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Polarimetry I

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Talk Outline

- Polarimetry I
  - What and How?
    - What is polarized light and ways to quantify it
    - Hardware to measure it
    - Calibration
    - Some advanced topics
- Polarimetry II - George
  - Why?
    - What astrophysics do we learn

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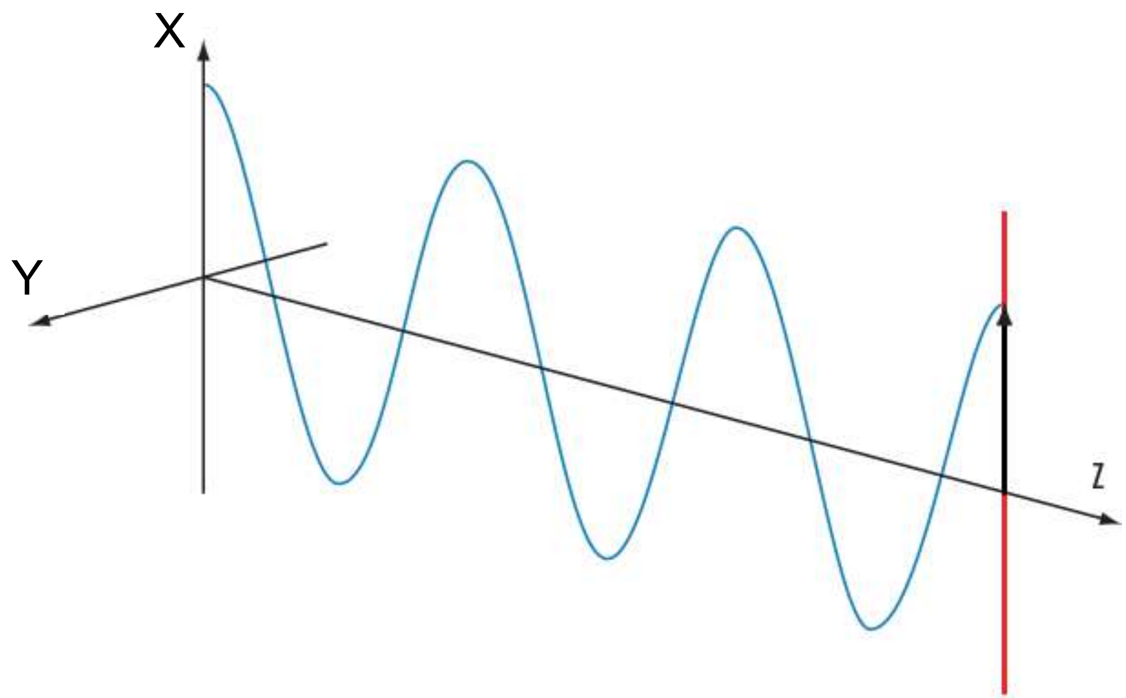
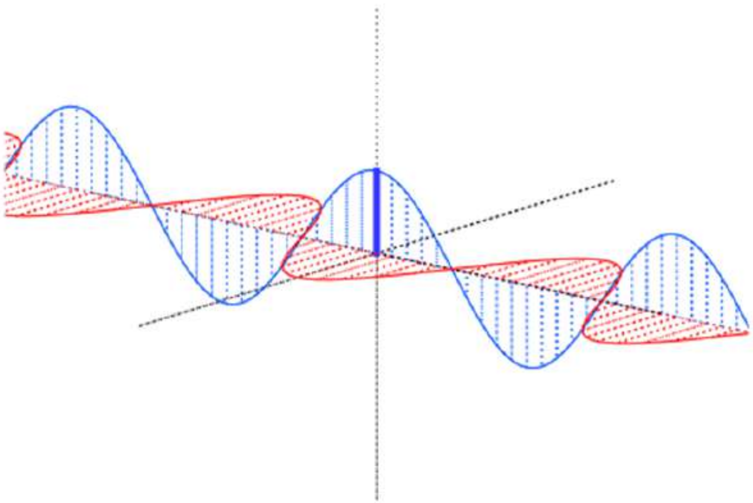
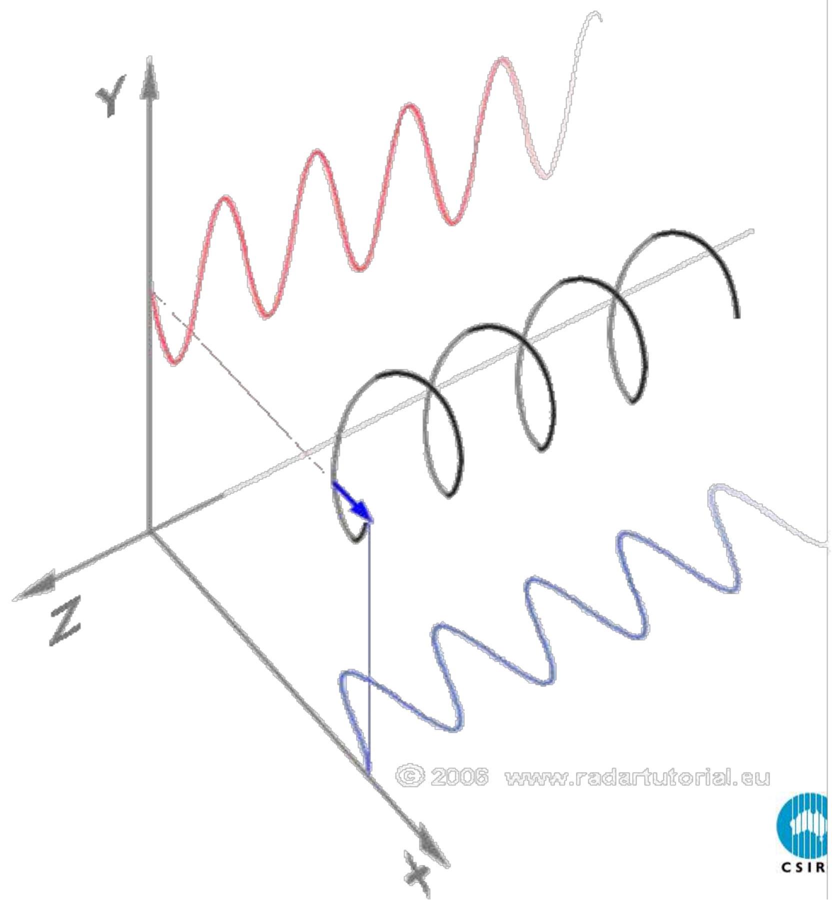
Polarized waves

$$
\begin{aligned}
\mathbf{E}_x(z,t) &= \hat{\imath} E_{0x} \cos(kz - \omega t) \\
\mathbf{E}_y(z,t) &= \hat{\jmath} E_{0y} \cos(kz - \omega t + \epsilon)
\end{aligned}
$$



Linear and circular





Partially polarized emission

- Three parameters are needed to describe a fully polarized wave (e.g. E_{0x}, E_{0y}, ε)
- Natural emission mostly randomly polarized or “unpolarised”
- ***Four*** parameters are needed to describe partially polarized emission

Parameterizing partially polarized emission

- Power quantities which are additive
- Compact and “neat”
- Independent of measurement hardware
- Readily measureable
- Stokes parameters

Stokes parameters

- I – total power – the sum of the power in any two orthogonal components …
- Q – difference in power between the vertical and horizontal components:

$$E_0^2 - E_{90}^2$$

- U – difference between $\pm 45^o$ components.

$$E_{45}^2 - E_{-45}^2$$

- V – difference between circularly polarized components.

$$E_{RCP}^2 - E_{LCP}^2$$

Stokes parameter properties

- Q,U,V are the differences of comparable powers. Just as likely to be negative as positive!
- Polarized fraction

$$p = \frac{\sqrt{Q^2 + U^2 + V^2}}{I}$$

- Angle of linear polarization

$$\theta = \frac{1}{2}\mathrm{atan2}(U, Q)$$

Hardware to measure it

- Polarimetry I
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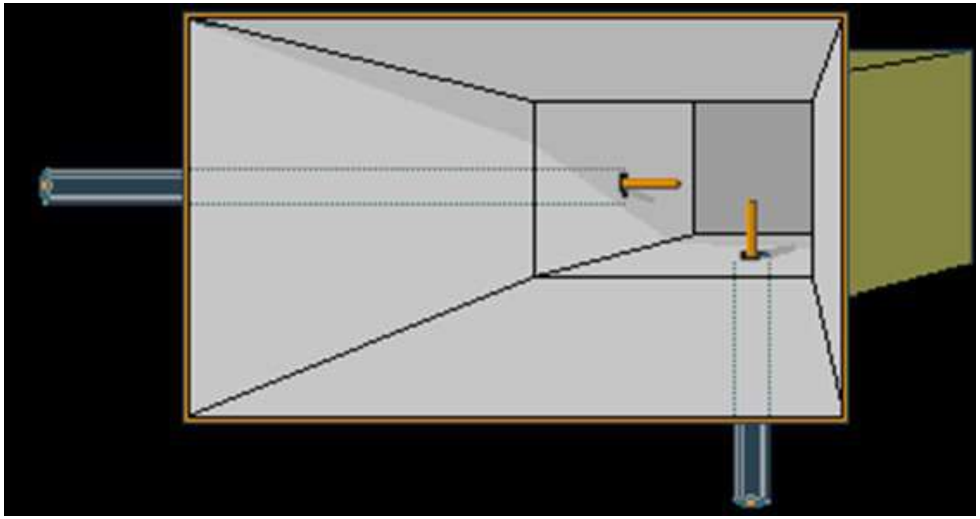
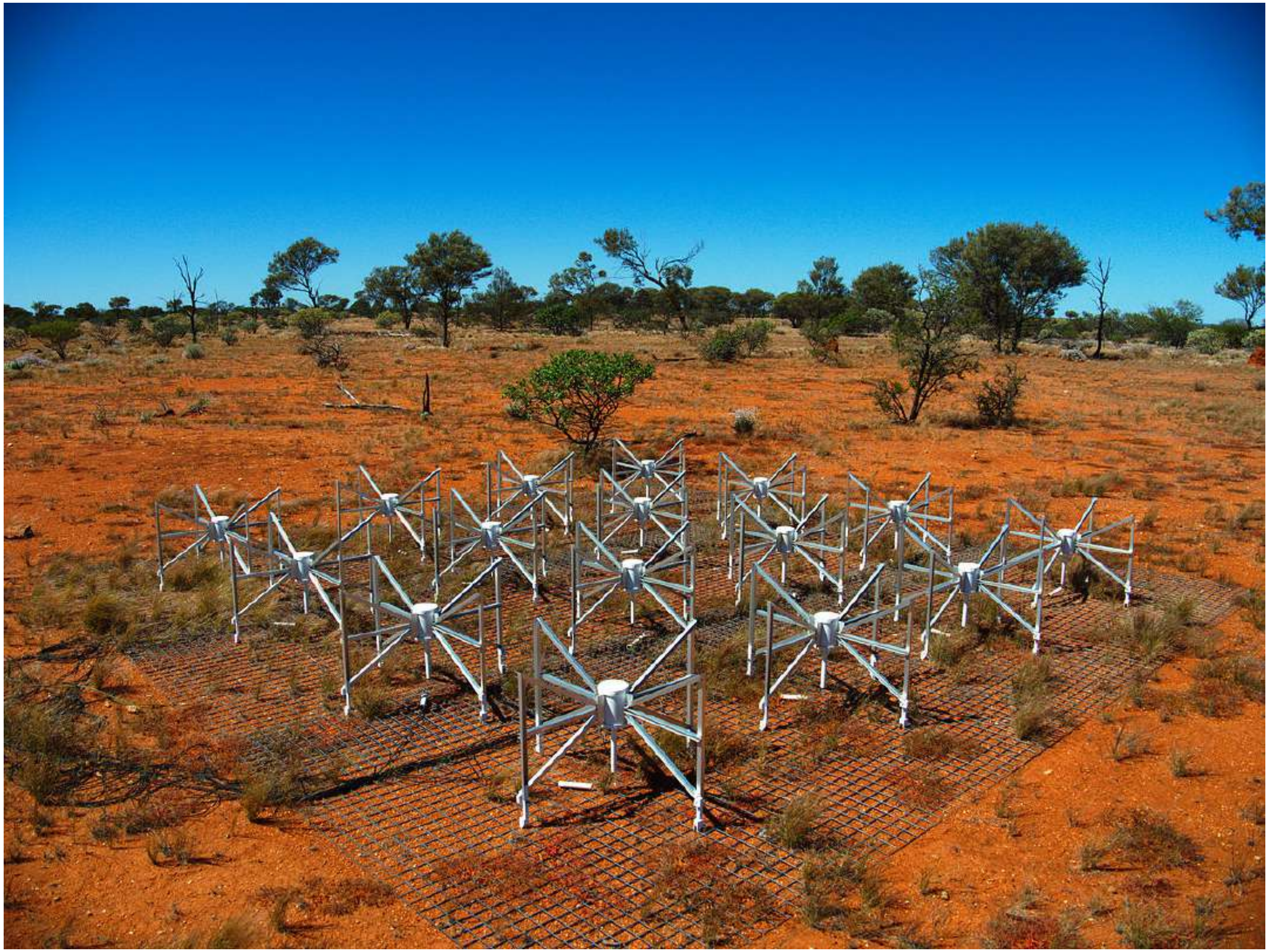
Feeds

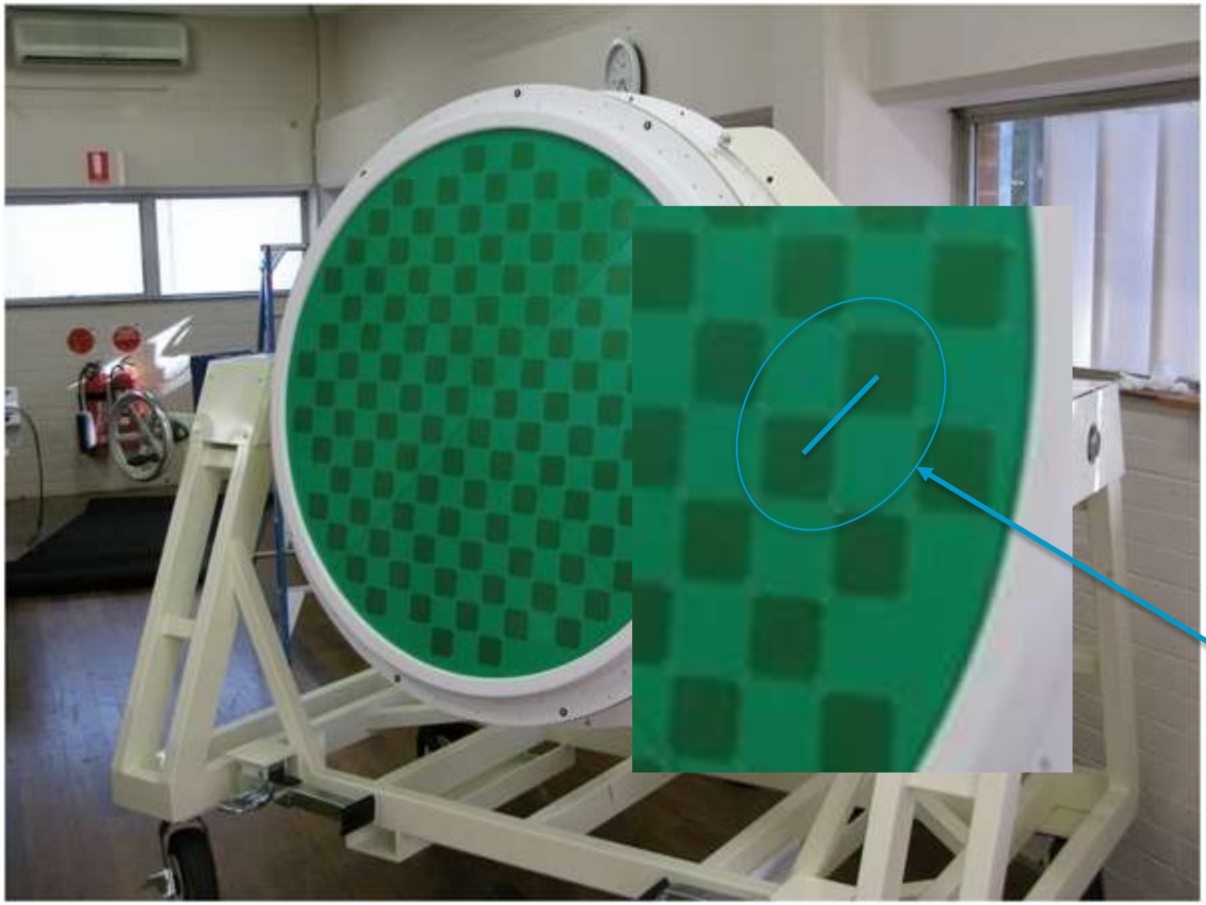
- Linearly-polarized feeds. Dipoles.
  - “Vertical” and “horizontal” - “X” and “Y” *(note X is vertical, Y is horizontal)*



MWA dipoles



ASKAP checkerboard



“Dipole”

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Circularly-polarized feeds

- Helical feeds.
  - "Right handed" and "left handed"

- Quadrature hybrids.
  - Circularly-polarized feeds synthesized by linears with 90 degree phase shifter (quadrature hybrid)

Measuring Stokes parameters in the radio

- Stokes parameters

$$
\begin{aligned}
I &= E_0^2 + E_{90}^2 \\
Q &= E_0^2 - E_{90}^2 \\
U &= E_{45}^2 - E_{-45}^2 \\
V &= E_{RCP}^2 - E_{LCP}^2
\end{aligned}
$$

- Stokes parameters

$$
\begin{aligned}
I &= XX + YY \\
Q &= XX - YY \\
U &= XY + YX \\
V &= i(YX - XY)
\end{aligned}
$$

where $X = E_0$, $Y = E_{90}$

Factors of 2!

Is Stokes I

XX + YY

or

(XX + YY)/2

Making Stokes images

- Each antenna measures two orthogonal polarizations - X and Y.
- For every baseline, form all four possible correlations - XX, YY, XY, YX.
- *Calibration etc*
- Appropriately combine the four correlations to get four Stokes “visibilities”.
- Perform standard imaging with these Stokes visibilities to make Stokes images.

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Rotation of the sky

Alt-az mount

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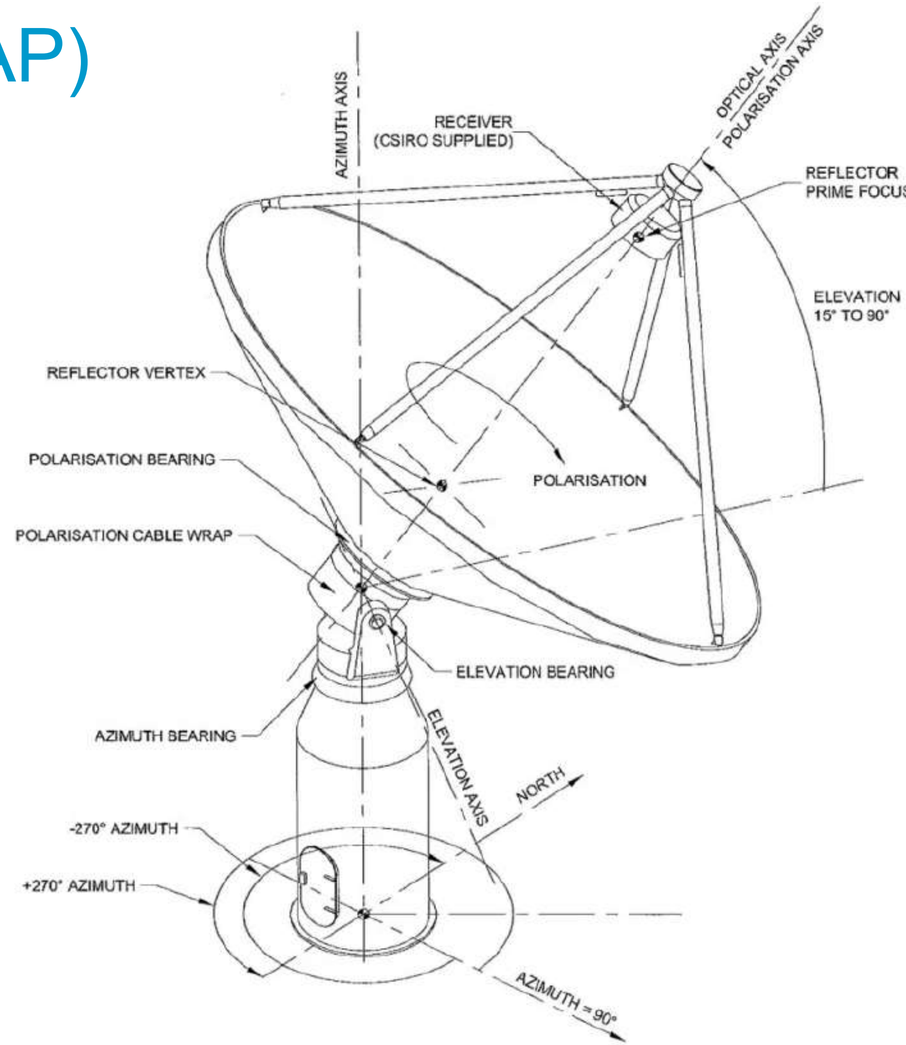
Equatorial mounts

Counteract the rotation

- “Parallactifiers”
- “Sky mount” (ASKAP)



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And others …

Calibration

- Polarimetry I
  - What and How?
    - What is polarized light and ways to quantify it
    - Hardware to measure it
    - Calibration
    - Some advanced topics

Calibration

- Calibration accounts for a number of effects
  - Rotation of the sky
  - Polarization leakage
  - XY phase
  - Ionospheric Faraday rotation
  - *Off-axis polarization response (Polarimetry II)*

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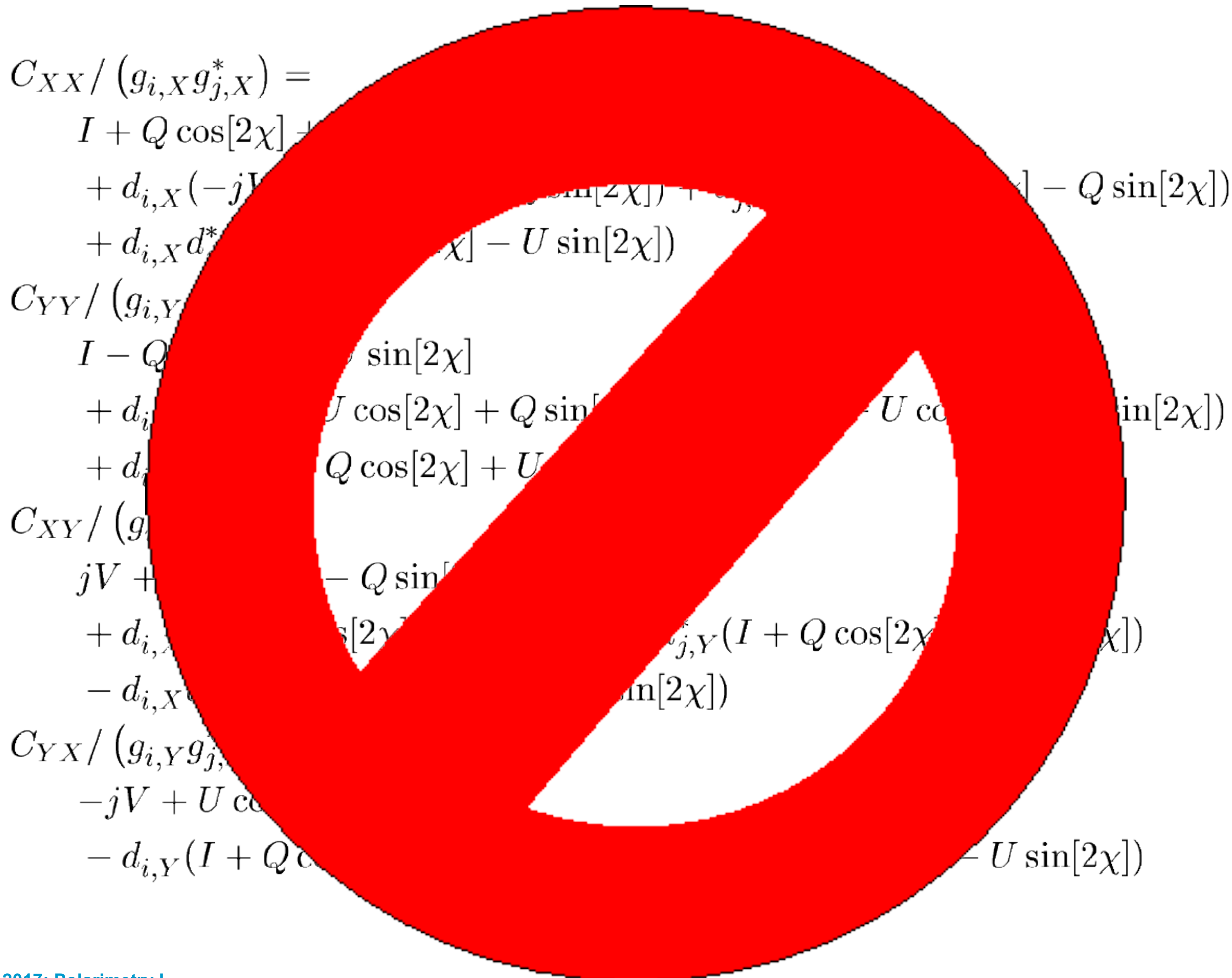
Just say no!



$C_{XX}/\left(g_{i,X}g^*_{j,X}\right) =$
$I + Q\cos[2\chi]$
$+ d_{i,X}(-jV$ … $] - Q\sin[2\chi])$
$+ d_{i,X}d^*$ … $\chi] - U\sin[2\chi])$
$C_{YY}/\left(g_{i,Y}\right.$
$I - Q$ … $\sin[2\chi]$
$+ d_i$ … $U\cos[2\chi] + Q\sin$ … $U\cos$ … $\sin[2\chi])$
$+ d_i$ … $Q\cos[2\chi] + U$
$C_{XY}/\left(g\right.$
$jV +$ … $- Q\sin$
$+ d_{i,}$ … $[2\chi$ … $_{j,Y}(I + Q\cos[2\chi$ … $\chi])$
$- d_{i,X}$ … $\sin[2\chi])$
$C_{YX}/\left(g_{i,Y}g_{j,}\right.$
$-jV + U\cos$
$- d_{i,Y}(I + Q\cos$ … $- U\sin[2\chi])$

Polarization leakage

- For real-world feeds, the feed will respond both to the desired polarization, and to a small degree the orthogonal polarization:

$$E'_X = E_X + d_x E_Y$$
$$E'_Y = E_Y + d_y E_X$$

The leakage (“d term”) is typically ~10^{-2}. It is caused by alignment error, feed ellipticity, etc. Generic linear model (will suit any system with a linear response).

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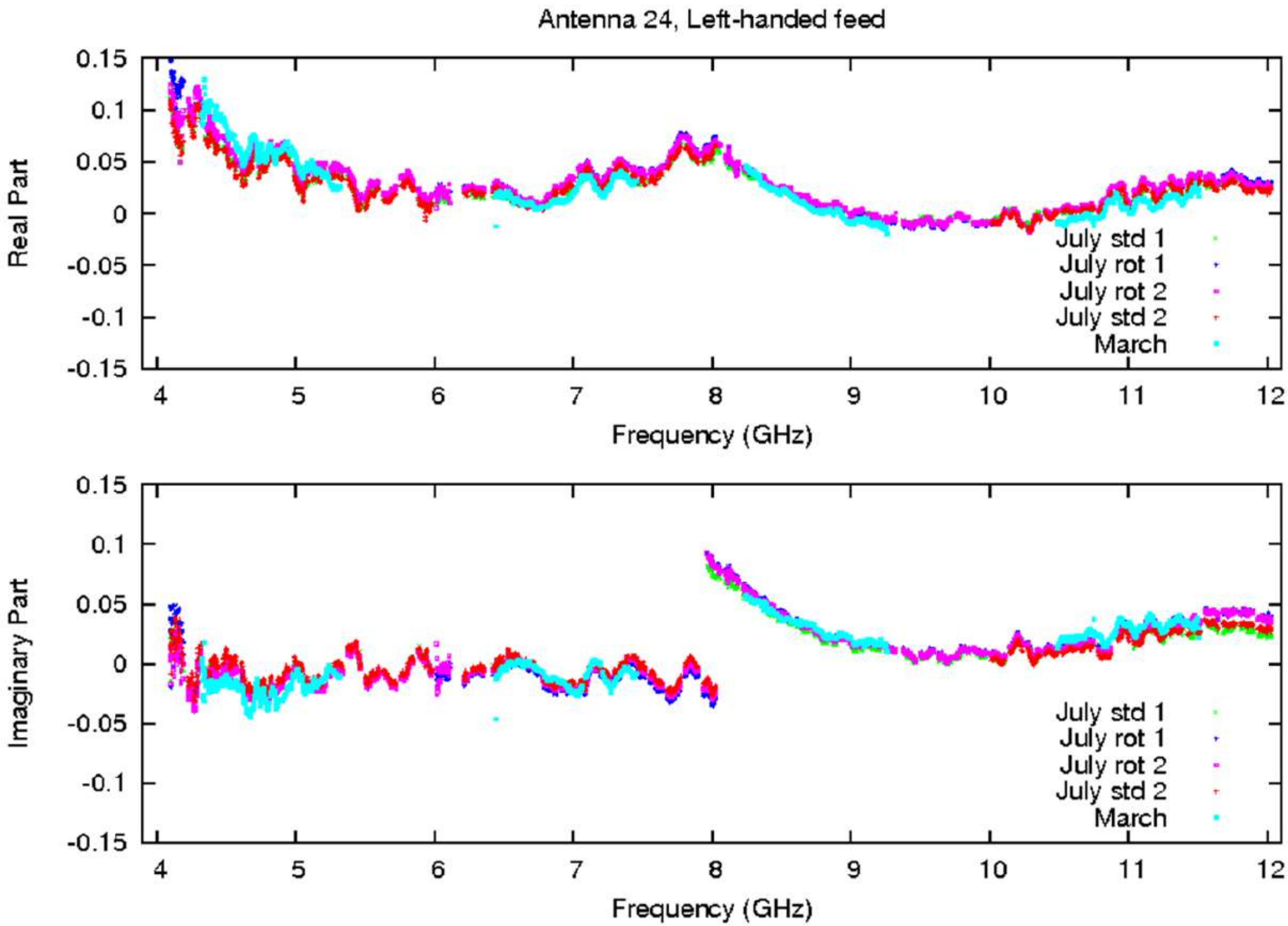
Polarization leakage (continued)

- Leakages a function of antenna, position in beam, frequency (?) and possibly time (??).
- ASKAP field centre leakage very low and independent of frequency. ASKAP leakage as a function of position in beam more an issue *(Widefield polarimetry in next lecture).*

VLA leakages



Polarization leakage (continued)

- Determine leakage terms.
  - ***"To good order",*** simple regular observations of "typical" calibrator is all that is needed to allow a solution for leakages to be made.
  - Q and U Stokes parameters of calibrator either
    - Must be known and accounted for, or
    - Decoupled from polarization leakage using rotation of the sky (resulting from antenna mount or ionosphere).
  - V should be know (usually safe to assume 0)

XY phase

- Instrumental phase difference between X and Y polarization channels can be poorly know.
- Correcting for XY phase is often the most important step for polarimetry.
- Calibration approaches
  - Injected calibration signal (continuous for the ATCA, under development for ASKAP, not available for VLA).
  - Observation of source with strong polarization (required for the VLA).
  - More exotic schemes.

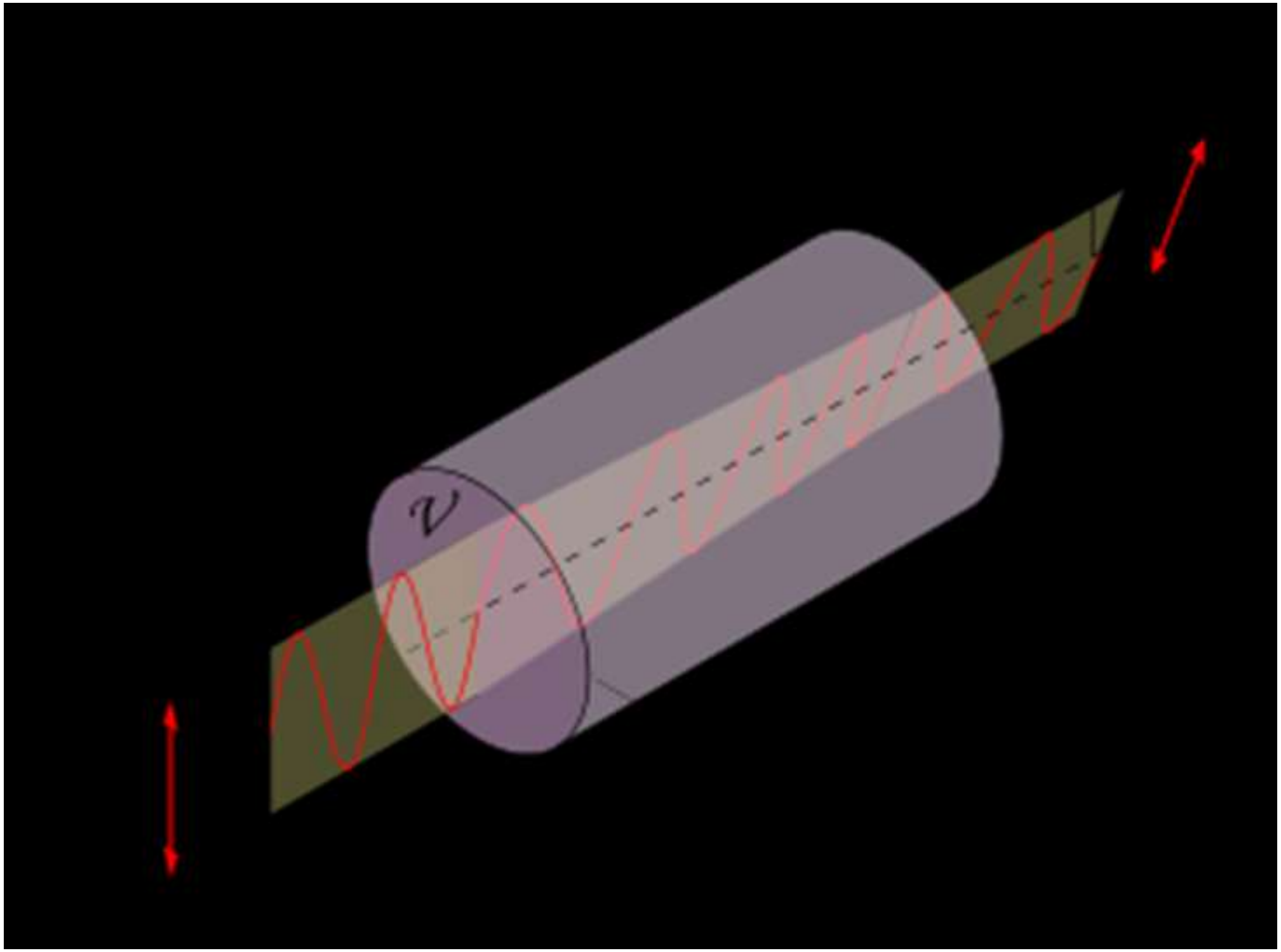
Ionospheric Faraday rotation

- The ionosphere is a significantly variable plasma layer 100-300 km in the upper atmosphere threaded by Earth's magnetic field.
- Propagation through this medium causes *Faraday rotation* – the rotation of the direction of linear polarization. For rotation measure *RM*, the angle of rotation is

$$\varphi = RM\ \lambda^2$$

- Ionospheric RM typically 1-10 rad/m^2

Faraday rotation



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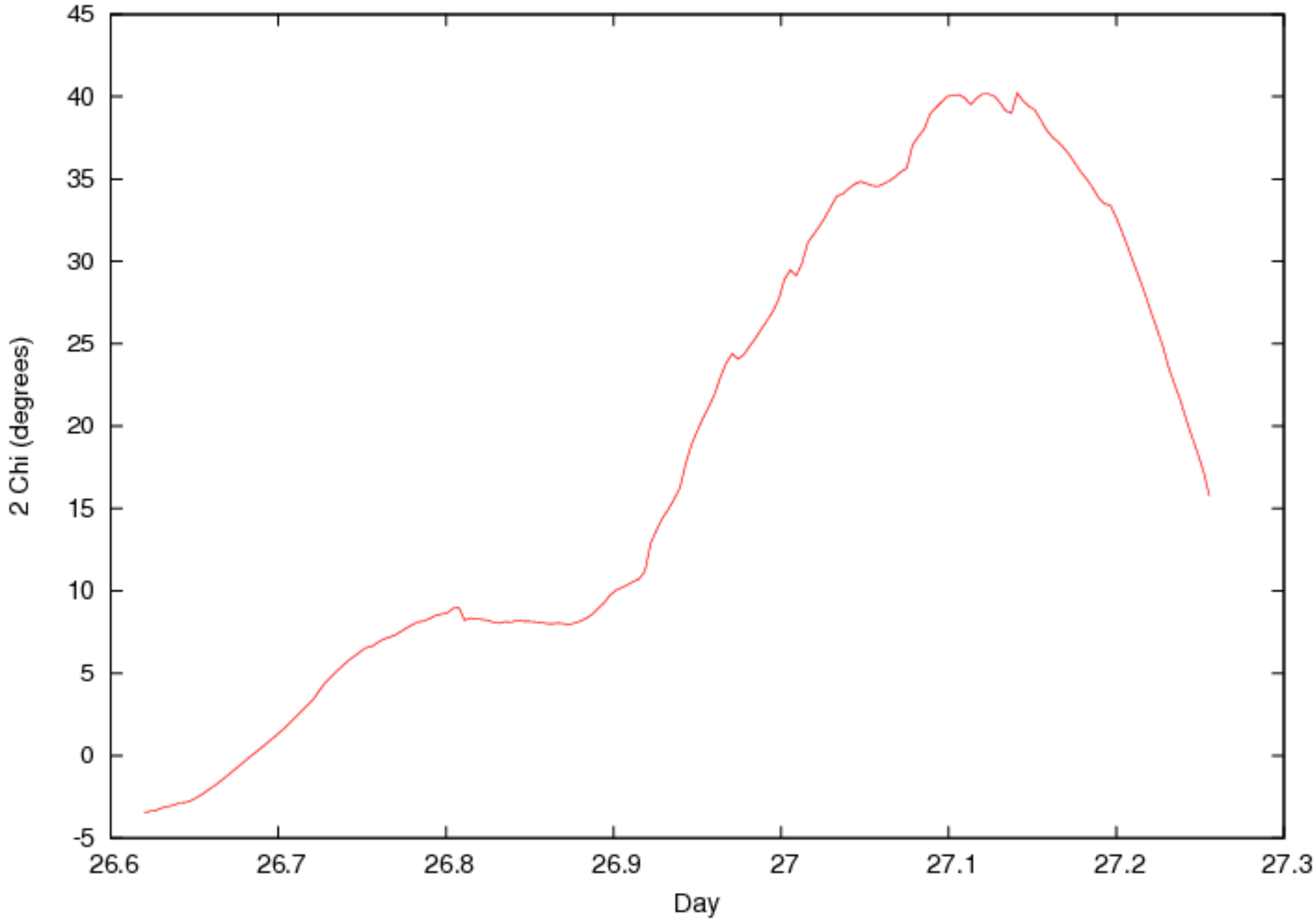
ASKAP example at 700 MHz



Ionospheric Faraday rotation (continued)

- Ionospheric Faraday rotation generally only an issue below 2 GHz. Probably needs to be accounted for below 1 GHz.
- Calibration:
  - GPS stations provide global continuous monitoring of the ionosphere. These data plus models give Faraday rotation accuracy of a ~0.2 rad/m^2.
  - Frequent monitoring of “position angle” calibrator also possible.

Jones matrices

- The response of an optical component to polarized light can described by a simple linear operation on the voltages:

$$E'_X = a_{11} E_X + a_{12} E_Y$$
$$E'_Y = a_{21} E_X + a_{22} E_Y$$

Leakage

$$\boxed{\begin{aligned} E'_X &= E_X + d_x E_Y \\ E'_Y &= E_Y + d_y E_X \end{aligned}}$$

Jones matrices

- The response of an optical component to polarized light can described by a simple linear operation on the voltages:

$$E'_X = a_{11}E_X + a_{12}E_Y$$

$$E'_Y = a_{21}E_X + a_{22}E_Y$$

or more simply, in matrix form – the "Jones" matrix

$$e' = Je$$

for a complex system, the overall Jones matrix will be the product of several matrices:

$$e' = J_1 J_2 J_3 \ldots J_n e$$

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Jones matrices continued …

- Jones matrices are “antenna based” and complex valued

- Antenna gain:
  (includes XY phase)

$$G = \begin{pmatrix} g_x & 0 \\ 0 & g_y \end{pmatrix}$$

- Polarization leakage: $D = \begin{pmatrix} 1 & d_x \\ d_y & 1 \end{pmatrix}$

- Rotation:

$$R = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

Polarimetric interferometry

The overall Jones matrix for an antenna will be

$$J = GDR_{\mathrm{s}}R_{\mathrm{i}}$$

then for baseline *k-l*, the response will be

$$V'_{kl} = J_k \otimes J_l^* V_{kl}$$

- J_k is the antenna Jones matrix.
- V_{kl} is a vector of 4 polarization correlations (e.g. XX, XY, YX and YY correlations).
- $\otimes$ represents Kronecker matrix product

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Depolarization

- Depolarization = loss of polarimetric signal.

- Caused by the system polarimetric response not being constant (i.e. varying spatially, with time, across bandwidth etc etc) smearing out the polarimetric signal.

- Calibrated interferometer arrays generally have very low depolarization *(in polarimetric jargon, a system with no depolarization is called "pure").*

Mueller matrices

A Muller matrix is a 4x4 matrix, *M*, that relates output Stokes parameters of a system to the inputs:

$$\begin{pmatrix} I' \\ Q' \\ U' \\ V' \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}$$

For a system with no depolarization, the Mueller matrix has an associated Jones matrix

$$M = S^{-1}(J \otimes J^*)S$$

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Some advanced topics

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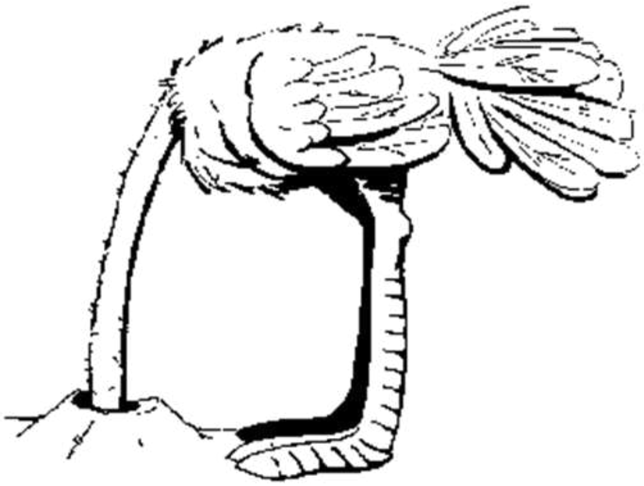
Ostrich calibration approach



Advanced calibration

- Polarimetric calibration converts an interferometer into a system with no depolarization. There are 7 parameters that are potentially ill-determined.

$$
\begin{aligned}
I &\Longleftrightarrow I \quad \text{(flux calibration)} \\
I &\Longleftrightarrow Q \\
I &\Longleftrightarrow U \\
I &\Longleftrightarrow V \\
Q &\Longleftrightarrow U \\
Q &\Longleftrightarrow V \\
U &\Longleftrightarrow V
\end{aligned}
$$

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Advanced calibration …

- "Standard" calibration removes leakage of Stokes I into Q,U,V. It does not calibrate out leakage of Q,U,V into each other. These leakages are second-order and often not important.
- Formally three independent "observations" of known calibrators are needed to fully solve for all corrupting/leakage terms. Astrophysical requirements and "rotation" can reduce this number.

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Off-axis polarimetry (more next lecture)

- Polarimetric response (and Jones matrices) generally a function of position within the primary beam. A good body of theory exists for dealing with position-variant polarimetric response.
- Fractional off-axis polarization (e.g. a the half power points or beyond) can be large if uncorrected. Treat with caution!

Linearly- vs circularly-polarized feeds

- Although this lecture has been framed for linearly-polarized feeds, similar situation applies for circularly-polarized feed systems (different details but same concepts).

- Some debate on whether linearly- (e.g. ATCA, ASKAP, WSRT, KAT) or circularly-polarized feeds (e.g. VLA) are “best”.

3D movie cinemas

Linears vs circulars

- Fundamentally both are equivalent in some sense. Both work in practice.
- The sky and engineering reality breaks the symmetry. “General” polarization calibration easier with circulars.
- Engineering tends to make linearly-polarized feeds more attractive. Precision polarization calibration easier with linears.

Sit on the fence

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Thank you

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