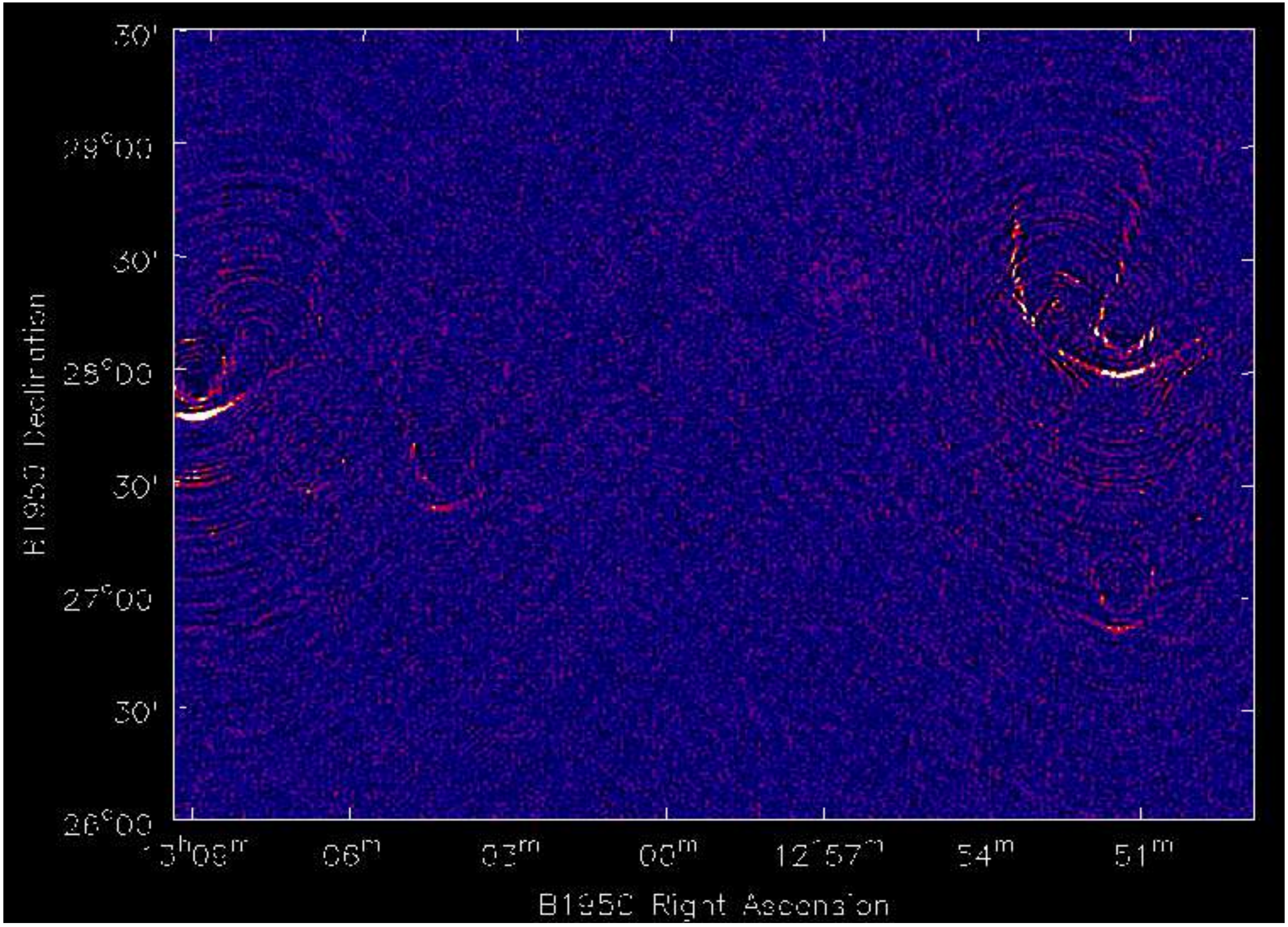
Wide field and wide band imaging

Tim Cornwell

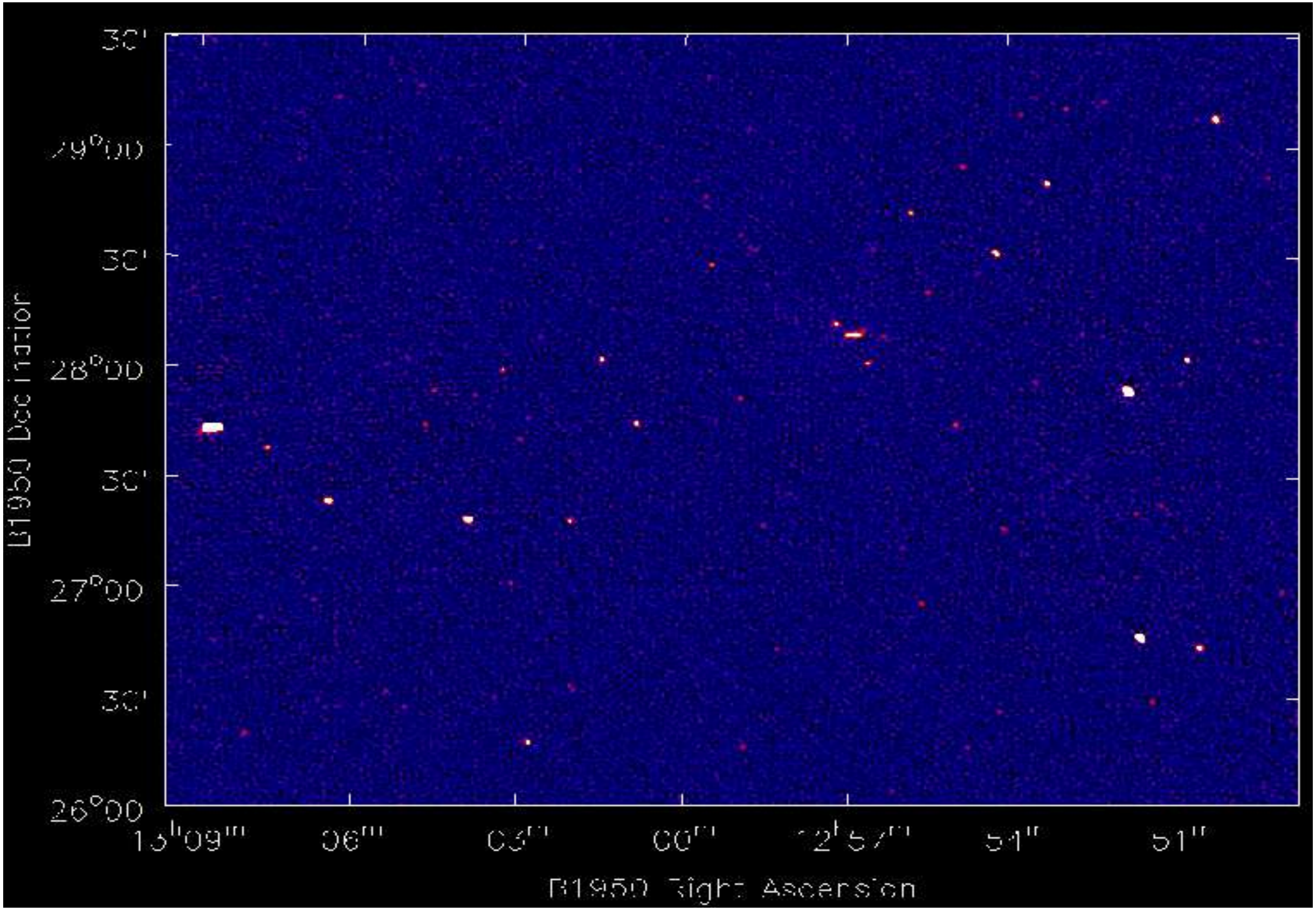
Overview

- Wide field effects in imaging
- Wide band effects in imaging (largely defer to Urvashi’s talks)

Standard 2D imaging

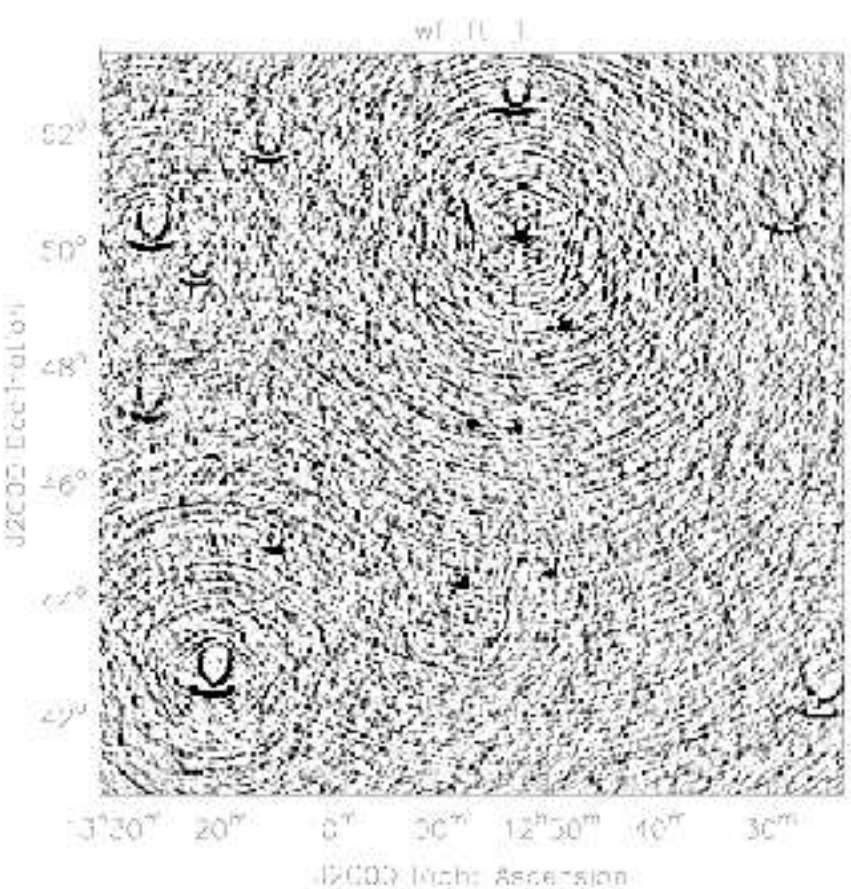


Faceted imaging

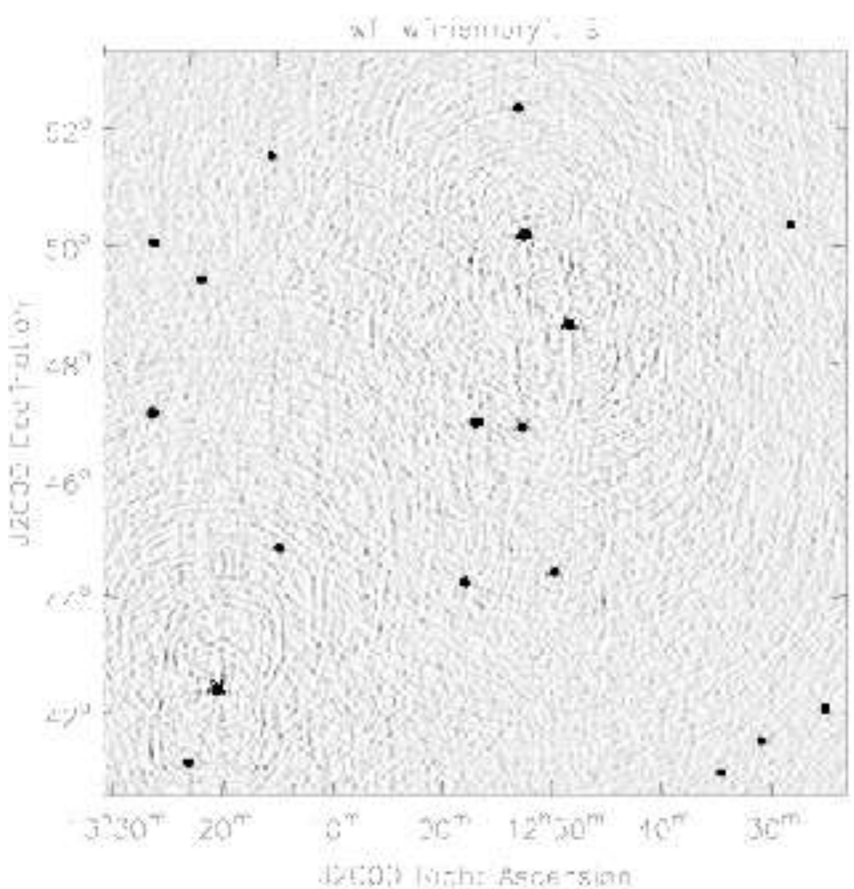


Comparison of 2D, faceted, w projection algorithms

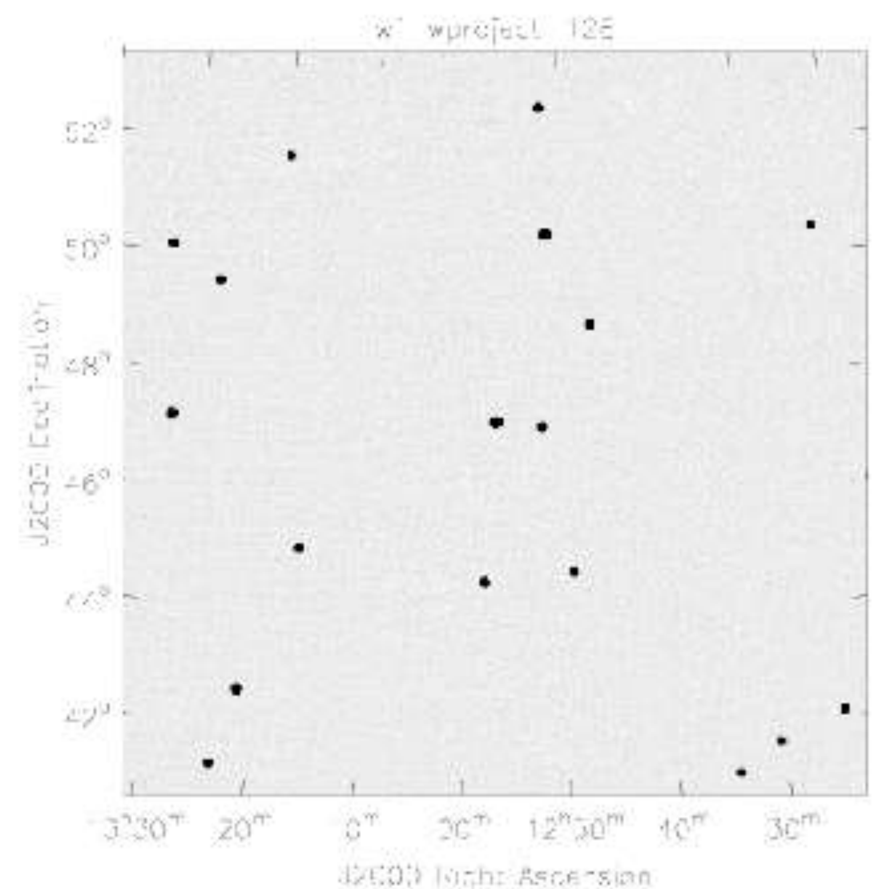
2D Fourier transform



Faceted Fourier transforms



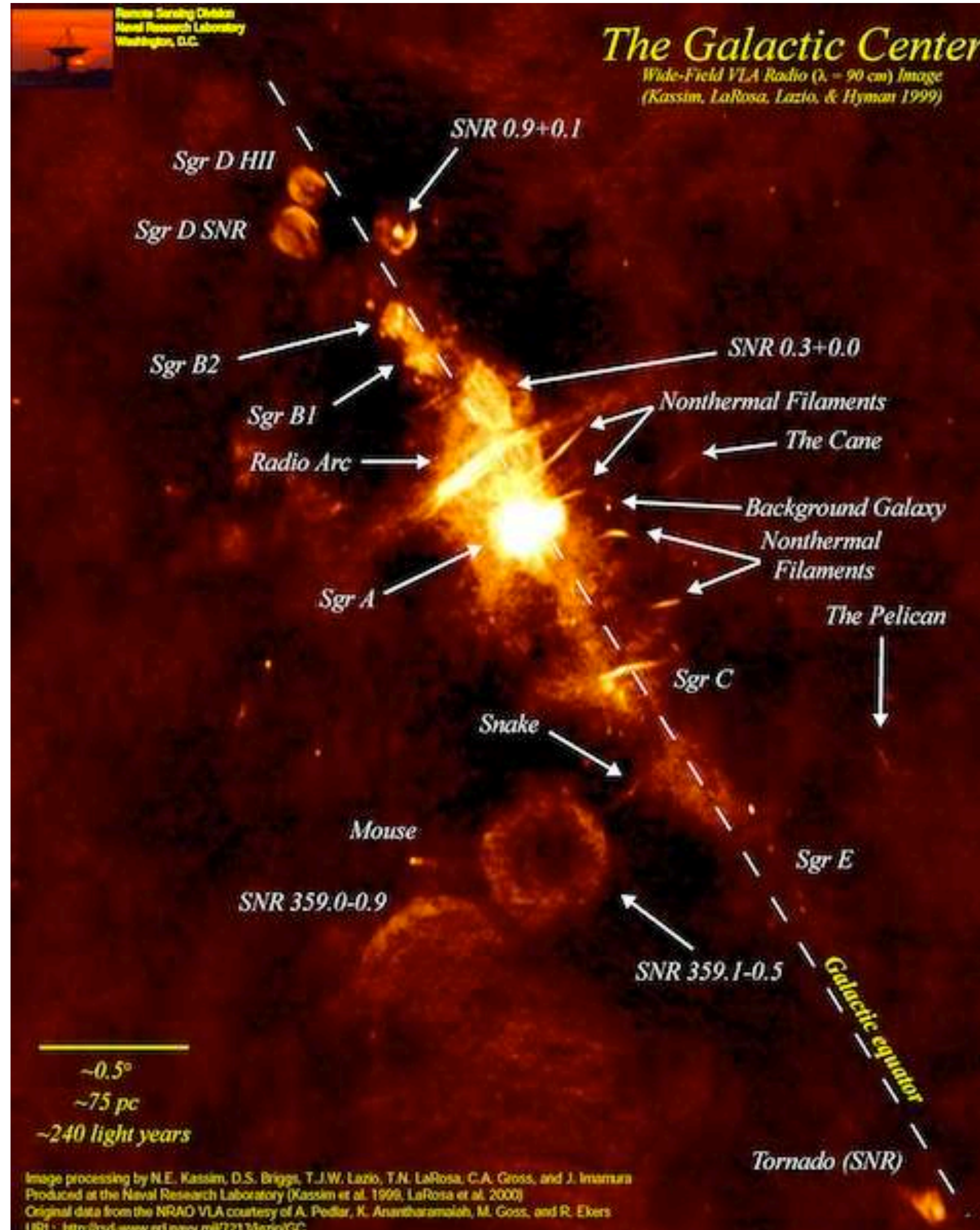
W projection



Simulation of VLA 74 MHz long integration

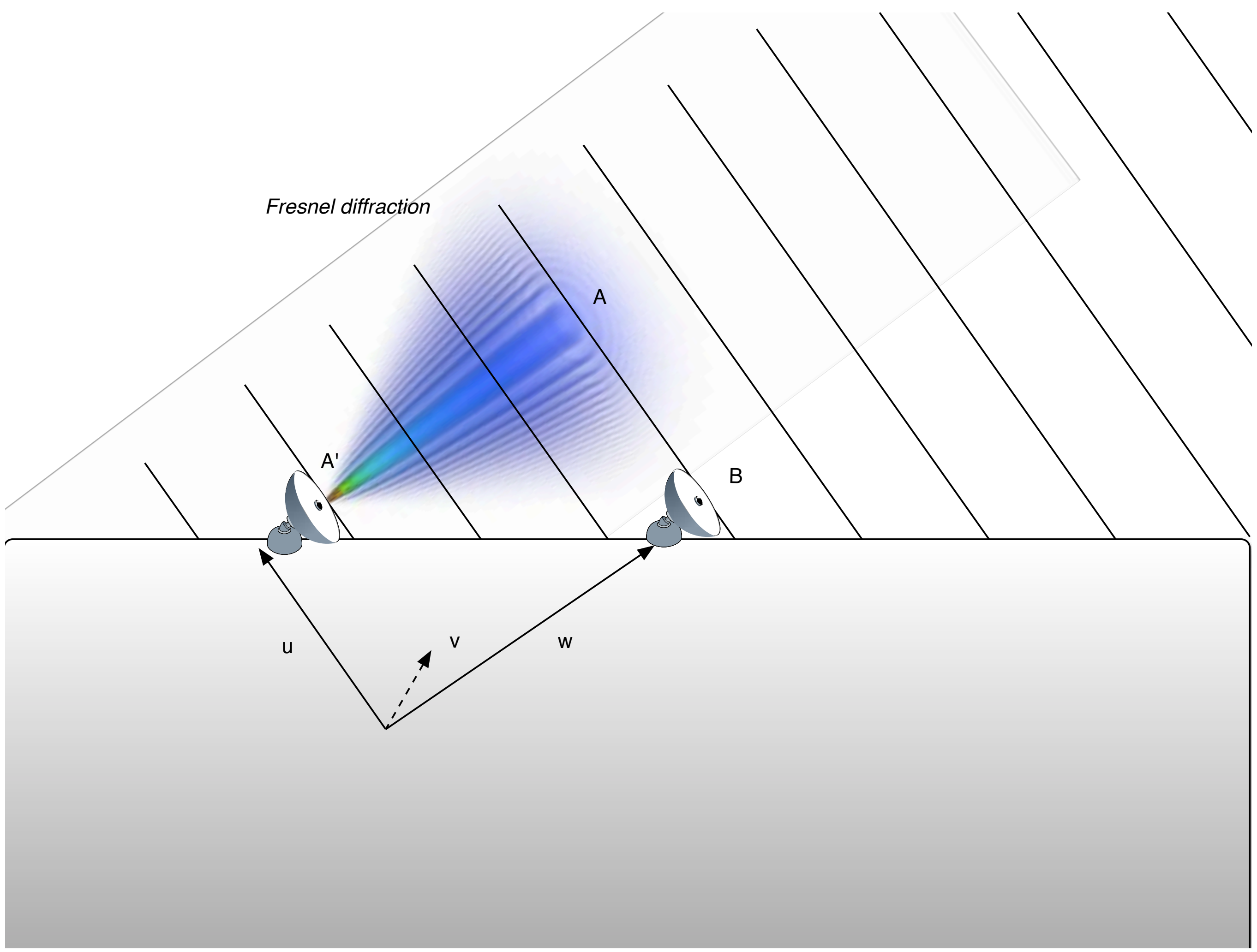
Galactic centre at 327 MHz

- VLA 330MHz image
- Only possible when w term could be corrected
- Early example of parallel processing
- Used Fortran + PVM on networked IBM RS-6000s
- Dan Briggs did this processing



Origin of w term effect

$$V(u,v,w)=\int \frac{I(l,m)}{\sqrt{1-l^2-m^2}} e^{2\pi j\left(ul+vm-w\left(\sqrt{1-l^2-m^2}-1\right)\right)} dldm$$



- Ron showed how to decompose single dish into antennas + delay lines
- Delay is a function of direction on sky
- First consequence: need to sample sufficiently in delay
- Second consequence: need to propagate to same wavefront

Can telescope design help?

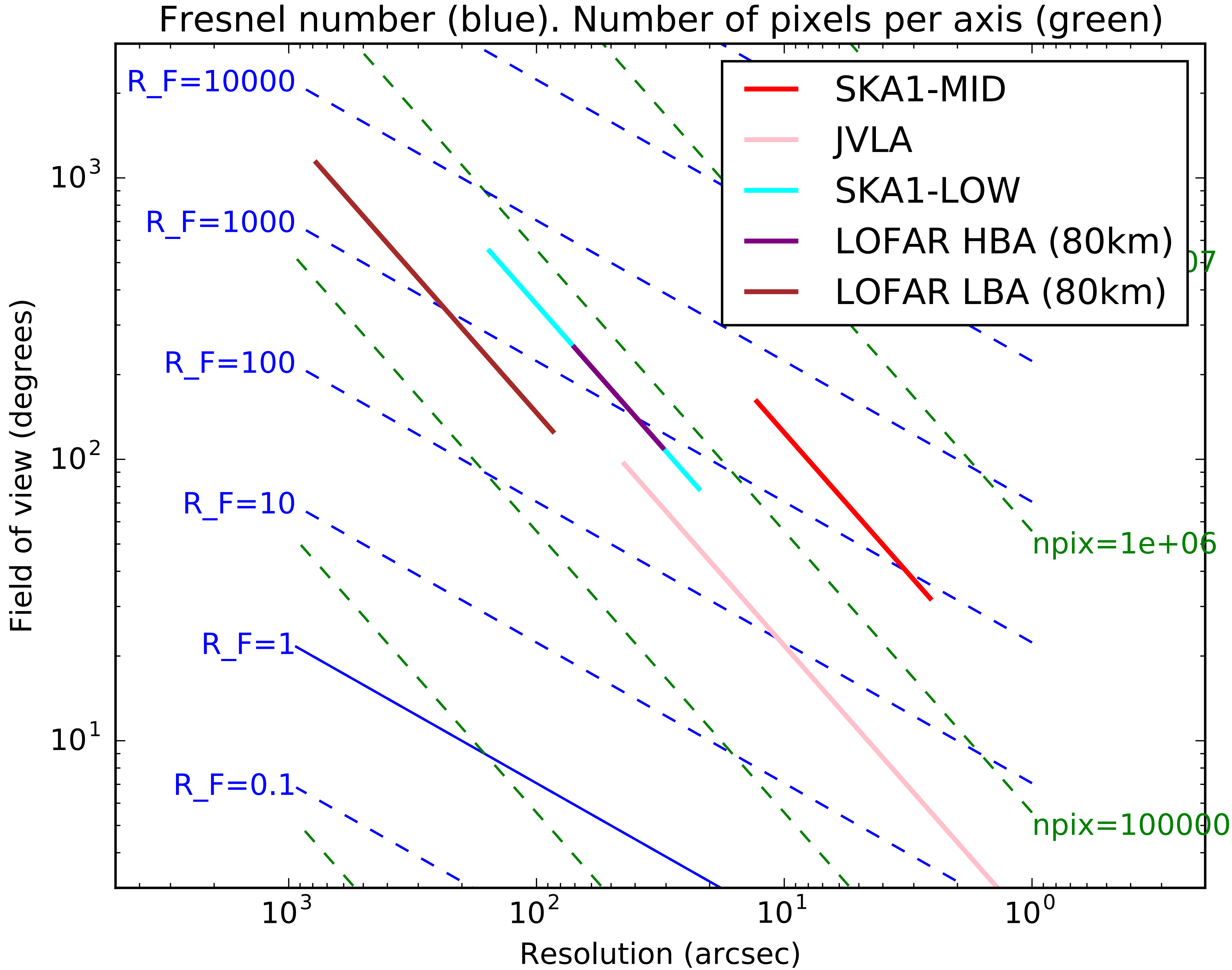
- Coplanar array e.g. WSRT, ATCA, MWA
- Large antennas/stations

Fresnel number

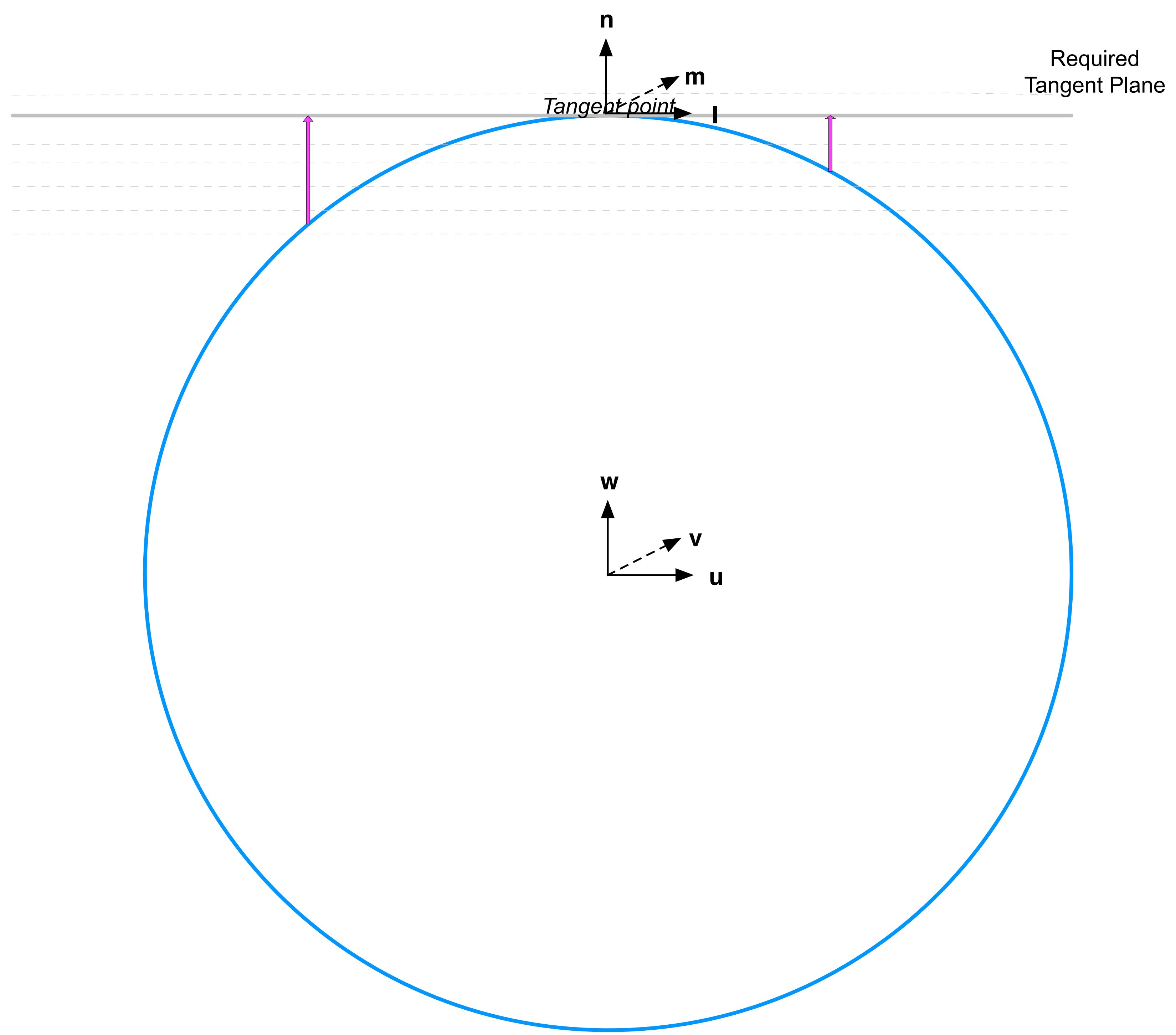
- Strength of phase term is measured by Fresnel number
- Worst at lower frequencies

$$R_F = \frac{\theta_{FOV}^2}{\theta_{synth}}$$

$$R_F \approx \frac{\lambda B}{D^2}$$


Fresnel number (blue). Number of pixels per axis (green)
R_F=10000
R_F=1000
R_F=100
R_F=10
R_F=1
R_F=0.1
npix=1e+06
npix=100000
SKA1-MID
JVLA
SKA1-LOW
LOFAR HBA (80km)
LOFAR LBA (80km)
Field of view (degrees)
Resolution (arcsec)
10^3
10^2
10^1
10^0

Coordinate systems



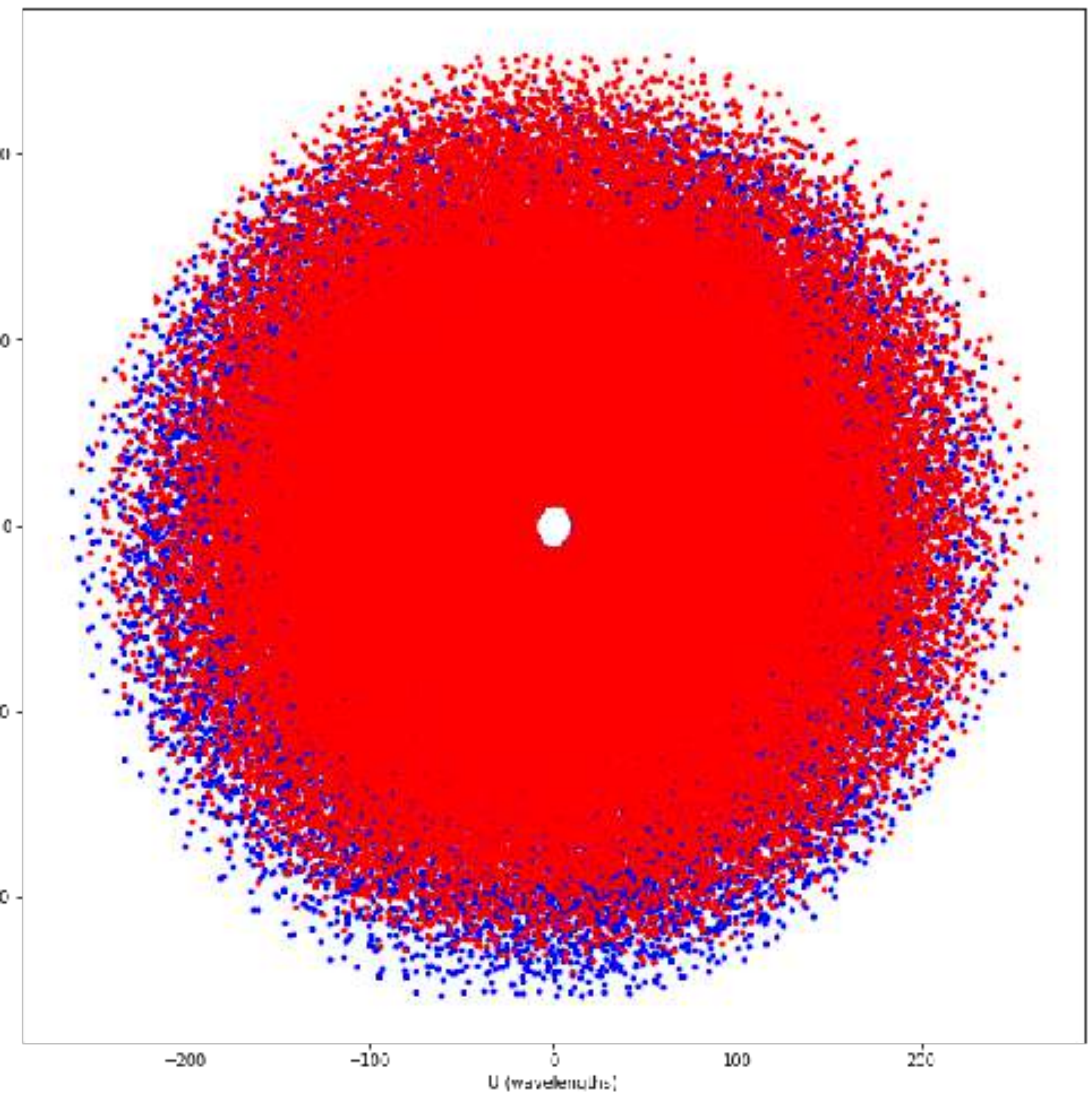
3D imaging

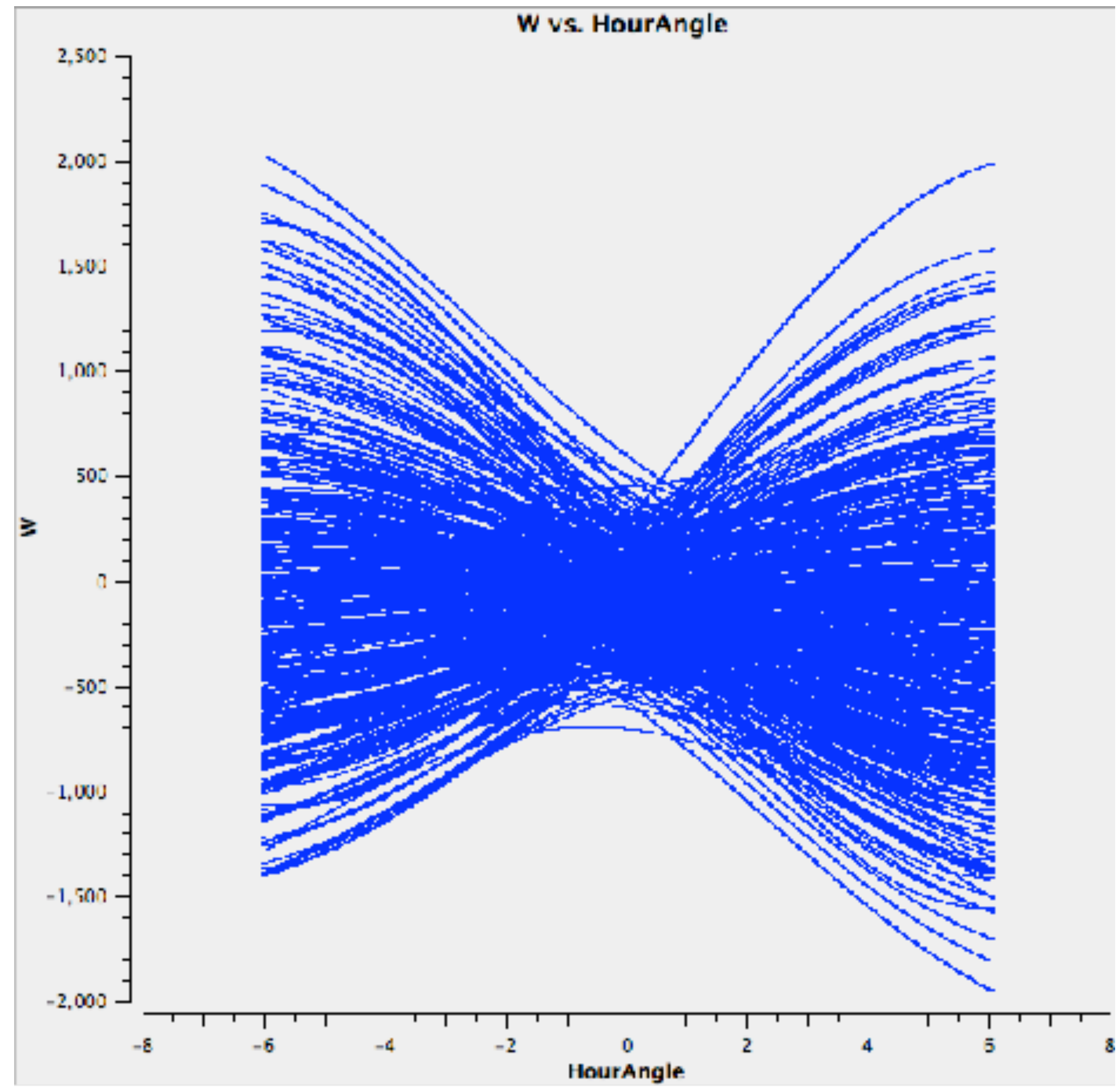
$$V(u,v,w)=\int \frac{I(l,m,n)}{n}\delta\left(l^2+m^2+n^2-1\right)e^{2\pi j\left(ul+vm+w\left(n-1\right)\right)}dldmdn$$

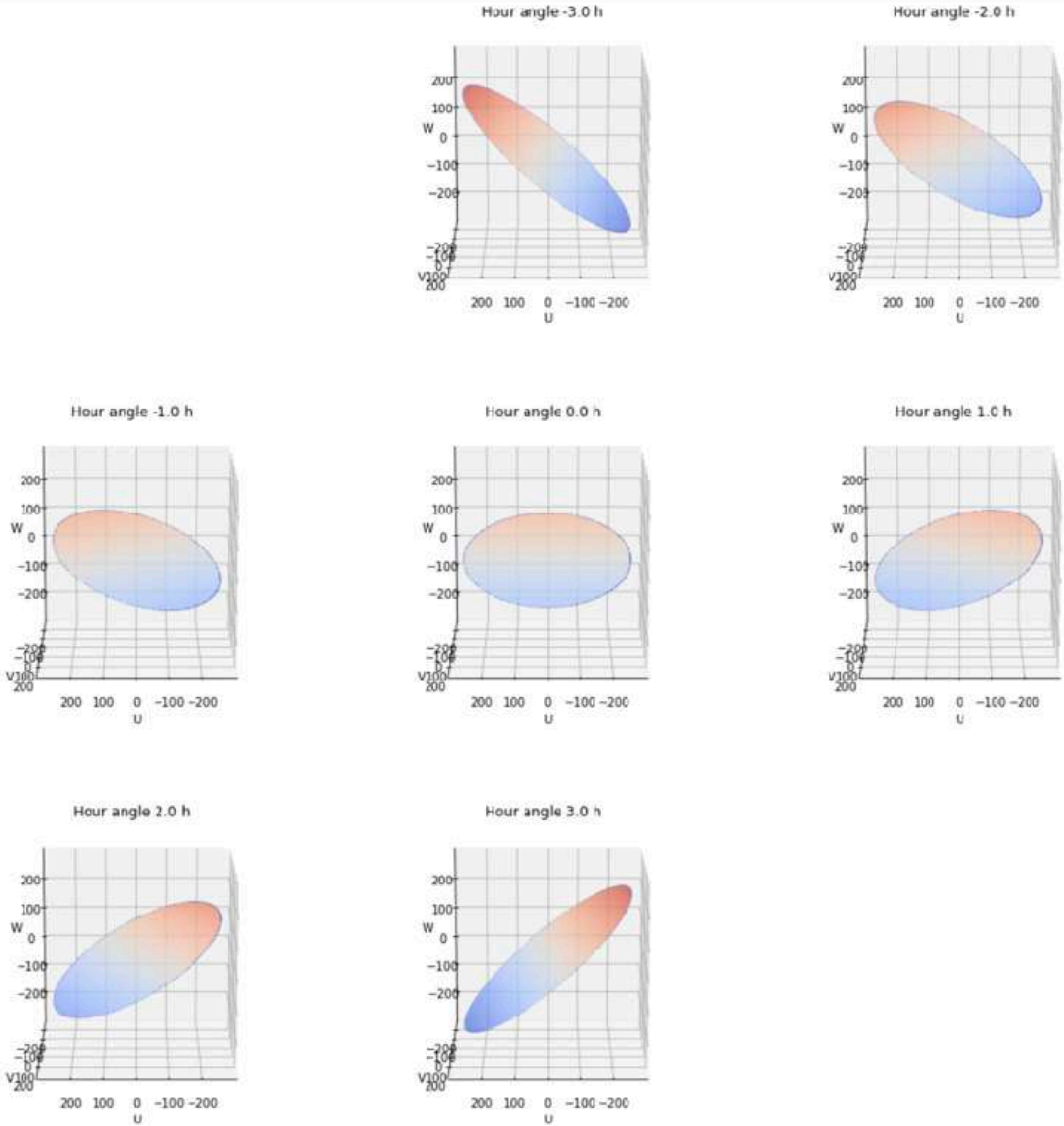
- Easy to do
- Obtain 3D convolution equation
- PSF in (l,m,n) space is shift-invariant
- Practical only for small cases
- Most of cube in (l,m,n) is empty

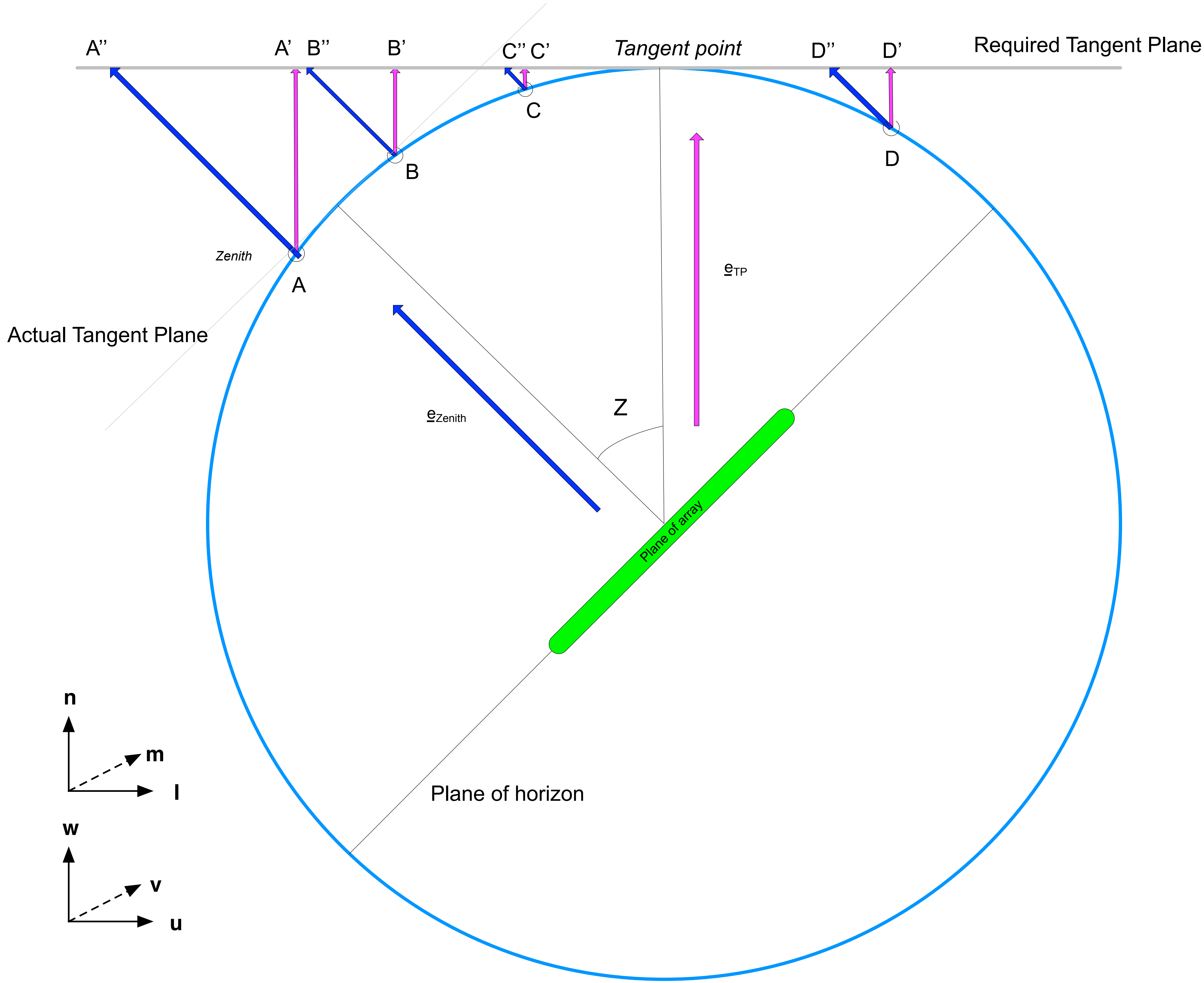
$$I^{D,3}=B^{D,3}\otimes I^{sky,3}$$

Simulation of SKA1-LOW core long observation






Hour angle -3.0 h
Hour angle -2.0 h
Hour angle -1.0 h
Hour angle 0.0 h
Hour angle 1.0 h
Hour angle 2.0 h
Hour angle 3.0 h
W
U
V
200
100
0
−100
−200

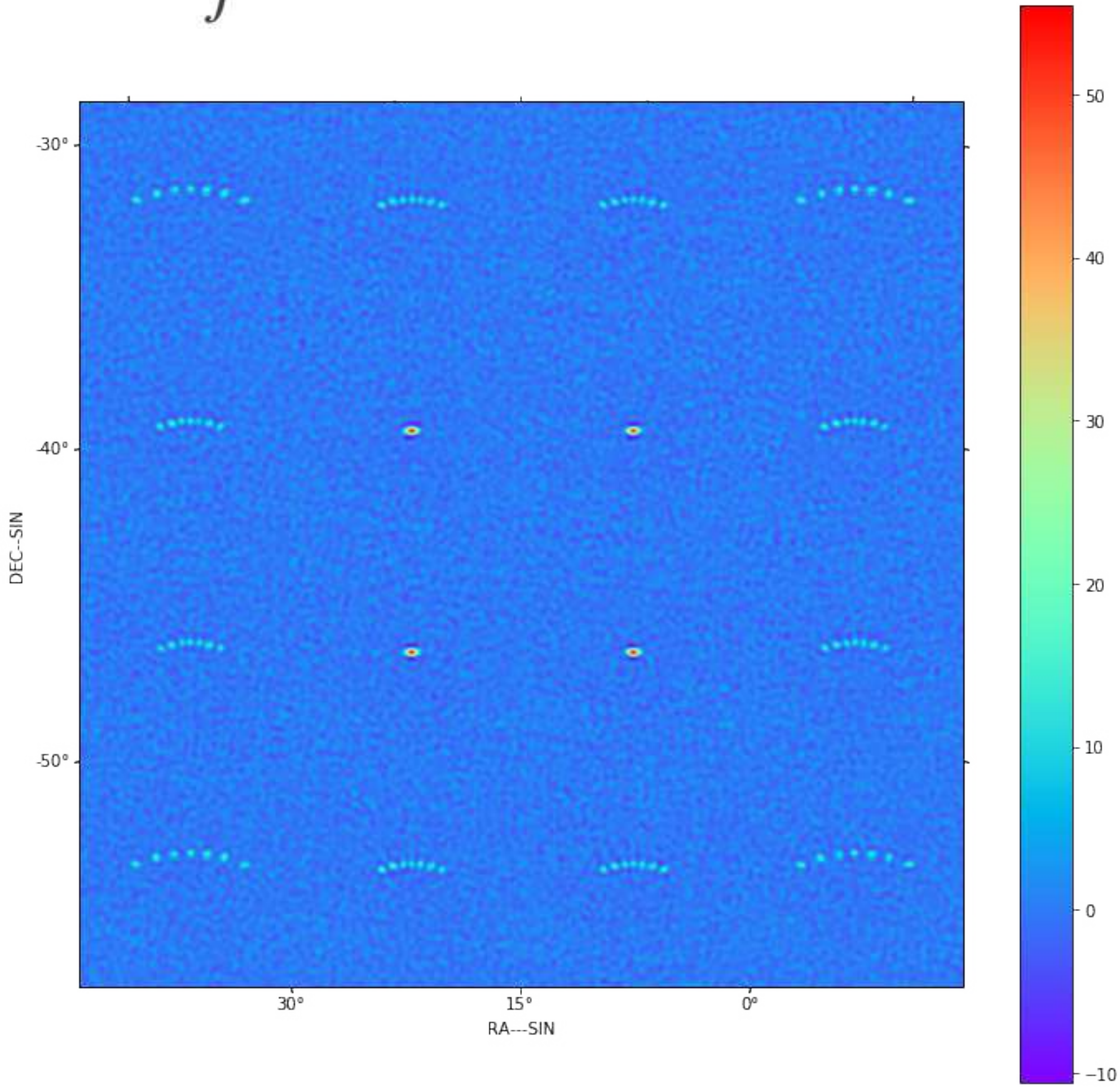


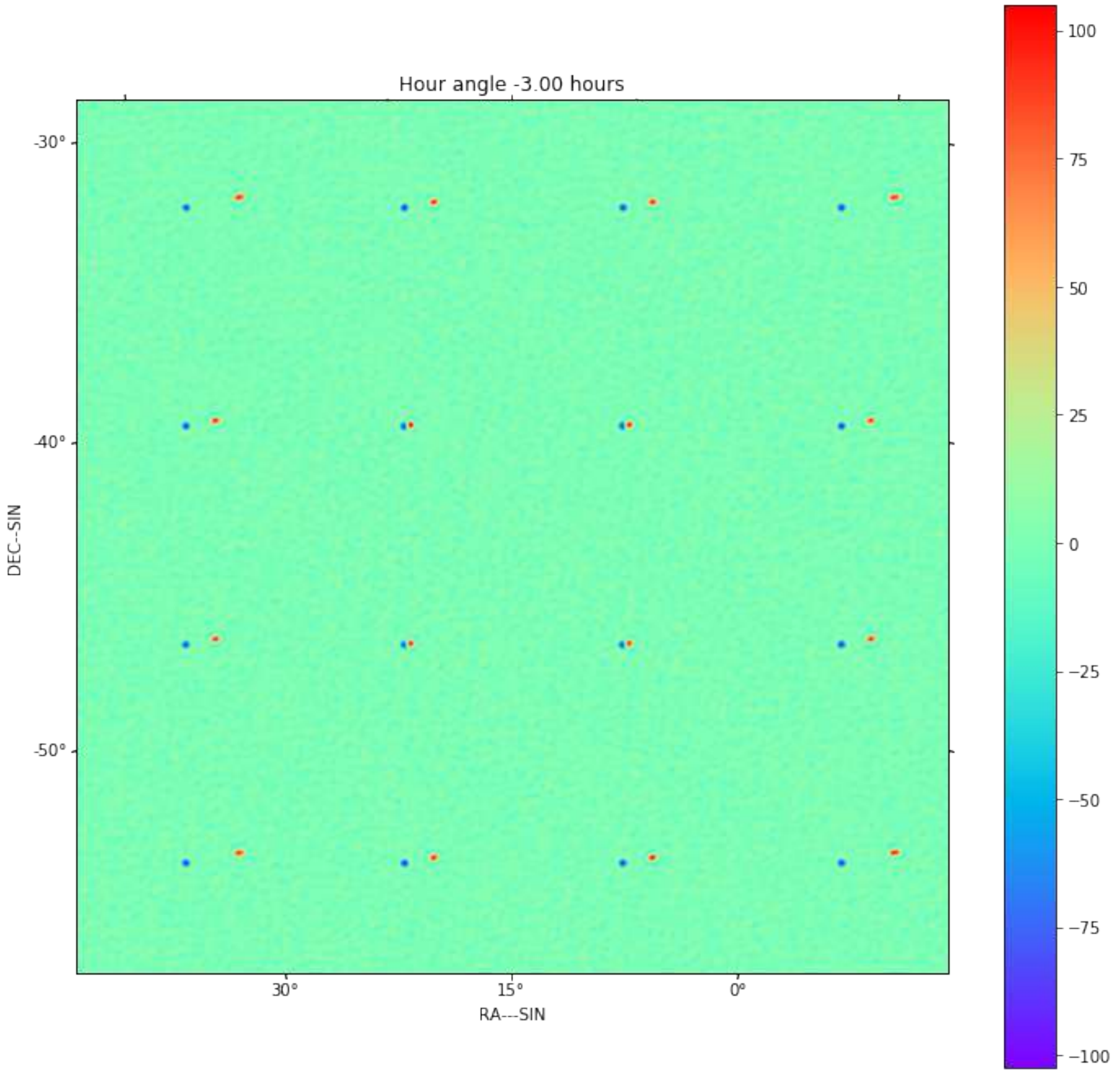
A''
A'
B''
B'
C'' C'
Tangent point
D''
D'
Required Tangent Plane
C
B
D
Zenith
A
Actual Tangent Plane
$\underline{e}_{TP}$
Z
$\underline{e}_{Zenith}$
Plane of array
Plane of horizon
n
m
l
w
v
u

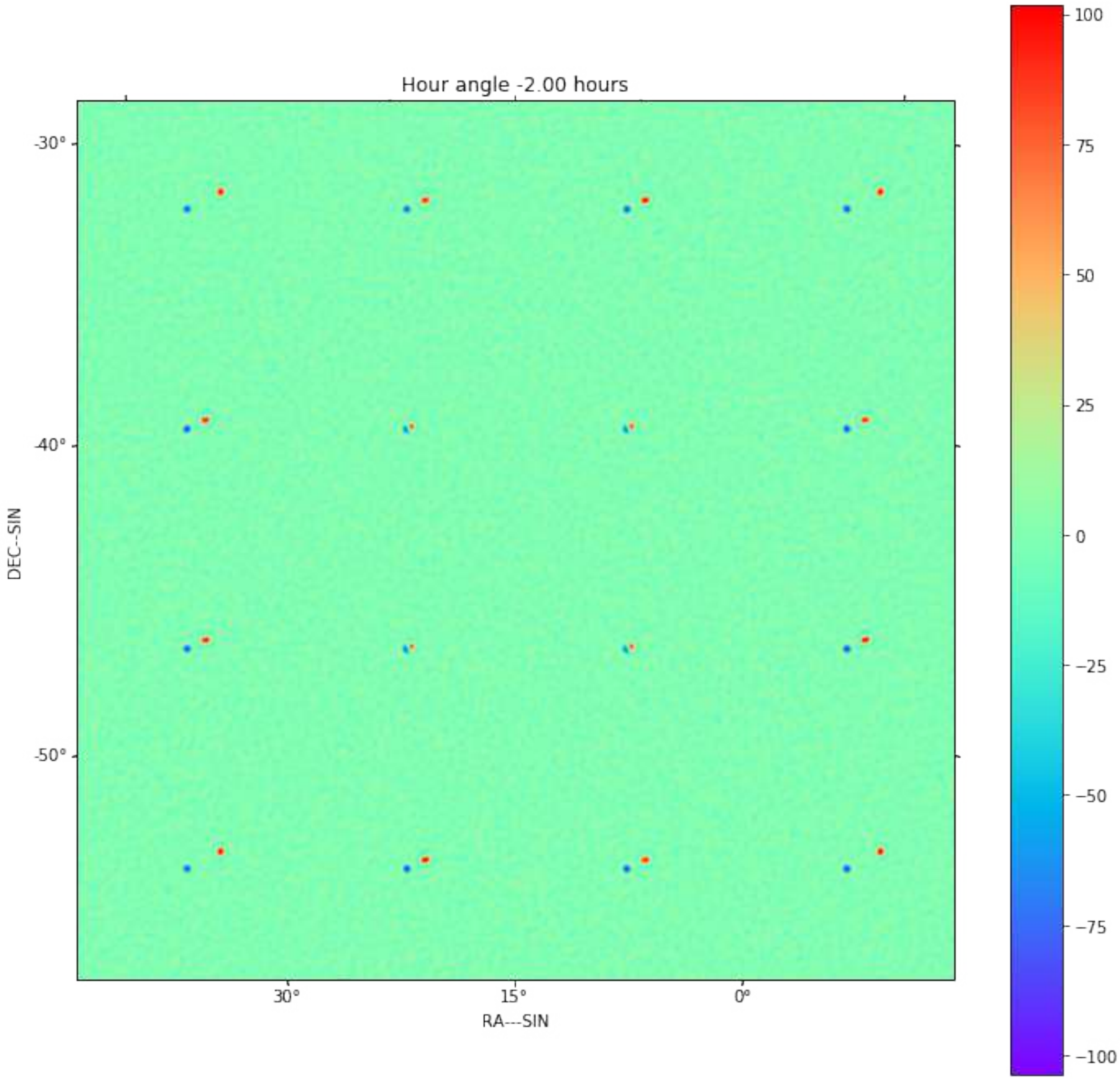
$$V(u, v, w) = \int I(l, m)e^{2\pi j(ul+um)}\, dl dm$$

2D transform for 7 hourangles

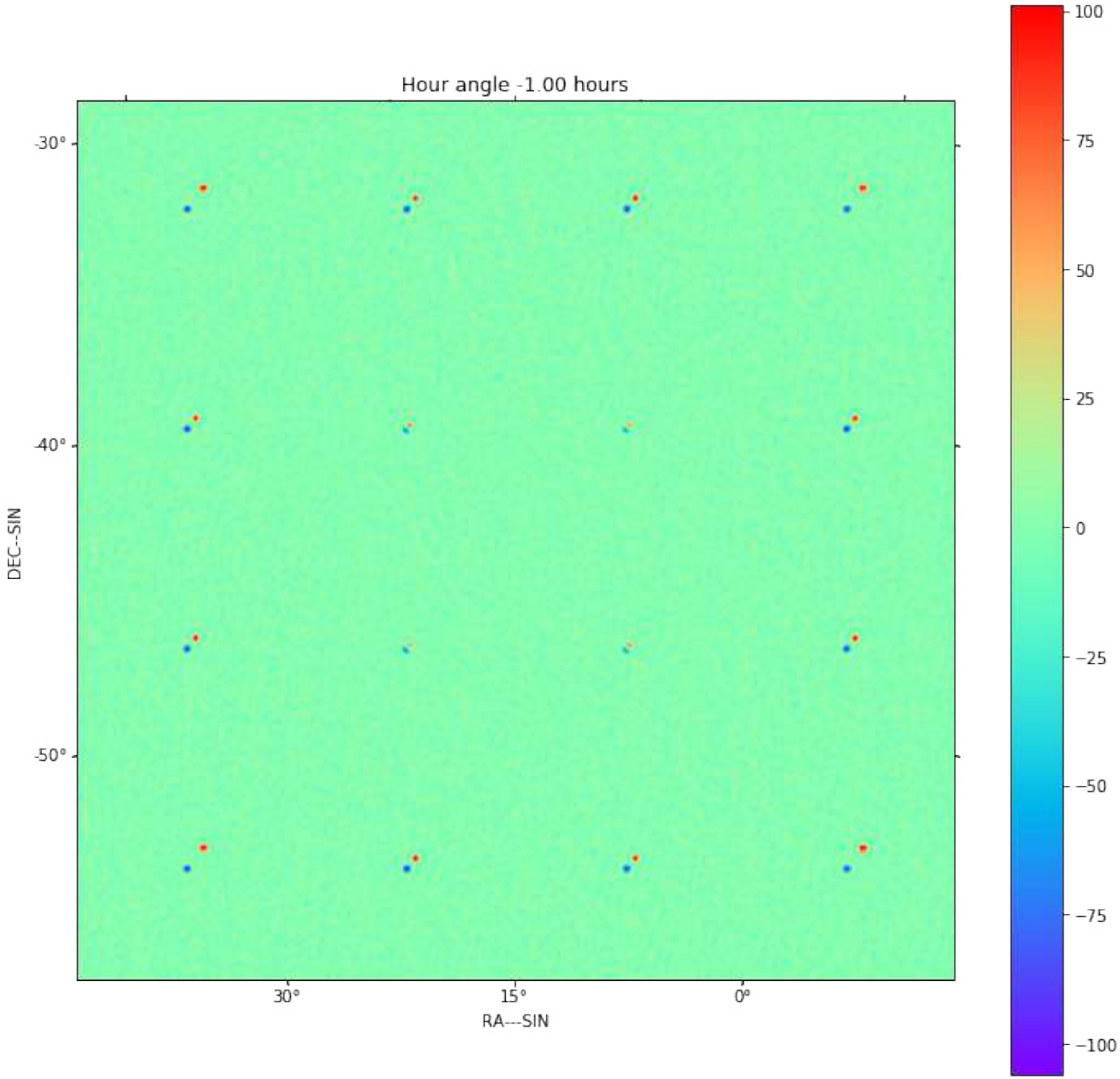
Each hour angle causes different position error

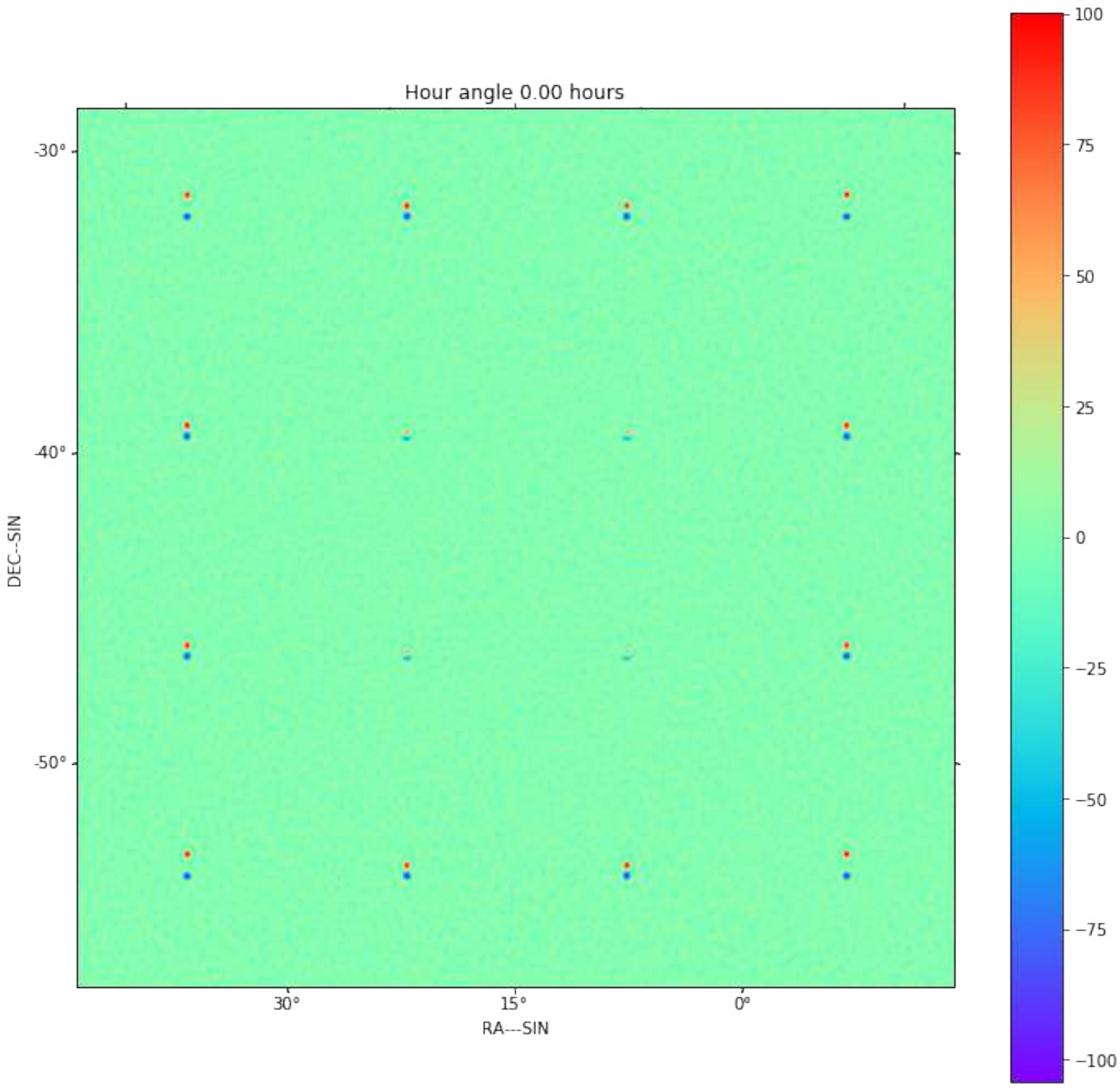



Hour angle -3.00 hours
-30°
-40°
-50°
DEC--SIN
30°
15°
0°
RA---SIN
100
75
50
25
0
−25
−50
−75
−100

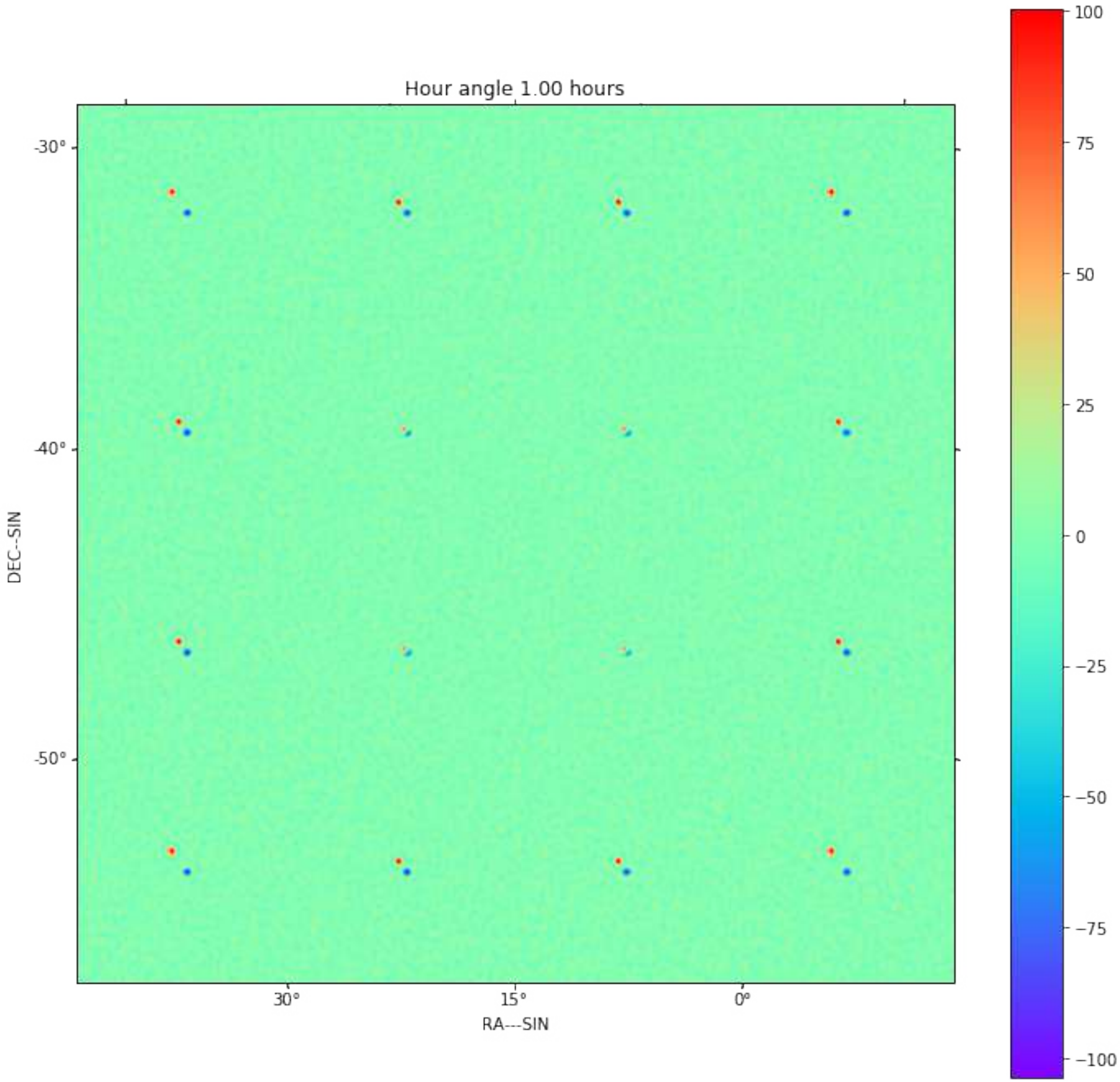


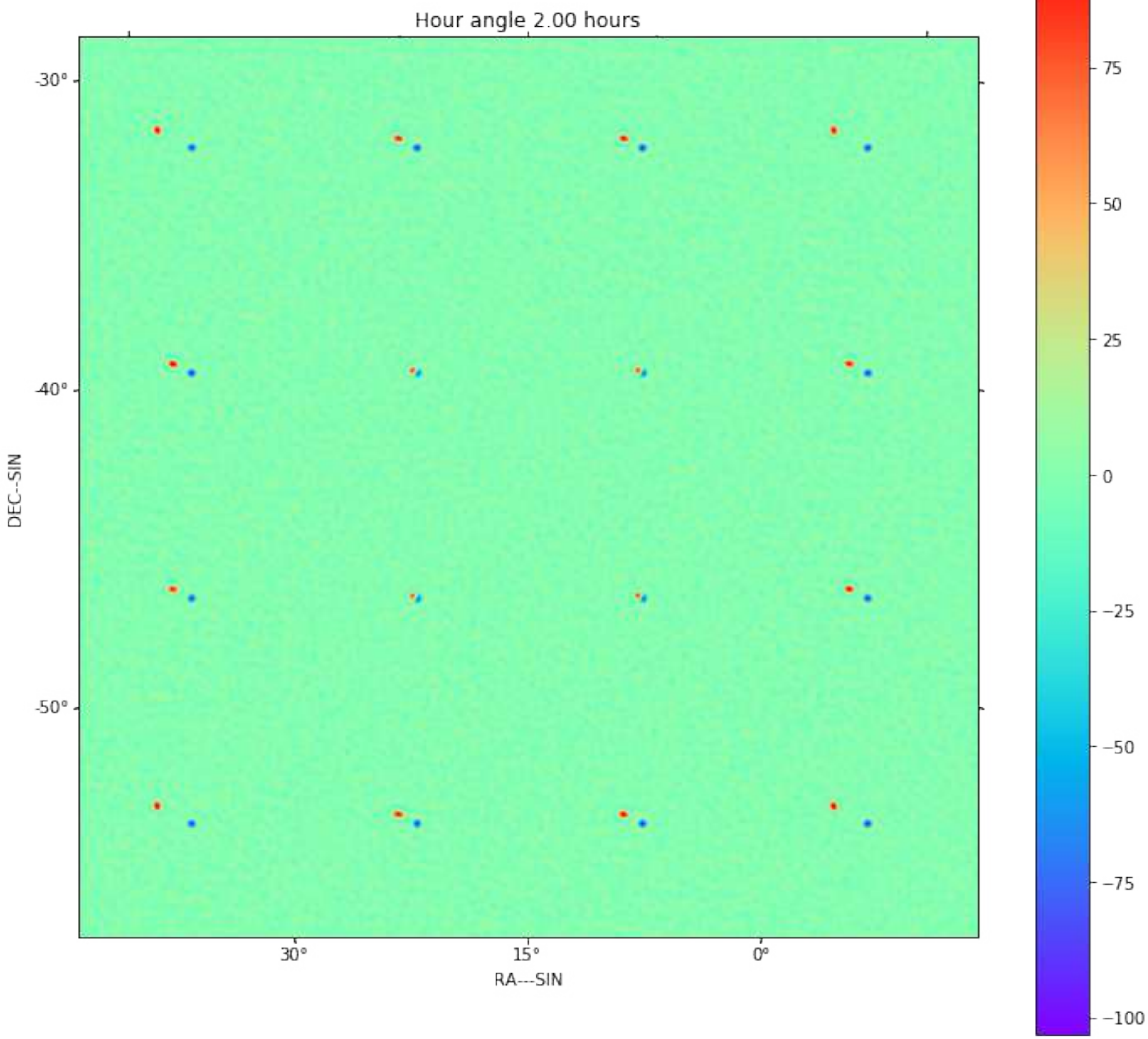
Hour angle -2.00 hours
-30°
-40°
-50°
DEC--SIN
30°
15°
0°
RA---SIN
100
75
50
25
0
−25
−50
−75
−100

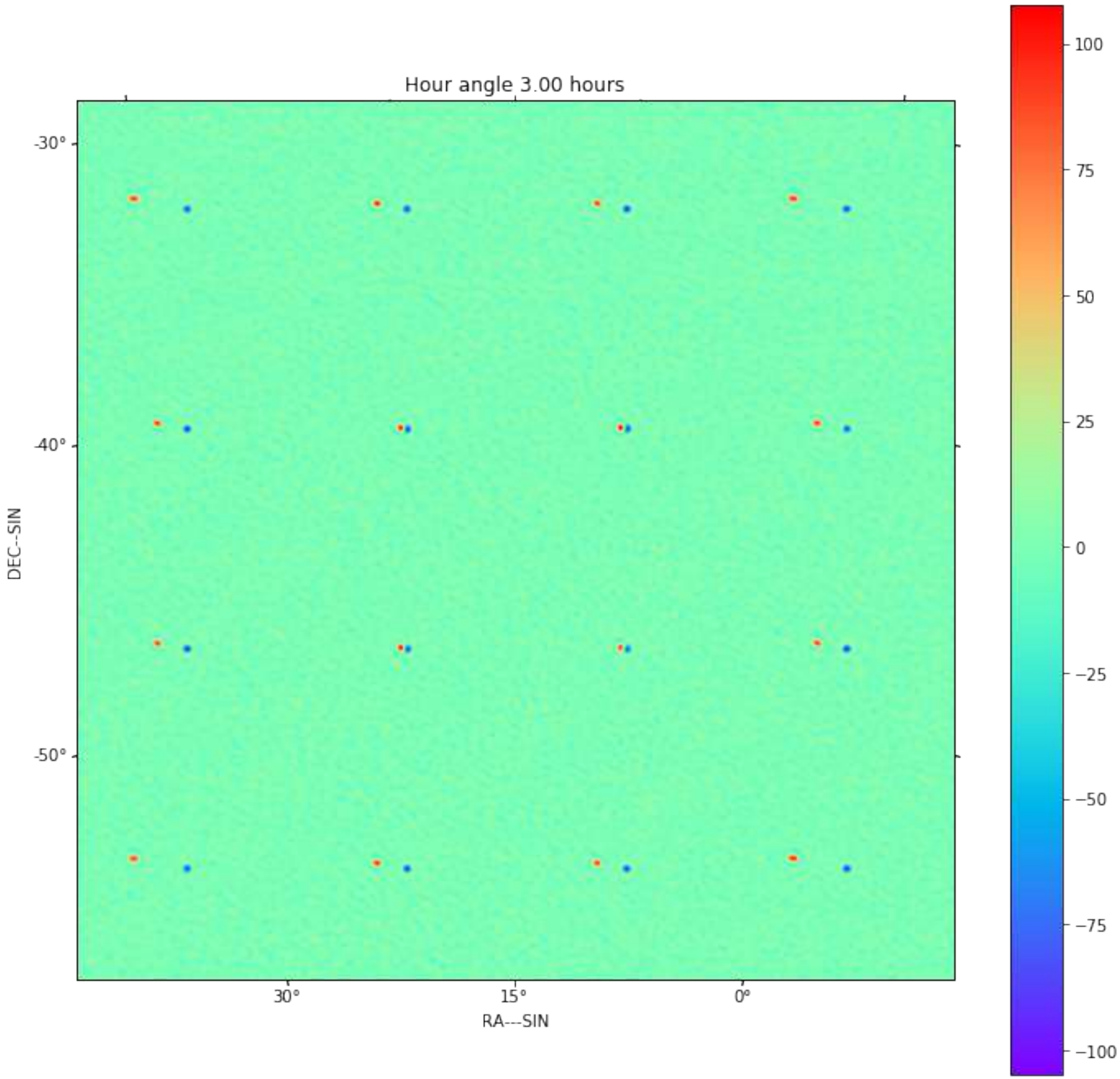

Hour angle -1.00 hours
-30°
-40°
-50°
DEC--SIN
30°
15°
0°
RA---SIN
100
75
50
25
0
−25
−50
−75
−100



Hour angle 0.00 hours
-30°
-40°
-50°
DEC--SIN
30°
15°
0°
RA---SIN
100
75
50
25
0
−25
−50
−75
−100


Hour angle 1.00 hours
-30°
-40°
-50°
DEC--SIN
30°
15°
0°
RA---SIN
100
75
50
25
0
−25
−50
−75
−100


Hour angle 2.00 hours
-30°
-40°
-50°
DEC--SIN
30°
15°
0°
RA---SIN
75
50
25
0
−25
−50
−75
−100

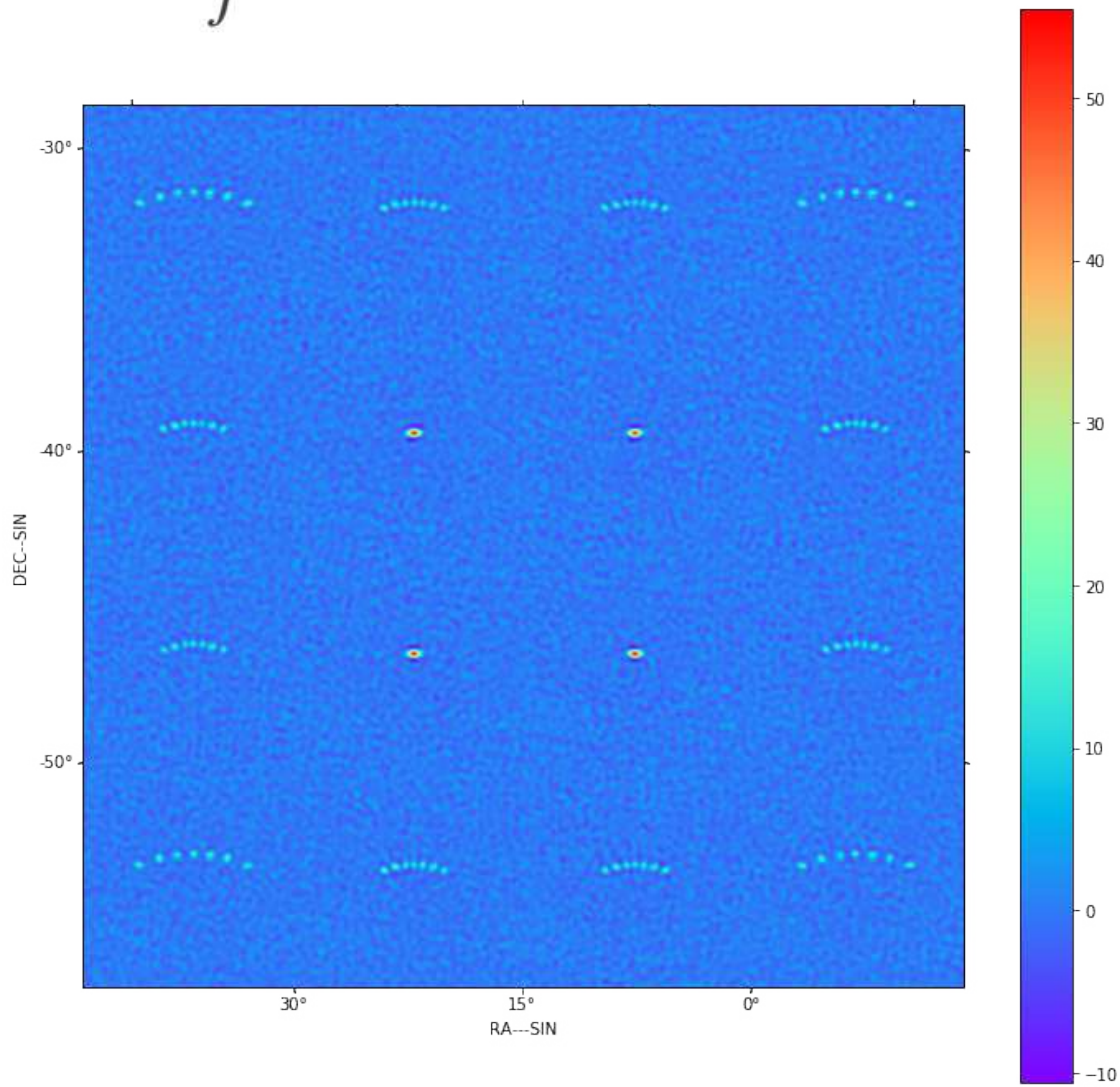


Hour angle 3.00 hours
-30°
-40°
-50°
DEC--SIN
30°
15°
0°
RA---SIN
100
75
50
25
0
−25
−50
−75
−100

$$V(u, v, w) = \int I(l, m) e^{2\pi j(ul+um)} \, dl dm$$

2D transform for 7 hourangles

Each hour angle causes different position error

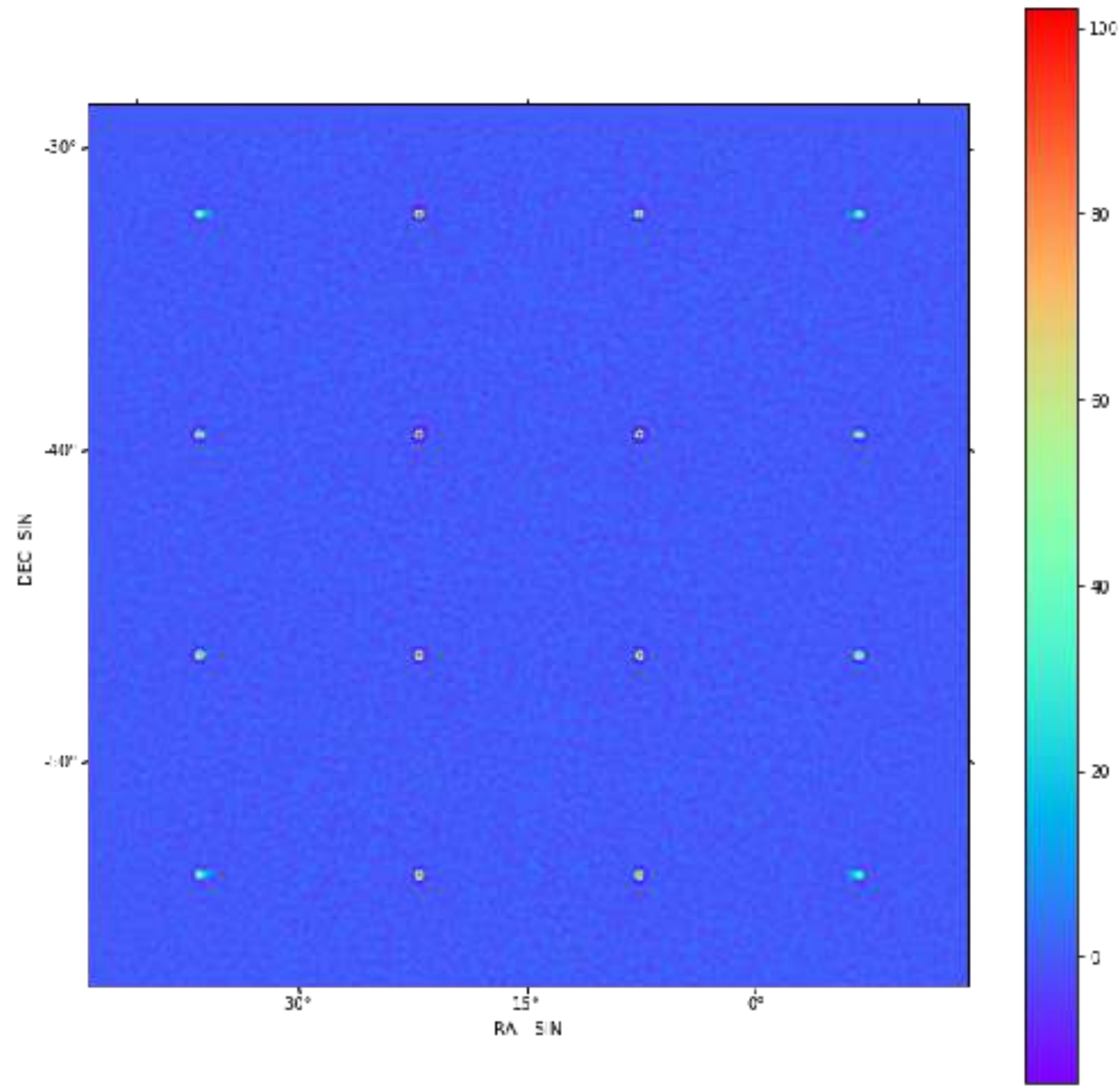


Time slice imaging

$$w = au + bv$$

$$l' = l + a(\sqrt{1 - l^2 - m^2} - 1))$$

$$m' = m + b(\sqrt{1 - l^2 - m^2} - 1))$$

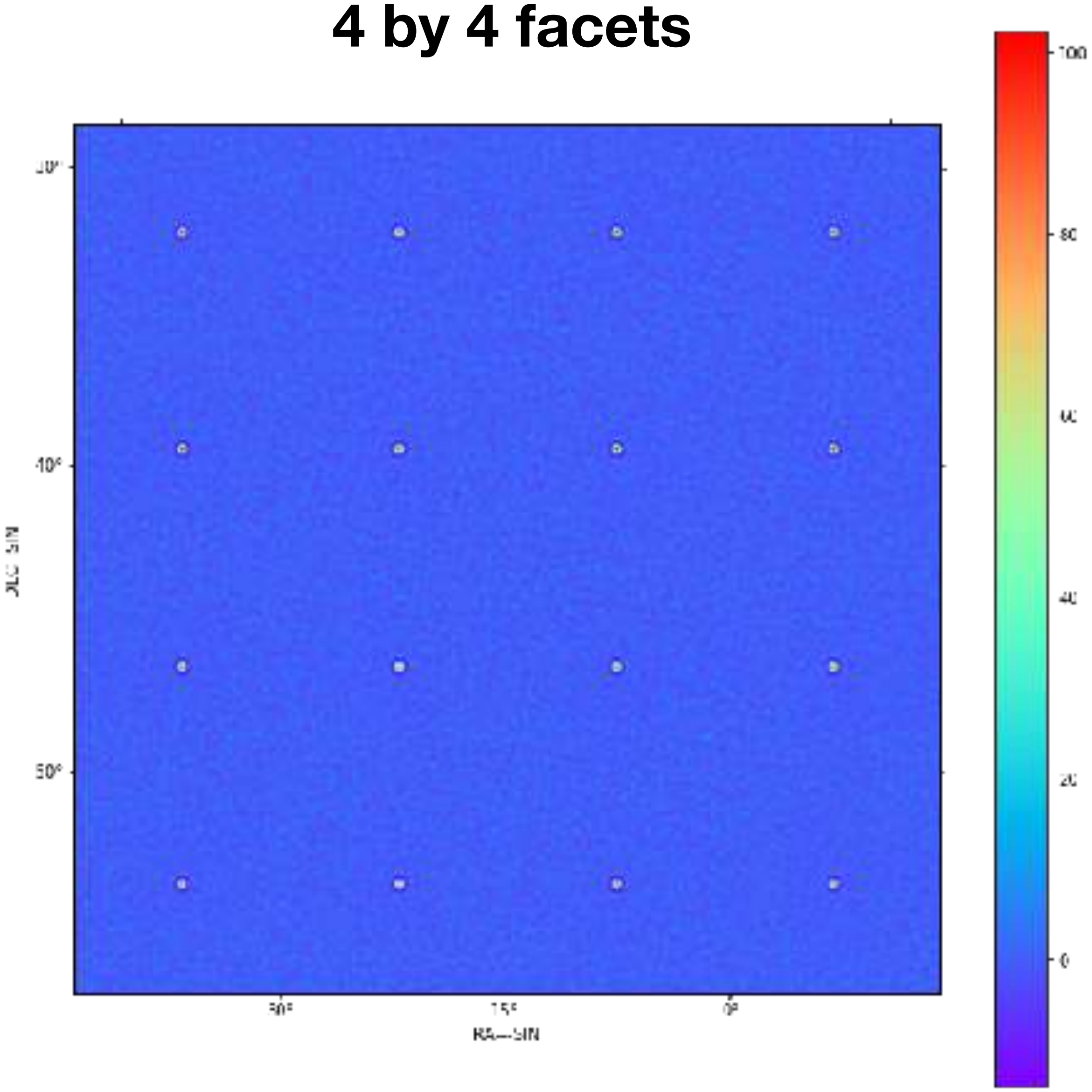


- Time-dependent coordinate distortion
- Known and easy to correct
- But accuracy for point sources is poor

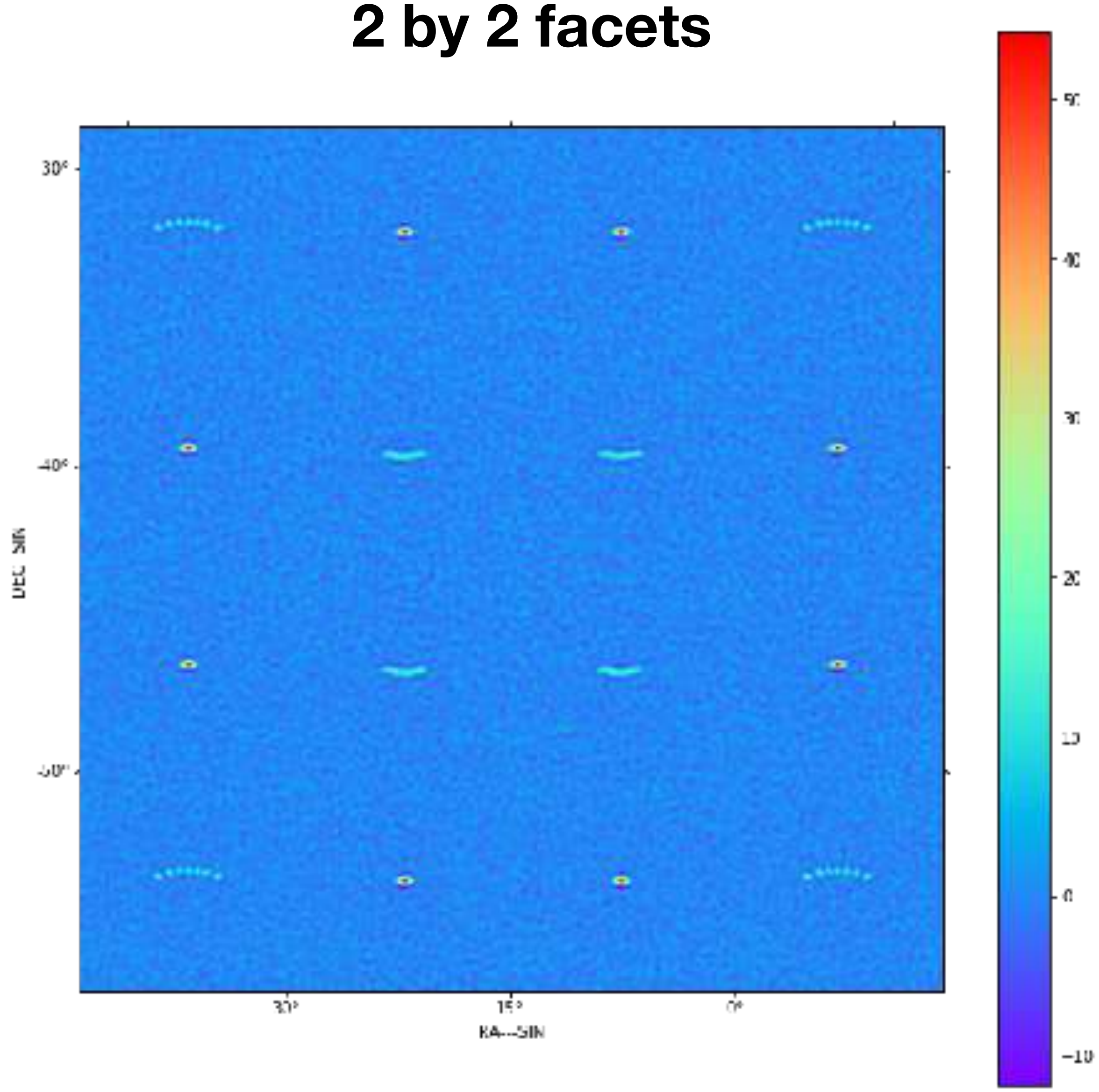
Faceted imaging

$$V(u, v, w) = \sum_{i,j} \frac{1}{\sqrt{1 - l_{i,j}^2 - m_{i,j}^2}} e^{-2\pi j(ul_{i,j} + um_{i,j} + w(\sqrt{1-l_{i,j}^2-m_{i,j}^2}-1))} \int I(\Delta l, \Delta m) e^{-2\pi j(u\Delta l_{i,j} + u\Delta m_{i,j})}\, dldm$$

4 by 4 facets



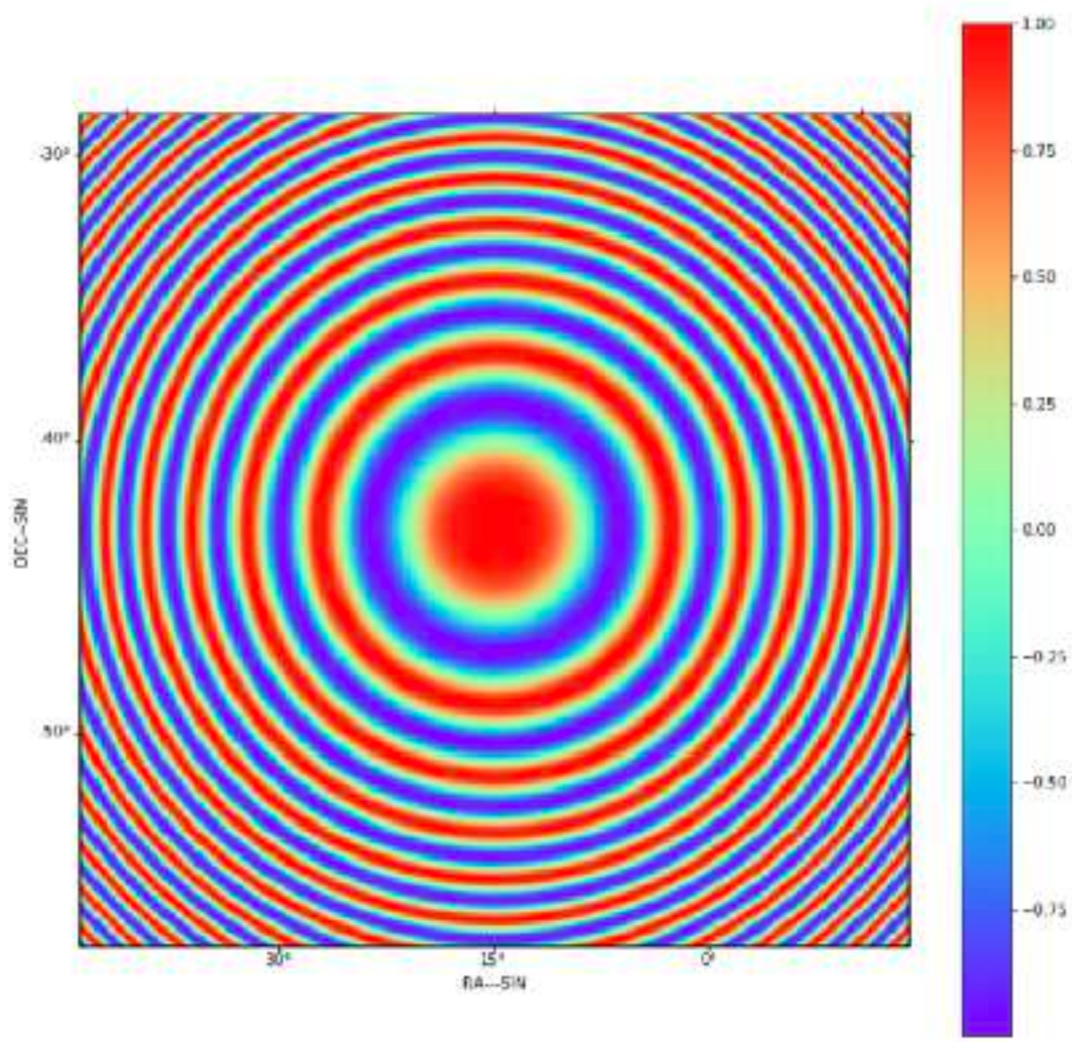
2 by 2 facets

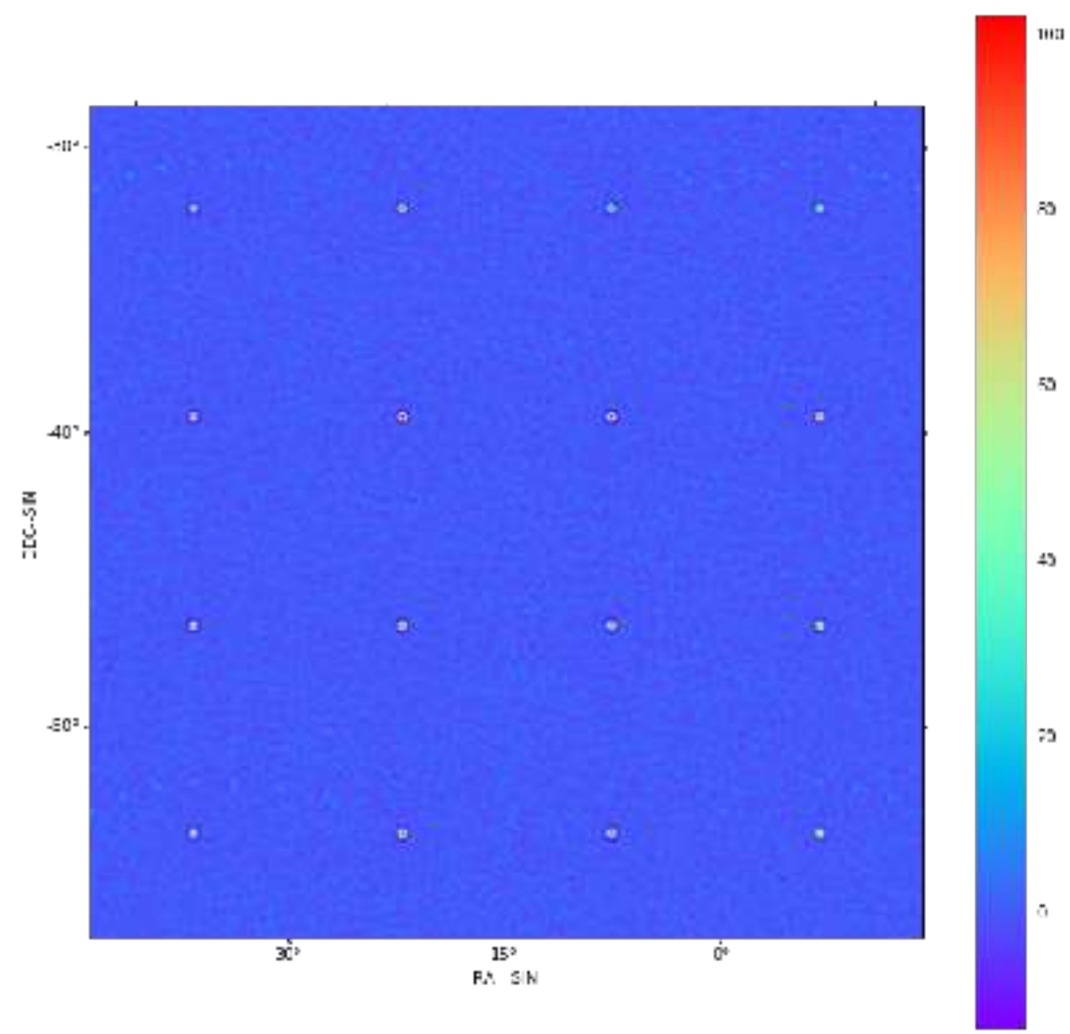


W stacking

$$V(u,v,w)=\sum_k \int G(l,m,w_k)I(l,m)e^{2\pi j\left(ul+vm\right)}dldm$$

$$G(l,m,w_k)=\frac{1}{\sqrt{1-l^2-m^2}}e^{2\pi jw_k\left(\sqrt{1-l^2-m^2}-1\right)}$$

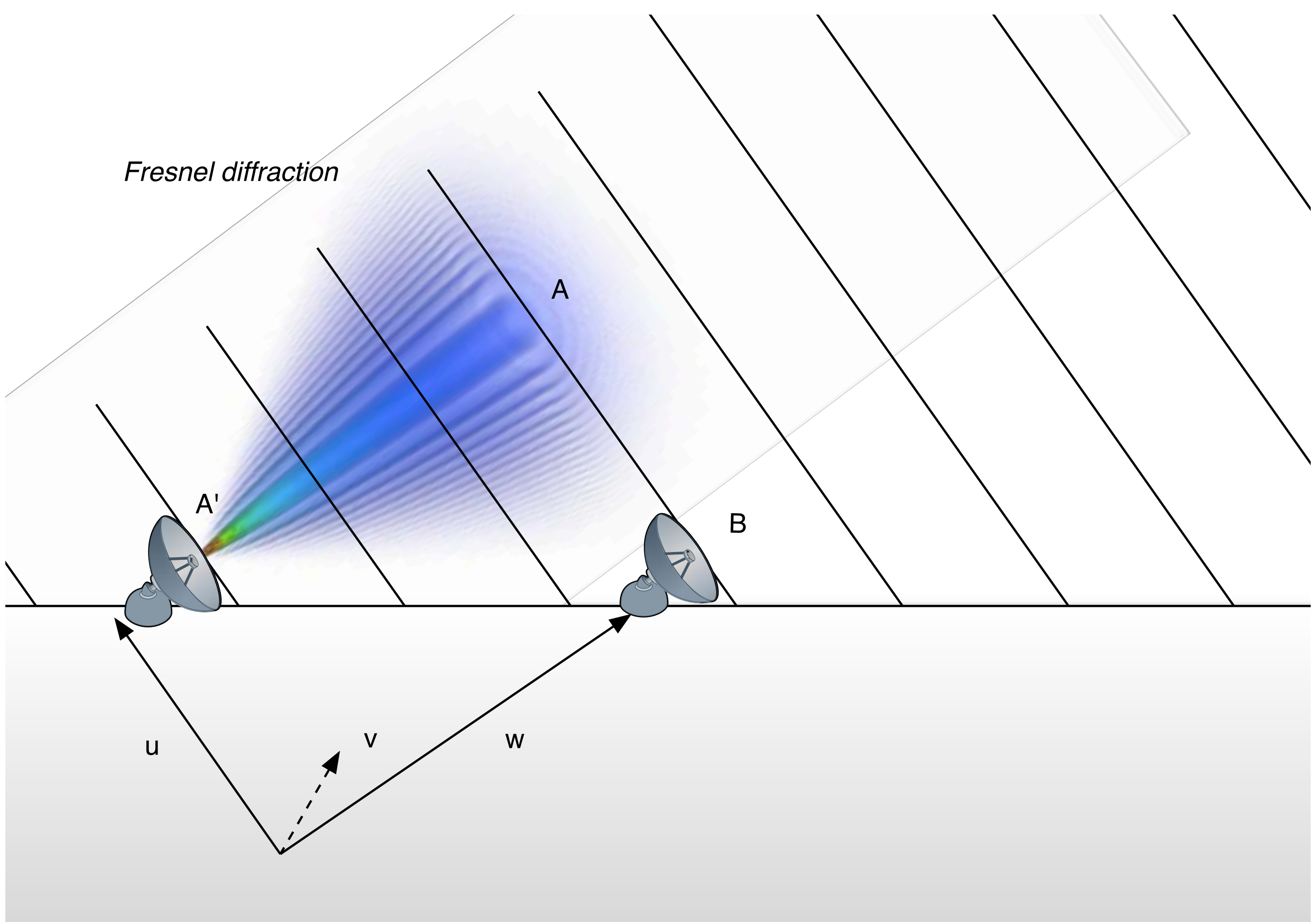


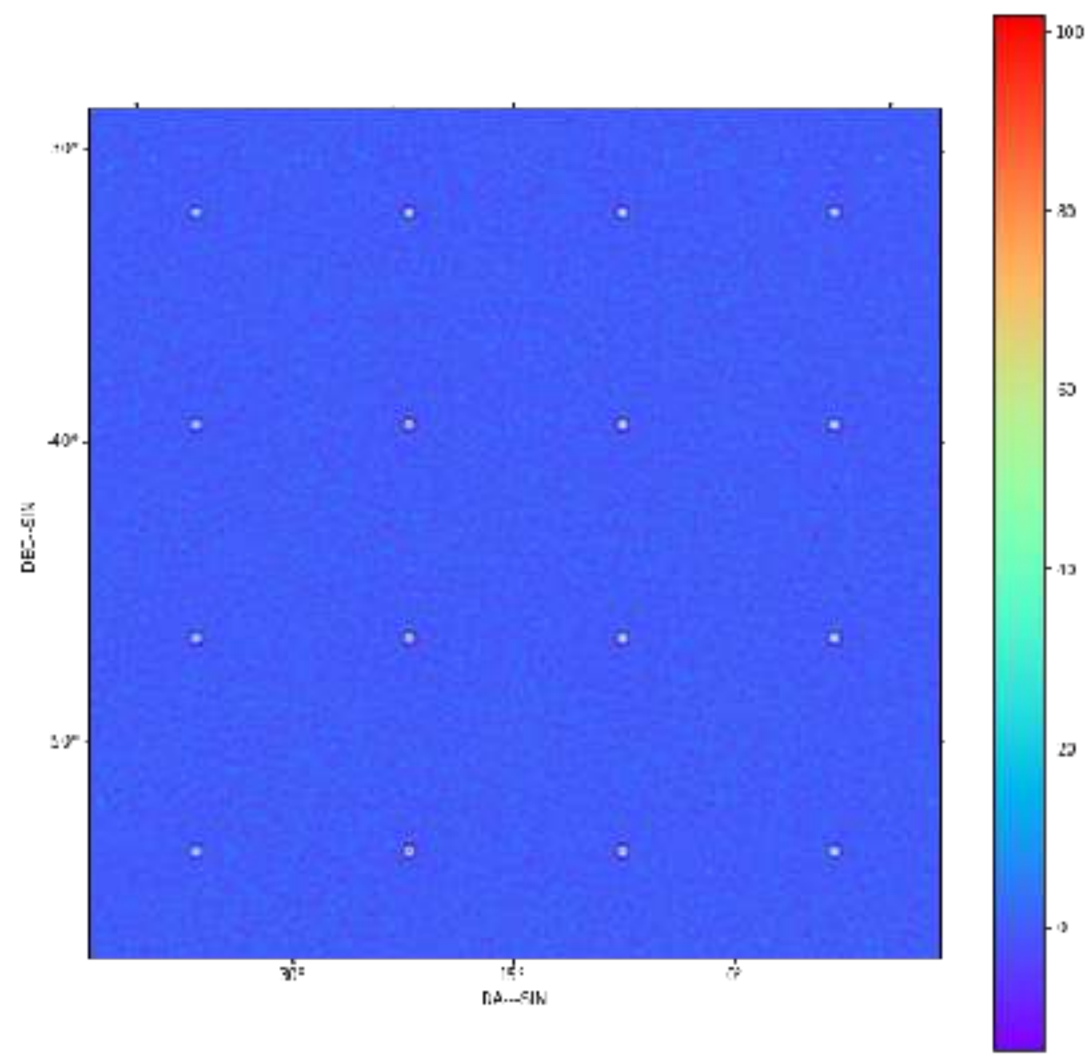


W Projection

$$V(u,v,w) = G(u,v,w) \otimes \int I(l,m) e^{2\pi j(ul+vm)} dl dm$$

$$G(u,v,w) = \int \frac{1}{\sqrt{1-l^2-m^2}} e^{2\pi j\left(ul+vm+w\left(\sqrt{1-l^2-m^2}-1\right)\right)} dl dm$$



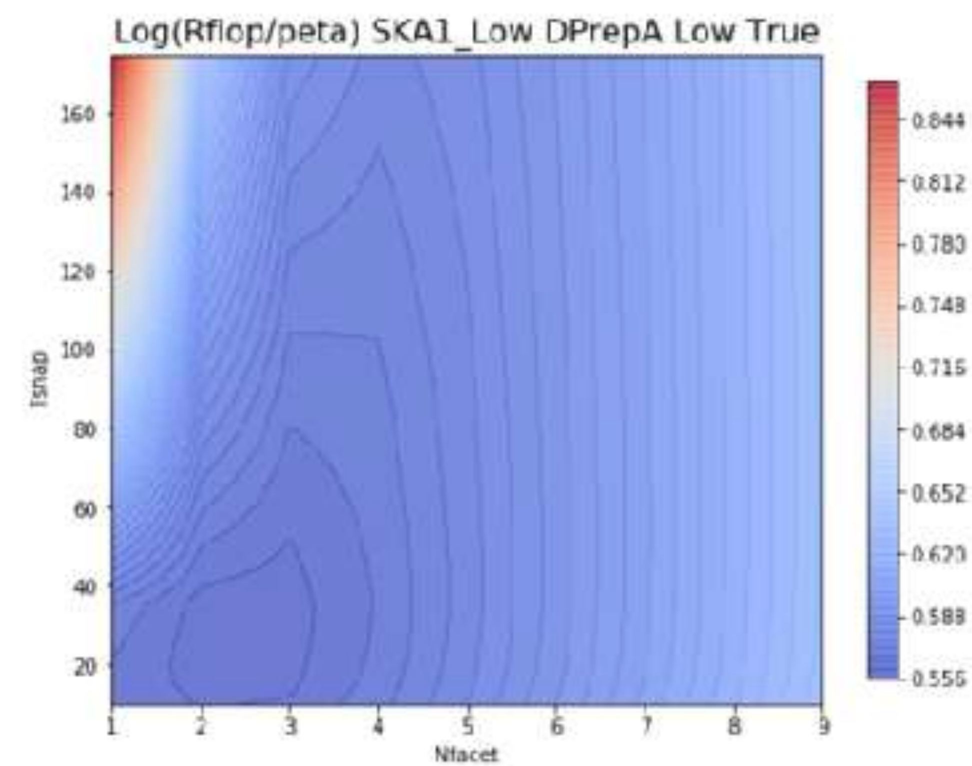


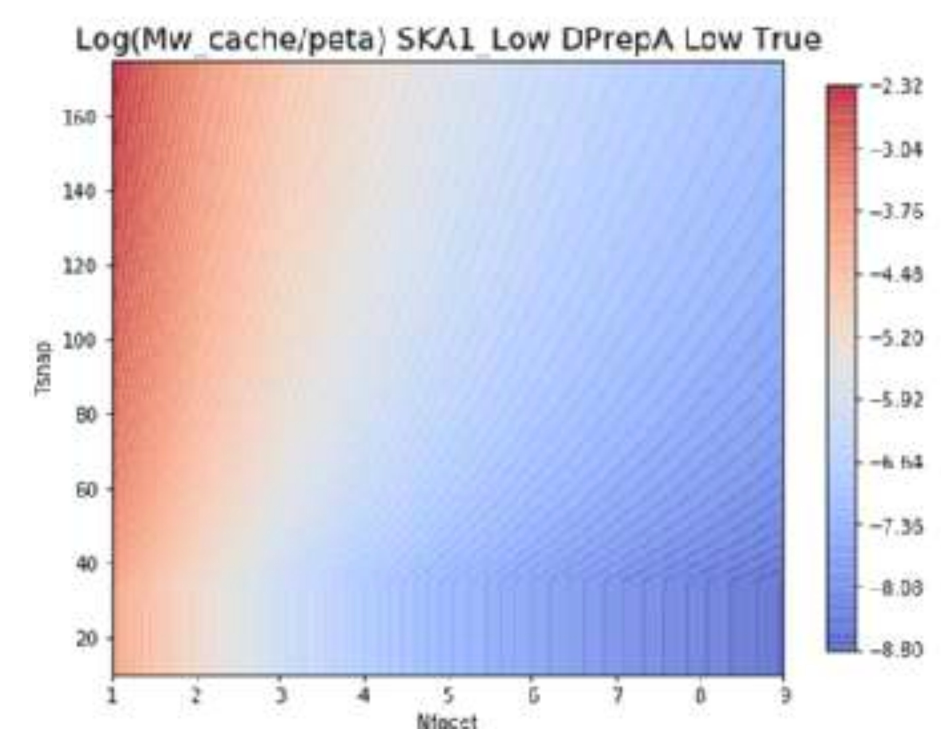
Processing Flops

• W projection	R_F^2
• Snapshots alone	R_F
• W projection + snapshots	$R_F^{2/3}$

Which algorithm?

- Many different algorithms available in different software
- Given sufficiently large resources all algorithms give the same answer
- Tradeoff between flops, memory, IO
- Optimum for SKA is wstacking + timeslice + facets
- Facets often useful as well for non-isoplanatic imaging





W sampling

- W sampling for maximum amplitude error

$$\Delta w = \frac{\sqrt{2\Delta A}}{N_{pixel}\theta_{FOV}^2}$$

- Number of w planes

$$N_w = \frac{\pi}{\sqrt{2\Delta A}} R_F$$

- Fresnel number

$$R_F = \frac{\theta_{FOV}^2}{\theta_{syn}}$$

The limit of wide field imaging

- Sampling in UV

$$\Delta uv = \frac{\sqrt{2\Delta A}}{2\pi\theta_{FOV}}$$

- Sampling in W

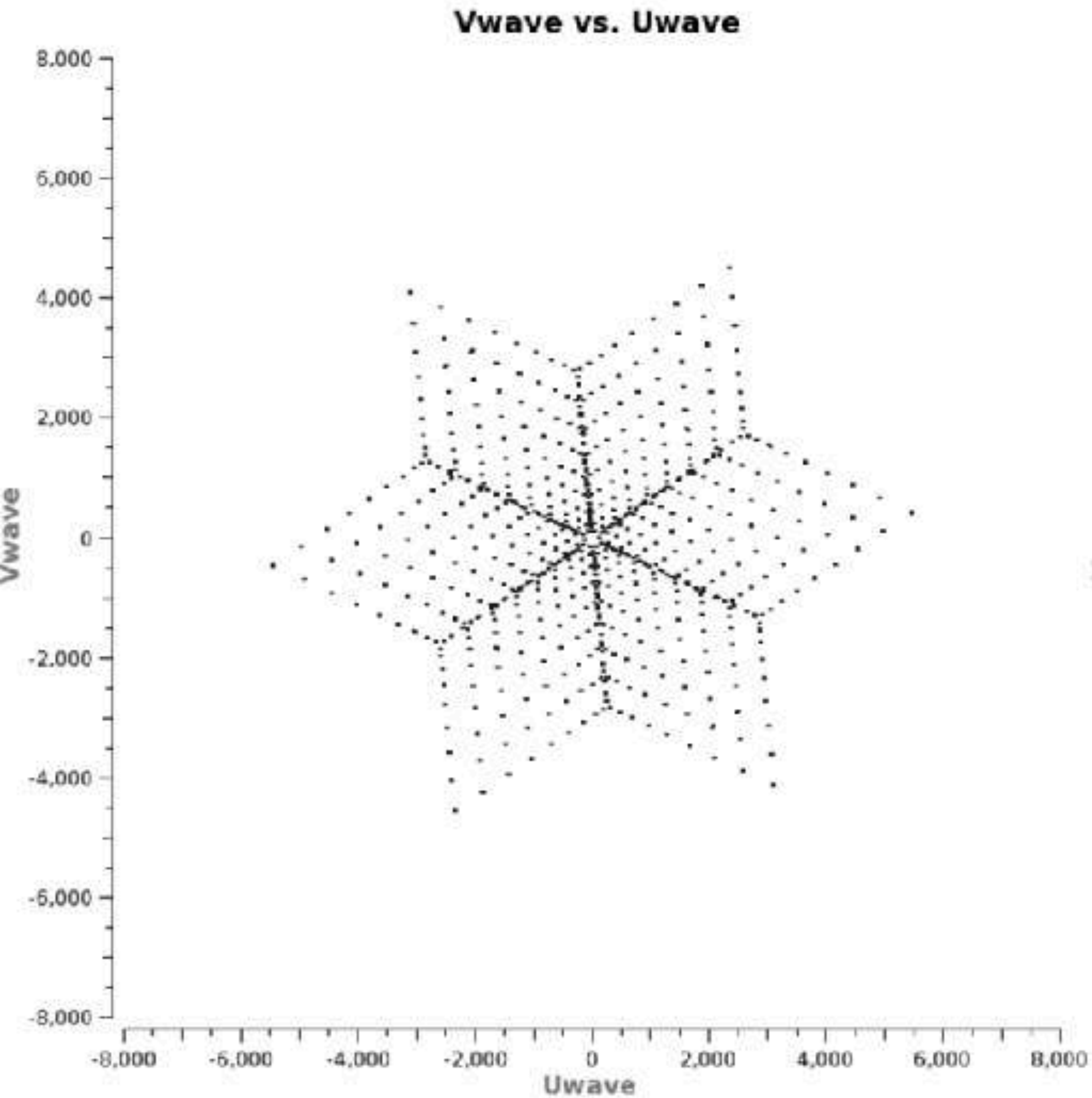
$$\Delta w = \frac{2\Delta uv}{\theta_{FOV}}$$

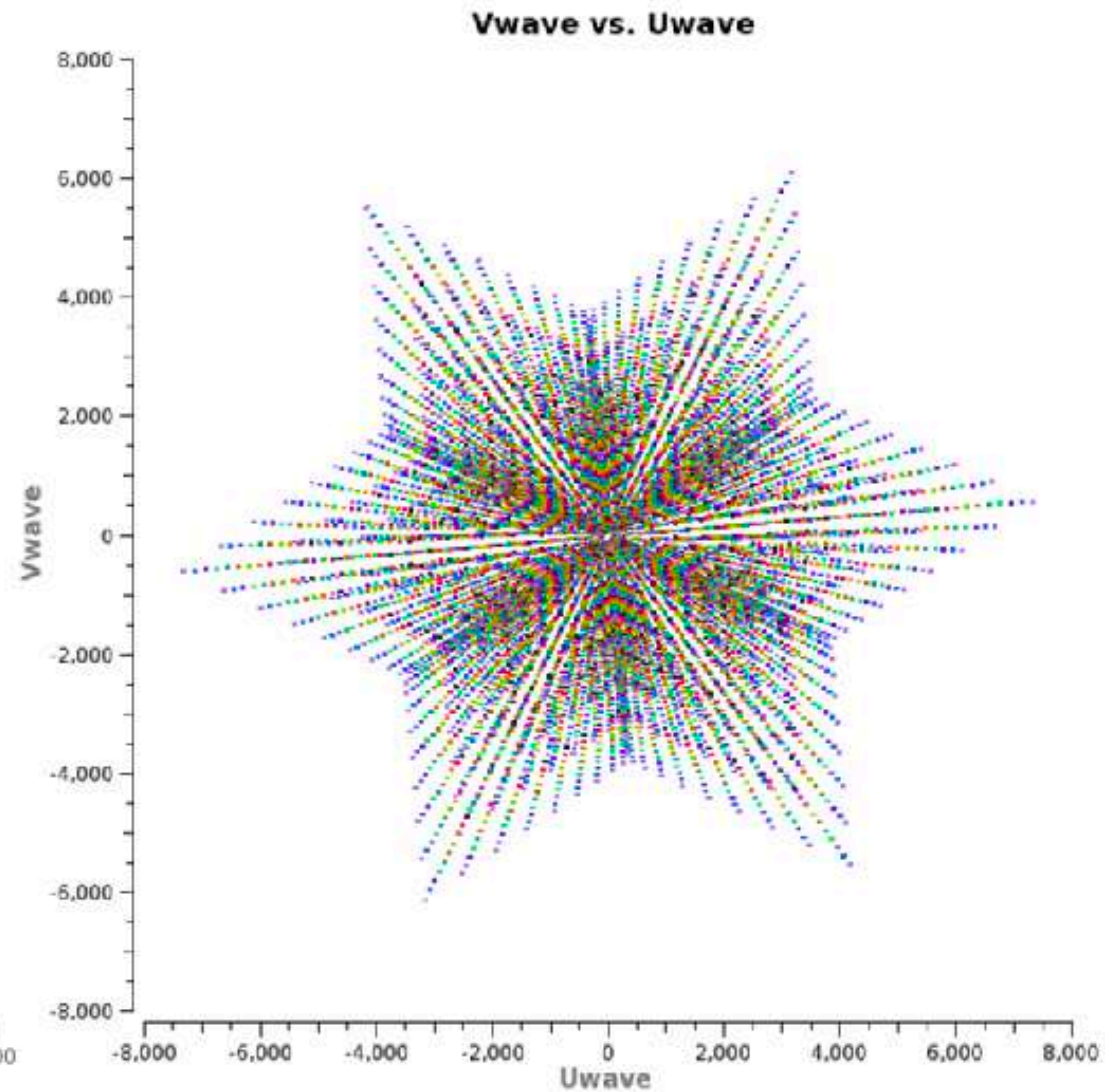
- As FOV expands, sampling must be same for U,V, and W
- Embed in 3D cube and use FFT's?

Wide bandwidth imaging

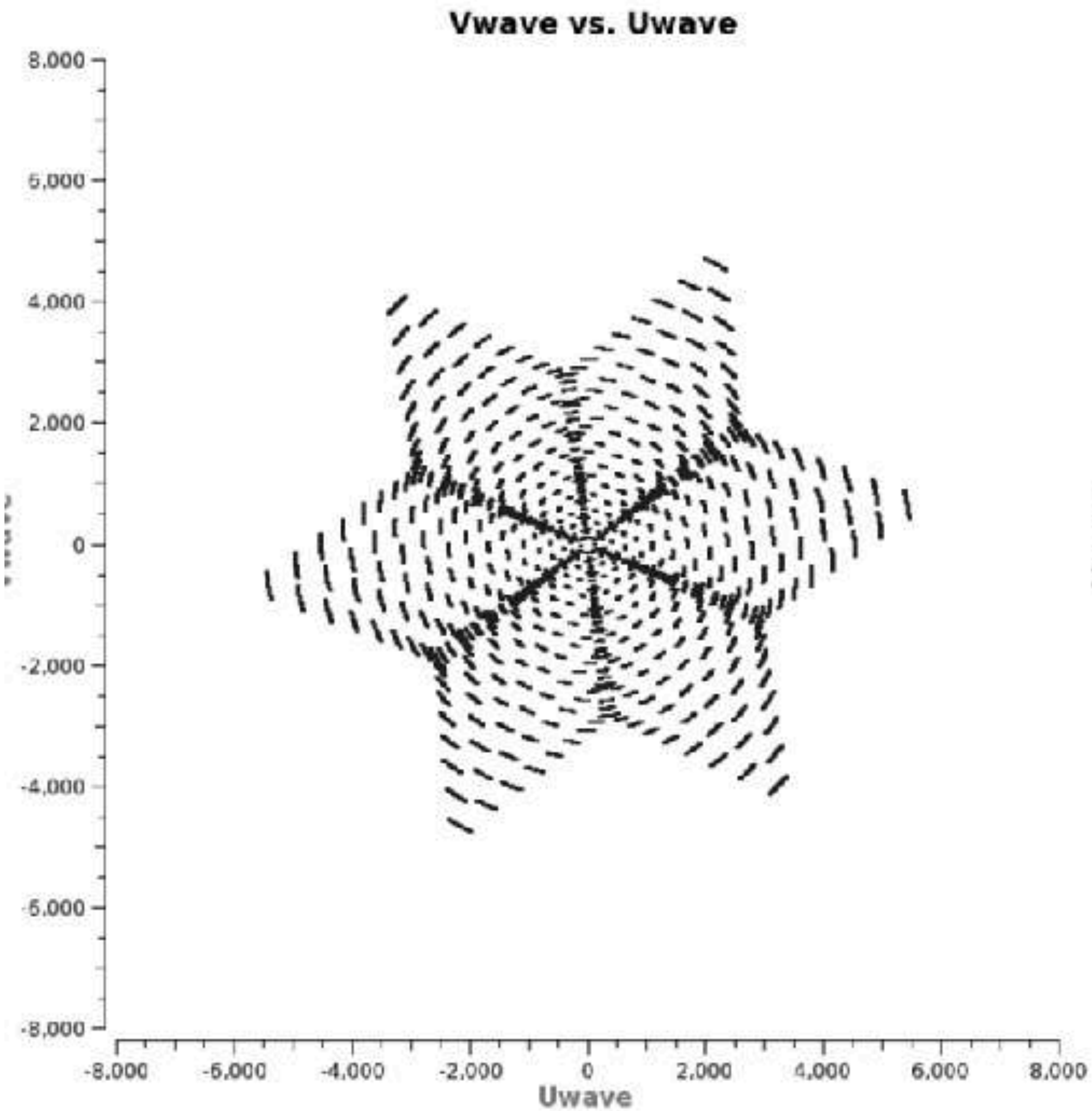
- Improving UV coverage improves the quality of imaging
- Both UV coverage and sensitivity scale with frequency
- But sources change with frequency
- And primary beam changes with frequency
- Need to solve for both image at some frequency plus change with frequency
- e.g. Taylor expansion in frequency

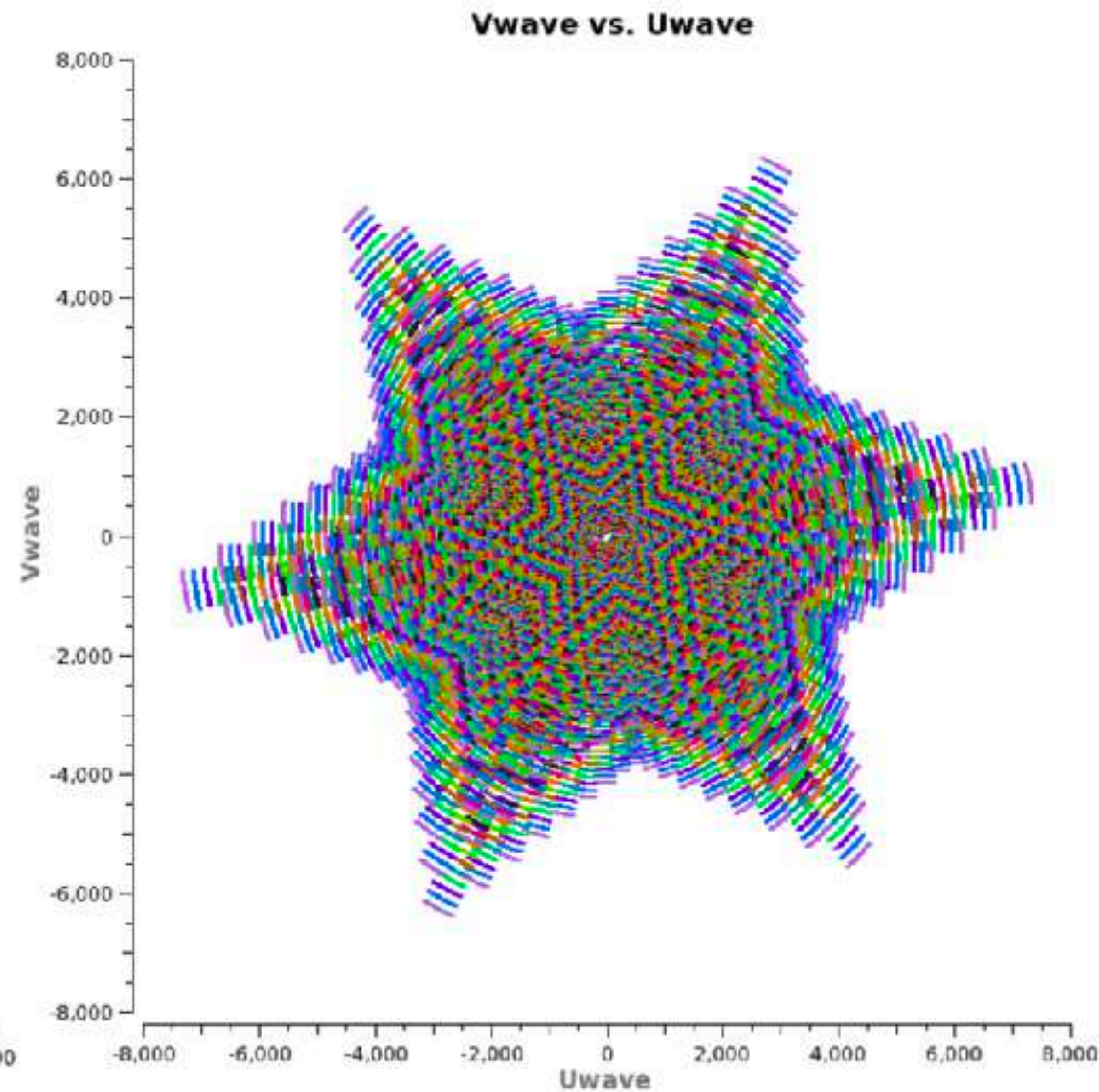
VLA snapshot





VLA short integration



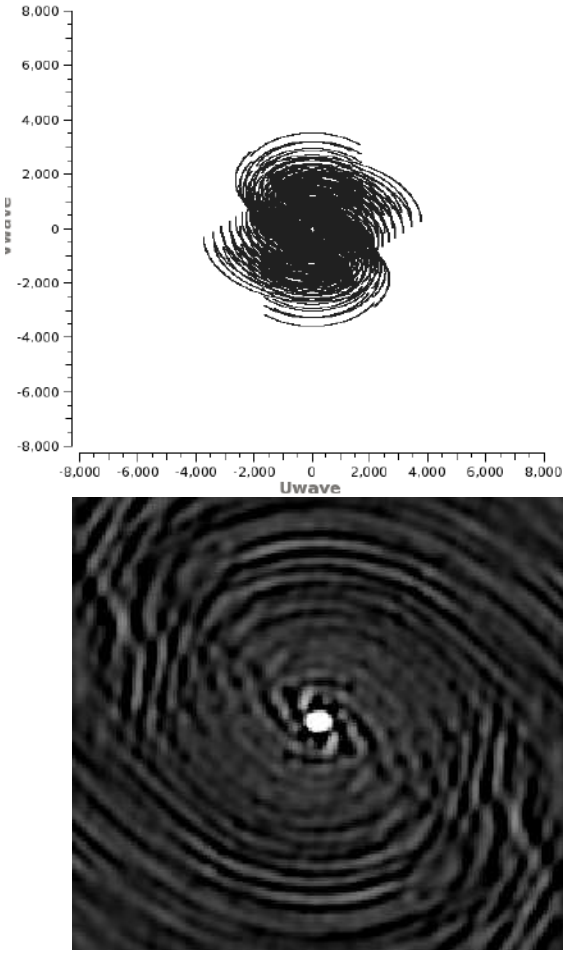


Imaging across a wide frequency range

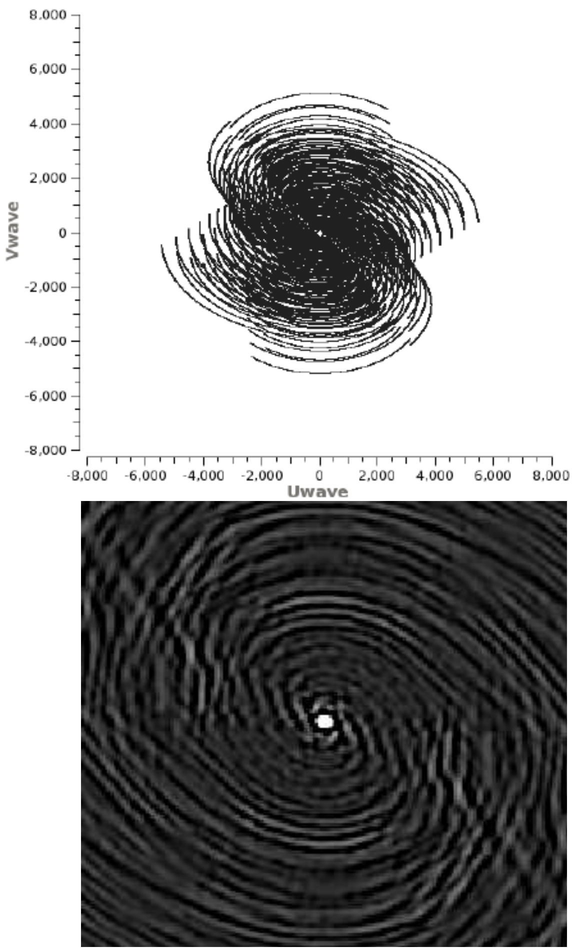
Large bandwidth => Increased 'instantaneous' imaging sensitivity $\sigma_{cont} = \frac{\sigma_{chan}}{\sqrt{N_{chan}}}$

- Angular-resolution increases at higher frequencies
- Sensitivity to large scales decreases at higher frequencies
- Wideband UV-coverage has fewer gaps => lower PSF sidelobe levels

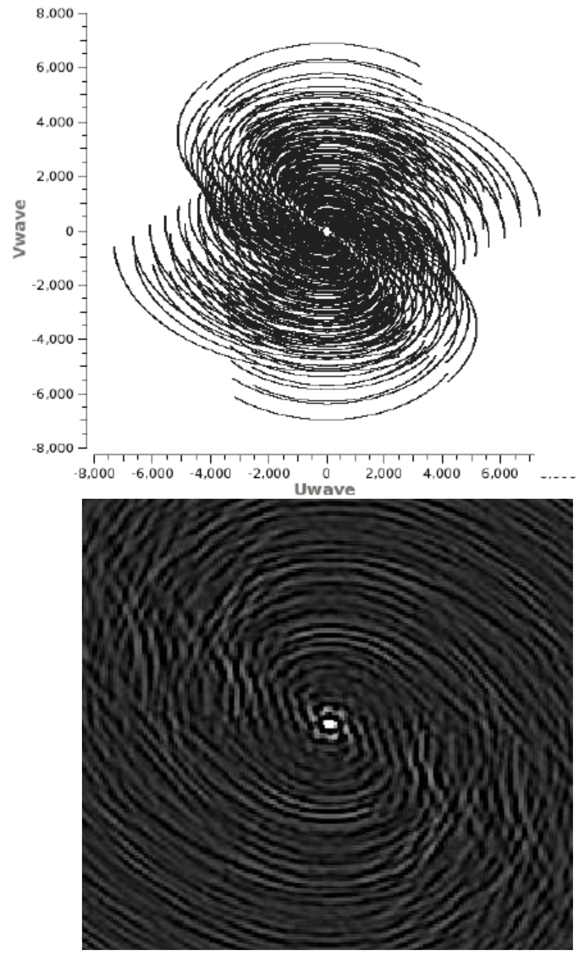
1.0 GHz



1.5 GHz

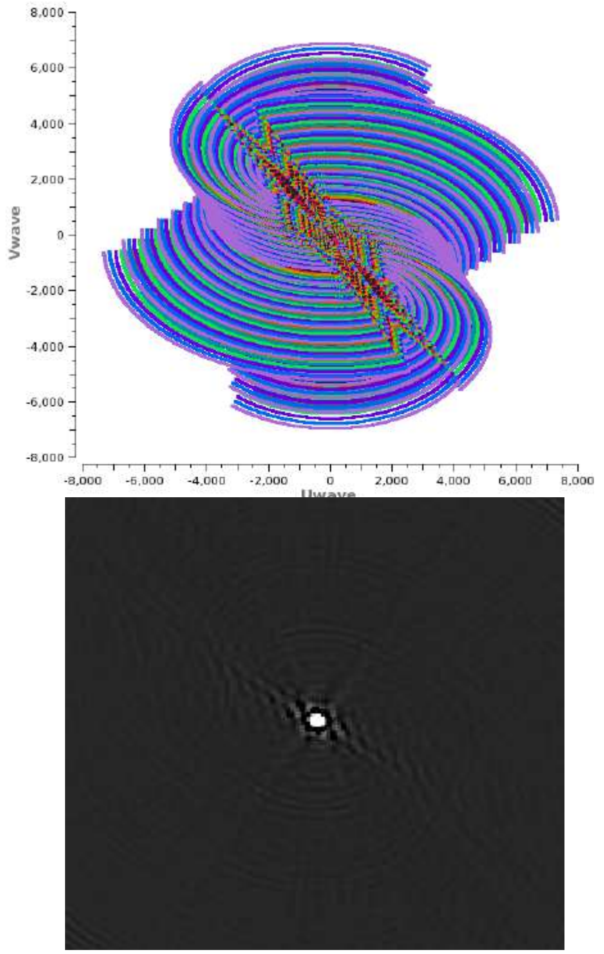


2.0 GHz



Observed image : $I_{\nu}^{obs} = I_{\nu}^{sky} * PSF_{\nu}$

1.0 - 2.0 GHz



$$I_{wb}^{obs} = \sum_{\nu} \left[I_{\nu}^{sky} * PSF_{\nu} \right]$$

Algorithms

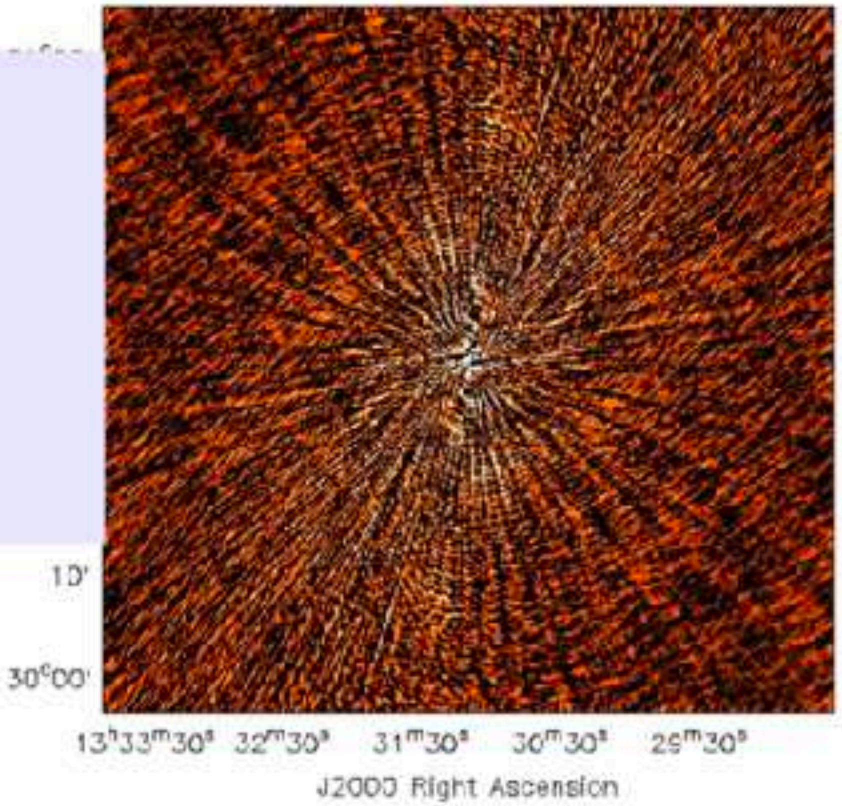
- Sault-Wieringa algorithm Clean, first order in frequency
- Rau algorithm, multiscale, n order expansion in frequency

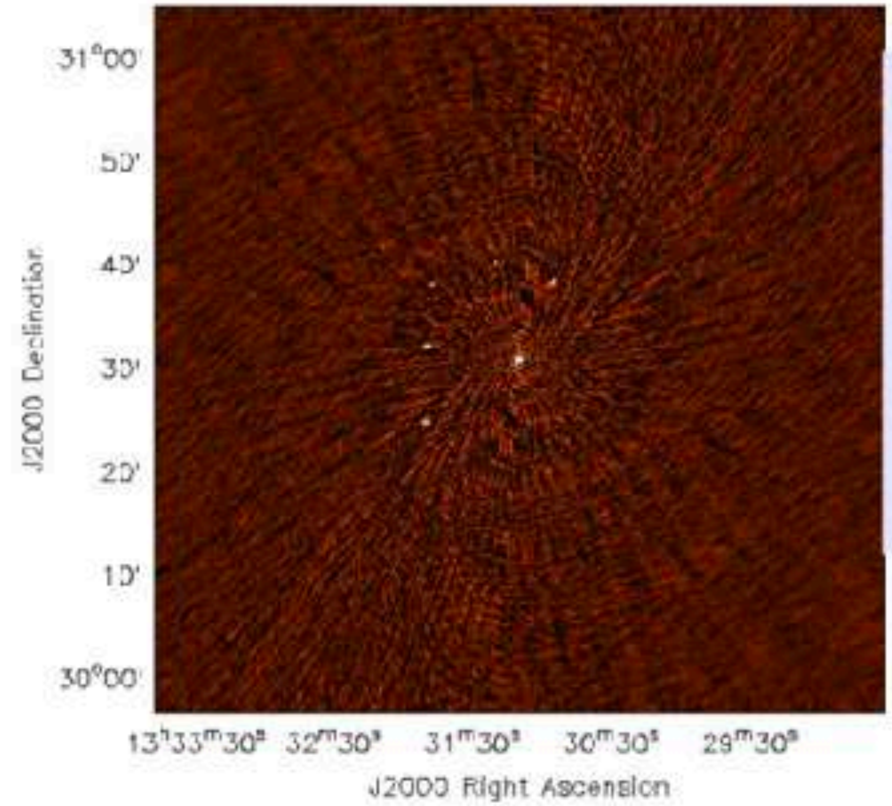
Dynamic-range with MS-MFS : 3C286 example : Nt=1,2,3,4

NTERMS = 1

Rms :
9 mJy -- 1 mJy

DR :
1600 -- 13000





NTERMS = 2

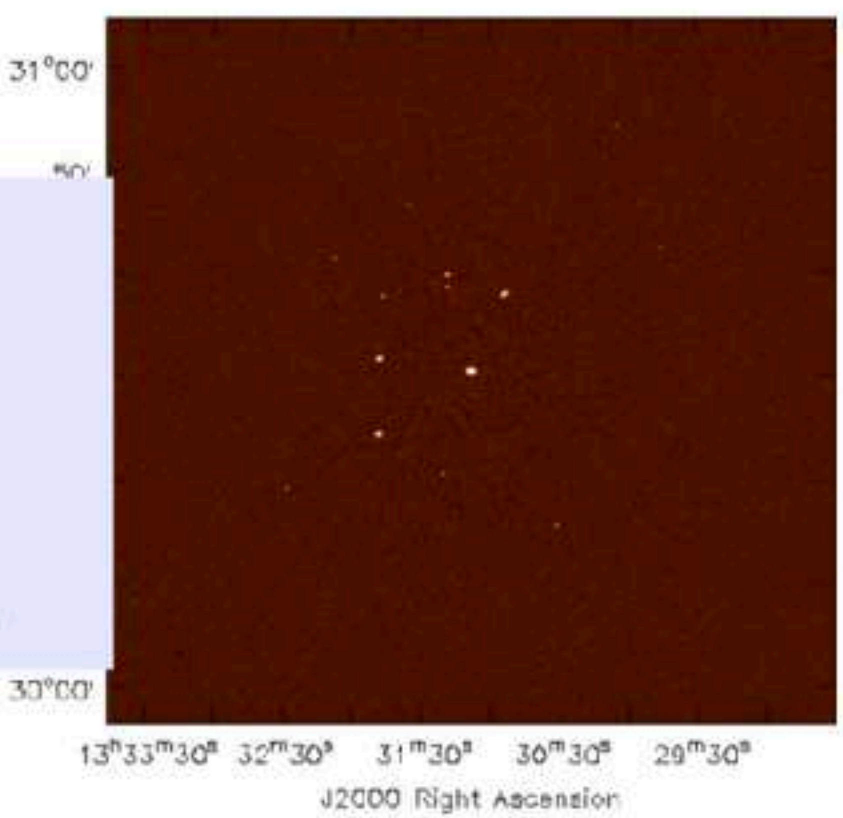
Rms :
1 mJy -- 0.2 mJy

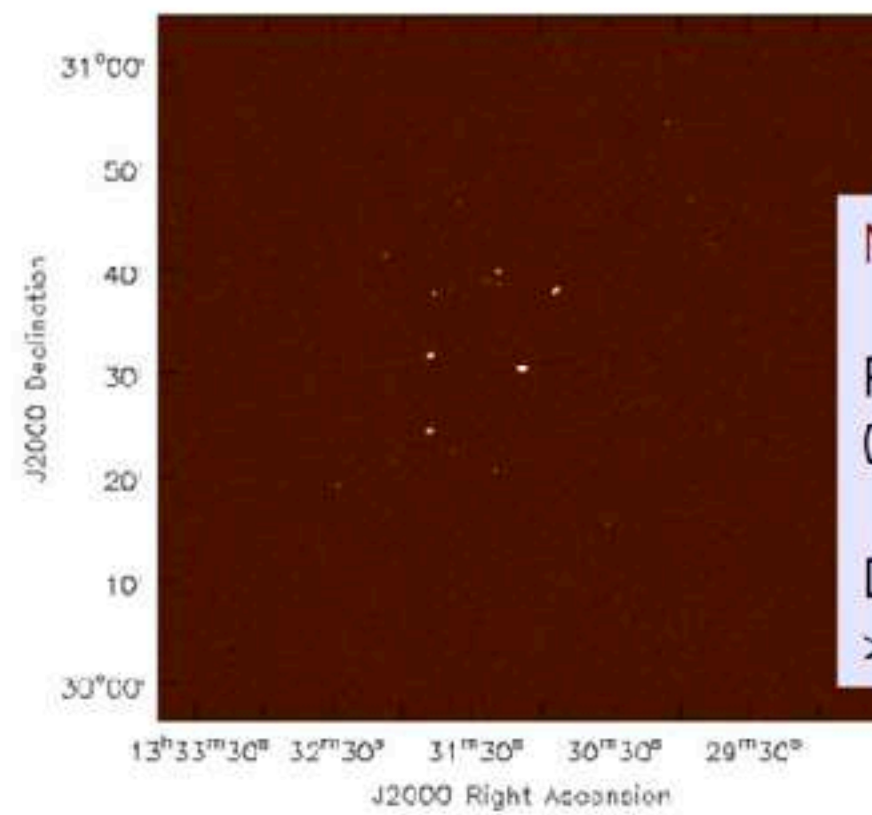
DR :
10,000 -- 17,000

NTERMS = 3

Rms :
0.2 mJy -- 85 uJy

DR :
65,000 -- 170,000





NTERMS = 4

Rms
0.14 mJy -- 80 uJy

DR :
>110,000 -- 180,000

More information

- Urvashi Rau Ph.D. thesis
- Urvashi Rau RAS talk, 2012