Wide-Field Imaging I:
Full-Beam Imaging & Surveys

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What is Wide-Field Imaging?

- "Narrow-Field" Imaging:
  - imaging of a single field well within Primary Beam (PB)
  - able to ignore (orientation dependent) PB effects
  - able to ignore non-coplanar array (w-term) effects
  - this will get you far, but sometimes you need…
- "Wide-Field" Imaging:
  - includes non-coplanar array (w-term) effects
  - includes orientation-dependent (polarized) PB effects
  - includes mosaciking outside PB
  - all these more complicated for wide-bandwidth imaging!
  - these effects limit dynamic range & fidelity even within PB

Outline of this Lecture

- The Imaging Equation revisited
- The W-Term
- The (Polarized) Primary Beam
- Mosaicking 101
- Imaging Techniques for Linear Mosaics
- Mosaic Sampling – Hex Grids & On-The-Fly
- Practical Mosaicking – Observing Preparation
- Setting up your Survey

Interferometer Equation

- Relates what we measure in a visibility *k* to what is on the sky and what our antennas and array are doing (no noise):

$$V_\nu(u_k, v_k, w_k) = \iint \frac{dl\,dm}{\sqrt{1-l^2-m^2}} S_\nu(l,m) A_{k\nu}(l,m) e^{-2\pi i\left(u_k l + v_k m + w_k\left(\sqrt{1-l^2-m^2}-1\right)\right)}$$

  – the visibility index *k* encapsulates the time, antenna pair, parallactic angle, pointing direction, phase center of the observation for **A**

- To do 2D Fourier transforms between the (l,m) and (u,v)

$$V_\nu(u_k, v_k, w_k) = \iint \frac{dl\,dm}{\sqrt{1-l^2-m^2}} S_\nu(l,m) A_{k\nu}(l,m) W(l,m,w_k) e^{-2\pi i(u_k l + v_k m)}$$

Data — Sky — Primary Beam — Geometry — Fourier kernel

- Write with linear operators: (includes noise term)

$$\underline{\mathbf{v}} = \mathbf{F}^{-1}\,\mathbf{W}\,\mathbf{A}\,\mathbf{s} + \underline{\mathbf{n}}$$

Linear operator notation: can represent integrals or discrete matrix equations!

From sky to *uv* plane

- Visibility equation (subsume **W** inside **A** for now):

$$\underline{v} = \mathbf{F}^{-1}\,\mathbf{A}\,\mathbf{s} + \underline{n}$$

- Fourier transformation of vector and matrix operators:

$$\underline{s} = \mathbf{F}^{-1}\,\mathbf{s} \qquad \underline{A} = \mathbf{F}^{-1}\,\mathbf{A}\,\mathbf{F}$$

- Equivalent Fourier (uv) domain equation (insert $\mathbf{F}\mathbf{F}^{-1}$):

$$\underline{v} = \underline{A}\,\underline{s} + \underline{n}$$

(this is just stating the Fourier convolution theorem)

- This is saying that the Fourier transform of the sky ($\mathbf{F}^{-1}$s) is convolved in the uv-plane with the transform of the Primary Beam (including any geometric w-term)

Visibility vector $\underline{v}$ is over distinct integrations and channels.

Sky image vector $\mathbf{s}$ is over pixels on sky.

A is not square and is non-invertible!

Image Reconstruction

- Visibilities and the Sky

$$\underline{v} = \underline{A}\,\underline{s} + \underline{n}$$

  - $\underline{A}$ known instrumental response, but is not invertible
    - true uv-plane convolved by aperture cross correlation $\underline{A}$
    - has finite support (2x dish dia.) in uv-plane (not including w-term)
  - $\underline{A}$ has support only where there is data in $\underline{v}$
    - incomplete sampling of uv-plane by visibilities
  - instrumental noise $\underline{n}$ is a random variable with covariance $\mathbf{N} = \langle nn^T \rangle$
- Maximum Likelihood Estimate (MLE) of sky:

$$\underline{s}_{MLE} = (\underline{A}^T \underline{N}^{-1} \underline{A})^{-1} \underline{A}^T \underline{N}^{-1} \underline{v} = \underline{R}^{-1} \underline{d} \qquad \underline{d} = \underline{H}\,\underline{v}$$

$$\underline{R} = \underline{A}^T \underline{N}^{-1} \underline{A} \qquad \underline{H} = \underline{A}^T \underline{N}^{-1}$$

$\underline{R}$ singular (at best ill-conditioned) so no inversion practical

The Dirty Map

Use of $\mathbf{N}^{-1}$ here in gridding is "natural weighting". Other choices give uniform or robust weighting!

- Grid onto sampled uv-plane

$$\underline{d} = \underline{\boldsymbol{H}}\, \underline{\mathbf{v}} = \underline{\boldsymbol{H}}\, \underline{s} + \underline{n}_d$$

  - $\underline{\boldsymbol{H}}$ should be close to $\underline{\boldsymbol{H}}_{MLE}$, e.g.

$$\underline{\boldsymbol{H}} = \underline{\mathcal{A}}^T\, \underline{\mathcal{N}}^{-1} \quad : \quad \underline{\mathcal{A}} \sim \underline{\mathbf{A}} \quad \underline{\mathcal{N}} \sim \underline{\mathbf{N}}$$

  - $\underline{\mathcal{A}}^T$ should sample onto suitable grid in uv-plane
  - reminder: need only be approximate for gridding = "A-projection"
    - include w-term geometry A → AG = "AW-projection" (later)
- Invert onto sky ➔ "dirty image"

$$\mathrm{d} = \mathbf{F}\, \underline{d} = \mathbf{R}\, \mathrm{s} + \mathrm{n}_d \qquad \mathbf{R} = \mathbf{F}\, \underline{\boldsymbol{R}}\, \mathbf{F}^{-1}$$

  - image is "dirty" as it contains artifacts
    - convolution by "point spread function" (columns of **R**) = PSF dirty beam
    - multiplication by response function (diagonal of **R**) = Primary Beam
    - noise (& calibration errors, etc.)

Imaging Equation Summary

- Our best estimation (MLE) of the sky is

$$s_{\mathrm{MLE}} = \mathbf{R}^{-1}\, \underline{\boldsymbol{A}}^{T}\, \underline{\boldsymbol{N}}^{-1}\, \underline{\boldsymbol{v}}$$

- An intermediate estimate is the dirty image

$$\mathbf{d} = \mathbf{F}\, \underline{\boldsymbol{H}}\, \underline{\boldsymbol{v}} = \mathbf{F}\, \underline{\boldsymbol{H}}\, (\, \mathbf{F}^{-1}\, \mathbf{A}\, \mathbf{s} + \underline{\boldsymbol{n}}\,) = \mathbf{R}\, \mathrm{s} + \mathrm{n}_{\mathrm{d}}$$

  – all the data, regardless of where the antennas and array were pointed or phase, goes into this image through $\underline{\boldsymbol{H}}$
    - *There is a single uv-plane*
  – can choose the gridding kernel $\underline{\boldsymbol{H}}$ to optimize image
    - e.g. $\underline{\boldsymbol{H}} = \underline{\mathcal{A}}^{T}\, \underline{\mathcal{N}}^{-1}$

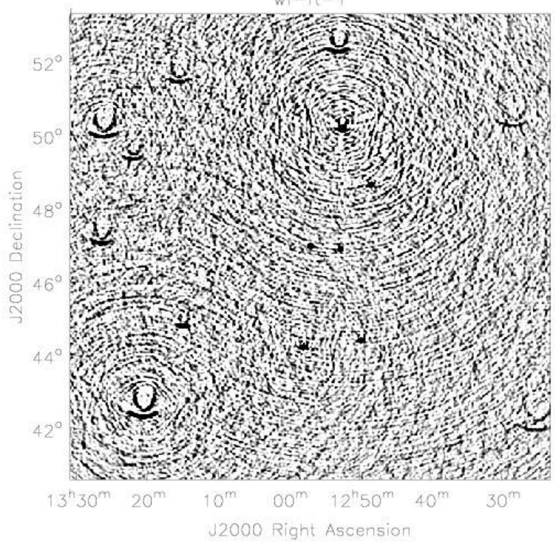
You can store **R** and **H** for later use!

  – **R** is the known relation between **d** and **s** (PSF & PB)
    - iterative methods (e.g. Cotton-Schwab Clean) perform well

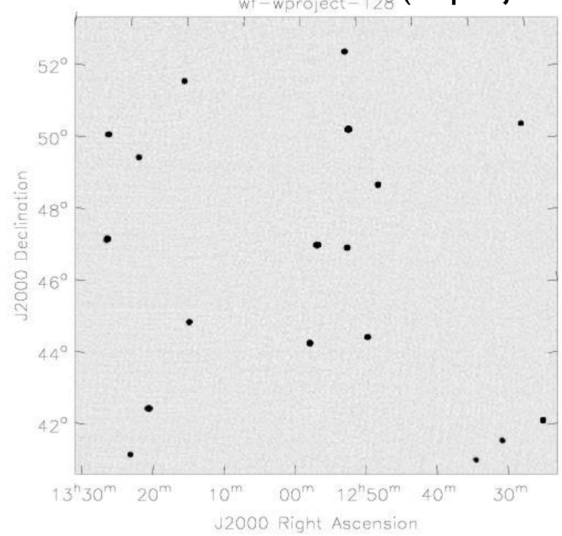
Wide-field Imaging: W-term

- 2D Fourier transform approximation of the imaging equation breaks down due to geometric term ("The W-term problem")

Without w-term correction



With w-term correction (w-projection)



For the Student:
Hey, these look like diffraction patterns! Is there some sort of diffraction at work here?

- Imaging dynamic range throughout the image is limited by deconvolution errors due to the sources away from the (phase) center.

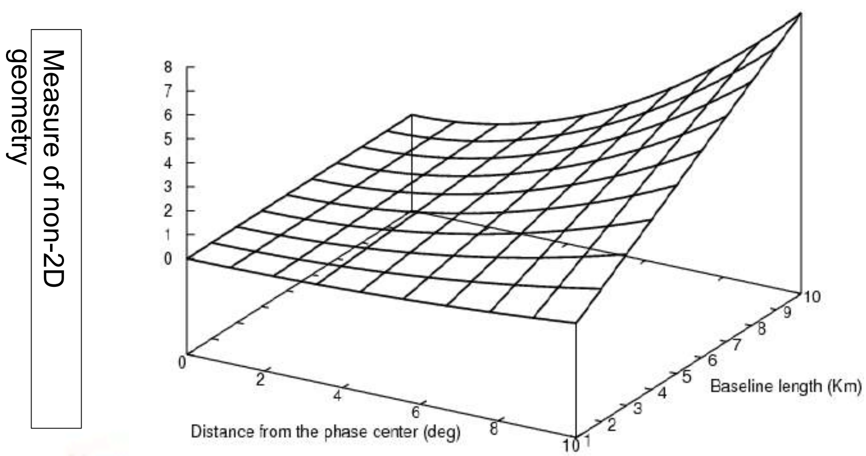
Wide-field Imaging: W-term

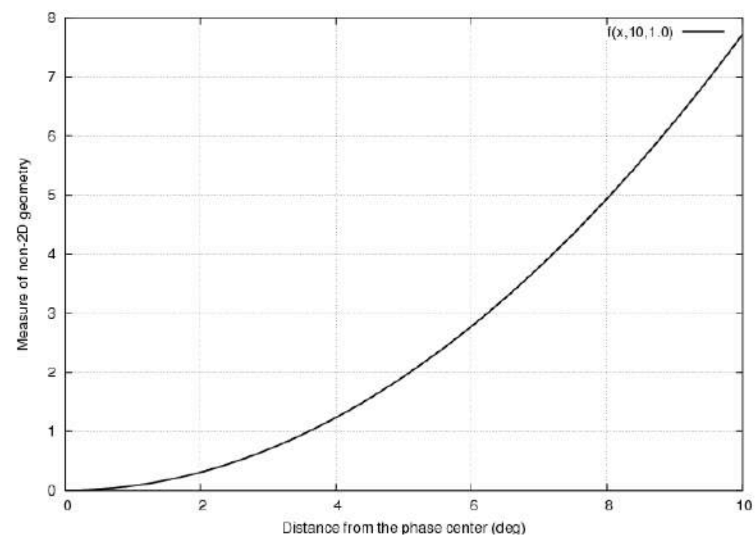
- Deviation from 2D geometry increases with FoV and baseline length.
- W-Term: convolution in the imaging equation (IE) by

$$W(l,m,w) = e^{-2\pi i w\left(\sqrt{1-l^2-m^2}-1\right)}$$

For the Student:

Can **W** be treated as an extra Fourier transform (3D)? If so, is this a practical implementation?





W-term: when does it matter?

- The geometric term in the IE is:

$$W(l,m,w) = e^{-2\pi i w\left(\sqrt{1-l^2-m^2}-1\right)}$$

- This is negligible when:
  - the Field-of-View (FoV) is small: $l^2 + m^2 << 1$
  - the array is nearly coplanar: $w << \sqrt{u_{\max}^2 + v_{\max}^2}$
  - duration of observations is short
    - "snapshot"

Remember: Earth rotation will cause non-coplanarity of baselines in 2D array!

Ref: Chapter 19

- Rule of thumb: ok when

$$\frac{\lambda_{\max} B_{\max}}{D^2} < 1$$

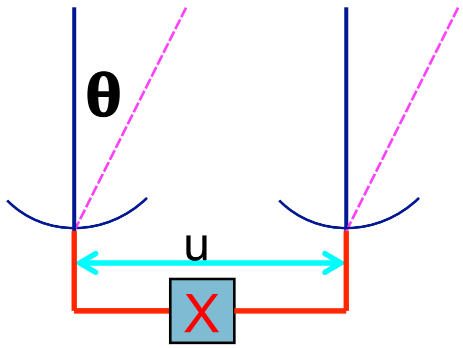
D = Dish diameter B_{max} = maximum baseline

Example: VLA A-config
λ_{max} < 2cm (15GHz) !

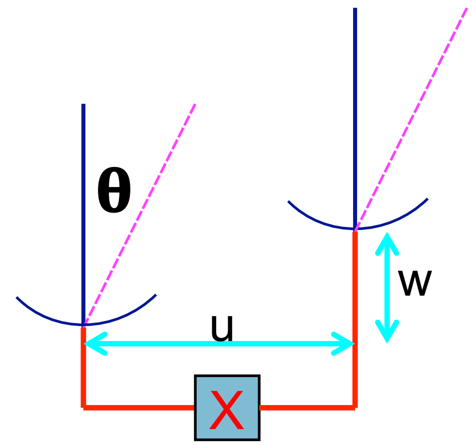
VLA: D=25m

VLA configurations:
(A) 36km (C) 3.4km
(B) 11km (D) 1.0km

W-term: geometric picture



$$l = \sin(\theta)$$
$$n = \cos(\theta)$$



- Phase φ of the visibilities for offset angle θ
  - For the interferometer in a plane (left):

$$\varphi = 2\pi u l$$

  - For the interferometer not in a plane (right) :

$$\varphi = 2\pi [ul + w(n-1)]$$

- ***W*** ≈ 1 (coplanar) only when: (1) w « u, or (2) $\theta \approx 0$

W-term: optics picture

- We want to measure:

$$V_{12}^{o} = \left\langle E_1^{'}(u,v,w) E_2^{*}(0,0,0) \right\rangle$$

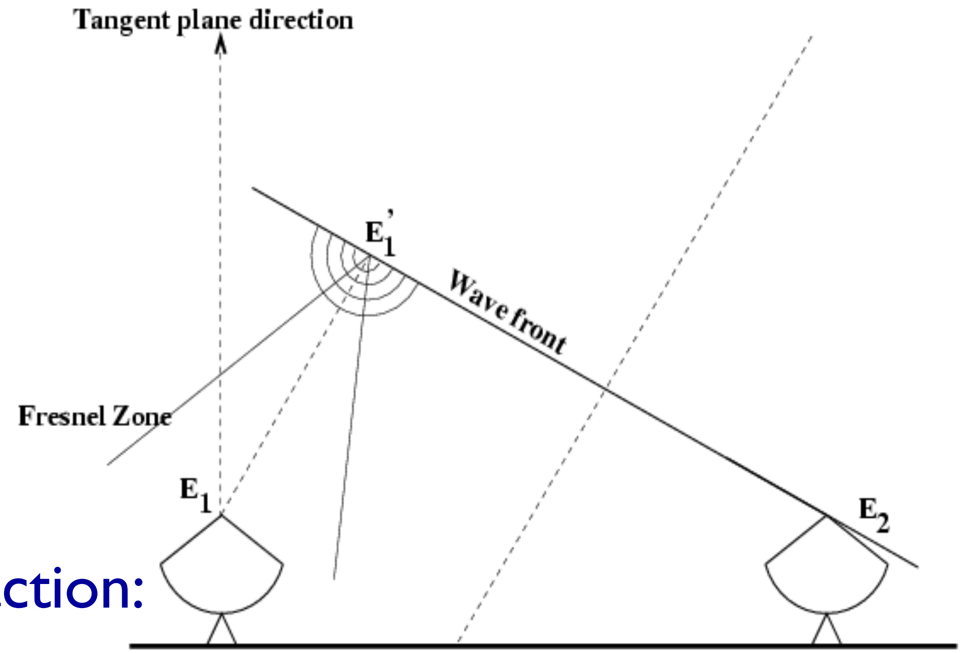
(in wavefront tangent plane)

- We actually measure:

$$V_{12} = \left\langle E_1(u,v,0) E_2^{*}(0,0,0) \right\rangle$$

(in imaging tangent plane)

- Propagate using Fresnel diffraction:



$$V_{12}^{o} = V_{12} * W(u,v,w)$$

$$W(u,v,w) = FT\left[e^{-2\pi i w\left(\sqrt{1-l^2-m^2}-1\right)} \right]$$

- so

$$V_{12}^{0} = \iint \frac{dl\,dm}{\sqrt{1-l^2-m^2}} S_\nu(l,m) A_{k\nu}(l,m) e^{-2\pi i(u_{12}l+v_{12}m)} e^{-2\pi i w_{12}\left(\sqrt{1-l^2-m^2}-1\right)}$$

For the Student:

What is the functional form of transform W?

W-term: image-plane faceting

- Interpret *S(l,m)* as emission on the surface of the Celestial Sphere of unit radius: $l^2+m^2+n^2=1$
  - Approximate the celestial sphere by a set of tangent planes – a.k.a. “facets” – such that 2D geometry is valid per facet
  - Use 2D imaging on each facet
  - Re-project and stitch the facet-images to a single 2D plane

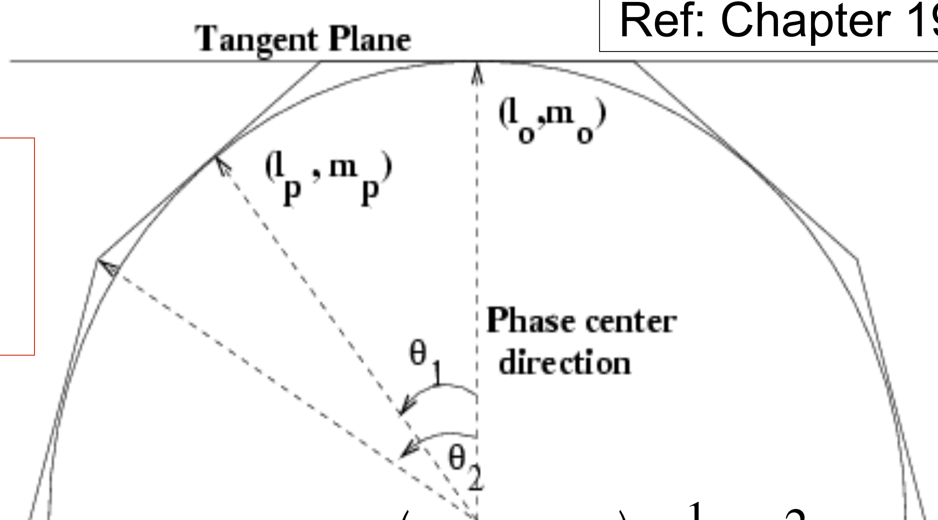
Ref: Chapter 19

- Number of facets required:

$$N_{poly} \approx 2\theta_{FoV}^2 \frac{B_{max}}{\lambda_{max}} = \frac{2B_{max}\lambda_{max}}{[fD]^2}$$

Example:
VLA A-config
1.0GHz
N>34

f=1 for critical sampling.
f<1 for high dynamic range



$$error: \sin(\theta_1)\left(1-\cos(\theta_2)\right) \approx \frac{1}{2}\theta_1\theta_2^2$$

W-term: uv-plane faceting

- Since shifting (rotating axes on sky) and summing image facets is a linear operation, and our Imaging Equation is linear, there must be an equivalent in the uv-plane to faceting:

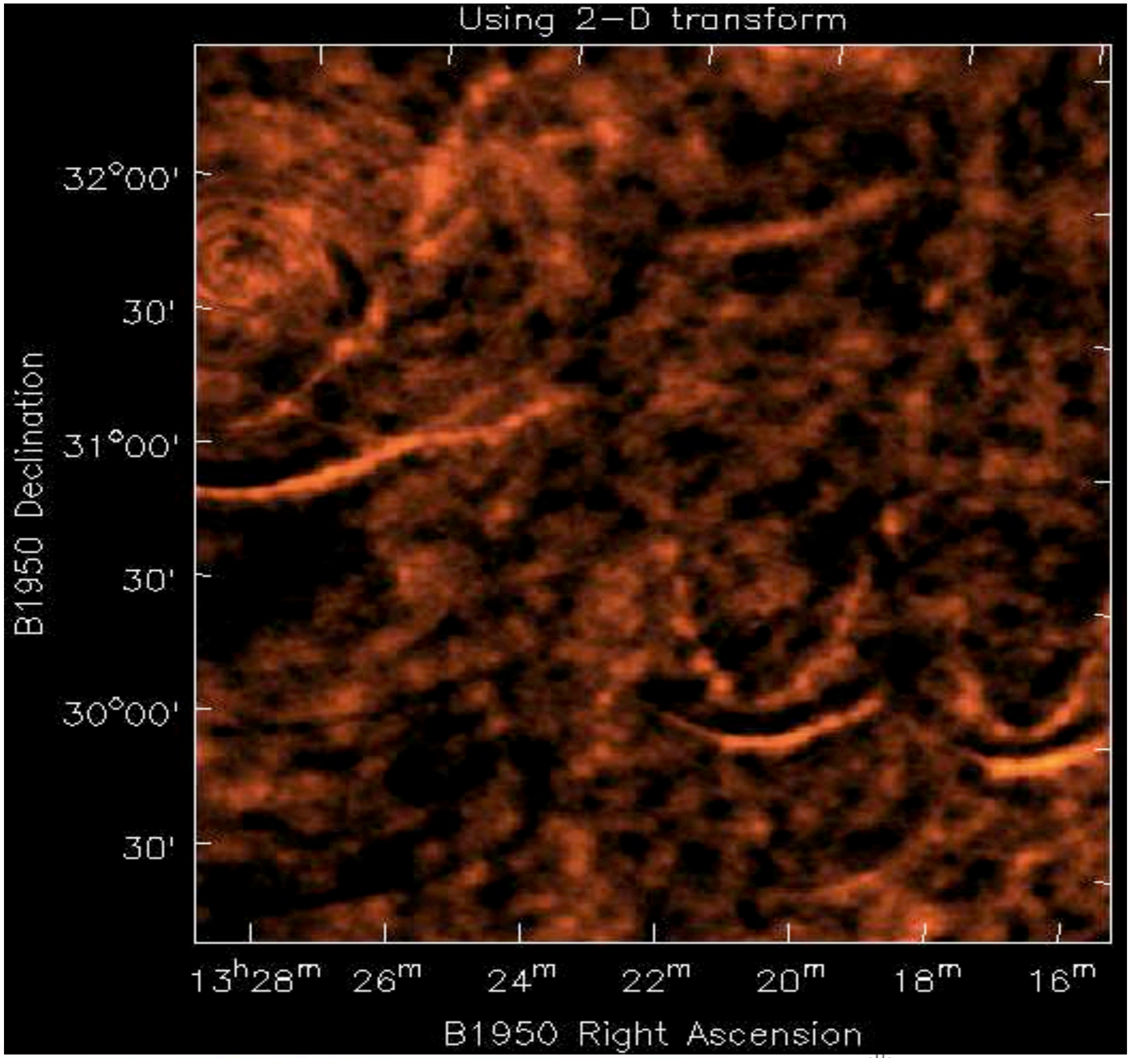
$$I(C\vec{\theta}) \rightarrow \left[\det(C)\right]^{-1} V\left(C^{-1T}\vec{u}\right)$$

  - C = image-plane coordinate transformation
  - θ and $\boldsymbol{u}$ are the image and uv plane coordinates respectively
- uv-plane faceting vs. image plane faceting
  - errors same as in image plane faceting
  - produces a single image (no edge effects)
  - global (single plane) deconvolution straightforward
  - use of advanced algorithms for extended emission possible
  - can be faster in some implementations

Note: uv-plane faceting used in CASA

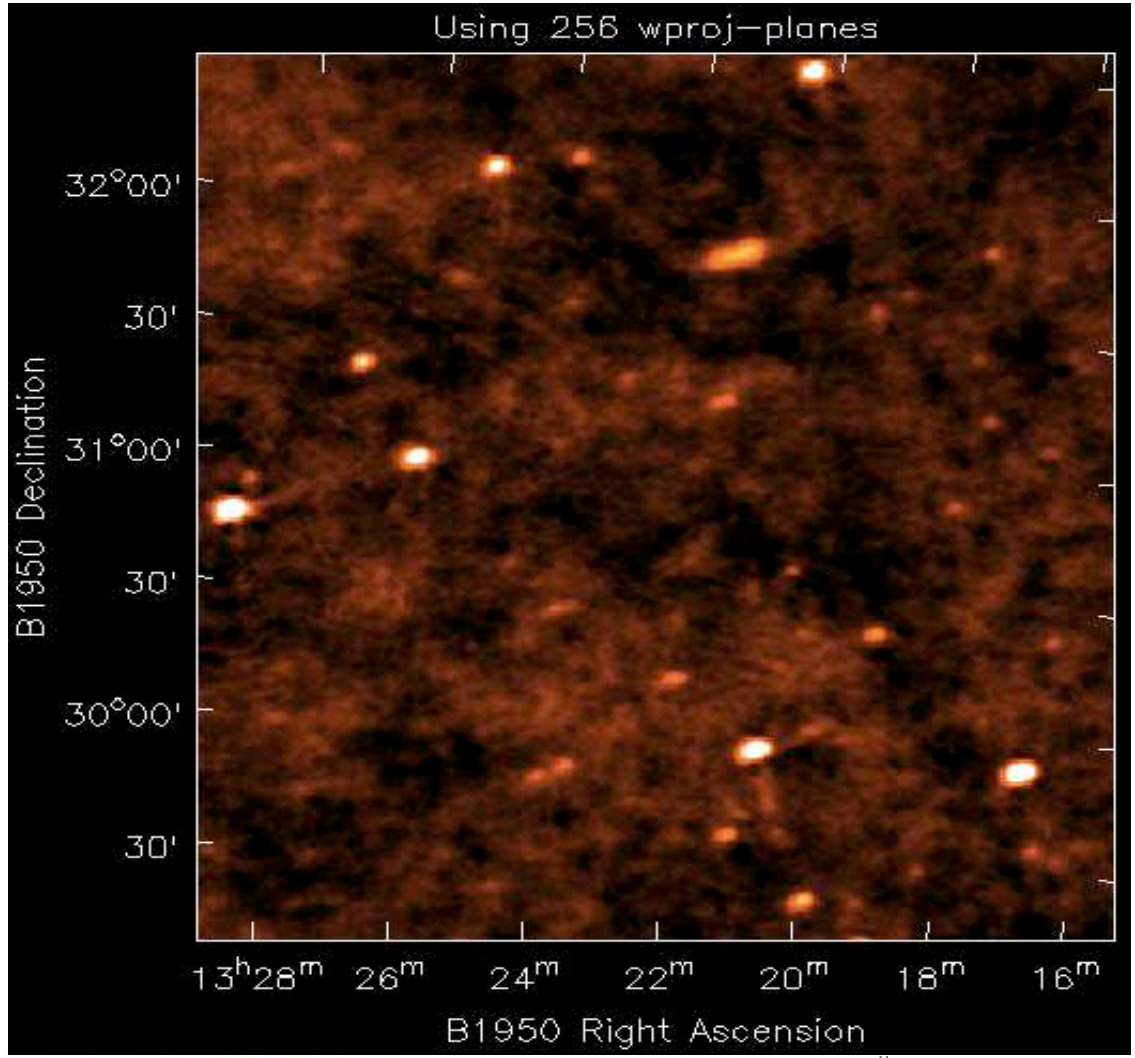
W-term: example

- No correction
  - W-term introduces a phase error
  - dependent on distance from center of image
  - dependent on baseline length and frequency (uv radius)
  - characteristic arc shapes



W-term: corrected!

- Correction applied
  - using CASA w-projection
  - 256 w planes



W-term: W-projection

- Go back to our Imaging Equation (including W):

$$\underline{v} = \mathbf{F}^{-1}\,\mathbf{W}\,\mathbf{A}\,\mathbf{s} + \underline{n}$$

- or in the uv domain:

$$\underline{v} = \underline{W}\,\underline{A}\,\underline{s} + \underline{n}$$

- which implies that we image using:

$$\underline{d} = \underline{H}\,\underline{v} \qquad \underline{H} = \underline{\mathcal{A}}^{T}\,\underline{\mathcal{W}}^{T}\,\underline{N}^{-1}$$

($\underline{\mathcal{W}} \approx \underline{W}$ using Hermitian transpose, $\underline{W}^{T}\,\underline{W} = 1$ unitary)

- Gridding using the W-kernel (in the uv-plane) is called "W-projection" (see Cornwell, Golap, Bhatnagar EVLA Memo 67). This is an efficient alternative / augmentation to faceting.

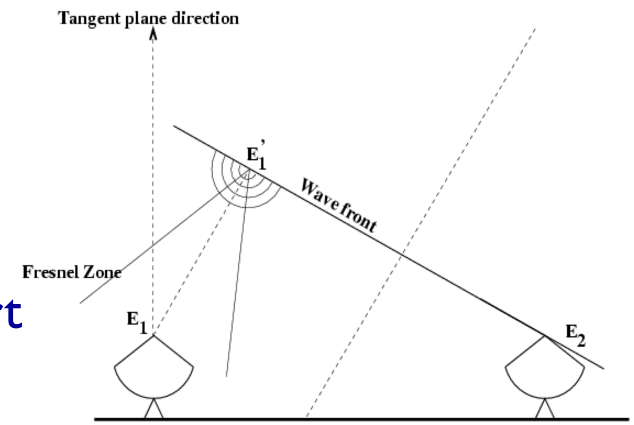
Ref: Cornwell et al. IEEE Special topics in SP, Vol. 2, No5, 2008 [arXiv:0807.4161]

W-term: W-projection

- As $\underline{\mathbf{W}}$ is unitary and $\underline{\mathcal{W}}^T\underline{\mathbf{W}}\approx I$ $\underline{\mathcal{W}}^T$ and $\underline{\mathbf{W}}$ can be used in the forward and inverse transforms to produce minimally distorted residual images as part of Cotton-Schwab (CS) clean

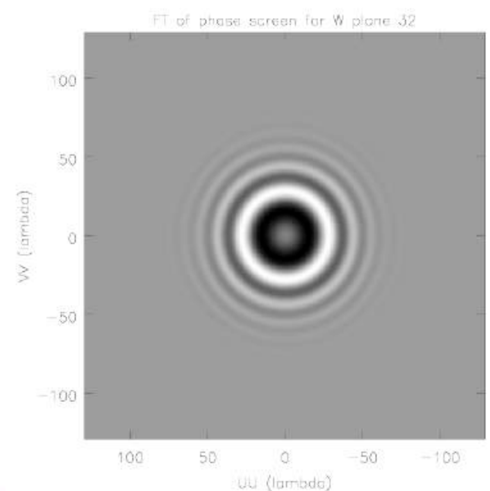
$$V^o(u,v,w) = V(u,v,0) * \underline{W}(u,v,w)$$

- Model prediction during major cycle:
  – compute 2D FFT of model image → $V(u,v,0)$
  – evaluate above convolution to get $V^o(u,v,w)$
  – subtract from visibilites to get residual
- Compute dirty residual image for minor cycle:
  – use $\mathcal{W}^T(u,v,w)$ on each $V^o(u,v,w)$ on a grid in w, sum to get $V^{proj}(u,v)$
  – this is just a modification of normal gridding CF!
  – make dirty image with 2D FFT-1 of V(u,v)



Radius of Fresnel zone:

$$\frac{r_f}{\lambda} \approx \sqrt{w}$$



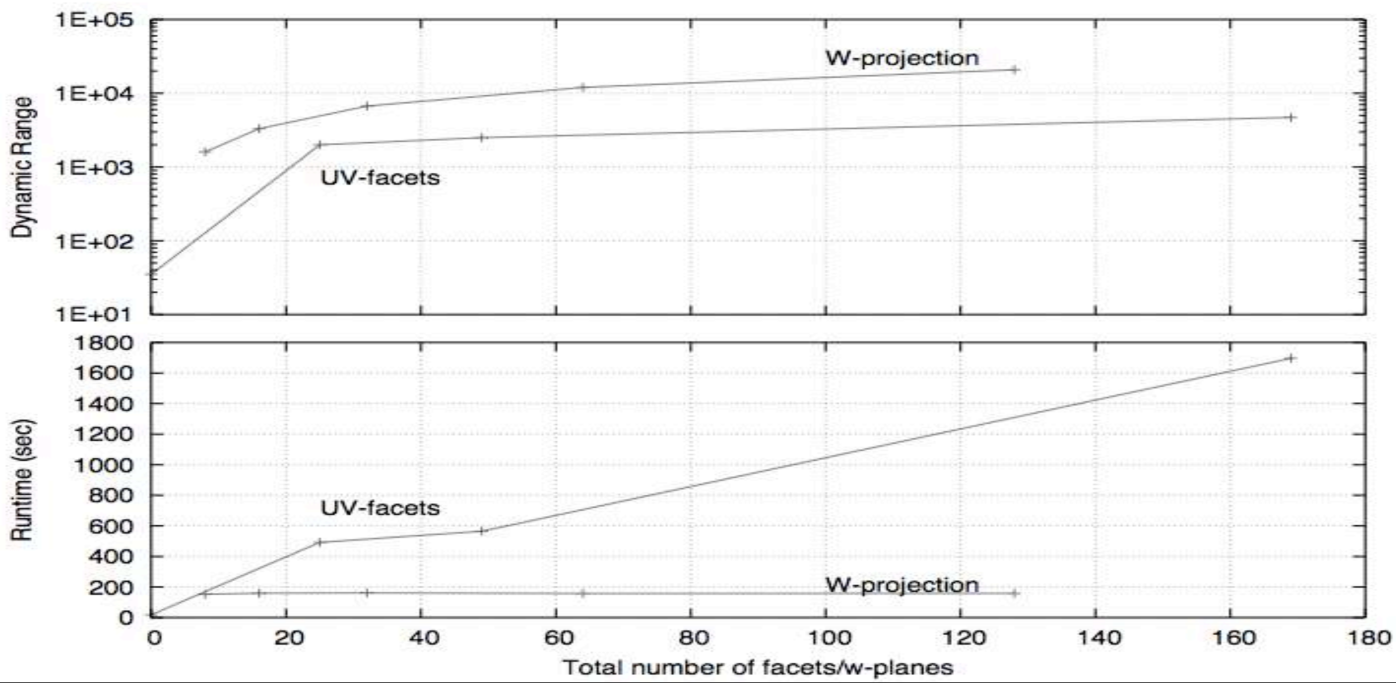
W-term: Performance

- Scaling
  - Faceted imaging: $(N^2_{fac} + N^2_{GCF})\ N_{vis}$
  - W-projection: $(N^2_{Wplanes} + N^2_{GCF})\ N_{vis}$
  - Ratio:

In practice W-projection works well for modest (128 or 256) number of w-planes. Best to combine with faceting for larger problems!

In practice W-Projection algorithm is about 10x faster

Size of G(u,v,w) increases with W



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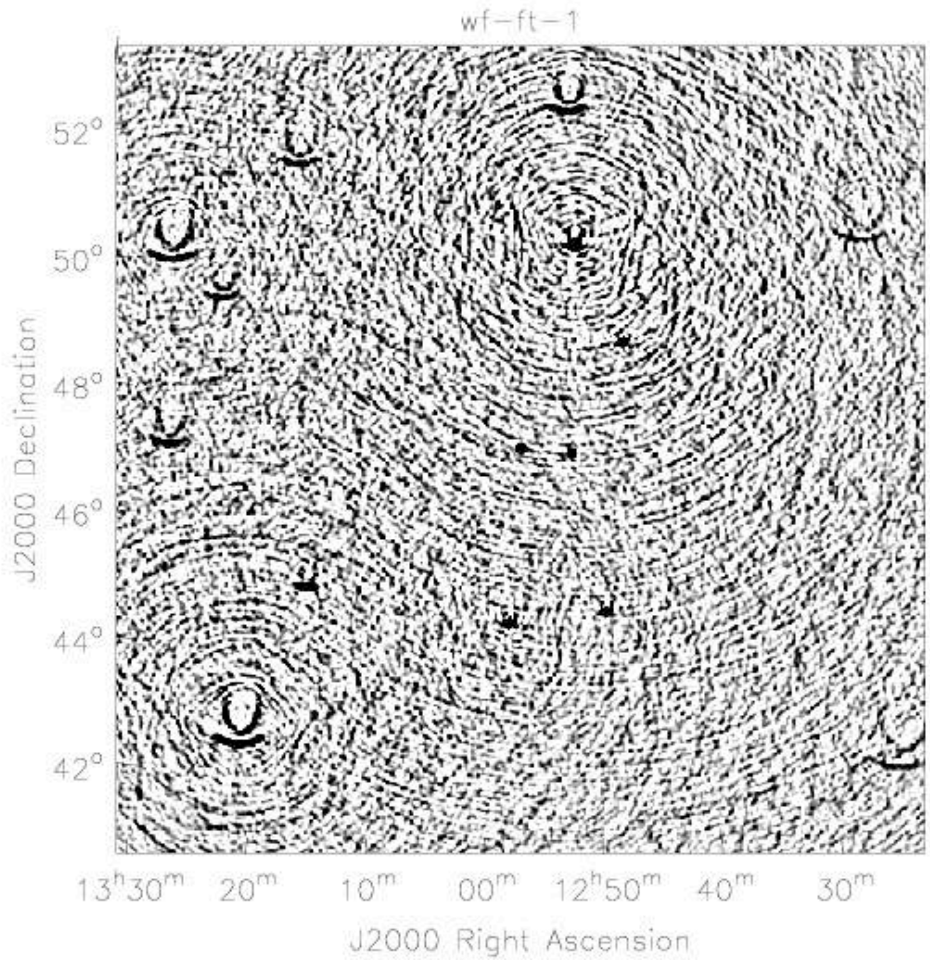
Example: 2D imaging uncorrected



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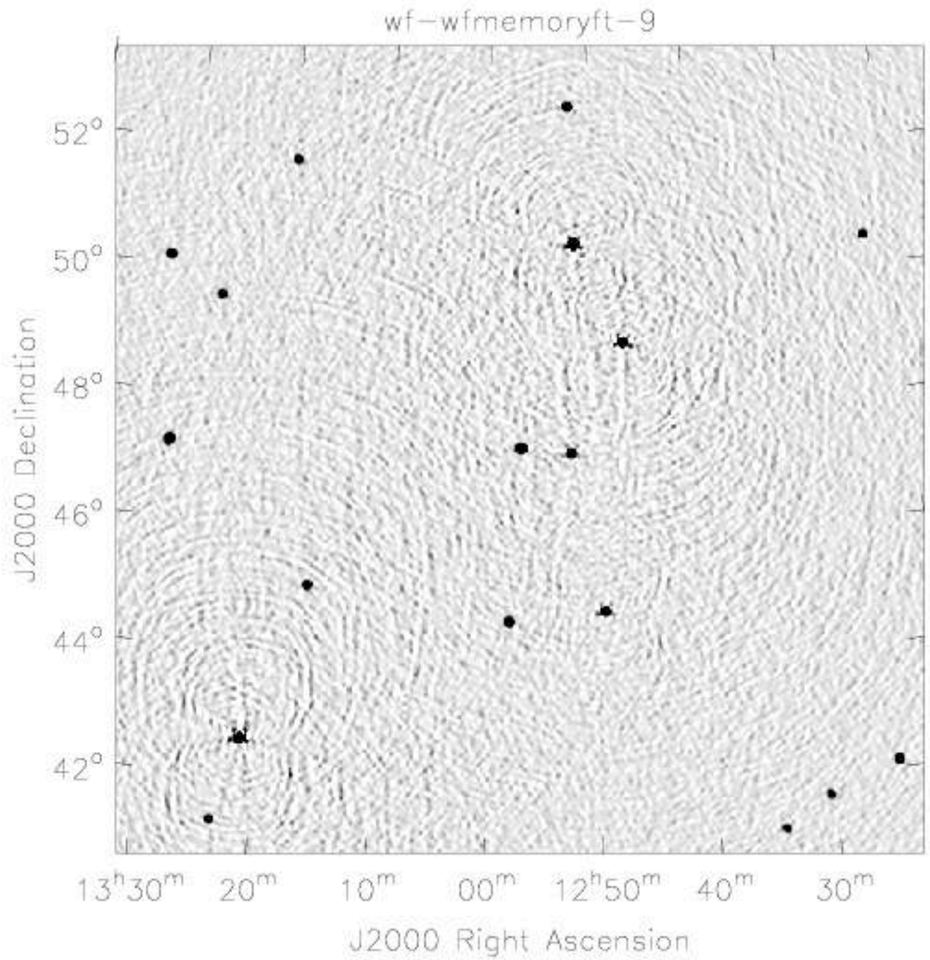

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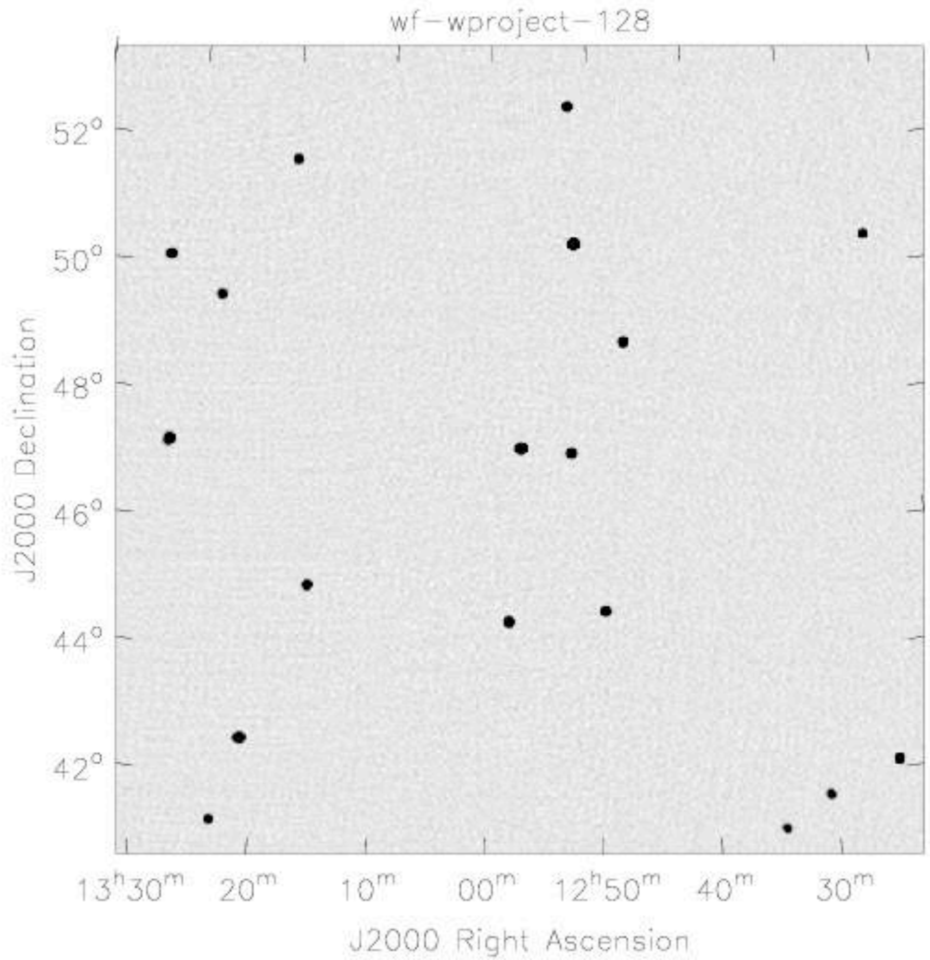
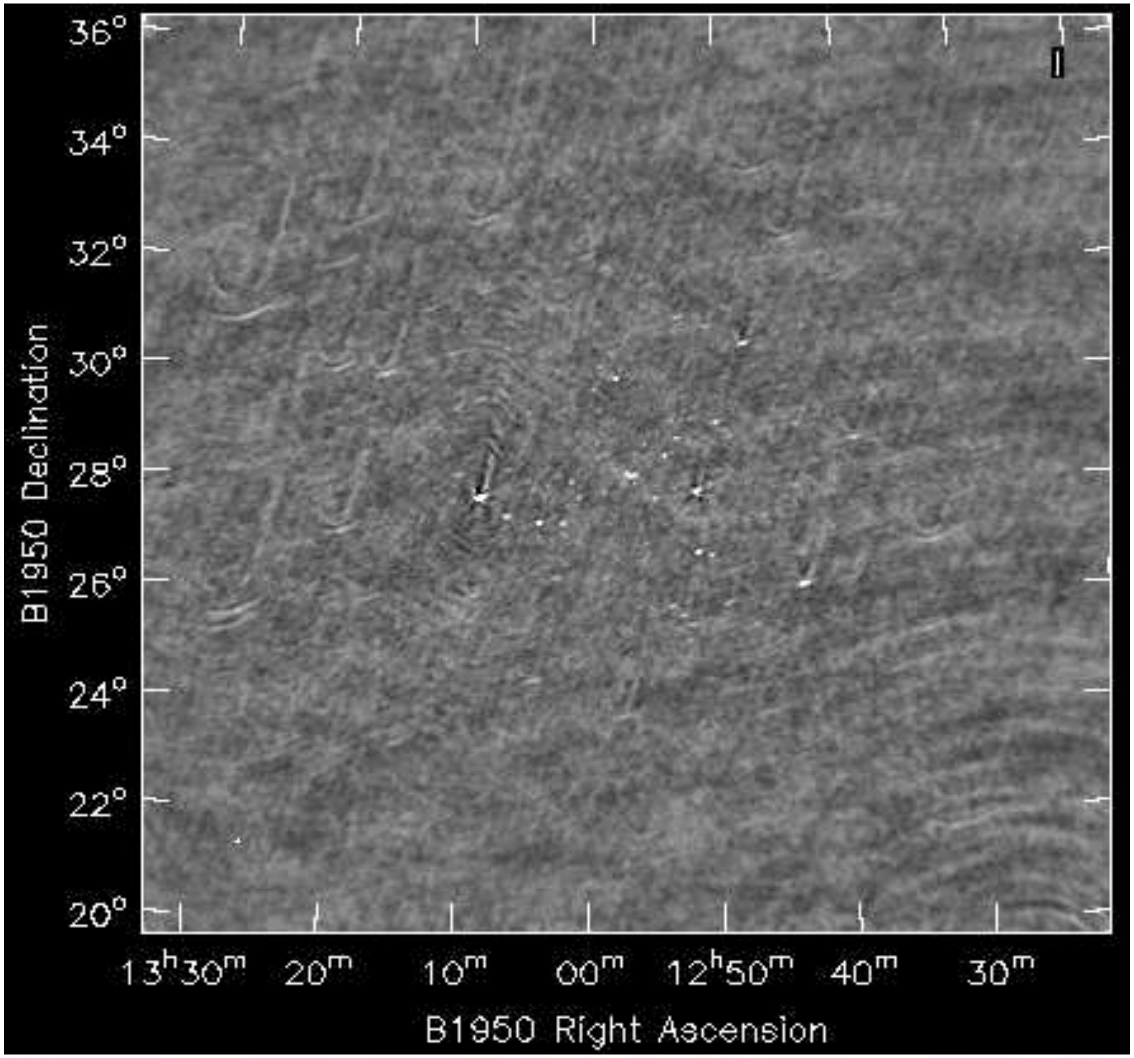


Example: VLA 74MHz before correction

Courtesy
Kumar Golap



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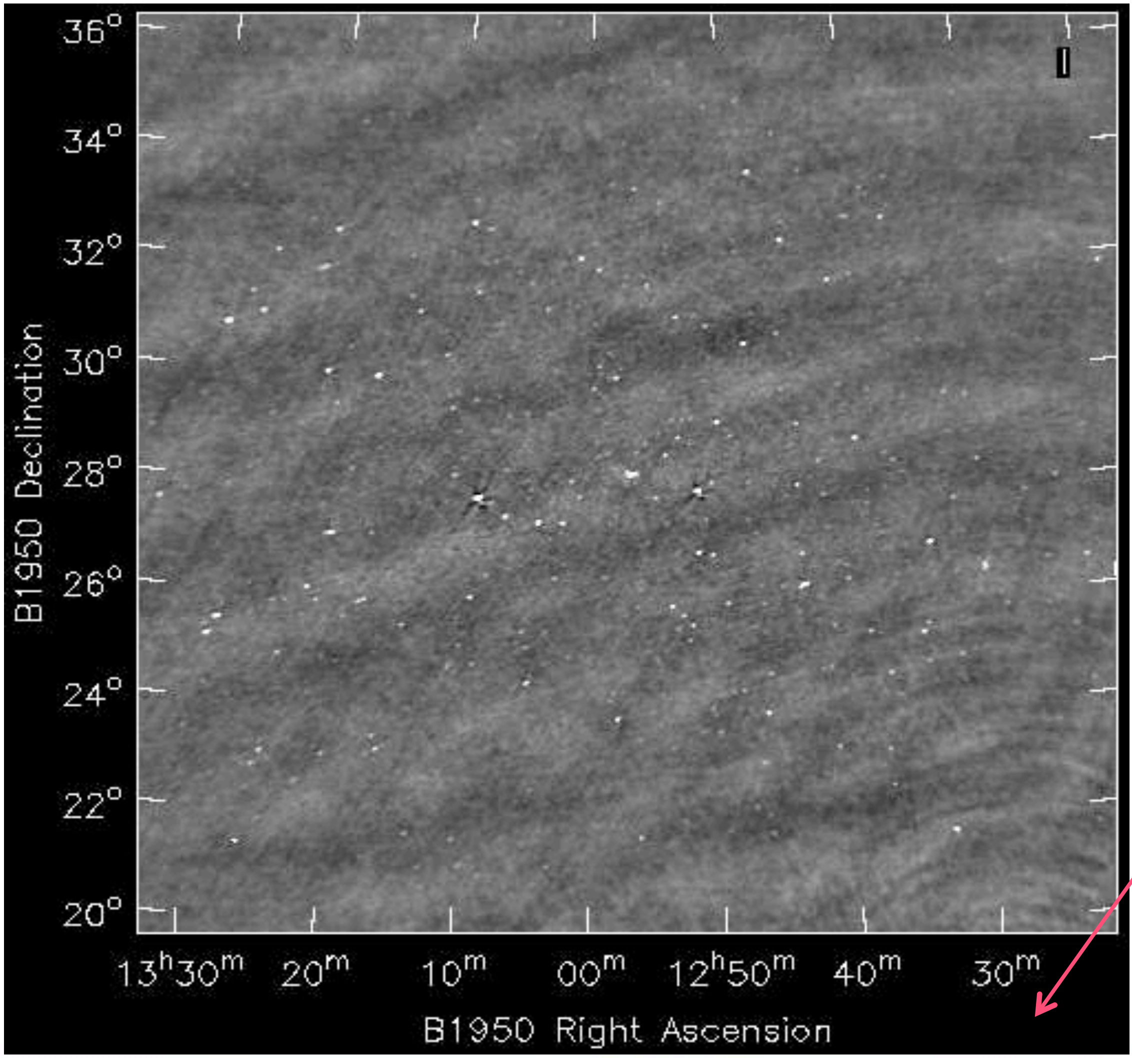
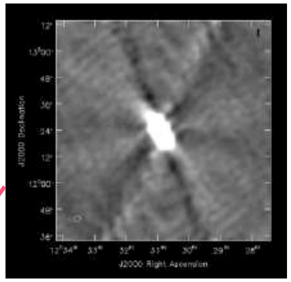


Sub-image of an "outlier" field. This bright source should be peeled out!



W-term: Practical Considerations

- Faceted imaging:
  - number of facets in l and m

$$n_{\text{facets}} = \frac{\pi\theta\sigma_w}{\sqrt{32\delta A}}$$

θ = FOV (radians) σ_w = rms w δA = max tolerable amp loss

  - $N_{\text{facets}} = n_{\text{facets}} \times n_{\text{facets}}$, $\theta \sim \lambda/D$ $\sigma_w \sim \sqrt{w_{max}} \sim \sqrt{B_{max}/\lambda}$

$$N_{\text{facets}} \approx \frac{\pi^2}{32\delta A}\left(\frac{\lambda}{D}\right)^2\left(\frac{B_{\max}}{\lambda}\right) \approx \frac{0.31}{\delta A}\left(\frac{B_{\max}\lambda}{D^2}\right)$$

- W-projection:
  - $N_{\text{Wplanes}} \sim N_{\text{facets}}$?? Choose 256? We are working on formula
  - space w-planes uniformly in $\sqrt{w}$

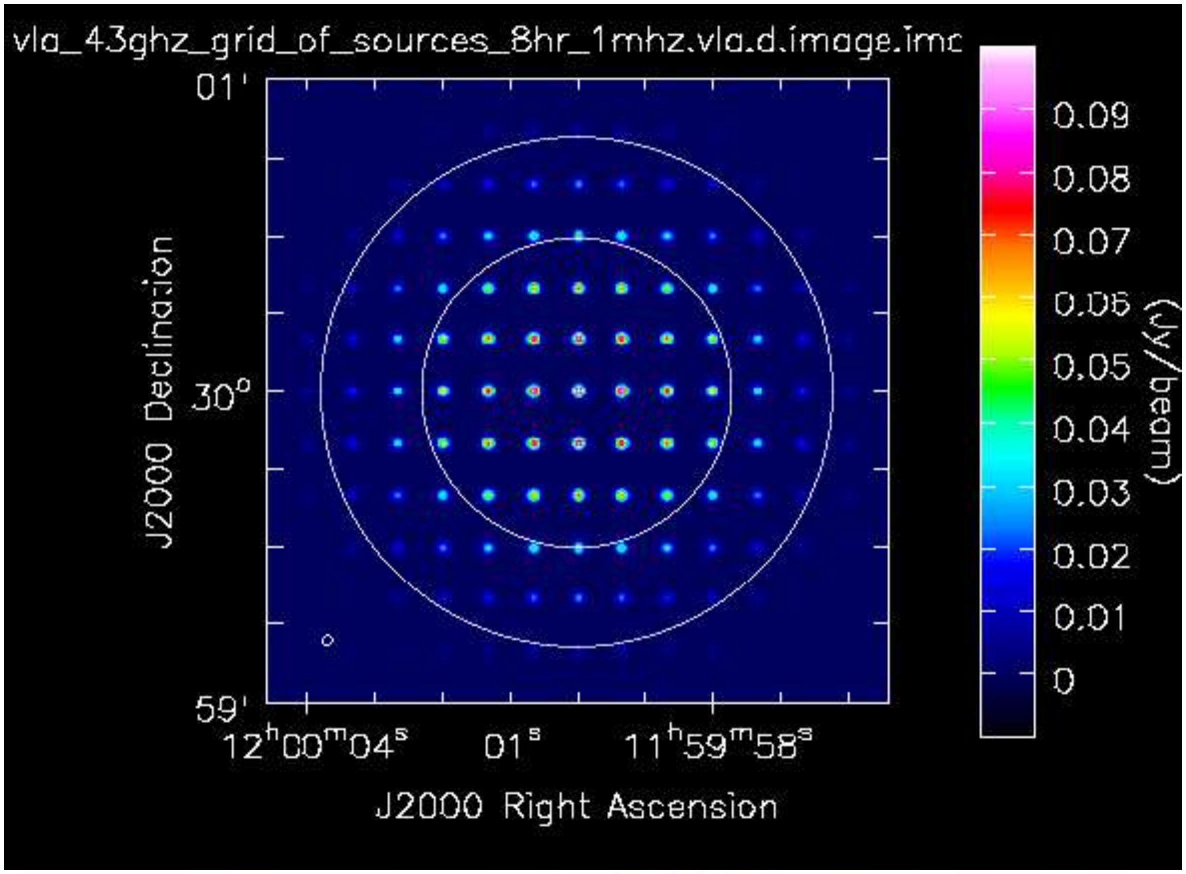
Ref: Cornwell et al. IEEE Special topics in SP, Vol. 2, No5, 2008 [arXiv:0807.4161]

Wide-field imaging: Primary Beam

- The "Primary Beam" pattern for an interferometric array (visibility from the correlation of a pair of antennas) contains effects from:
  - amplitude fall-off (with characteristic FWHM or Gaussian dispersion) due to geometric mean of diffraction patterns from the antenna elements (including optics, blockage)
  - phase pattern due to diffraction and optics (focus, etc.)
  - polarization pattern due to optics (reflection from dish surface, feed legs, location of feed (off-axis) in focal plane, any secondary or tertiary mirrors)
  - large (angle) scale sidelobes and scattered power due to surface errors (e.g. misaligned panels) [T.Hunter talk]

Primary Beam – responses

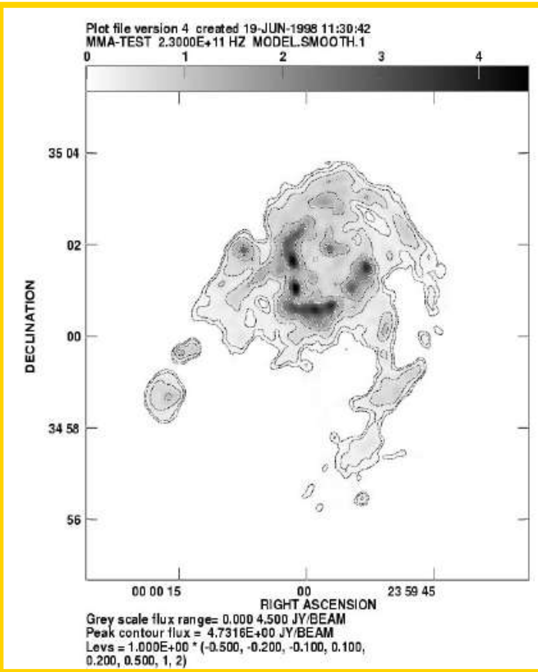
- Example: response to a grid of point sources within the primary beam (courtesy T. Hunter):



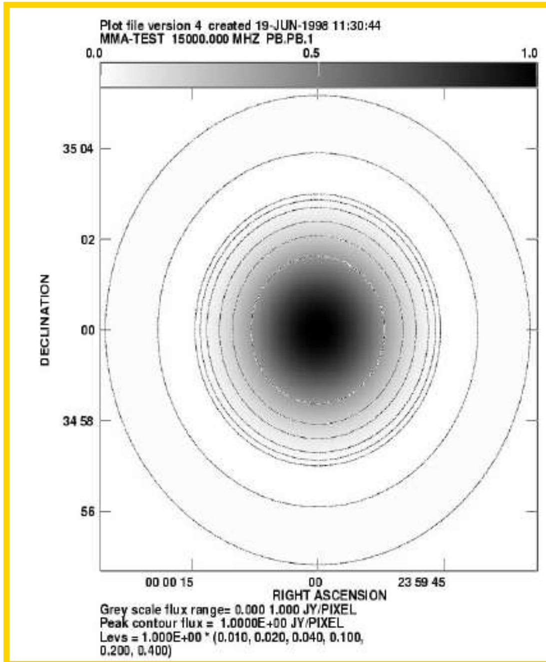
Primary Beam – example 1

CASA tip: pbcor=True during clean or divide by .flux image. Uses Airy or Gaussian beam model ☹

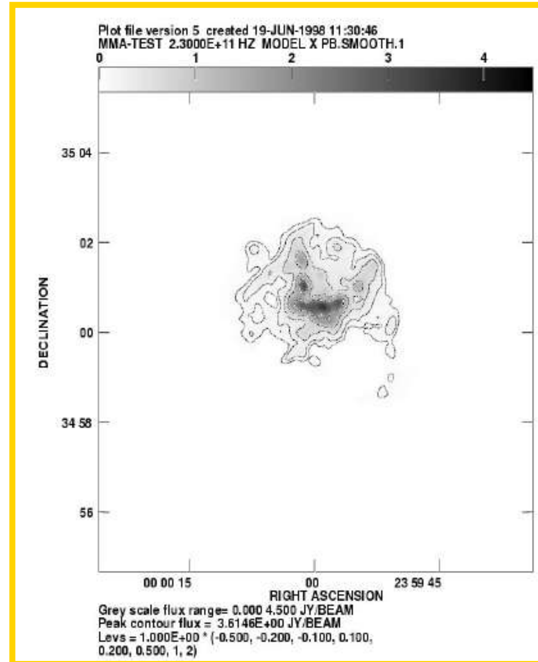
- Primary effect – apodization of image by PB amplitude pattern
  - – suppression of emission far from pointing center



Emission structure larger than PB



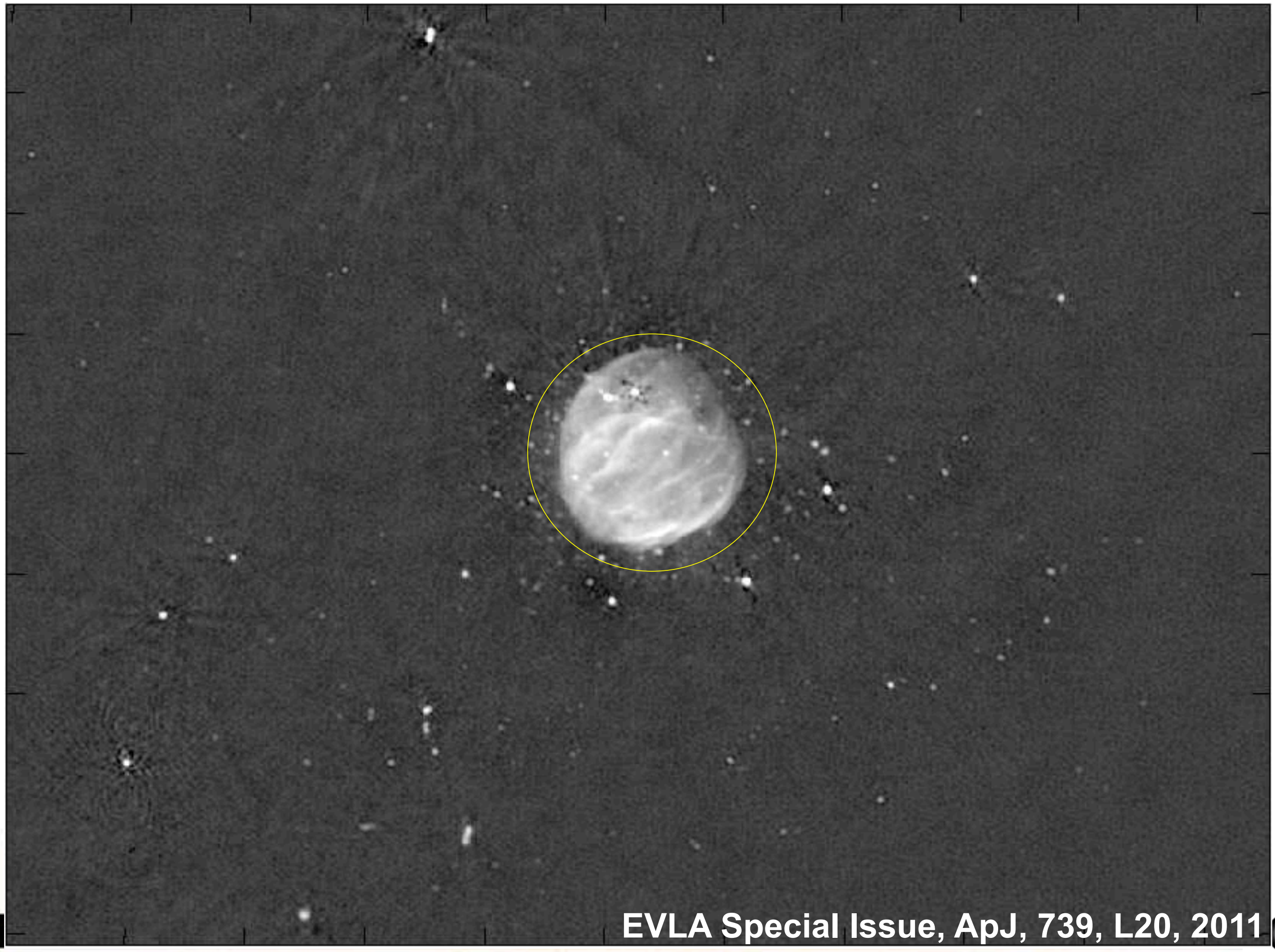
PB sensitivity pattern on sky (circular symmetry assumed)



PB applied: sensitive to center emission only

Primary Beam – example 2

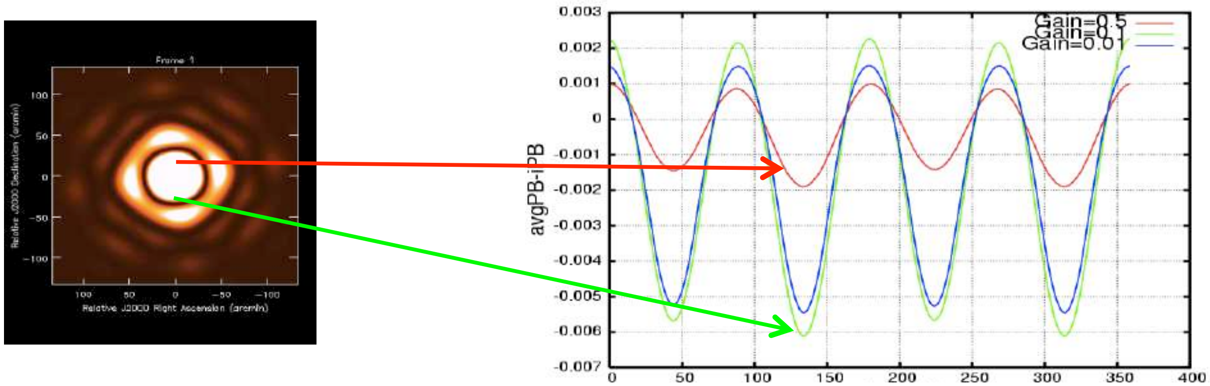
- another example…



Primary Beam Time-dependence

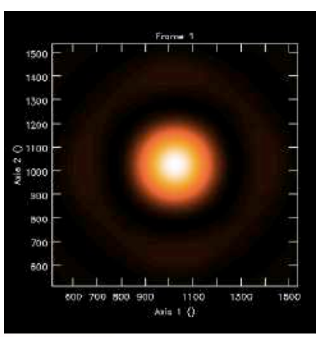
- Full-beam imaging: Antenna Primary Beam (PB) asymmetry cannot be ignored. "Sidelobe" pattern very sensitive to orientation.

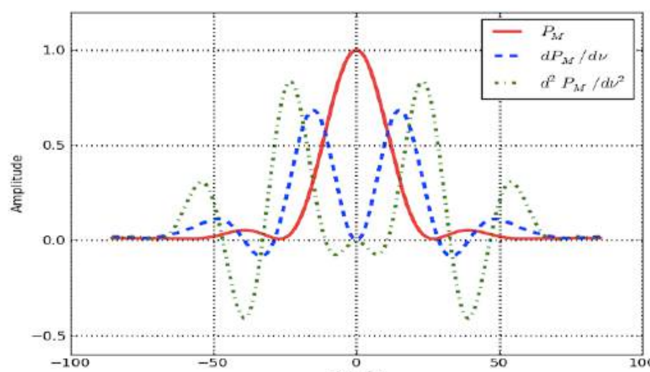
PB(ν,t): *Scales with frequency and changes with time/orientation*



For the Student:

What happens if there is a time-variable antenna-dependent pointing error?





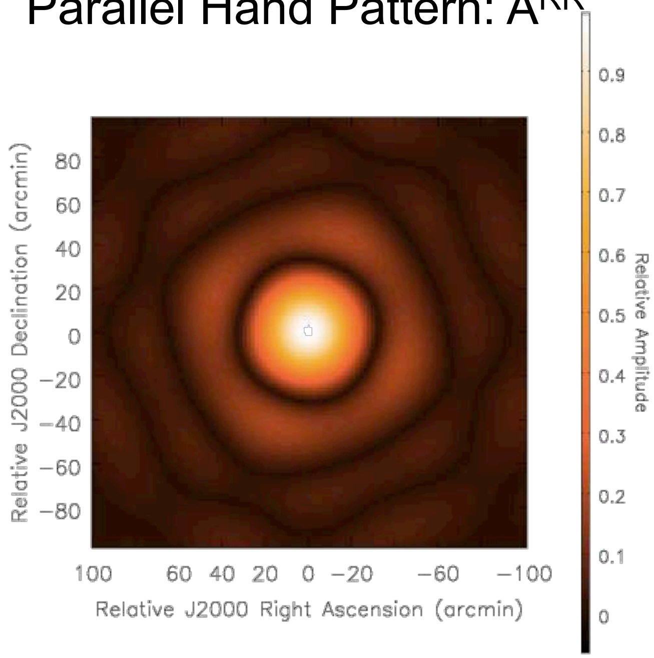
Antenna PB = autocorrelation of voltage patterns $E_i{*}E_i^*$

Visibility "vPB" = cross-correlation $E_i{*}E_j^*$ of voltage patterns. If not identical vPB will be complex

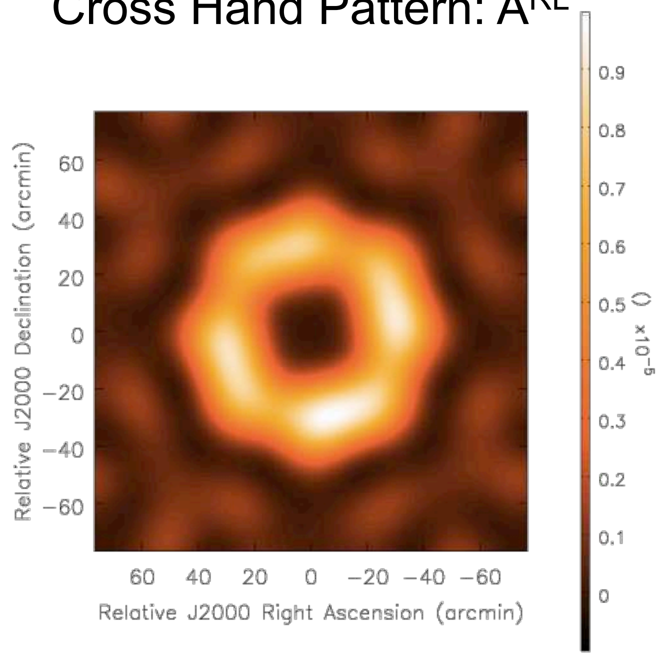
Primary Beam: Polarized

- The aperture response functions have polarization dependence
  - cross-correlation of complex antenna voltage patterns (R,L or X,Y)

Parallel Hand Pattern: A^{RR}



Cross Hand Pattern: A^{RL}



Imperative – you MUST correct (at some level) for (polarized) primary beam effects during imaging if you want accurate wide-field images!

Primary Beam Mapping

Now underway for JVLA (Perley, Cotton, Jagannathan)

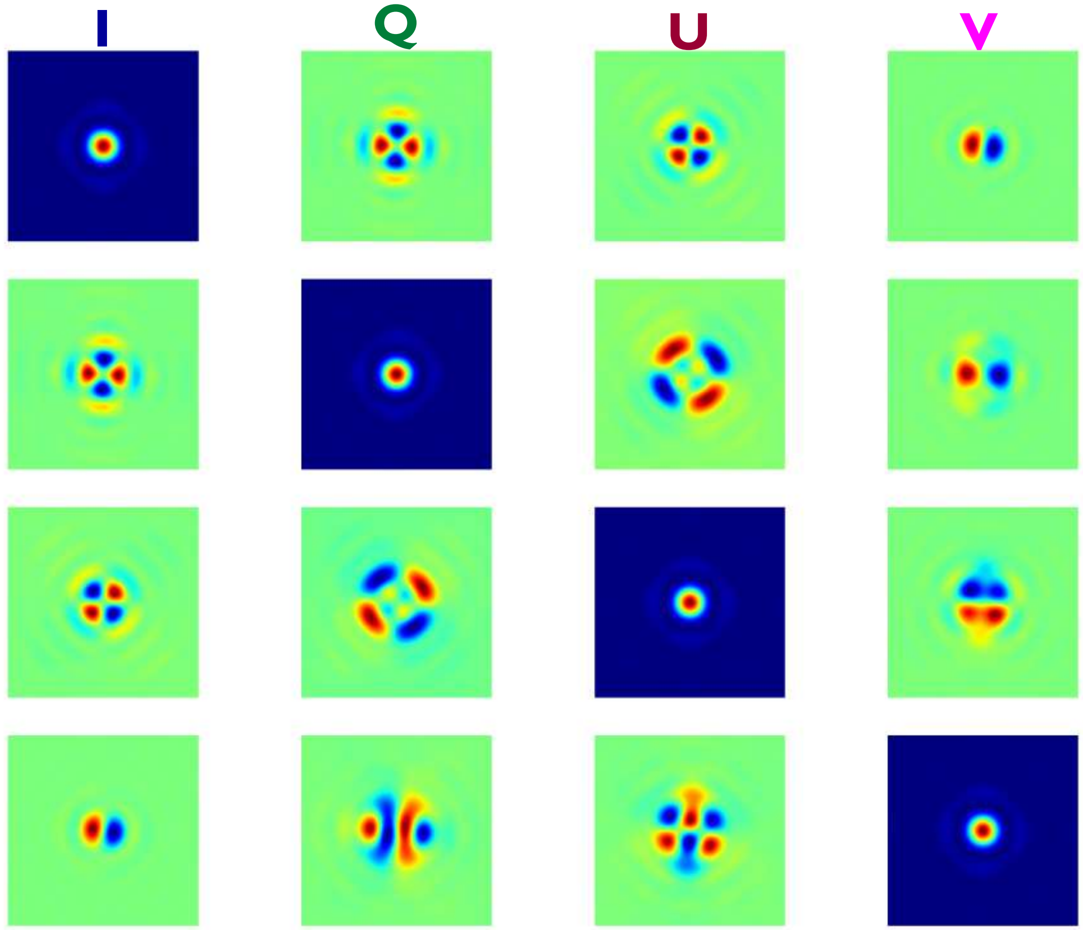
For the Student:

What do the leakages QI UI VI imply?

This row: II IQ IU IV

Q and U get quadrupolar patterns of induced cross-polarization. Note purity is good on-axis (after calibration).

Direction-dependent. Must correct during imaging.



IV leakage showing R/L squint.

"Squint": Due to VLA feeds off-axis on feed ring, R and L are displaced (phase gradients in X and Y). This causes apparent V signature in unpolarized sources!

Mueller matrix (IQUV) showing leakages in L-band. Data taken this past weekend!

Primary Beam Summary

- The primary beam response of the antennas in the array must be corrected for during imaging to get accurate intensities (and polarizations) for source outside the core of the beam.
- Due to various optics effects, the primary beam is asymmetric and rotates with respect to the source as the sky rotates (in parallactic angle).
- During imaging can be corrected approximately in gridding (A-projection) and accurately in de-gridding (major cycles).
- Accurate time-dependent beam correction is expensive!
- The primary beam structure depends on frequency, so wideband imaging is harder (see Wide Bandwidth Imaging lecture by Urvashi on Monday!)

Mosaicking

- Accurate imaging of emission covering areas larger than the primary beam requires the combination of multiple pointings. Example: Large area sky surveys

- This is called "Mosaicking"
- Simple mosaicking captures the distribution of compact structures (each << primary beam in size) = panorama
- If you have good measurements on the shortest baselines (which probe the center of the uv-plane) then mosaicking can reconstruct spacing on sub-aperture scales!

Mosaicking Options

- Classic – Linear Image-plane Mosaicking
  - Form separate images from each pointing (using normal clean), then form weighted sum with PB correction to form larger image. Example: NVSS and FIRST
- Modern – Joint Deconvolution
  - Linear: At minor cycles form linear mosaic of residual images.
  - Gridded: Use Fourier shift theorem to combine pointings in the uv-plane during gridding (A-projection) with application of the phase gradient from phase center offsets of each pointing. Example: CBI (Myers et al. 2003)

of course there is a third path…

- Post-modern – use Joint Mosaicking for subsets of nearby pointings, and Image plane Mosaicking to combine sub-mosaics.
  - Combine with "peeling" of sources outside of beams.

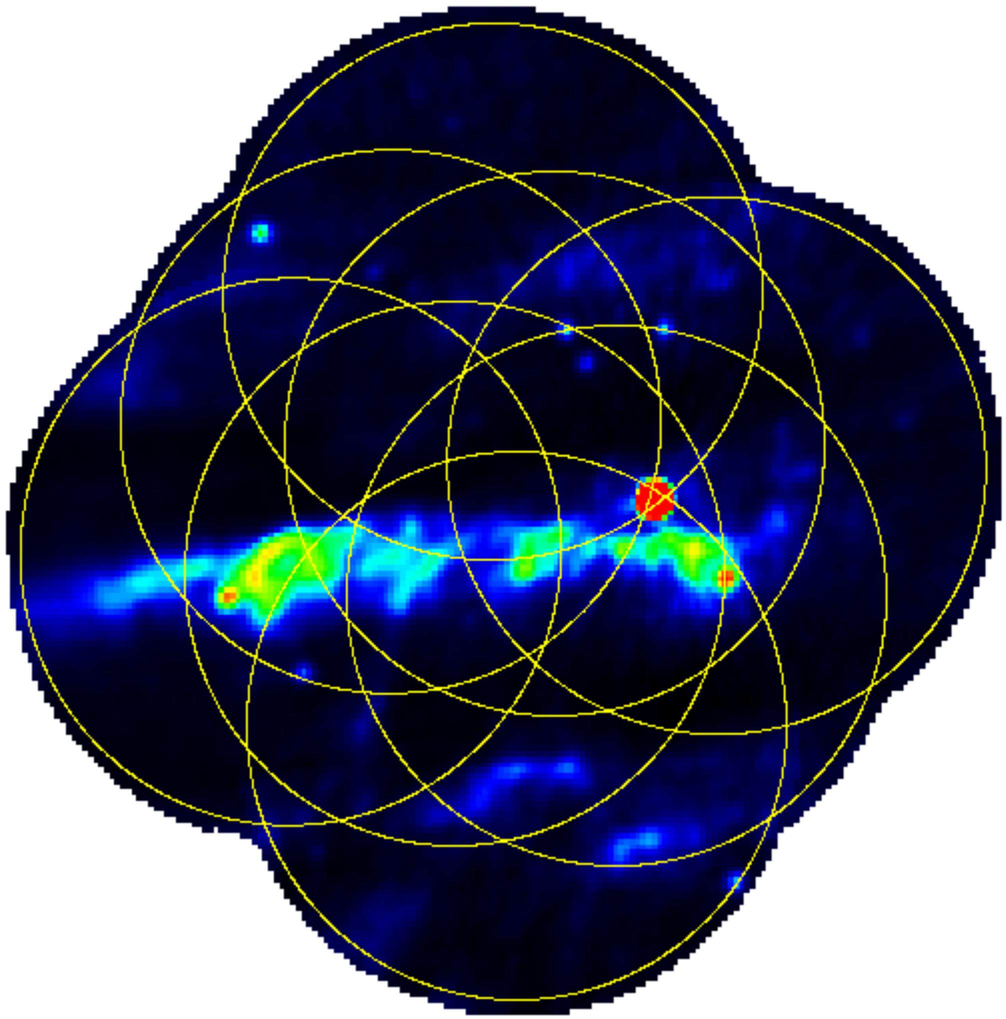
Linear Mosaicking

- Form a linear combination of the individual pointings p of the individual deconvolved images I_p on a pixel by pixel basis

$$I(\vec{m}) = B(\vec{m}) \frac{\sum_p A(\vec{m}-\vec{m}_p)\sigma_p^{-2} I_p(\vec{m})}{\sum_p A^2(\vec{m}-\vec{m}_p)\sigma_p^{-2}}$$

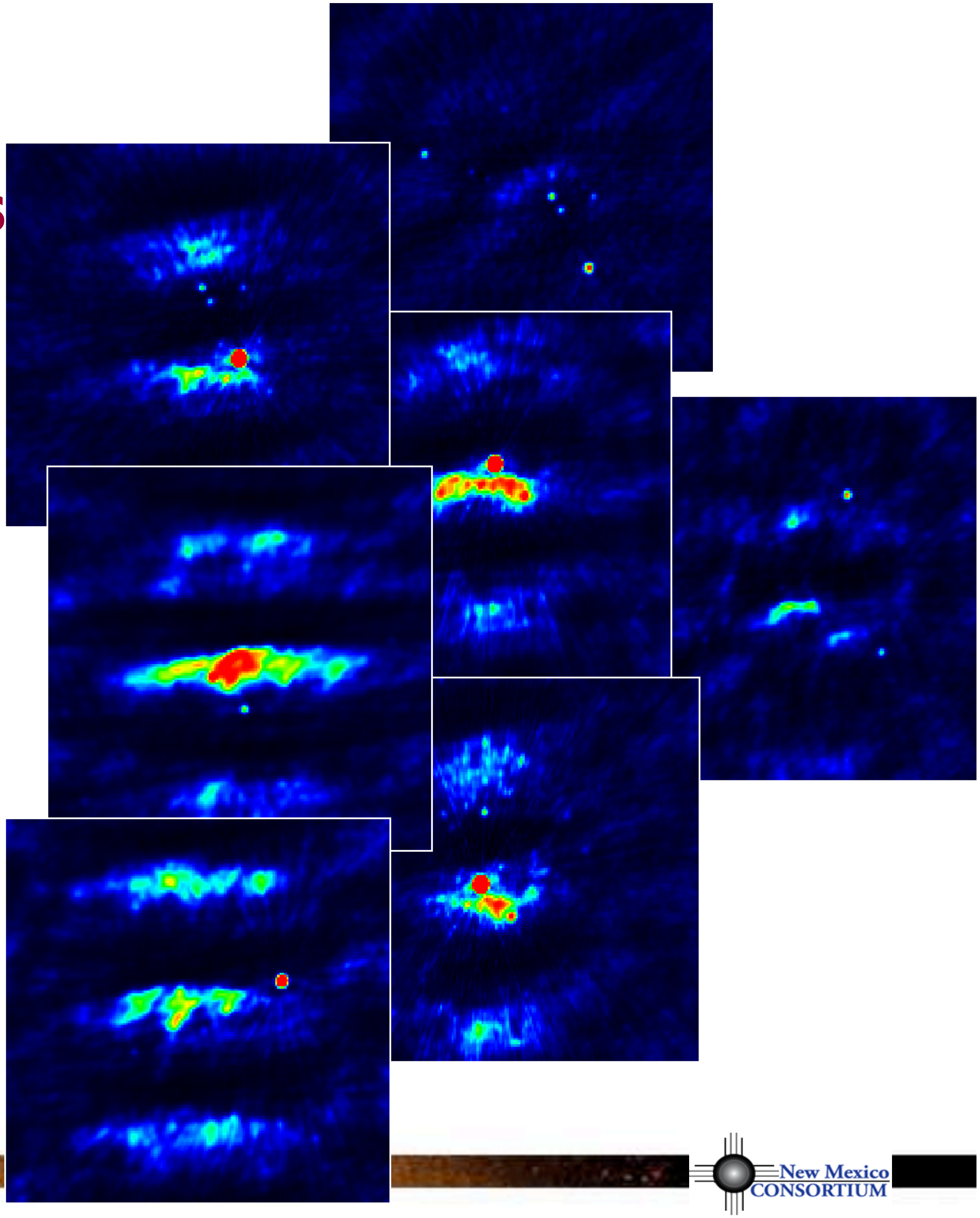
- Here σ^{-2}_p (diagonals of $\mathbf{N}^{-1}$) is the inverse noise variance of an individual pointing and **A**(**m**) is the primary response function of an antenna (primary beam)
- W(**m**) is a weighting function that suppresses noise amplification at the edge of mosaic
- This linear weighted mosaic can also be computed for the residual dirty image at minor cycles to carry out a joint linear mosaic deconvolution.

Linear Mosaic – observe pointings

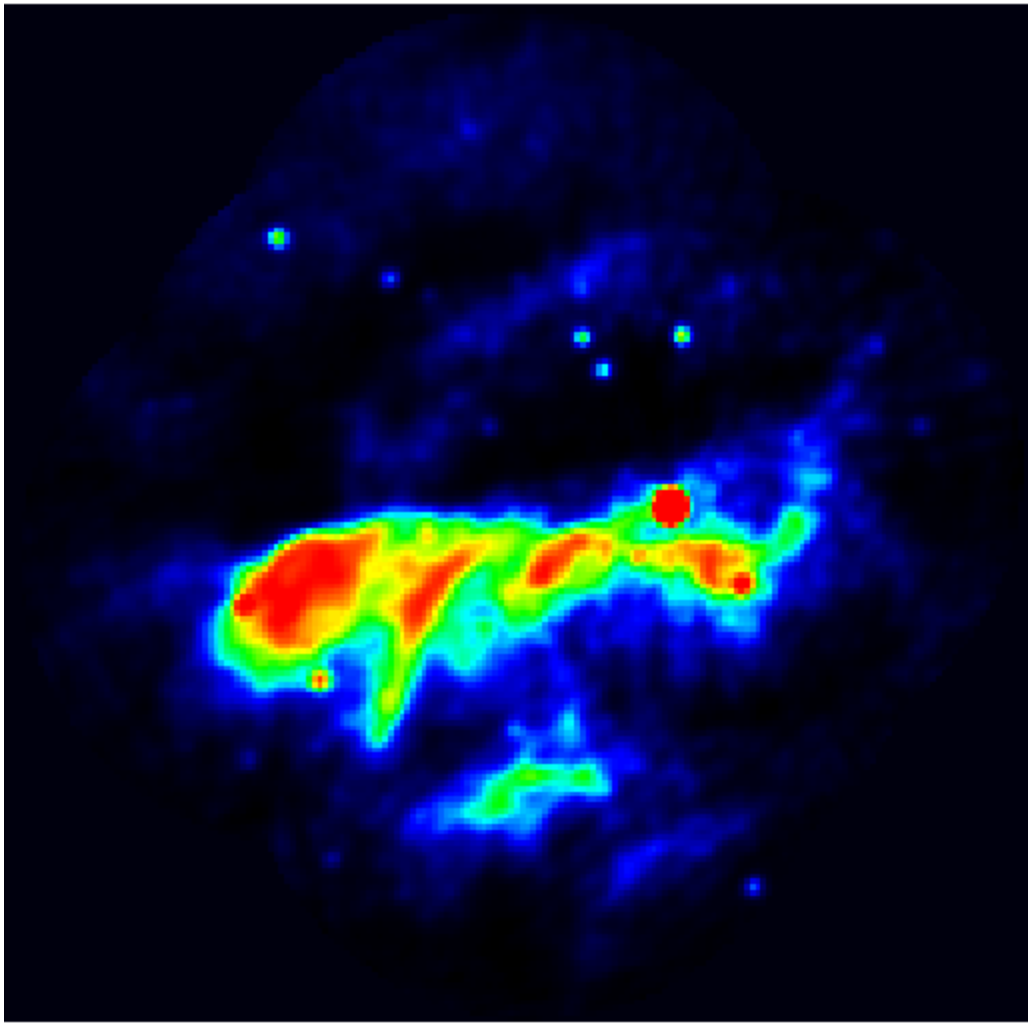


Linear Mosaic – individual images

- Treat each pointing separately
- Image & deconvolve each pointing
- Stitch together linearly with optimal pointing weights from noise and primary beam

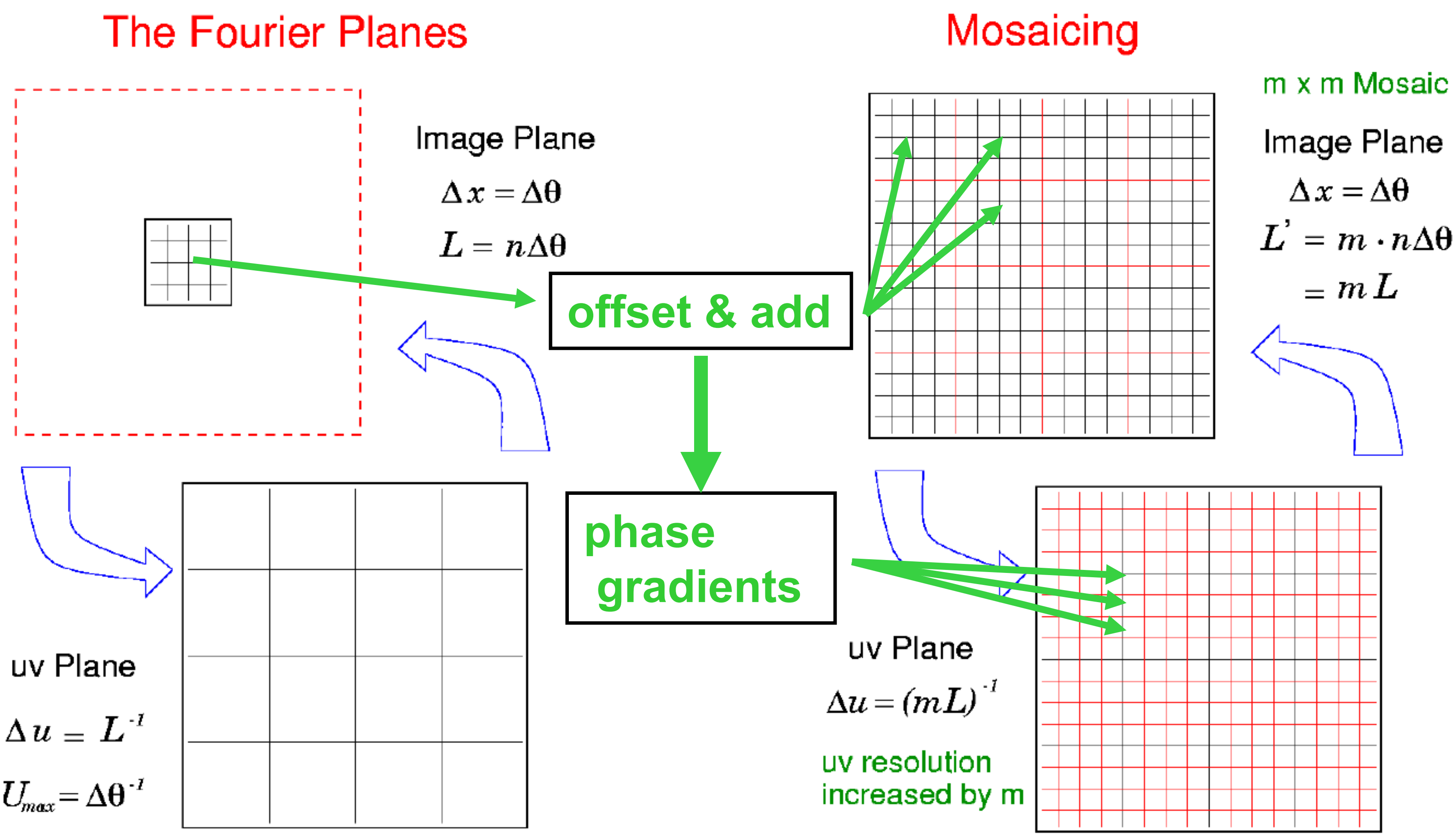


Linear Mosaic – combine pointings



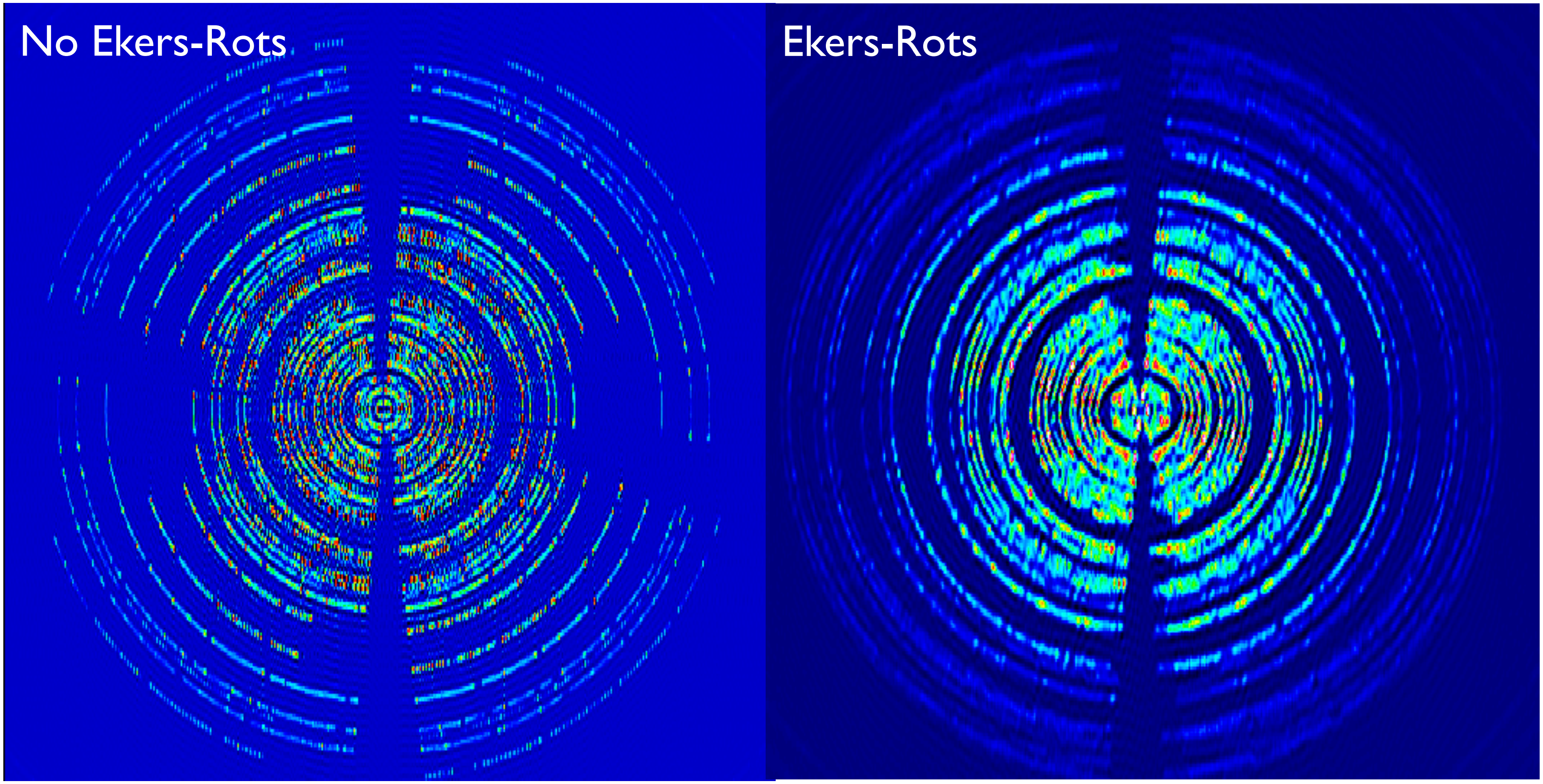
Mosaicking 101

Ekers & Rots (1979) – mosaicking can synthesize short spacings. Effectively is a FT of the mosaic grid.



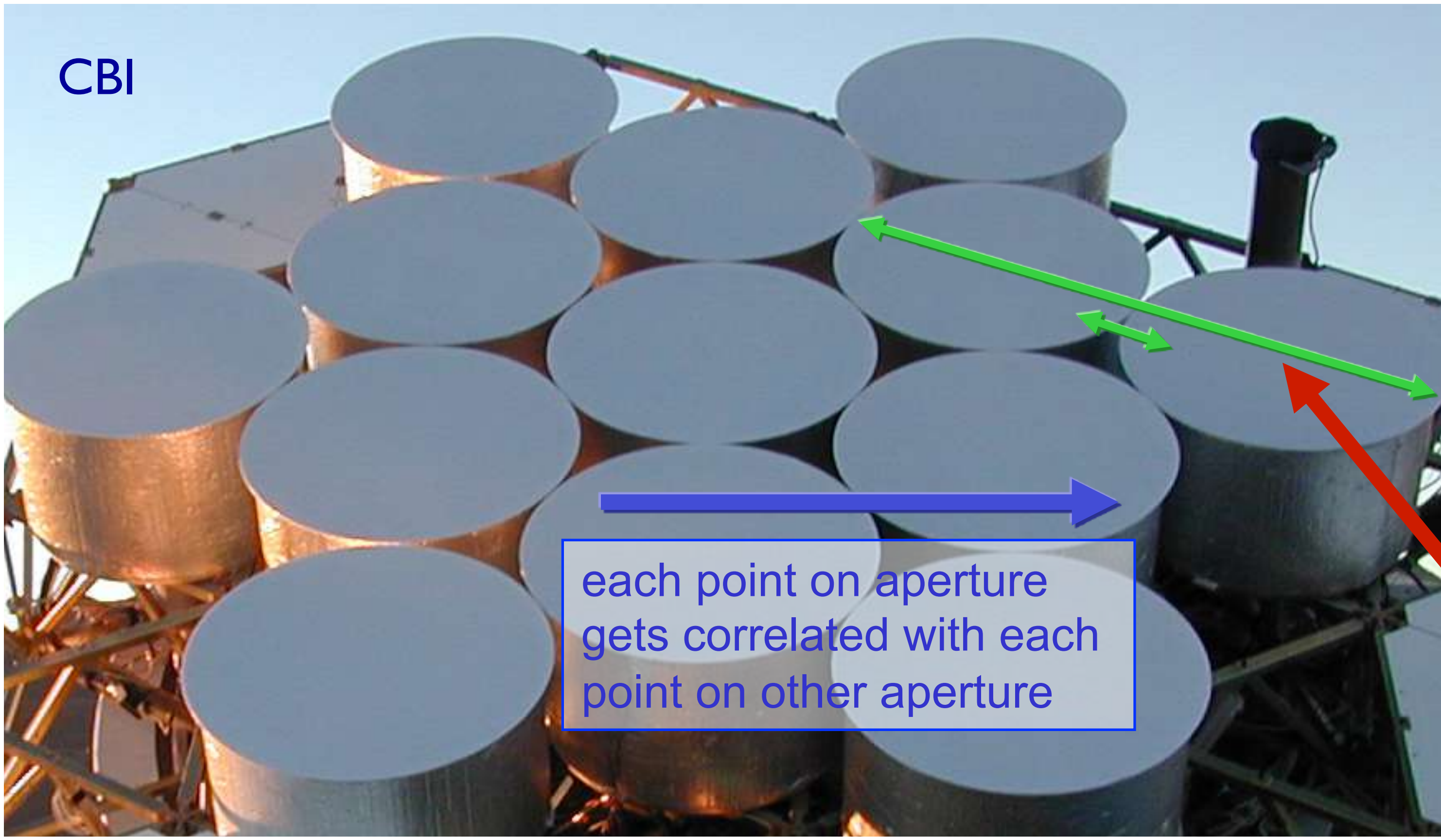
Mosaicking increases resolution in ALL parts of the uv-plane!

Mosaicking and the uv-plane



The Aperture Plane

- See "Widefield Imaging II" tomorrow (Brian Mason) for more on recovery of short spacing information.



visibility contains range of baselines from closest to furthest parts of apertures

autocorrelations measure uv spacings inside D/λ

interferometer cannot measure "zero-spacing" w/o autocorrelations

Mosaicking Options – comparison

- Individual Deconvolution, Linear Mosaic:
  - Disadvantages:
    - Deconvolution only possible to depth of individual pointing
    - Overlap regions rather noisy when not drastically oversampled
  - Advantage:
    - Each pointing can be treated and calibrated separately for best results. Can be an advantage for high-dynamic range imaging where calibration effects need to be treated with great care. Easy to integrate with "peeling".
- Joint (Gridded) Approach:
  - Advantages:
    - Uses all uv info per overlap → better sensitivity & beam
    - More large-scale structure due to PB convolution in uv-plane
  - Disadvantages:
    - Requires a good model for the primary beam response
    - WA-projection can be expensive

Mosaicking - equations

- Use our Imaging Equation
  - Visibility *k* for the phase center ϕ and pointing center *p* where **A** is the primary beam pattern (ignoring w)

$$V_\nu(\vec{\mathbf{u}}_k) = \iint d^2\mathbf{m}\, S_\nu\left(\vec{\mathbf{m}}\right) A_{k\nu}\left(\vec{\mathbf{m}} - \vec{\mathbf{m}}_p\right) e^{-2\pi i \vec{\mathbf{u}}_k \cdot (\vec{\mathbf{m}} - \vec{\mathbf{m}}_\phi)}$$

$$\vec{\mathbf{u}} = (u, v) \quad \vec{\mathbf{m}} = (l, m) \quad d^2\mathbf{m} = \frac{dl\,dm}{\sqrt{1 - l^2 - m^2}}$$

  - express convolution integral in uv-plane (shift theorem)

$$V_\nu(\vec{\mathbf{u}}_k) = e^{2\pi i \vec{\mathbf{u}}_k \cdot \vec{\mathbf{m}}_\phi} \iint d^2\mathbf{u}\, \underline{S}_\nu\left(\vec{\mathbf{u}}\right) \underline{A}_{k\nu}\left(\vec{\mathbf{u}}_k - \vec{\mathbf{u}}\right) e^{2\pi i (\vec{\mathbf{u}} - \vec{\mathbf{u}}_k) \cdot \vec{\mathbf{m}}_p}$$

This tells you how to apply the phase offsets and gradients from the pointing and phase centers – these centers are not necessarily the same in a mosaic!

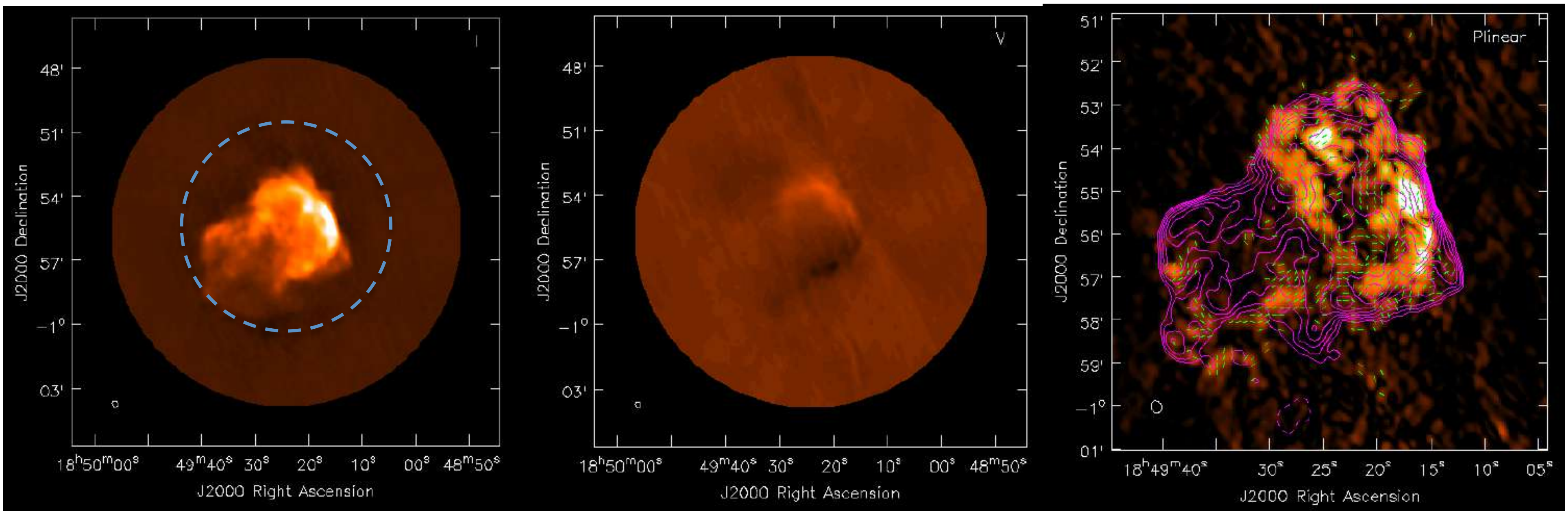
Mosaicing – joint imaging example

From JVLA casaguide for Supernova Remnant 3C391 at 4.6GHz:

Stokes I image showing extended SNR emission.

Stokes V image showing artifacts from R/L squint.

Linear polarization vectors, on-axis leakage corrected.



Hexagonal mosaic, 7 pointings. FWHM is 9.8’ at 4.6GHz.

Mosaicking Limitations

- Your ability to combine observations from different pointings is limited by your knowledge of the primary beam pattern(s)
- Denser field sampling gives more uniform coverage, but requires more processing for imaging.
- If you use fields taken with different uv-coverages (e.g. observed at different hour angles) then the PSF will vary over the mosaic (it will be a weighted sum of field PSFs).
- Flagging can cause unanticipated gaps and variations across the mosaic.
- Bright sources can cause problems for "nearby" fields, self-calibration can help, and possibly "peeling"
- Pointing errors will induce errors into mosaic (via PB).

Mosaicking in CASA

- Calibrate as you would do for a single pointing (e.g. pipeline)
- Use the **clean** task with your favorite parameters
- In *imagermode* use *'mosaic'*
  - Use *ftmachine='ft'* for joint linear deconvolution, *'mosaic'* for the joint gridded imaging (preferred, faster)
  - will always use Cotton-Schwab (major/minor cycle) algorithm
  - Use *psfmode='clark'* (default) or *'hogbom'* (for poor psf)
  - Fill in *'multiscale'* parameters (scales) for MS Clean
- Linear mosaicking of cleaned images only available from the CASA toolkit (`im.linearmosaic`) currently. [AIPS FLATN]
- Contributed tasks for mosaicking field setup (*makeschedule*)
  - also check ALMA OT and JVLA OPT (e.g. for OTF)

Practical Mosaicking

- So you want to mosaic over a large area?

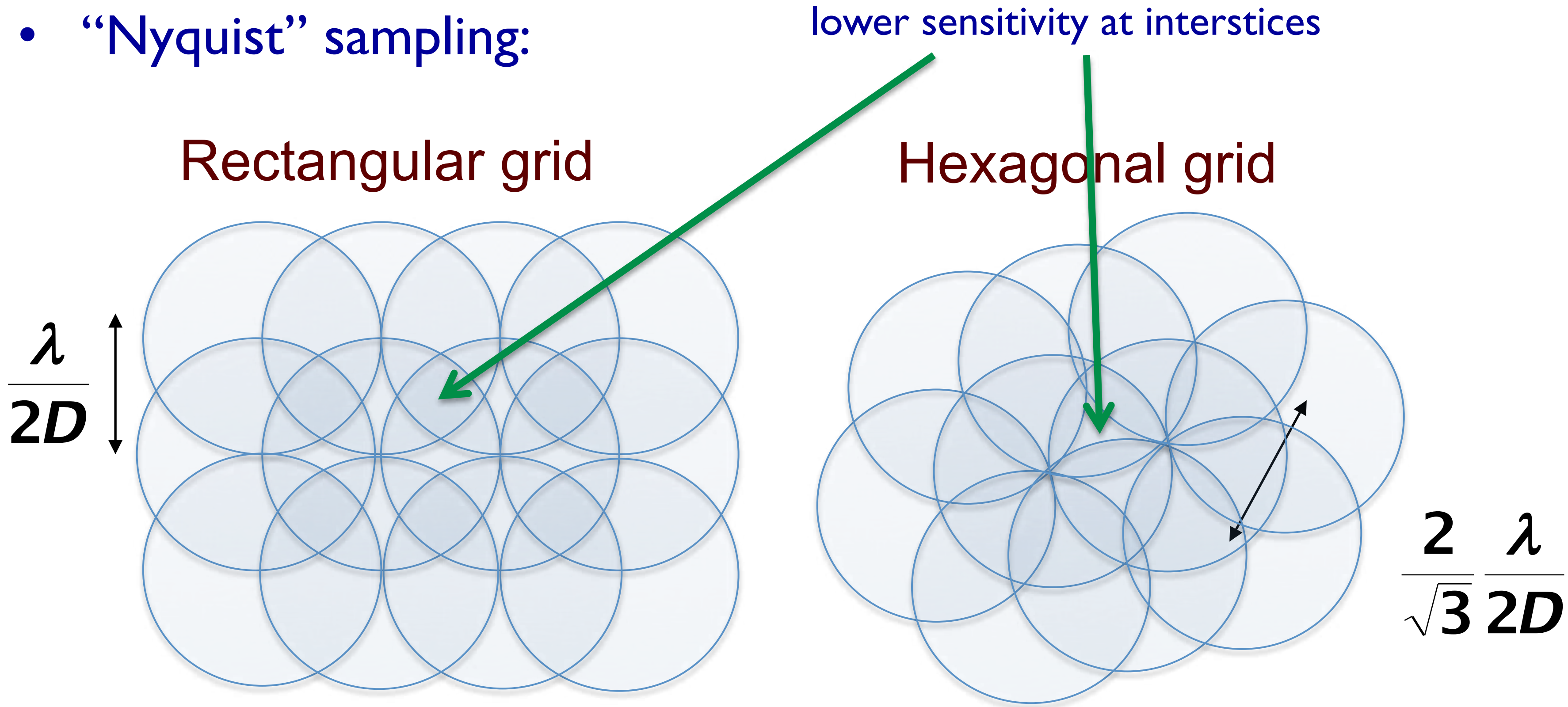
- What field center pattern to use?
- How often to come back to a individual pointing
- Slew time of Antennas
- Change of atmospheric conditions

- For more information, see the Guide to VLA Observing section on mosaicking:

https://science.nrao.edu/facilities/vla/docs/manuals/obsguide/modes/mosaicking

Practical Mosaicking – pattern

- Different ways to lay out the field centers on the sky:
- "Nyquist" sampling:

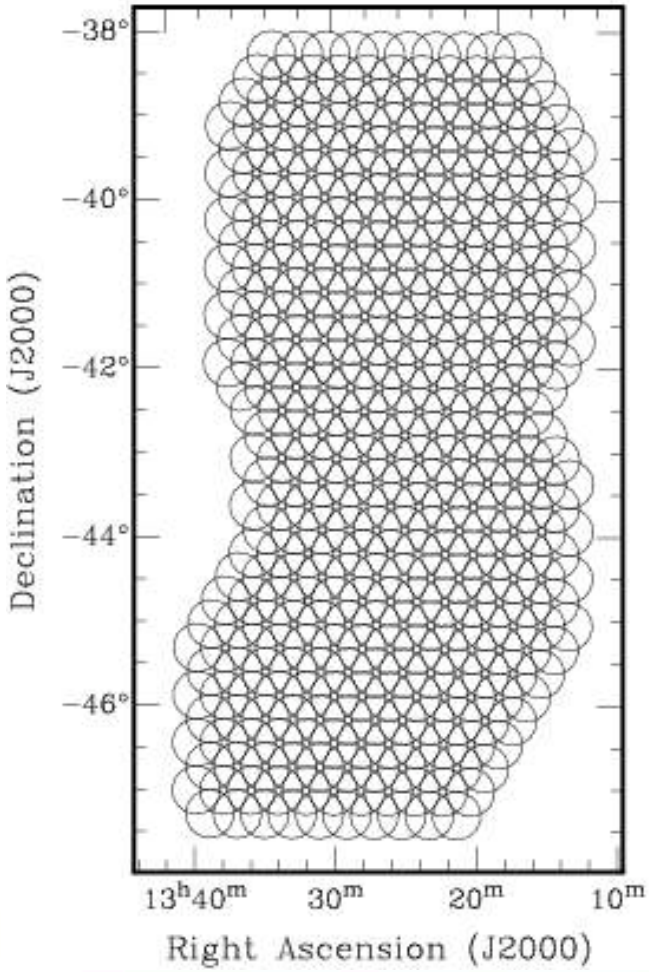


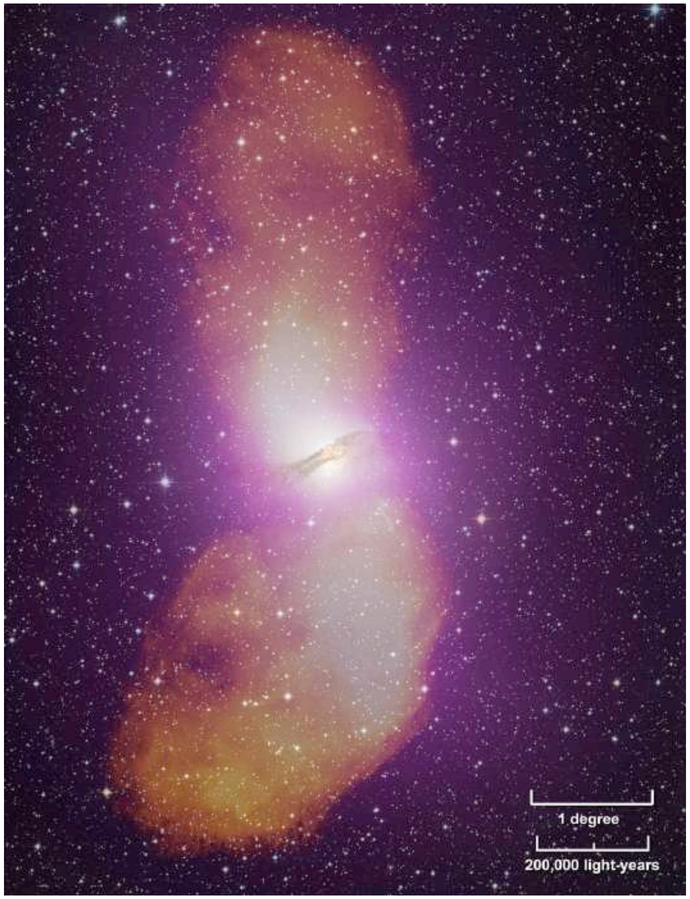
Minimum "Nyquist" for structure information recovery

Most efficient coverage with minimal non-uniformity (centers on equilateral triangles). <u>Preferred!</u>

Example: Centaurus A – 406 pointings

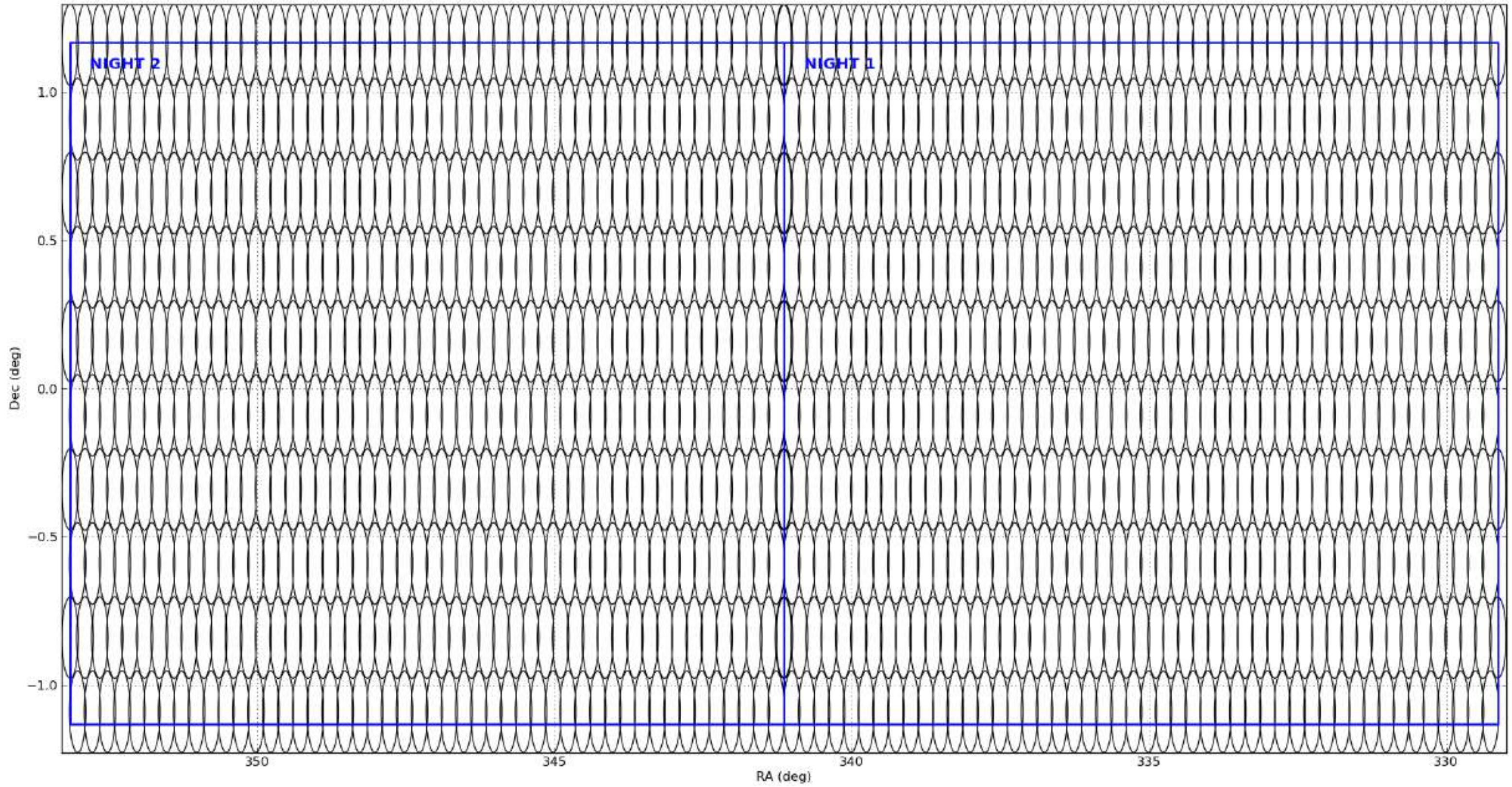
- Feain et al. 2011





Example – JVLA 2-4GHz Stripe82

- Mooley, Hallinan, et al. (Caltech) – 50 sq.deg. pilot, B-config



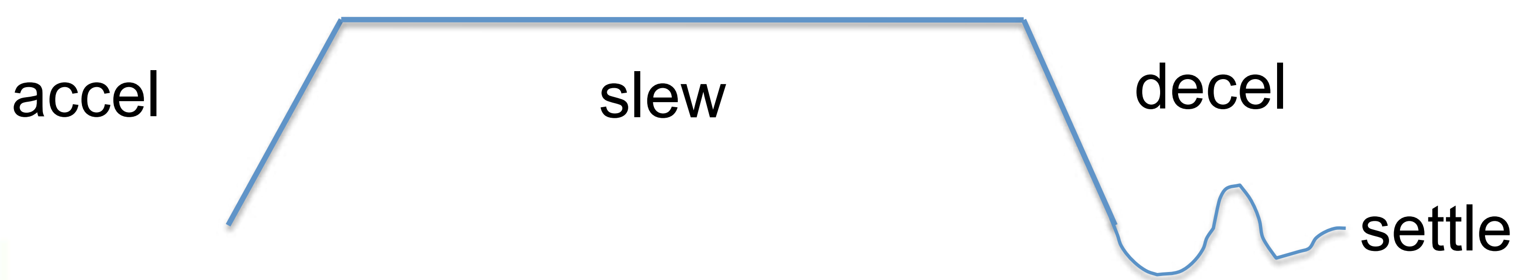
Hexagonal mosaic with 970 pointings (*makeschedule*)
Observed in 3 epochs. Coadded rms 50 μJy

Practical Mosaicking: Antenna Slew

- Telescope slew times are calculated by:
  - Acceleration
  - Constant Slew velocity
  - Deceleration
  - Settling time

Good news! An Antenna Control Unit (ACU) upgrade has started on the JVLA . This will give us better control and greatly reduce settling times!

- Some telescopes may have variations in Az and El
- JVLA: acceleration: 2.2 deg s^{-2}, slew rate: 20 deg min^{-1} in El, 40 in Az
  - Settling time: around 6-7s min, ~1-3s shorter in El, longer in Az
- ALMA: acceleration: 24 deg s^{-2}, slew rate 180 deg min^{-1} in El, 360 in Az = super fast!



Practical Mosaicking: Time is your Enemy

- The water vapor content of the atmosphere can change on small timescales
- In particular there can be large variations in individual cells
  - Changing sky brightness
  - Changing opacity
  - Increased phase noise
- Delay variations due to ionosphere are possible at low frequencies

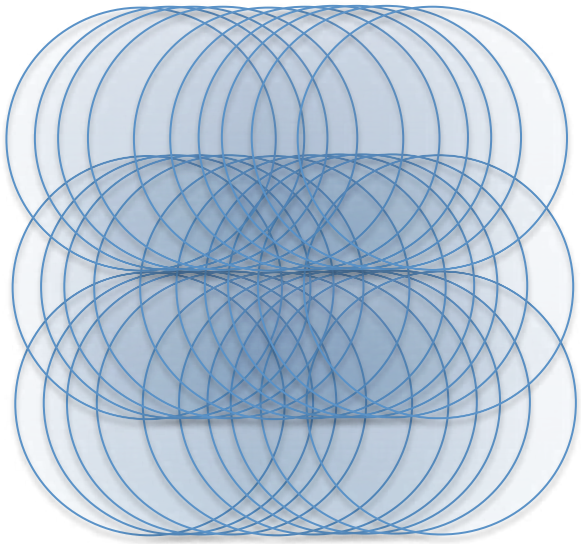
→ Try to cover the full mosaic fast but more frequently

This will make the map (and uv-coverage) more uniform, but it can increase your overhead

Other Patterns

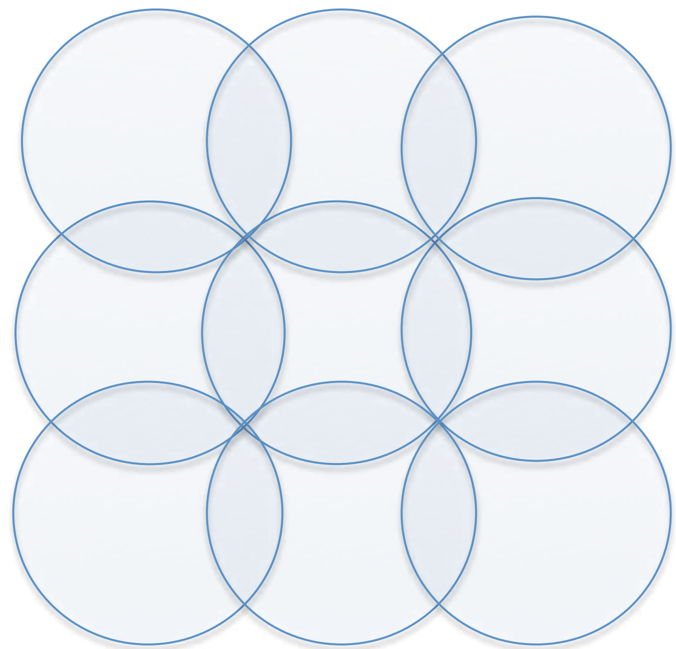
- On-The-Fly (OTF) Interferometry – continual scanning
- Sub-Nyquist sampling

OTF



Antenna scan does not stop, fast dumping of data, influences the primary beam shape, produces lots of data but reduces overhead

Non-Nyquist



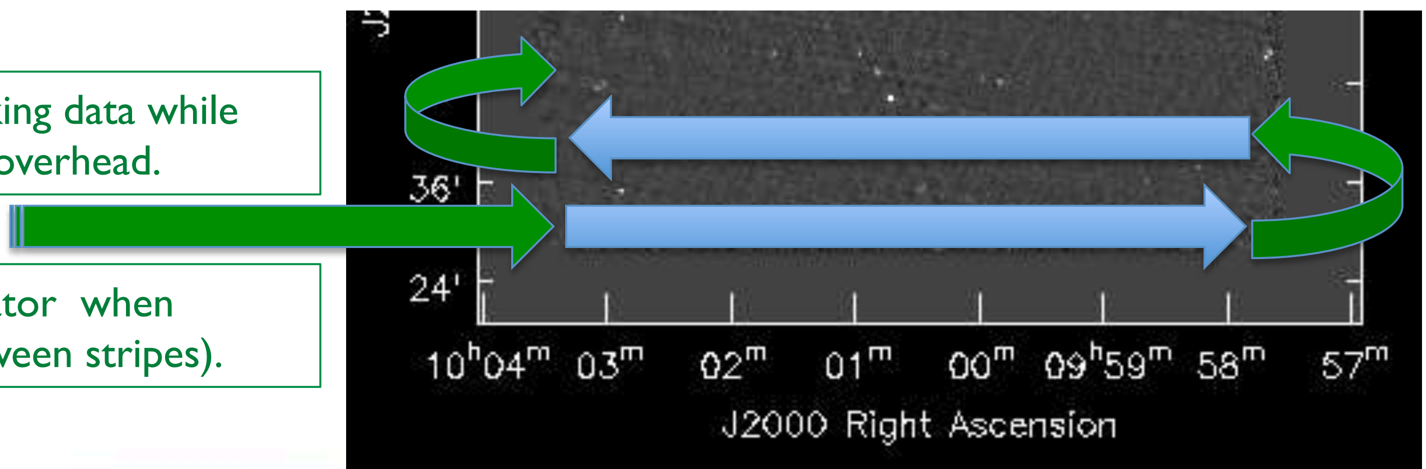
Fast sky coverage, non-uniform

On-the-fly (OTF) scanning

- Available for JVLA (currently shared risk) and ALMA
- JVLA Atomic element: "OTF Scan" of length Duration in time
  - StartPosition (RA,Dec) → EndPosition (RA,Dec)
    - COSMOS Field example: 2 square degrees = 85'x85'
    - COSMOS 6GHz : 85' in 150sec = 34''/sec (2x sidereal)
  - Usually scan at fixed Dec, weave in RA alternate stripes
    - Step phase center every N_{int} integrations (or fix at stripe center)
    - Scan stripes FWHM/$\sqrt{2}$ separation (at ν_{max}) for uniformity

Efficiency: taking data while scanning, no overhead.

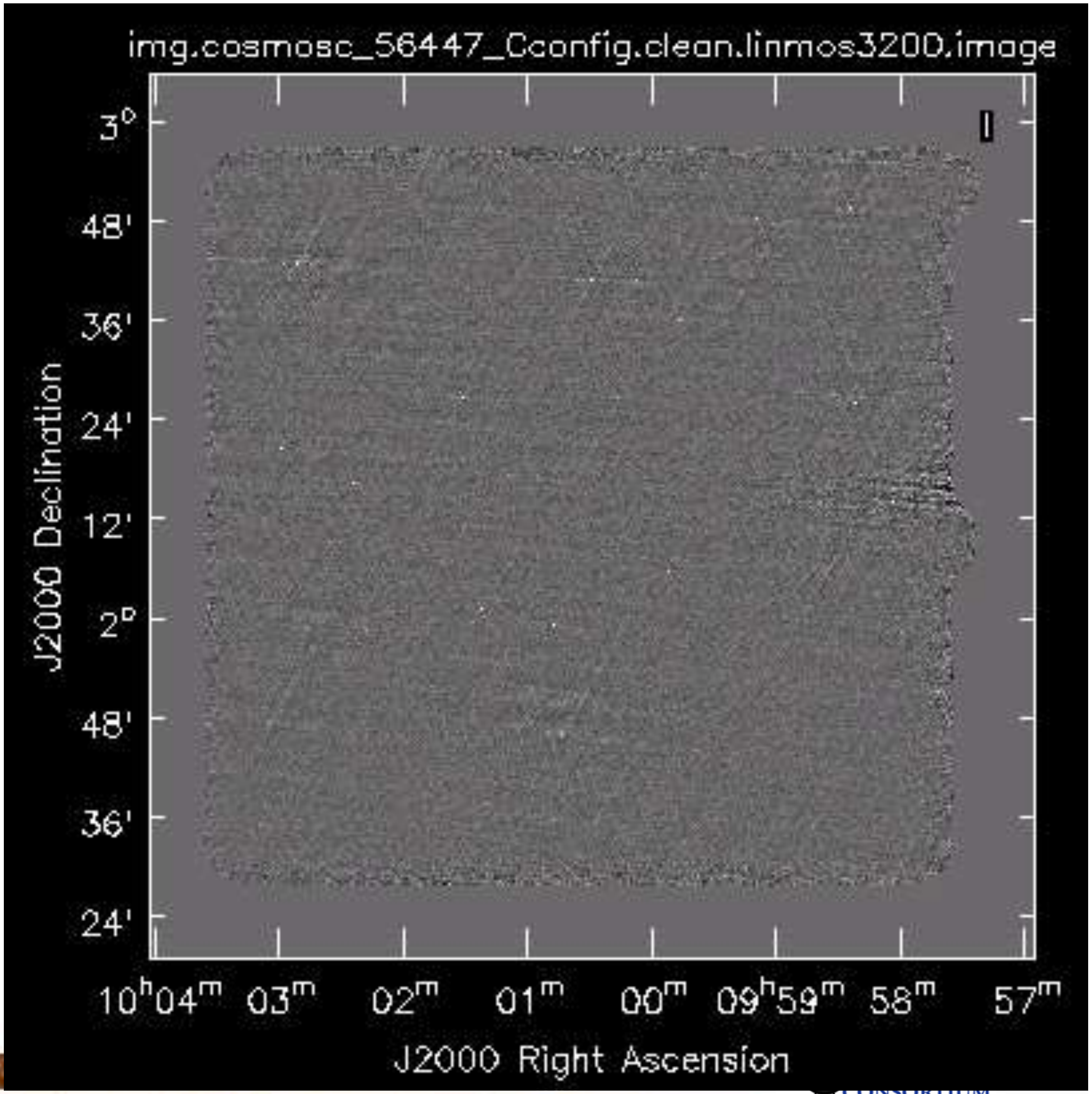
Go to calibrator when needed (between stripes).



OTF Now –VLA COSMOS 6GHz C-config

- C-config C-band OTF
  - 3200x3200 2” cells
  - 8bit 2GHz, 84μJy rms
  - <30m on-src
  - Now: simple linear mosaic after clean of each 4s “field” (CASA)
  - Striping/defects: RFI, missing data, unboxed cleaning, spectral index

COSMOS field
13A-362 (Myers)
C-band 1hr SB
4.2-5.2 + 6.5-7.5GHz
2 square degrees
OTF scans in RA
432 phase centers
Repeat bi-monthly.



OTF Surveys

We are crazy enough to do this! The VLA Sky Survey (VLASS) is being planned. See https://science.nrao.edu/science/surveys/vlass

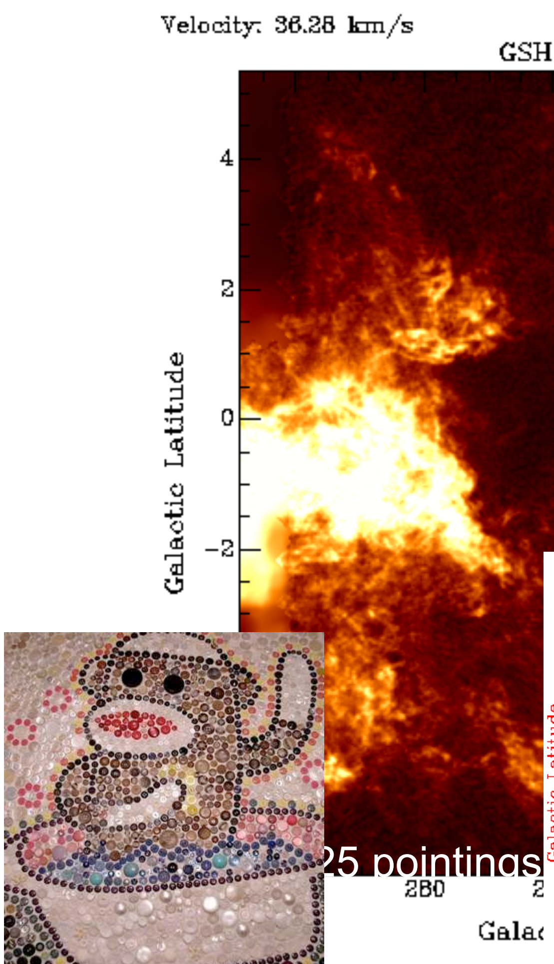
- Goal: Cover a fixed area Ω to a rms image sensitivity σ
  - how long does it take? how many pointings? what scan rate?
  - effective beam area (area under square of beam) Ω_B=0.5665θ^2_{FWHM}
  - example: JVLA 3GHz (15' FWHM) Ω_B = 0.0354 deg^2
    - 34000 deg^2 (full sky from VLA) will have 960282 beam areas!
  - use JVLA exposure calculator for integration time t_{int} to reach σ
    - 2-4GHz (1500MHz bandwidth) σ=100μJy gives t_{int} = 7.7sec
    - survey speed SS = Ω_B/t_{int} = 16.5 deg^2/hour (or arcmin^2/sec)
    - our full-sky survey will need 7.4Msec = 2054 hours integration
  - stripe (row) spacing FWHM/$\sqrt{2}$ (at 4GHz) θ_{row} = 7.96'
    - scan speed SS/θ_{row} = 2.1 arcmin/sec (0.19 FWHM/sec at 4GHz!)
    - need 0.5sec integrations to minimize PB smearing in integration
  - change phase center every ~7sec = 1 million phase centers!

https://science.nrao.edu/facilities/vla/docs/manuals/obsguide/modes/mosaicking

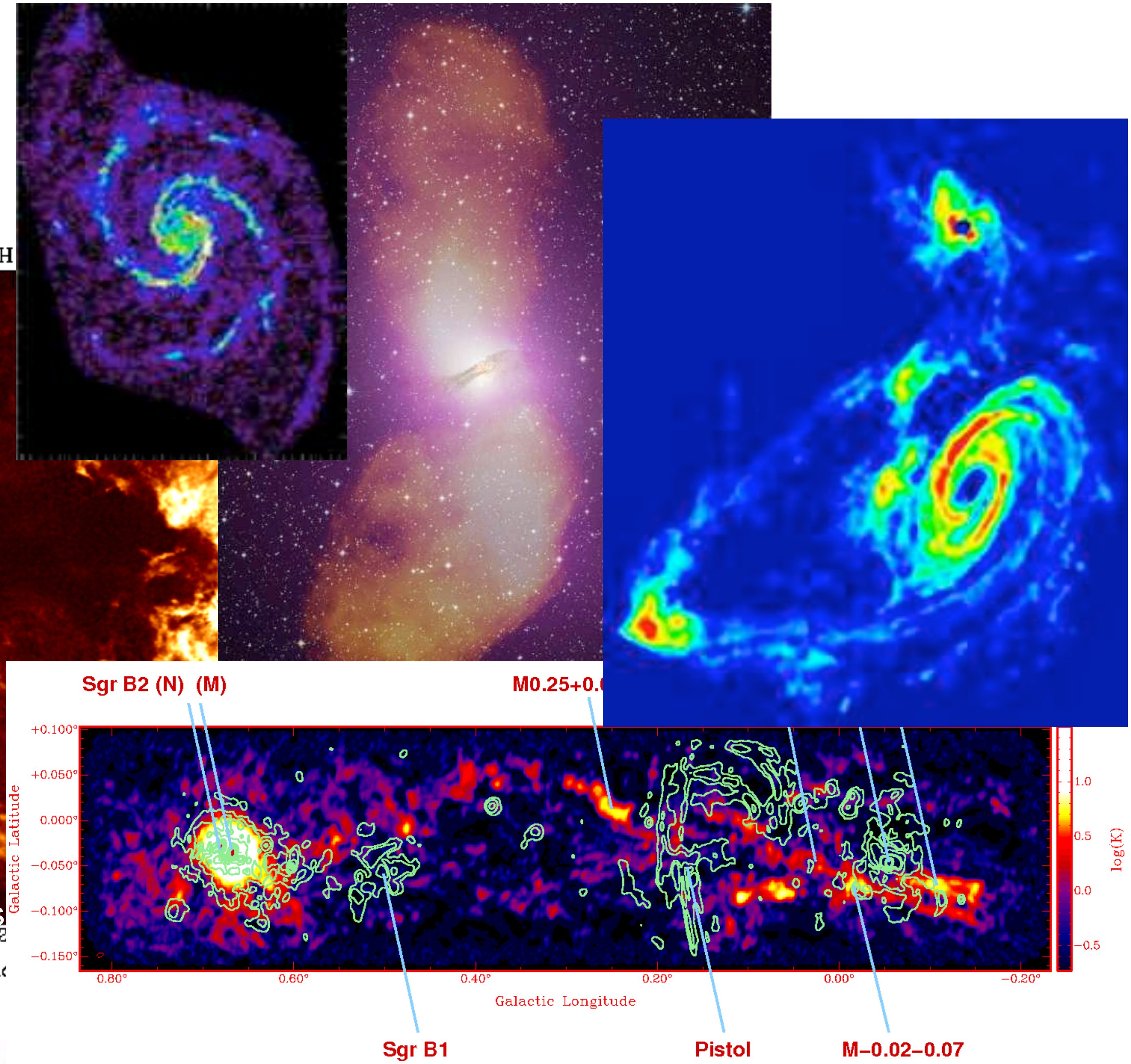
Summary

- Imaging of fields larger than the central area of the primary beam requires attention to special issues
  - determined by imaging equation
  - non-coplanar array geometry (W-term)
  - (polarized) primary beam pattern (optics)
- Carrying out sky surveys or accurate observations of fields and structures larger than the main beam of the array requires multiple pointings of the array, which are combined in imaging using Mosaicking
  - linear and/or joint mosaic deconvolution algorithms
- Carrying out mosaicking requires control of antenna motion and correct calculation of fields and integration times.

The End?



Velocity: 36.28 km/s
GSH
Galactic Latitude
4
2
0
−2
25 pointings
280



Sgr B2 (N) (M)
+0.100°
+0.050°
0.000°
−0.050°
−0.100°
−0.150°
Galactic Latitude
0.80°
0.60°
0.40°
0.20°
0.00°
−0.20°
Galactic Longitude
1.0
0.5
0.0
−0.5
log(K)
Sgr B1
Pistol
M−0.02−0.07

References for W-term & PB

- Interferometry and Synthesis in Radio Astronomy, 2nd Ed.: Thompson, Moran and Swenson
- Synthesis Imaging in Radio Astronomy: II – The "White Book"
- W-Projection: IEEE Journal of Selected Topics in Signal Processing, Vol. 2, No. 5, 2008
- A-Projection: A&A, 487, 419, 2008 (arXiv:0808.0834)
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- NRAO Algo. R&D Page: https://safe.nrao.edu/wiki/bin/view/Software/Algorithms/WebHome
- Home pages of SKA Calibration and Imaging Workshops (CALIM), 2005, 2006, 2008, 2009
- Home Pages of: EVLA, ALMA, ATA, LOFAR, ASKAP, SKA, MeerKat

References for Mosaicking & Surveys

- Interferometry and Synthesis in Radio Astronomy, 2nd Ed.: Thompson, Moran and Swenson
- Synthesis Imaging in Radio Astronomy: II – The "White Book"
- A-Projection: A&A, 487, 419, 2008 (arXiv:0808.0834)
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  - http://sundog.stsci.edu/top.html
- VLASS: https://science.nrao.edu/science/surveys/vlass