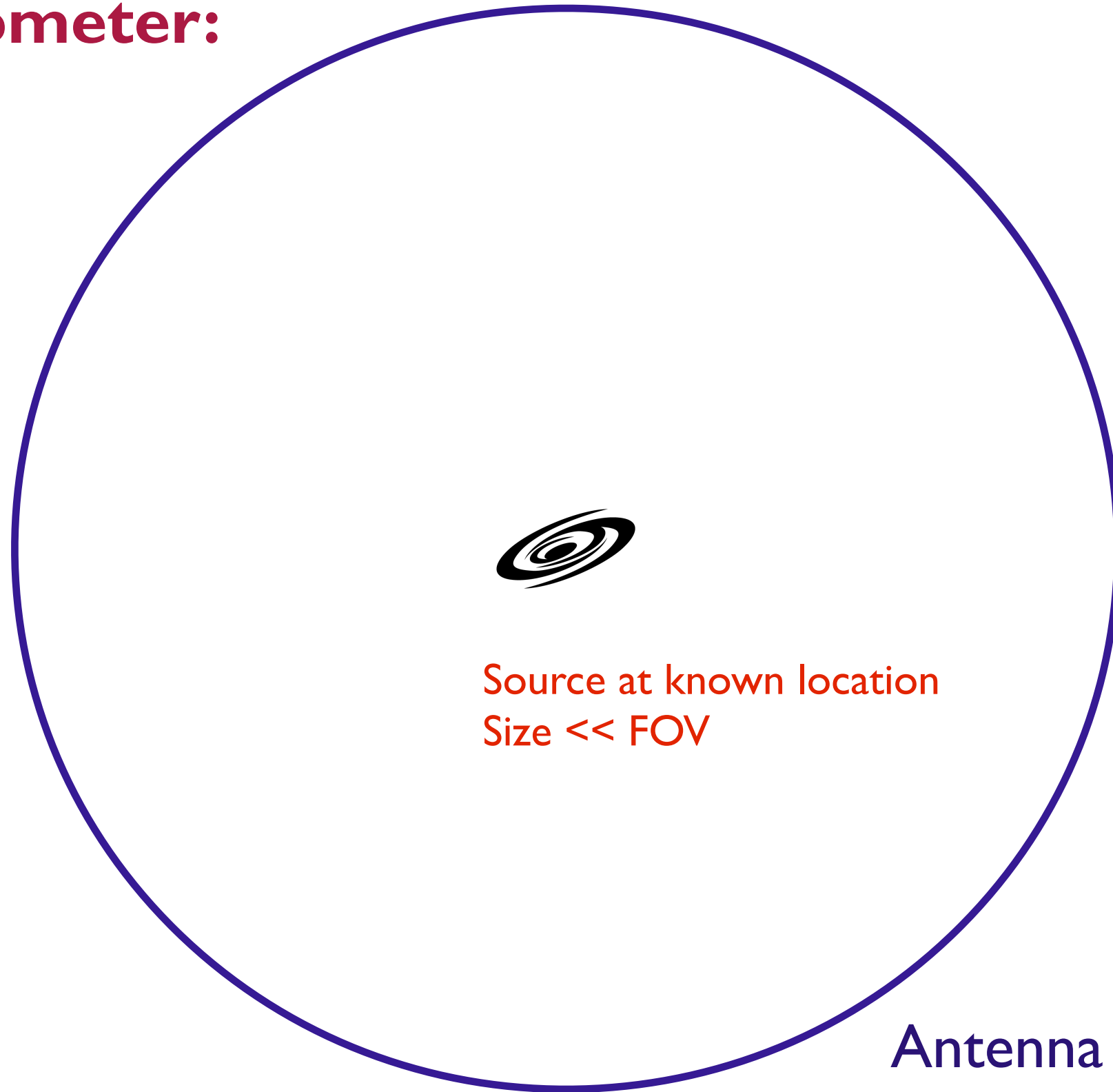


# Mosaicking II: Large-Scale Imaging & Short Spacing Corrections

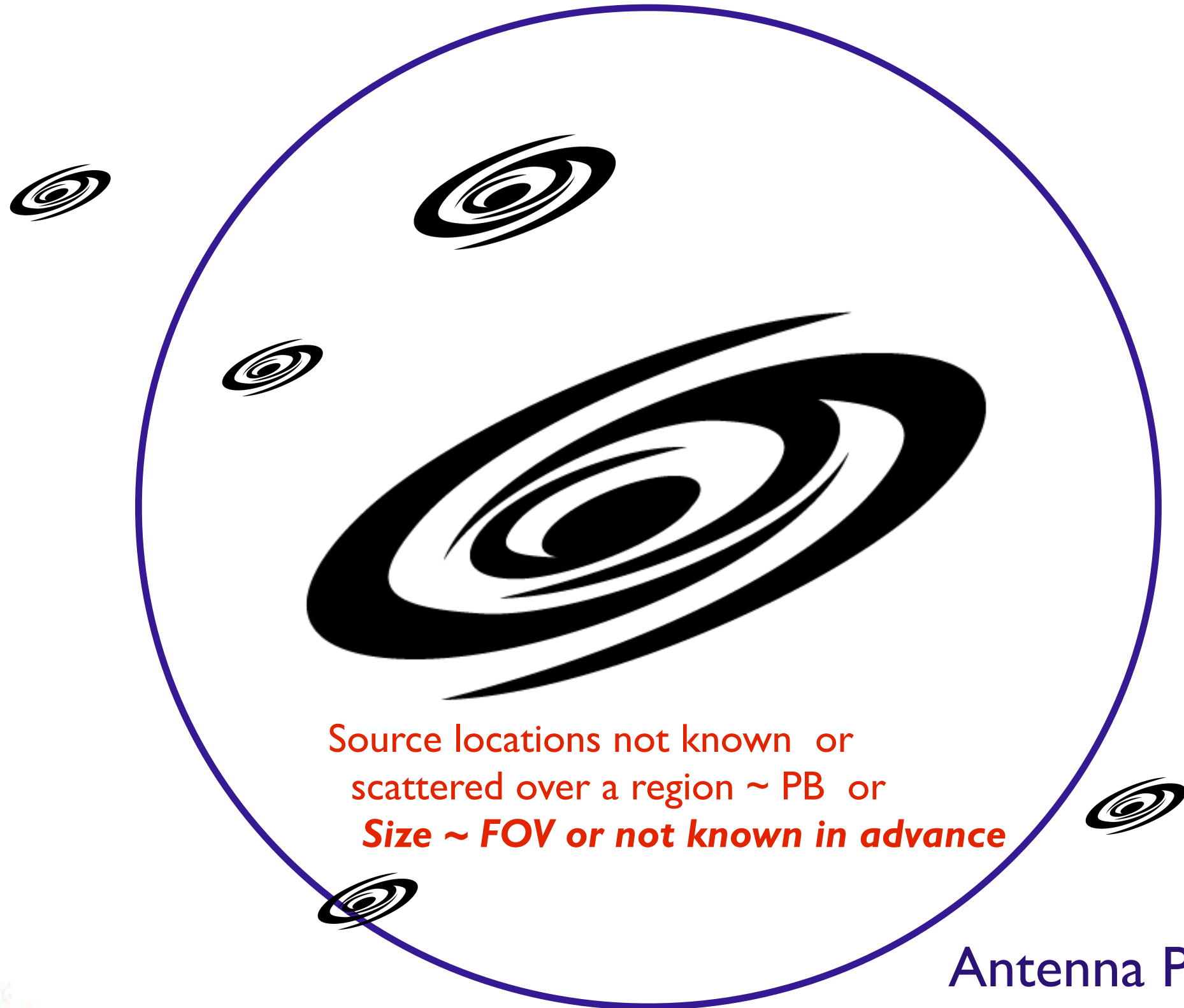


**Brian S. Mason**  
**NRAO**

# The simplest observing scenario for an interferometer:



But that's often not the case...



**But that's often not the case...**

**You need to mosaic!**

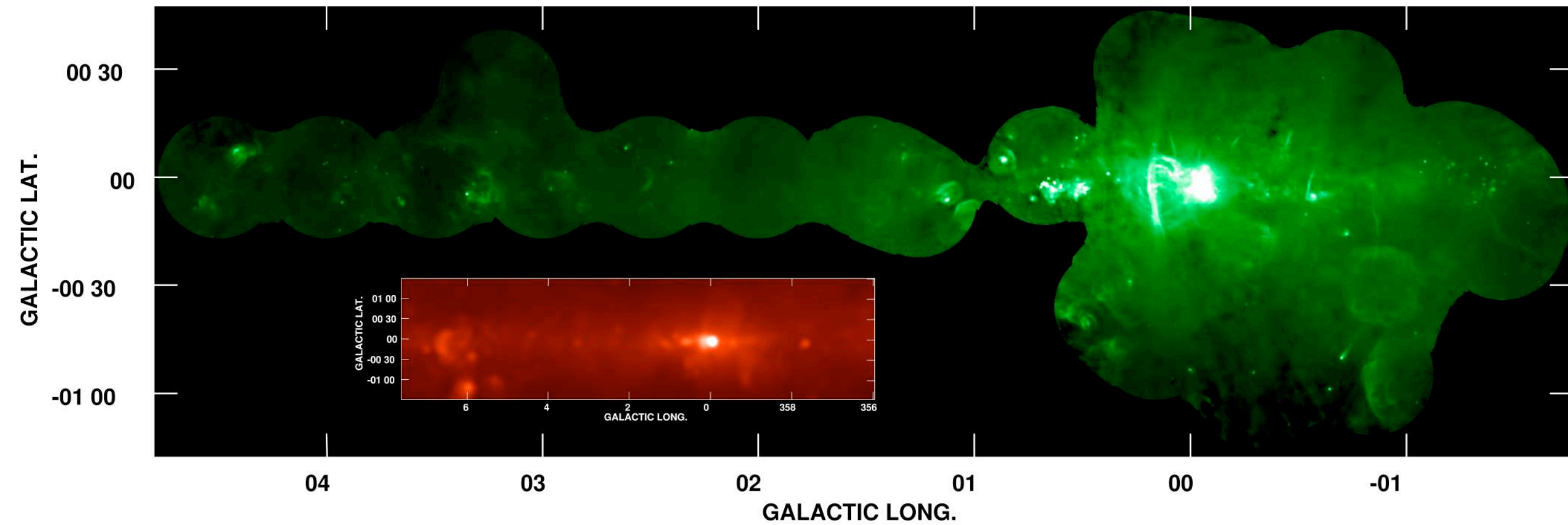
**Recovers flux on angular scales comparable to the primary beam**

**For larger scales you may need to add single dish data to your map.**

Source locations not known or  
scattered over a region  $\sim$  PB or  
**Size  $\sim$  FOV or not known in advance**

Antenna Primary Beam

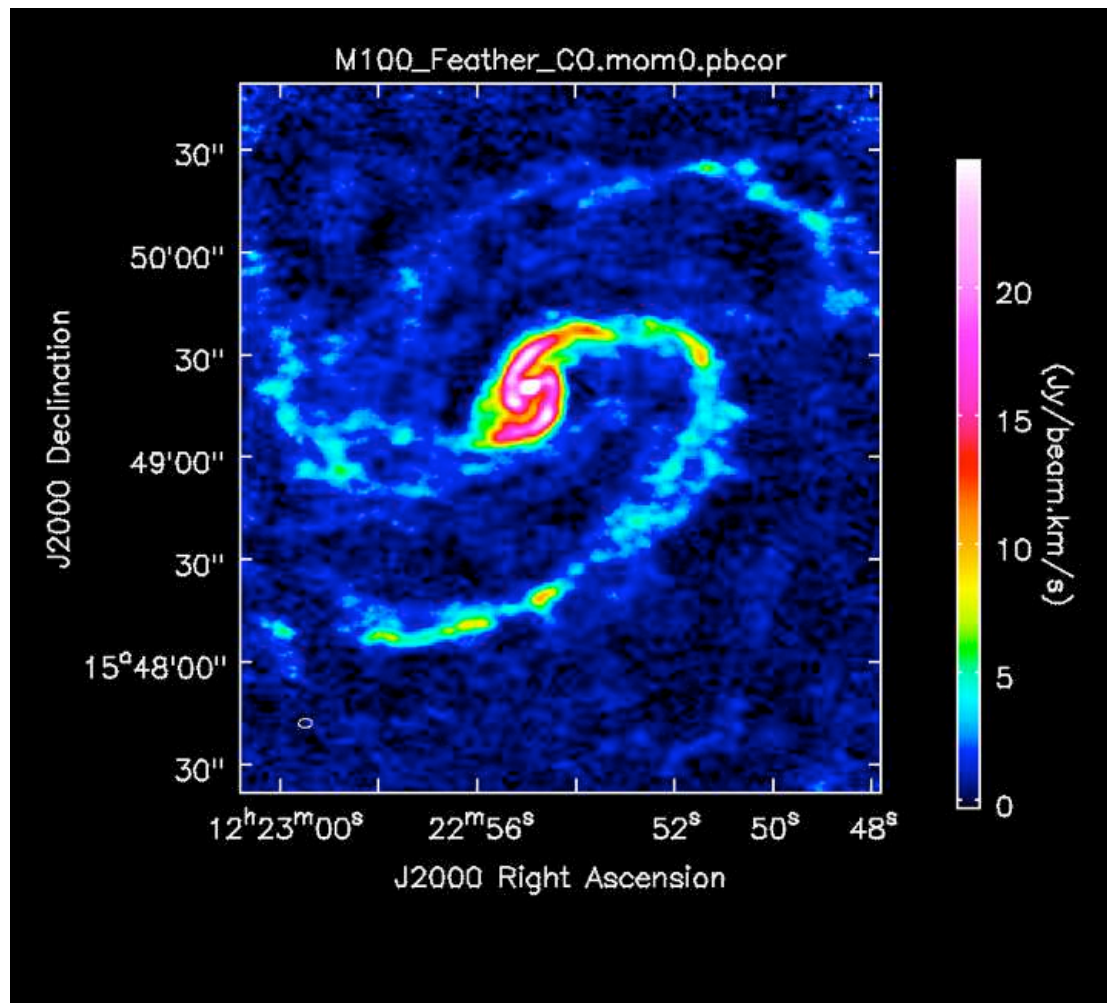




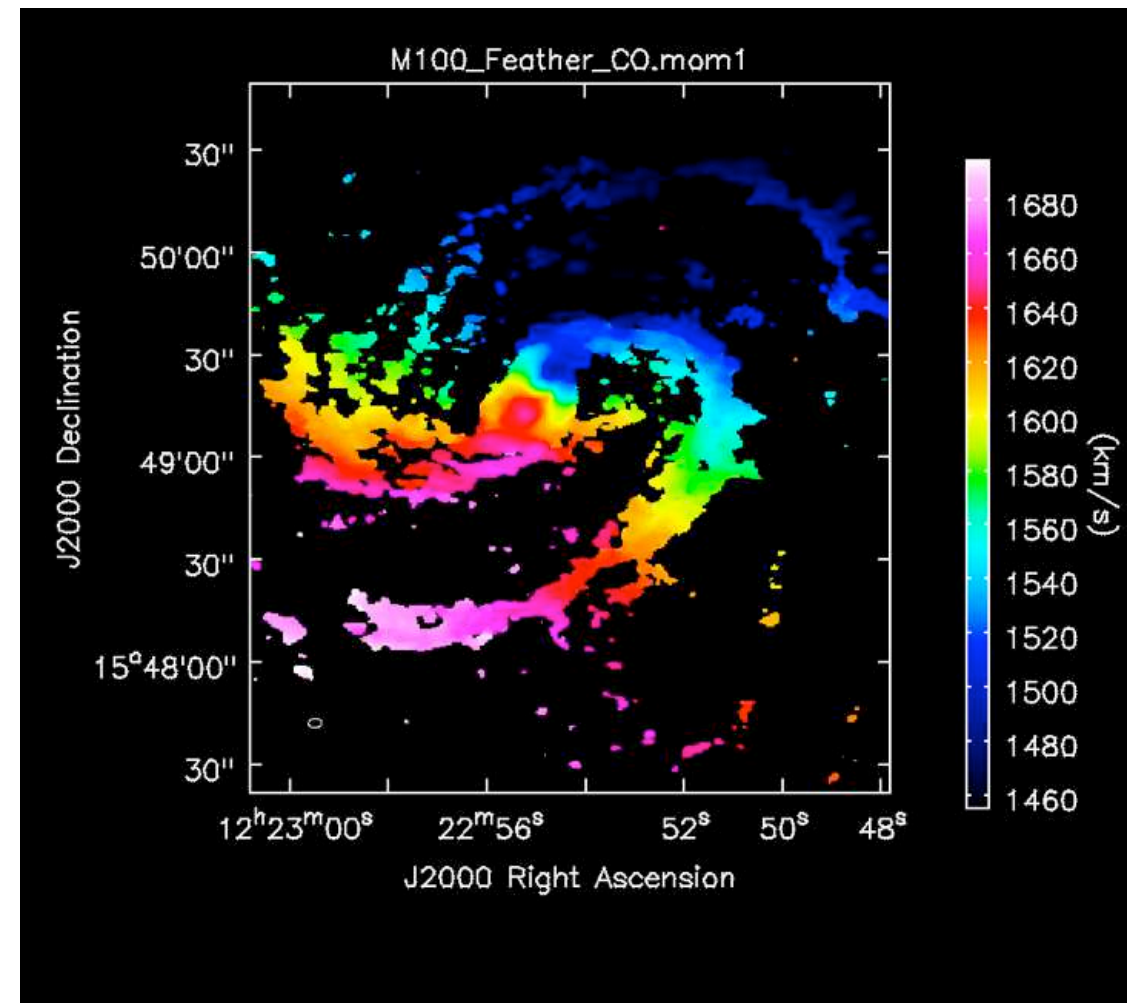
*20cm VLA Mosaic+GBT Single Dish (green) (red inset :GBT only)*

*Law, Yusef-Zadeh, & Cotton (2008)*

# ALMA Science Verification: M100

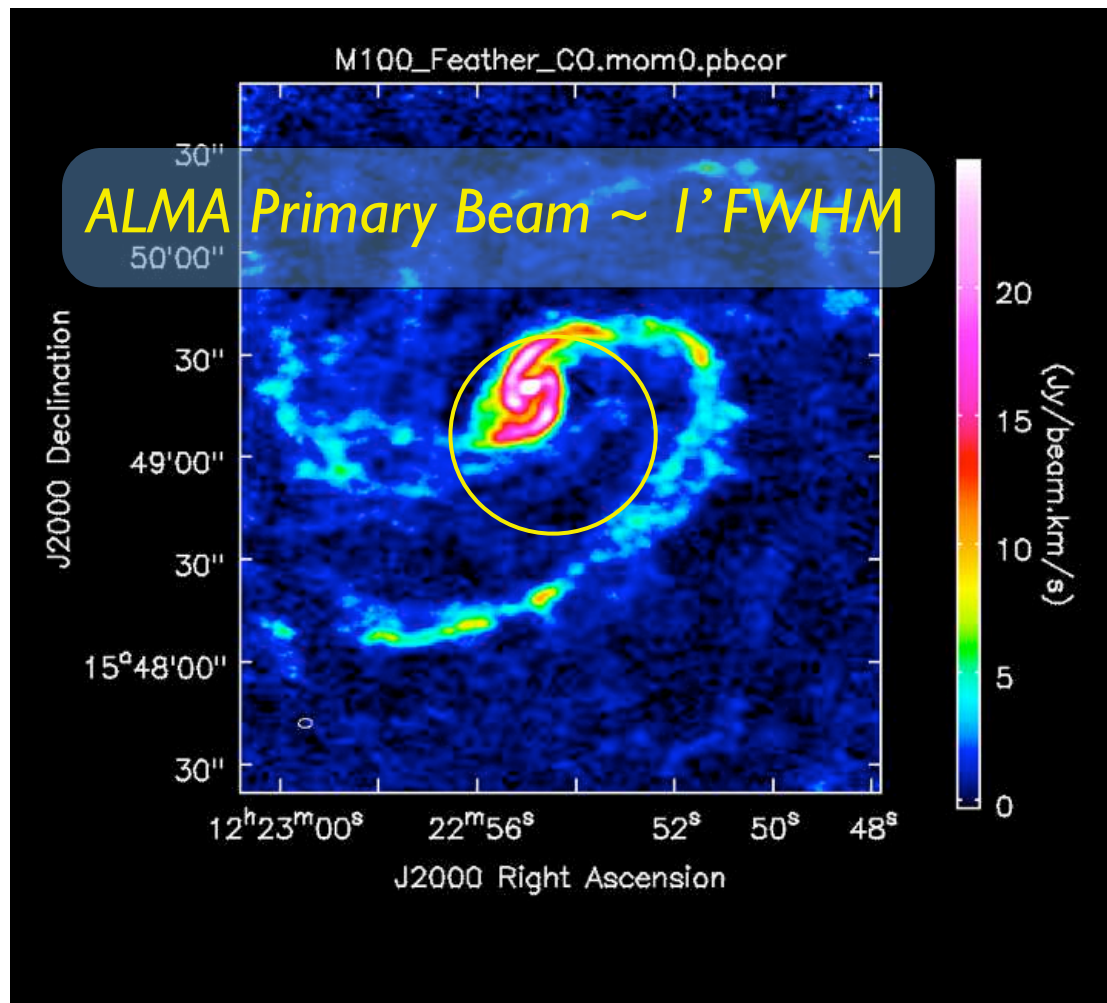


*Integrated CO line intensity  
Band 3 (115 GHz, ~2.6mm)*

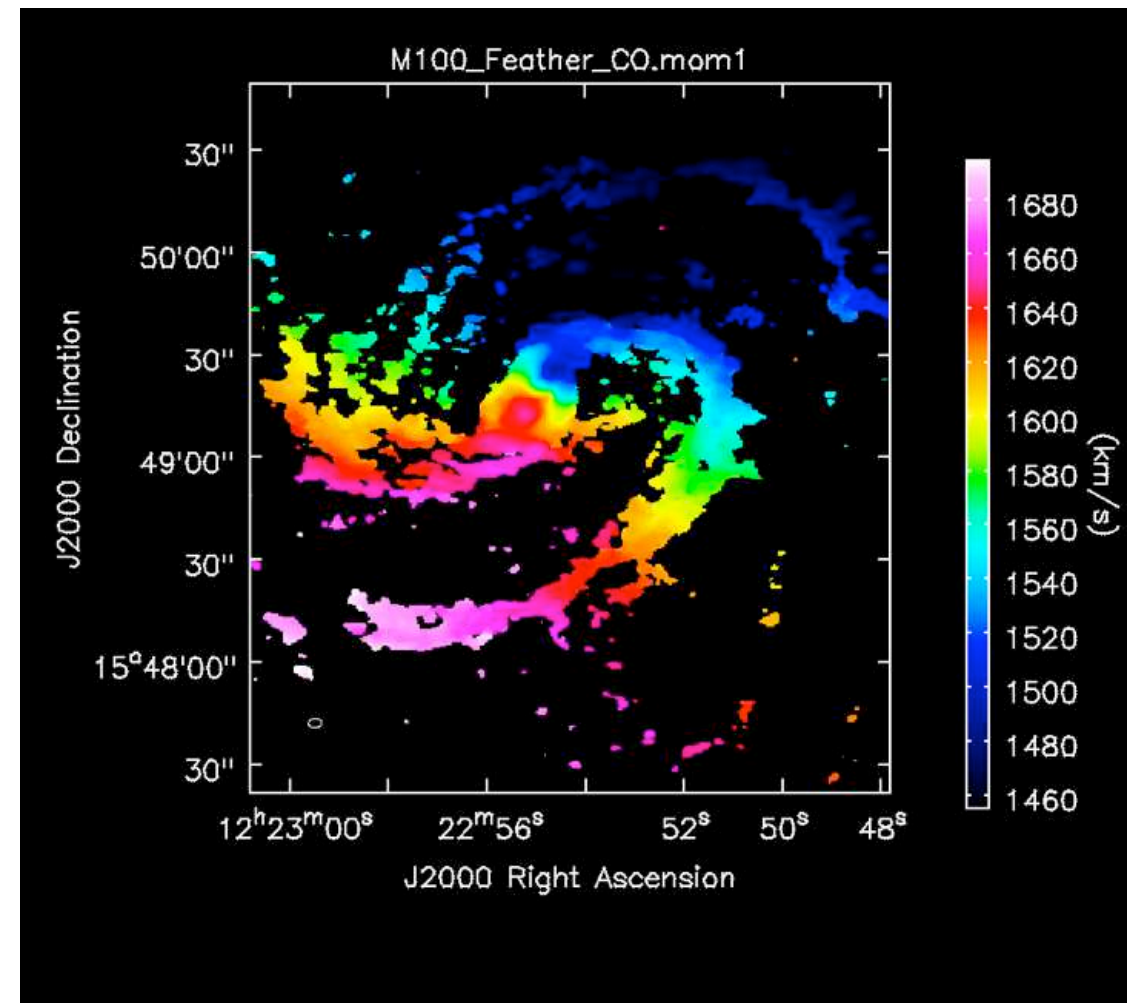


*1st moment map (velocity field of CO line)*

# ALMA Science Verification: M100



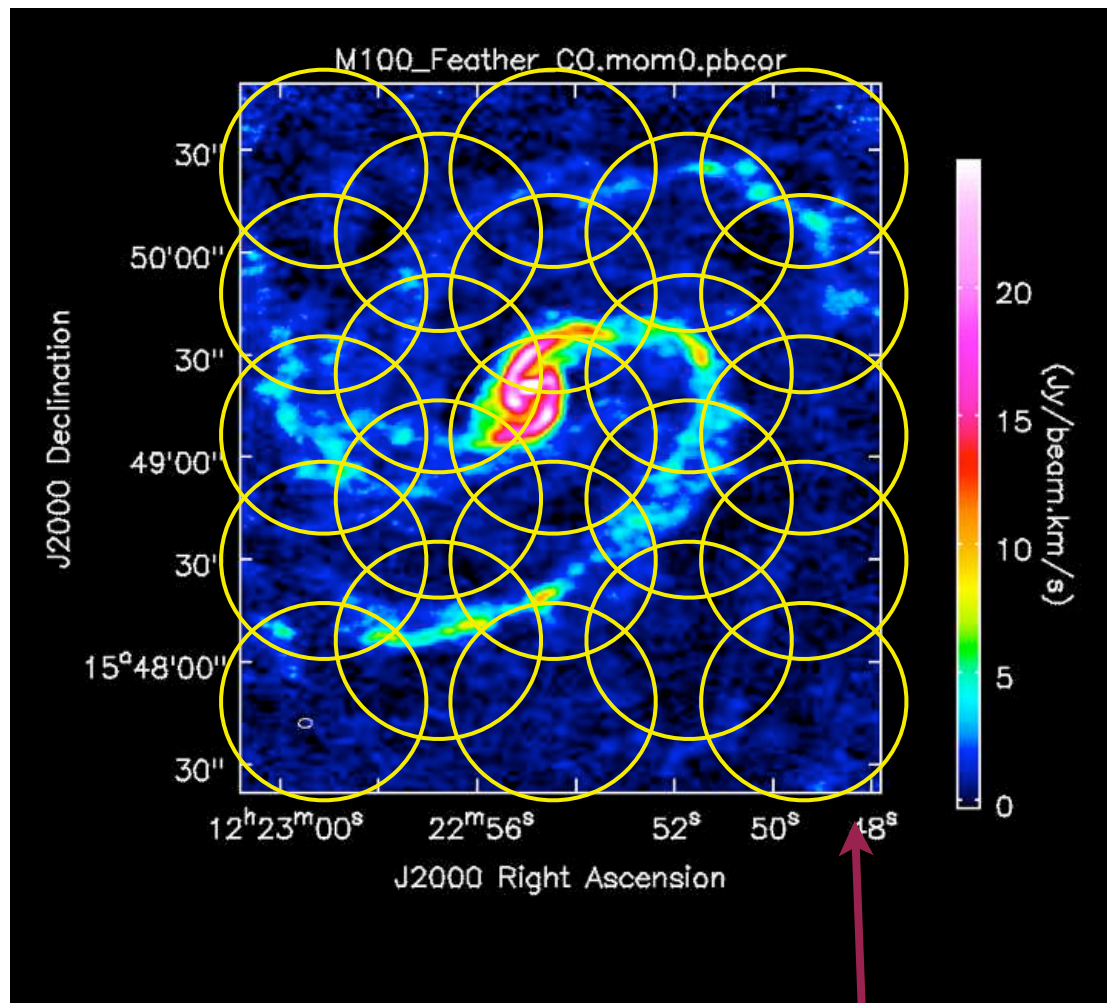
*Integrated CO line intensity  
Band 3 (115 GHz,  $\sim 2.6$  mm)*



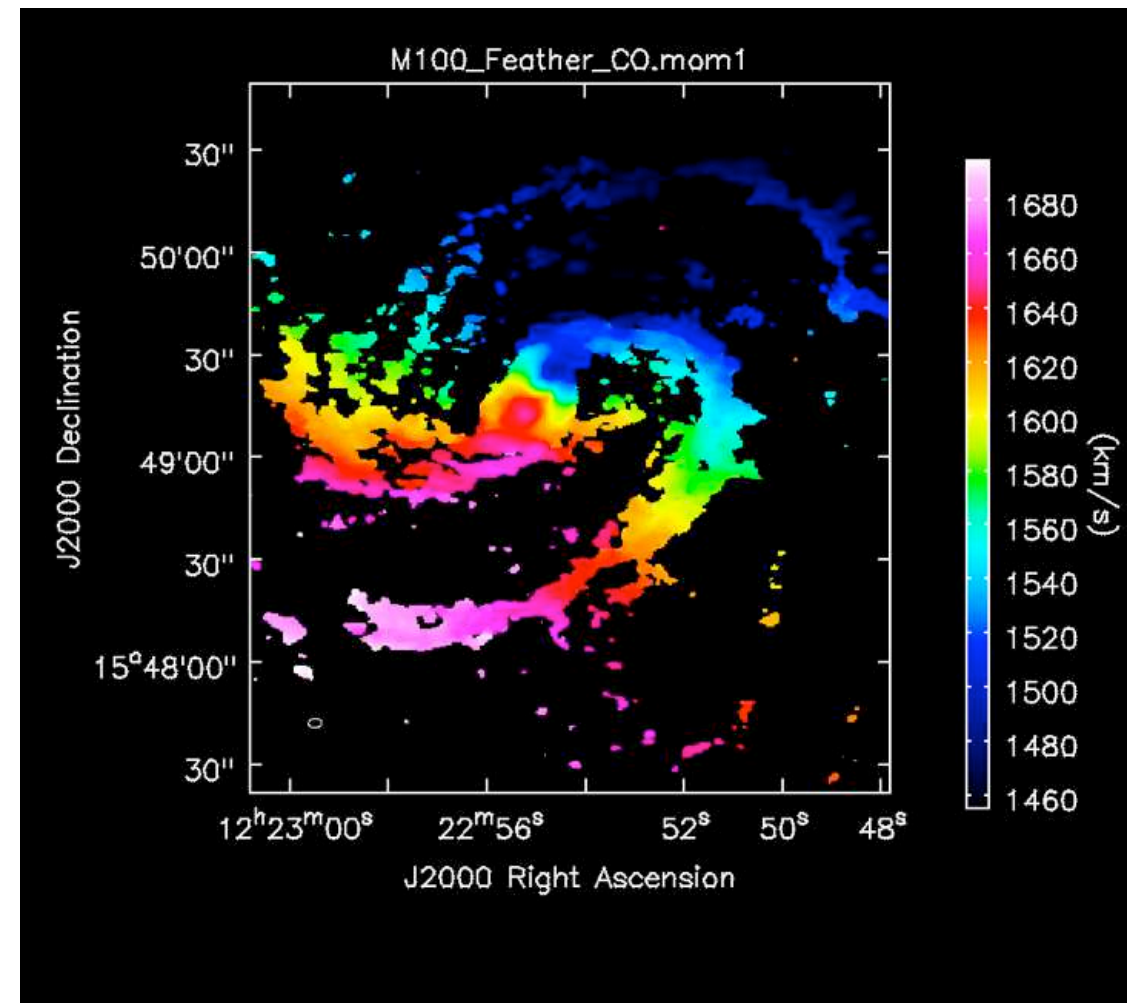
*1st moment map (velocity field of CO line)*



# ALMA Science Verification: M100



*Integrated CO line intensity  
Band 3 (115 GHz, ~2.6mm)*



*1st moment map (velocity field of CO line)*

**At short wavelengths, mosaicking is very commonly required**



# Limiting Angular Scales for an Interferometer

$$\theta_{PB} = (1.03 \rightarrow 1.2) \times \frac{\lambda}{D}$$

~ the diameter of the area imaged by one pointing of the interferometer (instantaneous field of view)

$$\theta_{LAS} = \frac{1}{2} \left( \frac{\lambda}{b_{min}} \right)$$

The “Spatial Period” of the largest angular scale Fourier component of the sky brightness measured by the interferometer

↑  
In practice, you only measure things \*half\* that big (say) very well.  
(even that might be optimistic)

*Exercise: you can quantify the LAS yourself using the “Gaussian Flux Loss” rule of thumb (D.Wilner lecture on deconvolution)*

# Limiting Angular Scales for an Interferometer

$$\theta_{PB} = (1.03 \rightarrow 1.2) \times \frac{\lambda}{D}$$

VLA: L-band (20cm) = 30'      Q-band (7mm) = 1'

ALMA(12m): Band3 (3mm) = 1'    Band9 (0.44mm) = 9''

$$\theta_{LAS} = \frac{1}{2} \frac{\lambda}{b_{min}}$$

VLA: L-band (20cm), D-array = 16'      Q-band (7mm), A-array = 1.2''  
                *30m*                                 *537m*

**ALMA(12m): Band3 (3mm), compact cfg = 20''      Band9 (0.44mm), ext.cfg= 1''**

*15m                                  43m*



*(based on currently advertised capabilities)*

# Limiting Angular Scales for an Interferometer

**If your region of interest is larger than this, you need to mosaic together many interferometer pointings.**

$$\theta_{PB} = (1.03 \rightarrow 1.2) \times \frac{\Delta}{D}$$

VLA: L-band (20cm) = 30'      Q-band (7mm) = 1'

ALMA(12m): Band3 (3mm) = 1'    Band9 (0.44mm) = 9''

$$\theta_{LAS} = \frac{1}{2} \frac{\lambda}{b_{min}} \quad \leftarrow$$

**If the structures you are interested in are larger than this, you need to mosaic and/or get data from a more compact configuration of the interferometer or single dish.**

VLA: L-band (20cm), D-array = 16'      Q-band (7mm), A-array = 1.2''  
                        *30m*                                 *537m*

**ALMA(12m): Band3 (3mm), compact cfg = 20''      Band9 (0.44mm), ext.cfg= 1''**

*15m                                  43m*



*(based on currently advertised capabilities)*



# Limiting Angular Scales for an Interferometer

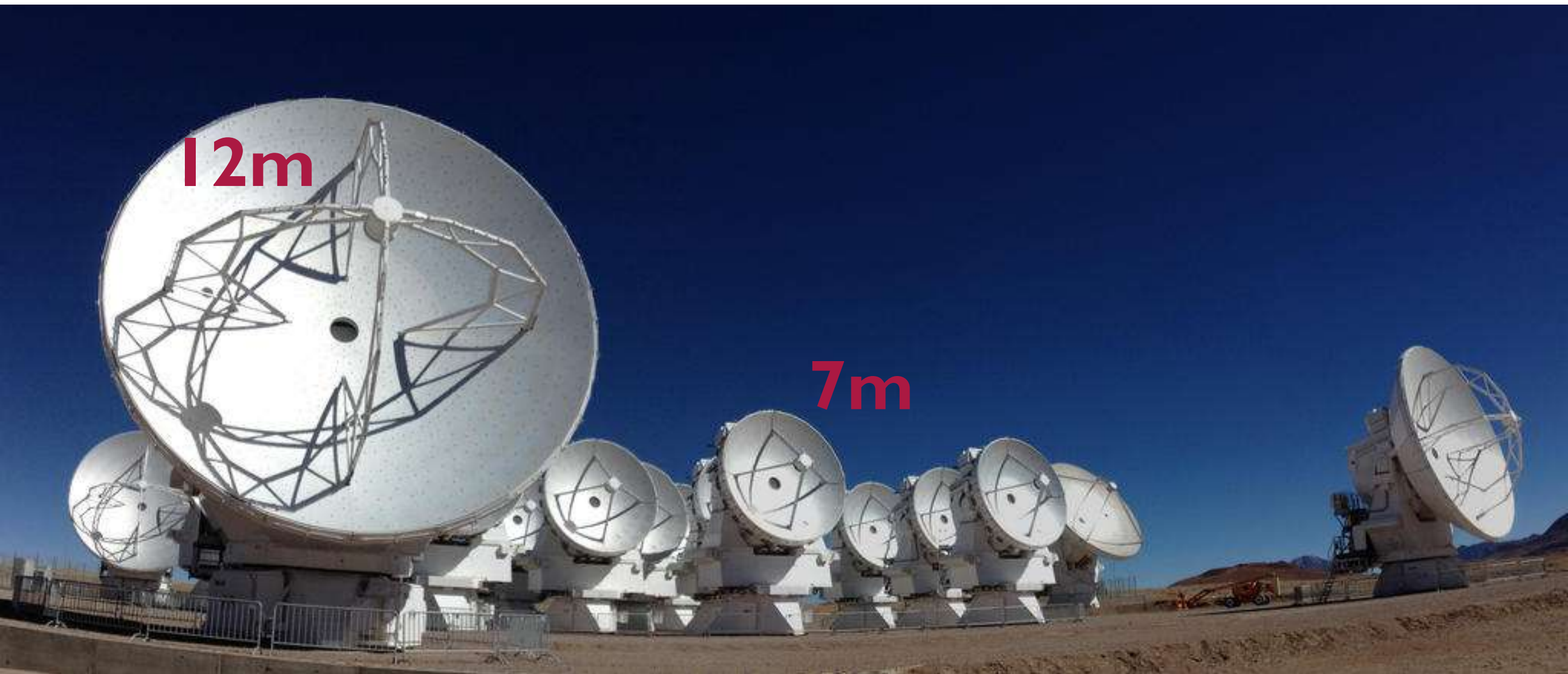
$$\theta_{PB} = (1.03 \rightarrow 1.2) \times \frac{\lambda}{D}$$

There is a limit to how compact a given interferometer can get

$$\theta_{LAS} = \frac{1}{2} \frac{\lambda}{b_{min}} \leq \frac{1}{2} \frac{\lambda}{D}$$

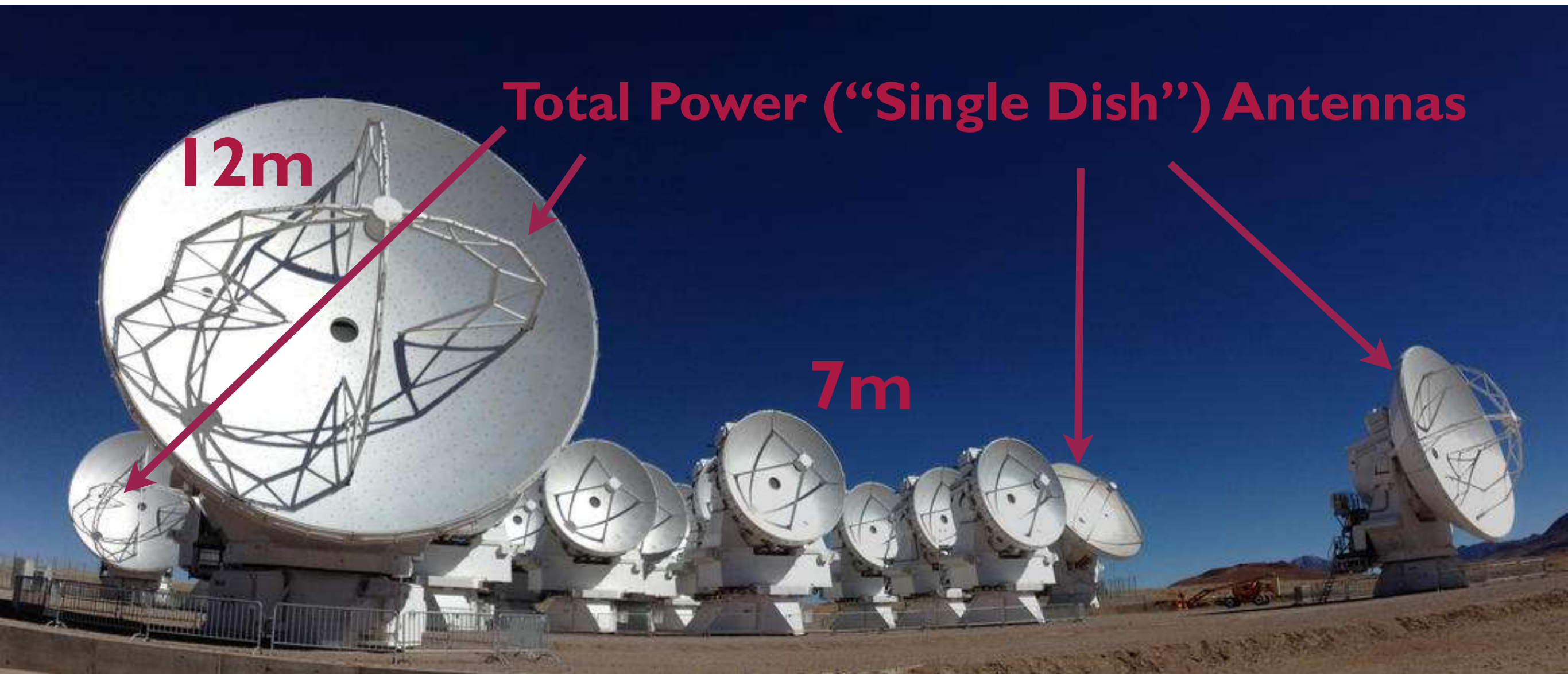
For angular scales much bigger than that you need smaller dishes, or data from a single dish telescope.

# The ALMA Compact Array (ACA)





# The ALMA Compact Array (ACA)



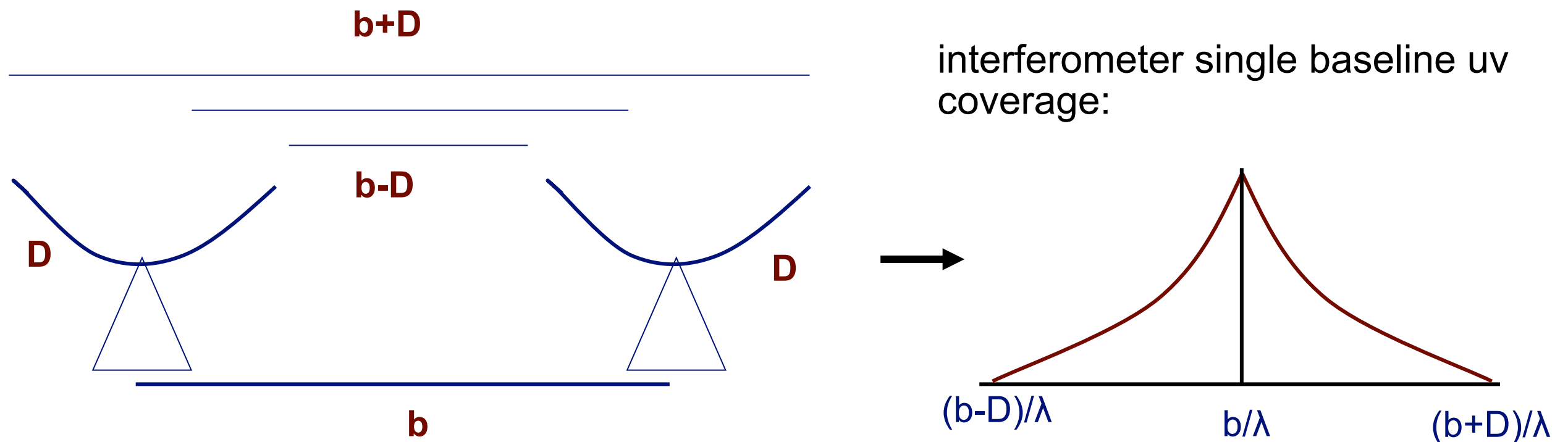


# CARMA



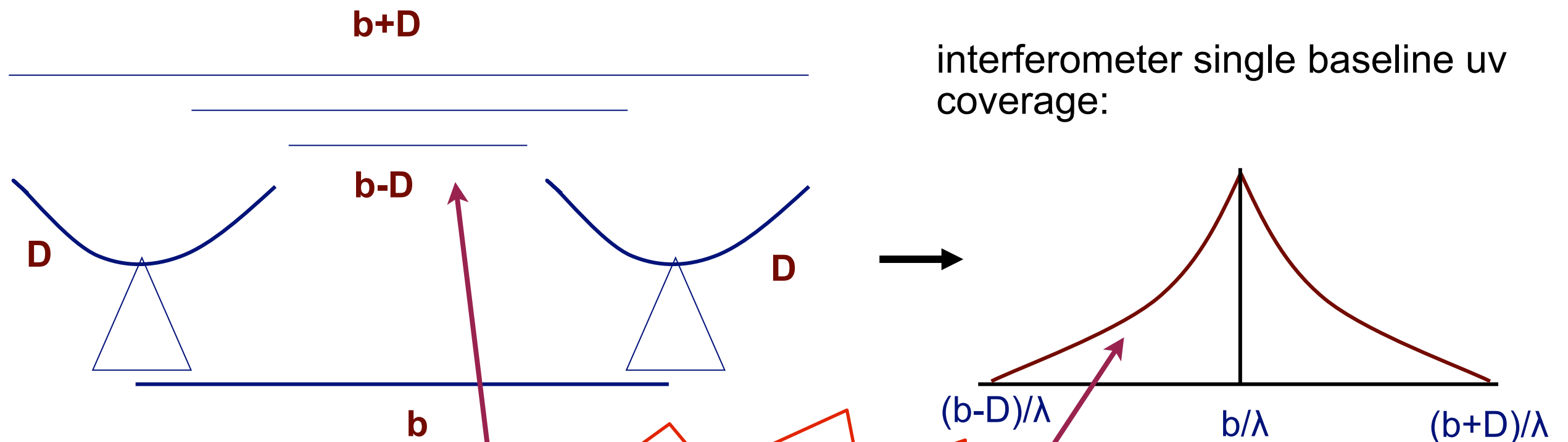
# Theory of Mosaicking: Ekers & Rots Theorem

An interferometer doesn't just measure angular scales  $\theta = \lambda/b$  it actually measures  $\lambda/(b-D) < \theta < \lambda/(b+D)$



# Theory of Mosaicking: Ekers & Rots Theorem

An interferometer doesn't just measure angular scales  $\theta = \lambda/b$  it actually measures  $\lambda/(b-D) < \theta < \lambda/(b+D)$



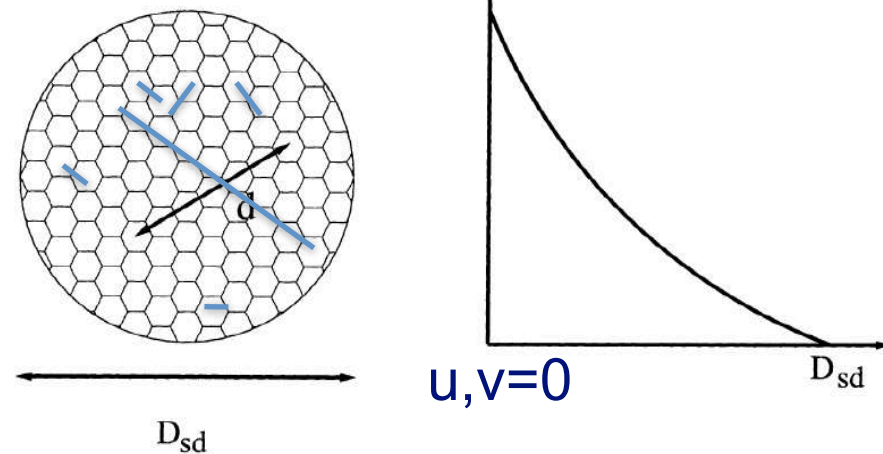
Information on scales larger than the shortest baseline



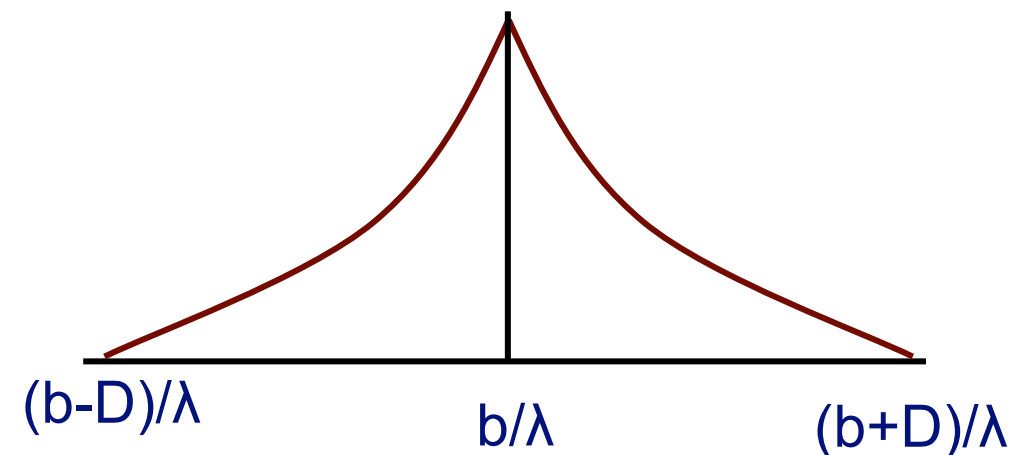
# Theory of Mosaicking: Ekers & Rots Theorem

Similarly: a single dish measures a range of baselines from spatial frequencies of \*zero\* (the mean level of the sky) up to (the dish diameter)/ $\lambda$

single dish “uv coverage”:



interferometer single baseline uv coverage:



# Theory of Mosaicking: Ekers & Rots Theorem

“An interferometer measures  $\lambda/(b-D) < \theta < \lambda/(b+D)$ ”

Motivation/Derivation:

$$\begin{aligned} V(u, v) &= \int \int d\ell \, dm \, A(\ell, m) I(\ell, m) e^{-2\pi i(u\ell + vm)} &= FT[A(\ell, m) I(\ell, m)] \\ & &= FT[A(\ell, m)] \otimes FT[I(\ell, m)] \end{aligned}$$

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$$= FT[A(\ell, m)] \otimes FT[I(\ell, m)]$$

$$A(\ell, m) = \left| \int \int_{\text{aperture}} du \, dv \, E(u, v) e^{-2\pi i(u\ell + vm)} \right|^2$$

$$= FT[E(u, v)] FT[E(u, v)]$$

$$= FT[E(u, v) \otimes E(u, v)]$$

$$\xleftrightarrow{FT} FT[A(\ell, m)] = E(u, v) \otimes E(u, v)$$



# Theory of Mosaicking: Ekers & Rots Theorem

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 V(u, v) &= \int \int d\ell \, dm \, A(\ell, m) I(\ell, m) e^{-2\pi i(u\ell + vm)} = FT[A(\ell, m) I(\ell, m)] \\
 &= FT[A(\ell, m)] \otimes FT[I(\ell, m)] \\
 &= [E(u, v) \otimes E(u, v)] \otimes FT[I(\ell, m)]
 \end{aligned}$$

$$\begin{aligned}
 A(\ell, m) &= \left| \int \int_{\text{aperture}} du \, dv \, E(u, v) e^{-2\pi i(u\ell + vm)} \right|^2 \\
 &= FT[E(u, v)] FT[E(u, v)] \\
 &= FT[E(u, v) \otimes E(u, v)]
 \end{aligned}$$

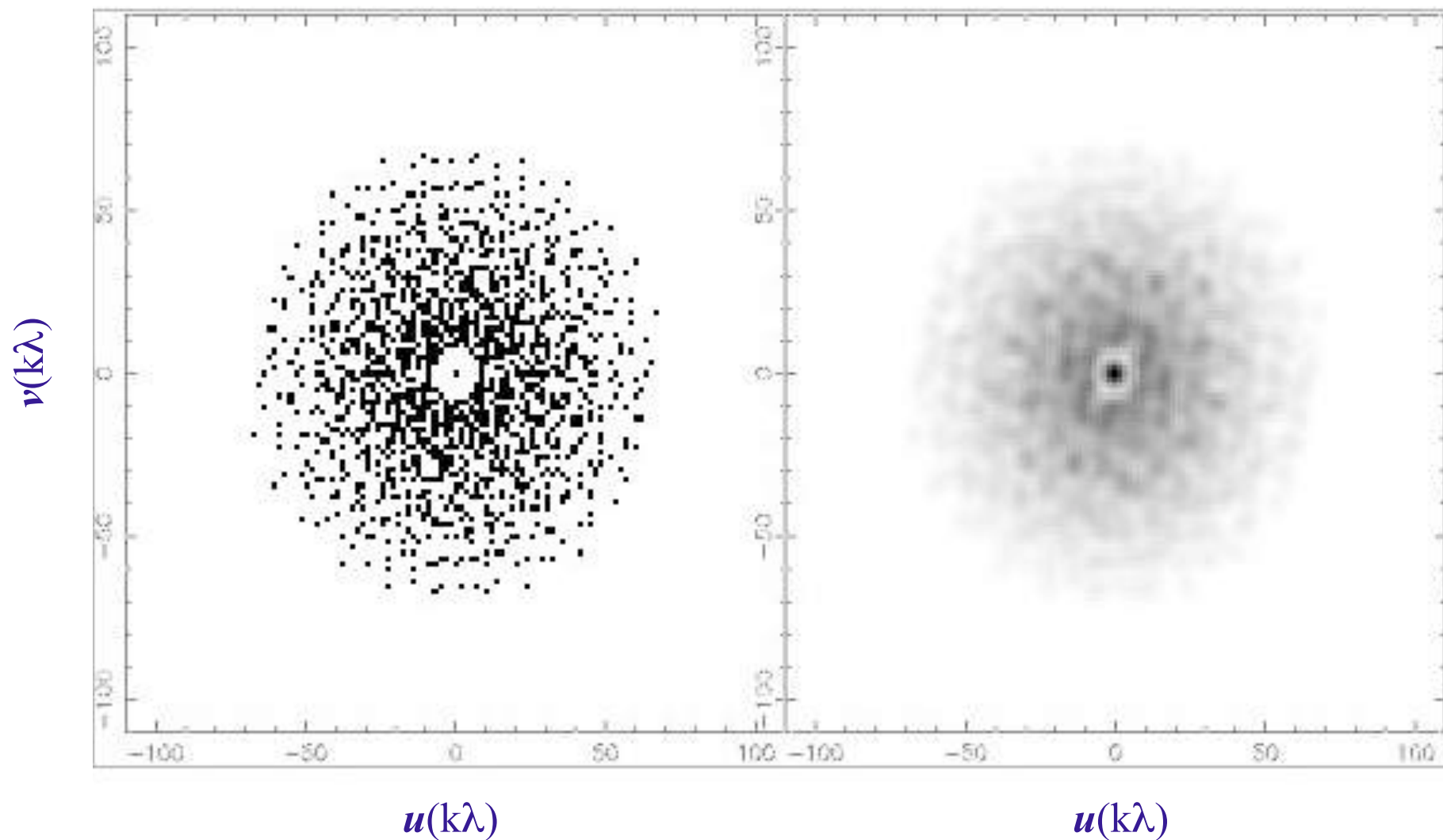
**Auto-correlation of aperture plane illumination function; support within  $r=(0,+D)$**

$$FT[A(\ell, m)] = E(u, v) \otimes E(u, v)$$

# Theory of Mosaicking: Ekers & Rots Theorem

nominal uv coverage: (baseline)/ $\lambda$

What you are really measuring:

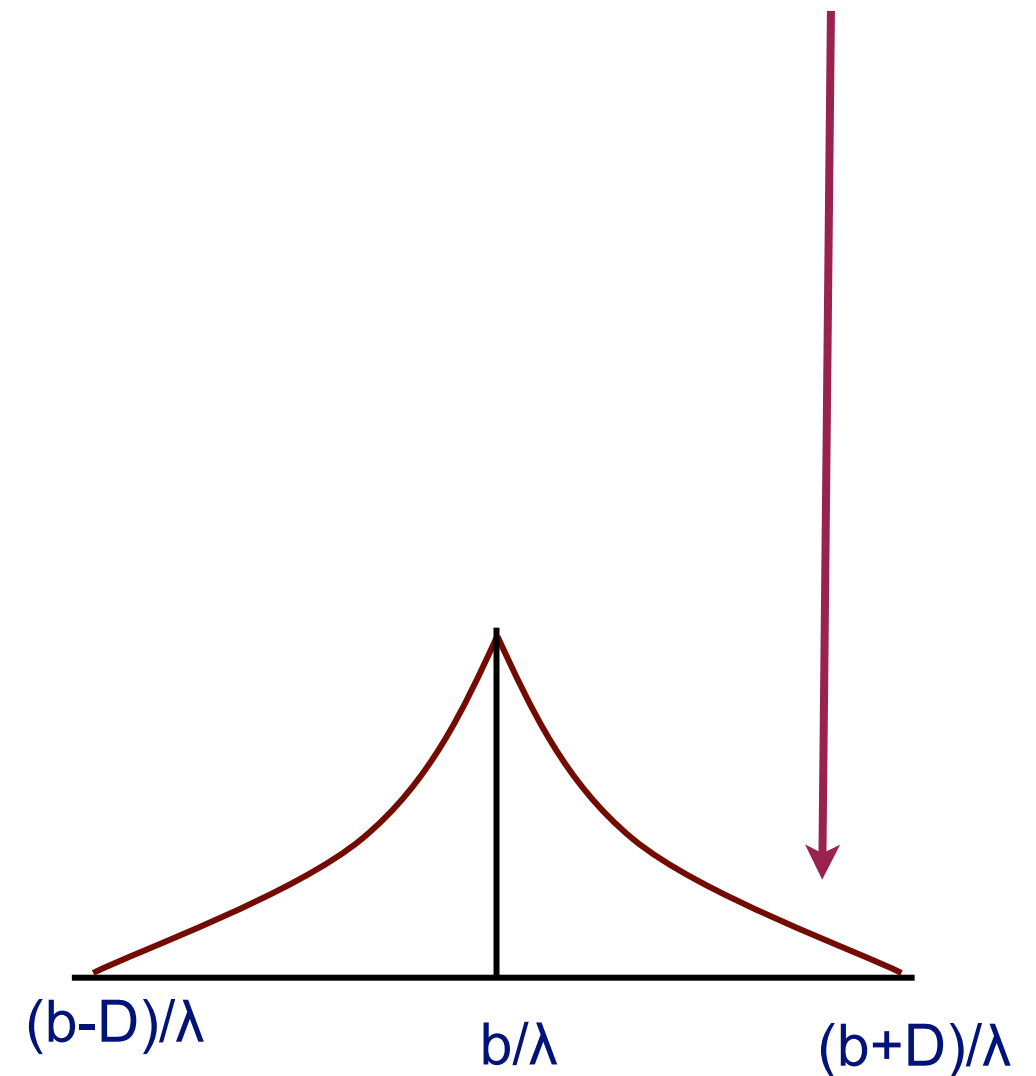


*Interferometer + Single Dish*

# The problem:

You want to separately estimate many Fourier component amplitudes between  $(b-D)/\lambda$  and  $(b+D)/\lambda$ , but you have measured only a single complex visibility!

*(a single dish has the same problem)*



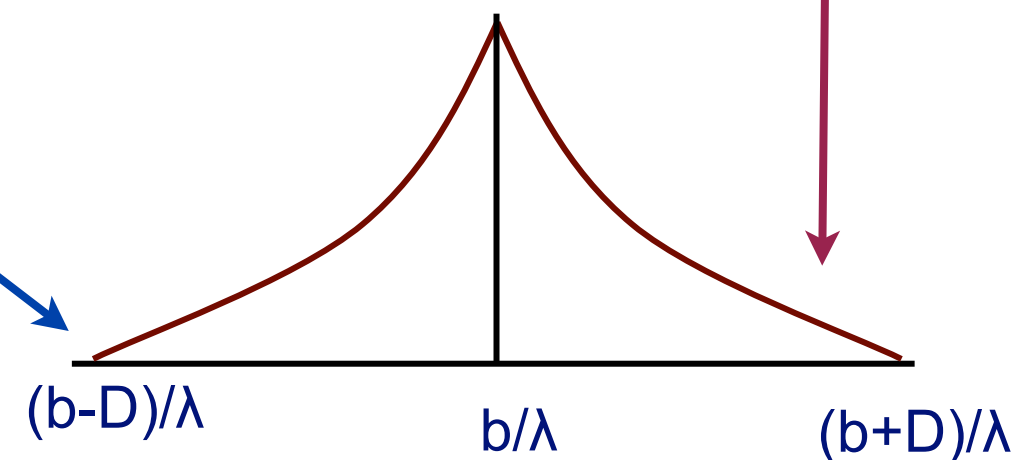


# The problem:

You want to separately estimate many Fourier component amplitudes between  $(b-D)/\lambda$  and  $(b+D)/\lambda$ , but you have measured only a single complex visibility!

**Solution:** scan the telescope over the sky and measure the visibility ( $V$ ) multiple times.

This allows you to separate out the the Fourier modes each measurement contains, increasing the maps' Fourier resolution & Largest (useful) Angular Scale.



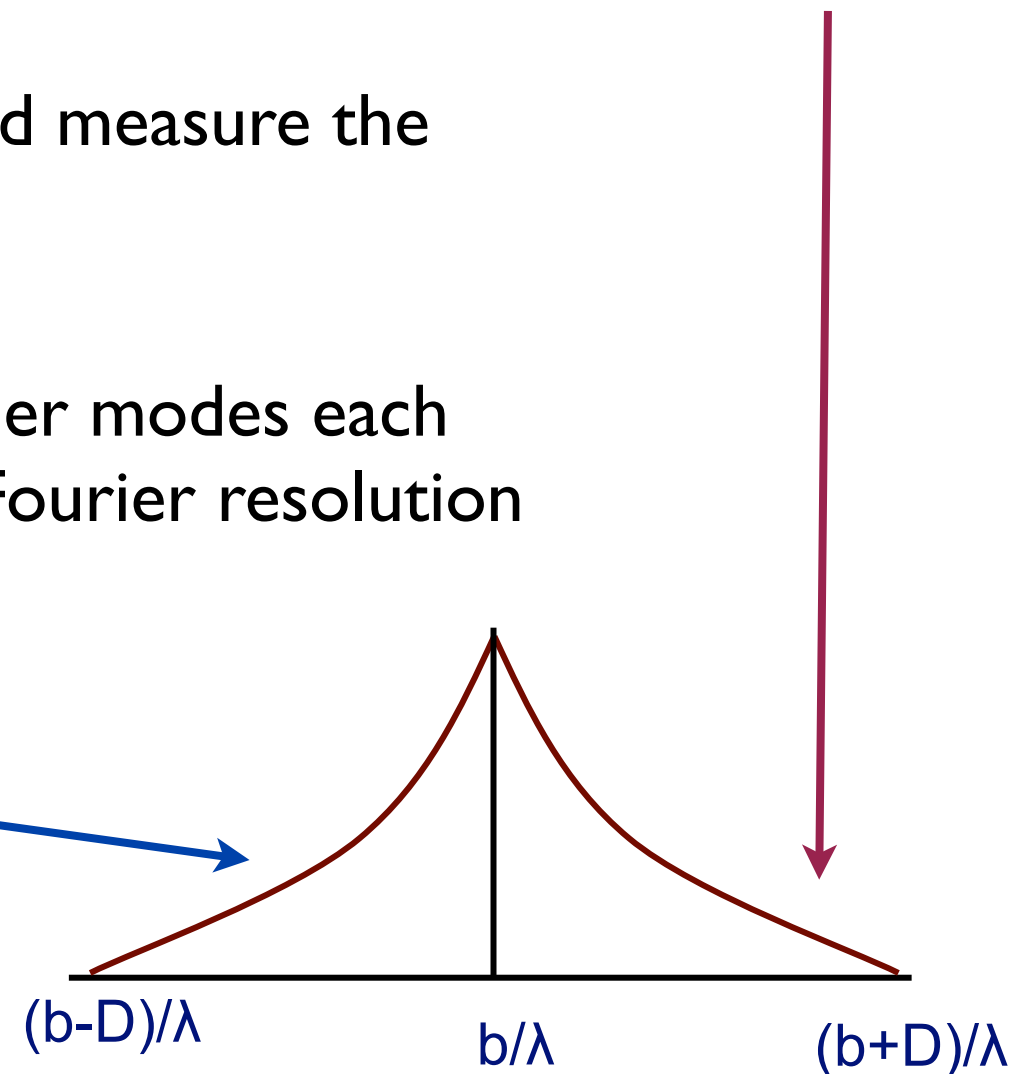
# The problem:

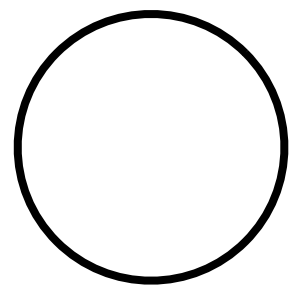
You want to separately estimate many Fourier component amplitudes between  $(b-D)/\lambda$  and  $(b+D)/\lambda$ , but you have measured only a single complex visibility!

**Solution:** scan the telescope over the sky and measure the visibility ( $V$ ) multiple times.

This allows you to separate out the the Fourier modes each measurement contains, increasing the maps' Fourier resolution & Largest (useful) Angular Scale.

**Caveat:** *signals away from  $b$  are attenuated so not measured as well.*

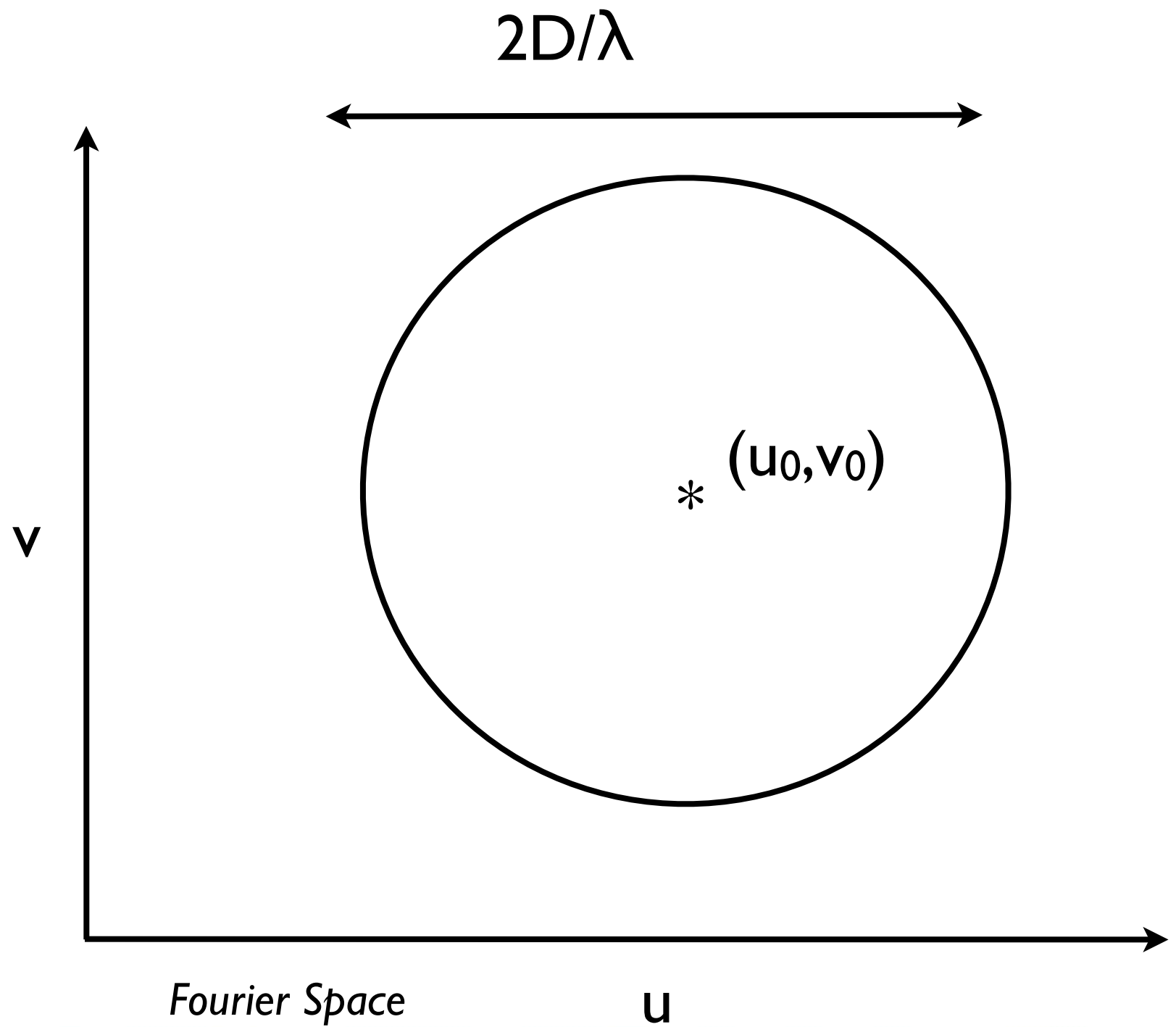




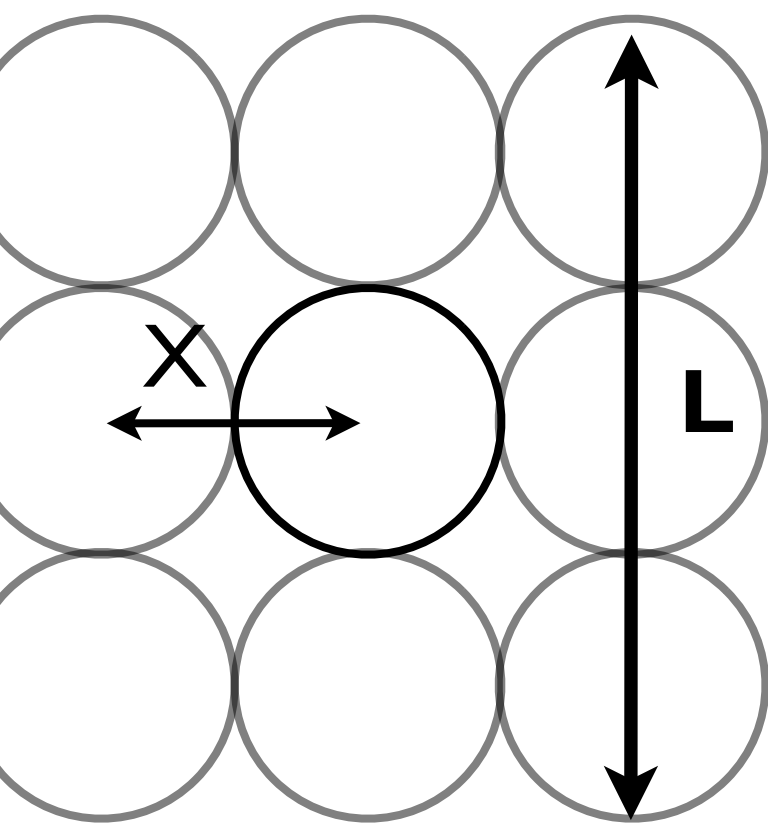
*antenna primary beam*

*Real Space*

*One Pointing of the interferometer*







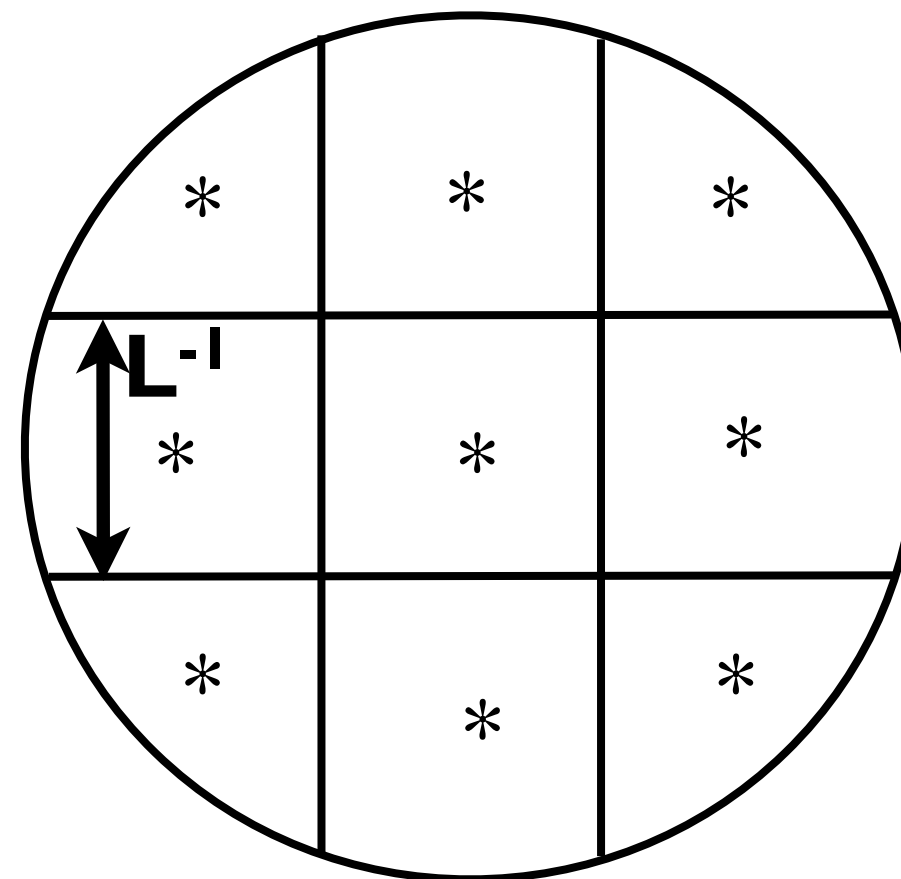
*Real Space*

*3x3 Pointings of the interferometer*

**N PB Areas = Nx more resolution  
in Fourier Space**

**v**

$2D/\lambda$



*Fourier Space*

**u**

ALIASES

$$X \leq \lambda/2D$$

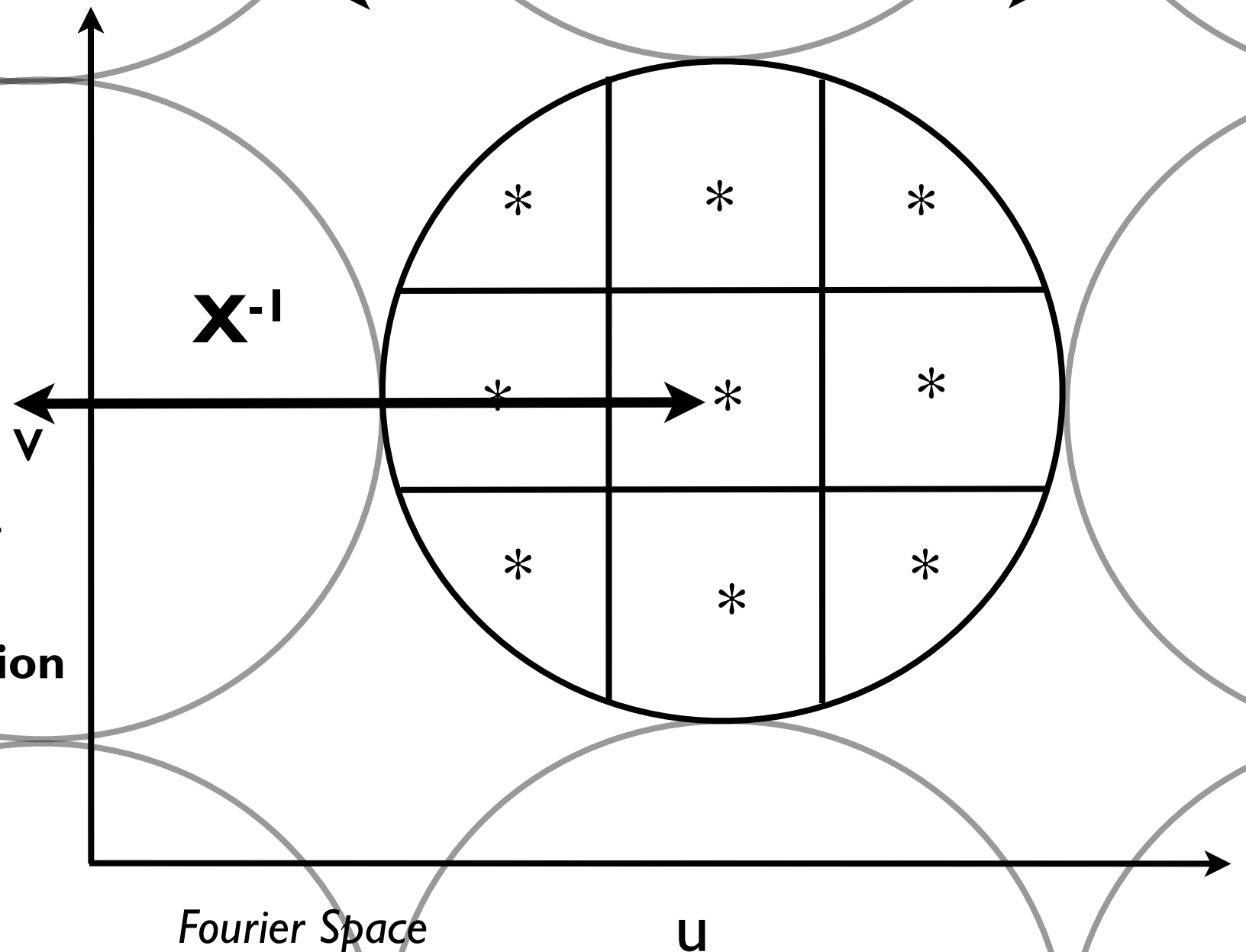
$$2D/\lambda$$

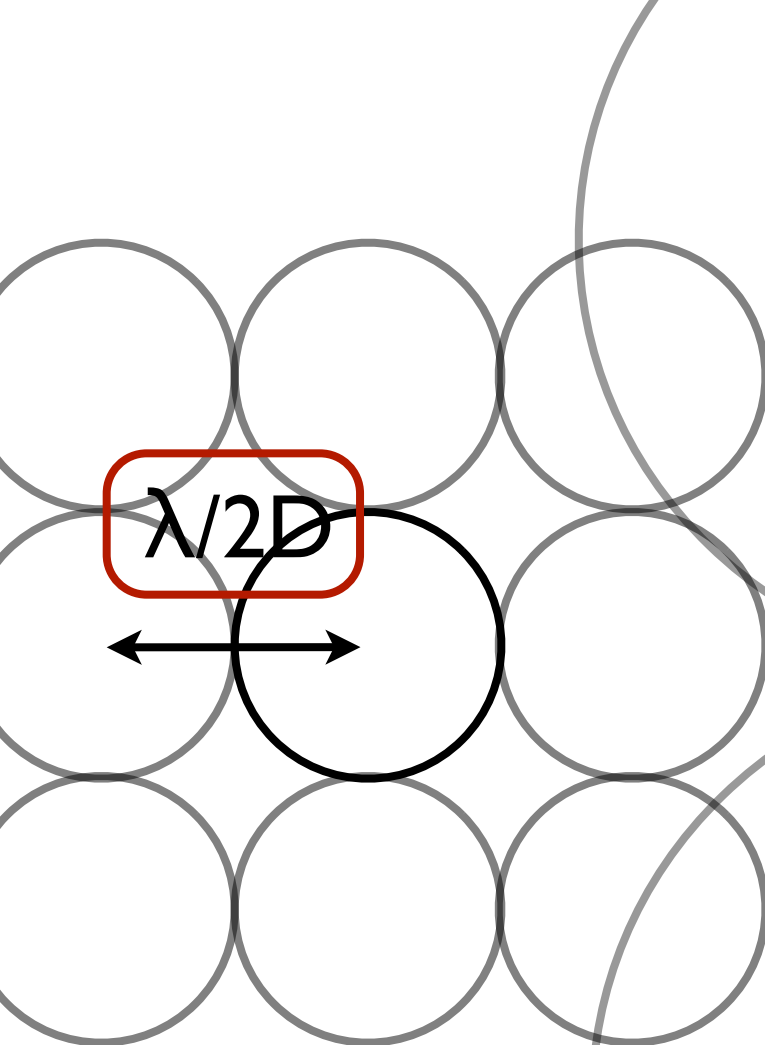


Real Space

3x3 Pointings of the interferometer

**N PB Areas = Nx more resolution  
in Fourier Space**





\*Pointing centers should be spaced by  $\lambda_{\text{shortest}}/2D$   
(not necessarily beam FWHM)

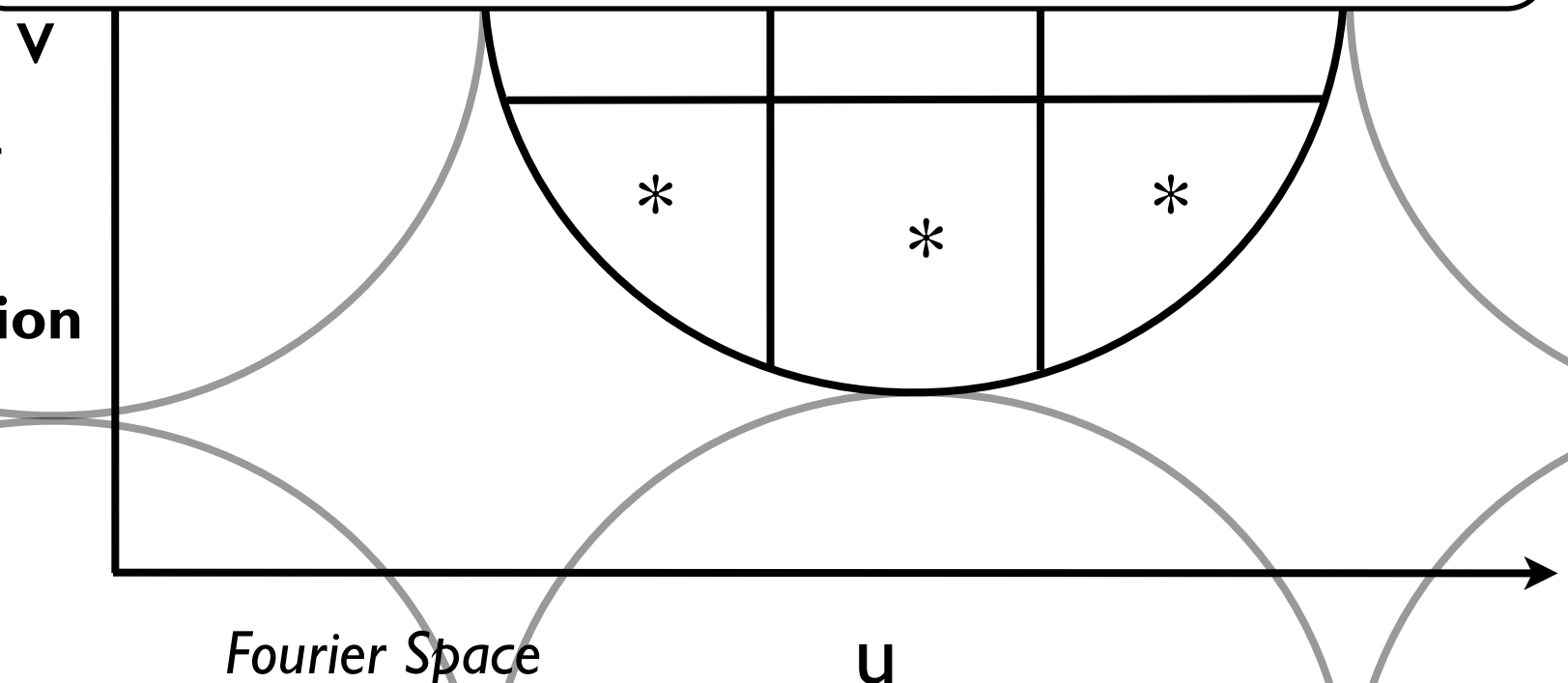
\*For round dishes: aperture plane more densely  
tiled without overlap by hexagonal mosaic  
*real space slightly more sparsely sampled*

\*Good sampling also provides closer to uniform  
noise in the image.

*Real Space*

*3x3 Pointings of the interferometer*

**N PB Areas = Nx more resolution  
in Fourier Space**



*Fourier Space*



# Stitching the Interferometer Maps together: Mosaic Imaging Algorithms in Practice

Widely-used methods for mosaic image reconstruction:

- **Linear combination**

*Make individual ptg dirty maps → deconvolve individually → combine deconv'd maps*

- **Joint deconvolution**

*Make individual ptg dirty maps → combine into one dirty map → deconvolve together  
(w/spatially varying PSF)*

- **Widefield Imaging by regridding of all visibilities before FFT into a single map**

*Combine visibilities from all pointings in uv-space → single dirty map → deconvolve*



# Stitching the Interferometer Maps together: Mosaic Imaging Algorithms in Practice

Widely-used methods for mosaic image reconstruction:

## ➤ Linear combination

*Make individual ptg dirty maps → deconvolve individually → combine deconv'd maps*

### Advantages:

- Careful imaging of each pointing separately
- Depends less on the exact knowledge of primary beam shape.
- Good when the field is crowded or confused (e.g., low frequency)

### Disadvantages:

- Deconvolution algorithm doesn't have access to Ekers-Rots information!
- “ “ has lower SNR data in overlap regions

***These are significant shortcomings for a nonlinear algorithm***

# Stitching the Interferometer Maps together: Mosaic Imaging Algorithms in Practice

Widely-used methods for mosaic image reconstruction:

## ➤ **Joint deconvolution / Wide field Imaging**

*Make individual ptg dirty maps → combine into one dirty map → deconvolve together  
[or co-grid all UV data]*

### **Advantages:**

- Deconvolution Algorithm** explicitly makes use of **Ekers-Rots** information (same “visibility” measured at different sky pointings):  
Improves Fourier resolution, **LAS**, and synthesized beam

### **Disadvantages:**

- Requires a good **PB** model

**Recommended unless special considerations guide you to another approach.**

**ftmachine='mosaic' in CASA clean() task.**



# Interferometric Mosaicking Issues

- Pointings are in a time sequence:
  - Each pointing has a different uv-coverage
  - Atmospheric water vapor/ionospheric variations from pointing to pointing
- Pointing is more critical than for non-mosaicked observation with an isolated source in the beam center
- Know your primary beam!

# Deconvolution

Mosaicking is typically done for **extended** sources.

***Deconvolution in this case is tricky.***

# Deconvolution

Mosaicking is typically done for **extended** sources.

***Deconvolution in this case is tricky.***

**CLEAN:** Preferably Cotton-Schwab, with small gain.

***You need to clean deeply ( $\sim 1\sigma$ ) for extended emission.***

**Justification:** in general the “CLEAN model” is not your best estimate of the sky; the reconvolved CLEAN model+residuals is.

- may require good uv coverage, a judiciously chosen clean box, & careful monitoring (interactive): beware of clean bias
- may take a long time for a spectral line cube.
- **Do not do this** if you are going to self-cal using the CLEAN model!

# Deconvolution

Mosaicking is typically done for **extended** sources.

***Deconvolution in this case is tricky.***

**CLEAN:** Issues to be aware of

- \* “CLEAN Bias”: constructive interference of synthesized beam sidelobes can make them appear higher than the main lobe of the synth. beam.
  - \* Reduces the apparent source fluxes recovered
  - \* most severe for extended sources
  - \* mitigated by good UV coverage (lower sidelobes), good masking.
  - \* see Condon et al. (1998) [NVSS survey paper]
- \* Mismatch of Clean & Dirty Beams: beam areas differ within relevant apertures, biasing integrated flux density values upward.
  - \* mitigated by deeper cleaning, correction factor
  - \* see Jorsater & VanMoorsel (1995) and Walter et al. (2008)



# Deconvolution

Mosaicking is typically done for **extended** sources.  
***Deconvolution in this case is tricky.***

## Multi-Scale CLEAN

- \* Generalize CLEAN to allow components of multiple sizes



- \* Obviously better suited to extended emission!
- \* Several parameters need to be chosen for it to work well (list of component scales, small-scale bias)
- \* Fully supported in CASA clean() task

# Deconvolution

Mosaicking is typically done for **extended** sources.

**Deconvolution in this case is tricky.**

## Maximum Entropy

\* Vary pixel values  $I_k$  in deconvolved image to **jointly**

\* Maximize match to a default image  $M_k$

$$\mathcal{H} = - \sum_k \left[ I_k \ln \left( \frac{I_k}{M_k e} \right) \right]$$

\* while maximizing the match of the FT of the model image to the visibility data (by minimizing  $\chi^2$ )

\* subject to  $I_k > 0$

$$\chi^2 = \sum_k \frac{|V(u_k, v_k) - V_{mod}(u_k, v_k)|^2}{\sigma_k^2}$$

\* If  $M_k = \text{const.}$  this tends to maximize the “smoothness” of the image

\* **A better choice: set  $M_k = (\text{single dish image})$**

\* Does well with extended emission but not bright compact sources embedded in extended emission.

\* Available at toolkit (expert) level in CASA; as VM and VTESS in AIPS

\* See Cornwell, Braun & Briggs (1999) & references therein.

“Entropy”  $\mathcal{H}$  maximized  
by  $I_k = M_k$



# Mosaicking in CASA

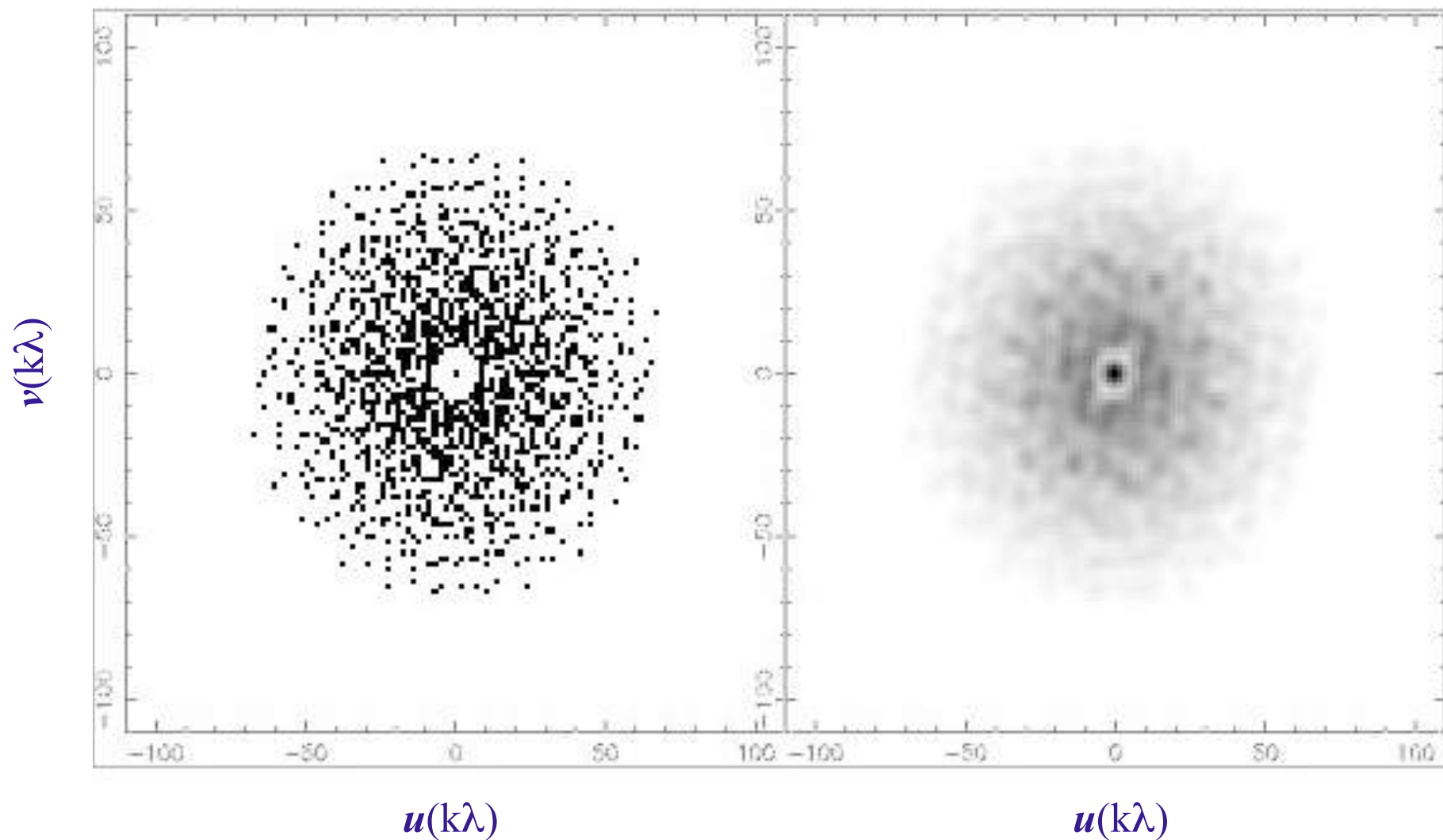
- Done in the imaging (deconvolution) stage; most of the fancy algorithms are implemented under the hood.
- Calibrate individual pointings as you would for a single pointing
- **CLEAN** supports joint deconvolution & widefield imaging (including w-term)
  - also offers multi-scale CLEANing
- *Image domain Linear Mosaicking & Maximum Entropy deconvolution are currently only available in the toolkit (lower-level, expert interface)*

# Effects of Missing Short & Zero Spacings

## *Interferometer + Single Dish*

nominal uv coverage: (baseline)/ $\lambda$

What you are really measuring:



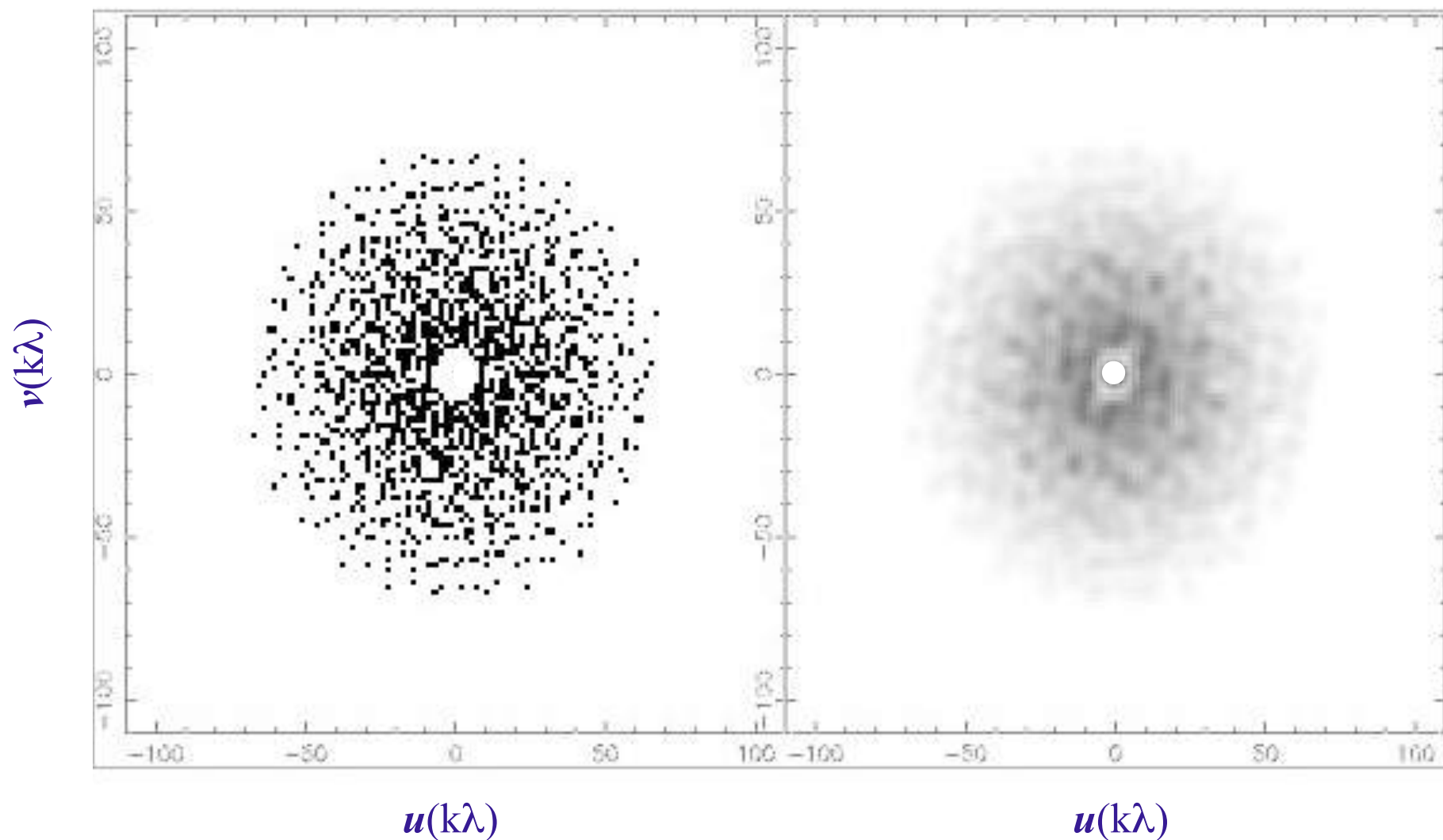


# Effects of Missing Short & Zero Spacings

Interferometer + ~~Single Dish~~

nominal uv coverage: (baseline)/ $\lambda$

What you are really measuring:



# Effects of Missing Short & Zero Spacings

Interferometer + ~~Single Dish~~

nominal uv coverage: (baseline)/ $\lambda$

What you are really measuring:

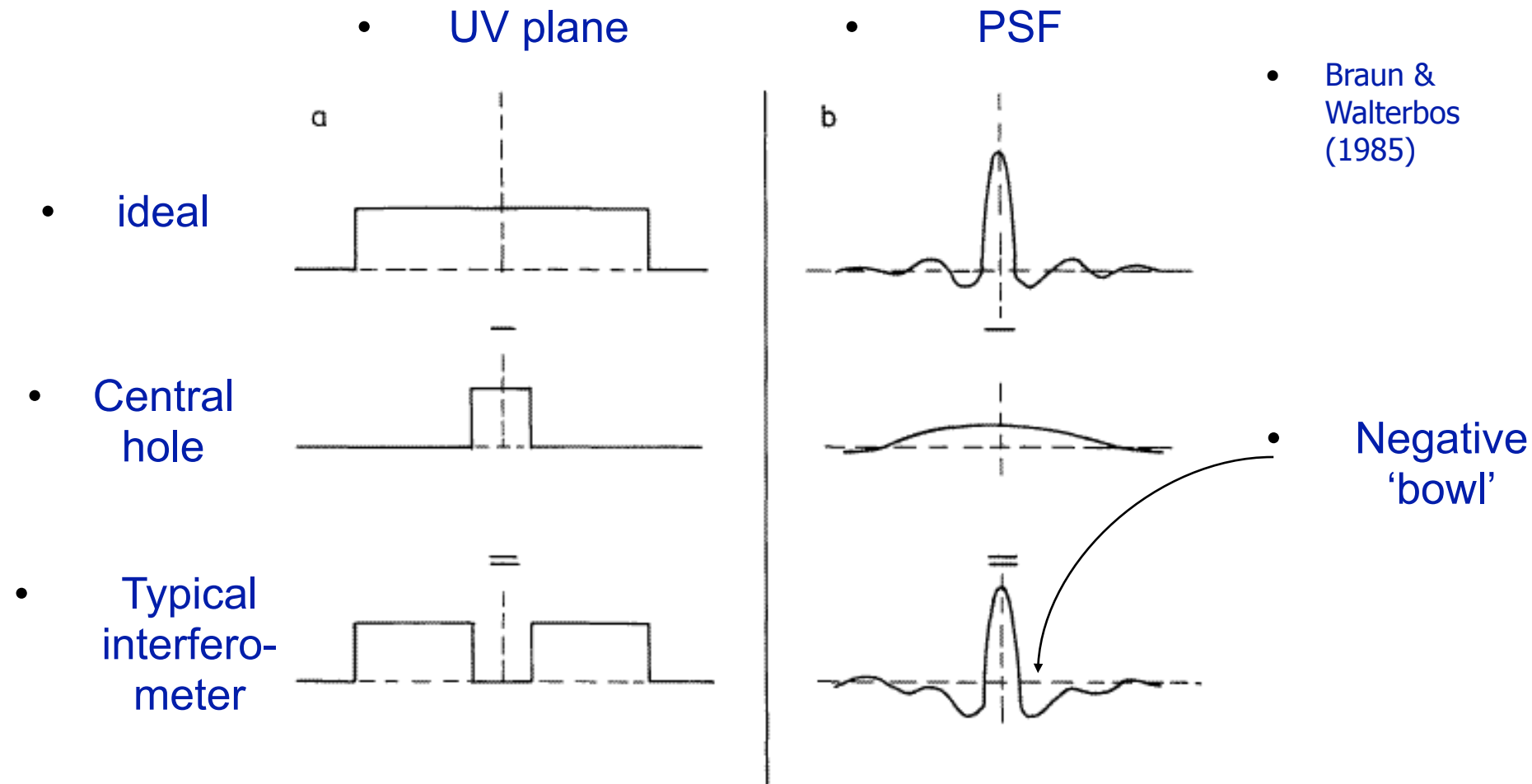
**The “background level” in your map is unmeasured / variable: this is a big problem for measuring the fluxes of individual objects or regions.**

**This matters because the science often comes from comparisons in different maps: the integrated line intensity in two transitions or lines; the continuum flux density at two widely separated frequencies.**

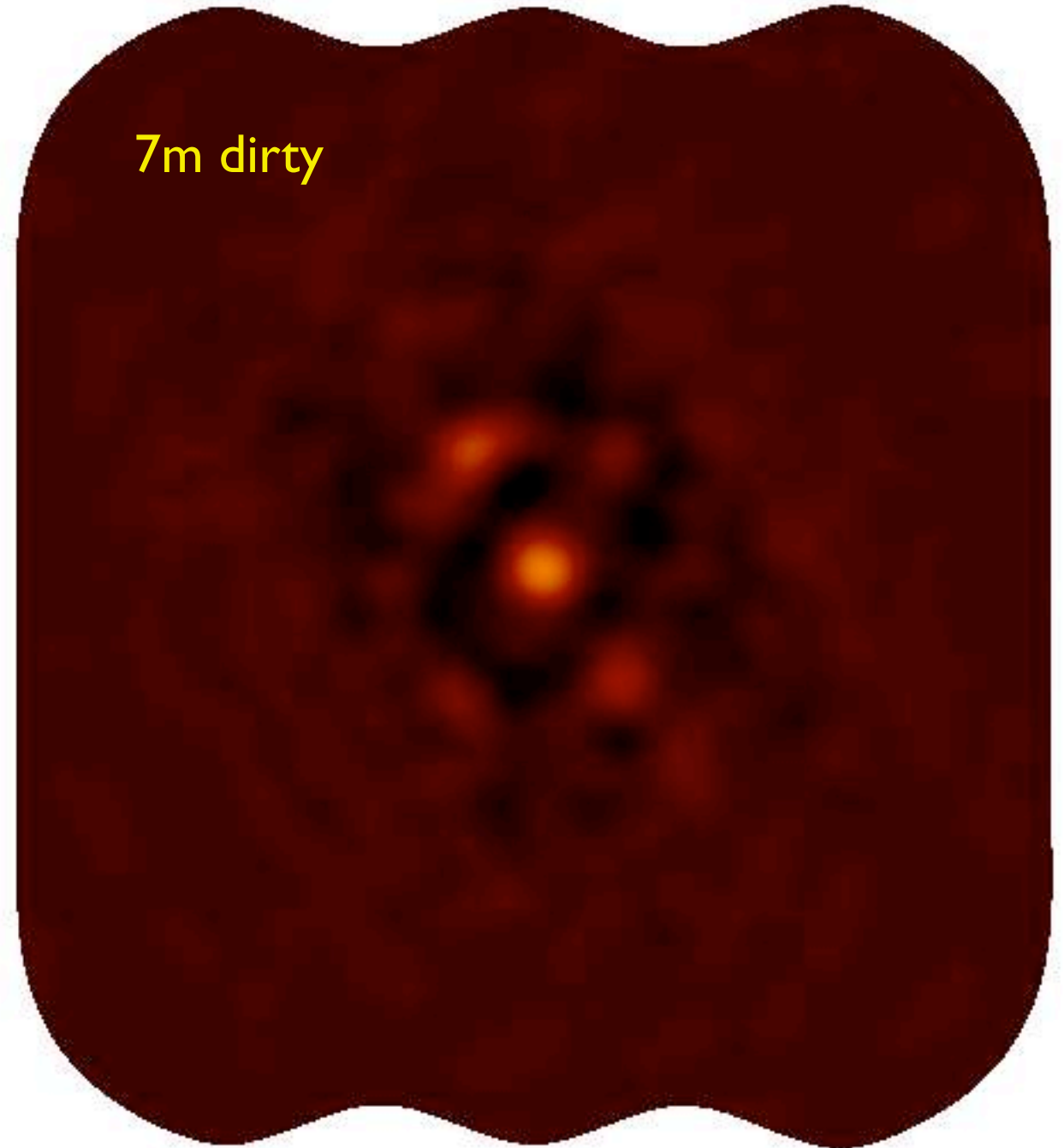
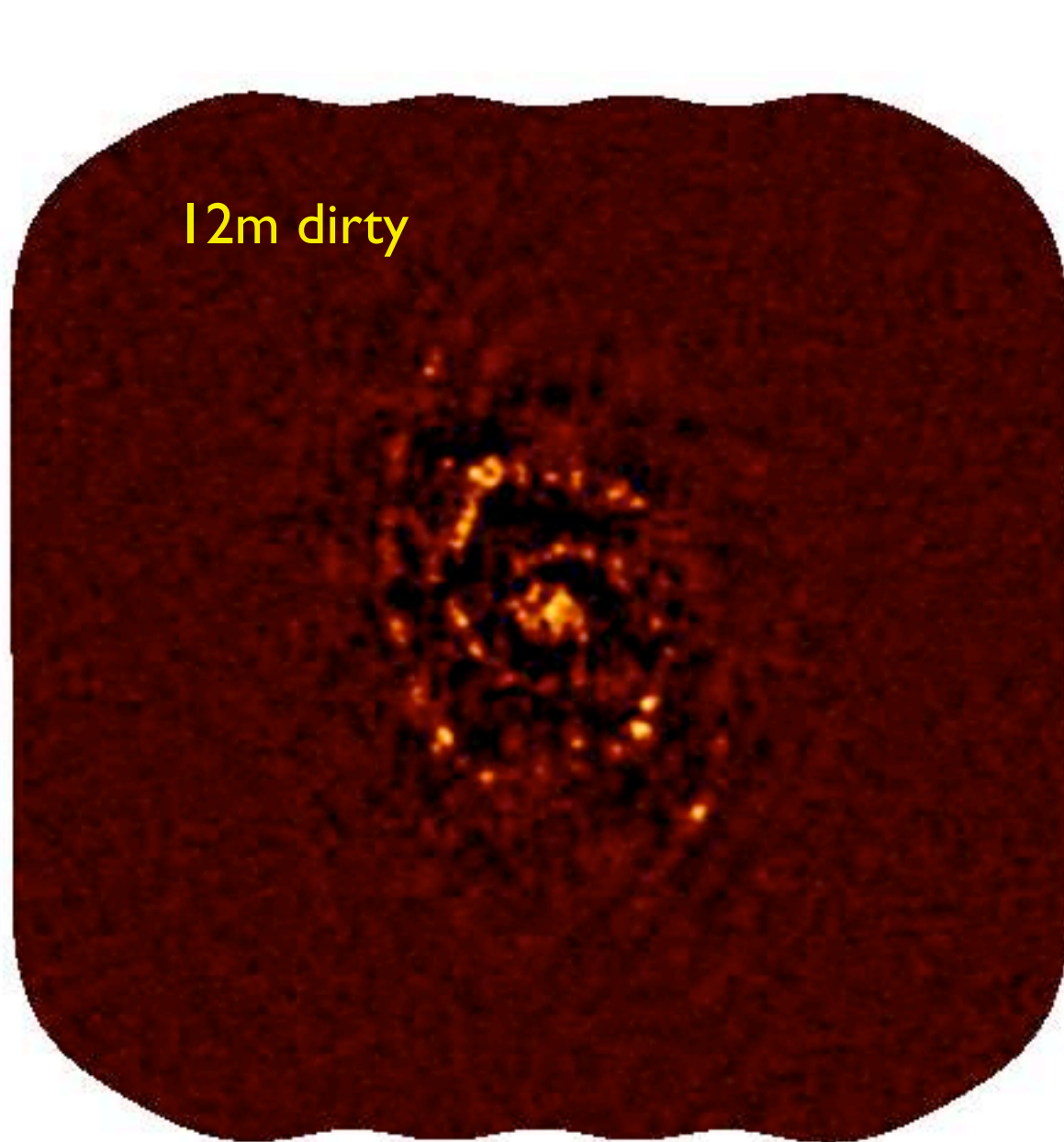
*(Often using data from completely different instruments...)*

# Effects of Missing Short & Zero Spacings

Interferometer + ~~Single Dish~~



# Effects of Missing Short & Zero Spacings

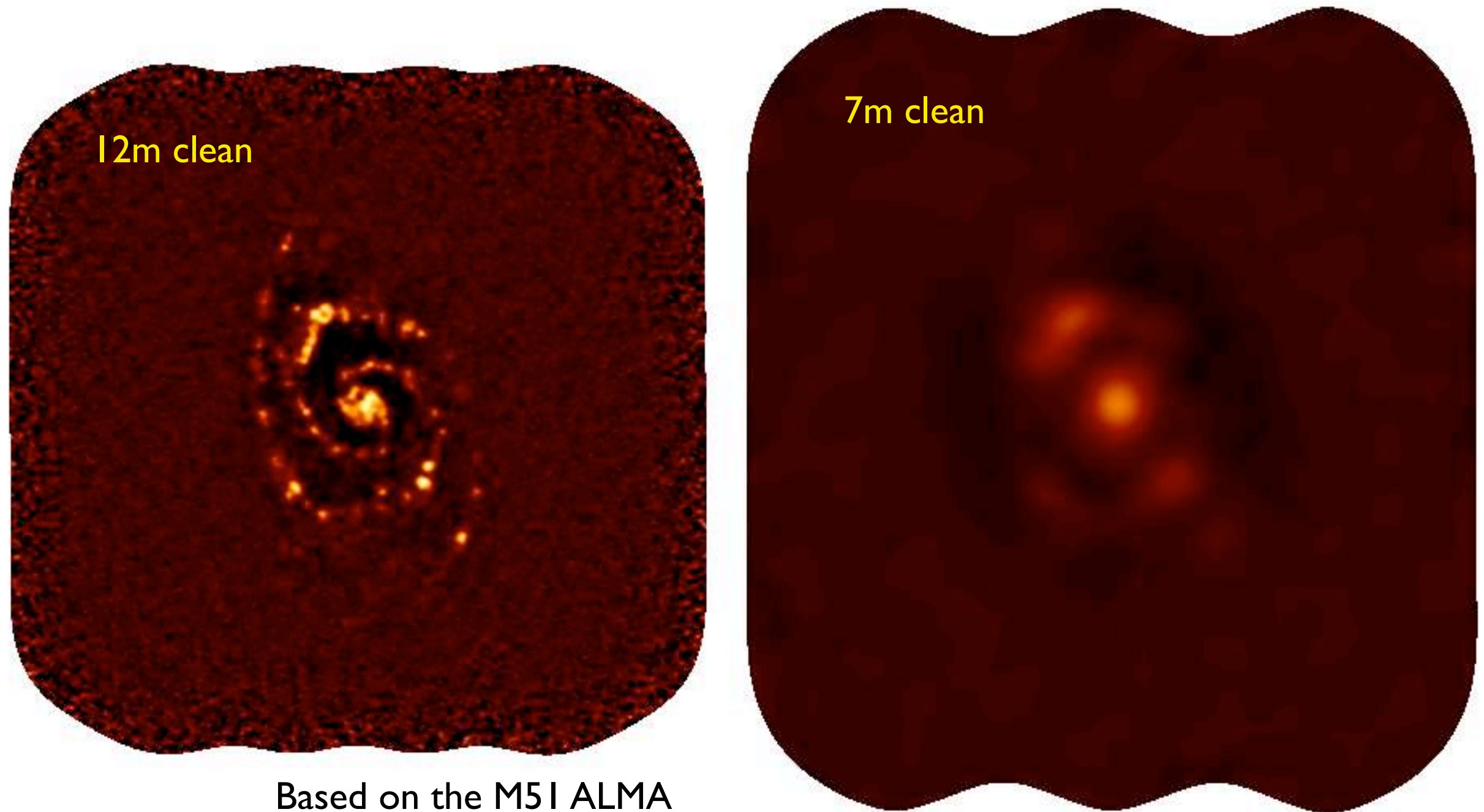


Based on the M51 ALMA  
simulation at

<http://casaguides.nrao.edu>



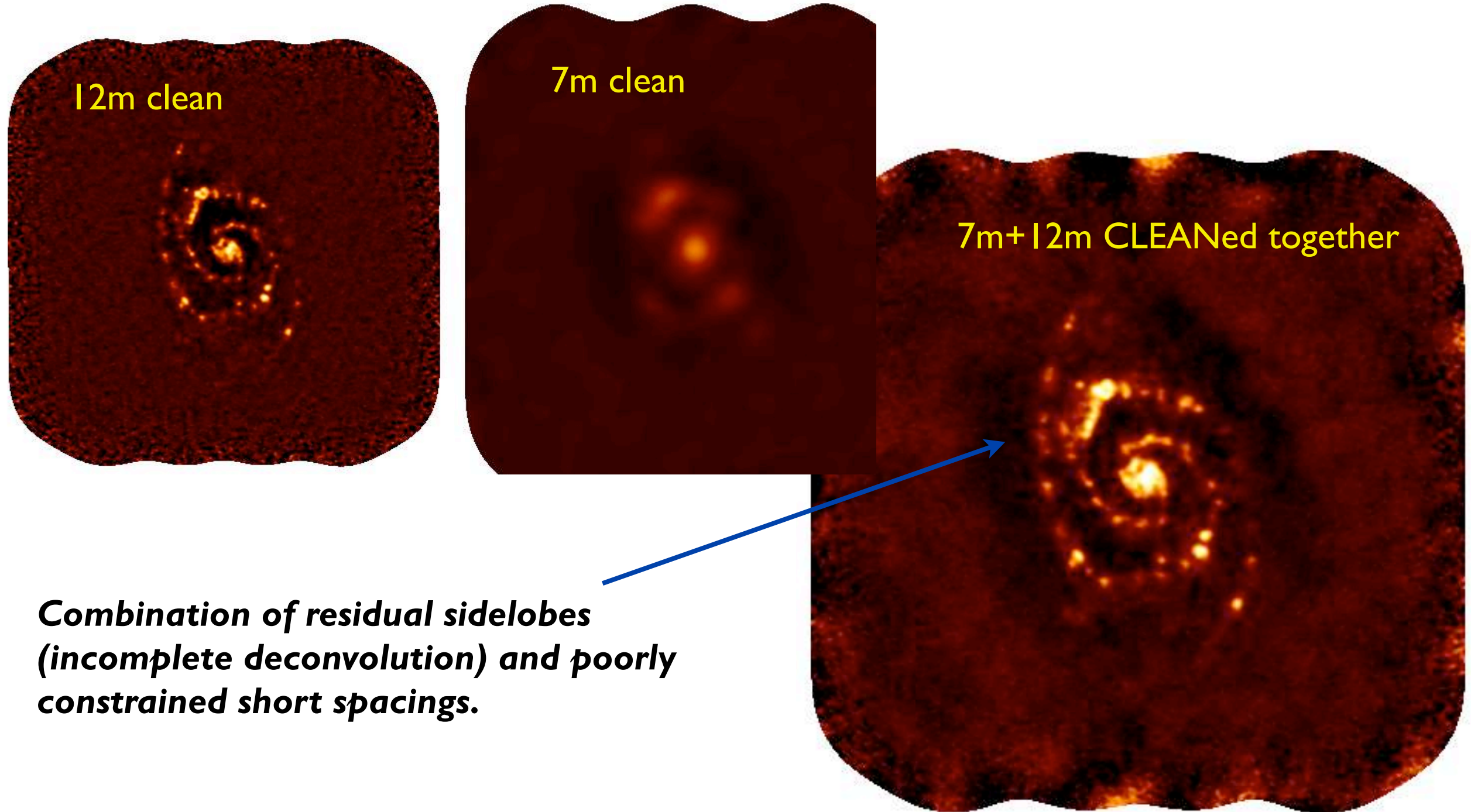
# Effects of Missing Short & Zero Spacings



Based on the M51 ALMA  
simulation at

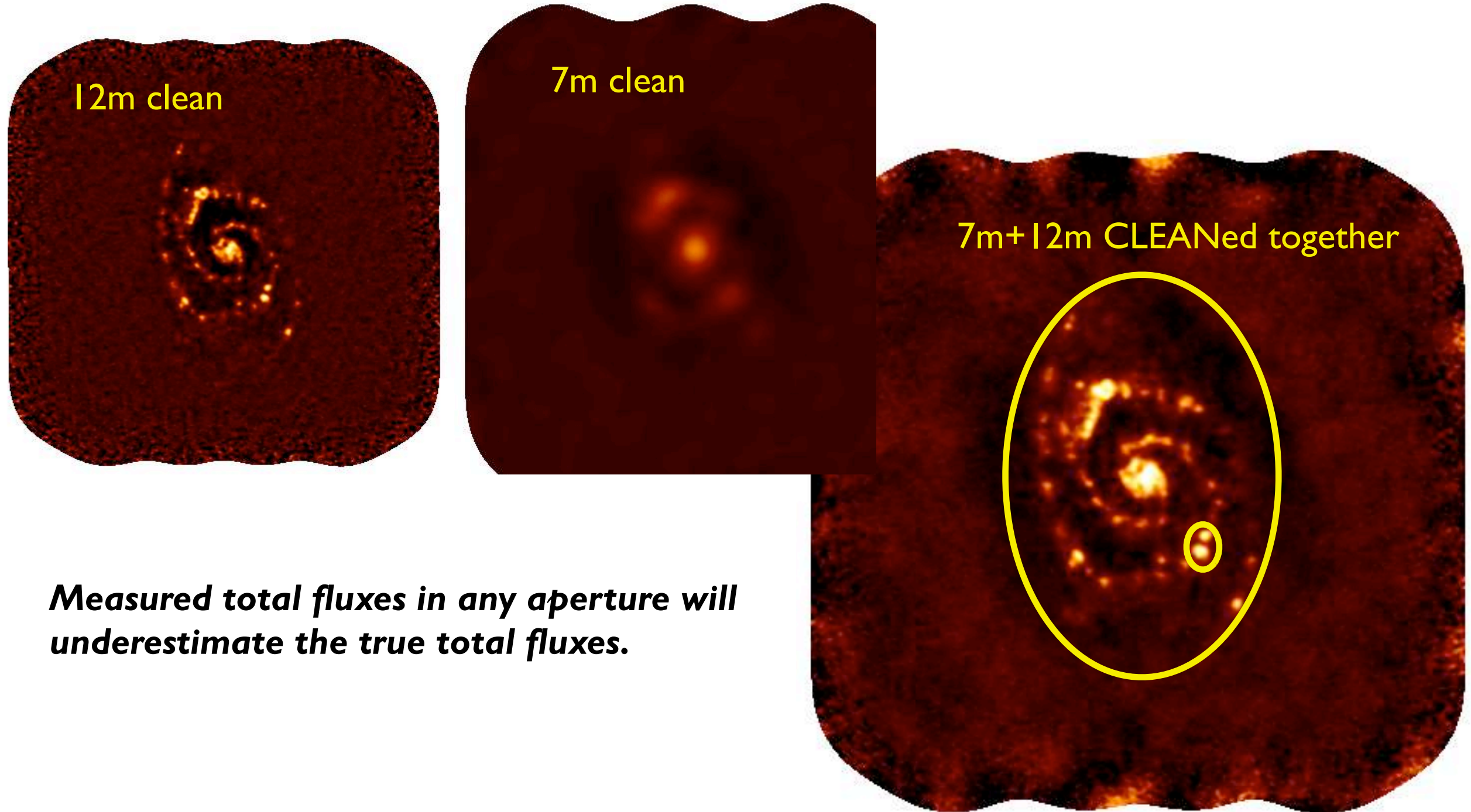
<http://casaguides.nrao.edu>

# Effects of Missing Short & Zero Spacings

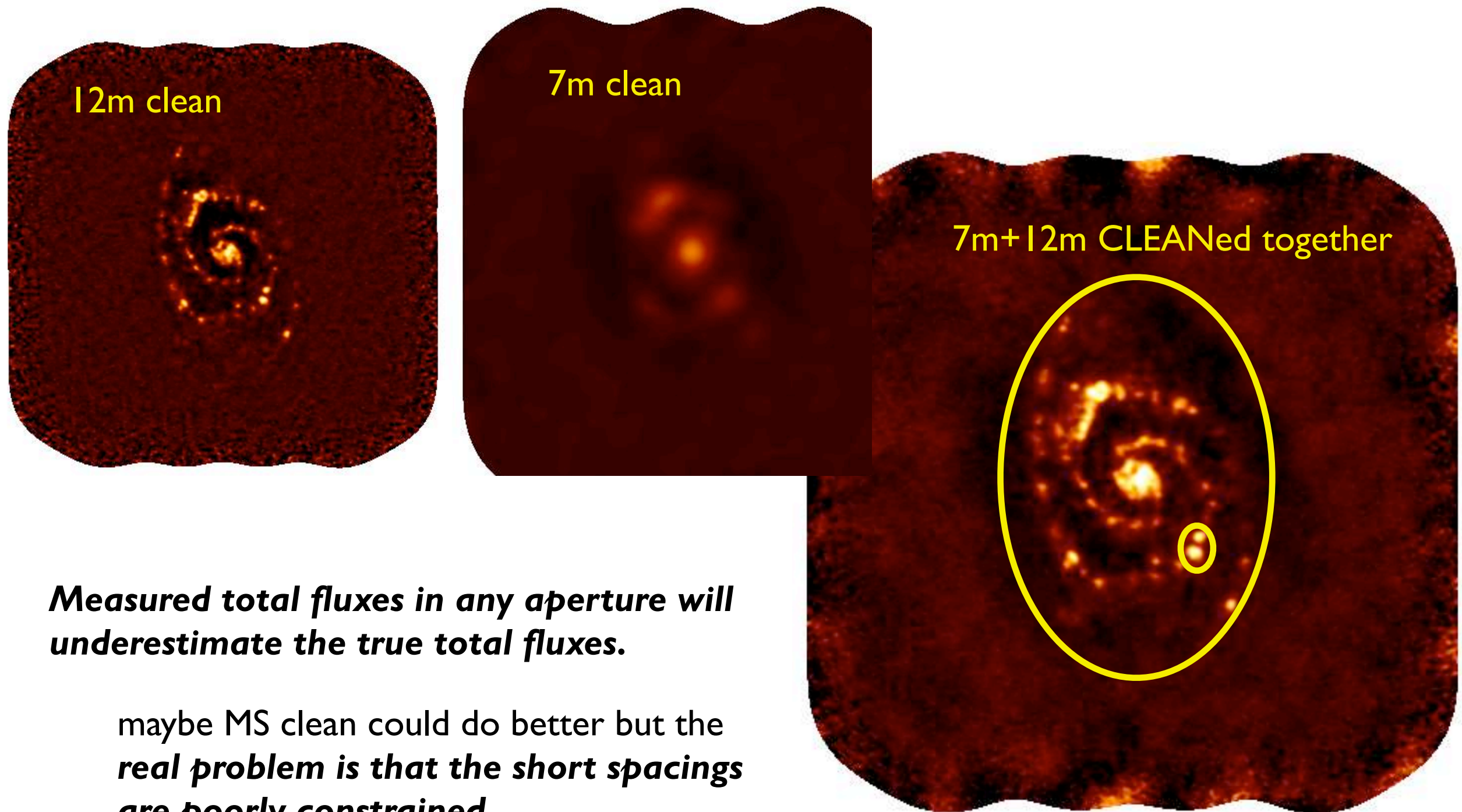




# Effects of Missing Short & Zero Spacings

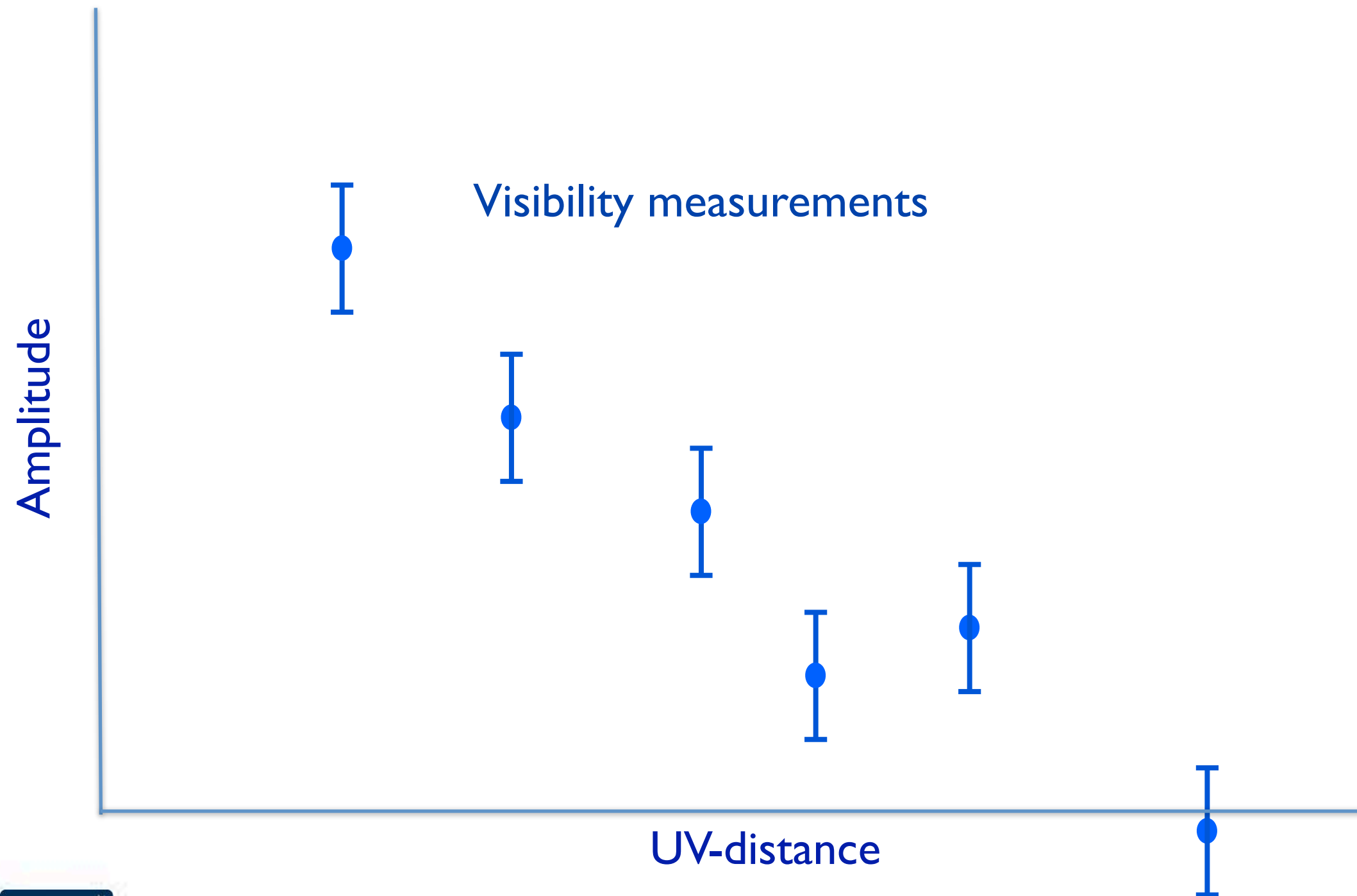


# Effects of Missing Short & Zero Spacings

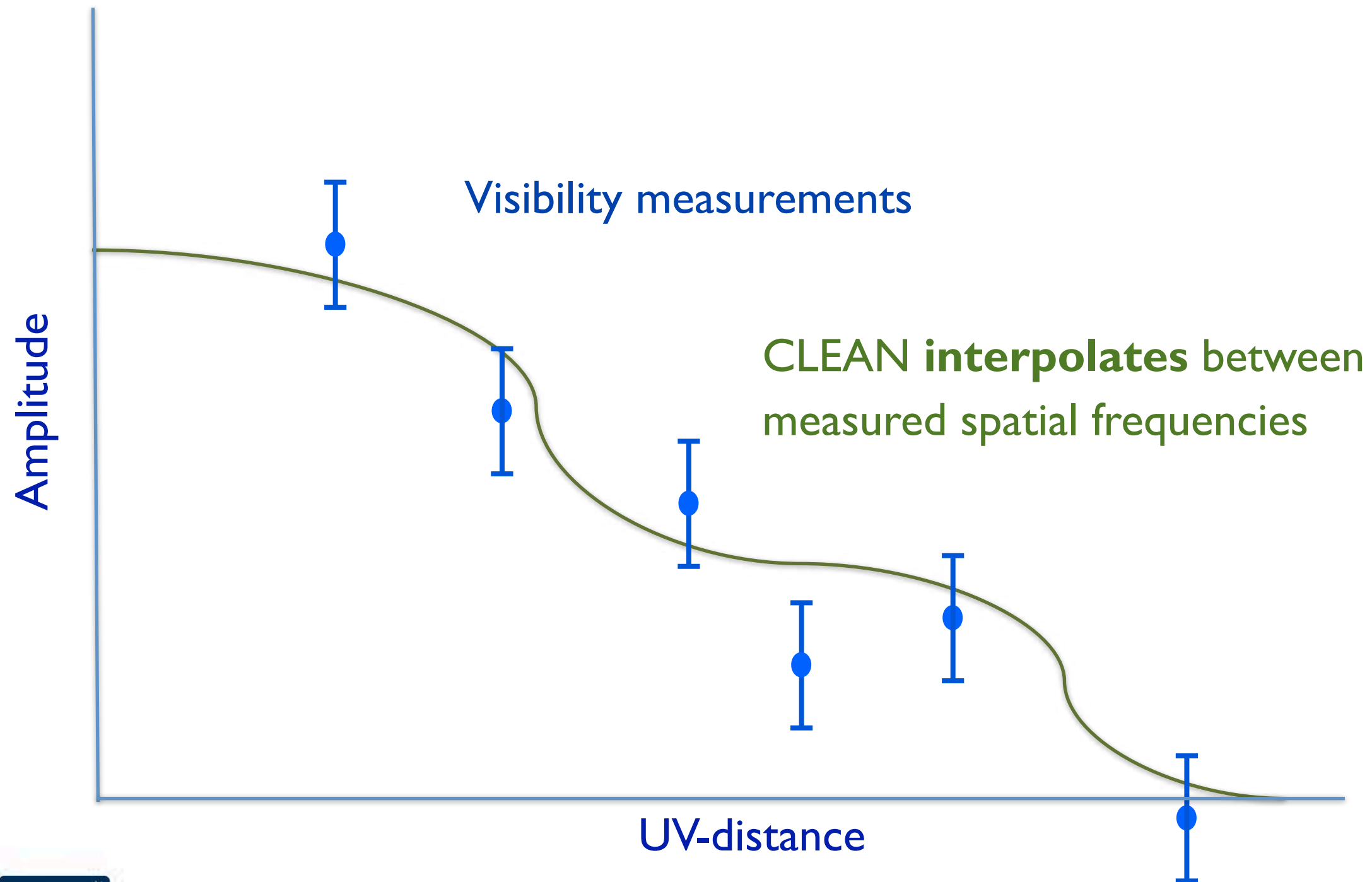




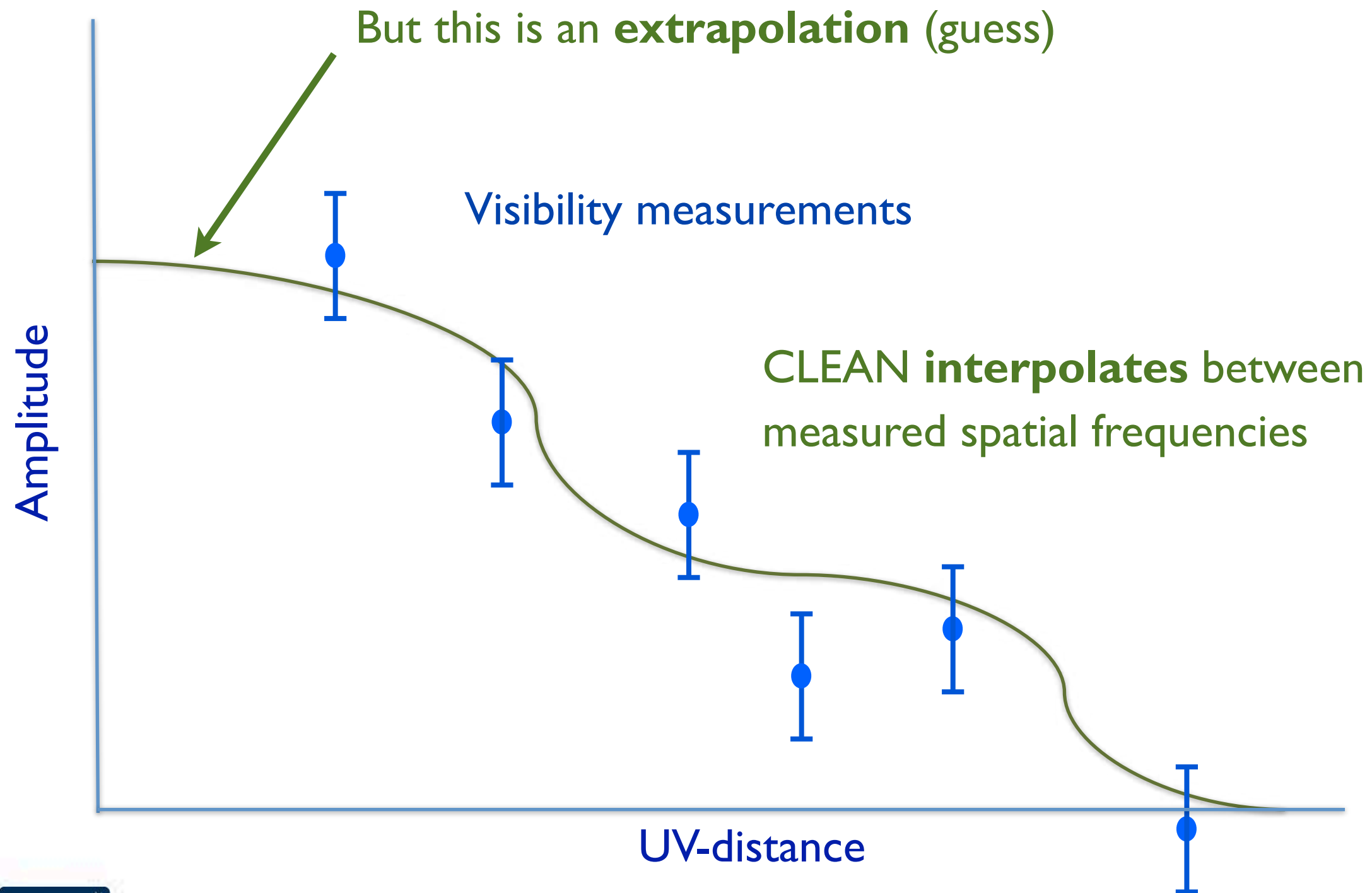
# Feathering



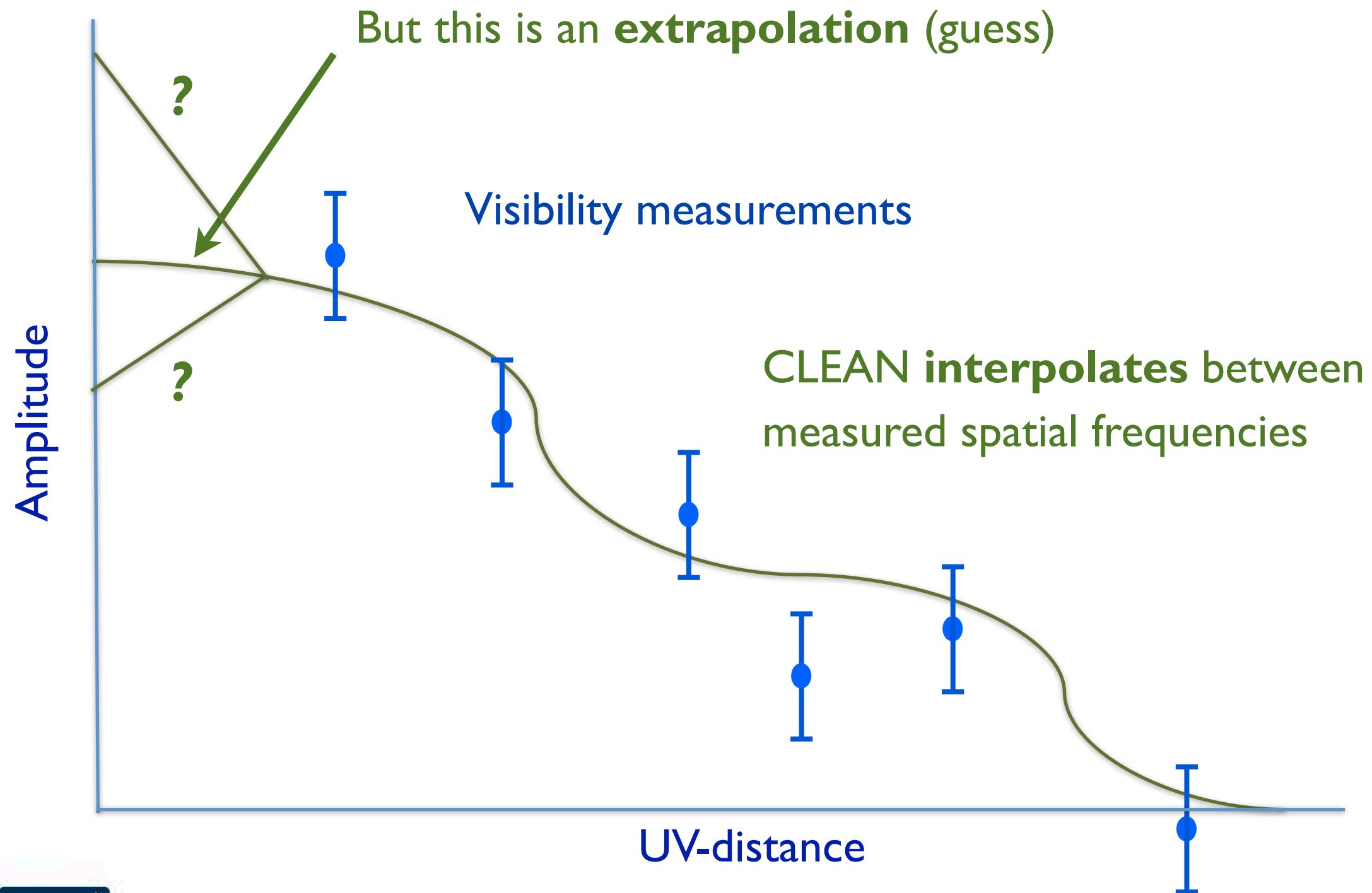
# Feathering



# Feathering

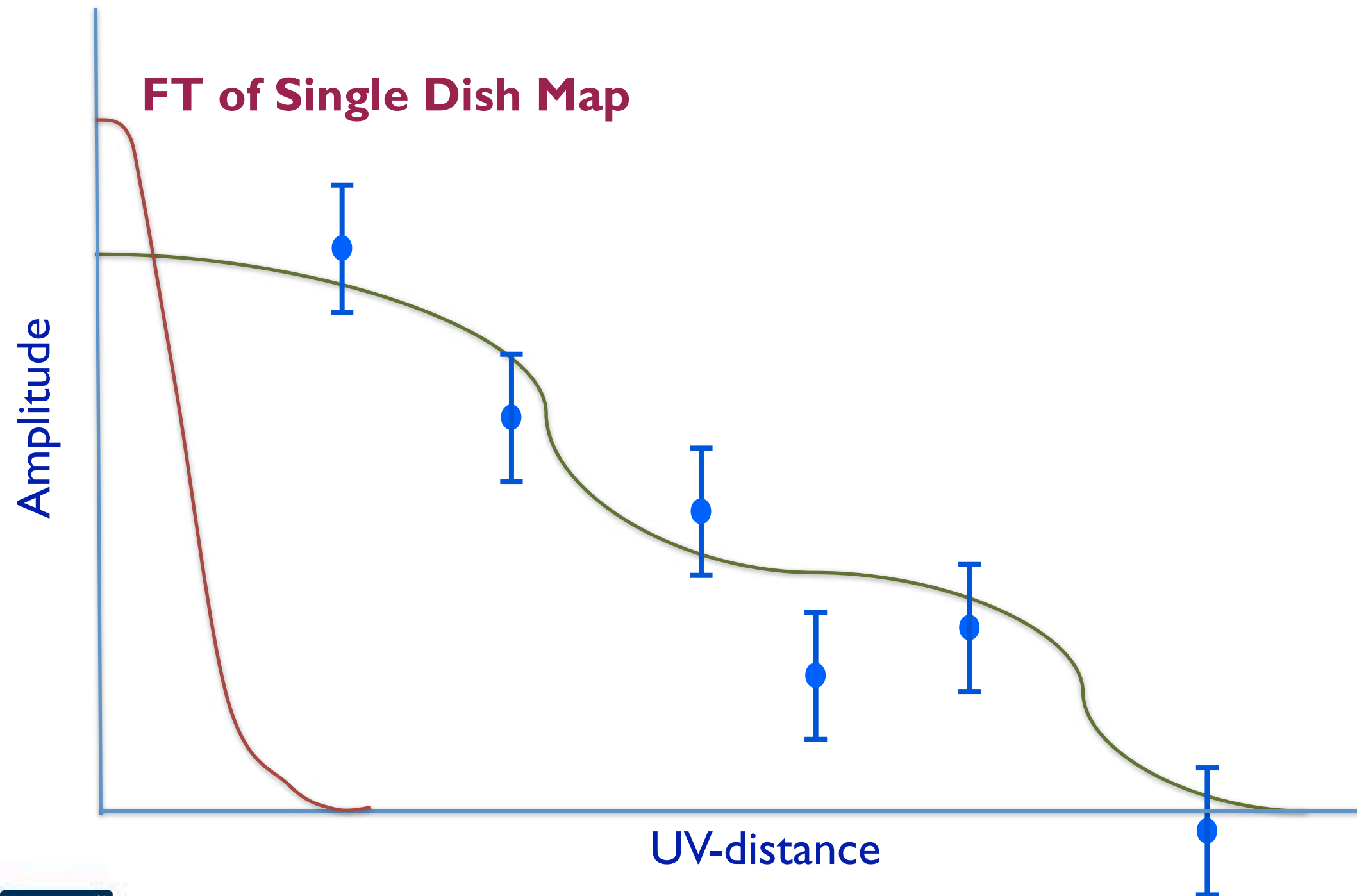


# Feathering

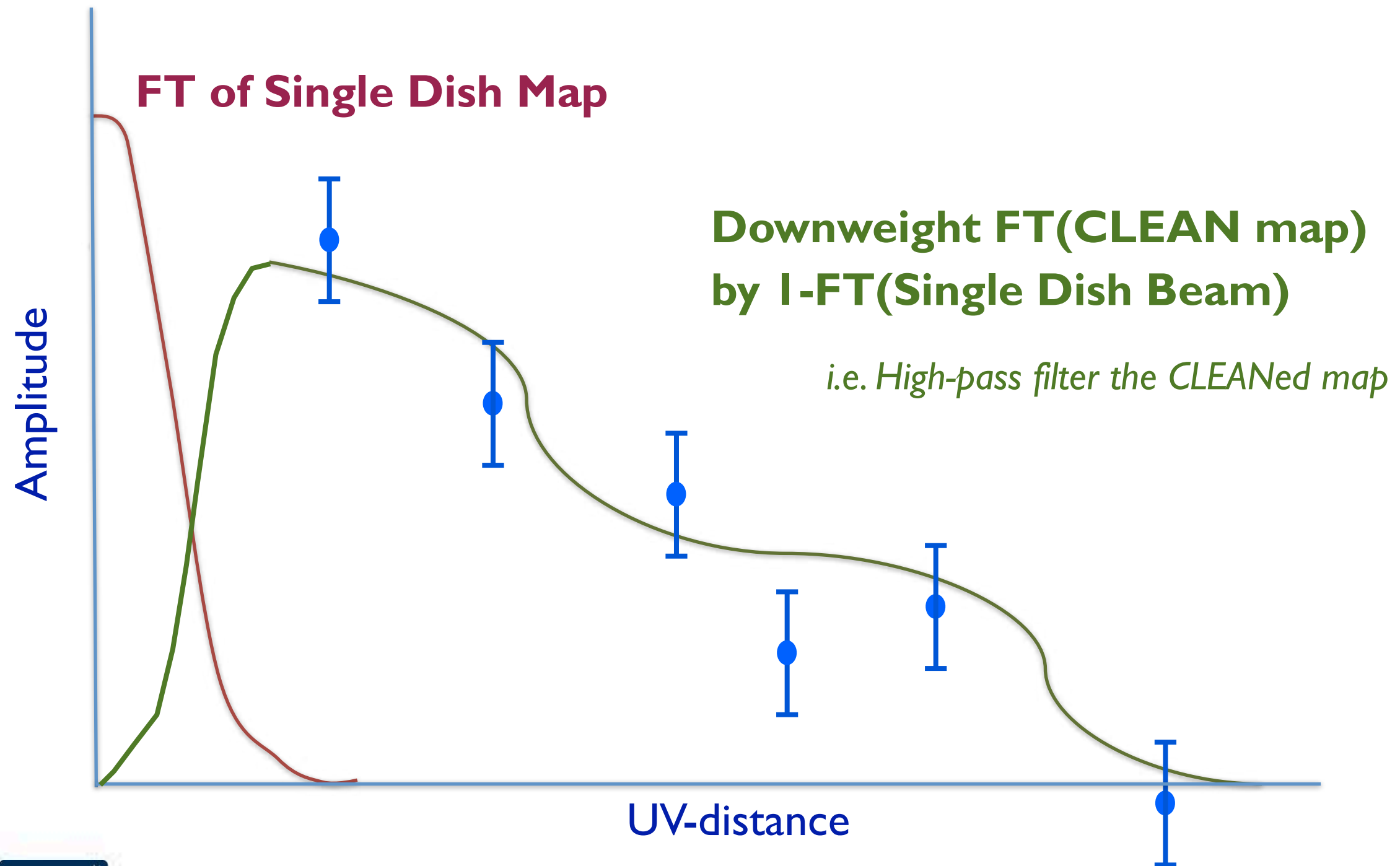




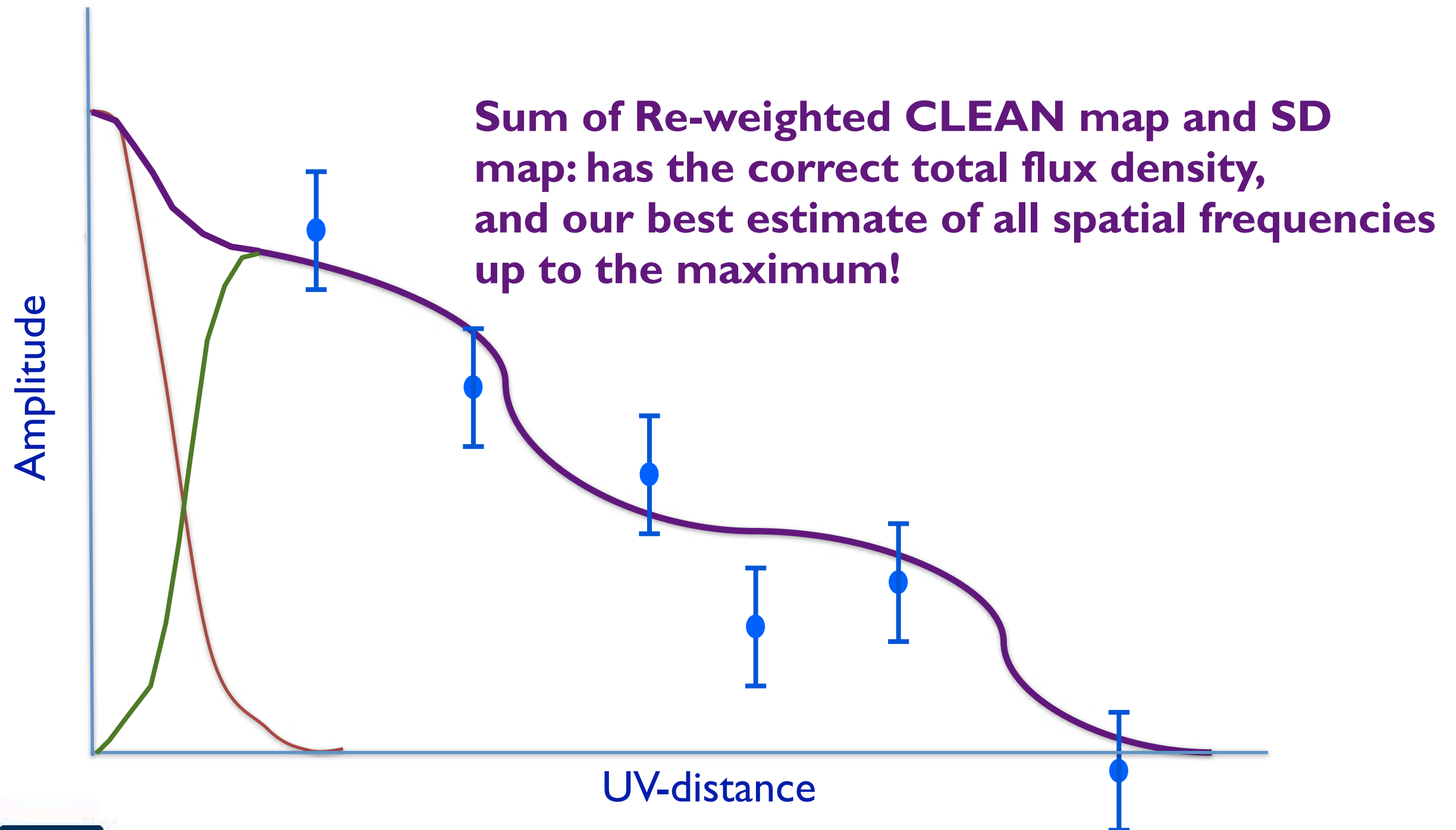
# Feathering

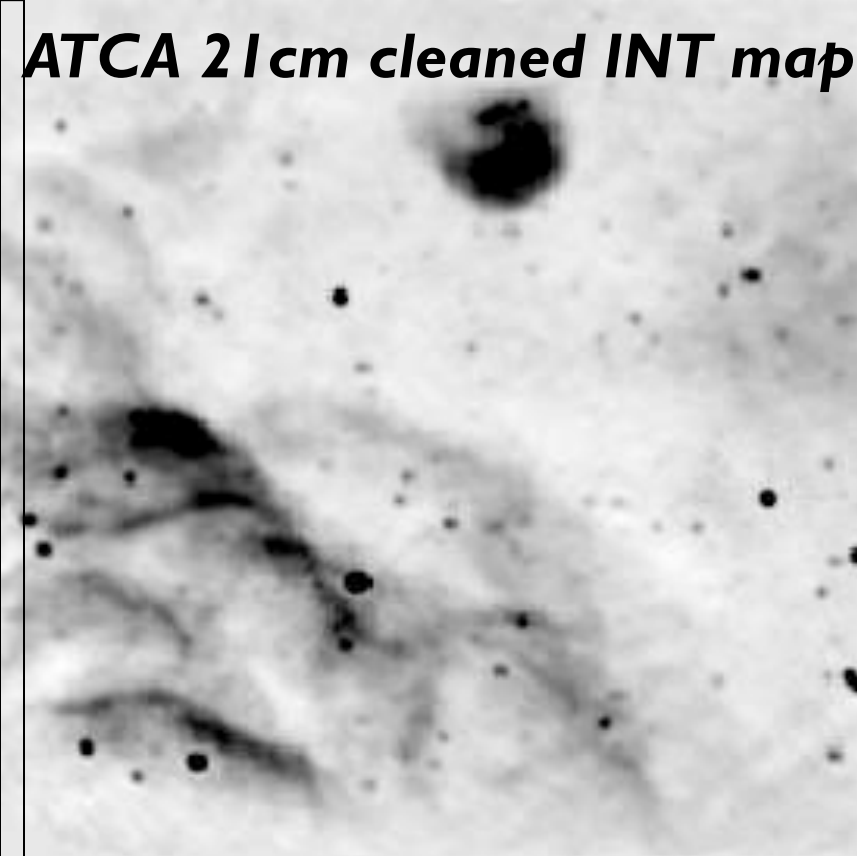


# Feathering

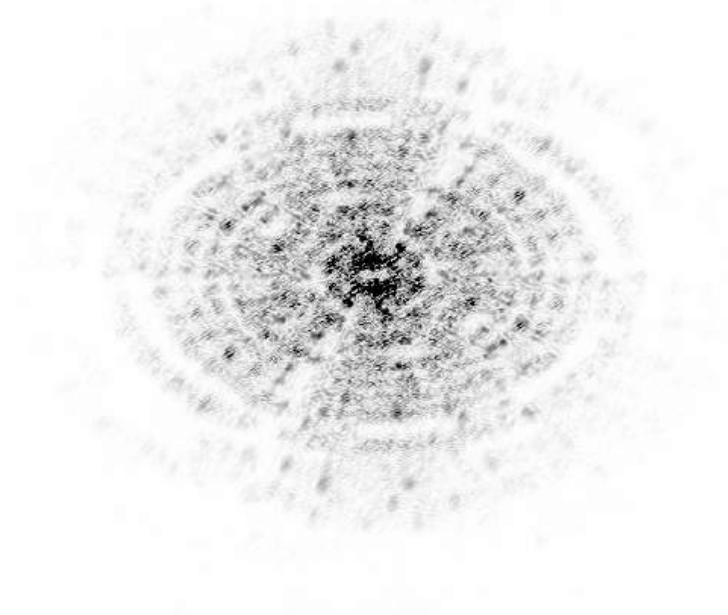


# Feathering



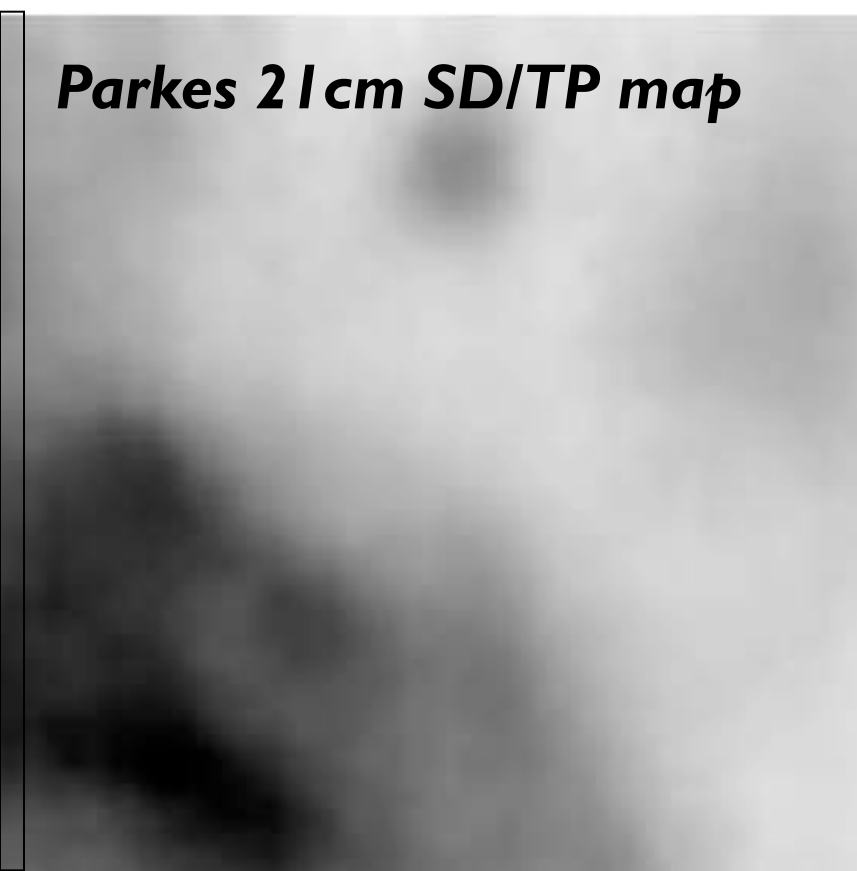
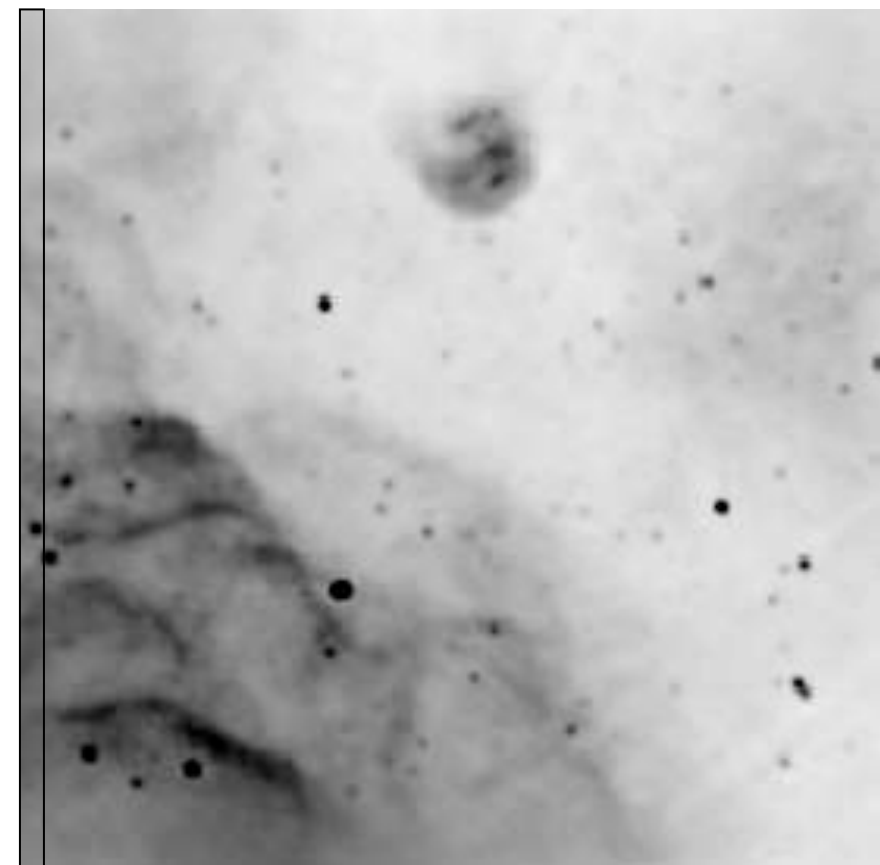


**FT**

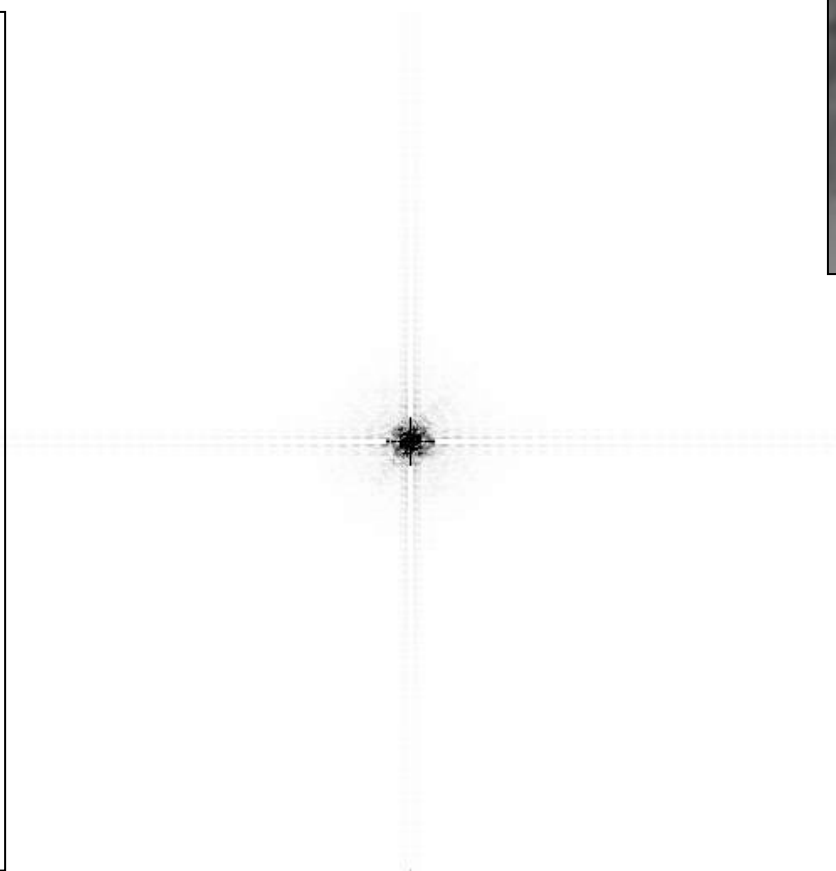


+

**FT<sup>-1</sup>=**



**FT**

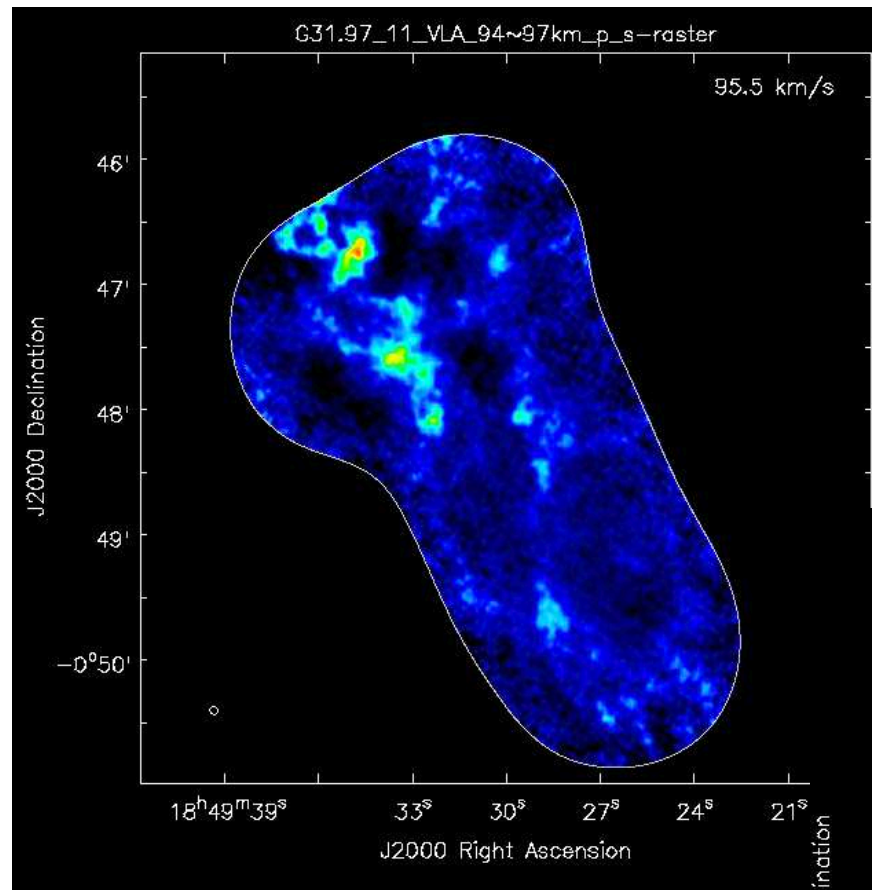


*McClure-Griffiths et al.*

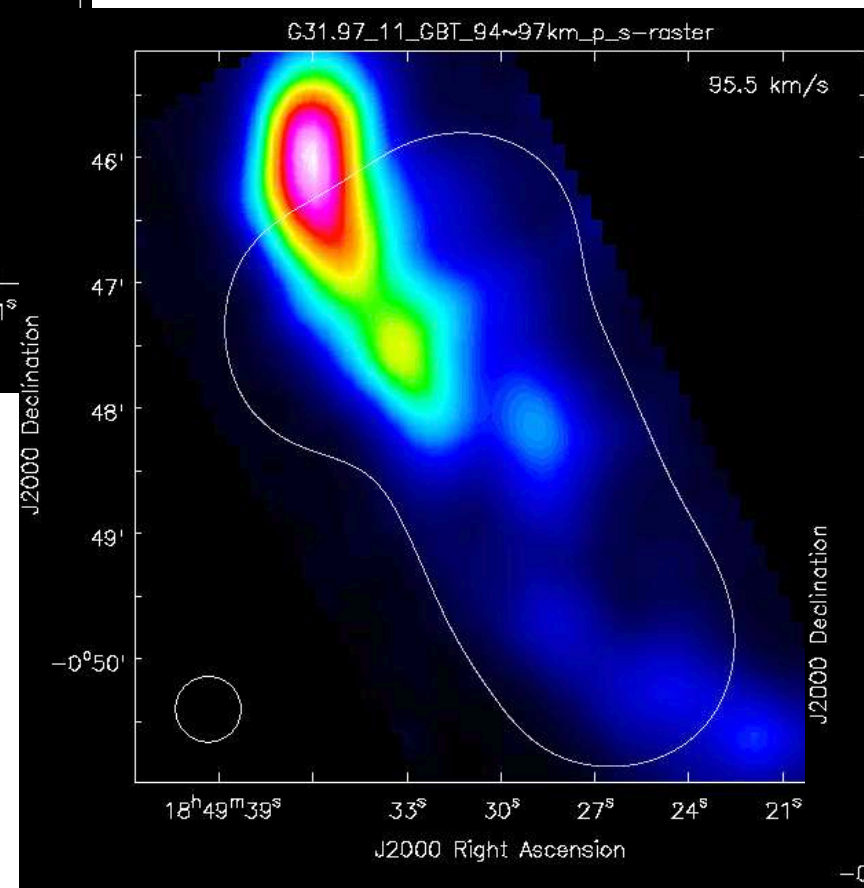




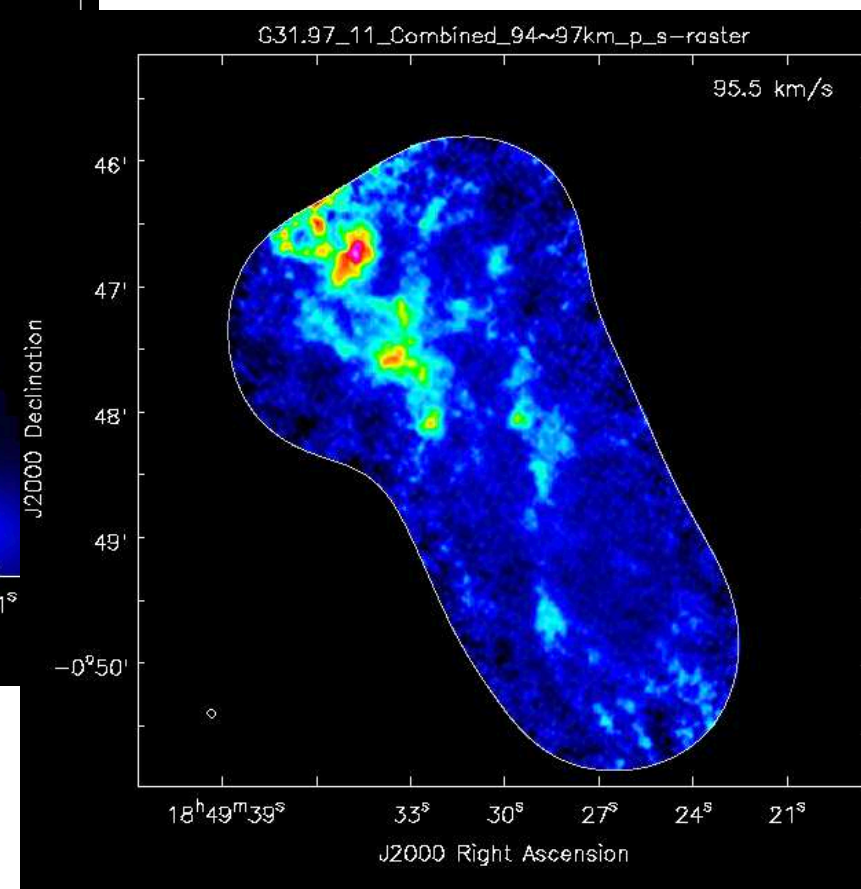
# EVLA $\text{NH}_3$ (multi-scale CLEANed)



**GBT  $\text{NH}_3$**

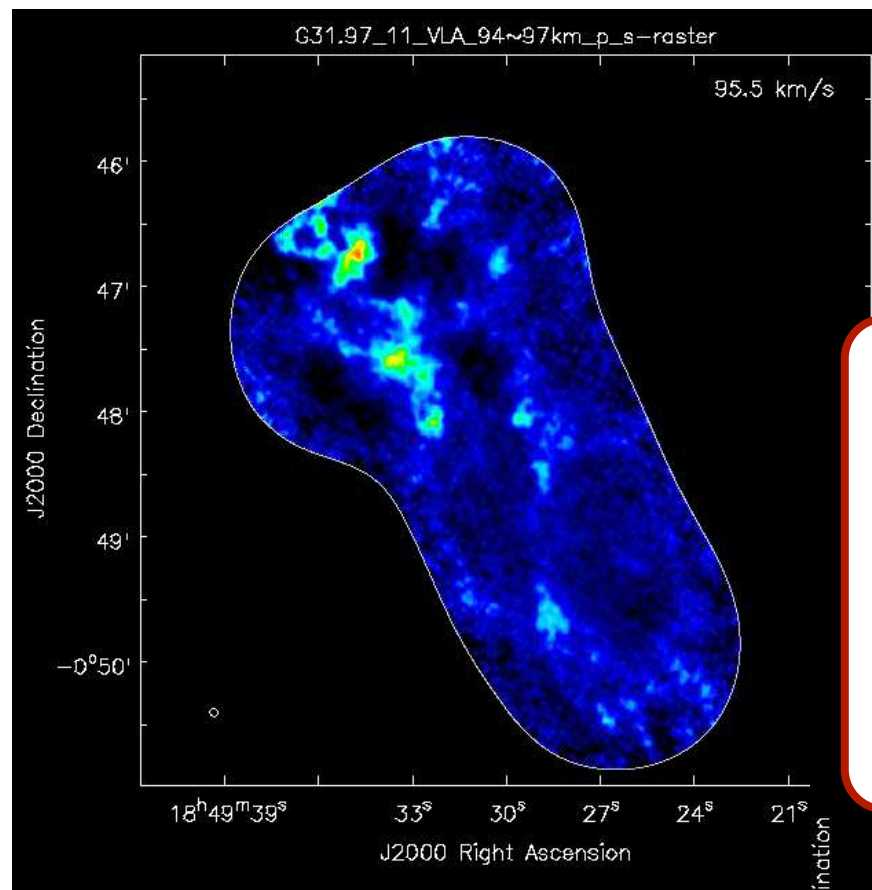


**Feathered**



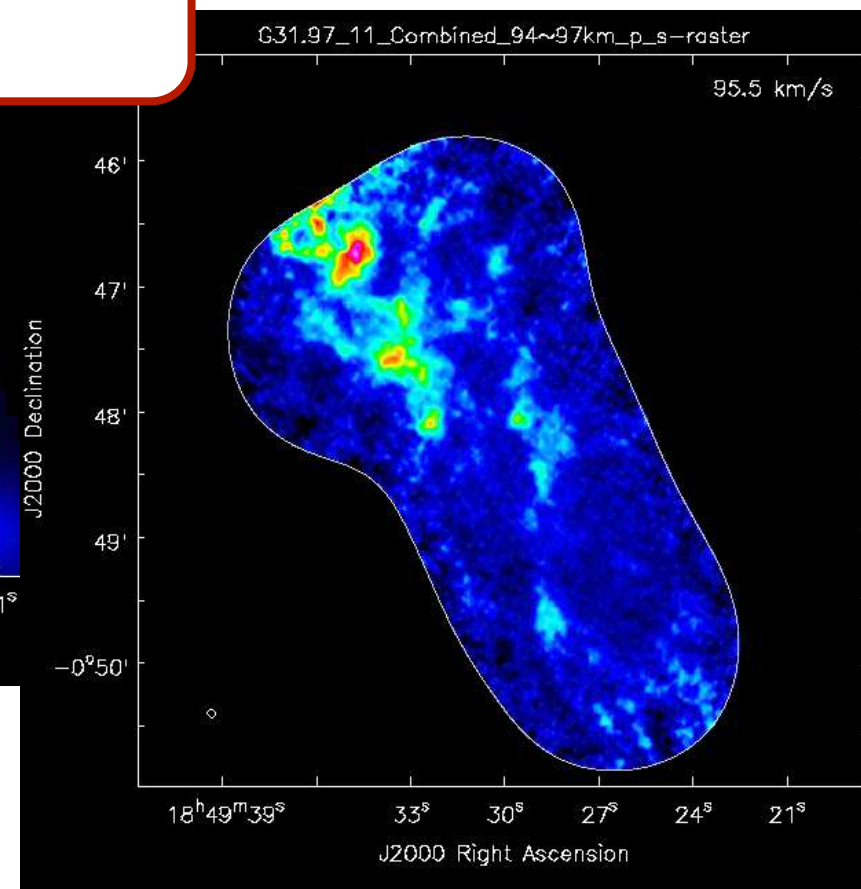
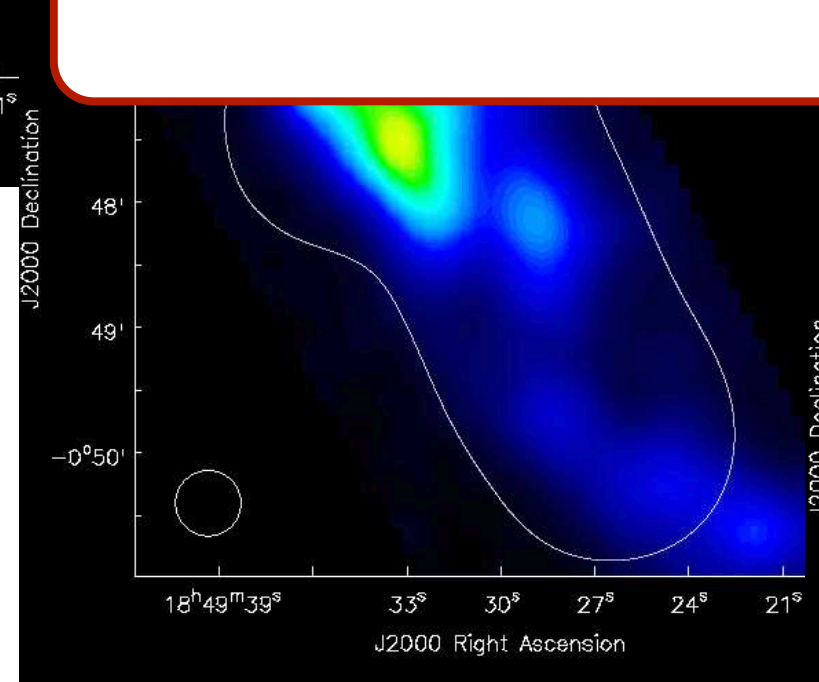
**Courtesy W. DiRienzo et al. (in prep.)**

## EVLA $\text{NH}_3$ (multi-scale CLEANed)

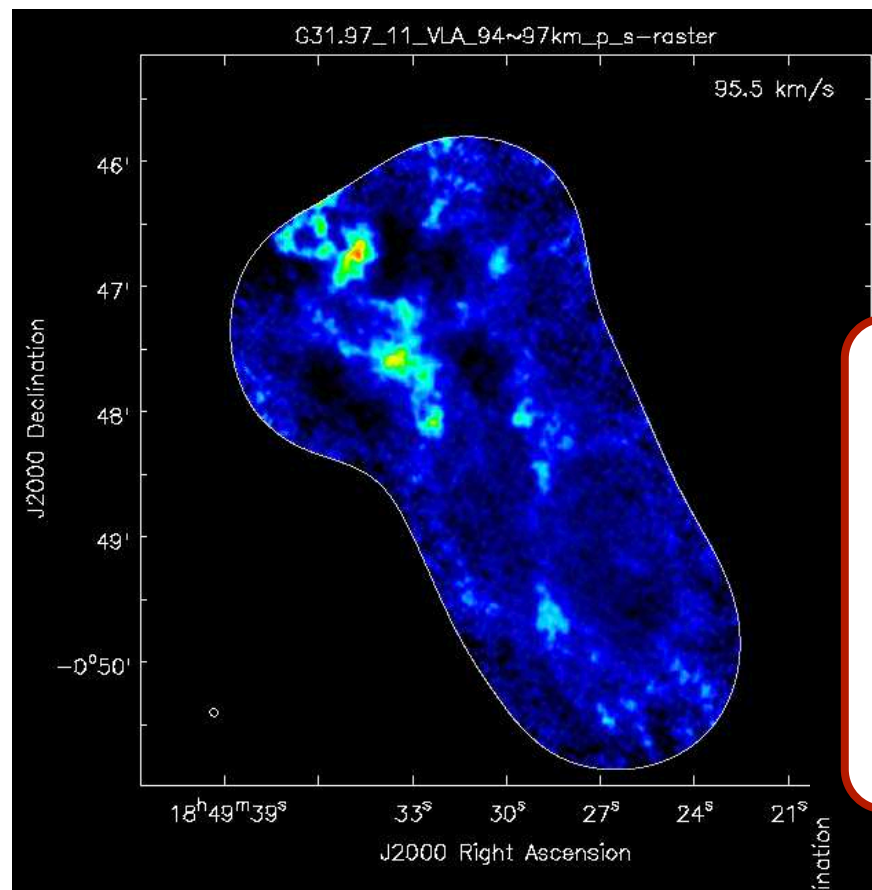


In CASA: Task feather()  
\*input low-res (SD) image  
\*high-res image  
\*SD calibration tweakable

**Feathered**



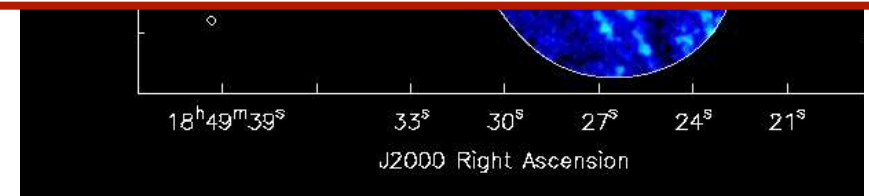
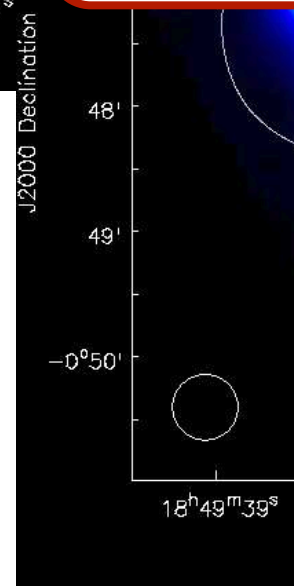
## EVLA $\text{NH}_3$ (multi-scale CLEANed)



In CASA: Task feather()  
\*input low-res (SD) image  
\*high  
\*SD

**Feathering is widely used and fairly robust ...  
but not the only way to do this!**

- \*MEM default image
- \*Turn SD into pseudo-visibilitys, jointly deconvolve together (e.g., Koda et al. 2011)
- \*See S. Stanimirovic article in *Single Dish Summer School Proceedings*



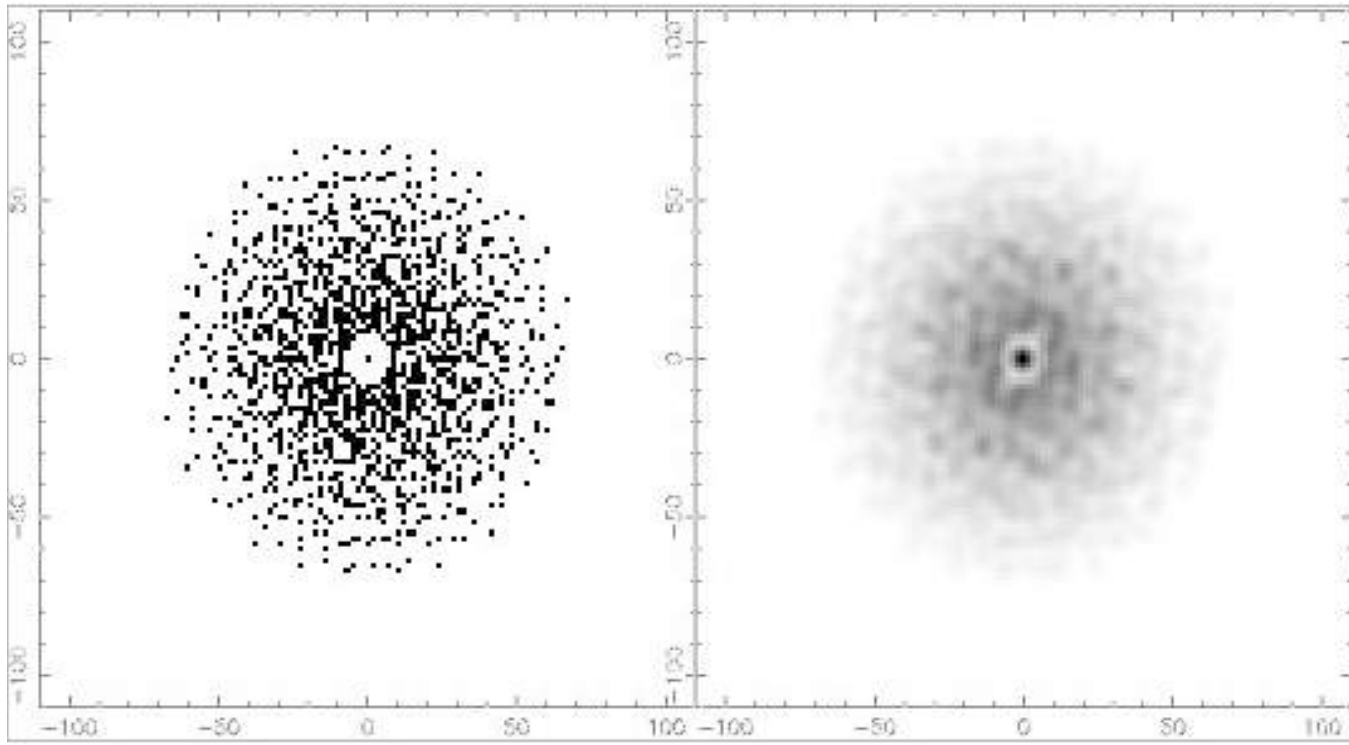


# Single Dish Issues

- Pointing errors
  - minimize; smooth to mitigate somewhat
- Striping
  - Scan rapidly and include signal-free “off” regions (spatial and/or spectral)
    - more of an issue for continuum than spectral line
  - use appropriate calibration & imaging algorithms.
- Sidelobes
  - if significant, you may need to deconvolve the single-dish data before combination (e.g., single-dish clean)
  - at short wavelengths, an “error beam” around the main beam is not uncommon
  - at long wavelengths, aperture blockage can be an issue (clear aperture is better)
- SD Image may *\*not\** have all spatial frequencies down to  $u=v=0$  (e.g., millimeter-wavelength continuum)
- Relative Calibration



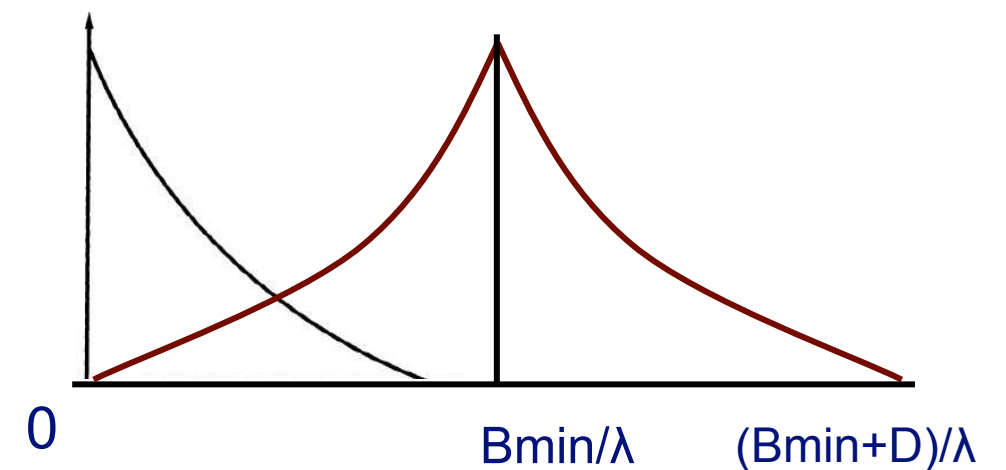
# What Single Dish Data do I Need?



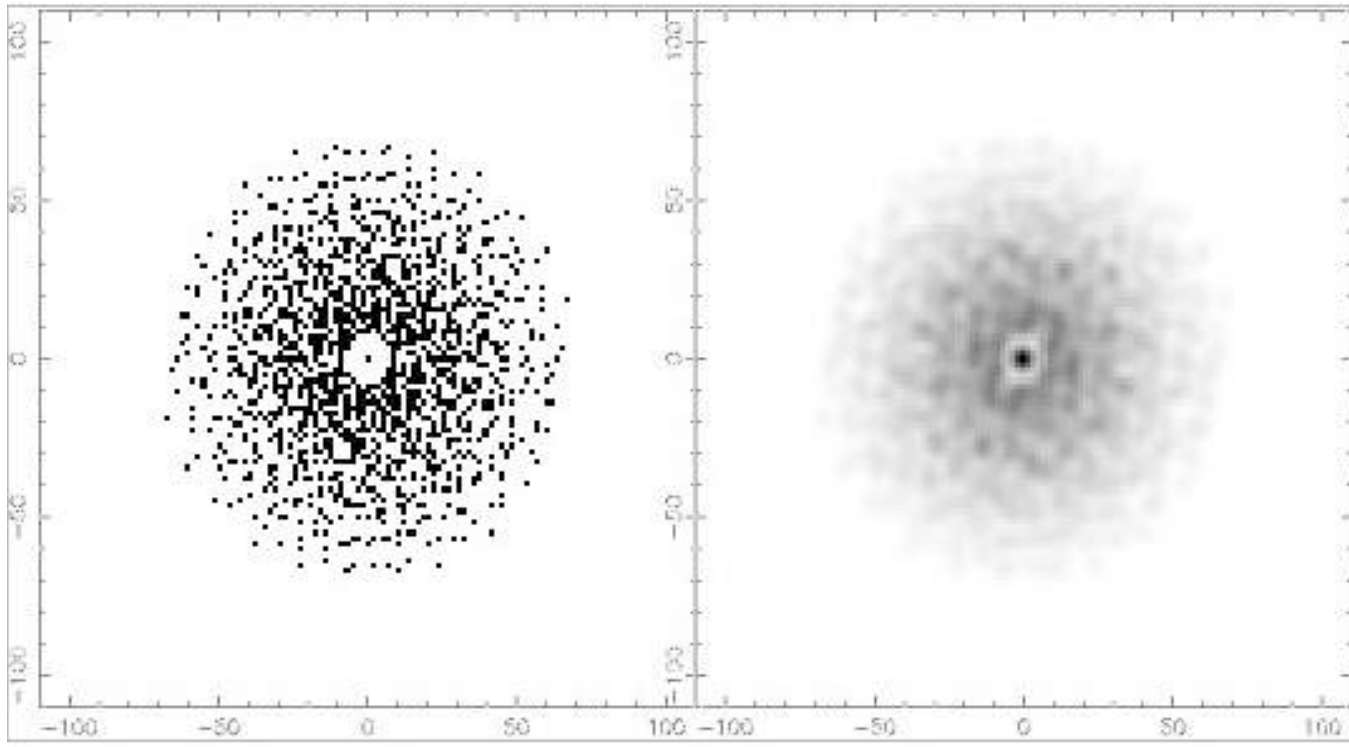
UV Space Sensitivities of  
interferometer (D)  
and single dish (D):

## Problems:

- \*You still have a “hole” between (0,0) and  $B_{\min}$*
- \*No common, well-measured spatial freq's*



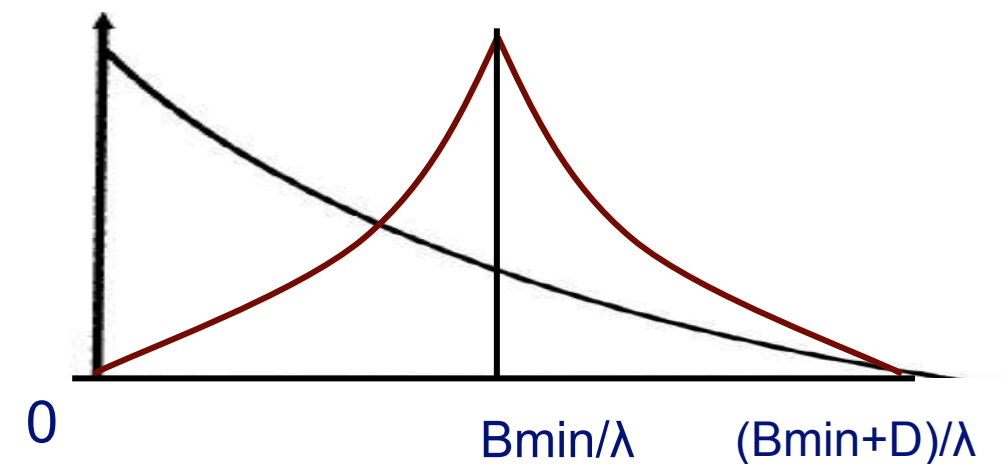
# What Single Dish Data do I Need?



UV Space Sensitivities of  
interferometer (D)  
and single dish (2D):

## Problems:

- \*You still have a “hole” between (0,0) and  $B_{min}$*
- \*No common, well-measured spatial freq’s*



**To maximize flux recovery and image quality, you want a single dish of  $D > 1.5 \times B_{min}$**

7m+12m CLEANed together

7m+12m CLEANed + 12m TP  
(feathered)

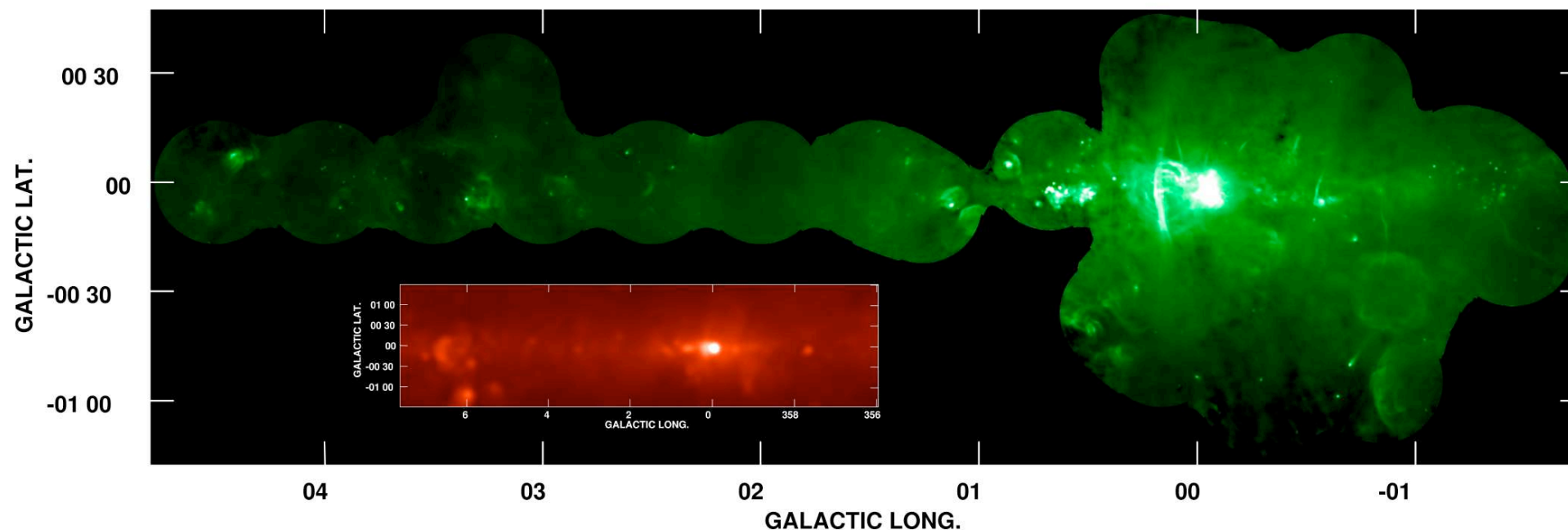
***Total flux is correct!***

0 0.05 0.1 0.15 0.2  
 $\text{mJy/arcsec}^2$



# Summary

- Each visibility of an interferometer measures a range of spatial frequencies
- By mosaicking, you can recover some of this information and make gorgeous, scientifically useful images!
  - Adding single dish data can make them even more useful
- There is no cookie-cutter approach to imaging extended emission with an interferometer, or to adding in single dish data
  - read up, experiment, and talk to some people who have done it before!





# References & Acknowledgements

- Synthesis Imaging Summer School proceedings
  - mosaicking article by M. Holdaway
  - deconvolution article by T.Cornwell
  - previous lectures by J.Ott, D.Shepherd
- Single Dish Summer School
  - article by S.Stanimirovic
- Theory of Mosaicking: Ekers & Rots (1979)
- Joint Deconvolution: Saul, Stavely-Smith, & Brouw (1996)
- CLEANing: Jorsater & VanMoorsel (1995); Walter et al. (2008); Condon et al. (1998); MS Clean: Cornwell (2008)
- Joint Mosaic UV Gridding: Myers et al. (2003)
- Example of Pseudo-Visibility Joint Deconvolution approach to SD+INT combo: Koda et al. (2011)
- Heterogeneous array / SD relative integration times:
  - Pety-Guth et al. (2008); Kuroono et al. (2009); Mason & Brogan (2013)
- Useful discussions with C.Brogan, U.Rao, J.Ott, & others

