Imaging and Deconvolution

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Sixteenth Synthesis Imaging Workshop

16-23 May 2018

thanks to
M. MacGregor
L. Matra

Overview

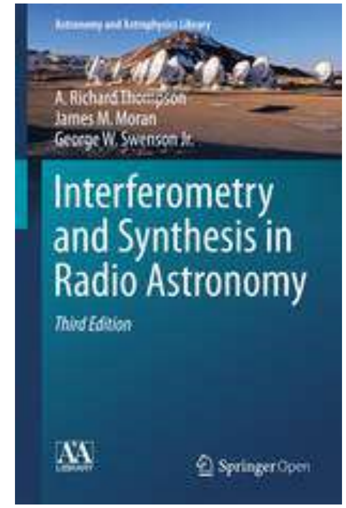
- gain intuition about interferometric imaging
- understand the need for deconvolution

topics

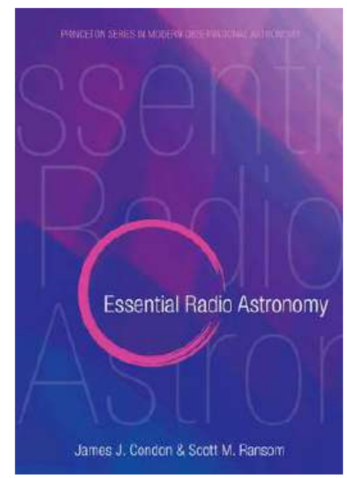
- get comfortable with Fourier Transforms
- review "visibility" concept and sampling the u,v plane
- formal description of imaging
- imaging in practice
- deconvolution and the clean algorithm

References

- Thompson, A.R., Moran, J.M. & Swensen, G.W. 2017, "Interferometry and Synthesis in Radio Astronomy" 3rd edition
  - open access: download pdf on link.springer.com (free!)

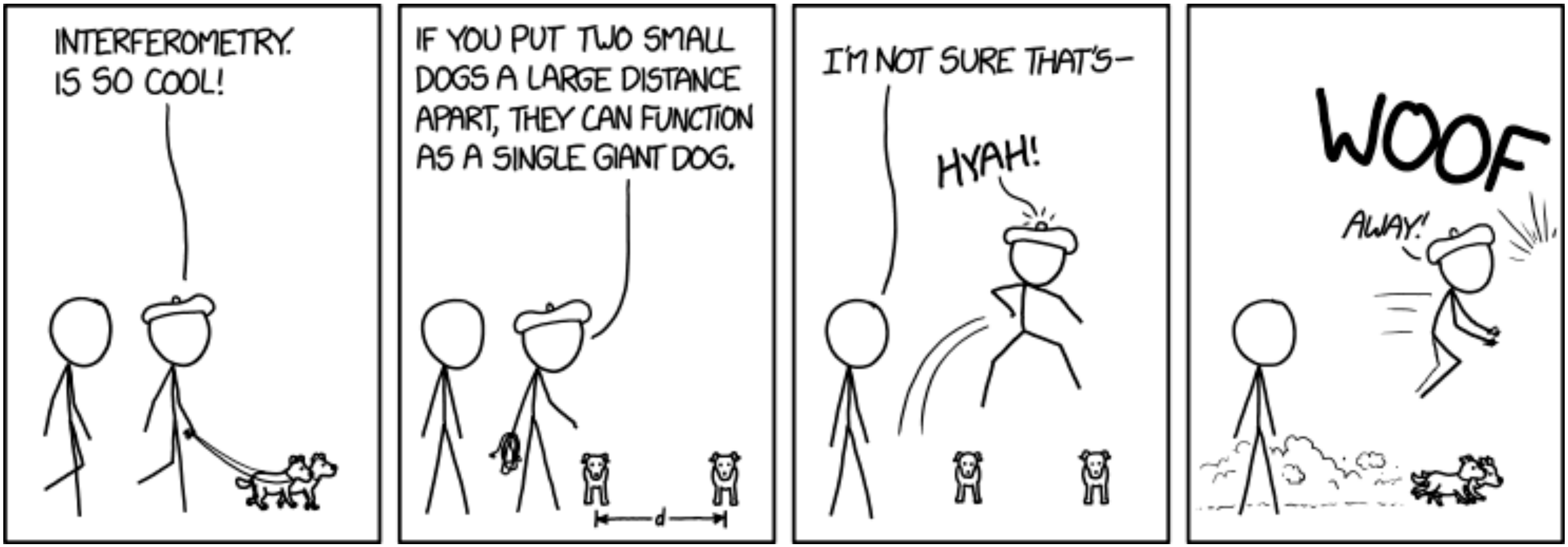


- past NRAO Synthesis Imaging Workshop proceedings
  - Perley, R.A., Schwab, F.R., Bridle, A.H., eds. 1989, ASP Conference Series 6, "Synthesis Imaging in Radio Astronomy"
  - lecture slides: *www.aoc.nrao.edu/events/synthesis*
- IRAM 2000 Interferometry School proceedings
  - *www.iram.fr/IRAMFR/IS/IS2008/archive.html*
- Condon, JJ & Ransom, S.M. 2016, "Essential Radio Astronomy", a complete one semester course, on-line at
  - *science.nrao.edu/opportunities/courses/era*



plus many other useful pedagogical presentations, e.g. ALMA Primer

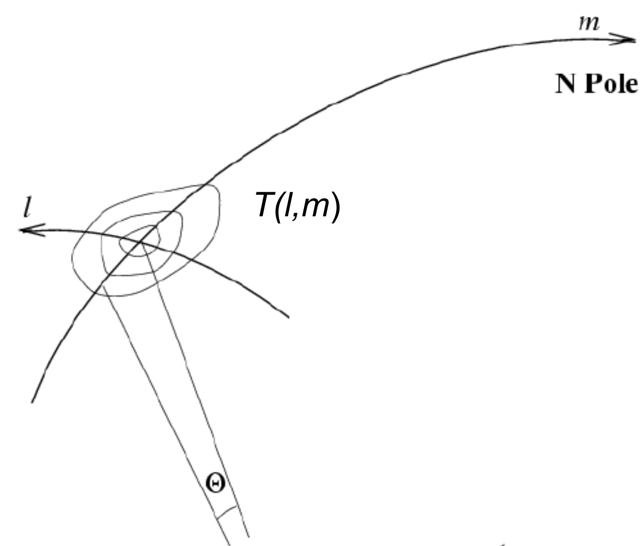
xkcd.com/1922/



It is important to note that while the size of dog can be arbitrarily large, it's not any more of a good dog than the two original dogs.

Visibility and Sky Brightness

V(u,v), the complex visibility function, is the 2D Fourier transform of T(l,m), the sky brightness distribution (for an incoherent source, small field of view, far field, etc.) [see TMS for derivation]



mathematically

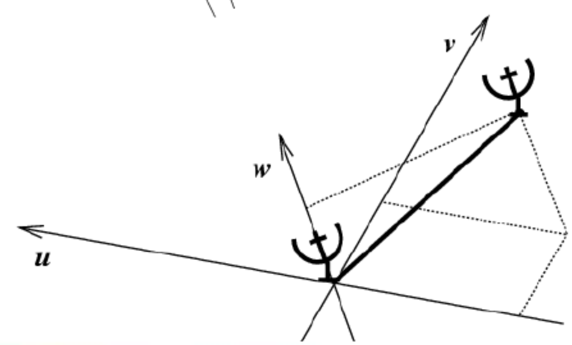
$$V(u,v) = \int\int T(l,m) e^{-i2\pi(ul+vm)} dl dm$$

$$T(l,m) = \int\int V(u,v) e^{i2\pi(ul+vm)} du dv$$

u,v are E-W, N-S spatial frequencies [wavelengths]

l,m are E-W, N-S angles in the tangent plane [radians]

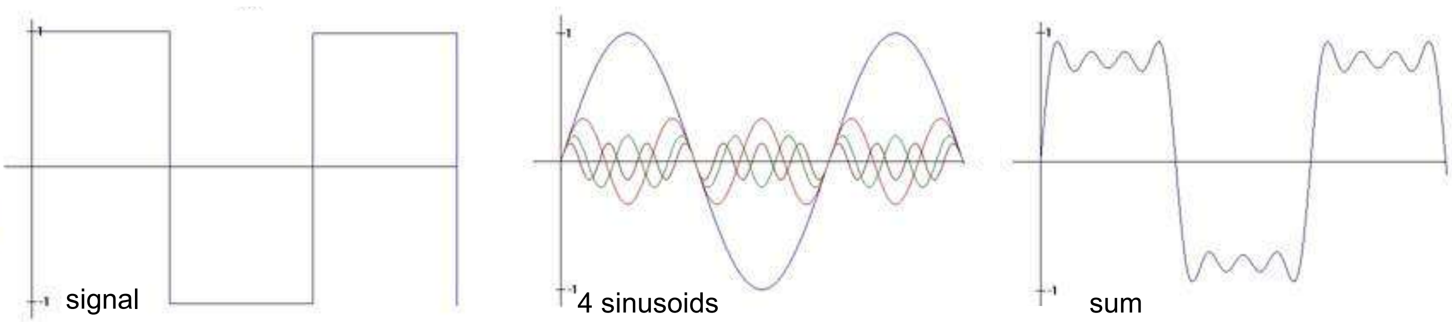
(recall $e^{ix} = \cos x + i \sin x$)



$$V(u,v) \xrightarrow{\mathcal{F}} T(l,m)$$

The Fourier Transform

- Fourier theory: any well behaved signal (including images) can be expressed as the sum of sinusoids



$$x(t) = \frac{4}{\pi}\left(\sin(2\pi f t) + \frac{1}{3}\sin(6\pi f t) + \frac{1}{5}\sin(10\pi f t) + \cdots\right)$$

- the Fourier transform is the mathematical tool that decomposes a signal into its sinusoidal components
- the Fourier transform contains *all* of the information of the original signal

Jean Baptiste Joseph Fourier 1768-1830

The Fourier Domain

- acquire some comfort with the Fourier domain
  - in older texts, functions and their Fourier transforms occupy *upper* and *lower* domains, as if "functions circulated at ground level and their transforms in the underworld" (Bracewell 1965)

- some properties of the Fourier transform $g(x) \xrightarrow{\mathcal{F}} G(s)$

  adding $g(x) + h(x) = G(s) + H(s)$

  scaling $g(\alpha x) = \alpha^{-1} G(s/\alpha)$

  shifting $g(x - x_0) = G(s) e^{i2\pi x_0 s}$

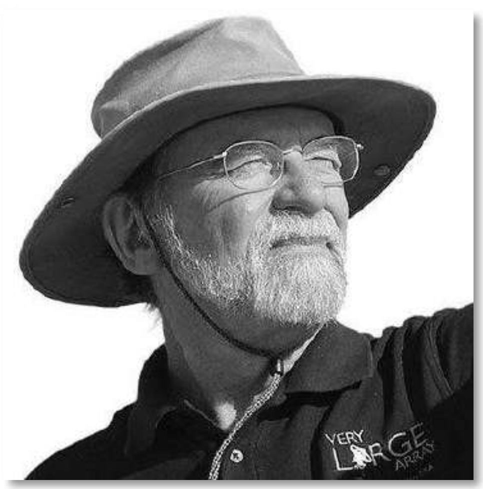
  convolution/multiplication $g(x) = h(x) * k(x) \quad G(s) = H(s)K(s)$

  Nyquist-Shannon sampling theorem $g(x) \subset \Theta$ completely determined if $G(s)$ sampled at $\leq 1/\Theta$

Visibilities

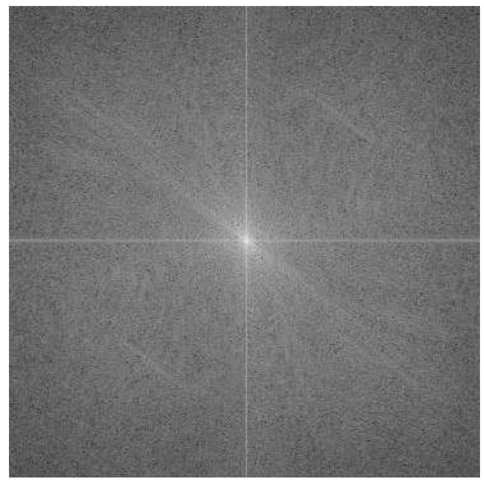
- each *V(u,v)* is a complex quantity
  - expressed as *(real, imaginary)* or *(amplitude, phase)*

T(l,m)

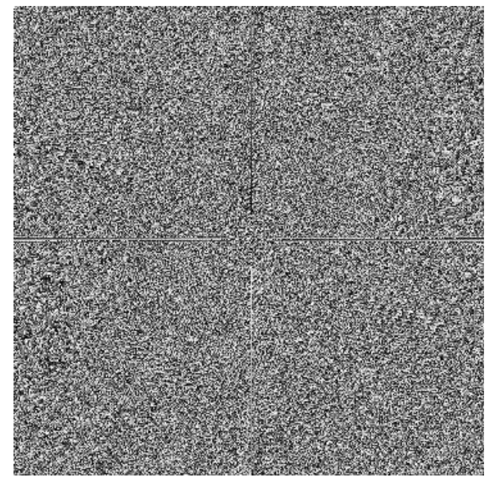


$\xrightarrow{\mathcal{F}}$

V(u,v) amplitude



V(u,v) phase

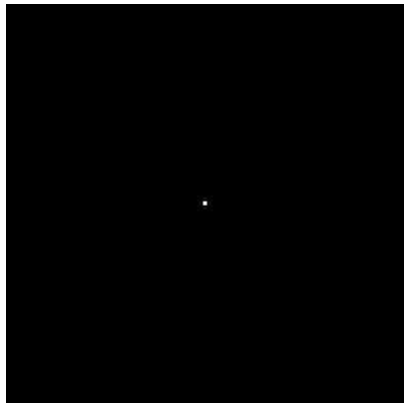


- each *V(u,v)* contains information on *T(l,m) everywhere*, not just at a given *(l,m)* coordinate or within a particular subregion

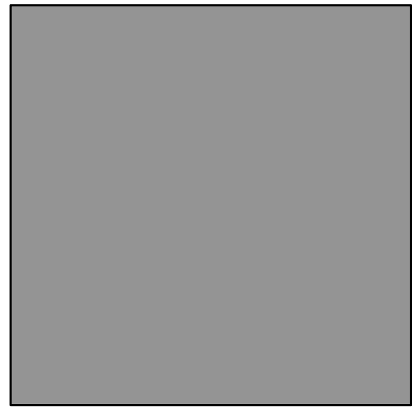
Example 2D Fourier Transforms

T(l,m) | *V(u,v) amplitude*

δ function

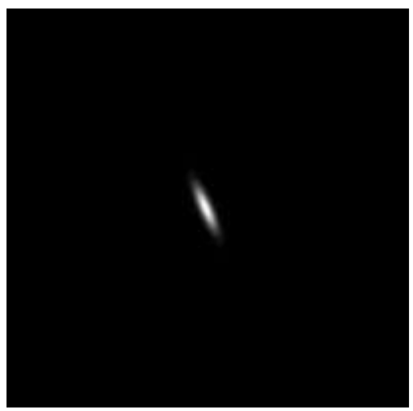


$\xrightarrow{\mathcal{F}}$

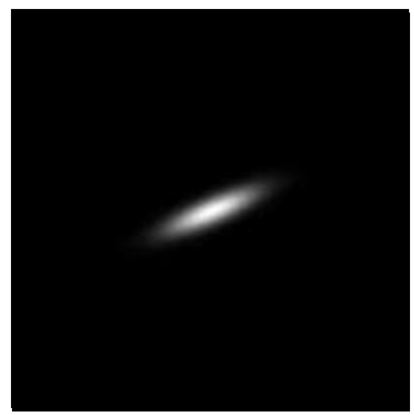


constant

elliptical Gaussian



$\xrightarrow{\mathcal{F}}$



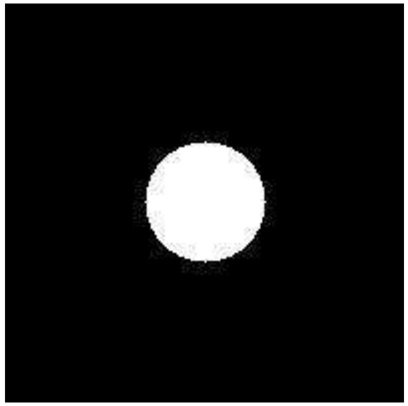
elliptical Gaussian

narrow features transform into wide features (and vice-versa)

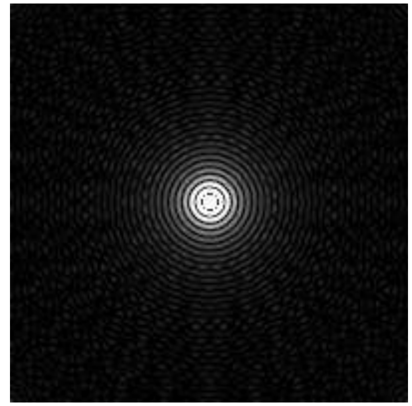
Example 2D Fourier Transforms

T(l,m) *V(u,v) amplitude*

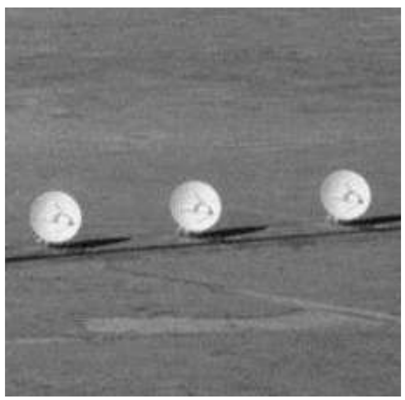
uniform disk



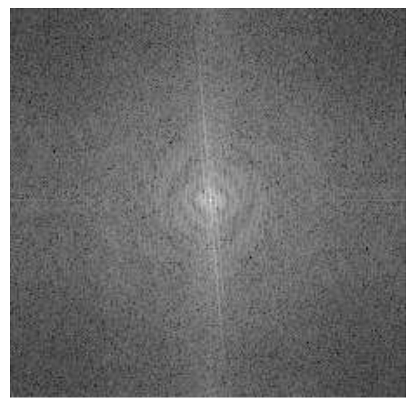
$\xrightarrow{\mathcal{F}}$



Bessel function



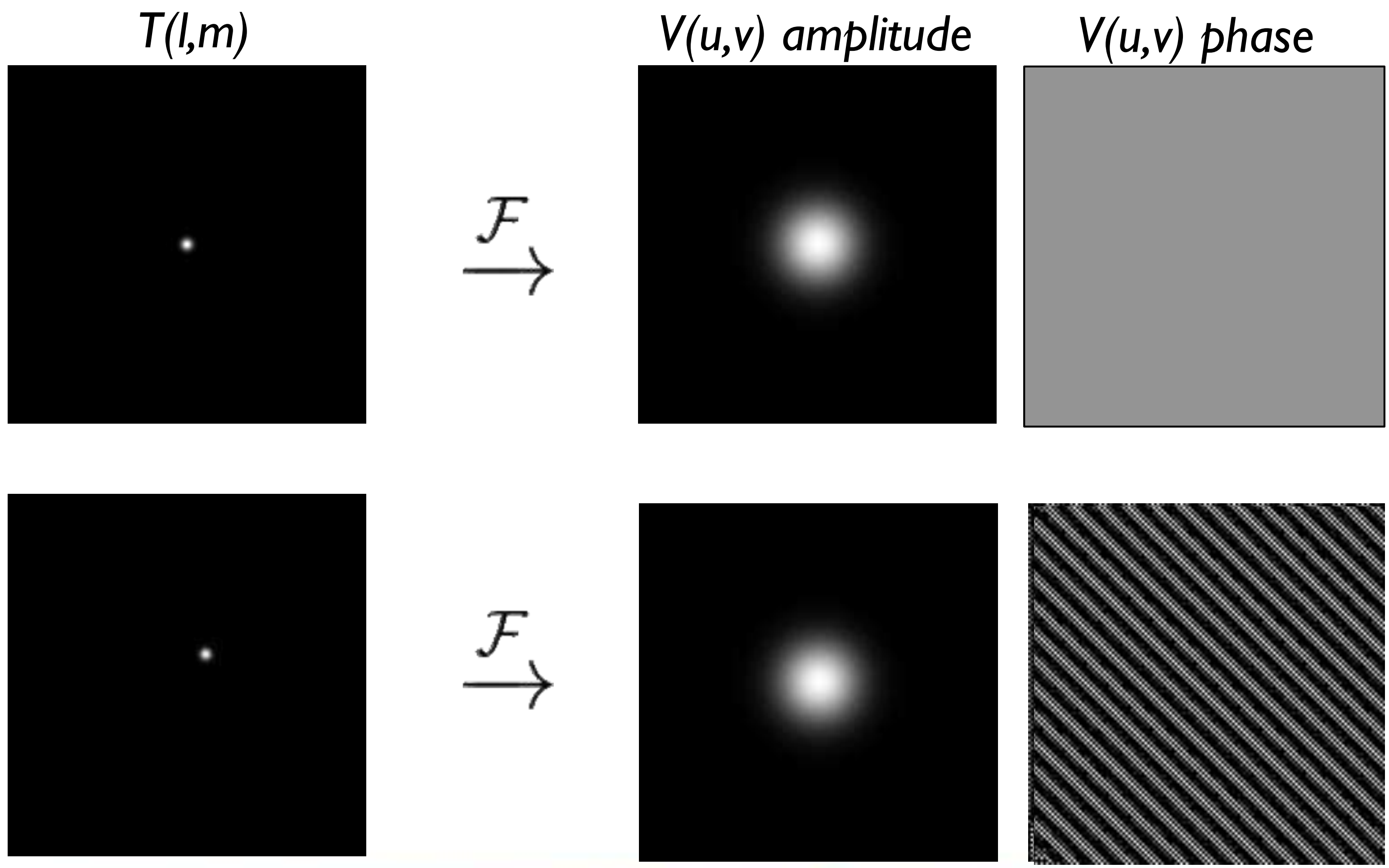
$\xrightarrow{\mathcal{F}}$



sharp edges result in many high spatial frequencies

Amplitude and Phase

- amplitude tells "how much" of a certain spatial frequency
- phase tells "where" this spatial frequency component is located



The Visibility Concept

$$V(u,v) = \int\int T(l,m)e^{-i2\pi(ul+vm)}dldm$$

- visibility as a function of baseline coordinates *(u,v)* is the Fourier transform of the sky brightness distribution as a function of the sky coordinates *(l,m)*

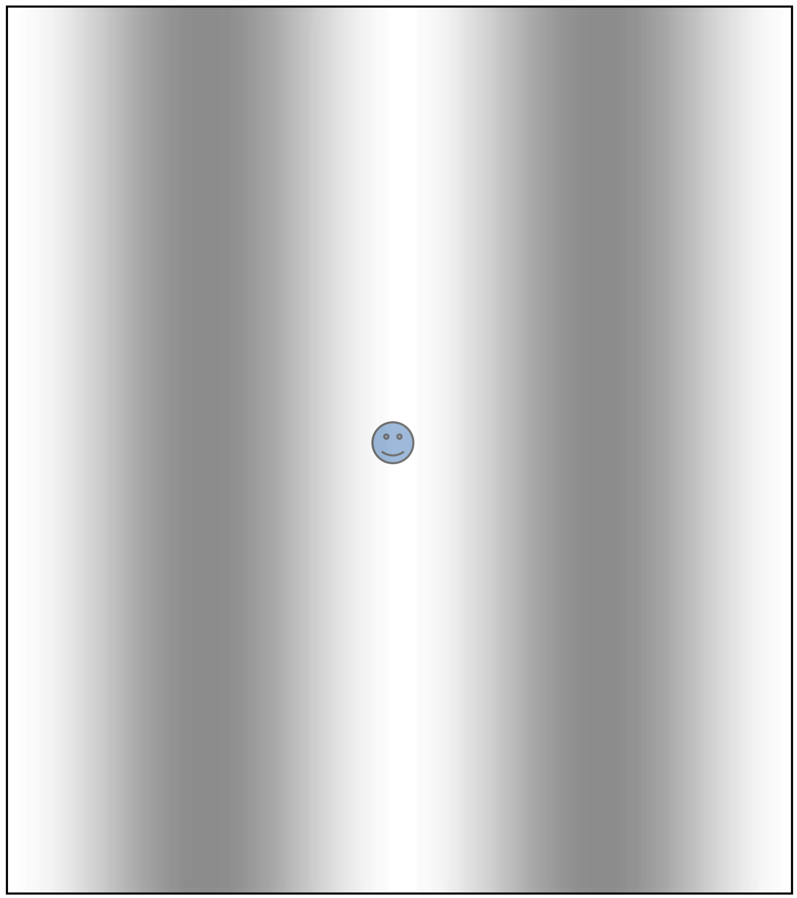
- since $T(l,m)$ is real, $V(u,v)$ is Hermitian and $V(-u,-v) = V^*(u,v)$ (get two visibilities for each *(u,v)* measurement)

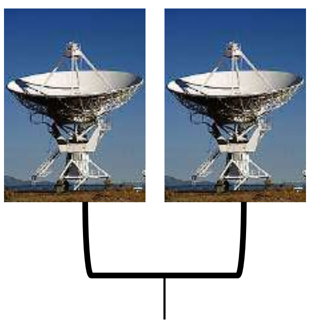
- $V(u=0,v=0)$ is the integral of $T(l,m)dldm$ = total flux density

NRAO

Small Source, Short Baseline

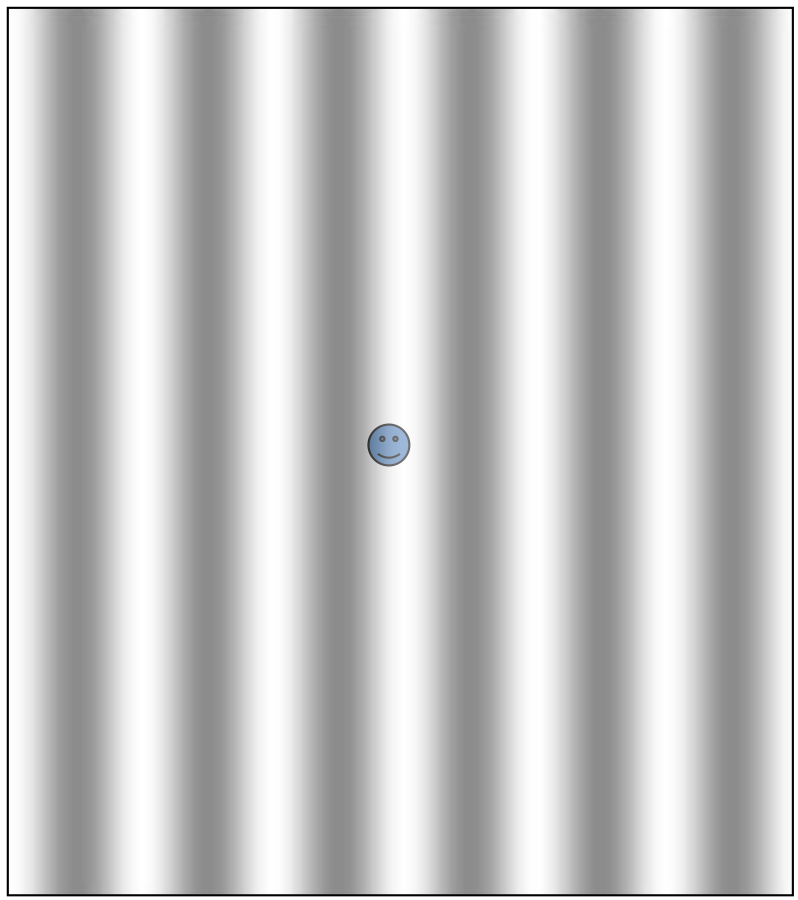
$$V(u,v) = \int\int T(l,m)e^{-i2\pi(ul+vm)}dldm$$

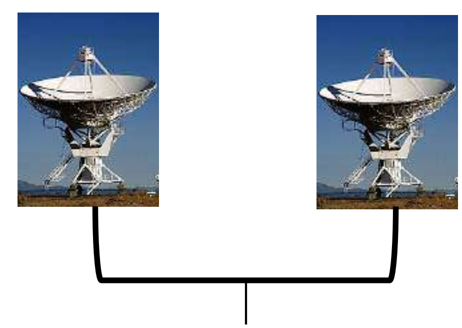




Small Source, Long Baseline

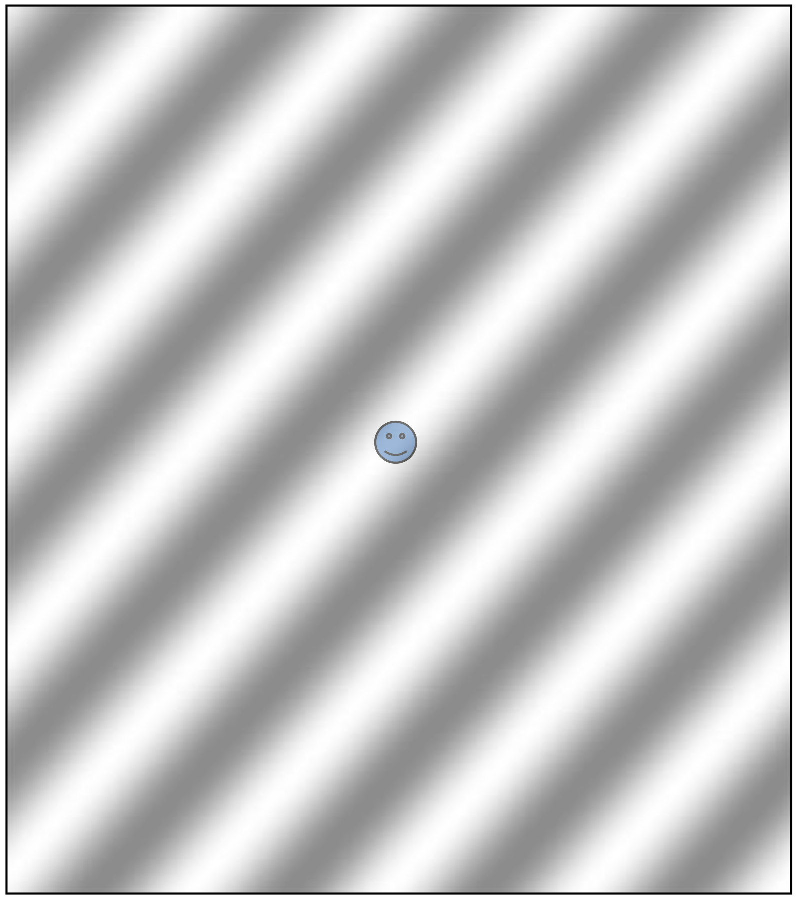
$$V(u,v) = \int\int T(l,m)e^{-i2\pi(ul+vm)}dldm$$

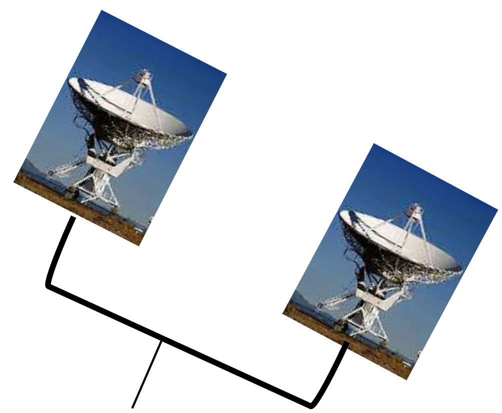




Small Source, Long Baseline

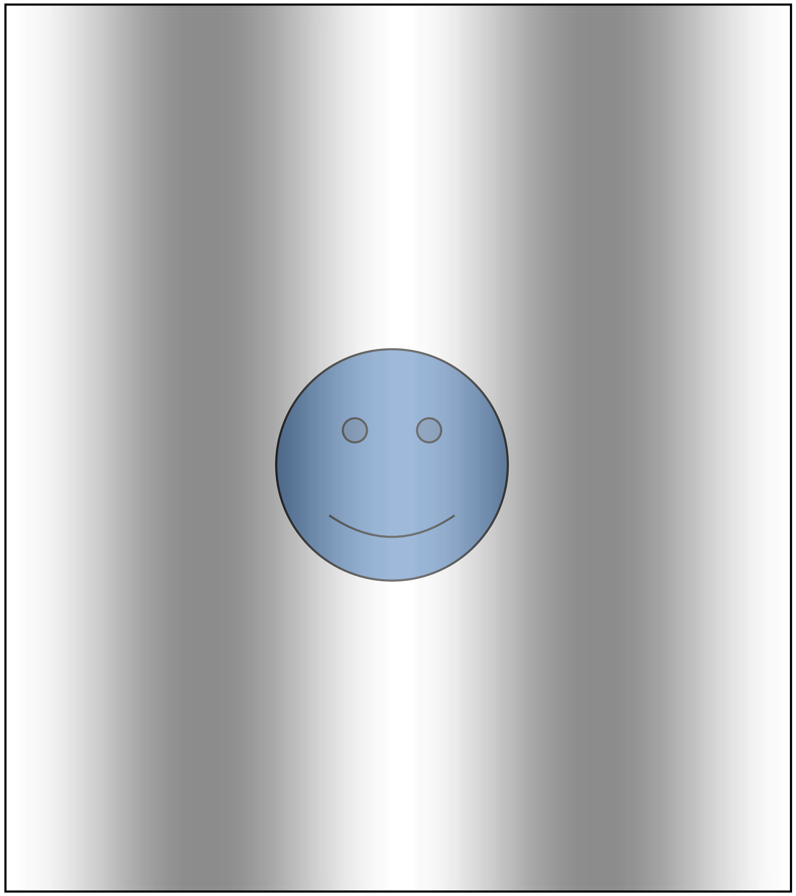
$$V(u,v) = \int\int T(l,m)e^{-i2\pi(ul+vm)}dldm$$

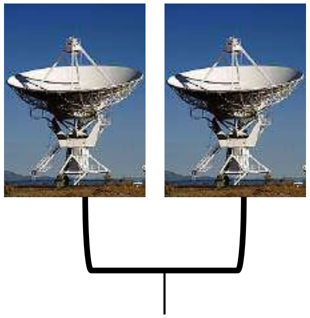




Extended Source, Short Baseline

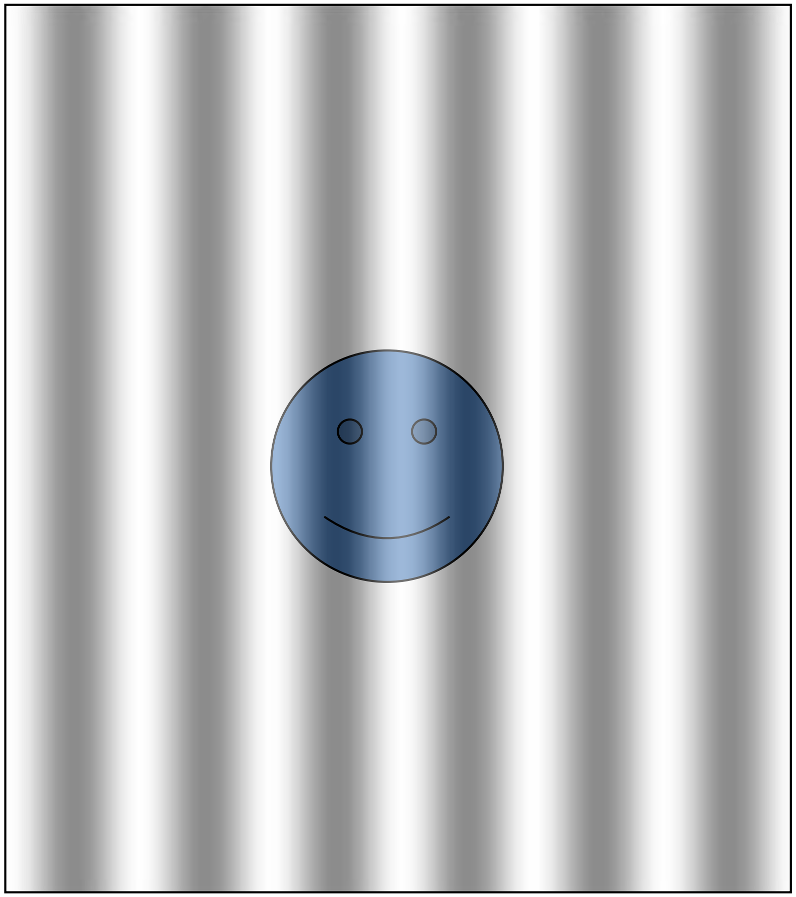
$$V(u,v) = \int\int T(l,m) e^{-i2\pi(ul+vm)} dl dm$$

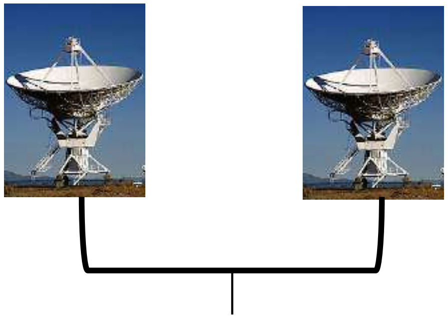




Extended Source, Long Baseline

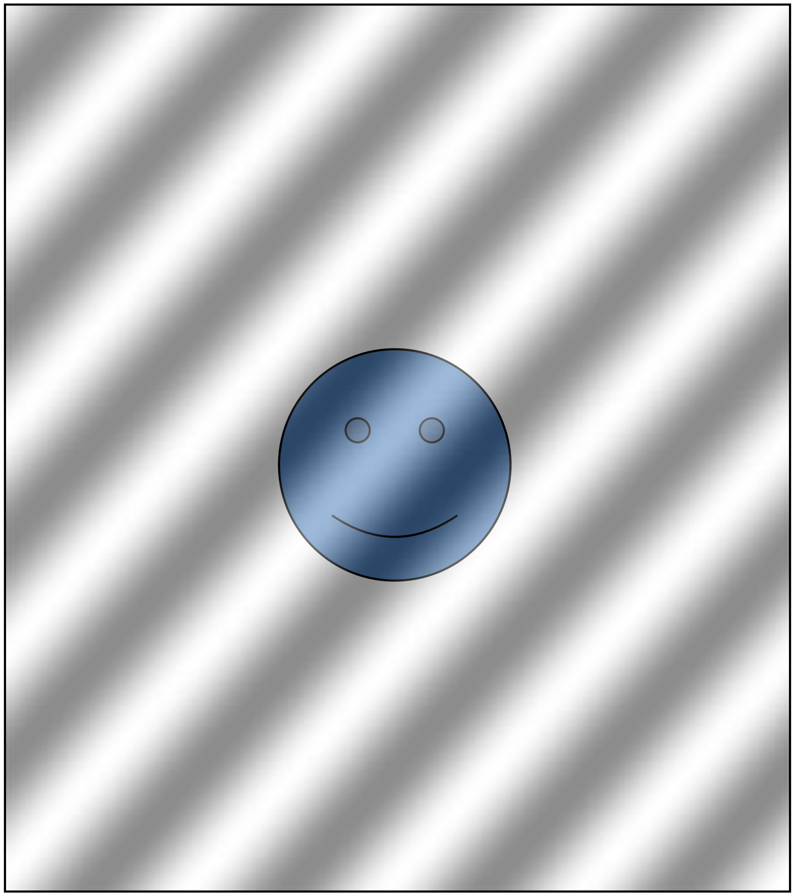
$$V(u,v) = \int\int T(l,m) e^{-i2\pi(ul+vm)} dl dm$$

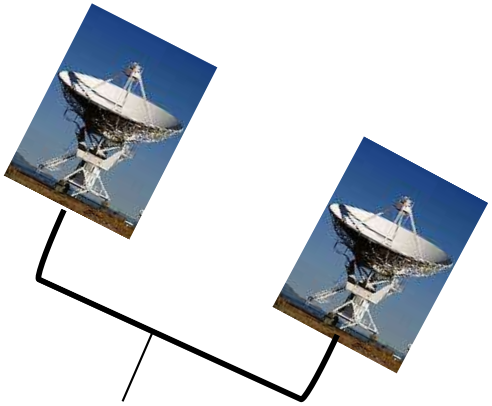




Extended Source, Long Baseline

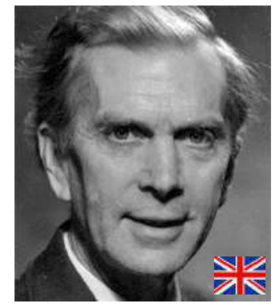
$$V(u,v) = \int\int T(l,m)e^{-i2\pi(ul+vm)}dldm$$





Aperture Synthesis

- basic idea: sample *V(u,v)* at enough *(u,v)* points using distributed small aperture antennas to synthesize a large aperture antenna of size (u_{max},v_{max})
- use more antennas for more samples
  - one pair of antennas = two *(u,v)* samples at a time
  - N antennas = N(N-1) samples at a time
  - reconfigure physical layout of N antennas for more
- use Earth rotation for more samples
  - fill in (u,v) plane over time
- use more wavelengths for more samples
  - need to determine source structure at some wavelength and the change with wavelength, e.g. Taylor expansion
  - "multi-frequency synthesis" for continuum imaging [Urvashi Rao,Monday]

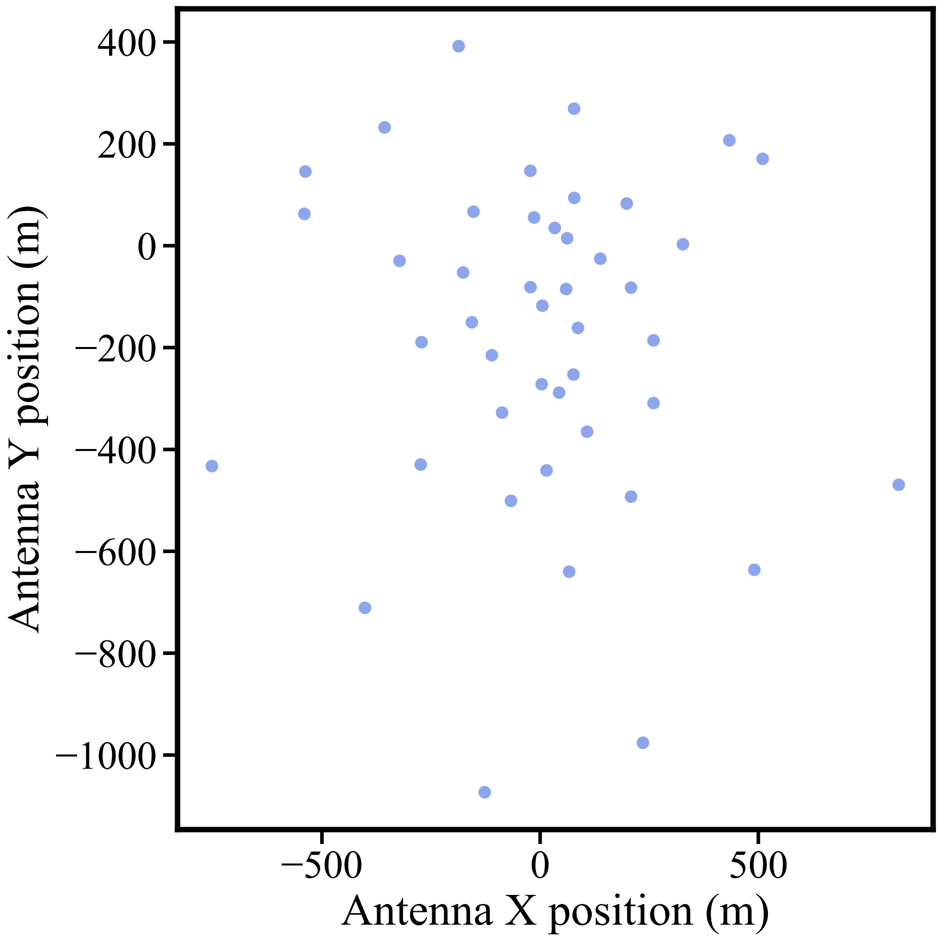


Sir Martin Ryle
1918-1984

1974 Nobel Prize in Physics

A few Aperture Synthesis Telescopes for Observations at Millimeter Wavelengths

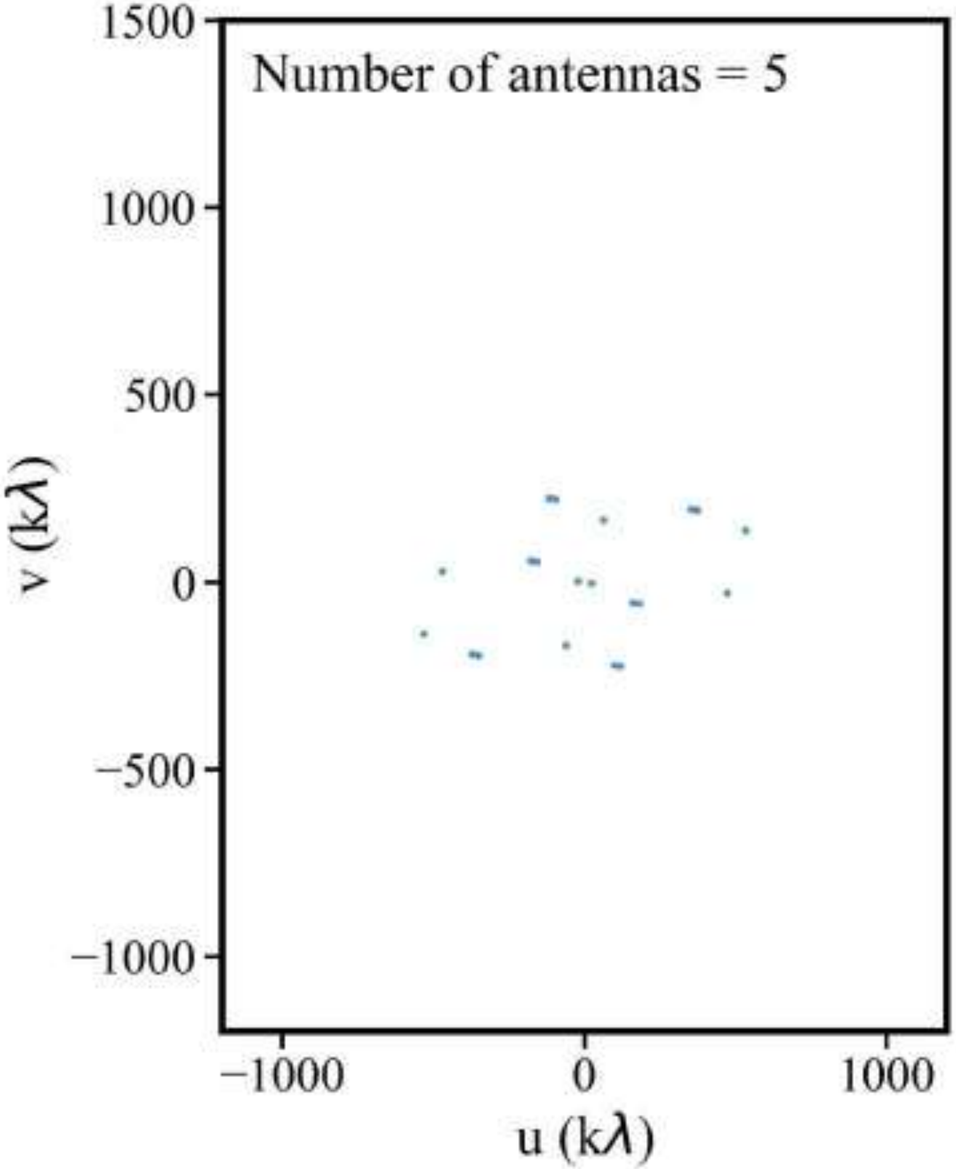
Example of (u,v) Plane Sampling

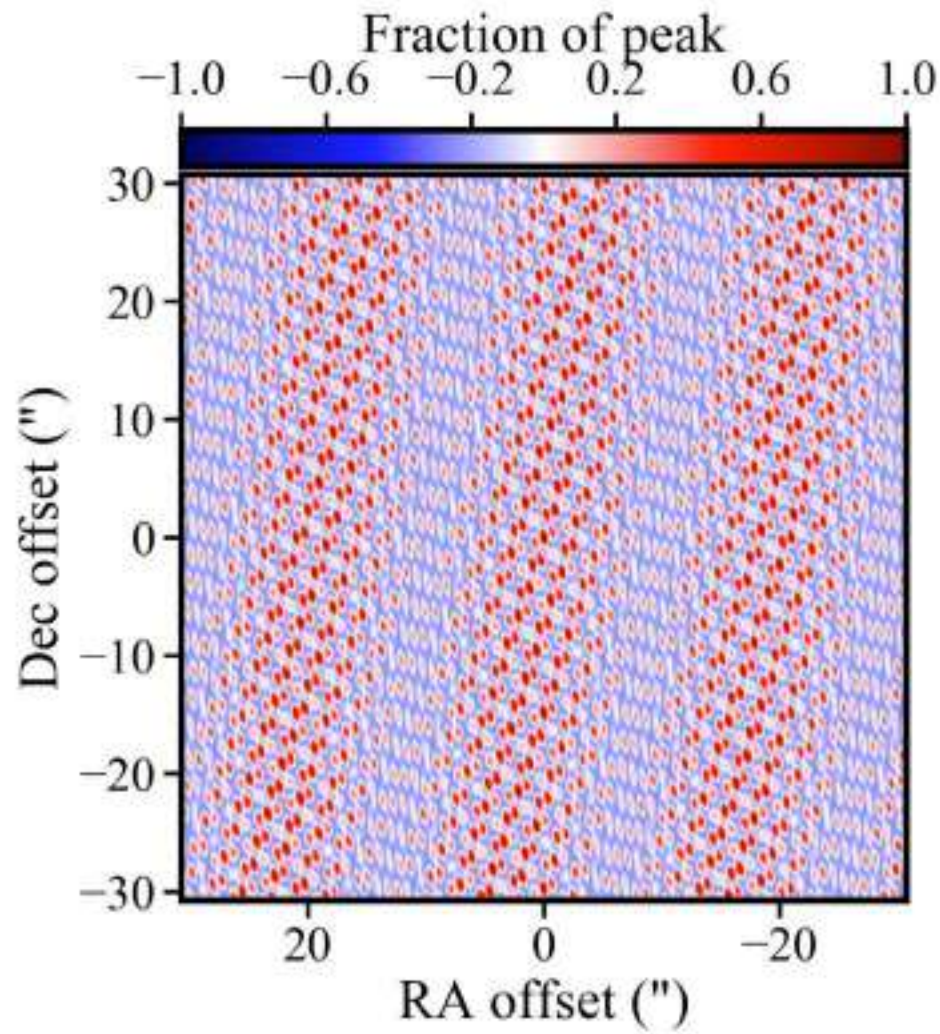


ALMA 12m antenna locations on August 8, 2015

[Array Design: Craig Walker, Monday]

(u,v) Plane Sampling: more Antennas

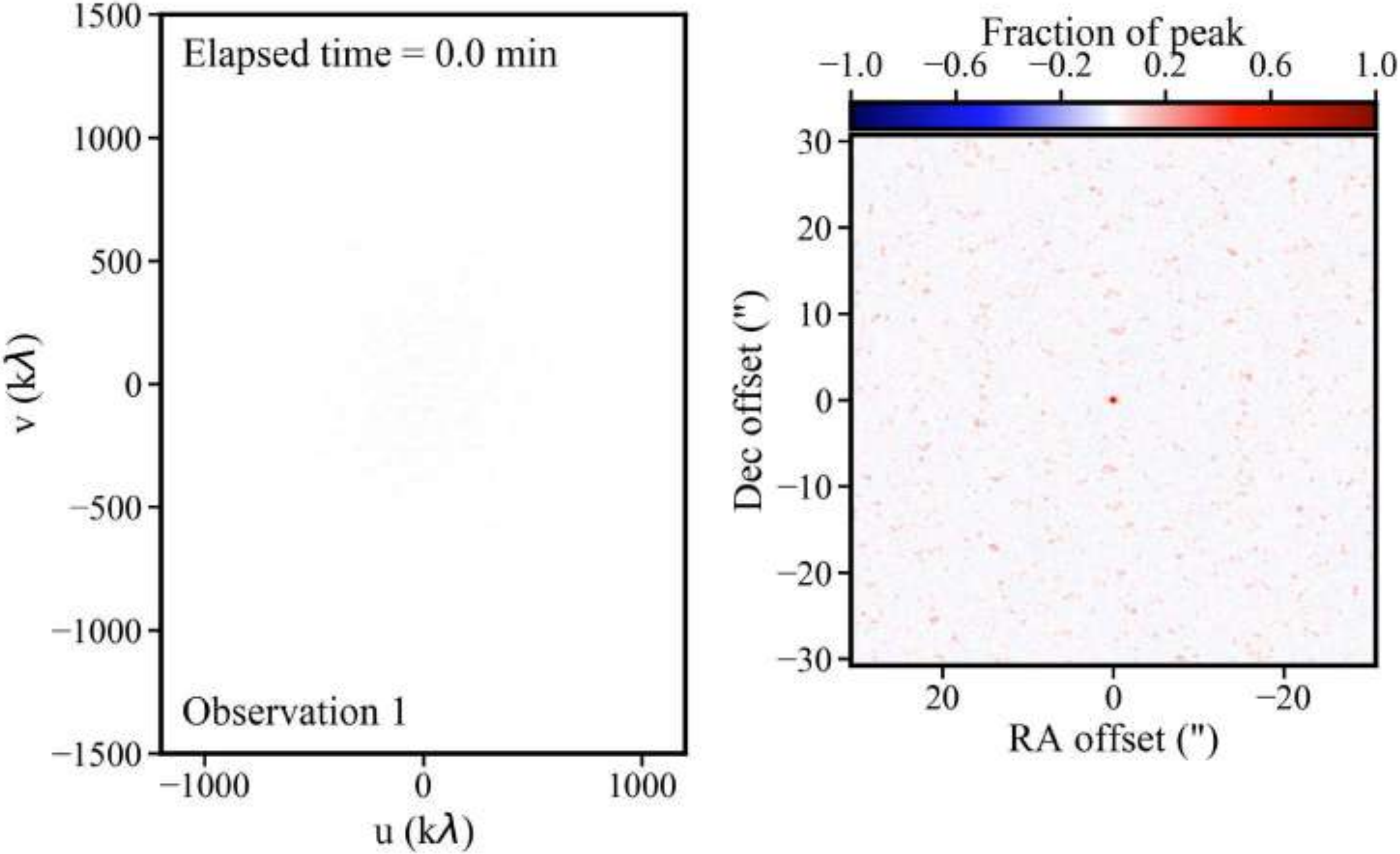




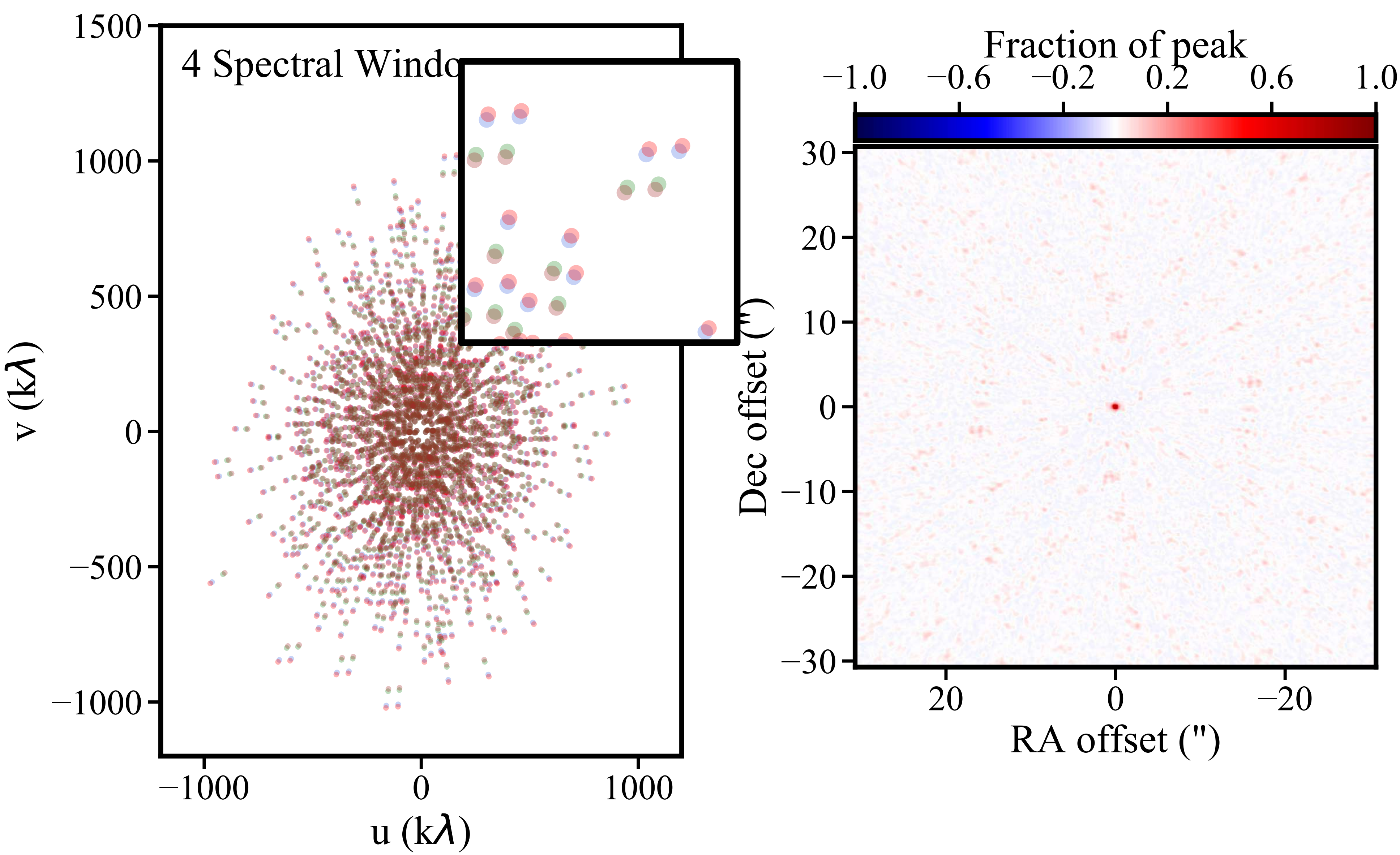
source dec -51 deg

ν=224.6 GHz (λ=1.3mm)

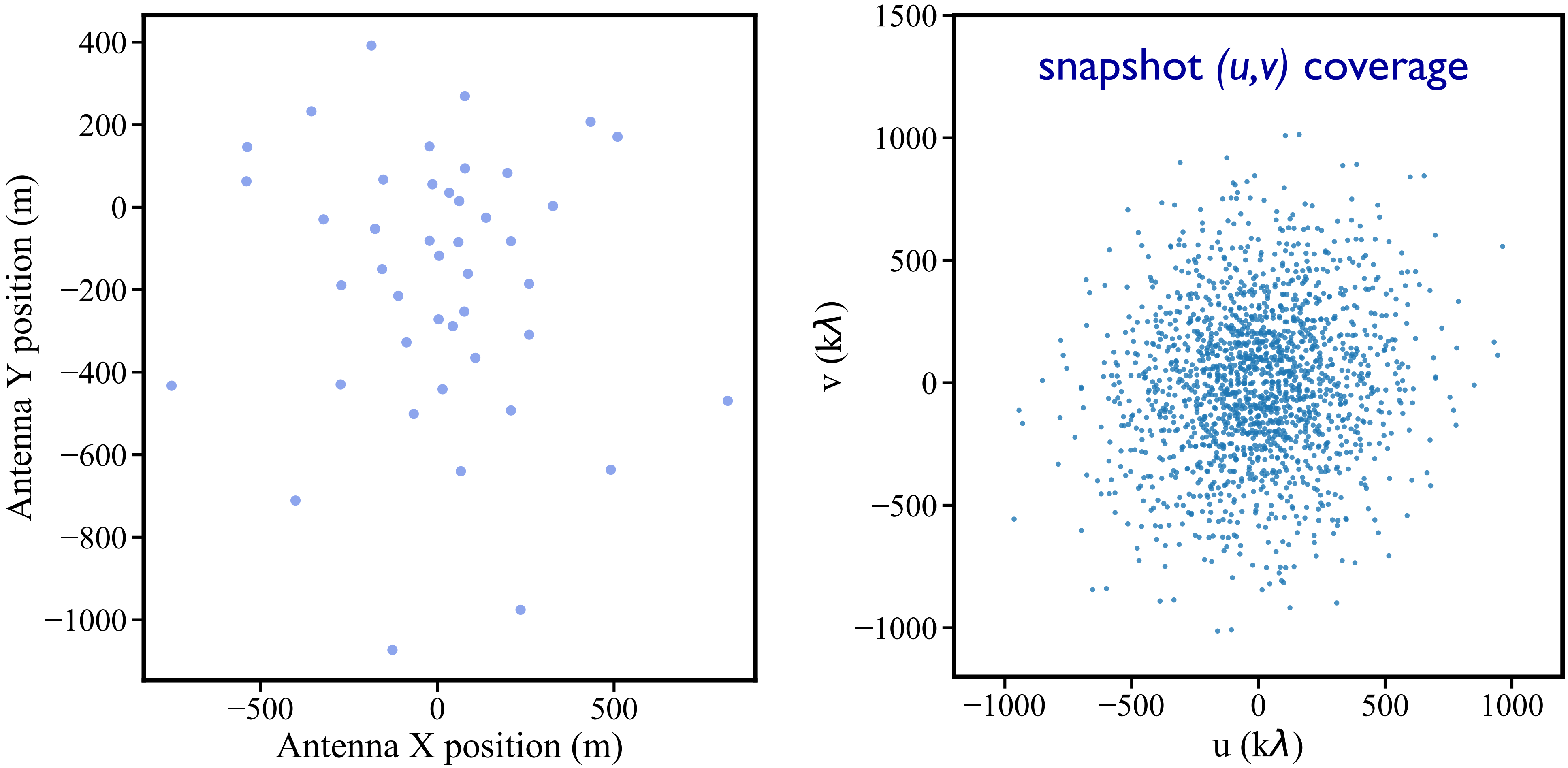
(u,v) Plane Sampling: Earth rotation



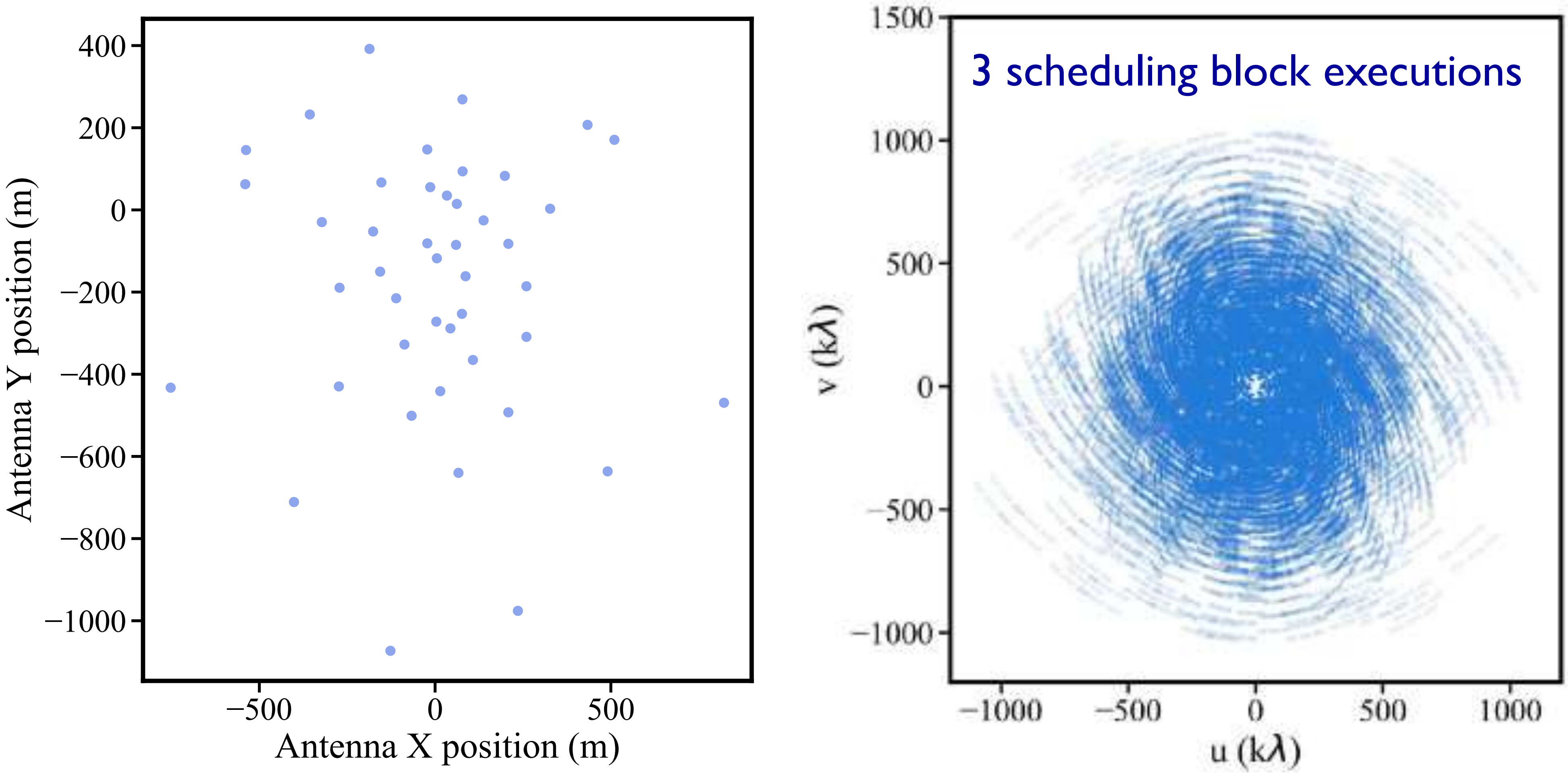
(u,v) Plane Sampling: more wavelengths



Example of (u,v) Plane Sampling

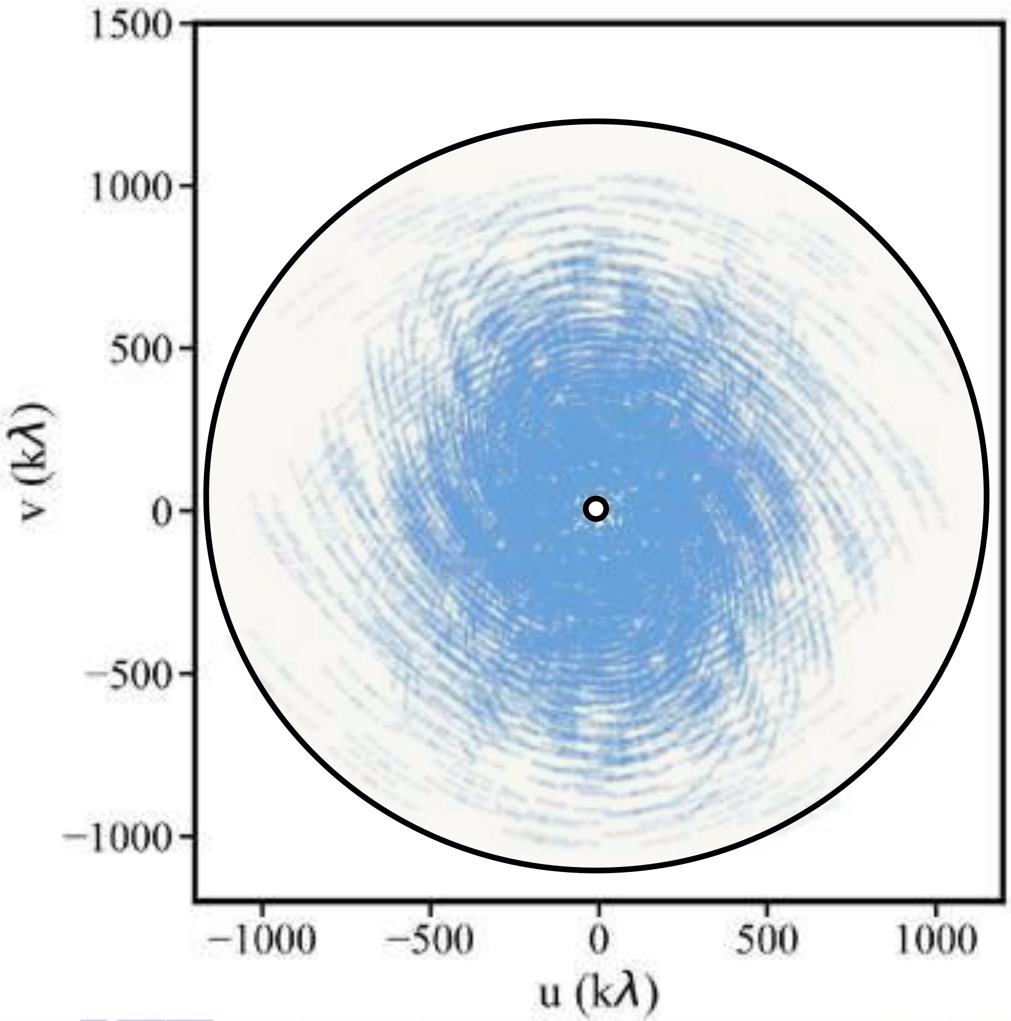


Example of (u,v) Plane Sampling



Implications of (u,v) Plane Sampling

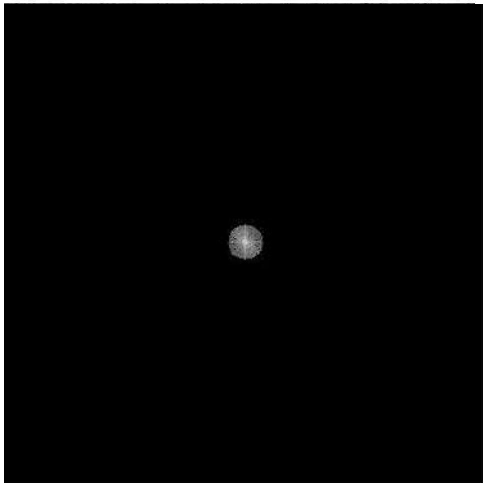
V(u,v) samples are limited by # antennas and Earth-sky geometry



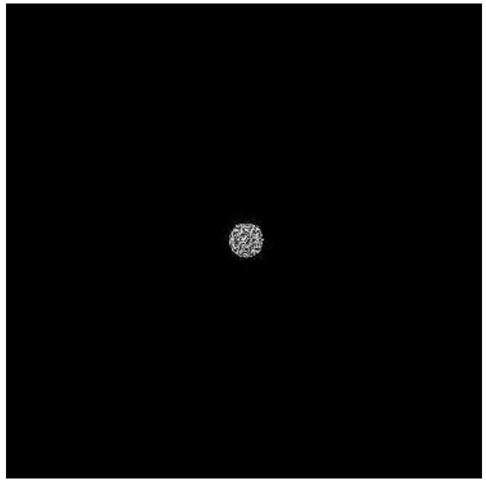
- *outer boundary*
  - no info on smaller scales
  - resolution limit
- *inner hole*
  - no info on larger scales
  - extended structures invisible
- *irregular sampling in between*
  - sampling theorem violated
  - information missing

Inner and Outer (u,v) Boundaries

V(u,v) amplitude



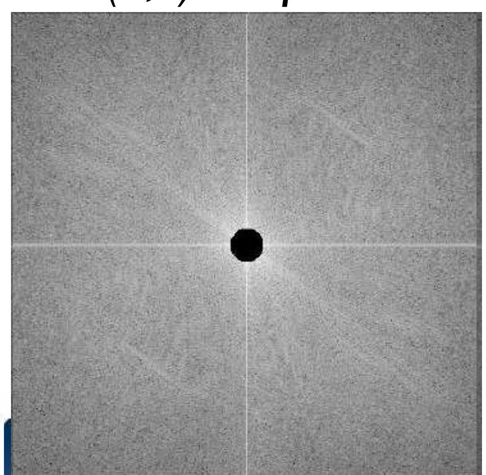
V(u,v) phase



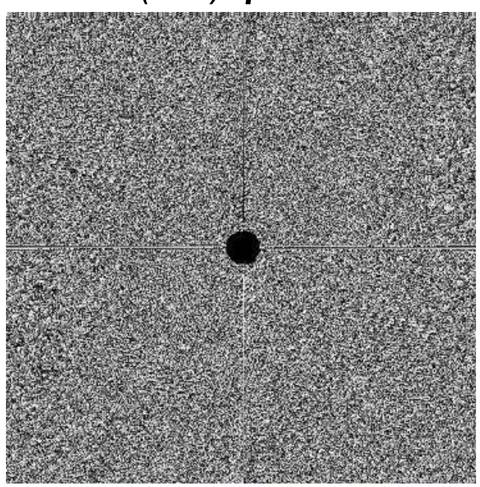
$\xrightarrow{\mathcal{F}}$

T(l,m)

V(u,v) amplitude



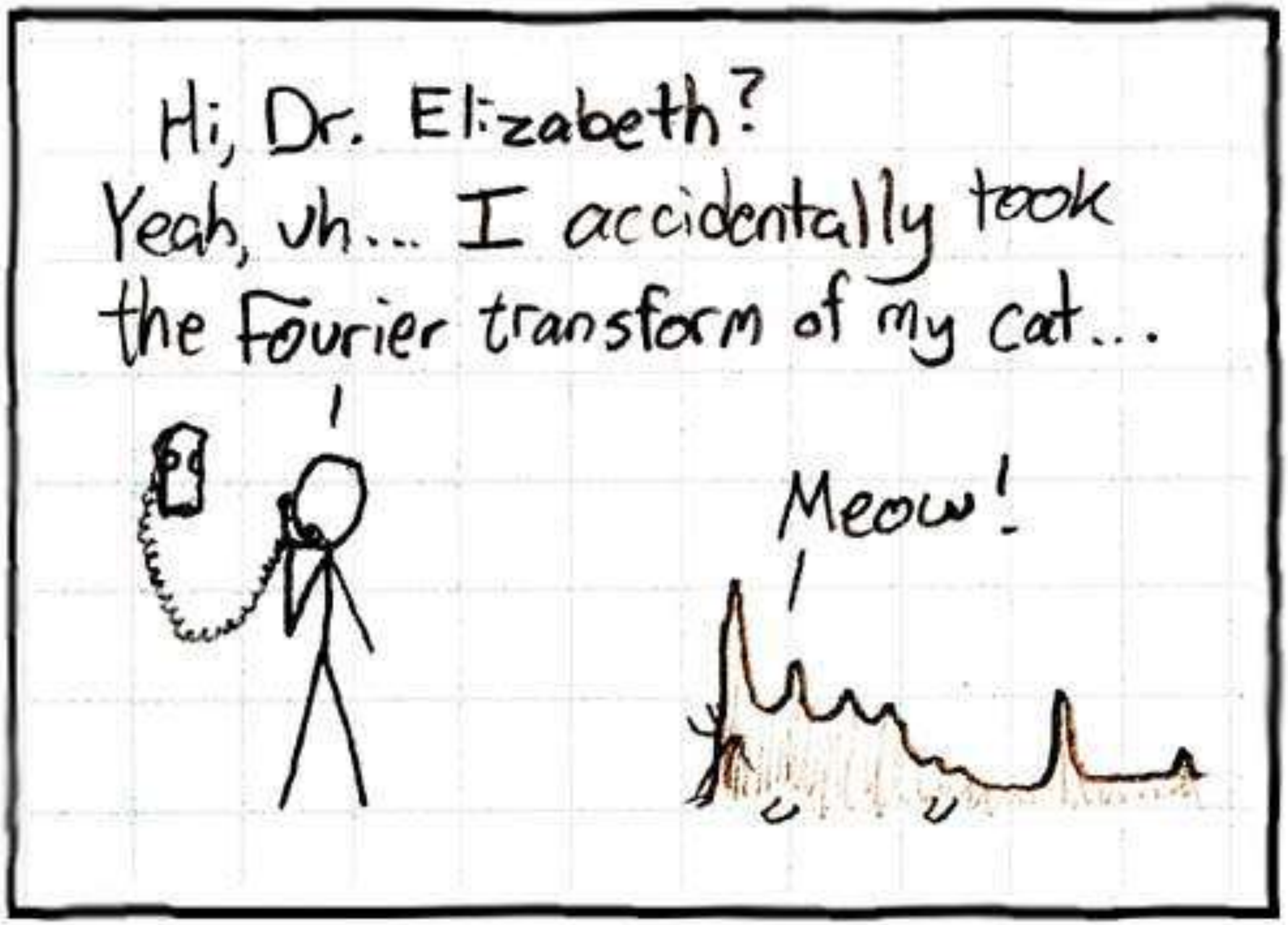
V(u,v) phase



$\xrightarrow{\mathcal{F}}$

T(l,m)

xkcd.com/26/



That cat has some serious periodic components.

Calibrated Visibilities… What's Next?

- analyze directly *V(u,v)* samples by model fitting
  - good for simple structures, e.g. point sources, symmetric disks
  - for a purely statistical description of sky brightness (e.g. fluctuations)
  - visibilities have well defined noise properties

[Greg Taylor, Tuesday]

- recover an image from the observed incomplete and noisy samples of its Fourier transform for analysis
  - Fourier transform *V(u,v)* to get $T^D(l,m)$,
    but difficult to do science with the dirty image $T^D(l,m)$
  - deconvolve $s(l,m)$ from $T^D(l,m)$ to determine a model of $T(l,m)$
  - work with the model of $T(l,m)$

CASA
`tclean`

Formal Description of Imaging

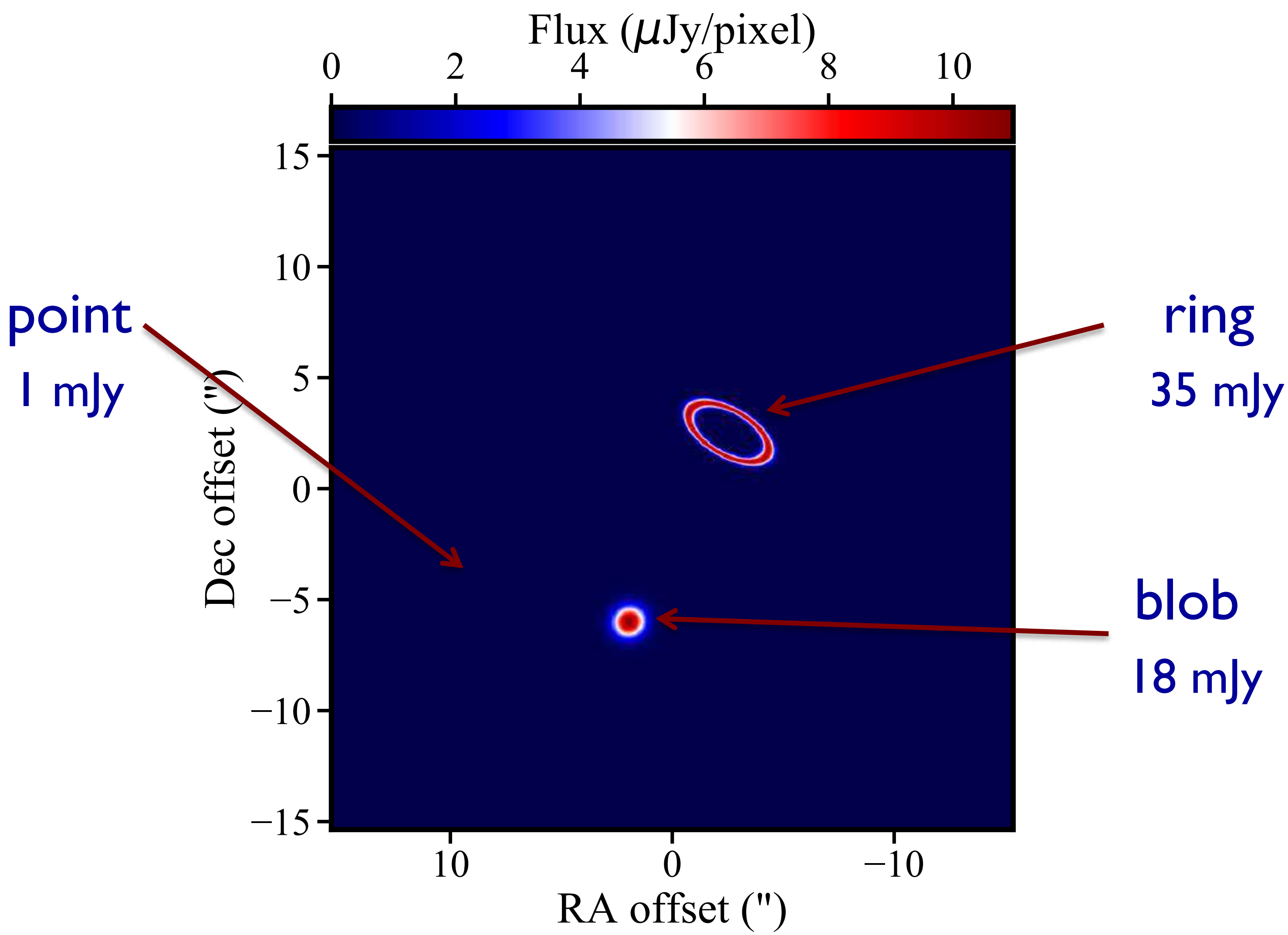
$$V(u,v) \xrightarrow{\mathcal{F}} T(l,m)$$

- sample Fourier domain at discrete points $S(u,v) = \sum_{k=1}^{M} \delta(u - u_k, v - v_k)$
- Fourier transform the sampled visibilities $V(u,v)S(u,v) \xrightarrow{\mathcal{F}} T^D(l,m)$
- apply the convolution theorem $T(l,m) * s(l,m) = T^D(l,m)$
  where the Fourier transform of the sampling pattern $s(l,m) \xrightarrow{\mathcal{F}} S(u,v)$ is the point spread function

the Fourier transform of the sampled visibilities yields the true sky brightness convolved with the point spread function

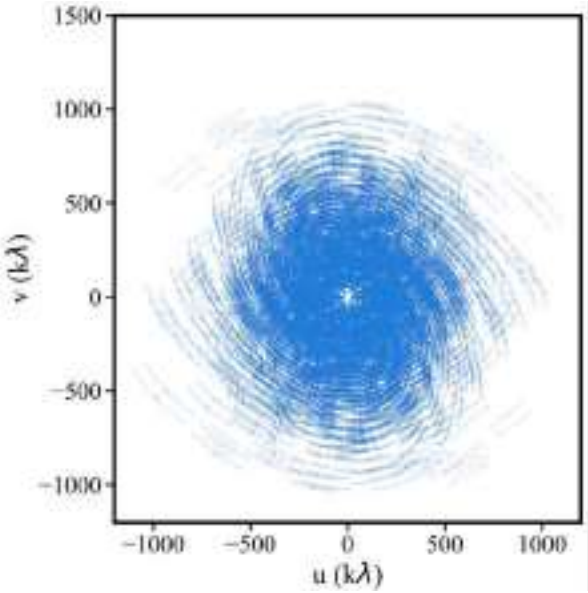
(radio jargon: "dirty image" is true image convolved with "dirty beam")

A model *T(l,m)* sky brightness distribution

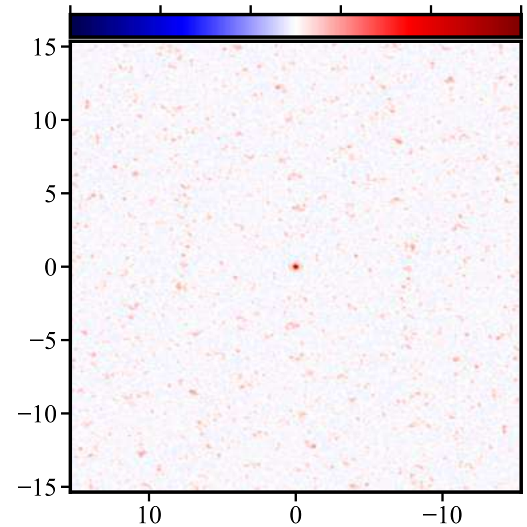


Dirty Beam and Dirty Image

S(u,v)



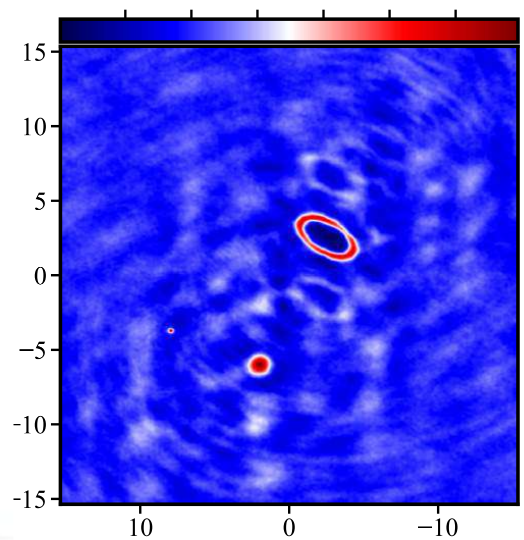
$\xrightarrow{\mathcal{F}}$



s(l,m)
"dirty beam"

$\ast$

T^D(l,m)
"dirty image"



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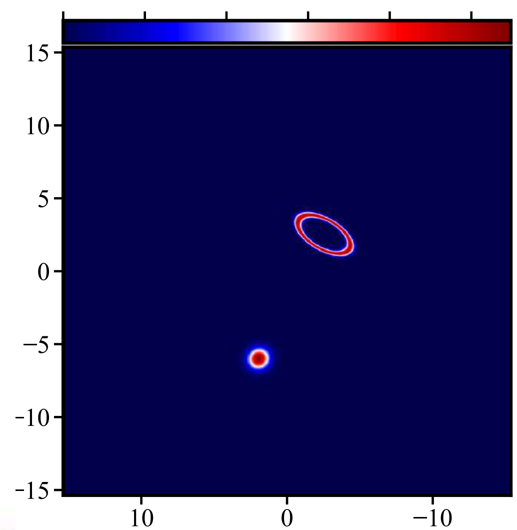
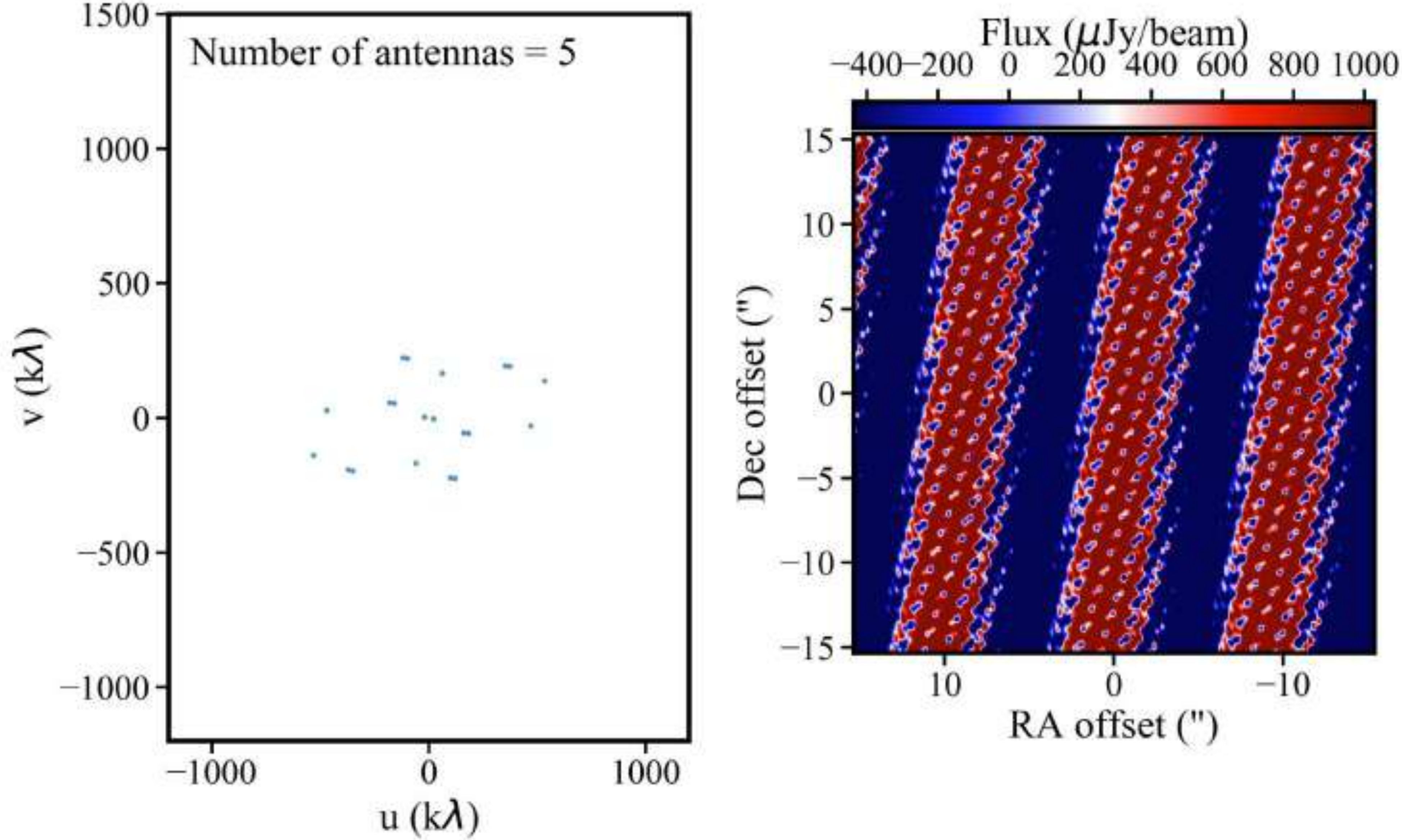
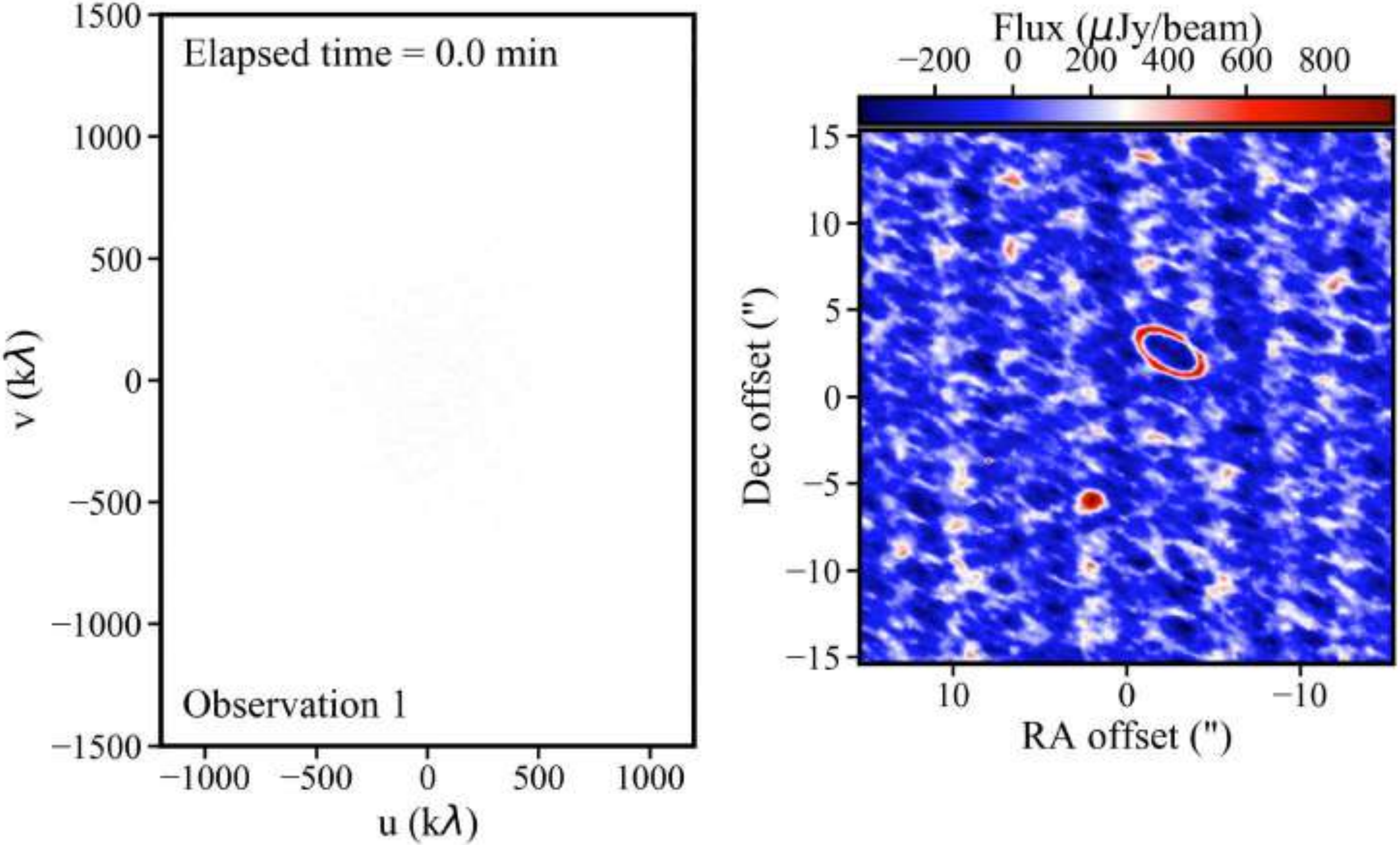


T(l,m)

(u,v) Plane Sampling: more antennas

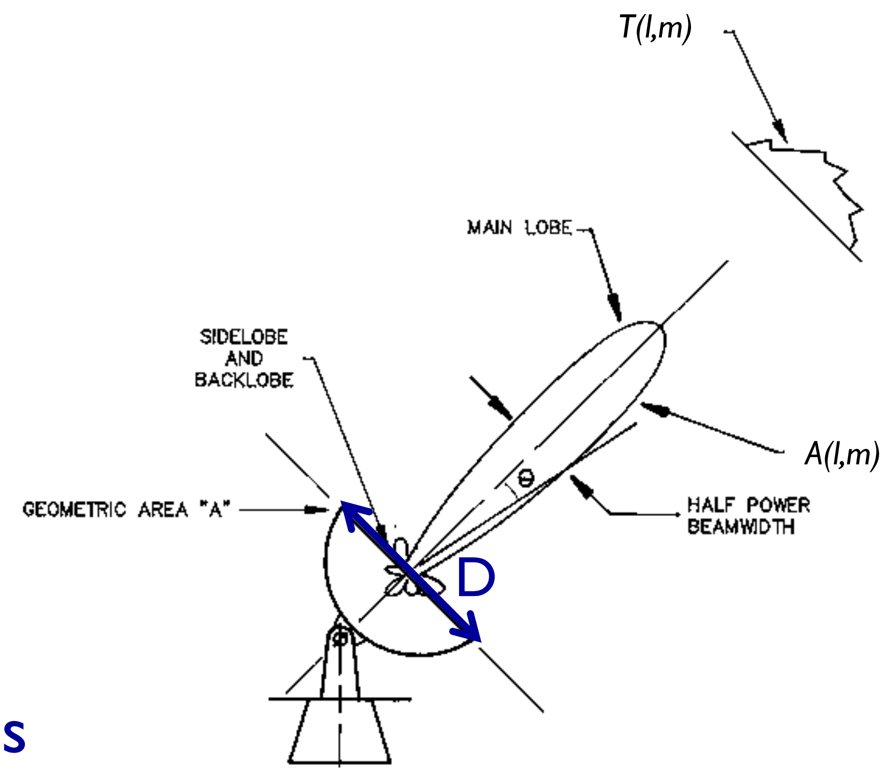


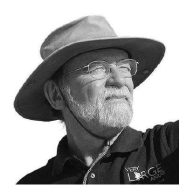
(u,v) Plane Sampling: Earth rotation

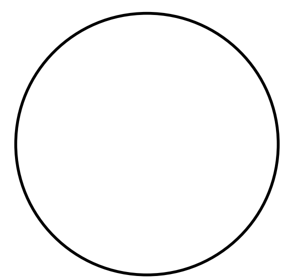


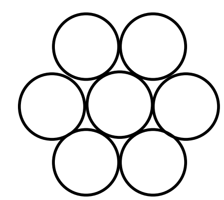
Field of View

- antenna response *A(l,m)* is not uniform across the entire sky
  - "primary beam" fwhm ~ λ/D
  - response beyond primary beam can be important ("sidelobes")
- antenna response *A(l,m)* modifies the sky brightness distribution
  - $T(l,m) \rightarrow T(l,m)A(l,m)$
  - can correct with division by *A(l,m)* in the image plane
  - large source extents require multiple pointings of antennas = mosaicking [Brian Mason, Friday]




T(l,m)


ALMA 12 m
115 GHz


ALMA 12 m
460 GHz

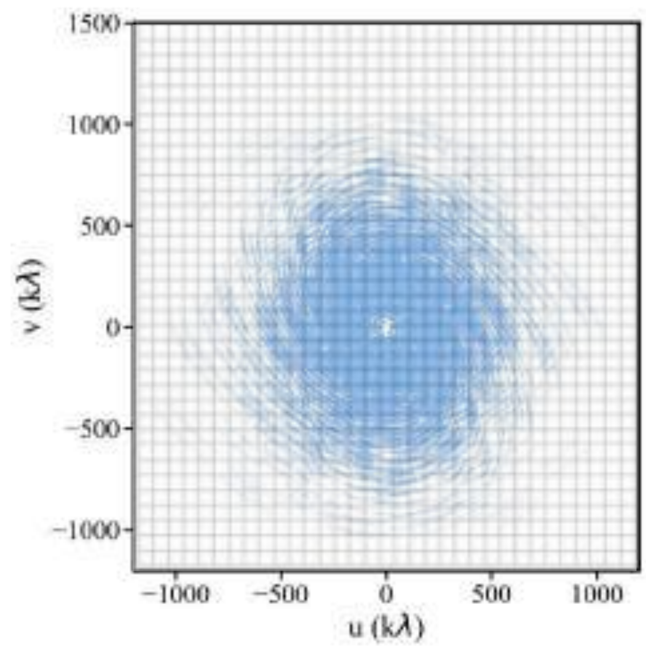
FFTs and Gridding

- "Fourier transform"
  - Fast Fourier Transform (FFT) algorithm is much faster than simple Fourier summation, $O(n\log n)$
  - FFT requires data on a regularly spaced grid
  - aperture synthesis does not provide *V(u,v)* on a regularly spaced grid, so…
- "gridding" used to resample *V(u,v)* for FFT
  - customary to use a convolution method
  - *(u,v)* cell ≈ 0.5D, where D = antenna diameter
  - special ("spheroidal") functions

    that minimize smoothing and aliasing

$$V^G(u,v) = V(u,v)S(u,v) * G(u,v)$$
$$\xrightarrow{F} T^D(l,m)g(l,m)$$

CASA `tclean` "gridder"



Pixel and Image Size

- pixel size: satisfy sampling theorem for longest baselines

$$\Delta l < \frac{1}{2u_{max}} \qquad \Delta m < \frac{1}{2v_{max}}$$

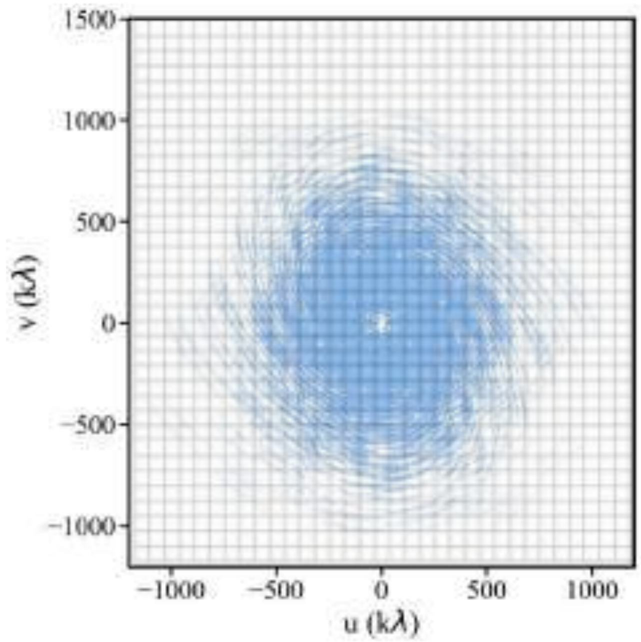
  - in practice, 3 to 5 pixels across dirty beam main lobe to aid deconvolution
  - e.g. ALMA at 1.3 mm, baselines to 1 km $\rightarrow$ pixel size < 0.13 arcsec
  - CASA `tclean` “cell”

- image size: natural choice often full primary beam *A(l,m)*
  - e.g. ALMA at 1.3 mm, 12 meter antennas $\rightarrow$ image size 2 x 27 arcsec
  - if there are bright sources in *A(l,m)* sidelobes, then the FFT will alias them into the image $\rightarrow$ make a larger image (or image outlier fields)
  - CASA `tclean` “imsize”

Visibility Weighting Schemes

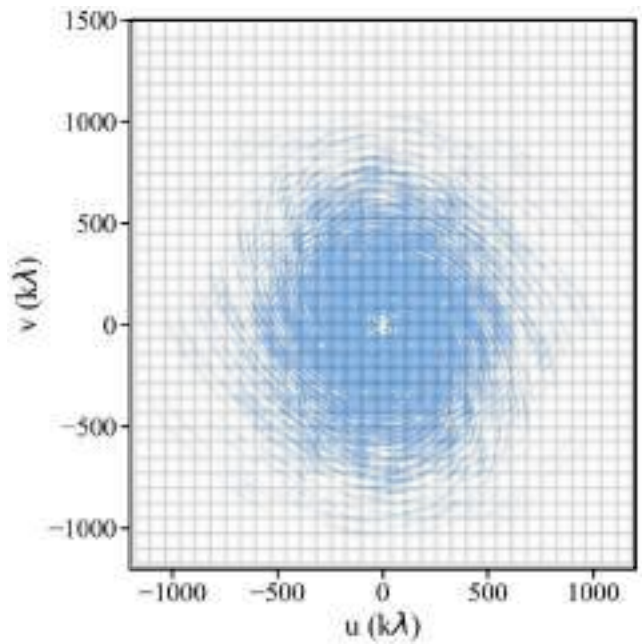
- introduce weighting function *W(u,v)*
- modifies sampling function
- $S(u,v) \rightarrow S(u,v)W(u,v)$
- changes *s(l,m)*, the dirty beam
- *W(u,v)* is gridded for FFT, too

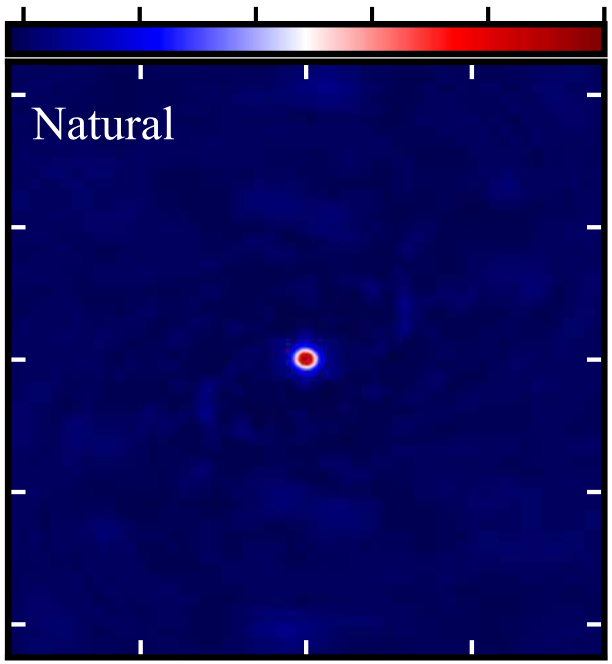
CASA `tclean` "weighting"



Natural Weighting

- $W(u,v) = 1/\sigma^2$ in occupied cells, where σ^2 is the noise variance
- generally gives more weight to short baselines, so the angular resolution is degraded
- maximizes point source sensitivity
- lowest rms in image

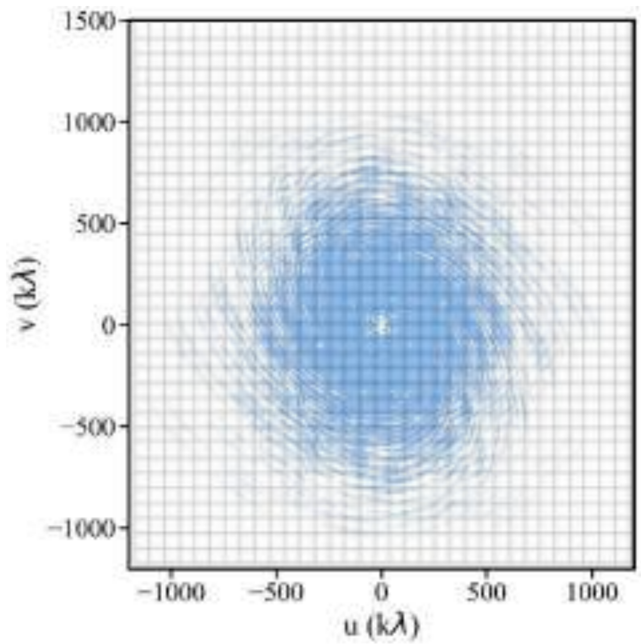


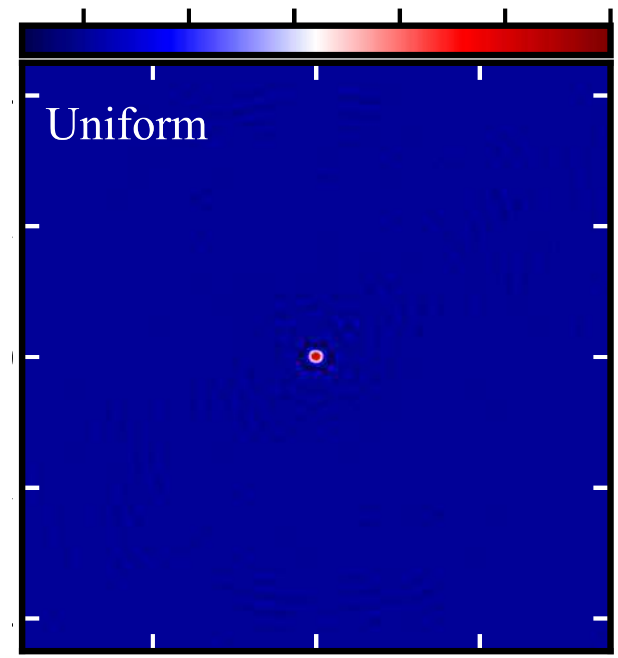


Uniform Weighting

- *W(u,v)* inversely proportional to local density of *(u,v)* samples
- weight for occupied cell = const

- fills *(u,v)* plane more uniformly so dirty beam sidelobes are lower
- gives more weight to long baselines, so angular resolution is enhanced
- downweights some data, so point source sensitivity is degraded
- n.b. can be trouble with sparse *(u,v)* coverage: cells with few samples have same weight as cells with many





Robust (Briggs) Weighting

- variant of uniform weighting that avoids giving too much weight to cells with low natural weight
- software implementations differ
- e.g. $$W(u,v) = \frac{1}{\sqrt{1 + S_N^2 / S_{thresh}^2}}$$

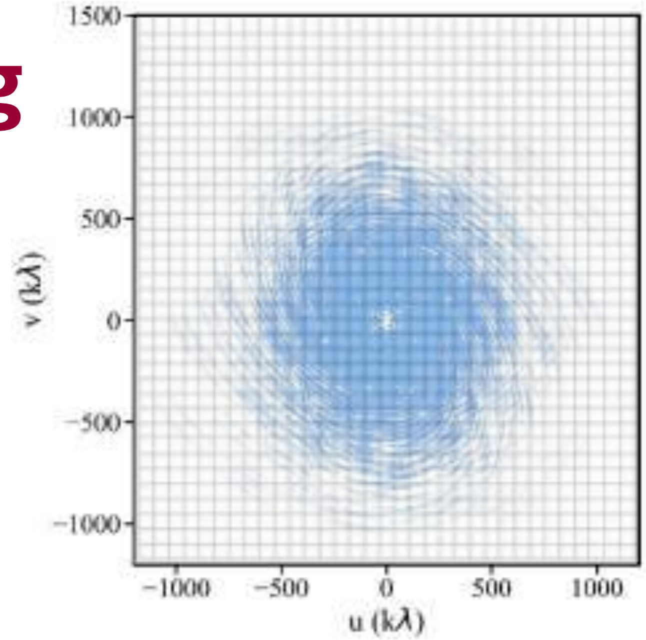
S_N is cell natural weight

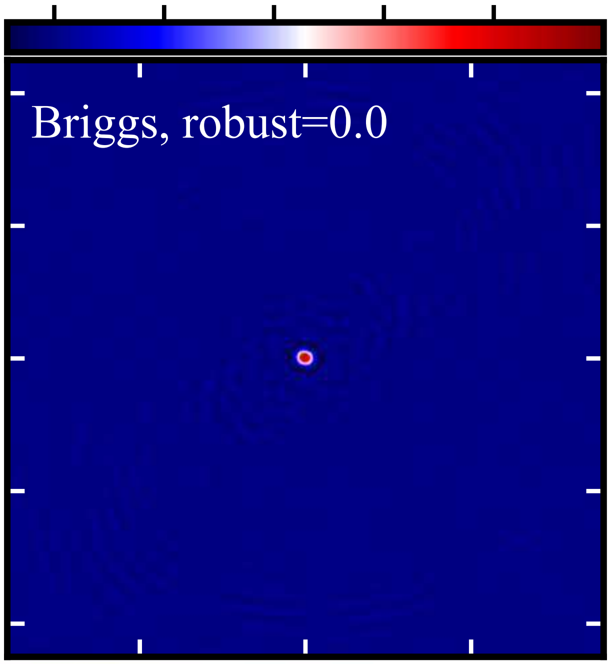
S_{thresh} is a threshold

high threshold $\rightarrow$ natural weight

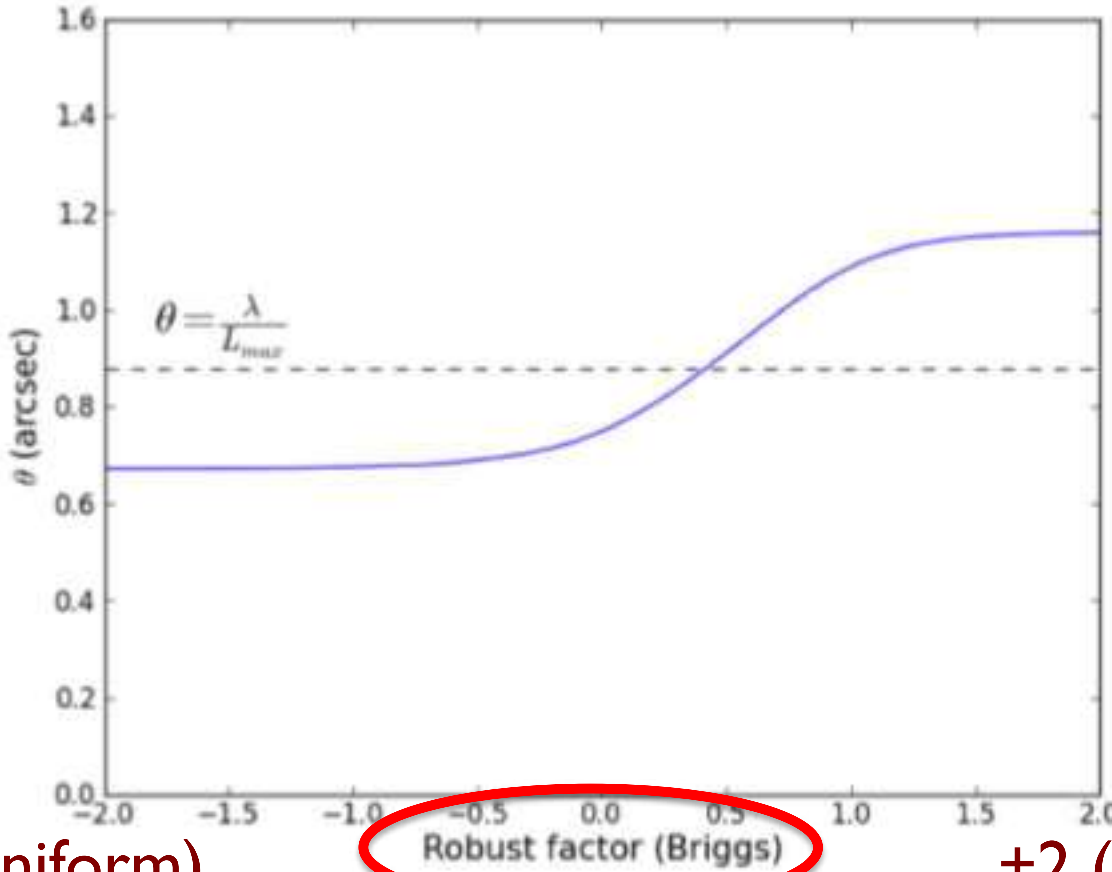
low threshold $\rightarrow$ uniform weight

an adjustable parameter that allows for continuous variation between maximum point source sensitivity and resolution





ALMA C43-4 Configuration Resolution



-2 (uniform)

+2 (natural)

CASA
`tclean`

Figure 7.6: Angular resolution achieved using different values of the CASA *robust* parameter for a 1-hour observation at 100 GHz and a declination of -23 deg in the C43-4 configuration. Note that *robust* $= -2$ is close to uniform weighting and *robust* $= 2$ is close to natural weighting. The dotted line corresponds to $k = 1$.

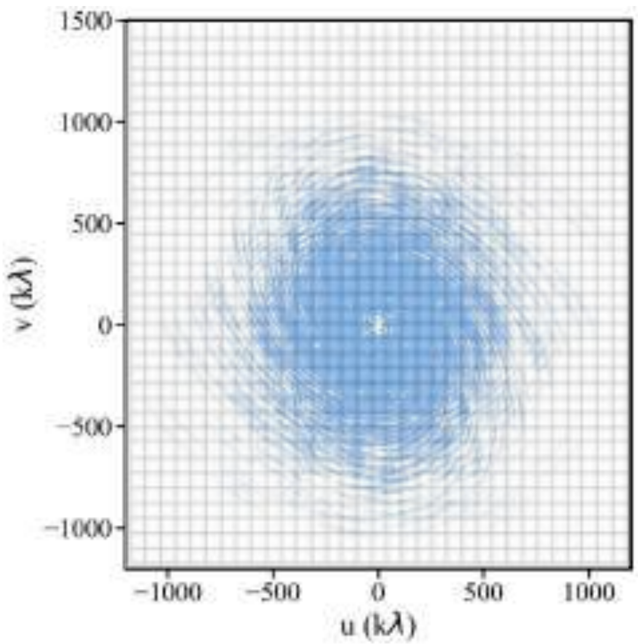
ALMA Cycle 6 Technical Handbook

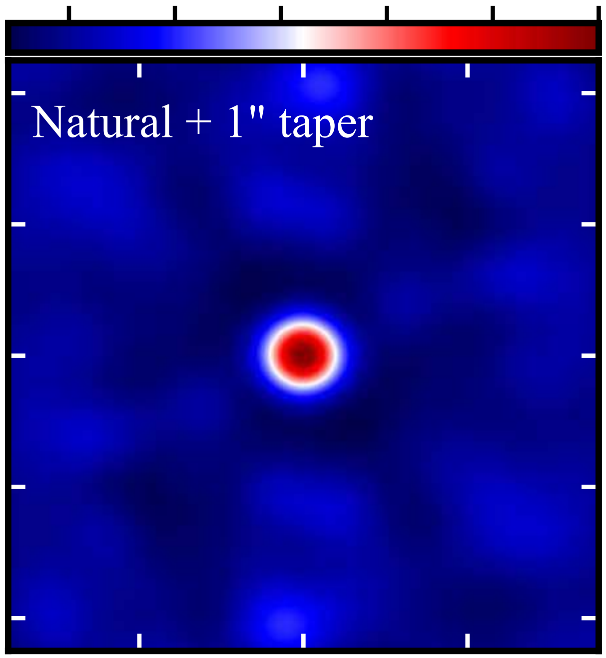
Tapering

- apodize *(u,v)* sampling by a Gaussian

$$W(u,v) = \exp\left(-\frac{(u^2+v^2)}{t^2}\right)$$

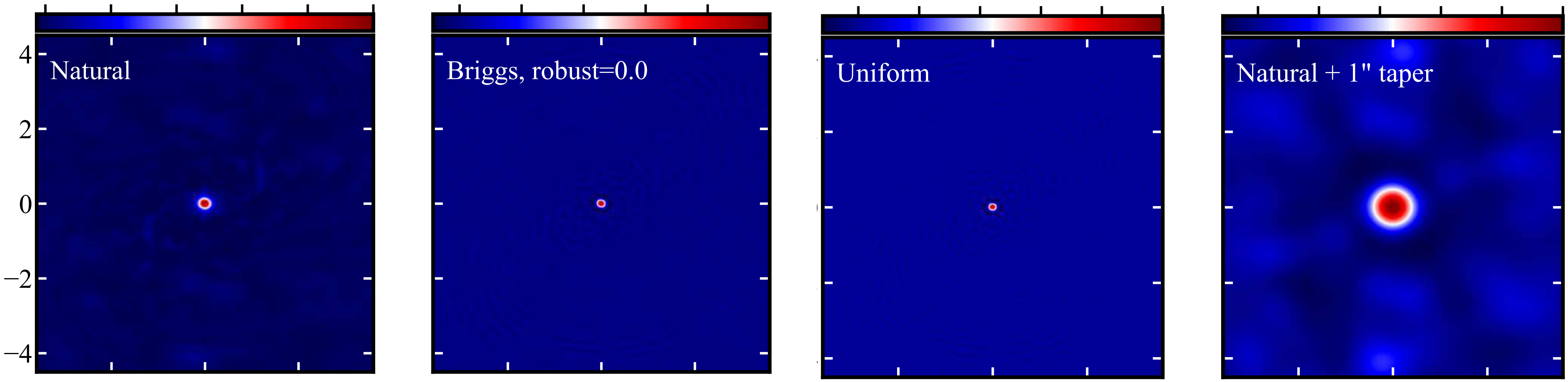
  t = adjustable tapering parameter
- like convolving image by a Gaussian
- downweights data at long baselines, so point source sensitivity degraded and angular resolution degraded
- may improve sensitivity to extended structure sampled by short baselines
- limits to usefulness





Weighting Schemes and Noise

- natural = equal weight for all visibilities [lowest noise]
- uniform = equal weight for filled *(u,v)* cells [highest noise]
- robust/Briggs = continuous variation between natural and uniform
- taper = decrease resolution, increase surface brightness sensitivity



natural (rms 10 μJy/beam) | robust=0 (rms 16 μJy/beam) | uniform (rms 28 μJy/beam) | natural + 1” taper (rms 23 μJy/beam)

Summary of Visibility Weighting Schemes

- imaging parameters provide a lot of freedom
- appropriate choices depend on science goals, e.g.
  - point source detection: natural weight
  - fine detail of strong source: uniform weight
  - complicated emission distribution: robust 0 to 1
  - weak and extended source: taper

	Robust/Uniform	Natural	Taper
resolution	higher	medium	lower
sidelobes	lower	higher	depends
point source sensitivity	lower	maximum	lower
extended source sensitivity	lower	medium	higher

Beyond the Dirty Image

- to keep you awake at night…
- $\exists$ an infinite number of $T(l,m)$ compatible with sampled $V(u,v)$, with "invisible" distributions $R(l,m)$ where $s(l,m) * R(l,m) = 0$
  - no data beyond u_{max}, v_{max} $\rightarrow$ unresolved structure
  - no data within u_{min}, v_{min} $\rightarrow$ limit on largest size scale
  - holes in between $\rightarrow$ synthesized beam sidelobes
- also noise $\rightarrow$ undetected/corrupted structure in $T(l,m)$
- no unique prescription to extract optimum estimate of $T(l,m)$

Deconvolution Philosophy

- use non-linear techniques to interpolate/extrapolate samples of *V(u,v)* into unsampled regions of the *(u,v)* plane

  (remove sidelobes of the dirty beam from the image)

- aim to find a sensible model of *T(l,m)* compatible with data
- requires *a priori* assumptions about *T(l.m)* to pick plausible "invisible" distributions to fill unsampled parts of *(u,v)* plane
- main assumption: real sky does not look like typical dirty beam
- "clean" deconvolution algorithm (and its variants) by far dominant in radio astronomy, though there are others in use
- a very active research area, e.g. compressed sensing

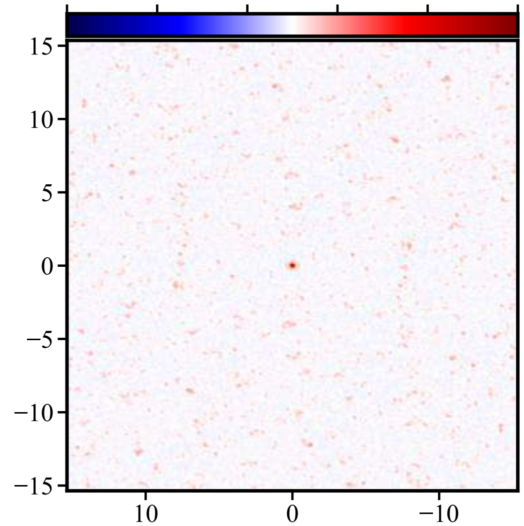
Classic Högbom (1974) clean Algorithm

- *a priori* assumption: *T(l,m)* is a collection of point sources

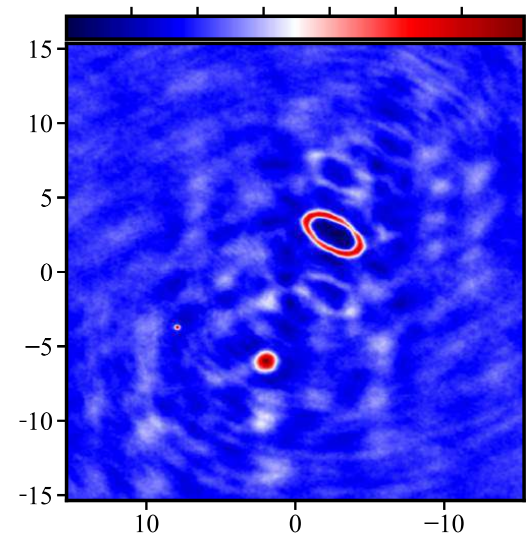
initialize a *clean component* list

initialize a *residual image* = dirty image

1. identify the highest peak in the *residual image* as a point source
2. subtract a scaled dirty beam *s(l,m)* x "loop gain" from this peak
3. add this point source location and amplitude to the *clean component* list
4. goto step 1 (an iteration) unless stopping criterion reached



s(l,m)



$T^D(l,m)$

Classic Högbom (1974) clean Algorithm

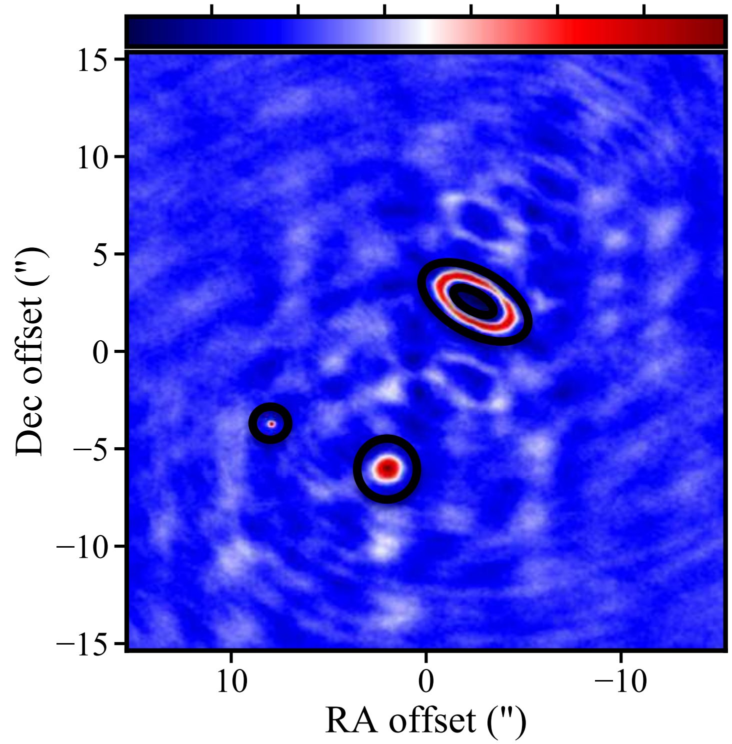
- stopping criterion
  - *residual map* maximum < threshold = multiple of rms , e.g. 2 x rms (if noise limited)
  - *residual map* maximum < threshold = fraction of dirty map maximum (if dynamic range limited)

- loop gain parameter
  - good results for g=0.1 (CASA `tclean` default)
  - lower values can work better for smooth and extended emission

- don't "overclean" to artificially low noise level
  - generally a problem only when *(u,v)* coverage is sparse

Classic Högbom (1974) clean Algorithm

- finite support
  - easy to include *a priori* information about where in the dirty map to search for *clean components*
  - implemented as image masks or clean boxes; CASA `tclean` "mask"
  - very useful, often essential for best results, but potentially dangerous
  - use with care
  - can be an arduous manual process; automasking under development



Classic Högbom (1974) clean Algorithm

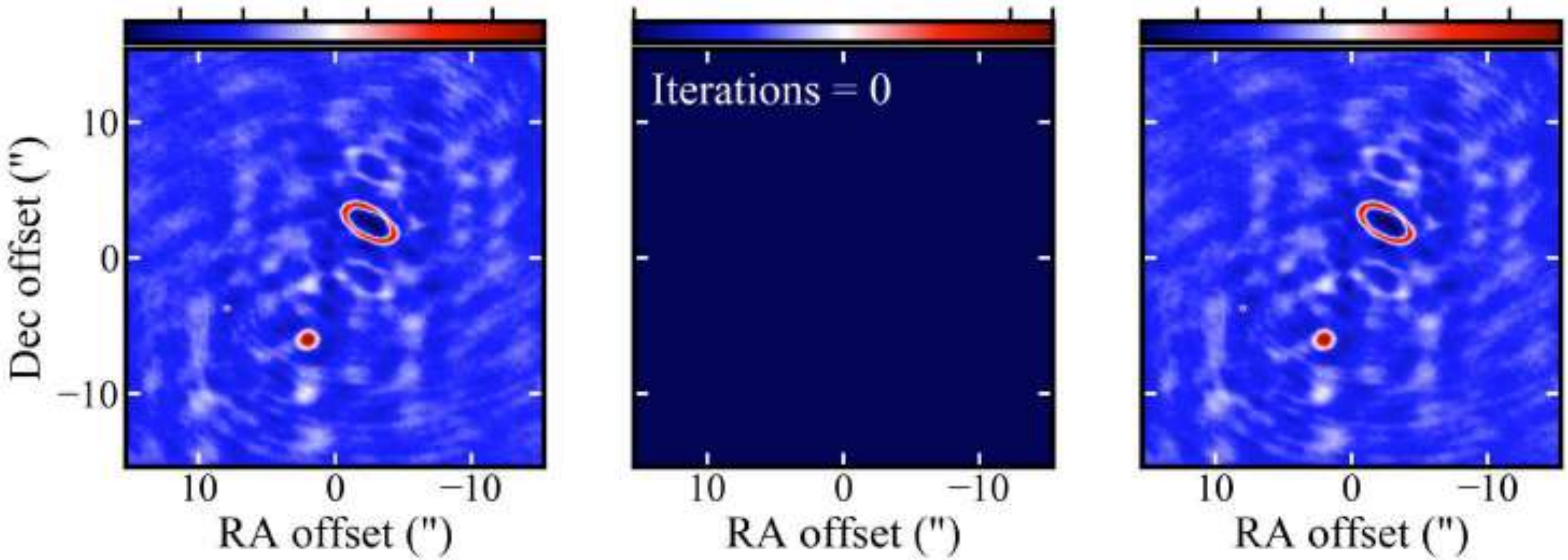
- last step is to create a final "restored" image
  - make a model image with all point source *clean components*
  - convolve point source model image with a "clean beam", an elliptical Gaussian fit to the main lobe of the dirty beam (avoids super-resolution of the point source component model)
  - add back *residual map* with noise and structure below the threshold
- restored image is an estimate of the true sky brightness $T(l,m)$
  - units of the restored image are (mostly) Jy per clean beam area = intensity, or brightness temperature

- Schwarz (1978) showed that clean is equivalent to a least squares fit of sinusoids to visibilities in the case of no noise

clean algorithm example

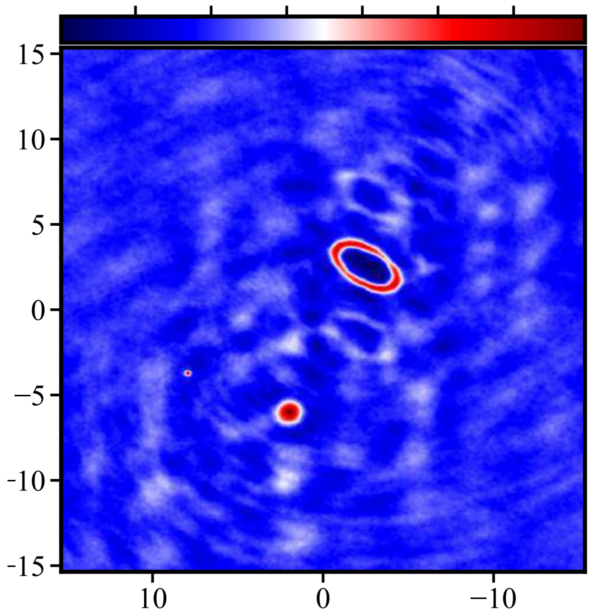
initialize

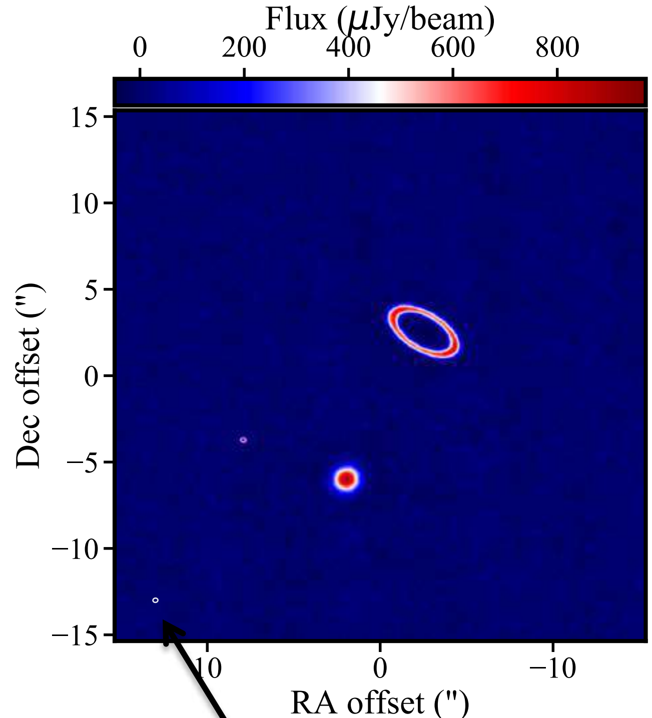
$T^D(l,m)$ *clean components* *residual map*



clean algorithm example: restored image

$T^D(l,m)$





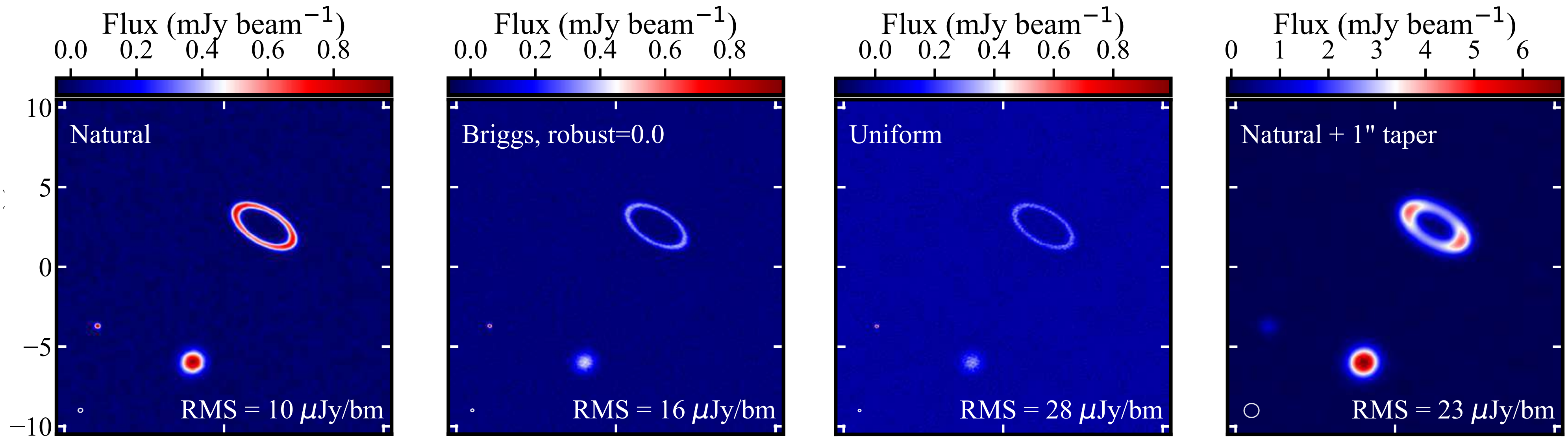
restored image

ellipse = clean beam Gaussian fwhm

final image depends on

imaging parameters (pixel size, visibility weighting scheme, gridding) and deconvolution (algorithm, iterations, masks, stopping criteria)

Results from Different Weighting Schemes



natural	robust=0	uniform	natural + 1” taper
0.29x0.25 p.a. -81	0.19x0.17 p.a. -78	0.17x0.15 p.a. -87	0.93x0.88 p.a. -86

Tune Imaging Parameters to Science

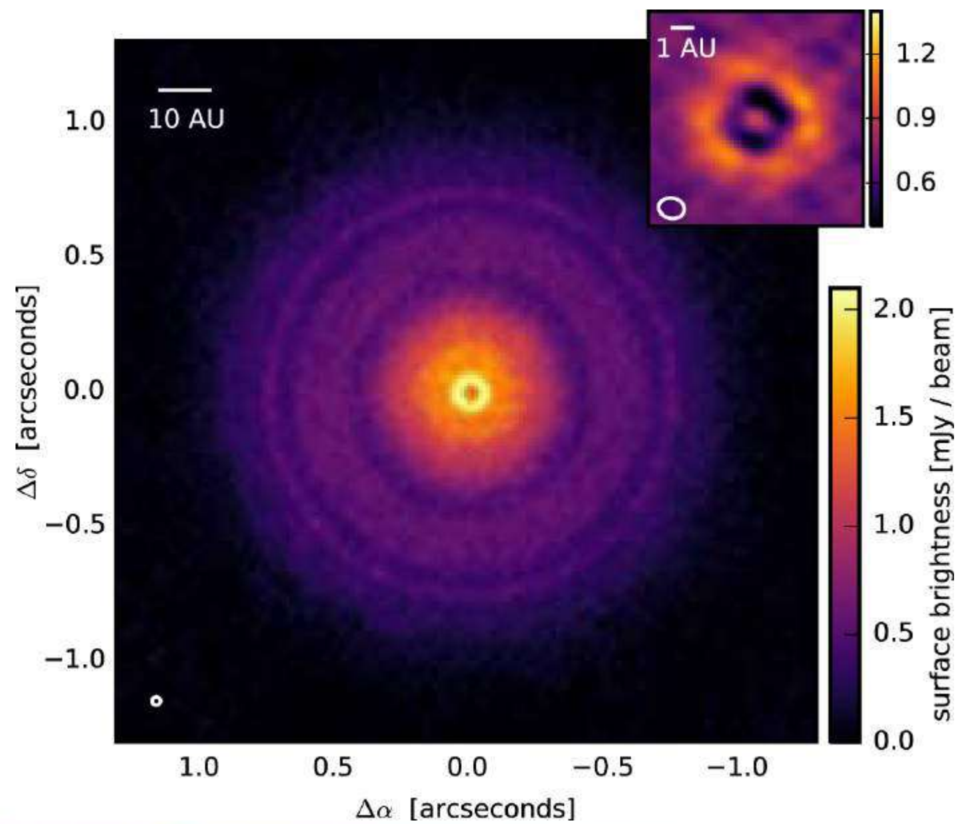
The Astrophysical Journal Letters, 820:L40 (5pp), 2016 April 1 doi:10.3847/2041-8205/820/2/L40

RINGED SUBSTRUCTURE AND A GAP AT 1 au IN THE NEAREST PROTOPLANETARY DISK

Sean M. Andrews[1], David J. Wilner[1], Zhaohuan Zhu[2], Tilman Birnstiel[3], John M. Carpenter[4], Laura M. Pérez[5], Xue-Ning Bai[1], Karin I. Öberg[1], A. Meredith Hughes[6], Andrea Isella[7], and Luca Ricci[1]

presents two images of 340 GHz emission from TW Hya protoplanetary disk from the same ALMA visibilities

- robust=0.5 + taper, for a circular 30 mas beam, to show the large scale structure of the disk
- inset: robust=0 for higher resolution, 0.24 x 0.18 mas, to highlight the gap at 1 AU radius



Variants on Basic clean Algorithm

- "Clark" clean
  - minor cycle

    Högbom clean with smaller beam patch, improves speed
  - major cycle

    clean components removed from *gridded* visibilities at once by FFT

- "Cotton-Schwab" clean
  - minor cycle

    Högbom clean with smaller beam patch, improves speed
  - major cycle

    clean components removed from *original* visibilities by FFT, then entire imaging process repeated to create residual image

see CASA `tclean` "deconvolver"

Scale Sensitive Deconvolution

- basic clean is scale-free, treats each pixel as independent
- adjacent pixels in an image may not be independent
  - resolution limit
  - intrinsic source size: an extended source covering 1000 pixels might be better characterized by a few parameters than by 1000 parameters, e.g. 6 parameters for a Gaussian distribution

- scale sensitive deconvolution algorithms employ fewer degrees of freedom to model plausible sky brightness distributions

CASA `tclean` "deconvolver=multiscale" and "scales"

– user must input appropriate scales

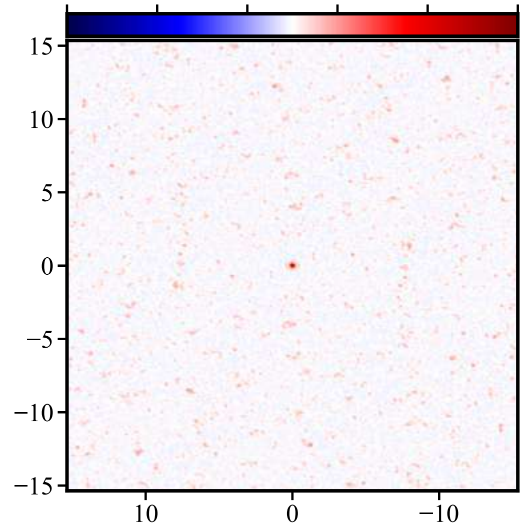
– typically a few (delta function, synthesized beam size, few times that)

CASA `tclean` filename extensions

- <*imagename*>.image
  - restored image
- <*imagename*>.psf
  - point spread function (= dirty beam)
- <*imagename*>.model
  - model image after deconvolution, e.g. clean components
- <*imagename*>.residual
  - residual image, e.g. after subtracting clean components
- <*imagename*>.mask
  - deconvolution mask
- <*imagename*>.pb
  - primary beam model
- <*imagename*>.sumwt
  - a single value sum of visibility weights [for natural weight, rms=(sumwt)$^{-0.5}$]

Maximum Entropy Algorithm

- *a priori* assumption: *T(l,m)* is smooth and positive



s(l,m)

maximize a measure of smoothness (entropy)*

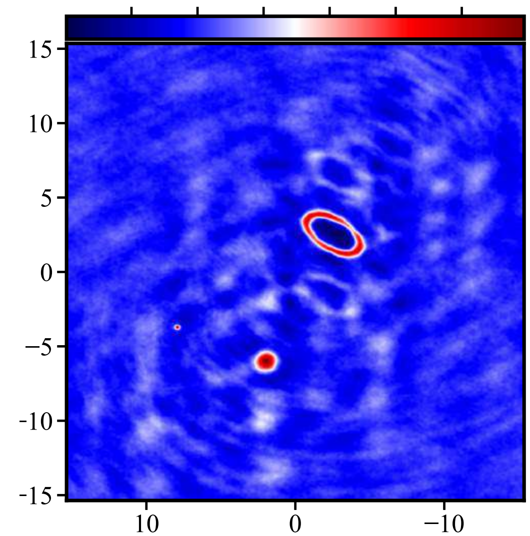
$$H = -\sum_k T_k \log\left(\frac{T_k}{M_k}\right)$$

subject to the constraints

$$\chi^2 = \sum_k \frac{|V(u_k,v_k) - \mathrm{FT}\{T\}|^2}{\sigma_k^2}$$

$$F = \sum_k T_k$$

where M is the "default image"



T^D(l,m)

fast (NlogN) solver, Cornwell & Evans (1983)

optional: convolve with Gaussian beam and add residual image to make final image

*vast literature about deep meaning of entropy as information content

Maximum Entropy Algorithm

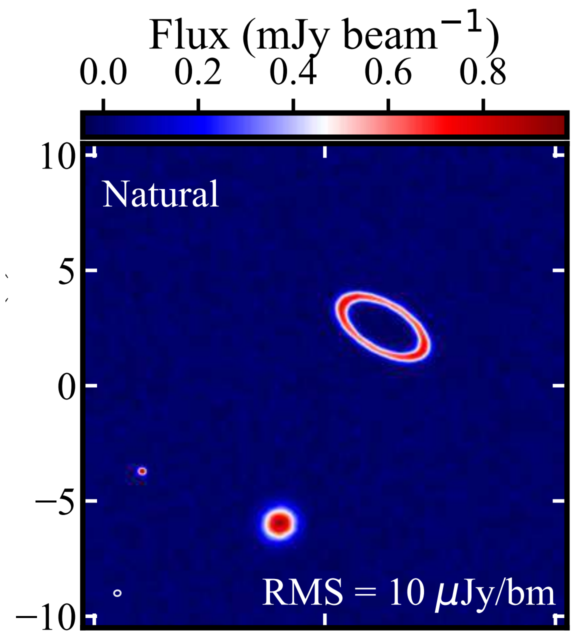
- easy to include *a priori* information with the default image
  - flat default is usual assumption if nothing known
  - a single dish image may be a good default
- straightforward to generalize χ^2 to combine different observations/telescopes to obtain optimal image
- many measures of entropy available
  - e.g. replace log with cosh $\rightarrow$ "emptiness" (does not enforce positivity)

- less robust and harder to drive than clean
- works best on smooth, extended emission
- can have trouble with point source sidelobes (could remove the point sources first with clean)

Measures of Image Quality

- *dynamic range*
  - ratio of peak brightness in image to rms noise in a region void of emission
  - easy way to calculate a lower limit to the error in brightness in a non-empty region
  - e.g. peak = 0.9 mJy/beam, rms = 10 μJy/beam

    $\rightarrow$ dynamic range = 90



- *fidelity*
  - difference between any produced image and the correct image
  - fidelity image = input model / difference

    = model * beam / abs(model * beam – reconstruction)

    = inverse of the relative error
  - need knowledge of the correct image to calculate
  - fidelity often much worse than dynamic range

“Invisible” Large Scale Structure

- important structure missed in central hole of *(u,v)* plane
- to estimate if lack of short baselines will be problematic
  - simulate the observations with a source model
  - check simple expressions for a Gaussian source or uniform disk

Homework Problem

- Q: By what factor is the central brightness reduced as a function of source size due to missing short spacings for a Gaussian characterized by fwhm $\theta_{1/2}$
- A: a Gaussian source central brightness is reduced 50% when

$$\theta_{1/2} = 18'' \left(\frac{\nu}{100\ GHz}\right)^{-1} \left(\frac{B_{min}}{15\ meters}\right)^{-1}$$

where B_{min} is the shortest baseline [meters], ν is the frequency [GHz] (derivation in appendix of Wilner & Welch 1994, ApJ, 427, 898)

ALMA "Maximum Recoverable Scale"

- adopted to be 10% of the total flux density of a uniform disk (not much!)

	Band	3	4	5	6	7	8	9	10
	Frequency (GHz)	100	150	185	230	345	460	650	870
Configuration									
7-m	θ_{res} (arcsec)	12.5	8.35	6.77	5.45	3.63	2.72	1.93	1.44
	θ_{MRS} (arcsec)	66.7	44.5	36.1	29.0	19.3	14.5	10.3	7.67
C43-1	θ_{res} (arcsec)	3.38	2.25	1.83	1.47	0.98	0.735	0.52	0.389
	θ_{MRS} (arcsec)	28.5	19.0	15.4	12.4	8.25	6.19	4.38	3.27
C43-2	θ_{res} (arcsec)	2.3	1.53	1.24	0.999	0.666	0.499	0.353	0.264
	θ_{MRS} (arcsec)	22.6	15.0	12.2	9.81	6.54	4.9	3.47	2.59
C43-3	θ_{res} (arcsec)	1.42	0.943	0.765	0.615	0.41	0.308	0.218	0.163
	θ_{MRS} (arcsec)	16.2	10.8	8.73	7.02	4.68	3.51	2.48	1.86
C43-4	θ_{res} (arcsec)	0.918	0.612	0.496	0.399	0.266	0.2	0.141	0.106
	θ_{MRS} (arcsec)	11.2	7.5	6.08	4.89	3.26	2.44	1.73	1.29
C43-5	θ_{res} (arcsec)	0.545	0.363	0.295	0.237	0.158	0.118	0.0838	0.0626
	θ_{MRS} (arcsec)	6.7	4.47	3.62	2.91	1.94	1.46	1.03	0.77
C43-6	θ_{res} (arcsec)	0.306	0.204	0.165	0.133	0.0887	0.0665	0.0471	0.0352
	θ_{MRS} (arcsec)	4.11	2.74	2.22	1.78	1.19	0.892	0.632	0.472
C43-7	θ_{res} (arcsec)	0.211	0.141	0.114	0.0917	0.0612	0.0459	0.0325	0.0243
	θ_{MRS} (arcsec)	2.58	1.72	1.4	1.12	0.749	0.562	0.398	0.297
C43-8	θ_{res} (arcsec)	0.096	0.064	0.0519	0.0417	0.0278	-	-	-
	θ_{MRS} (arcsec)	1.42	0.947	0.768	0.618	0.412	-	-	-
C43-9	θ_{res} (arcsec)	0.057	0.038	0.0308	0.0248	-	-	-	-
	θ_{MRS} (arcsec)	0.814	0.543	0.44	0.354	-	-	-	-
C43-10	θ_{res} (arcsec)	0.042	0.028	0.0227	0.0183	-	-	-	-
	θ_{MRS} (arcsec)	0.496	0.331	0.268	0.216	-	-	-	-

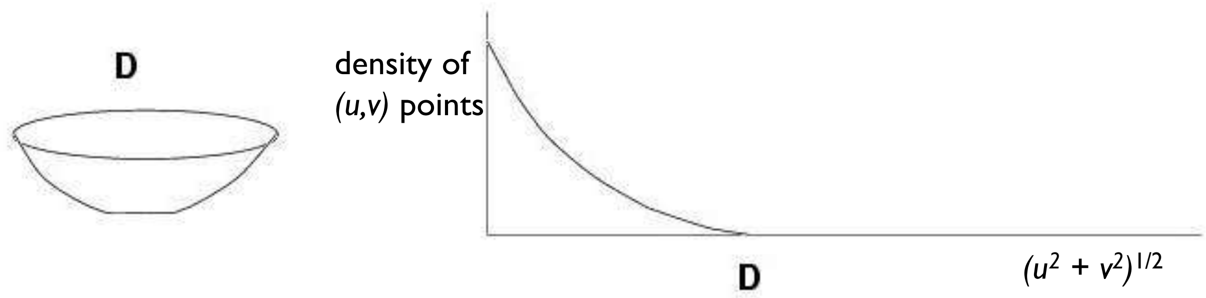
Table 7.1: Resolution (θ_{res}) and maximum recoverable scale (θ_{MRS}) for the 7-m Array and 12-m Array configurations available during Cycle 6 as a function of a representative frequency in a band. The value of θ_{MRS} is computed using the 5^{th} percentile baseline (L05) from Table 7.2 and Equation 7.7. The value of θ_{res} is the mean size of the interferometric beam obtained through simulation with CASA, using Briggs (u, v) plane weighting with *robust=0.5*. The computations were done for a source at zenith; for sources transiting at lower elevations, the North-South angular measures will increase proportional to 1/sin(ELEVATION).

ALMA Cycle 6 Technical Handbook

NRAO

Techniques to Obtain Short Spacings (1)

use a large single dish telescope



- *all* Fourier components from 0 to D sampled, where D is dish diameter (weighting depends on illumination)
- scan single dish across sky to make an image $T(l,m) * A(l,m)$
  where $A(l,m)$ is the single dish response pattern
- Fourier transform single dish image, $T(l,m) * A(l,m)$, to get $V(u,v)a(u,v)$
  and then divide by $a(u,v)$ to estimate $V(u,v)$ for baselines < D
- choose D large enough to overlap interferometer samples of $V(u,v)$
  and avoid using data where $a(u,v)$ becomes small
- example: VLA and GBT

Techniques to Obtain Short Spacings (II)

use a separate array of smaller antennas

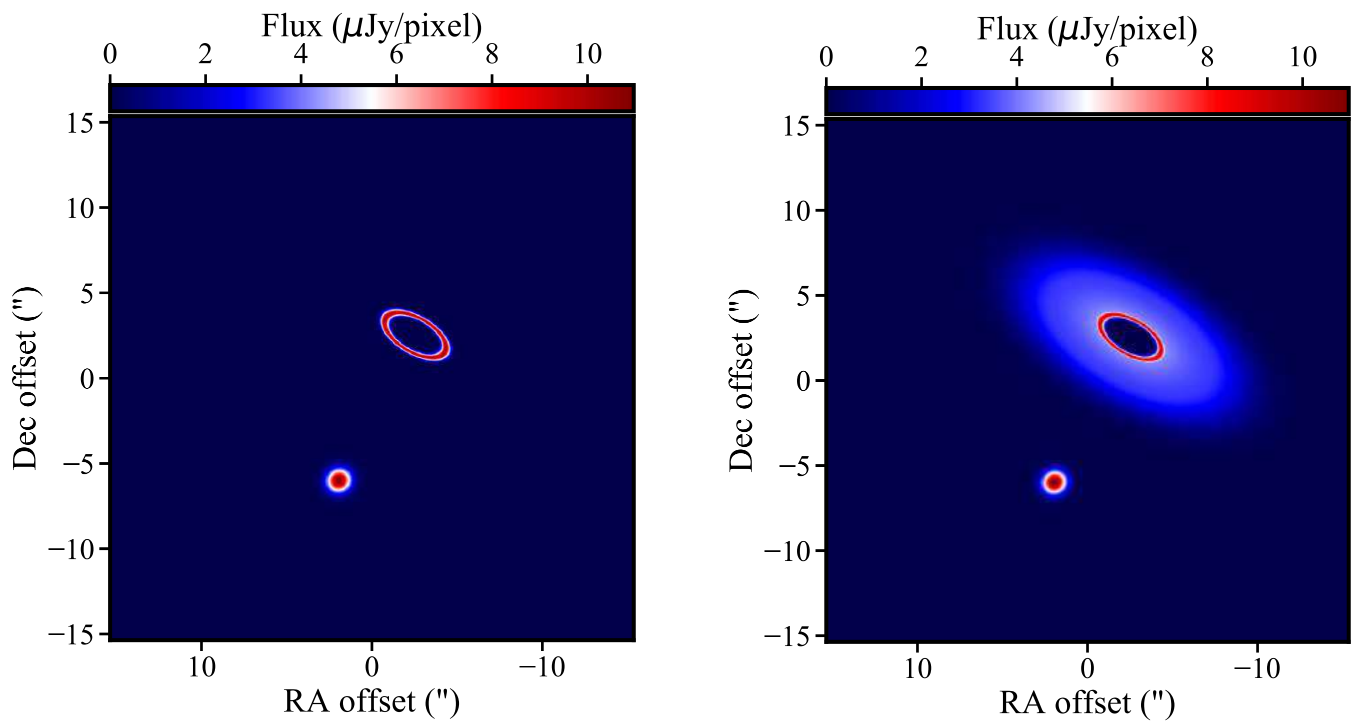
- small antennas can observe short baselines inaccessible to larger ones
- the larger antennas can be used as single dish telescopes to make images with Fourier components not accessible to the smaller antennas
- example: ALMA main array + ACA

main array
50x12m: 12m to 16km

ACA
12 x 7m: covers 7 to12m+
4 x 12m single dishes: 0 to 7m

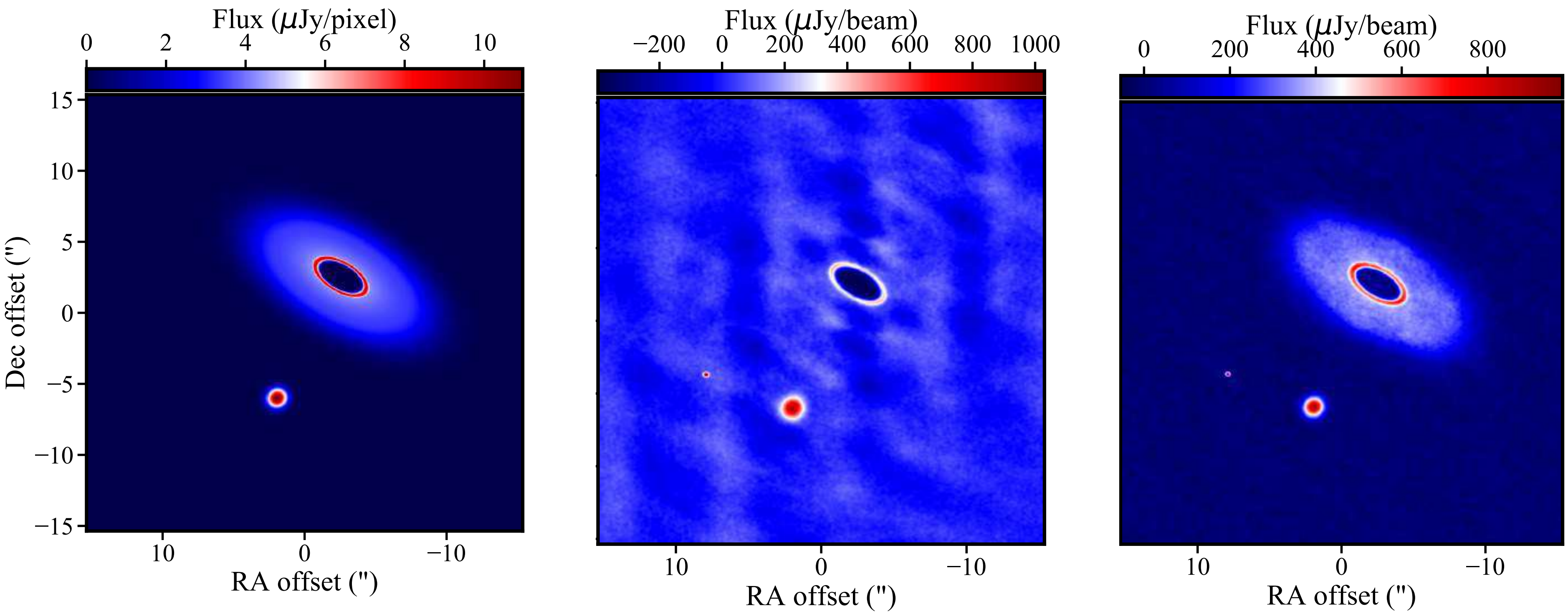
Missing Short Baselines: Demonstration

- Do the visibilities observed in our example discriminate between these two models of sky brightness *T(l,m)*?



Missing Short Baselines: Demonstration

T(l,m) *ALMA 12m array* *ALMA 12m array + ACA*



n.b. clean does not reach theoretical rms due to poorly sampled extended structure

much improved

Example: Missing Short Baselines

The Astrophysical Journal, 855:56 (10pp), 2018 March 1
https://doi.org/10.3847/1538-4357/aaacd7

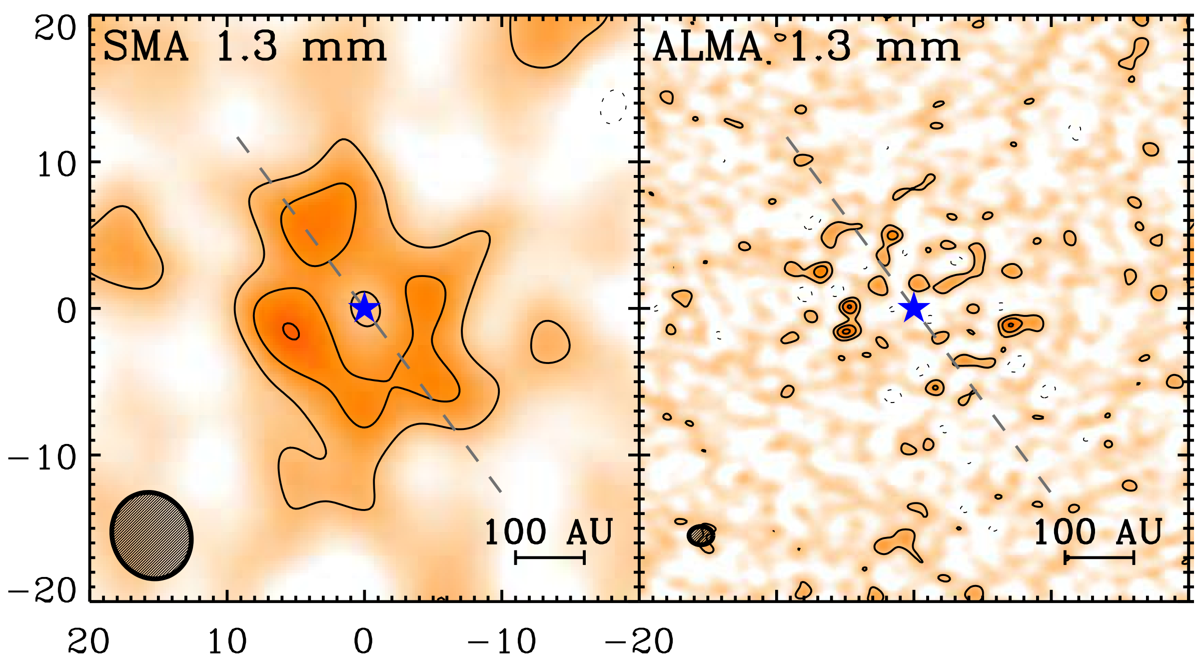
CrossMark

Resolved Millimeter Observations of the HR 8799 Debris Disk

David J. Wilner[1], Meredith A. MacGregor[1,2,6], Sean M. Andrews[1], A. Meredith Hughes[3], Brenda Matthews[4], and Kate Su[5]

SMA 8x6m

Booth et al. 2016
ALMA 38x12m



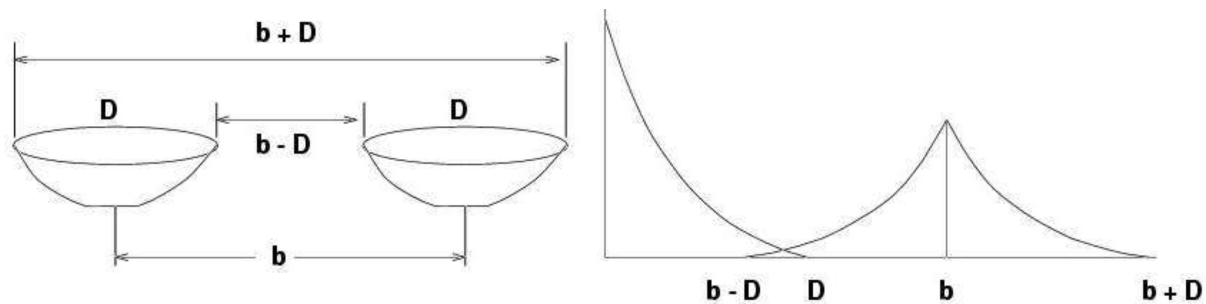
beam 6.1x5.6 arcsec
rms 180 μJy/beam

beam 1.7x1.2 arcsec
rms 16 μJy/beam

Techniques to Obtain Short Spacings (III)

mosaic with a homogeneous array

- recover a range of spatial frequencies around the nominal baseline b using knowledge of A(l,m), shortest spacings from single dishes (Ekers & Rots 1979)



- *V(u,v)* is a linear combination of baselines from b-D to b+D
- depends on pointing direction (l_0,m_0) as well as on (u,v)

$$V(u,v;l_0,m_0) = \int\int T(l,m)A(l-l_0,m-m_0)e^{i2\pi(ul+vm)}dldm$$

- Fourier transform with respect to pointing direction (l_0,m_0)

$$V(u-u_0,v-v_0) = \left(\int\int V(u,v;l_0,m_0)e^{i2\pi(u_0l_0+v_0m_0)}dl_0dm_0\right)/a(u_0,v_0)$$

Self Calibration

- *a priori* calibration from external calibrators must be interpolated from different time and sky direction from source, which leaves errors
- self calibration corrects for antenna based phase and amplitude errors *together with imaging* to create an improved source model
- why should this work?
  - at each time, measure N complex gains and N(N-1)/2 visibilities
  - source structure can be represented by small number of parameters
  - a highly overconstrained problem if N large and source simple
- in practice, an iterative, non-linear relaxation process
  - assume source model $\rightarrow$ solve for time dependent gains $\rightarrow$ form new source model from corrected data using clean $\rightarrow$ solve for new gains
  - requires sufficient signal-to-noise at each solution interval

- loses absolute phase from calibrators and therefore position information
- dangerous with small N arrays, complex sources, low signal-to-noise

Concluding Remarks

- interferometry samples Fourier components of sky brightness
- make an image by Fourier transforming sampled visibilities
- deconvolution attempts to correct for incomplete sampling

- remember
  - there are an infinite number of images compatible with the visibilities
  - missing (or corrrupted) visibilities affect the entire image
  - astronomers must use judgement in imaging and deconvolution
- it's fun and worth the trouble $\rightarrow$ high resolution images!

many, many issues not covered in this talk, see references

END