFT (shape channel response, contain RFI)

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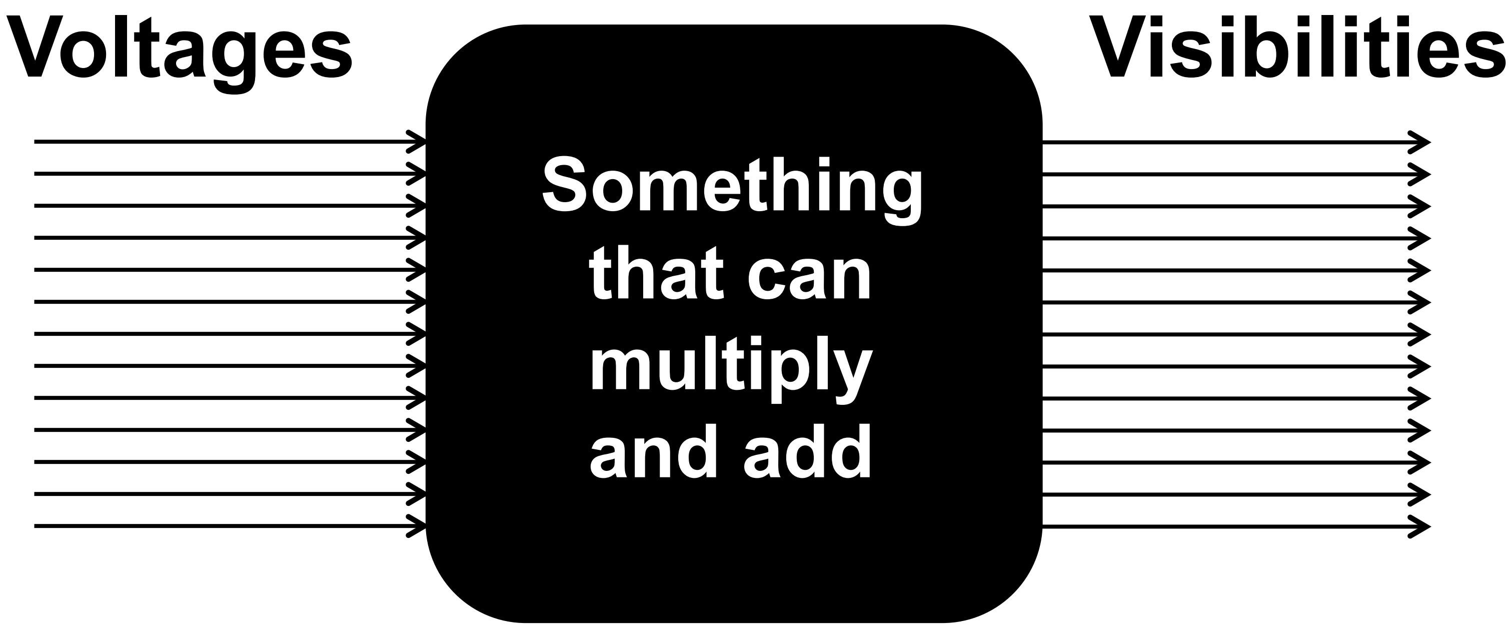
Correlator platforms

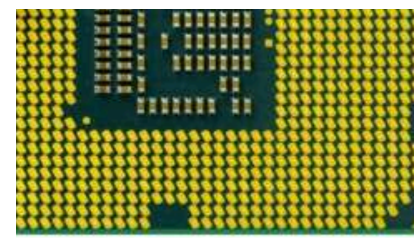


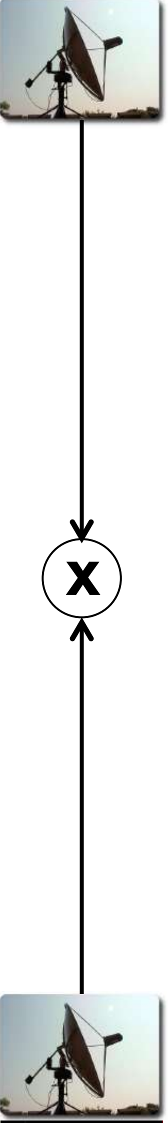
Correlators on CPUs

```
status = vectorFFT_CtoC_cf32(complexunpacked, fftd, pFFTSpecC, fftbuffer);
if(status != vecNoErr)
  csevere << startl << "Error doing the FFT!!!" << endl;

     ...

status = vectorAddProduct_cf32(vis1, vis2, &(scratchspace->threadcrosscorrs[resul
```





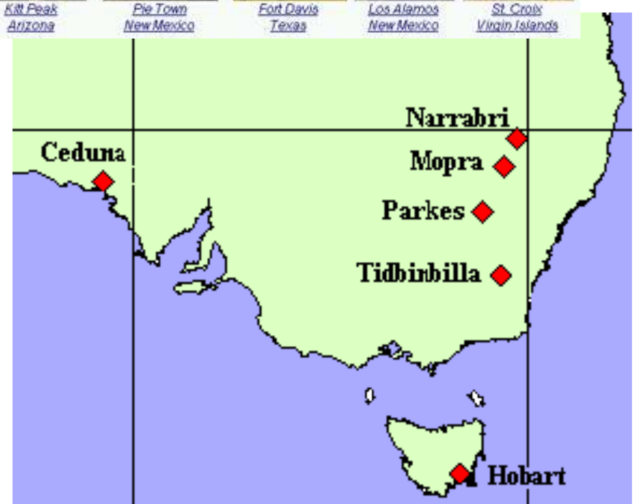
Correlators on CPUs

- Many positive points:
  - Can implement in “normal” code (e.g., C++); maintainable, many skilled coders
  - Development effort transferrable across generations of hardware
  - Incremental development is trivial
  - Natively good at floating point (good for FX), no cost to do high precision
- One major disadvantage:
  - CPUs not optimised for correlation; big system like JVLA would take **many** CPUs.

Correlators on CPUs

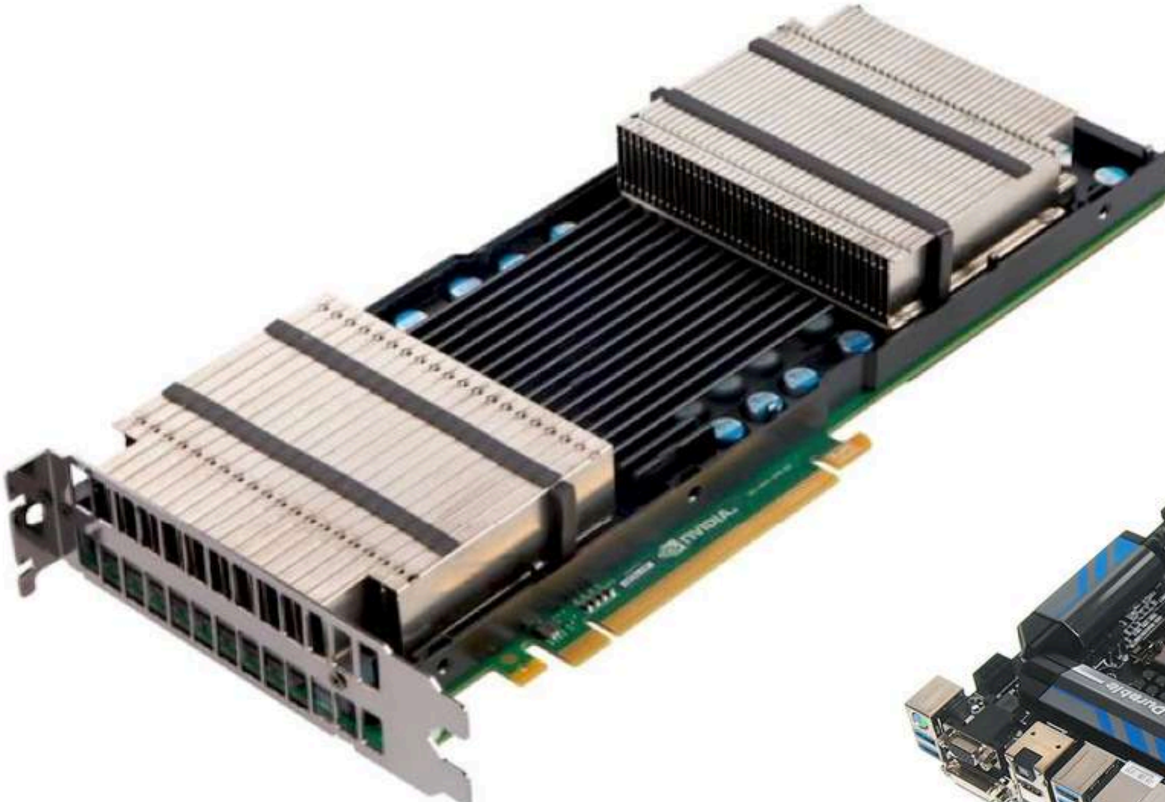
The Very Long Baseline Array, 10 stations

The European VLBI Network, ~20 stations



The Long Baseline Array, Australia, ~6 stations

Correlators on GPUs



Like CPUs, GPUs are mounted
on a standard motherboard

Correlators on GPUs

- Advantages:
  - More powerful and more efficient than CPUs
  - Also good at floating point
- Disadvantages:
  - Writing code is more difficult (GPUs are more specialized, less flexible: need to carefully manage data transfers)
  - Fewer trained GPU programmers available
  - Transfer-ability of code across hardware generations harder (capabilities change faster, need new code to use)

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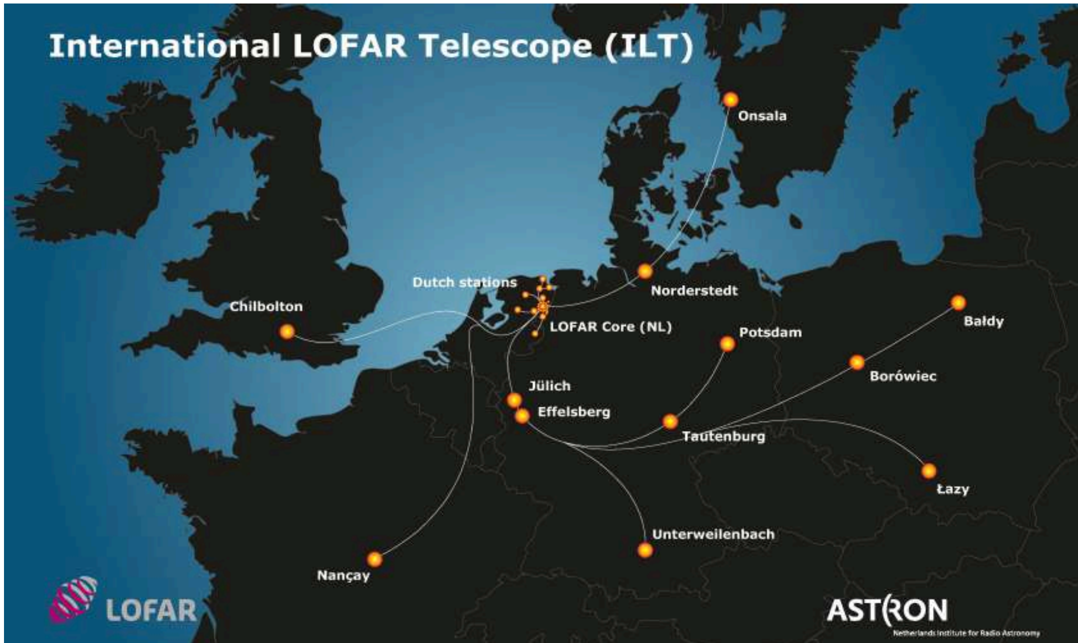
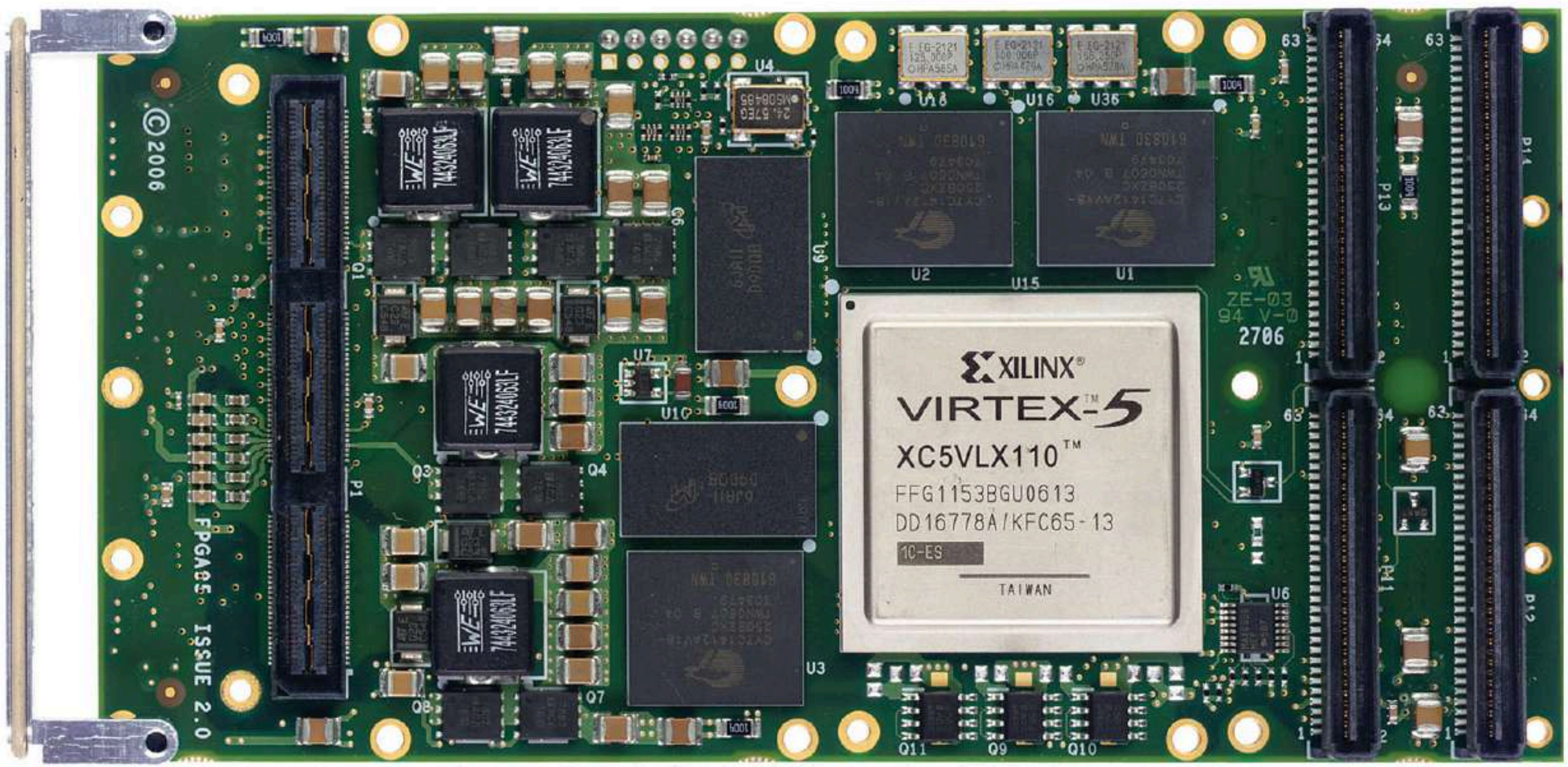
Correlators on GPUs

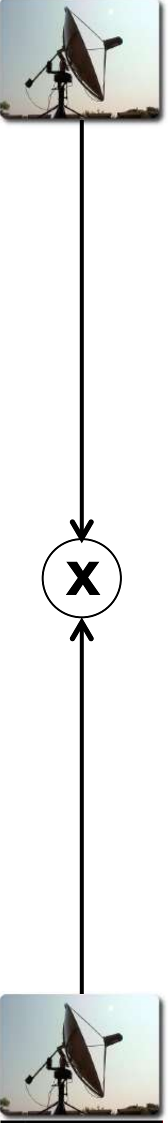


The Low Frequency Array (LOFAR), 73 stations

GMRT, India, 30 stations

Correlators on FPGAs





Correlators on FPGAs

- Advantages:
  - More efficient than CPUs or GPUs, particularly for integer multiplication
- Disadvantages:
  - Programming is harder again (especially debugging), yet fewer trained people
  - Transfer-ability across hardware generations even more limited
  - Synchronous (clocked) system, less robust to perturbations c.f. CPUs/GPUs

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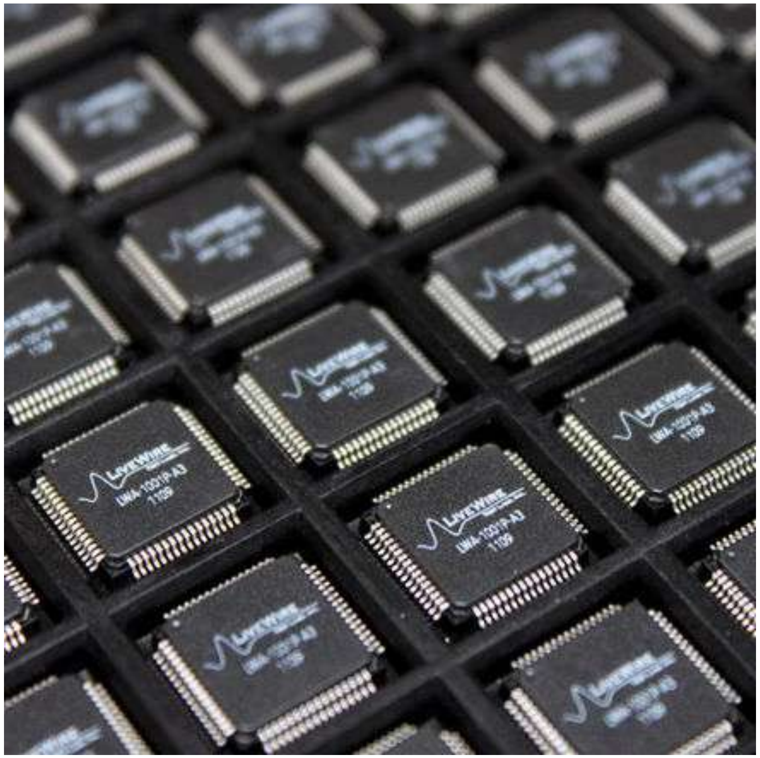
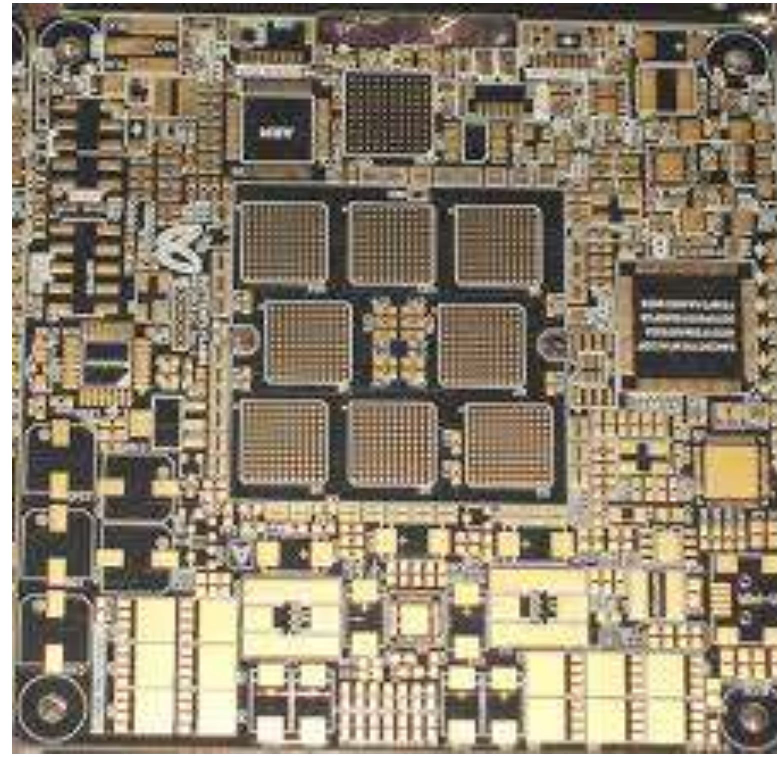
Correlators on FPGAs

“Roach” reconfigurable FPGA board used for correlation

MeerKAT, 64 dishes, under construction

Correlators on ASICs





As with FPGAs, ASICs are mounted on boards

Correlators on ASICs

- Advantages:
  - Highest possible efficiency, low per-unit cost
- Disadvantages:
  - Highest development cost (time and manufacturing setup)
  - Specialized knowledge required
  - Can't be changed / very difficult to upgrade during lifetime

Correlators on ASICs

The Westerbork Synthesis
Radio Telescope, Netherlands

The Very Large Array,
New Mexico

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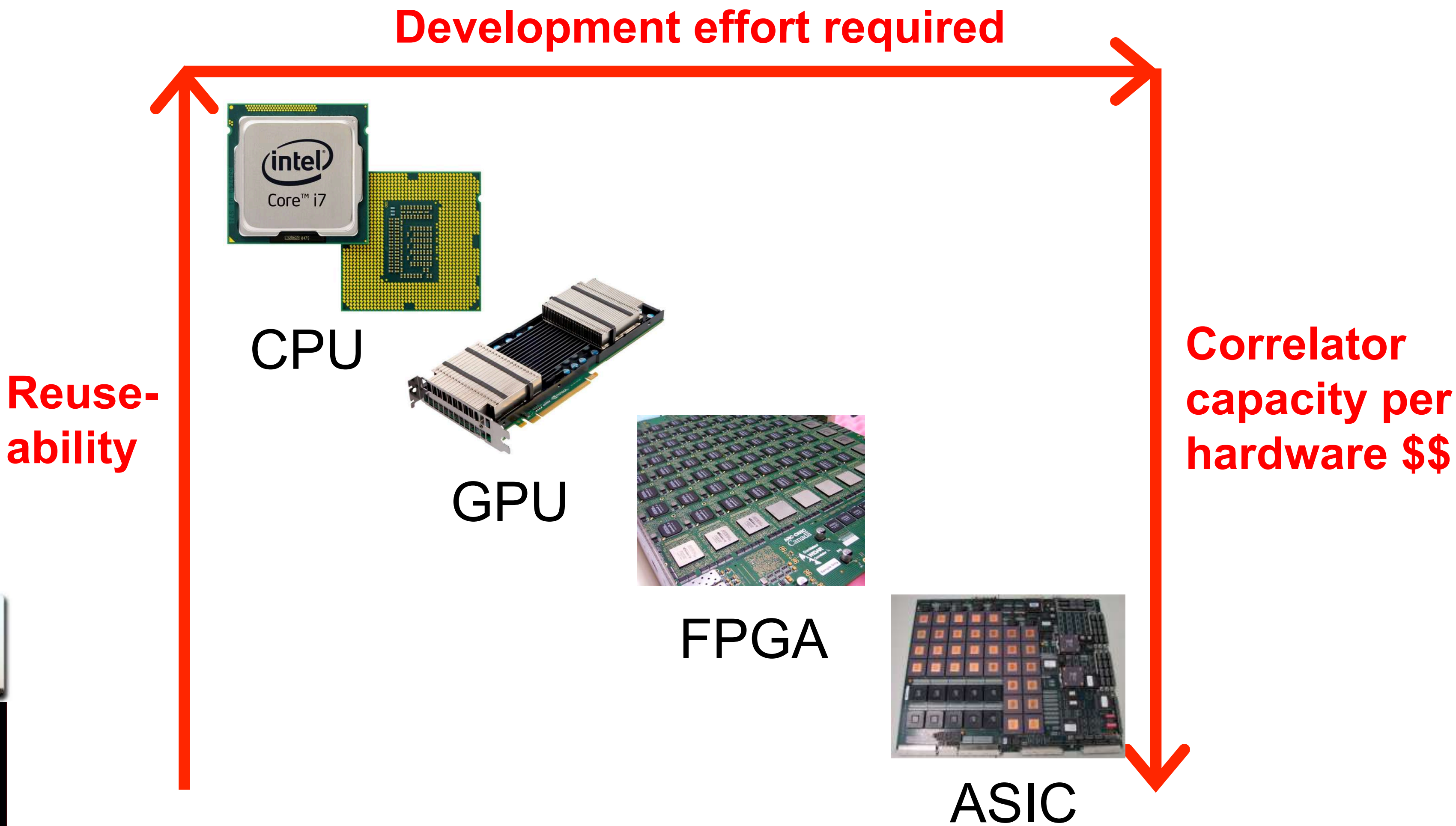
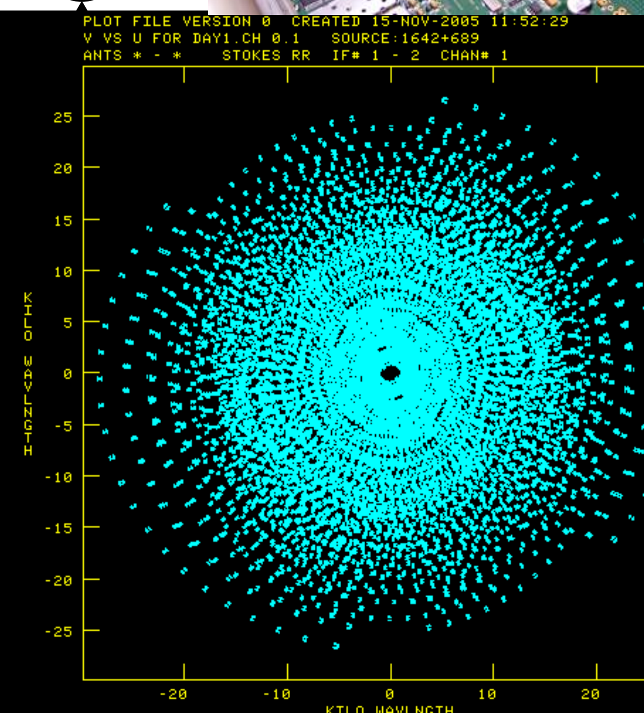
Correlator platform overview



The end



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