Calibration

George Moellenbrock, NRAO

Sixteenth Synthesis Imaging Workshop

16-23 May 2018

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References

- ~Theory
  - Interferometry and Synthesis in Radio Astronomy (2^{nd} or 3^{rd} ed. Thompson, Moran, & Swenson)
  - Tools of Radio Astronomy (6^{th} ed., Wilson, Rohlfs, & Huettemeister)
- "Practical Thinking"
  - ***These talks!***
  - Synthesis Imaging in Radio Astronomy II (Editors: Taylor, Carilli, & Perley)
- "Practical Doing"
  - VLA: https://science.nrao.edu/facilities/vla/docs/manuals/obsguide
  - VLBA: https://science.nrao.edu/facilities/vlba/other/intro
  - ALMA: https://almascience.nrao.edu
  - Tutorials!

Calibration I

George Moellenbrock, NRAO

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Synopsis

- Calibration I
  - Why do we have to calibrate?
  - Review Idealistic formalism → Realistic practice
  - Fundamental Calibration Principles
    - Practical Calibration Considerations
    - Baseline-based vs. Antenna-based Calibration
    - Solving for calibration
  - An example Visibility dataset
    - Flagging
- Calibration II
  - Scalar Calibration Example
  - Generalizations & Specializations
    - Full Polarization
    - A Dictionary of Calibration Effects
  - Calibration Heuristics and 'Bootstrapping'
  - New Calibration Challenges
  - Summary

Why Calibration?

- Synthesis radio telescopes, though exquisitely well-designed, are not perfect (e.g., surface accuracy, receiver noise, polarization purity, gain stability, geometric model errors, etc.)
- Need to accommodate deliberate engineering (e.g., frequency down-conversion, analog/digital electronics, filter bandpass, etc.)
- Hardware or control software occasionally fails or behaves unpredictably
- Scheduling/observation errors sometimes occur (e.g., wrong source positions)
- Atmospheric conditions not ideal
- Radio Frequency Interference (RFI)

Determining *instrumental and environmental properties* (calibration)
is a prerequisite to
determining *radio source properties*

From Idealistic to Realistic

- Formally, we wish to use our interferometer to obtain the visibility function:

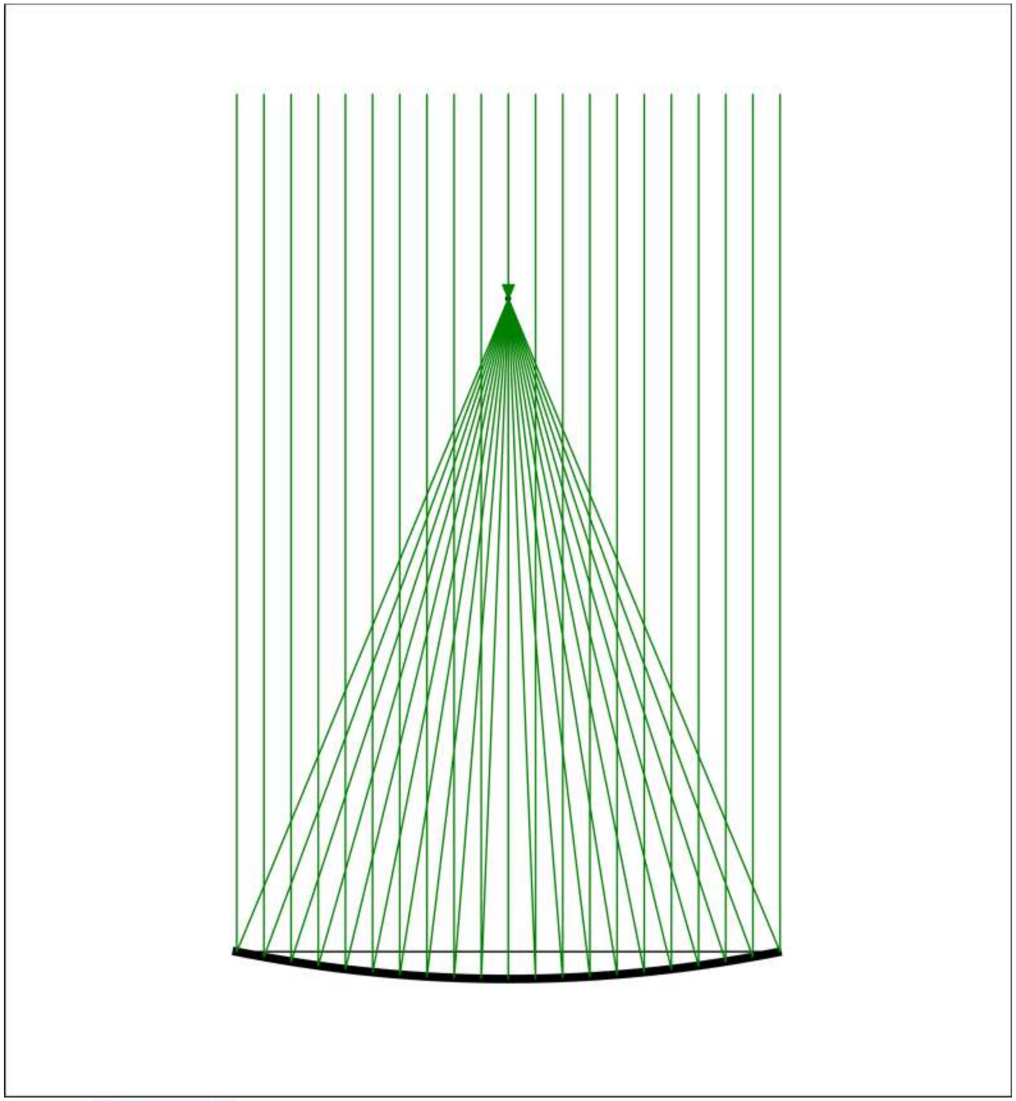
$$V(u,v) = \int_{sky} I(l,m) e^{-i2\pi(ul+vm)} dl dm$$

- ….a Fourier transform which we intend to invert to obtain an image of the sky:

$$I(l,m) = \int_{uv} V(u,v) e^{i2\pi(ul+vm)} du dv$$

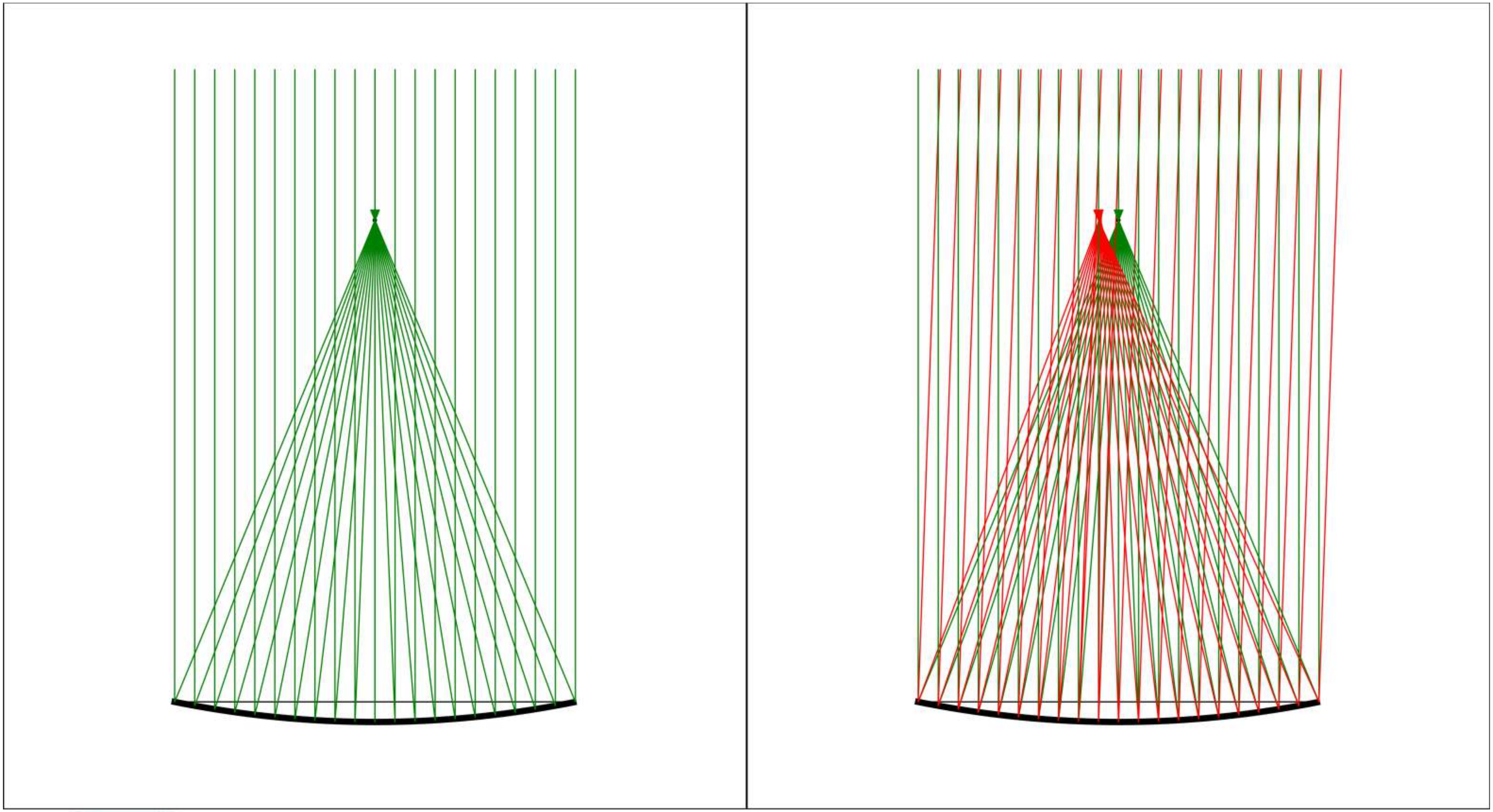
  – *V(u,v)* describes the amplitude and phase of 2D sinusoids that add up to an image of the sky (a direction-dependent average)
    - Amplitude: "~how much & ~how concentrated?"
    - Phase: "~where?"
    - c.f. Young's Double-Slit Interference Experiment (1804)
- To develop an intuitive feel for calibration, let's review: What are the *V(u,v)* and how do we measure them?

A Filled Aperture

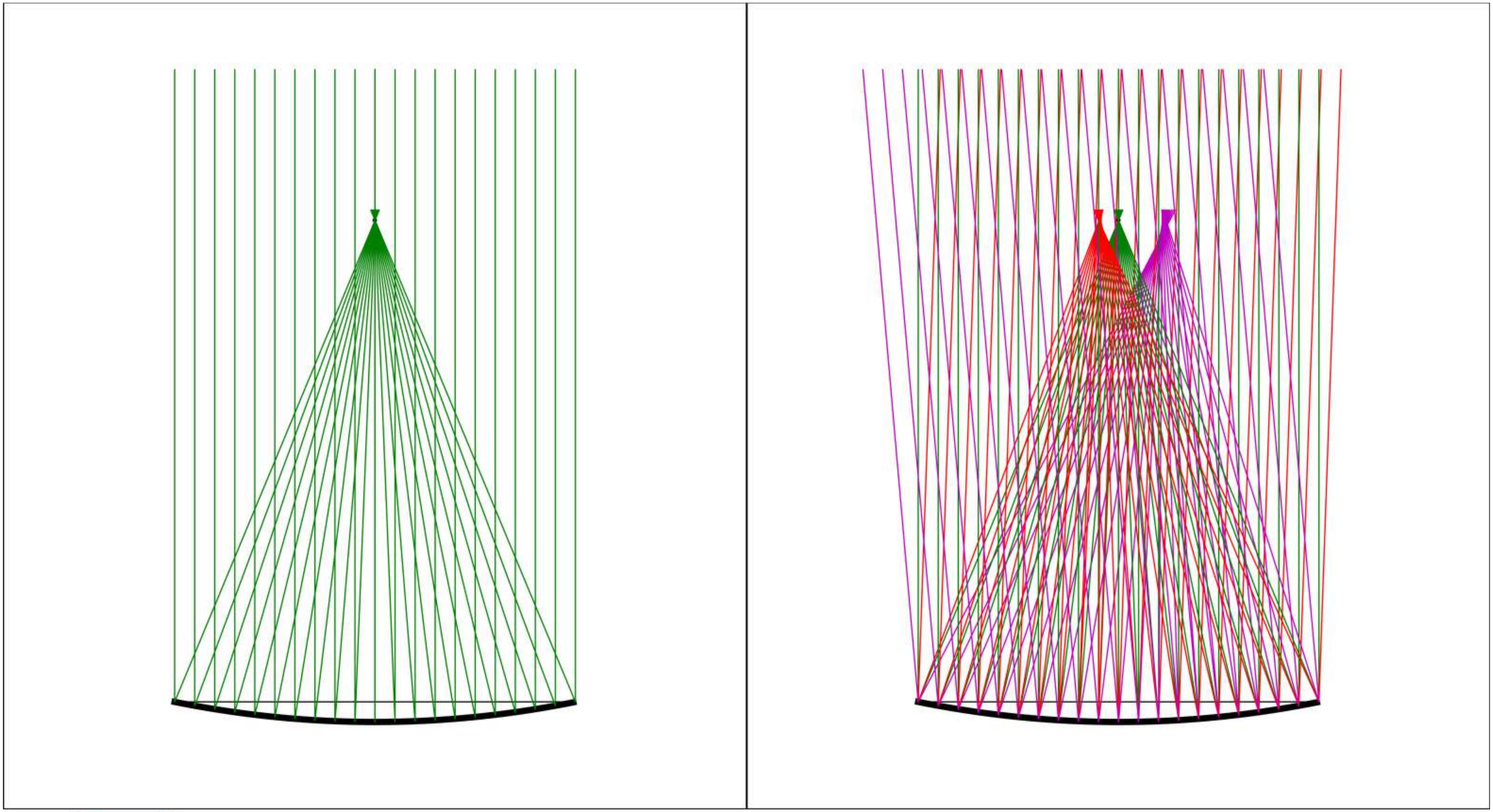


- EM wave focusing instruments
  - Your eye
  - A camera
  - A conventional telescope
- Properties
  - Gathering power (collecting area)
  - Resolution

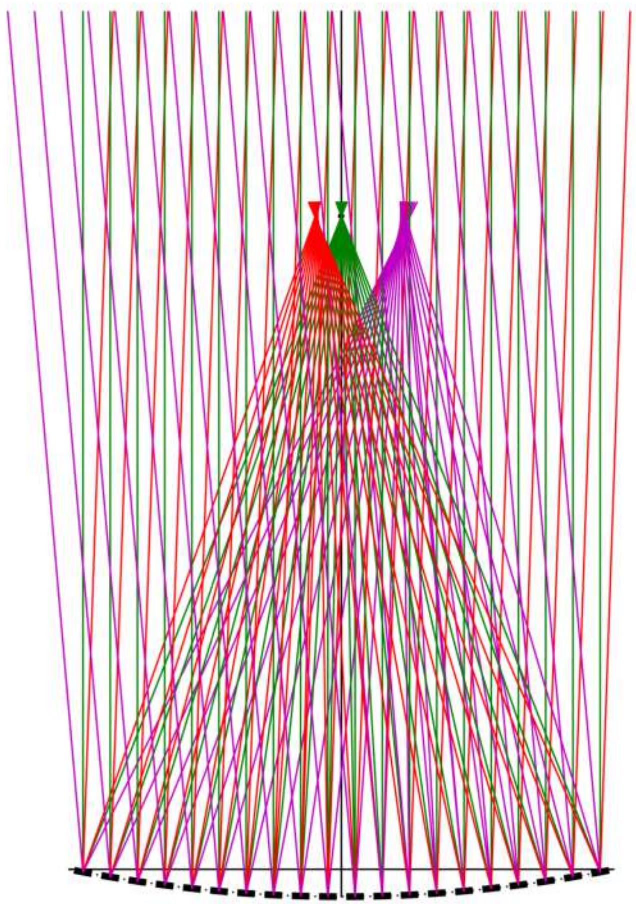
A Filled Aperture



A Filled Aperture

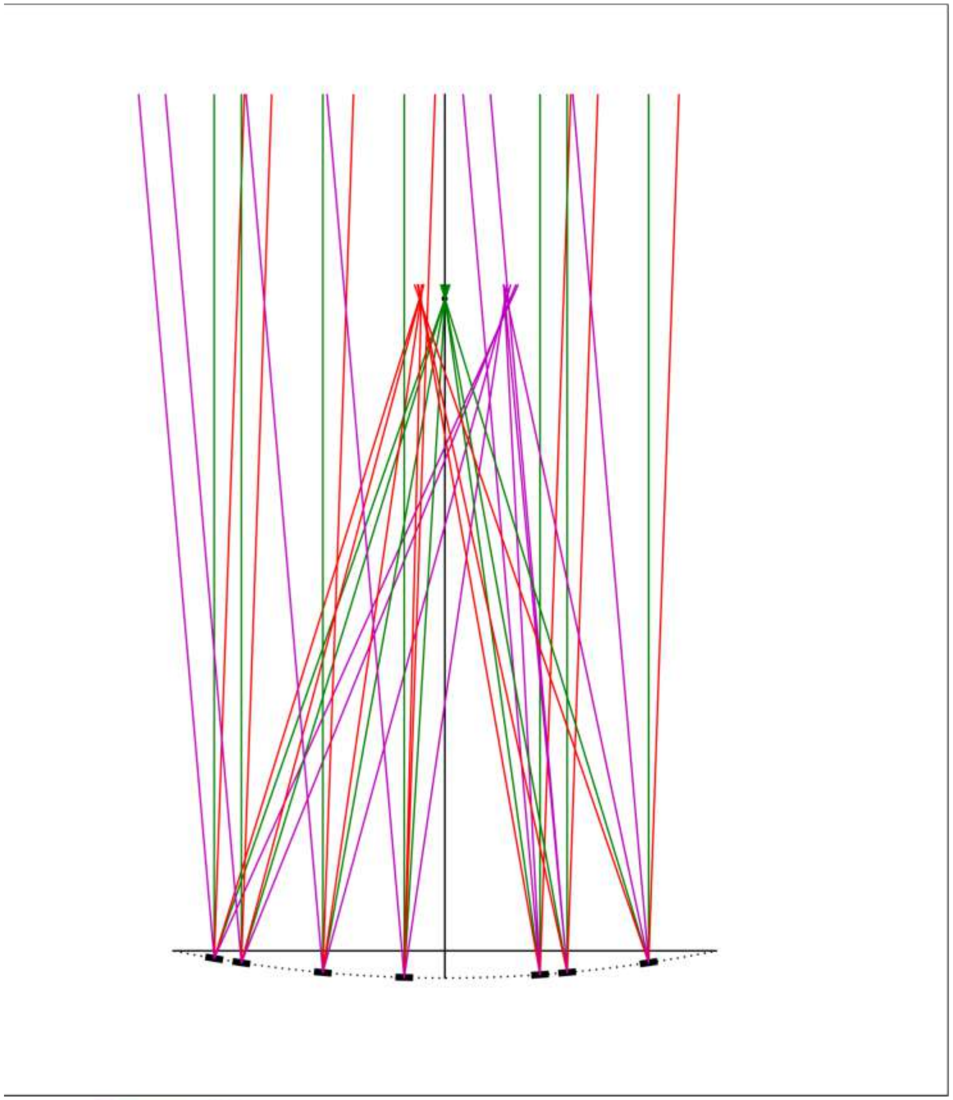


A Segmented (filled) Aperture



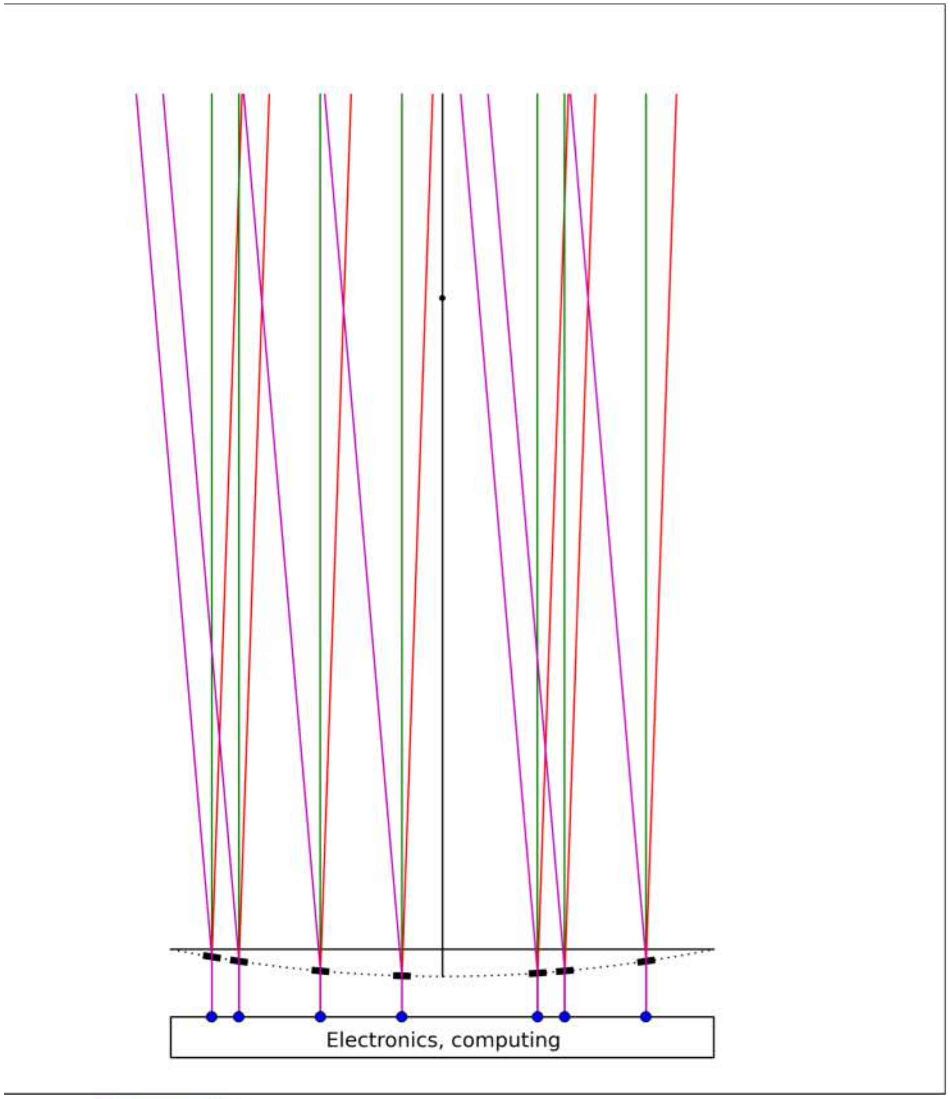
- Each segment 'gathers' EM field disturbances arriving from whole sky *(field superposition)*
- Parabolic figure redirects the net field and concentrates it in the focal plane
- *Diffraction* (EM *waves*!) dictates that each segment contributes complex (w/ phase) field to whole focal plane *(field superposition)*
- *Power* is *detected*: mean *square* of complex field sums per pixel: many *cross-products*…
  - Field disturbances from different directions (sources) are *independent*; no net contribution
  - Each surviving cross-product paints a sinusoid (*a "fringe"*) across focal plane, per source, per segment pair (*baseline*)
  - Per baseline source distribution sets the *"fringe visibility" (fringe superposition)*
- Global *Fringe Superposition* localizes direction-dependent source power at each pixel yielding a sensible image

An Unfilled Aperture



- Fewer segments, and pairs thereof
  - Less total collecting area
  - Uglier diffraction pattern
- Still, a sensible, if more modest image

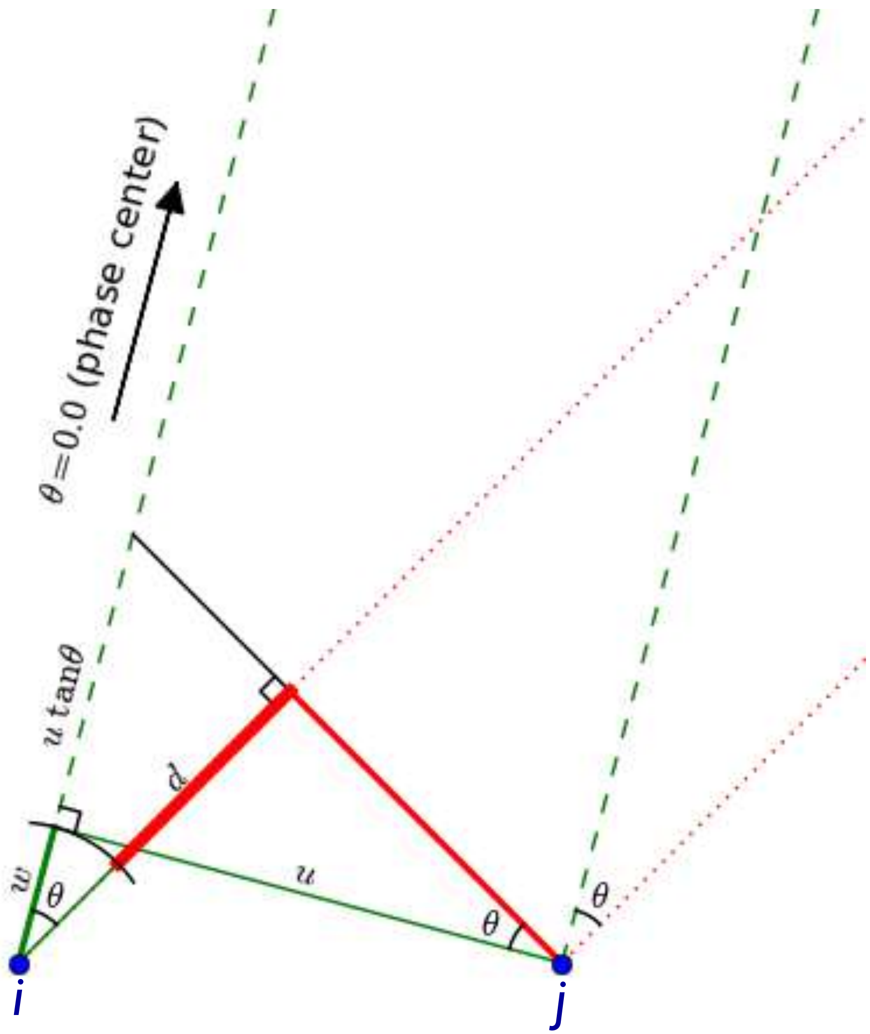
An Unfilled Aperture – virtual focus



- Fewer segments, and pairs thereof
  - Less total collecting area
  - Uglier diffraction pattern

- Synthesis Interferometry:
  - Cross-products *explicitly* formed electronically
  - "Focus" is formed by computation, through correlation and imaging

The Geometry of Interferometry



- Consider direction-dependent arrival geometry for E-field disturbance reception at two points, i and j, relative to the phase center direction

$$d = \left(w_\lambda + u_\lambda \tan\theta\right)\cos\theta - w_\lambda$$
$$= u_\lambda \sin\theta + w_\lambda\left(\cos\theta - 1\right)$$
$$d(l) = u_\lambda l + w_\lambda\left(\sqrt{1-l^2} - 1\right) \quad \text{(1D)}$$
$$\left(\sin\theta = l;\ \cos\theta = \sqrt{1-l^2}\right)$$

$$d(l,m) = u_\lambda l + v_\lambda m + w_\lambda\left(\sqrt{1-l^2-m^2} - 1\right) \quad \text{(2D)}$$
$$\approx u_\lambda l + v_\lambda m \qquad (l,m << 1)$$ (small angles)

Direction-dependent signals: $s_j = s_i e^{i2\pi d(l,m)}$

What are the *V(u,v)* that we form?

- Correlate the net E-field disturbances, x_i & x_j arriving at spatially separate sensors, *i* & *j*
  - delay-aligned for the phase-center
  - s_i & s_j are the direction-dependent E-field disturbances
- Direction integral and product can be reversed, because the E-field disturbances from different directions don't correlate *(***finite bandwidth***)*
- s_i and s_j (for a specific direction) differ only by a phase factor given by the arrival geometry, *d*
- $<|s_i|^2>$ is proportional to the brightness distribution, *I(l,m)*

$$V_{ij}^{obs} = \left\langle x_i \cdot x_j^* \right\rangle_{\Delta t}$$

$$= \left\langle \int_{sky} s_i\, dl_i\, dm_i \cdot \int_{sky} s_j^*\, dl_j\, dm_j \right\rangle_{\Delta t}$$

$$= \left\langle \int_{sky} s_i s_j^*\, dl\, dm \right\rangle_{\Delta t}$$

$$= \int_{sky} \left\langle \left| s_i \right|^2 \right\rangle e^{-i2\pi d(l,m)}\, dl\, dm$$

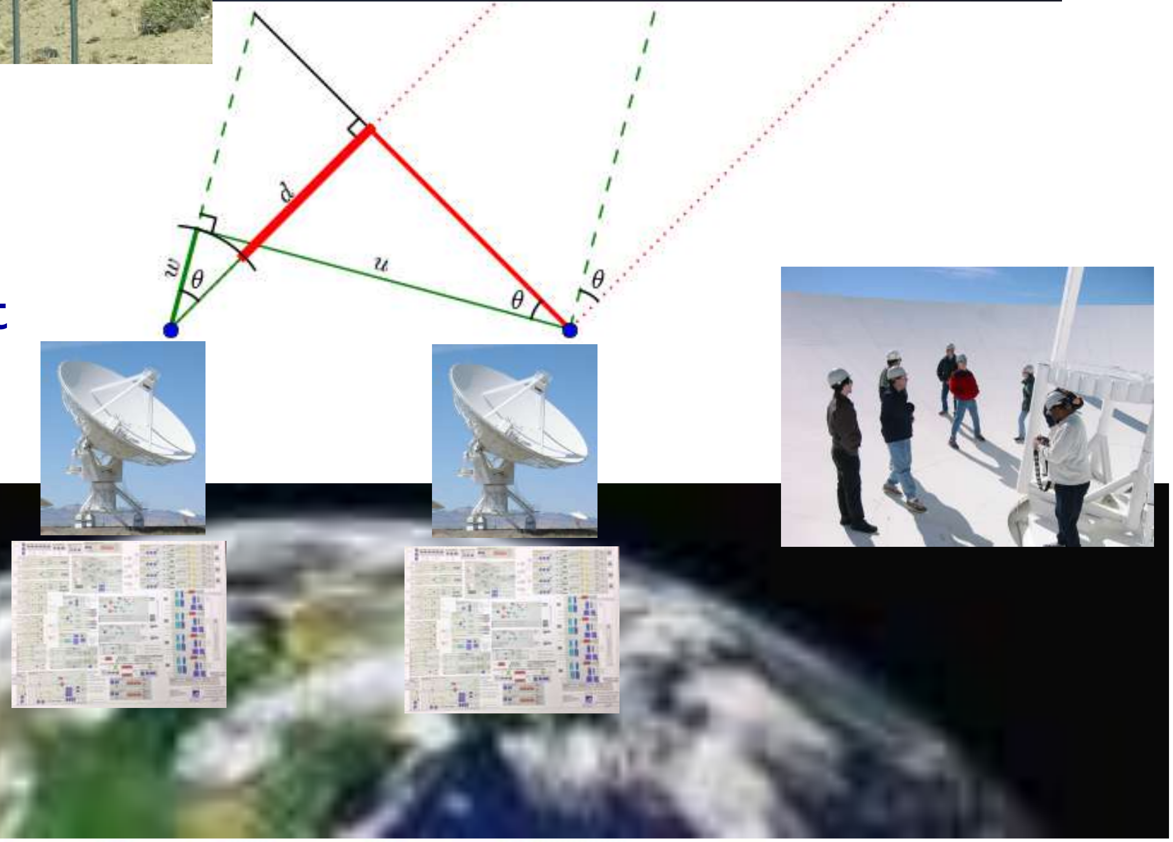
$$= \int_{sky} I(l,m)\, e^{-i2\pi d(l,m)}\, dl\, dm$$

$$= \int_{sky} I(l,m)\, e^{-i2\pi(ul+vm)}\, dl\, dm$$

But in reality…

- Weather
- Realistic Antennas
- Electronics…
- Digital correlation
- Finite noise
- …and the whole thing is moving!
- “Effective” geometry is not ideal

Drew Medlin



Realistic Visibility

- In practice, we obtain an imperfect visibility measurement per antenna pair:

$$V_{ij}^{obs}(u,v) = \left\langle x_i(t) \cdot x_j^*(t) \right\rangle_{\Delta t}$$
$$= J_{ij} V_{ij}^{true}(u,v)$$

  - x_i & x_j are mutually delay-compensated for the phase center
  - Averaging duration is set by the expected timescales for variation of the correlation result (~seconds)
- J_{ij} is a generalized *operator* characterizing the *net* effect of the observing process for antennas *i* and *j* on baseline *ij*, which we must *calibrate*
  - Includes any required scaling to physical units
- Sometimes J_{ij} corrupts the measurement irrevocably, resulting in data that must be *edited* or *"flagged"*

Realistic Visibility: Noise

- Normalized (fractional) visibility (Nyquist):

$$\sigma_{ij} = \frac{1}{\sqrt{2\Delta\nu\Delta t}}$$

- Absolute visibility:

$$\sigma_{ij} = \frac{\sqrt{T_i T_j}}{\sqrt{2\Delta\nu\Delta t}}$$

  - T_i, T_j are the system temperatures (total sampled powers), in whatever units the corresponding visibility data are in (K or Jy)
  - (The numerator, as measured by the correlator, is the factor by which visibilities are typically normalized, e.g. ALMA)

- Formal Visibility Weights:

$$w_{ij} = \frac{1}{\sigma_{ij}^2}$$

  - The fundamental measure of statistical information content
  - Uniform for normalized visibilities $(2\Delta\nu\Delta t)$

Practical Calibration Considerations

- Observatory housekeeping (optimizing default performance)
  - Nominal antenna positions, earth orientation and rate, clock(s), frequency reference
  - Antenna pointing/focus, voltage pattern, gain curve
  - Calibrator coordinates, flux densities, polarization properties
- Absolute *engineering* calibration (dBm, K, volts)?
  - Amplitude: episodic (ALMA) or continuous (EVLA/VLBA) T_{sys} or switched-power monitoring to enable calibration to nominal K (or Jy, with antenna efficiency information)
  - Phase: Water Vapor Radiometry (ALMA), otherwise practically impossible (relative antenna phase)
  - Traditionally, we concentrate instead on ensuring *effective* instrumental *stability* on adequate timescales
- ***Cross-calibration*** a better practical choice
  - Observe strong astronomical sources near science target against which calibration (J_{ij}) can be solved, and interpolate solutions onto target observations
  - Choose appropriate calibrators; usually ***point sources*** because we can easily predict their visibilities (Amp ~ constant, phase ~ 0)
  - Choose appropriate timescales for calibration

"Absolute" Astronomical Calibrations

- Flux Density Calibration
  - Radio astronomy flux density scale set according to several "constant" radio sources, and planets/moons
  - Use resolved models where appropriate
- Astrometry
  - Most calibrators come from astrometric catalogs; sky coordinate accuracy of target images tied to that of the calibrators
  - Beware of resolved and evolving structures, and phase transfer biases due to troposphere (especially for VLBI)
- Polarization
  - Usual flux density calibrators also have significant stable linear polarization position angle for registration
  - Calibrator circular polarization usually assumed zero (?)

- Relative calibration solutions (and dynamic range) insensitive to errors in these "scaling" parameters

Baseline-based Cross-Calibration

$$V_{ij}^{obs} = J_{ij} V_{ij}^{mod}$$

- Simplest, most-obvious calibration approach: measure complex response of *each baseline* on a standard source, and scale science target visibilities accordingly
  - "Baseline-based" Calibration: $J_{ij} = \left\langle V_{ij}^{obs} / V_{ij}^{mod} \right\rangle_{\Delta t}$
- Only option for single baseline "arrays"
  - historical one-at-a-time visibilities
- Calibration precision same as calibrator visibility sensitivity (on timescale of calibration solution). Improves only with calibrator strength.
- Calibration accuracy sensitive to departures of calibrator from assumed structure
  - Un-modeled calibrator structure transferred (in inverse) to science target!

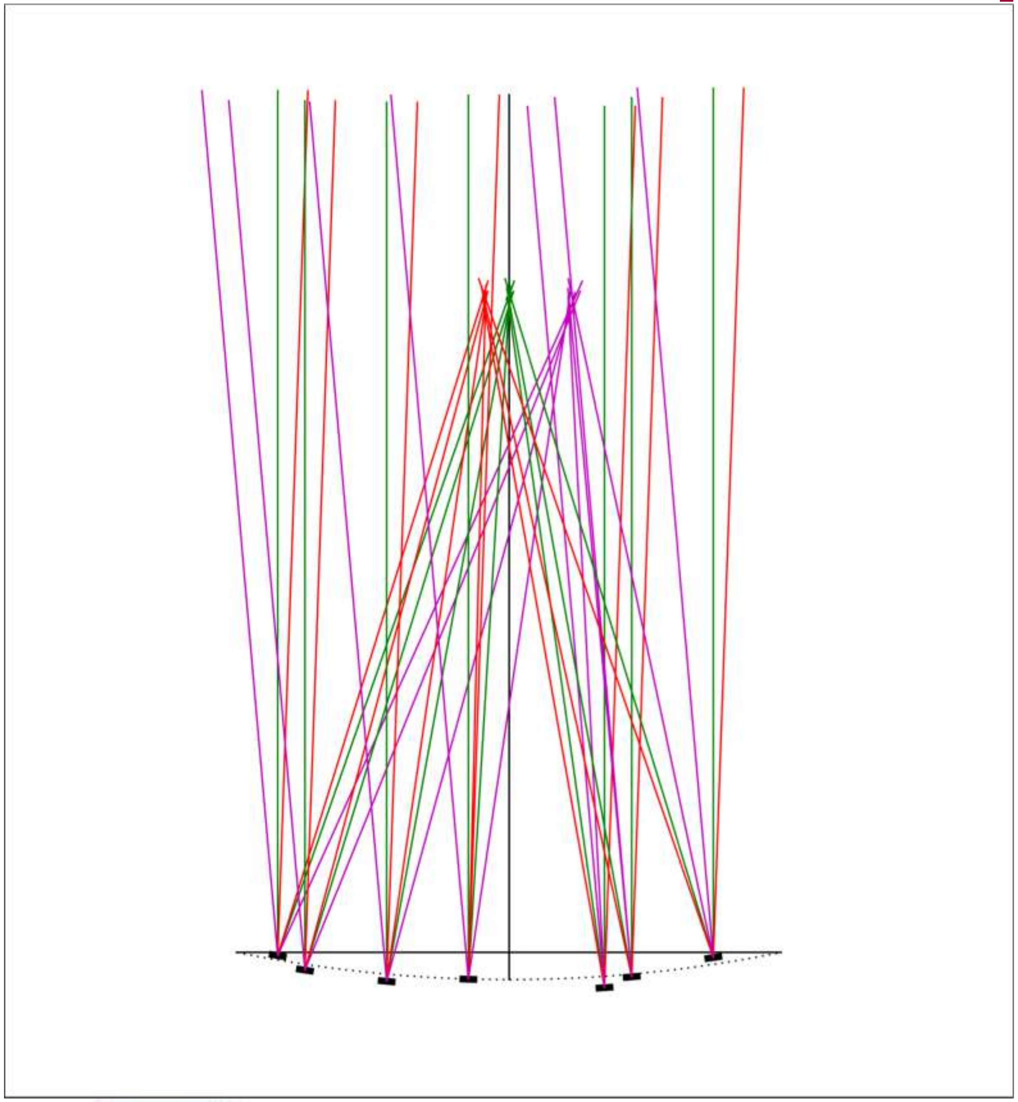
Antenna-based Cross Calibration

- Measured visibilities are formed from a product of *antenna-based* signals. Can we take advantage of this fact?

$$J_{ij} = J_i J_j^*$$

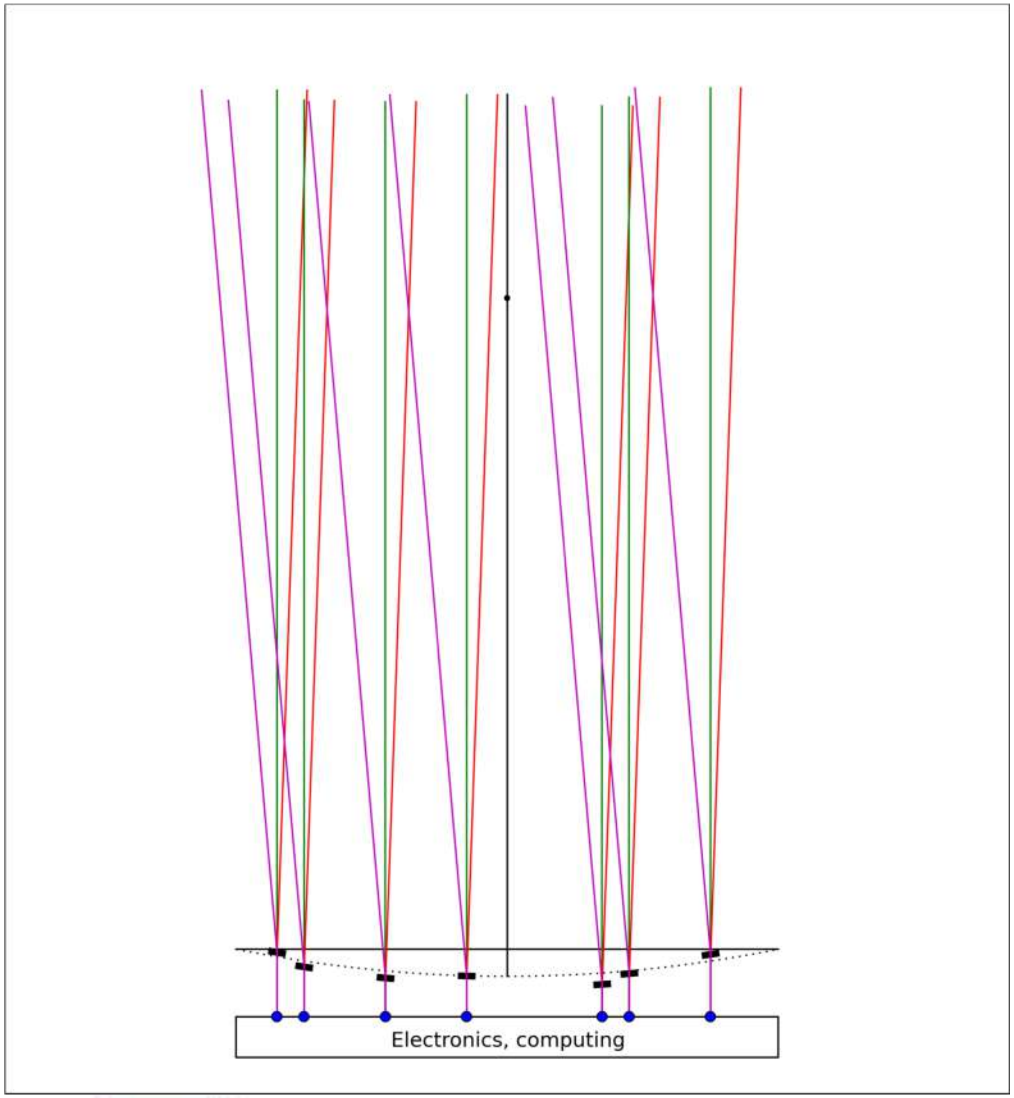
- *This is the fundamental insight that enabled the spectacular success of synthesis interferometers over the past 40 years.*
  - *Ryle (Nobel Prize in 1974 for developing aperture synthesis) very skeptical that atmospheric errors could be overcome…*

Distorted Unfilled Apertures



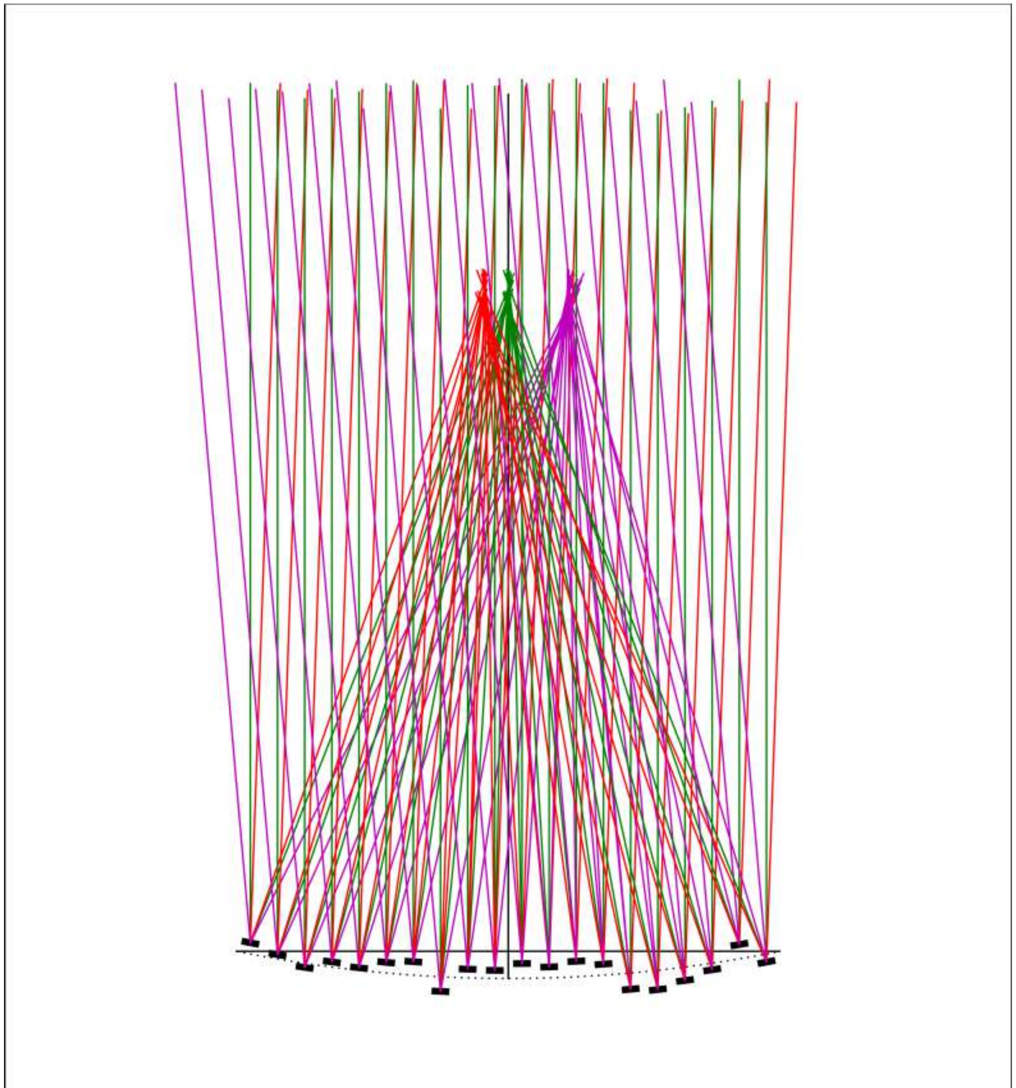
- Each unfilled aperture segment (antenna) has its own distinct properties that uniformly affect all correlations formed with other segments
  - E.g., unmodelled location and electronic path-length errors, atmosphere (delay errors)
  - Complex "Gain" (scale)

Distorted Unfilled Apertures



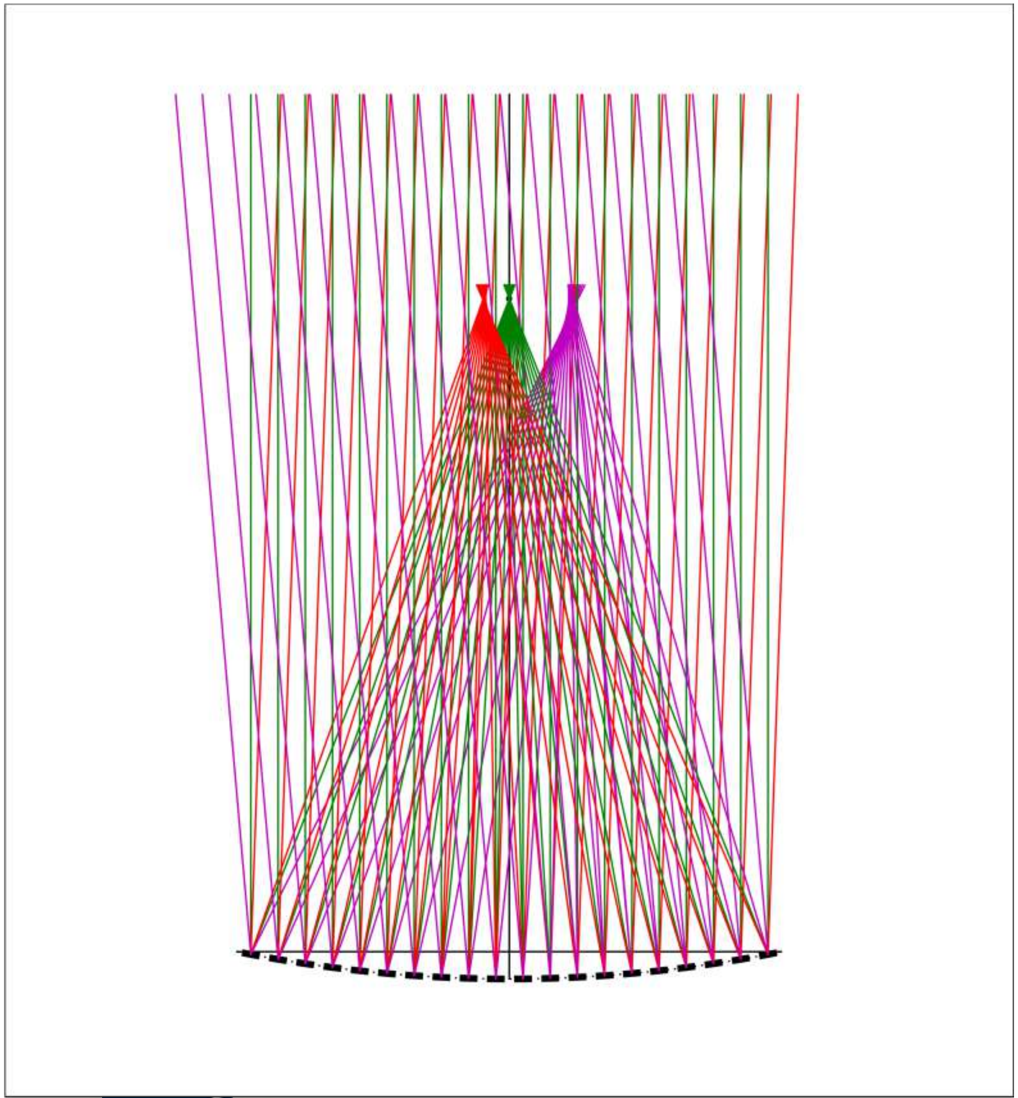
- Each unfilled aperture segment (antenna) has its own distinct properties that uniformly affect all correlations formed with other segments
  - E.g., unmodelled location and electronic path-length errors, atmosphere (delay errors)
  - Complex "Gain" (scale)
- Explicit formation and fine sampling of antenna-pair cross-products provides a post-observation—but *"pre-focus"*—opportunity to correct errors
  - I.e., to *calibrate*

Aside: Distorted Filled Apertures: AO



- Adaptive optics: Real-time, active correction of effective aperture geometry errors
  - Reflector figure (gravitational deflection, etc.)
  - Atmospheric propagation
- HST
  - Spherical aberration (constant)
  - Real-time "calibration" by introducing optical elements that correct wavefront *before* reaching the focus (otherwise, it was a *deconvolution* problem)
- Eyeglasses!
  - Calibration on an ~annual timescale…

Aside: Distorted Filled Apertures: AO



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- Eyeglasses!
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Antenna-based Cross Calibration

- The net time-dependent E-field signal sampled by antenna *i*, $x_i(t)$, is a combination of the desired signal, $s_i(t,l,m)$, corrupted by a factor $J_i(t,l,m)$ and integrated over the sky (l,m), and diluted by noise, $n_i(t)$:

$$x_i(t) = \int_{sky} J_i(t,l,m)s_i(t,l,m)dldm + n_i(t)$$

$$= s_i'(t) + n_i(t)$$

- $x_i(t)$ is sampled (complex) voltage provided to the correlator input
- $J_i(t,l,m)$ is the product of a series of effects encountered by the incoming signal
- $J_i(t,l,m)$ is an *antenna-based* (one index) complex number
  - Amplitude: "gain" (also units)
  - Phase: geometry/directional distortion
- Usually, $|n_i|^2 >> |s_i'|^2$ (i.e., noise power dominates)

Correlation of Realistic Signals

- The correlation of two realistic (aligned for a specific direction) signals from different antennas:
- Noise correlations have zero expectation—even if $|n_i|^2 >> |s_i|^2$
  – the correlation process isolates desired signals amidst zero-mean noise
- Same analysis as before, except we carry J_i, J_j terms
  – J_i's time- and frequency-dependence (and field-of-view) set the required timescale and frequency resolution

$$
\begin{aligned}
\left\langle x_i \cdot x_j^* \right\rangle_{\Delta t} &= \left\langle \left(s_i' + n_i\right) \cdot \left(s_j' + n_j\right)^* \right\rangle_{\Delta t} \\
&= \left\langle s_i' \cdot s_j'^* \right\rangle_{\Delta t} + \left\langle s_i' \cdot n_j^* \right\rangle_{\Delta t} + \left\langle n_i \cdot s_j'^* \right\rangle_{\Delta t} + \left\langle n_i \cdot n_j^* \right\rangle_{\Delta t} \\
&= \left\langle s_i' \cdot s_j'^* \right\rangle_{\Delta t}
\end{aligned}
$$

$$
\begin{aligned}
&= \left\langle \int_{sky} J_i s_i \, dl_i \, dm_i \cdot \int_{sky} J_j^* s_j^* \, dl_j \, dm_j \right\rangle_{\Delta t} \\
&= \left\langle \int_{sky} J_i J_j^* s_i s_j^* \, dl \, dm \right\rangle_{\Delta t} \\
&= \int_{sky} J_i J_j^* I(l,m) e^{-i2\pi(ul+vm)} \, dl \, dm
\end{aligned}
$$

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The Scalar Measurement Equation

$$V_{ij}^{obs} = \int_{sky} J_i J_j^* I(l,m) e^{-i2\pi(u_{ij}l+v_{ij}m)} \, dl \, dm$$

- First, isolate non-direction-dependent effects, and factor them from the integral:

$$= \left(J_i^{vis} J_j^{vis*}\right) \int_{sky} \left(J_i^{sky} J_j^{sky*}\right) I(l,m) e^{-i2\pi(u_{ij}l+v_{ij}m)} \, dl \, dm$$

- Next, we recognize that over small fields of view, it is often possible to assume $J^{sky}=1.0$, and we have a relationship between ideal and observed Visibilities:

$$= J_i J_j^* \int_{sky} I(l,m) e^{-i2\pi(u_{ij}l+v_{ij}m)} \, dl \, dm$$

$$V_{ij}^{obs} = J_i J_j^* V_{ij}^{true}$$

- Standard calibration of most existing arrays reduces to solving this last equation for the J_i, assuming a visibility model V_{ij}^{mod} for a calibrator
- Visibilities corrupted by *difference of antenna-based phases*, and *product of antenna-based amplitudes*

Aside: Auto-correlations and Single Dishes

- The *auto*-correlation of a signal from a *single* antenna:

$$\left\langle x_i \cdot x_i^* \right\rangle_{\Delta t} = \left\langle \left(s_i' + n_i\right) \cdot \left(s_i' + n_i\right)^* \right\rangle_{\Delta t}$$
$$= \left\langle s_i' \cdot s_i'^* \right\rangle + \left\langle n_i \cdot n_i^* \right\rangle$$
$$= \left\langle \int_{sky} \left|J_i\right|^2 \left|s_i\right|^2 dldm \right\rangle_{\Delta t} + \left\langle \left|n_i\right|^2 \right\rangle$$
$$= \int_{sky} \left|J_i\right|^2 I(l,m) dldm + \left\langle \left|n_i\right|^2 \right\rangle$$

  - This is an integrated (sky) power measurement plus *non-zero-mean* noise, i.e., the T_{sys}
  - Desired signal *not* simply isolated from noise
  - Noise usually dominates the power
  - Scalar calibration (c.f. single-baseline calibration)
- Single dish radio astronomy calibration strategies rely on switching (differencing) schemes to isolate desired signal from the noise

Solving for the J_i

- Observe point-like calibrator for which we know true visibilities, and…
- We can write:

$$V_{ij}^{obs} - J_i J_j^* V_{ij}^{mod} = 0$$

- …and define chi-squared:

$$\chi^2 = \sum_{\substack{i,j \\ i \neq j}} \left| V_{ij}^{obs} - J_i J_j^* V_{ij}^{mod} \right|^2 w_{ij} \qquad \left(w_{ij} = \frac{1}{\sigma_{ij}^2} \right)$$

- …and minimize chi-squared w.r.t. each J_i^* $\left(\frac{\partial \chi^2}{\partial J_i^*} = 0 \right)$, yielding:

$$J_i = \sum_{j \neq i} \left(V_{ij}^{obs} J_j V_{ij}^{mod*} w_{ij} \right) \Big/ \sum_{j \neq i} \left(\left| J_j \right|^2 \left| V_{ij}^{mod} \right|^2 w_{ij} \right)$$

$$= \sum_{j \neq i} \left(\frac{V_{ij}^{obs}}{J_j^* V_{ij}^{mod}} \right) W_{ij}' \Big/ \sum_{j \neq i} W_{ij}' \qquad \left(W_{ij}' = \left| J_j \right|^2 \left| V_{ij}^{mod} \right|^2 w_{ij} \right)$$

- (Requires iteration to solve the ensemble)

Solving for J_i (cont)

- Formal errors:

$$\sigma_{J_i} = \sqrt{\frac{1}{\sum_{j \neq i} \left|V_{ij}^{mod}\right|^2 \left|J_j\right|^2 / \sigma_{ij,\Delta t}^2}}$$

- For a ~uniform array (~same sensitivity on all baselines, ~same calibration magnitude on all antennas) and point-like calibrator:

$$\sigma_{J_i} \approx \frac{\sigma_{ij,\Delta t}}{\left|V^{mod}\right| \sqrt{\left\langle \left|J_j\right|^2 \right\rangle \left(N_{ant} - 1\right)}}$$

- Calibration error decreases with increasing calibrator strength *and* square-root of N_{ant} (c.f. baseline-based calibration).
- Other properties of the antenna-based solution:
  - Minimal degrees of freedom (N_{ant} factors, $N_{ant}(N_{ant}-1)/2$ measurements)
  - Net calibration for a baseline involves a phase difference, so *absolute* directional information is lost ($N_{ant}-1$ phases)
  - Closure…

Antenna-based Calibration and Closure

- Success of synthesis telescopes relies on antenna-based calibration
  - Fundamentally, any information that can be factored into antenna-based terms, could *be* antenna-based effects, and not true source visibility information
  - For $N_{ant} > 3$, non-trivial source visibility information cannot be *entirely* obliterated by any antenna-based calibration

- Observables *independent* of antenna-based errors: closure
  - Closure Phase (3 baselines)
  - Closure Amplitude (4 baselines)

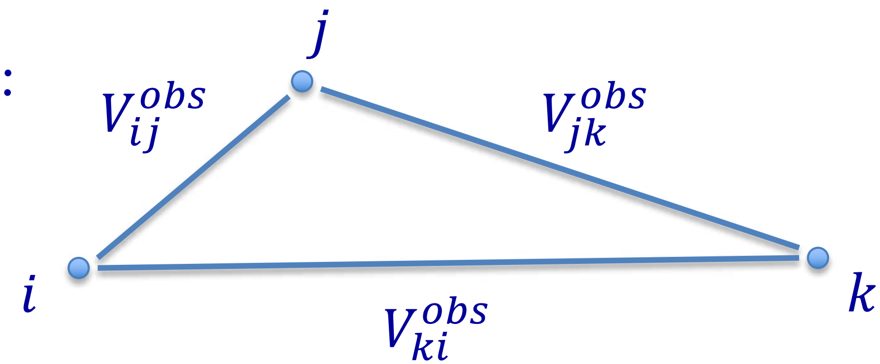
- Baseline-based calibration formally violates closure!

Closure Phase

$$V_{ij}^{obs} = A_{ij}^{obs} e^{i\boldsymbol{\phi}_{ij}^{obs}} \quad = G_i G_j^* \; V_{ij}^{true}$$

$$= g_i e^{i\boldsymbol{\theta}_i} \; g_j e^{-i\boldsymbol{\theta}_j} A_{ij} e^{i\boldsymbol{\phi}_{ij}^{true}} = g_i g_j A_{ij}^{true} e^{i\left(\boldsymbol{\phi}_{ij}^{true} + \boldsymbol{\theta}_i - \boldsymbol{\theta}_j\right)}$$

- Form total phase around three baselines:



$$\phi_{ij}^{obs} + \phi_{jk}^{obs} + \phi_{ki}^{obs} = \left(\phi_{ij}^{true} + \theta_i - \theta_j\right) + \left(\phi_{ij}^{true} + \theta_i - \theta_j\right) + \left(\phi_{ij}^{true} + \theta_i - \theta_j\right)$$
$$= \phi_{ij}^{true} + \phi_{jk}^{true} + \phi_{ki}^{true}$$

  - Closure phase is independent of antenna-based phase errors
  - $(N_{ant} - 1)(N_{ant} - 2)/2$ independent closure phases

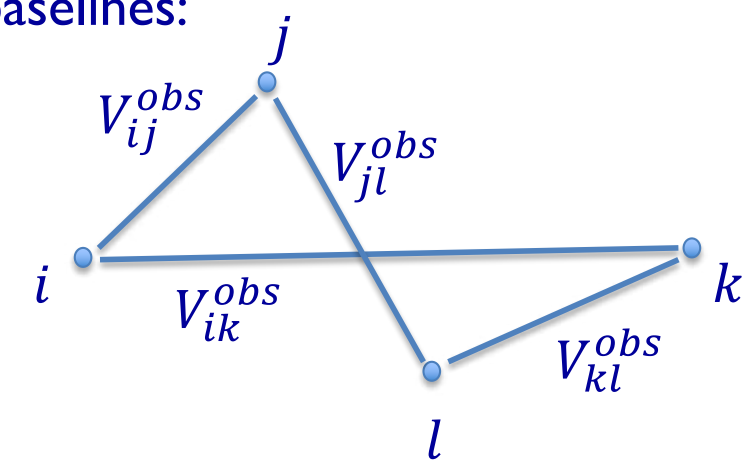
    - Baseline-based calibration formally violates closure!

Closure Amplitude

$$V_{ij}^{obs} = \boldsymbol{A_{ij}^{obs}} e^{i\phi_{ij}^{obs}} = G_i G_j^* \, V_{ij}^{true}$$

$$= \boldsymbol{g_i} e^{i\theta_i} \, \boldsymbol{g_j} e^{-i\theta_j} \boldsymbol{A_{ij}} e^{i\phi_{ij}^{true}} = \boldsymbol{g_i g_j A_{ij}^{true}} e^{i\left(\phi_{ij}^{true} + \theta_i - \theta_j\right)}$$

- Form ratios of amplitude products from four baselines:



$$\frac{A_{ij}^{obs} \; A_{kl}^{obs}}{A_{ik}^{obs} \; A_{jl}^{obs}} = \frac{g_i g_j A_{ij}^{true} g_k g_l A_{kl}^{true}}{g_i g_k A_{ik}^{true} g_j g_l A_{jl}^{true}} = \frac{A_{ij}^{true} \, A_{kl}^{true}}{A_{ik}^{true} \, A_{jl}^{true}}$$

  - Closure amplitude is independent of antenna-based amplitude errors
  - $N_{ant}(N_{ant} - 3)/2$ independent closure amplitudes
    - Baseline-based calibration formally violates closure!

Reference Antenna

- Since the "antenna-based" phase solution is derived from antenna phase *differences*, we do not measure phase absolutely
  - *relative* astrometry (only as good as assumed calibrator astrometry)
- Phase solutions typically referred to a specific antenna, the refant, which is assumed to have constant phase (zero, in both polarizations)
  - refant typically near array center
  - The refant's phase variation distributed to all other antennas' solutions
  - Asserts unambiguous phase continuity, for adequate time sampling, thereby ensuring reliable interpolation of phase (c.f. arbitrary phase offsets between solutions)
  - Asserts stable cross-hand phase frame (which must be calibrated)
- Problems:
  - A single good refant not always available over whole observation (time, frequency), due to flagging, etc.
  - Effective cross-hand phase of refant (or over multiple refant changes) may not, in fact, be stable…

Corrected Visibility

- Visibility…

$$V_{ij}^{obs} = J_i J_j^* V_{ij}^{true} \quad \rightarrow \quad V_{ij}^{cor} = J_i^{-1} J_j^{*-1} V_{ij}^{obs}$$

- …and weights!
  - calibrate the sigmas!

$$w_{ij}^{cor} = w_{ij}^{obs} \left|J_i\right|^2 \left|J_j\right|^2 = \frac{\left|J_i\right|^2 \left|J_j\right|^2}{\sigma_{ij}^2}$$

  - Statistical information content becomes baseline-dependent
  - Imaging will be a non-trivially-weighted direction-dependent average of the visibilities…

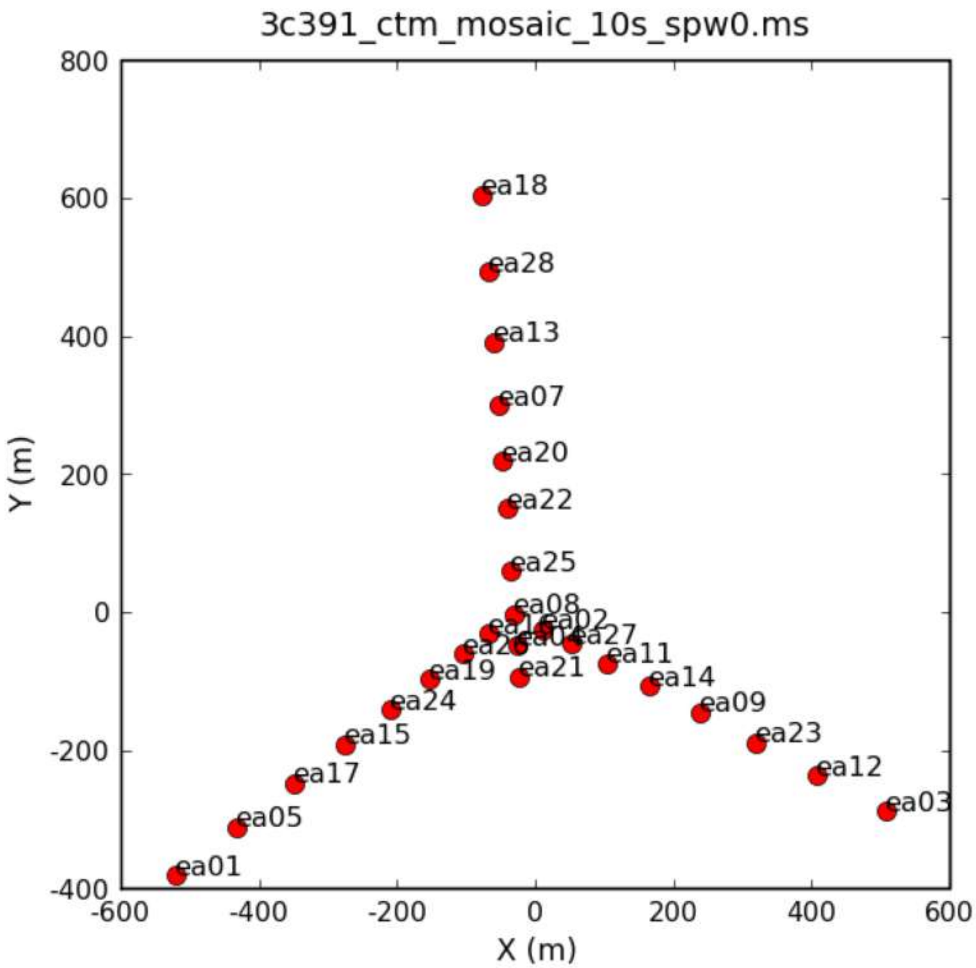
What Is Delivered by a Synthesis Array?

- An *enormous* list of complex visibilities! *(Enormous!)*
  - At each timestamp (~1-10s intervals): N(N-1)/2 baselines
    - EVLA: 351 baselines
    - VLBA: 45 baselines
    - ALMA: 1225+ baselines
  - For each baseline: up to 64 Spectral Windows ("spws", "subbands" or "IFs")
  - For each spectral window: tens to thousands of channels ($\Delta\nu$<10 MHz)
  - For each channel: 1, 2, or 4 complex correlations (polarizations)
    - EVLA or VLBA: RR or LL or (RR,LL), or (RR,RL,LR,LL)
    - ALMA: XX or YY or (XX,YY) or (XX,XY,YX,YY)
  - With each correlation, a weight value and a flag (T/F)
  - Meta-info: Coordinates, antenna, field, frequency label info
- $N_{total} = N_t \times N_{bl} \times N_{spw} \times N_{chan} \times N_{corr}$ visibilities
  - ~few $10^6 \times N_{spw} \times N_{chan} \times N_{corr}$ vis/hour $\rightarrow$ 10s to 100s of GB per observation

A Typical Dataset

- Array:
  - EVLA D-configuration (Apr 2010)
- Sources:
  - Science Target: 3C391, a SNR (7 mosaic pointings)
  - Near-target calibrator: J1822-0938 (~11 deg from target)
  - Flux Density calibrator: 3C286
  - Instrumental Polarization Calibrator: 3c84
- Signals:
  - RR,RL,LR,LL correlations
  - One spectral window centered at 4600 MHz, 128 MHz bandwidth, 64 channels

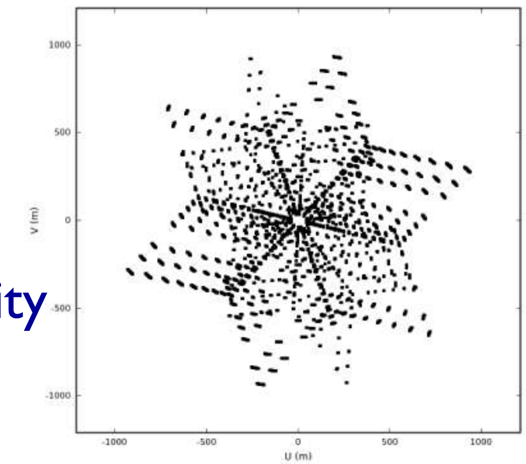
The Array

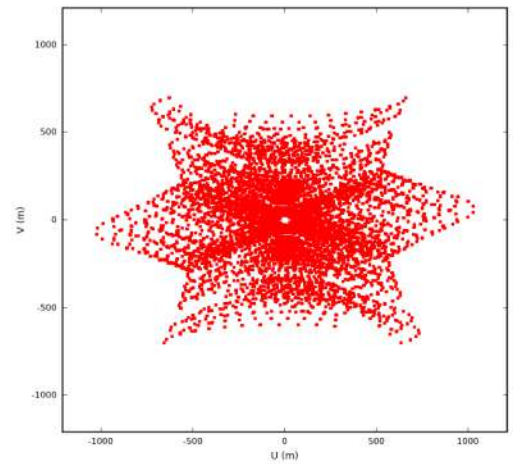

3c391_ctm_mosaic_10s_spw0.ms
Y (m)
X (m)
800
600
400
200
0
-200
-400
-600
-400
-200
0
200
400
600
ea18
ea28
ea13
ea07
ea20
ea22
ea25
ea08
ea27
ea11
ea19
ea21
ea14
ea24
ea09
ea15
ea23
ea12
ea17
ea03
ea05
ea01

UV-coverages

3C286

Flux Density



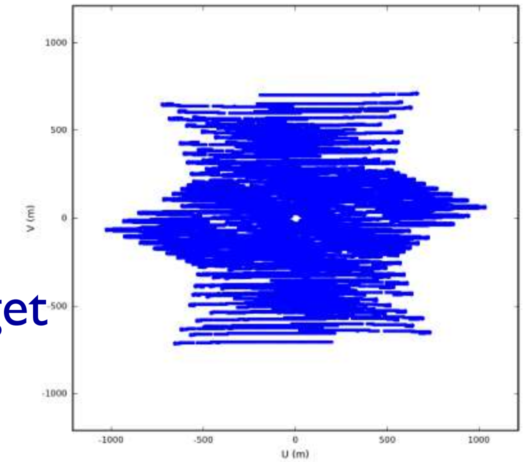


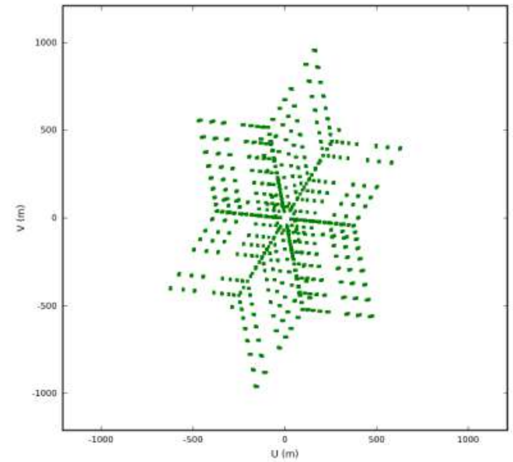
J1822-0938

Gain Calibrator

3C391

Science Target

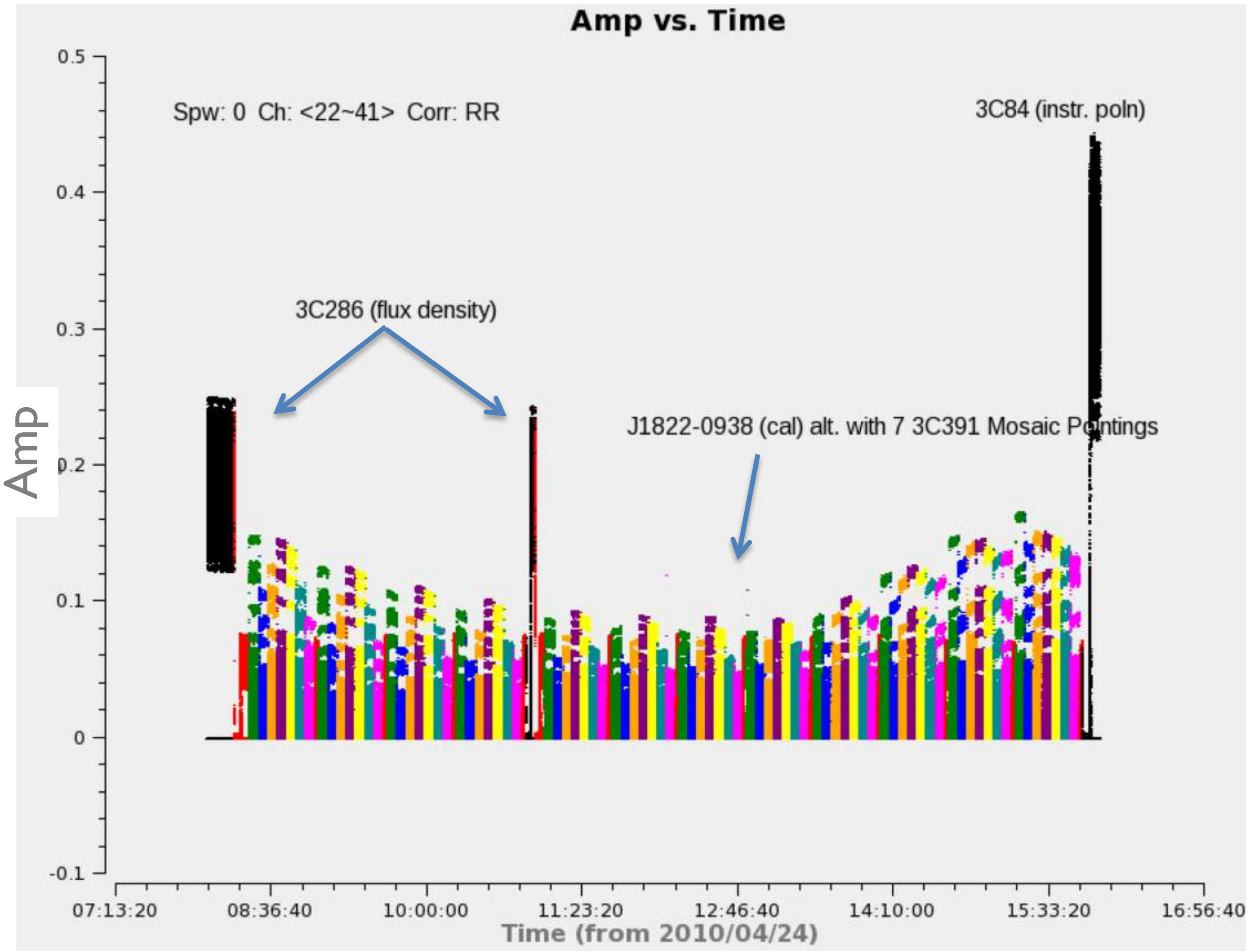




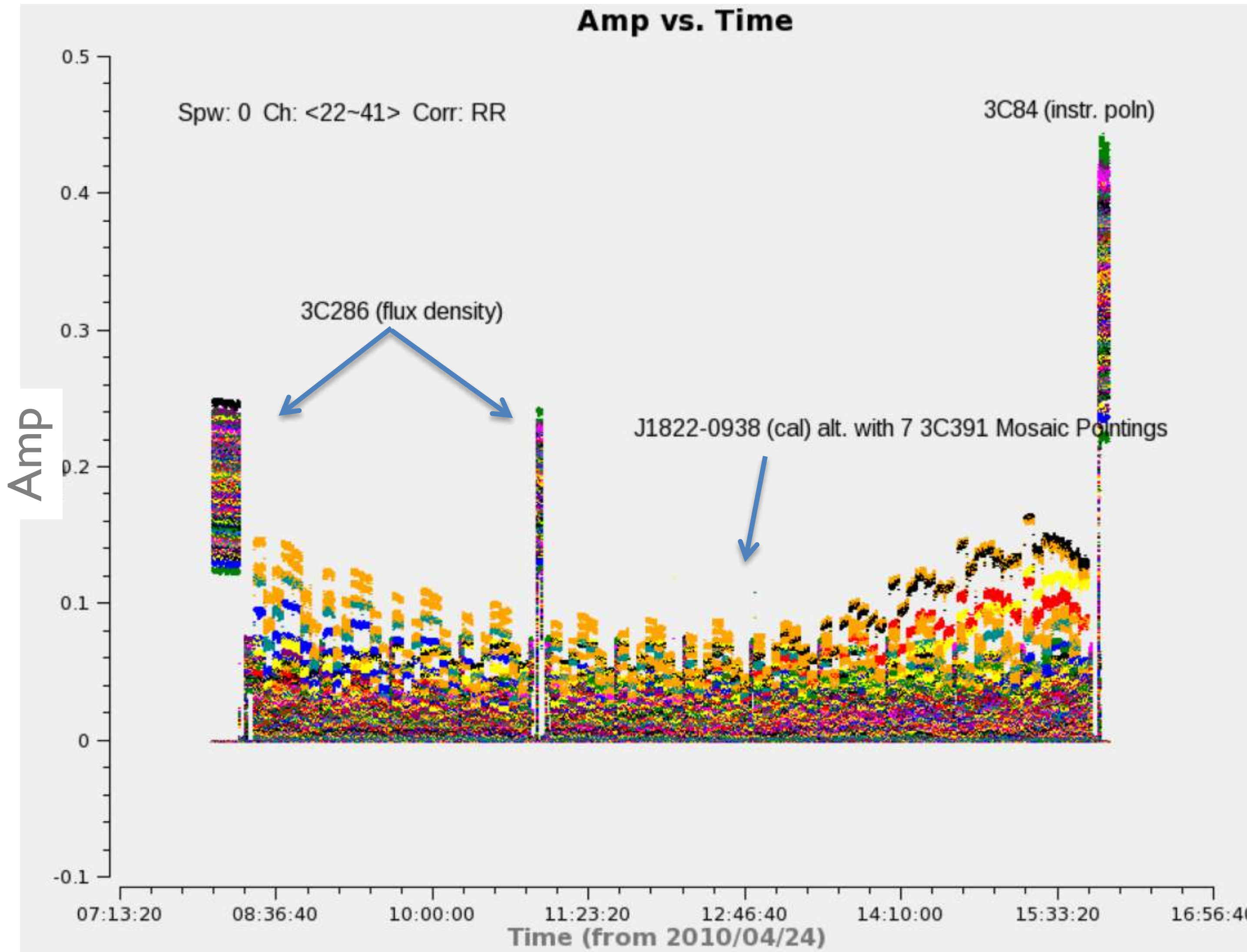
3C84

Instr. Poln Calibrator

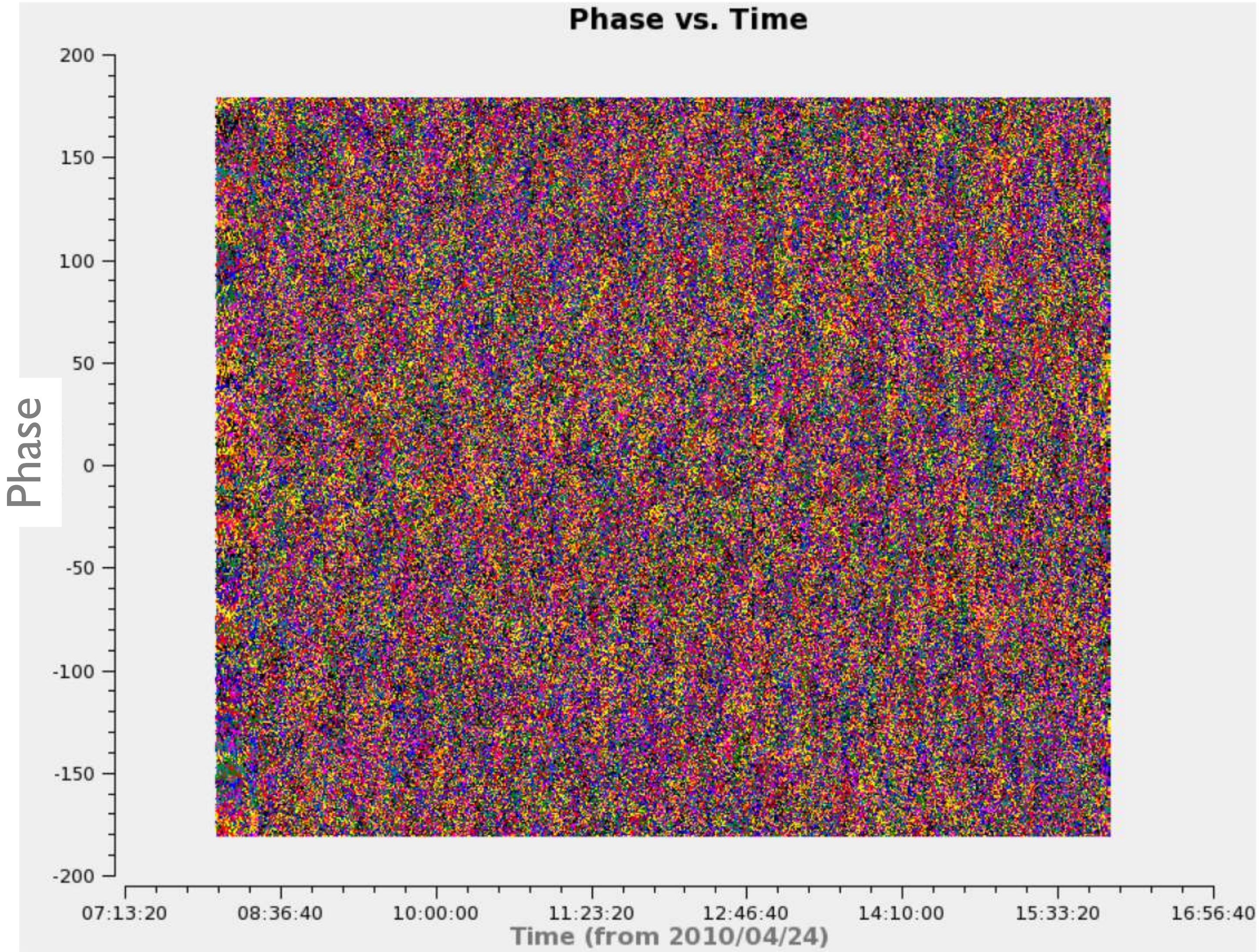
The Visibility Data (source colors)



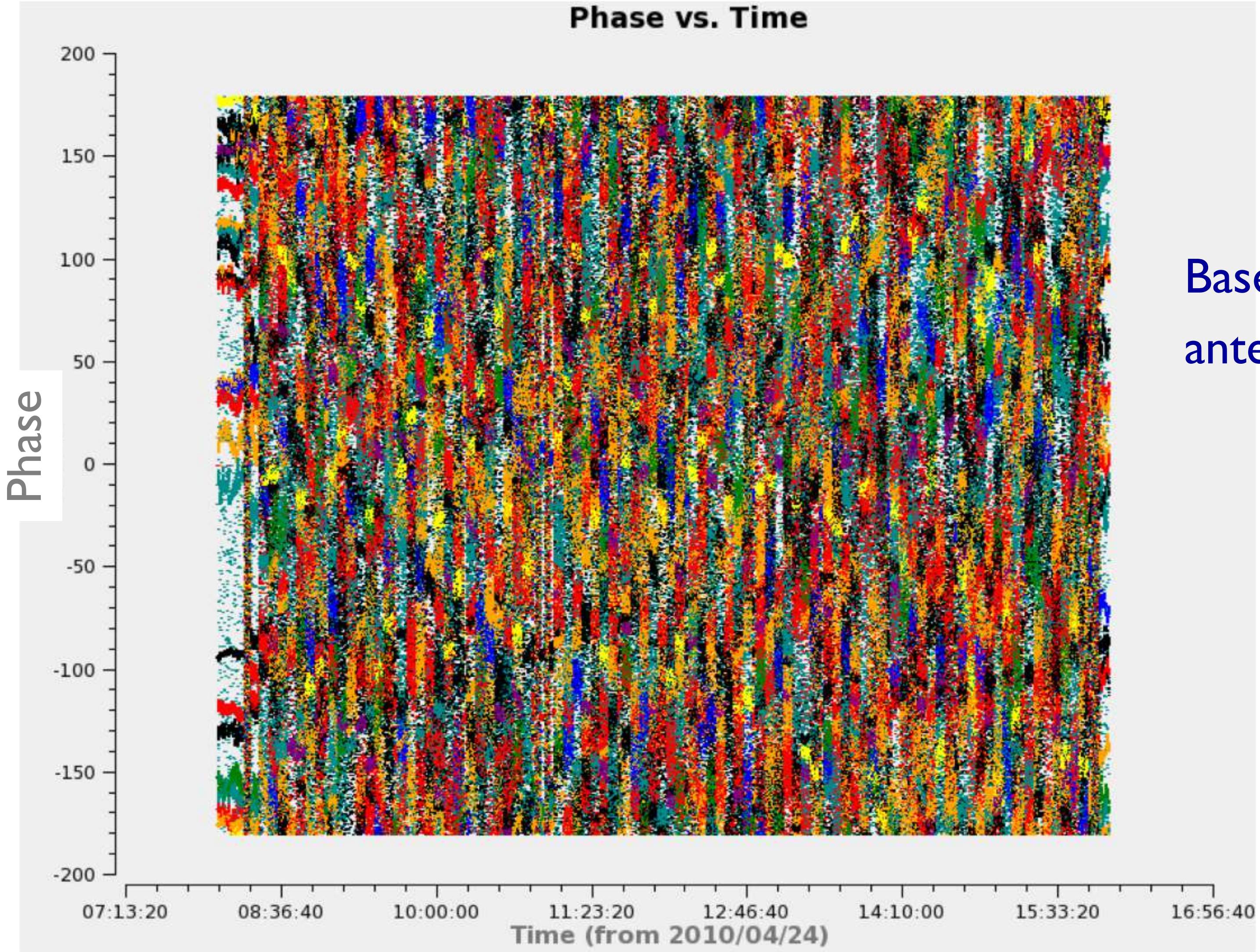
The Visibility Data (baseline colors)



The Visibility Data (baseline colors)

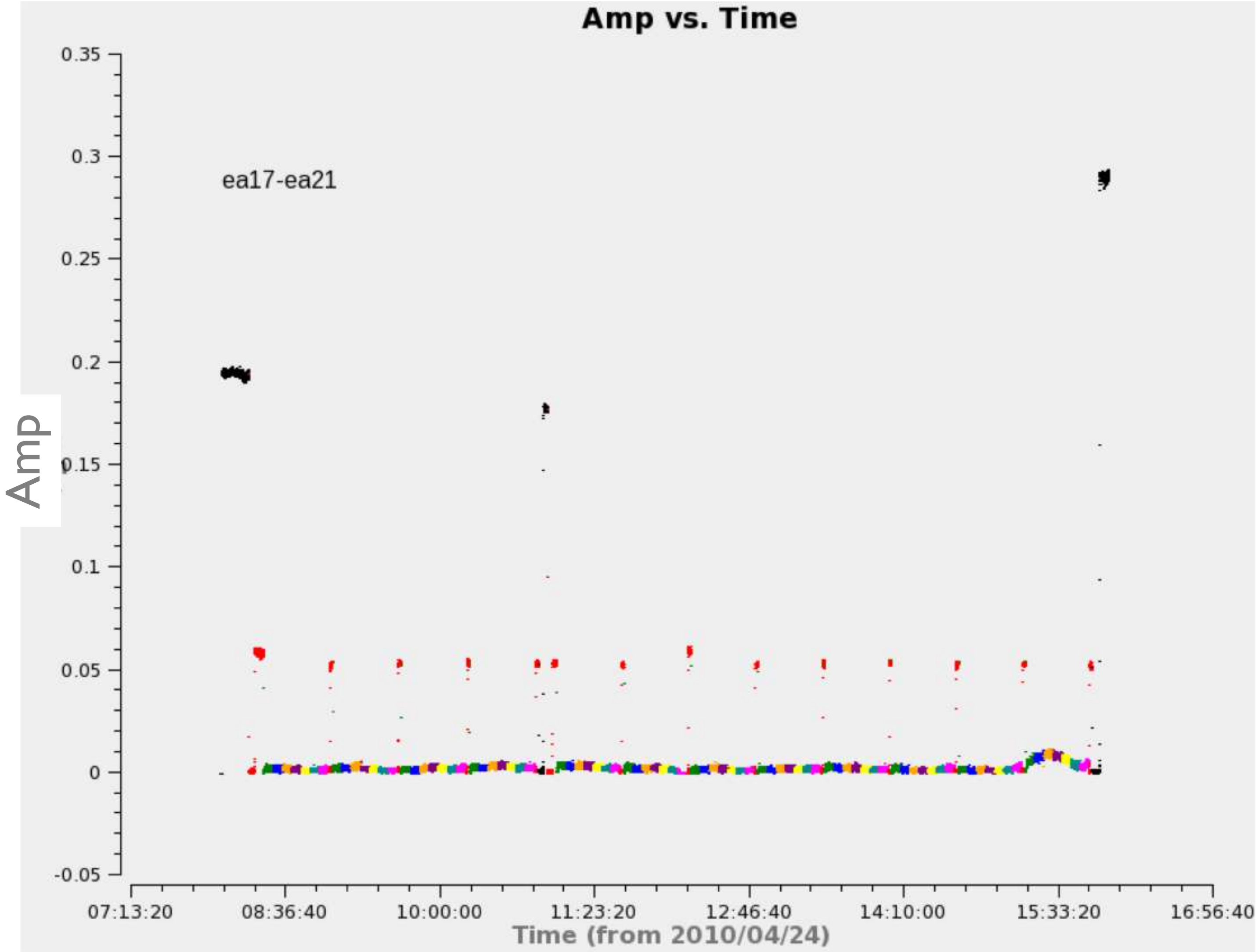


The Visibility Data (baseline colors)

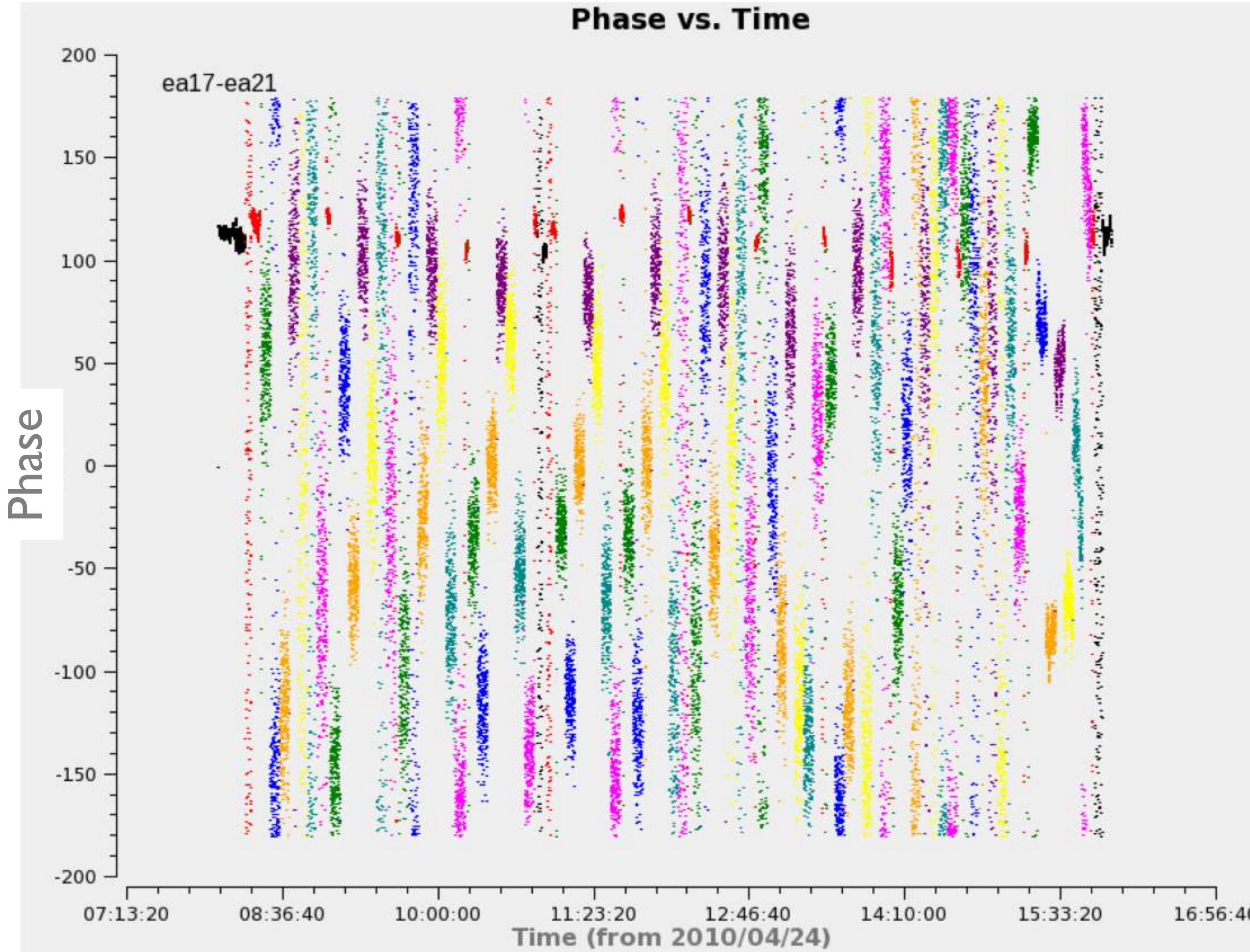


Baselines to antenna ea21

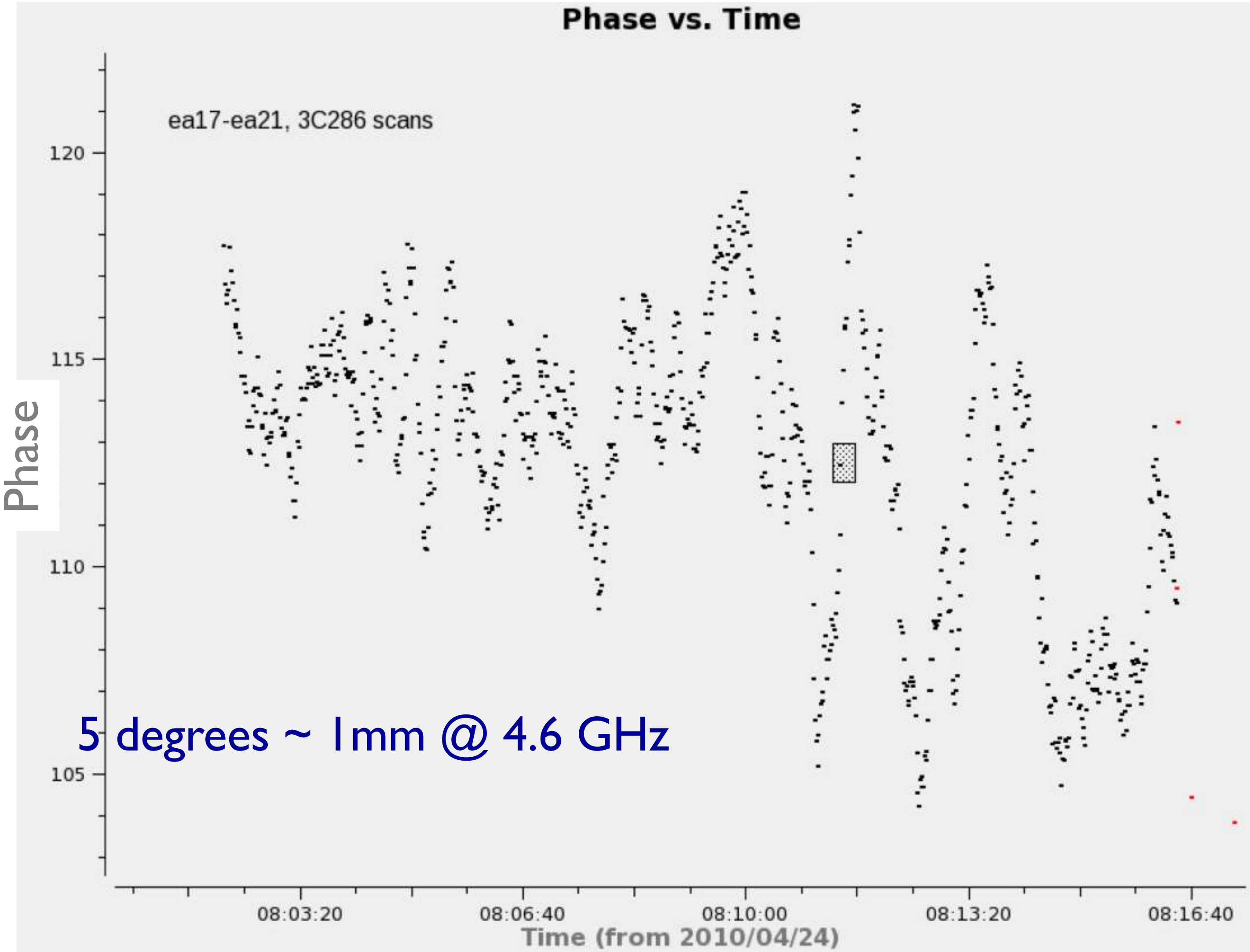
A Single Baseline – Amp (source colors)



A Single Baseline – Phase (source colors)

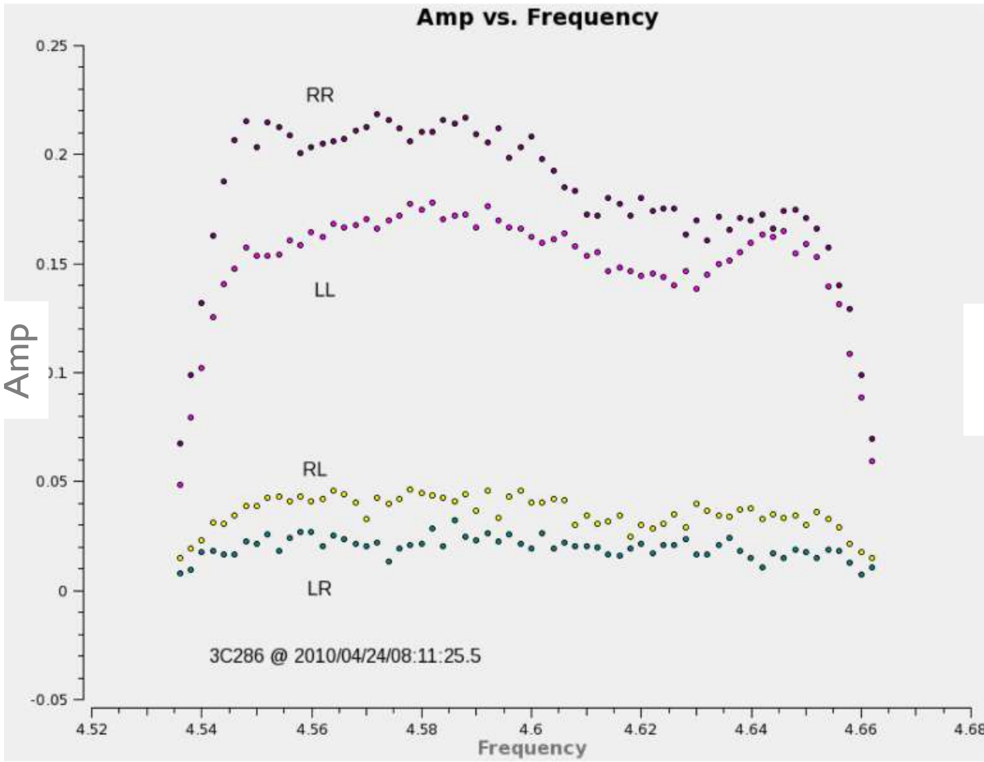


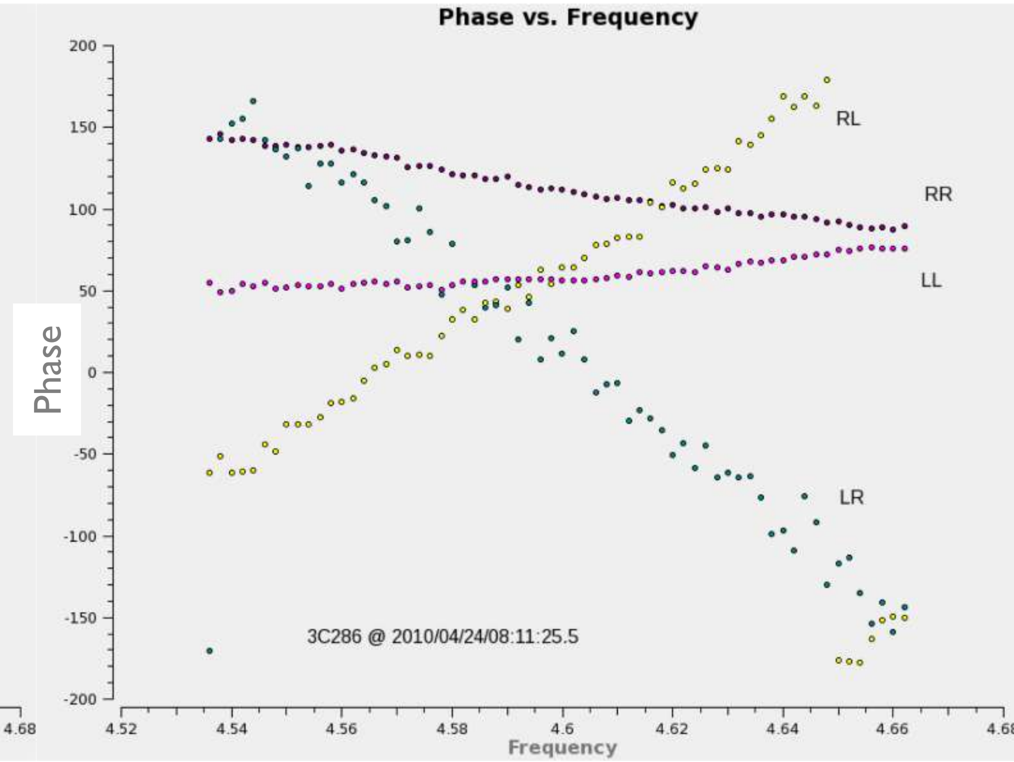
A Single Baseline – 2 scans on 3C286



5 degrees ~ 1mm @ 4.6 GHz

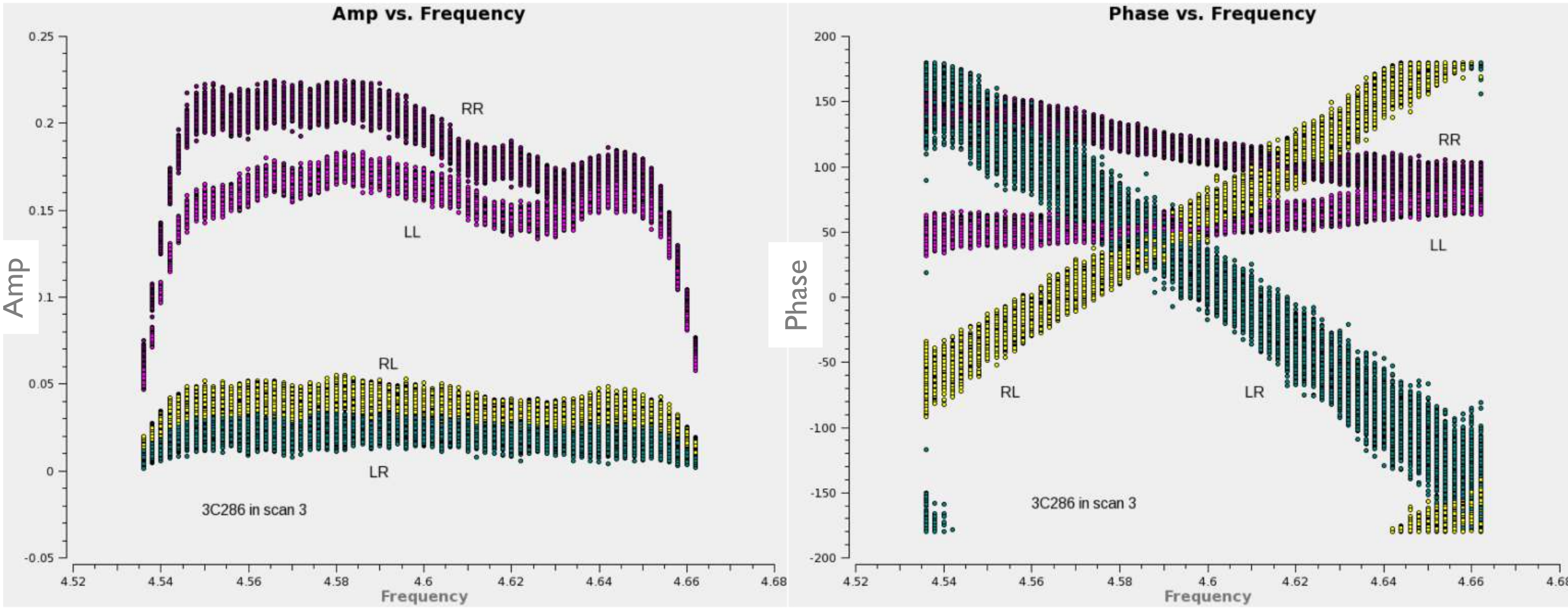
Single Baseline, Single Integration Visibility Spectra (4 correlations)





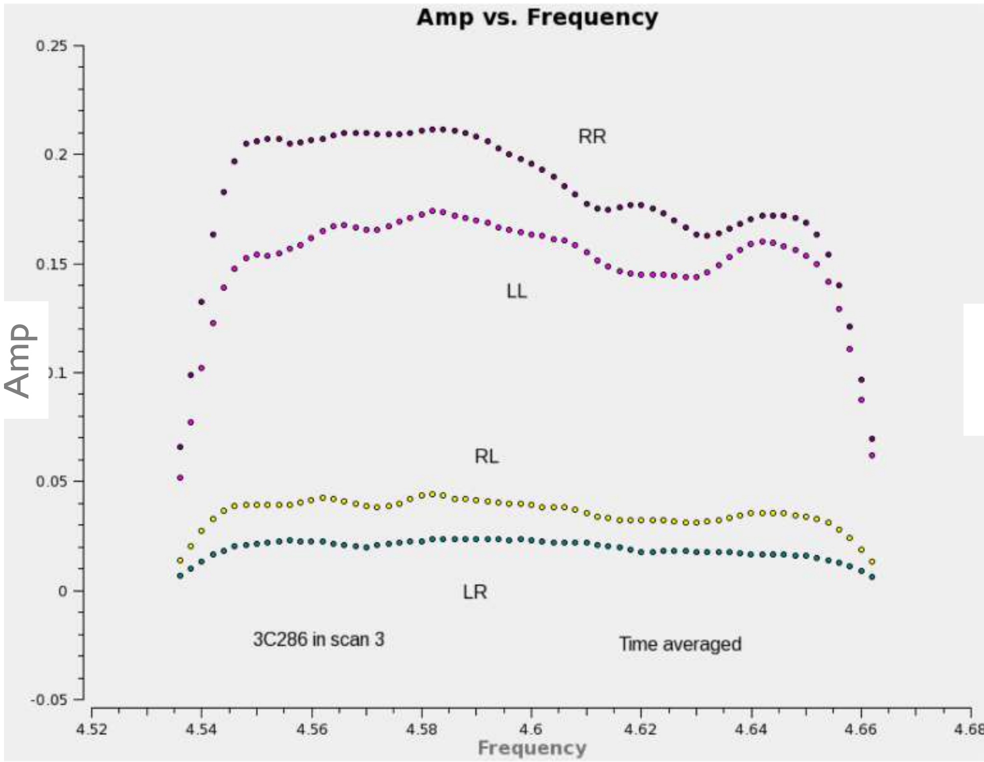
baseline ea17-ea21

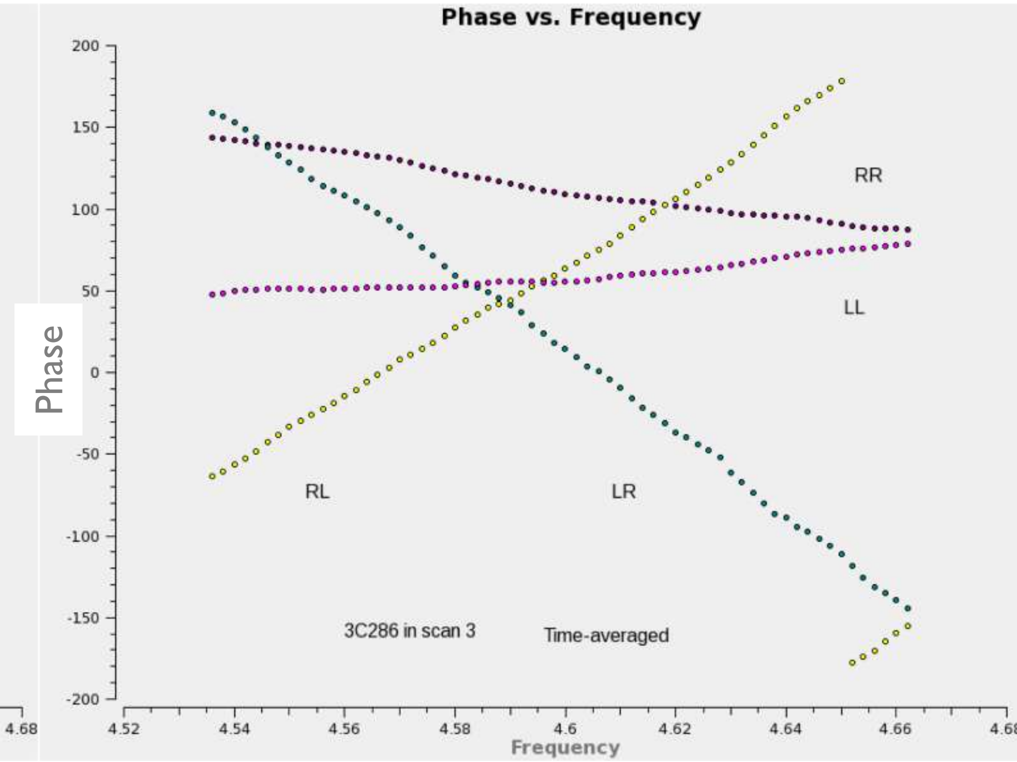
Single Baseline, Single Scan Visibility Spectra (4 correlations)



baseline ea17-ea21

Single Baseline, Single Scan (time-averaged) Visibility Spectra (4 correlations)



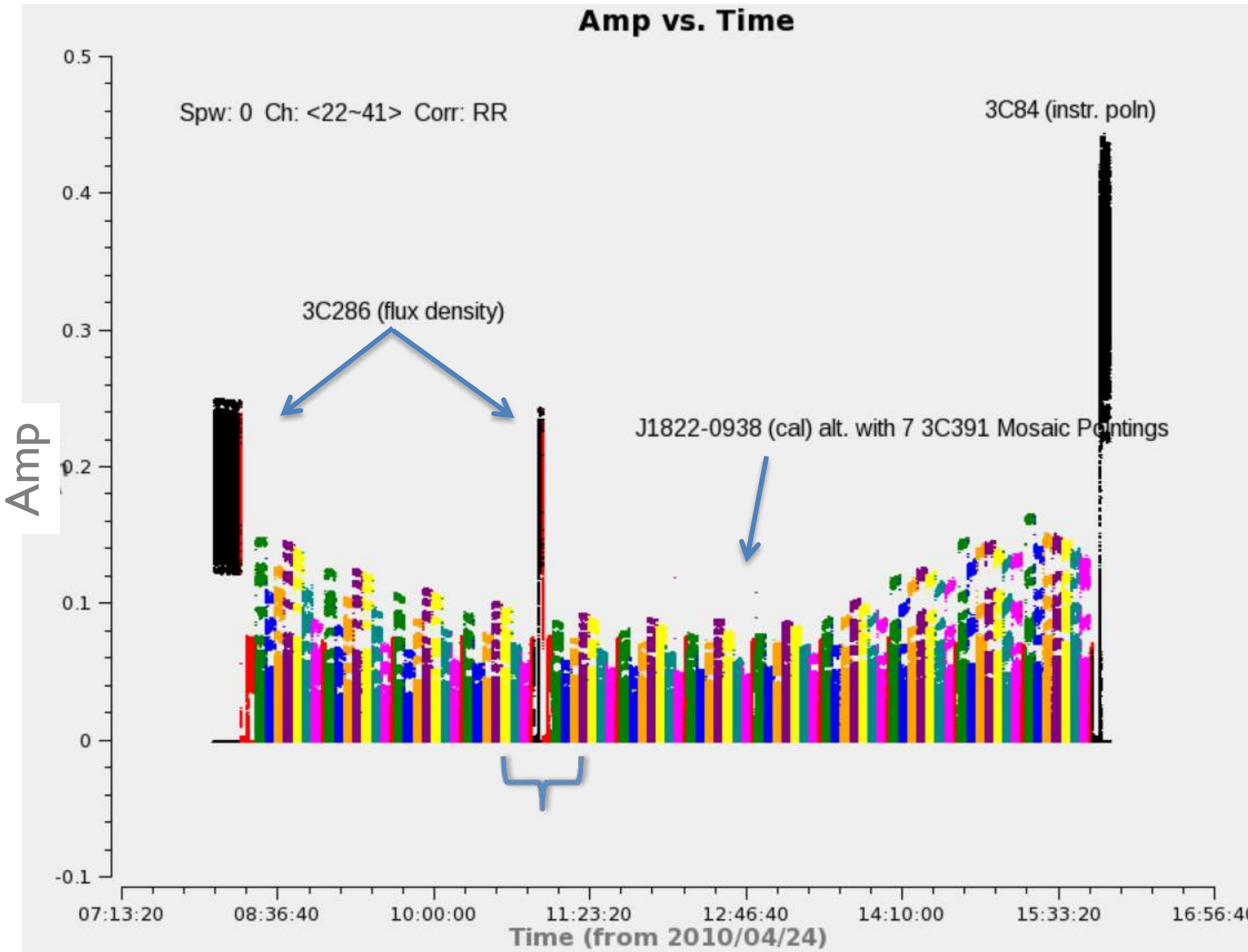


baseline ea17-ea21

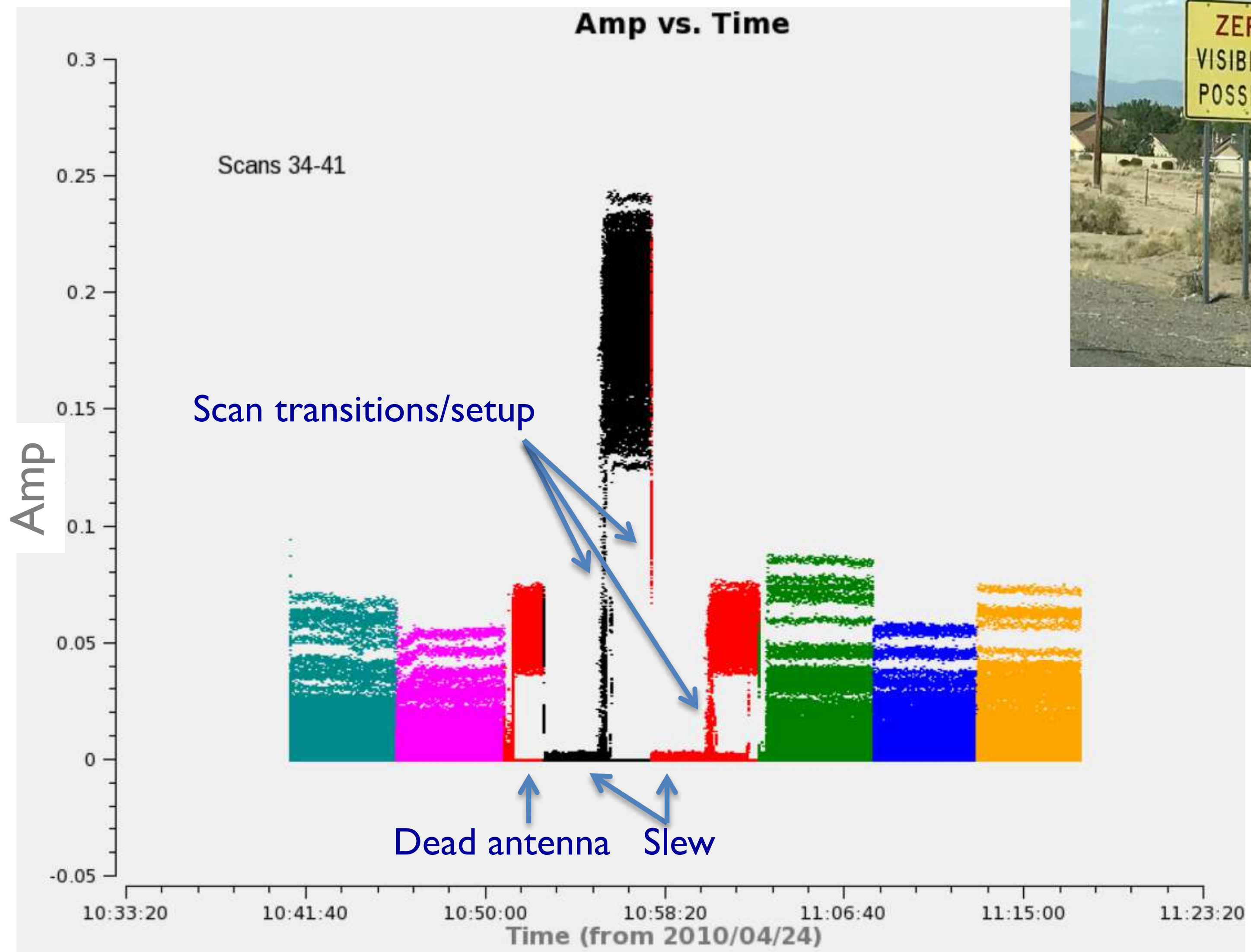
Data Examination and Editing

- After observation, initial data examination and editing very important
  - Will observations meet goals for calibration and science requirements?
- What to edit (much of this is now automated):
  - Some real-time flagging occurred during observation (antennas off-source, LO out-of-lock, etc.). Any such bad data left over? (check operator's logs)
  - Any persistently 'dead' antennas (check operator's logs)
  - Periods of especially poor weather? (check operator's log)
  - Any antennas shadowing others? Edit such data.
  - Amplitude and phase should be continuously varying—edit outliers
  - Radio Frequency Interference (RFI)?
- Caution:
  - Be careful editing noise-dominated data.
  - Be conservative: those antennas/timeranges which are obviously bad on calibrators are probably (less obviously) bad on weak target sources—edit them
  - Distinguish between bad (hopeless) data and poorly-calibrated data. E.g., some antennas may have significantly different amplitude response which may not be fatal—it may only need to be calibrated
  - Choose (phase) reference antenna wisely (ever-present, stable response)
- Increasing data volumes increasingly demand automated editing algorithms…

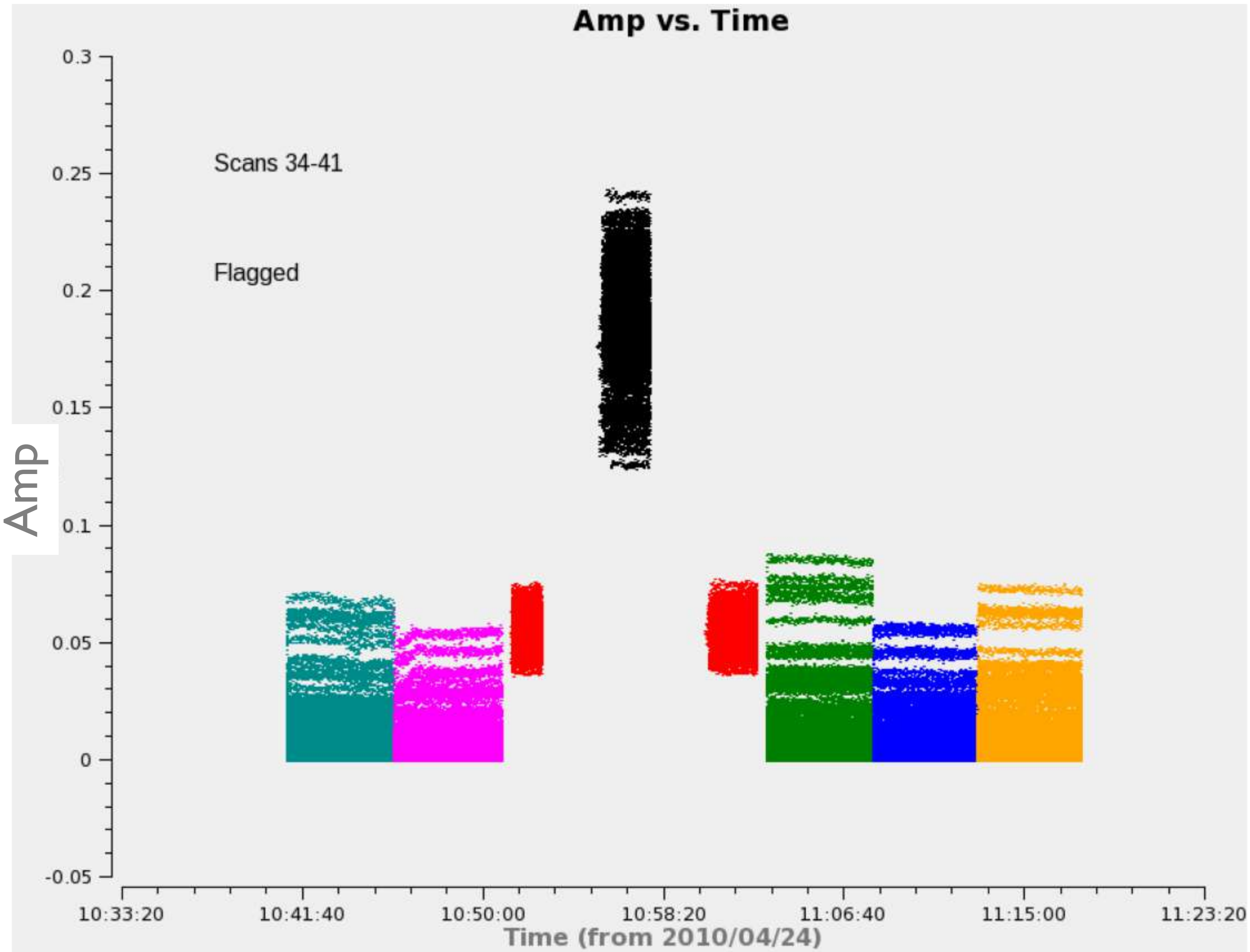
Editing Example



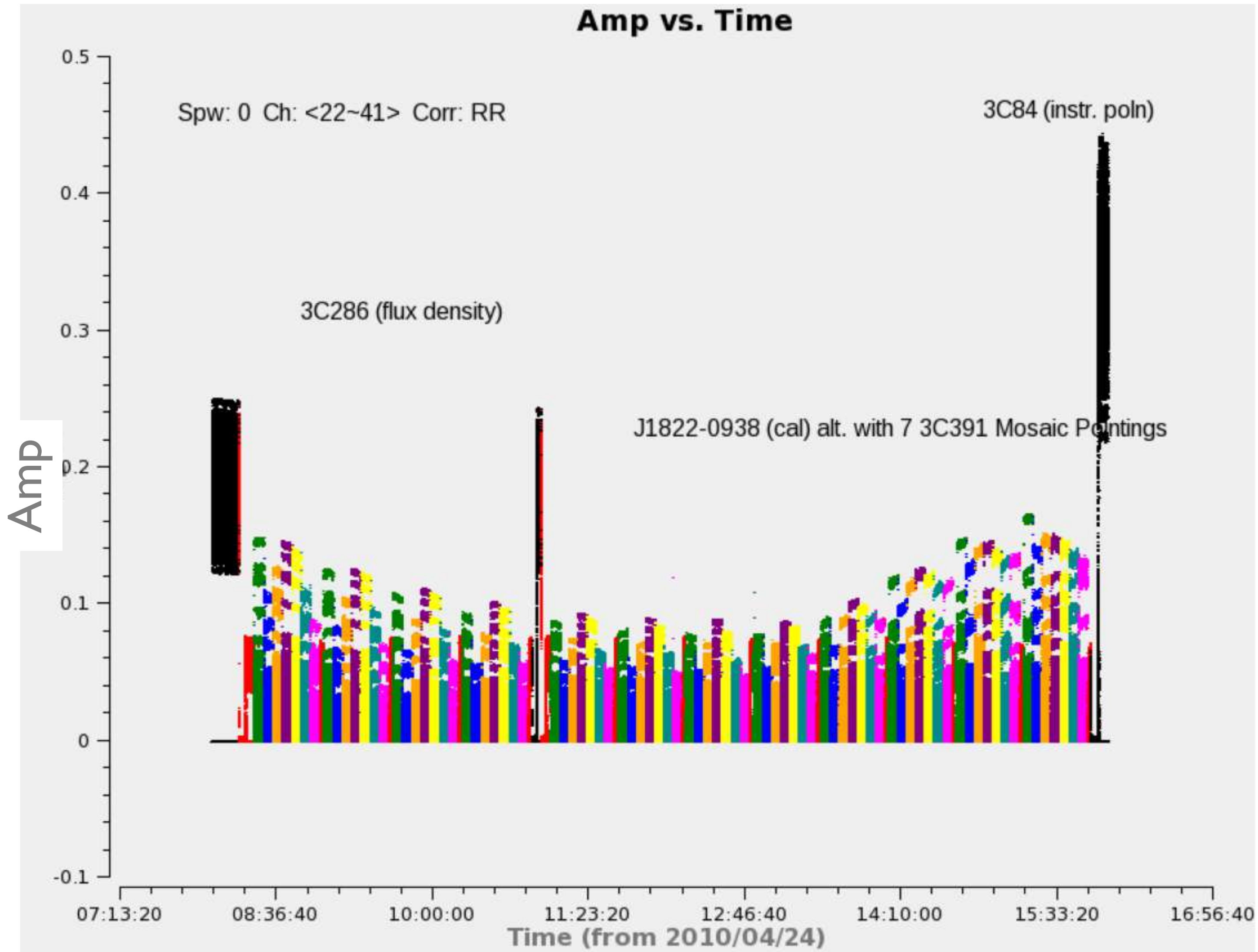
Editing Example



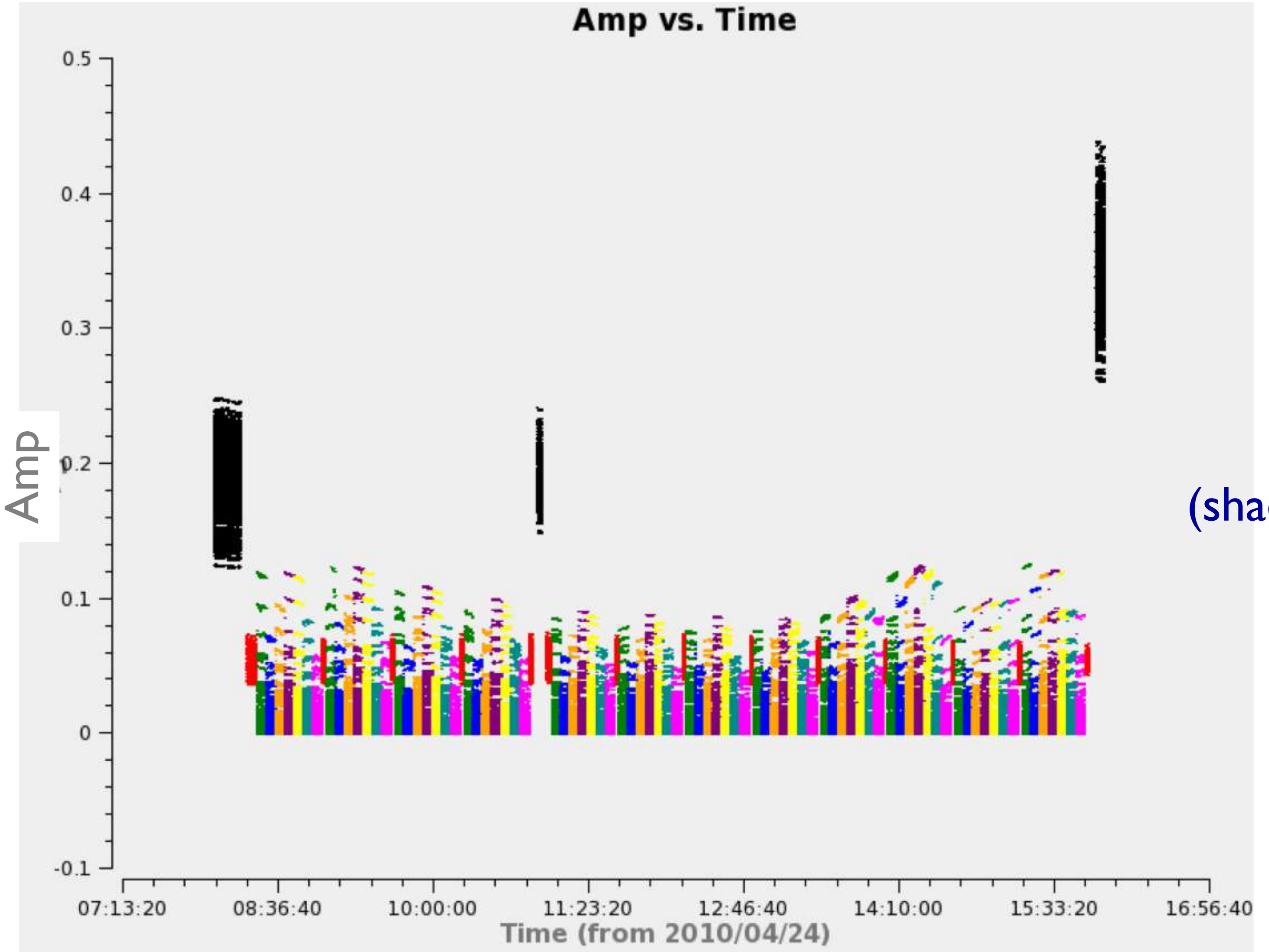
Editing Example



Editing Example (before)



Editing Example (after)



(shadowing)

Calibration II

George Moellenbrock, NRAO

Sixteenth Synthesis Imaging Workshop

16-23 May 2018

Synopsis

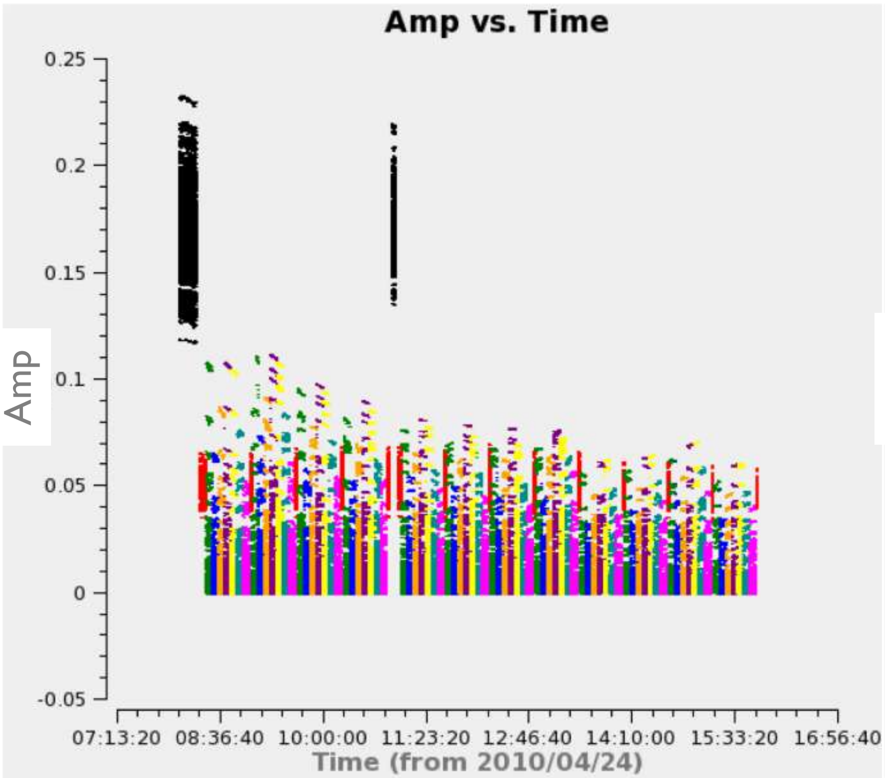
- Calibration I
  - Why do we have to calibrate?
  - Review Idealistic formalism → Realistic practice
  - Fundamental Calibration Principles
    - Practical Calibration Considerations
    - Baseline-based vs. Antenna-based Calibration
    - Solving for calibration
  - An example Visibility dataset
    - Flagging
- Calibration II
  - Scalar Calibration Example
  - Generalizations & Specializations
    - Full Polarization
    - A Dictionary of Calibration Effects
  - Calibration Heuristics and 'Bootstrapping'
  - New Calibration Challenges
  - Summary

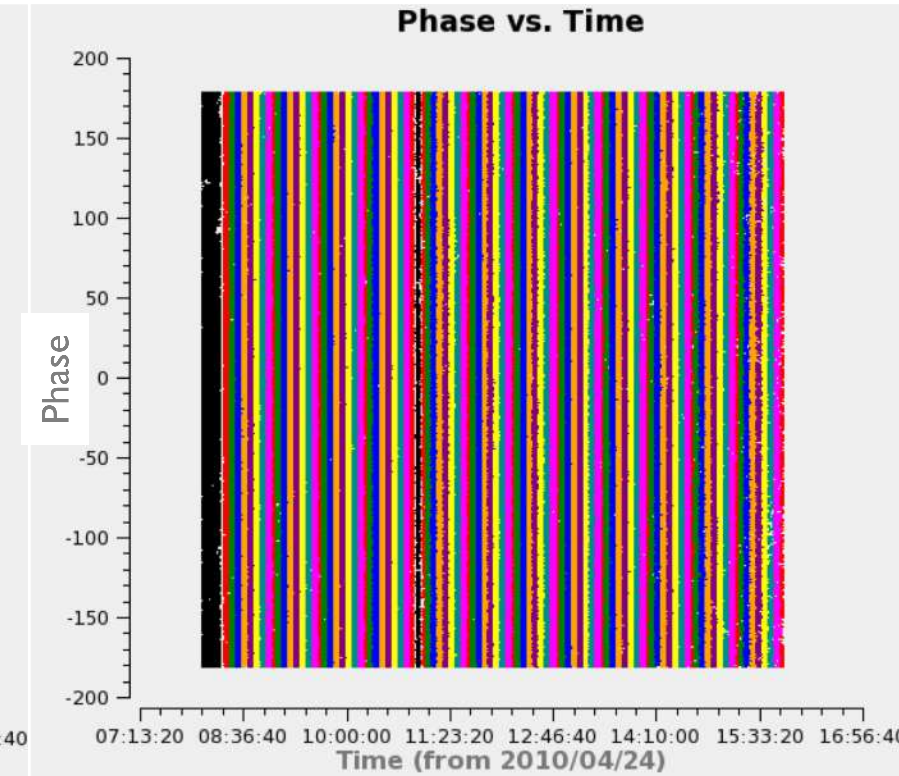
Simple Scalar Calibration Example

- Array:
  - EVLA D-configuration (Apr 2010)
- Sources:
  - Science Target: 3C391, a SNR (7 mosaic pointings)
  - Near-target calibrator: J1822-0938 (~11 deg from target; unknown flux density, assumed 1 Jy)
  - Flux Density calibrator: 3C286 (7.747 Jy, essentially unresolved)
- Signals (simplified for this example):
  - RR correlation only for this illustration (total intensity only)
  - One spectral window centered at 4600 MHz, 128 MHz bandwidth
  - 64 observed spectral channels averaged with normalized bandpass calibration applied (this illustration considers only the time-dependent 'gain' calibration)
  - (extracted from a continuum polarimetry mosaic observation)

Views of the Uncalibrated Data

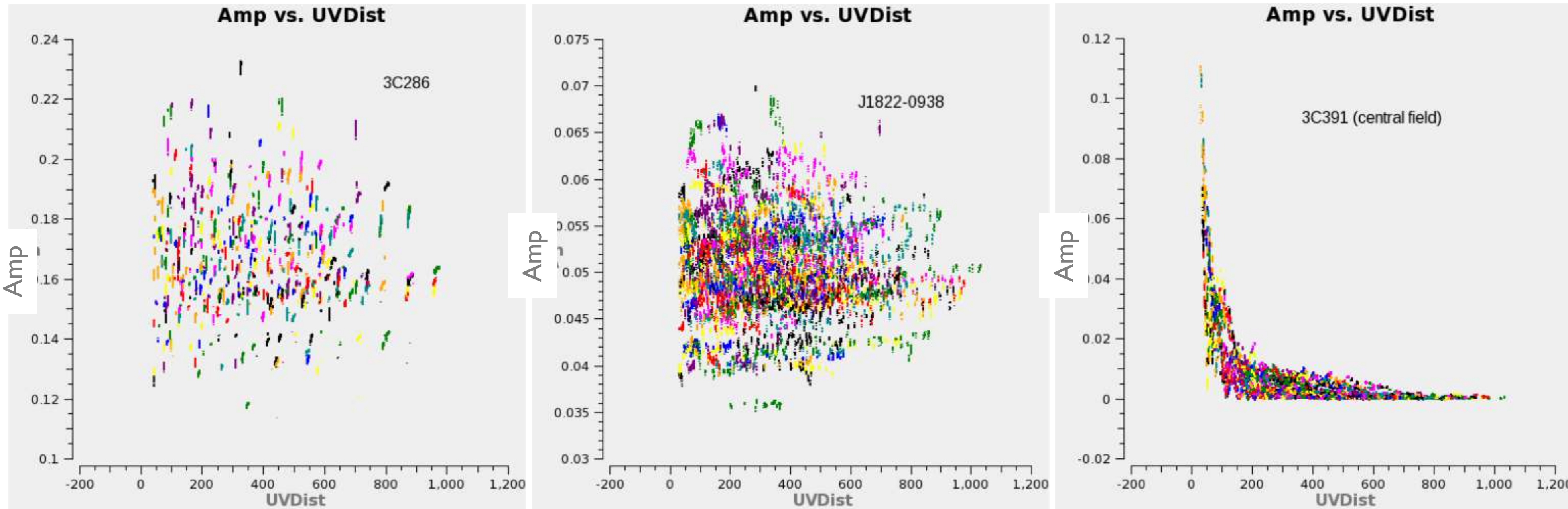
Field colors





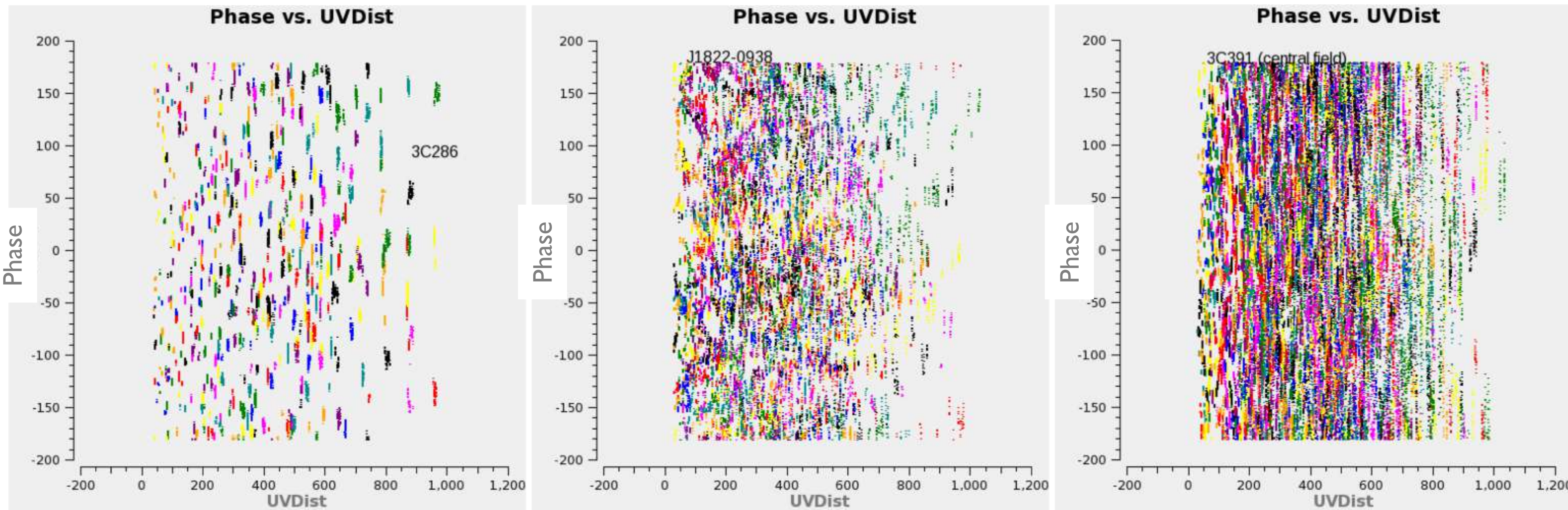
Views of the Uncalibrated Data

Baseline colors

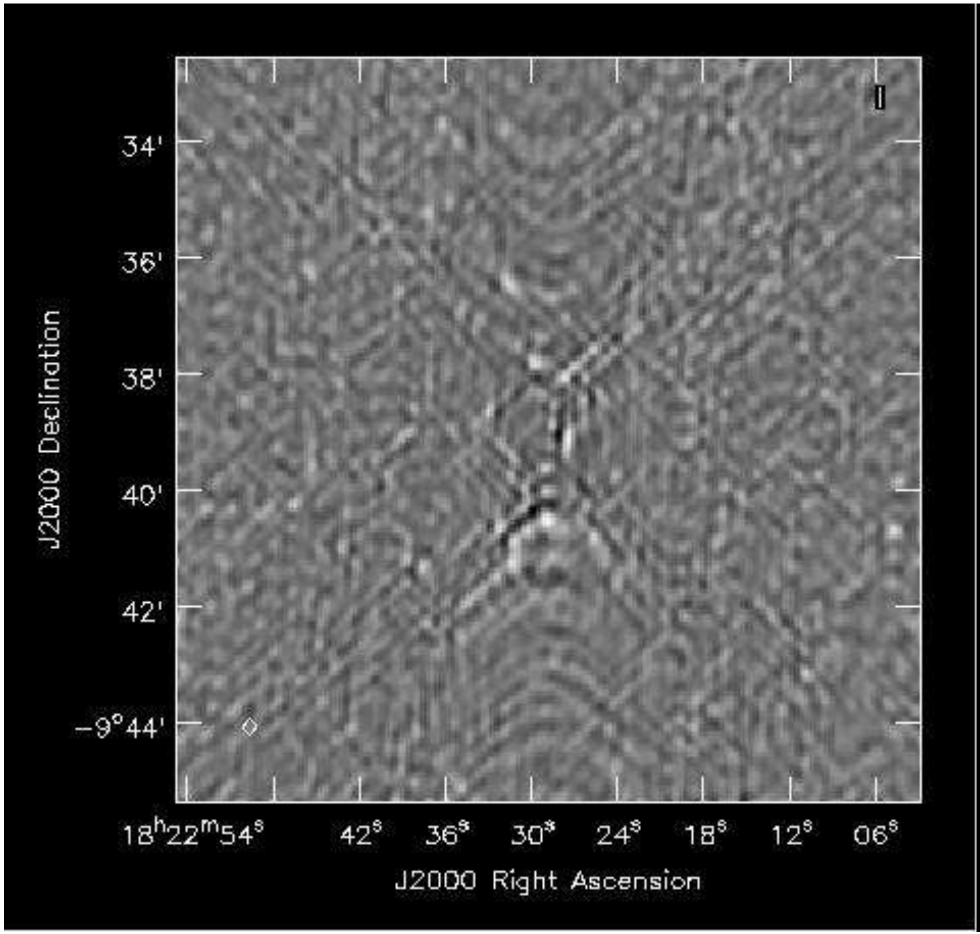


Views of the Uncalibrated Data

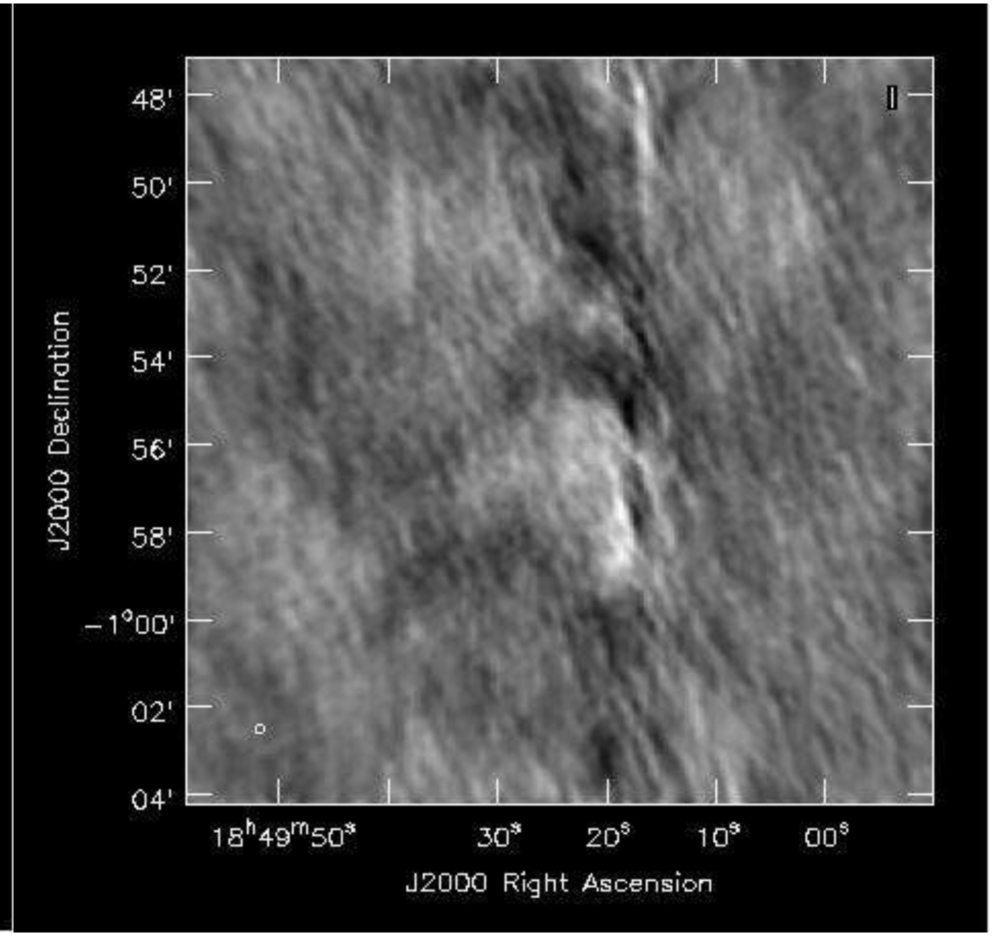
Baseline colors



Uncalibrated Images

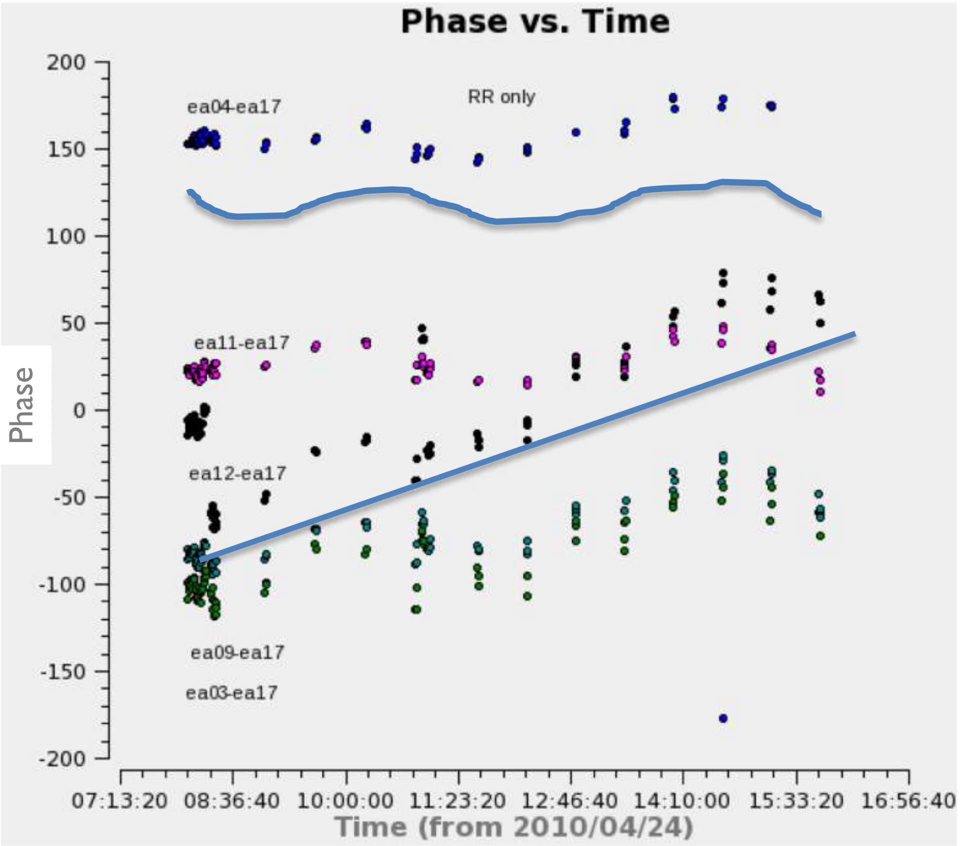


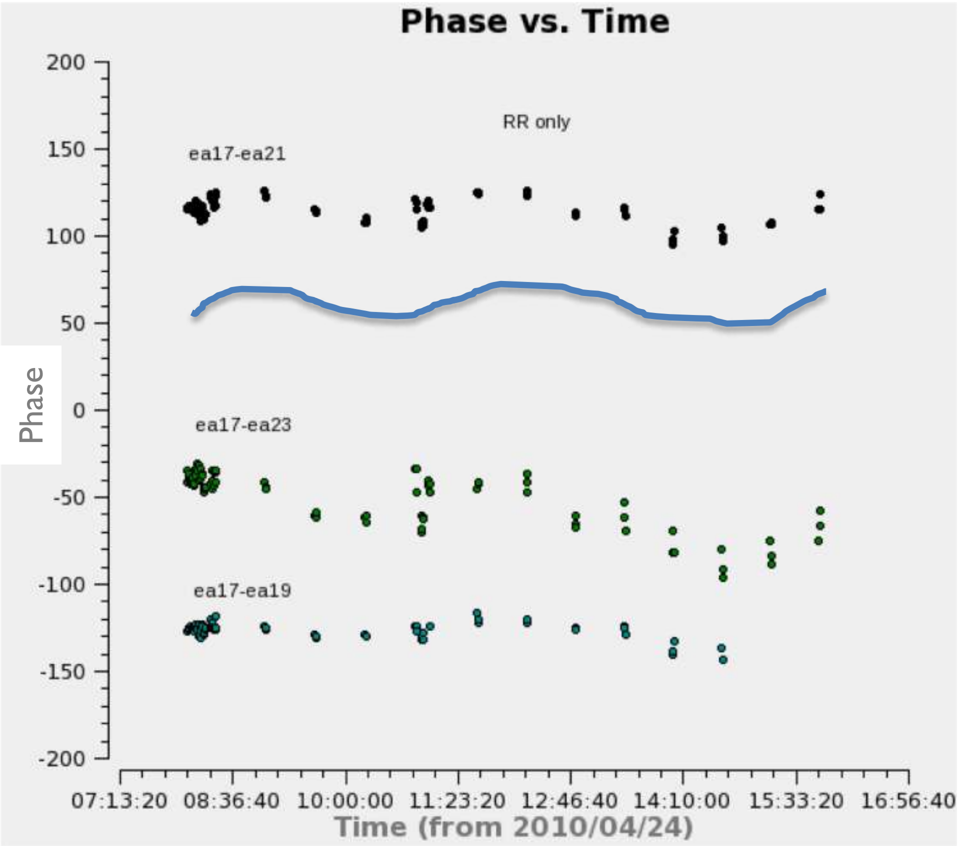
J1822-0938
(calibrator)



3C391
(science)

Rationale for Antenna-based Calibration





The Calibration Process

- Solve (LS) for antenna-based gain factors for each scan on all calibrators (*V^{mod}=S* for f.d. calibrator; *V^{mod}=1.0* for others) :

$$V_{ij}^{obs} = G_i G_j^* \; V_{ij}^{mod}$$

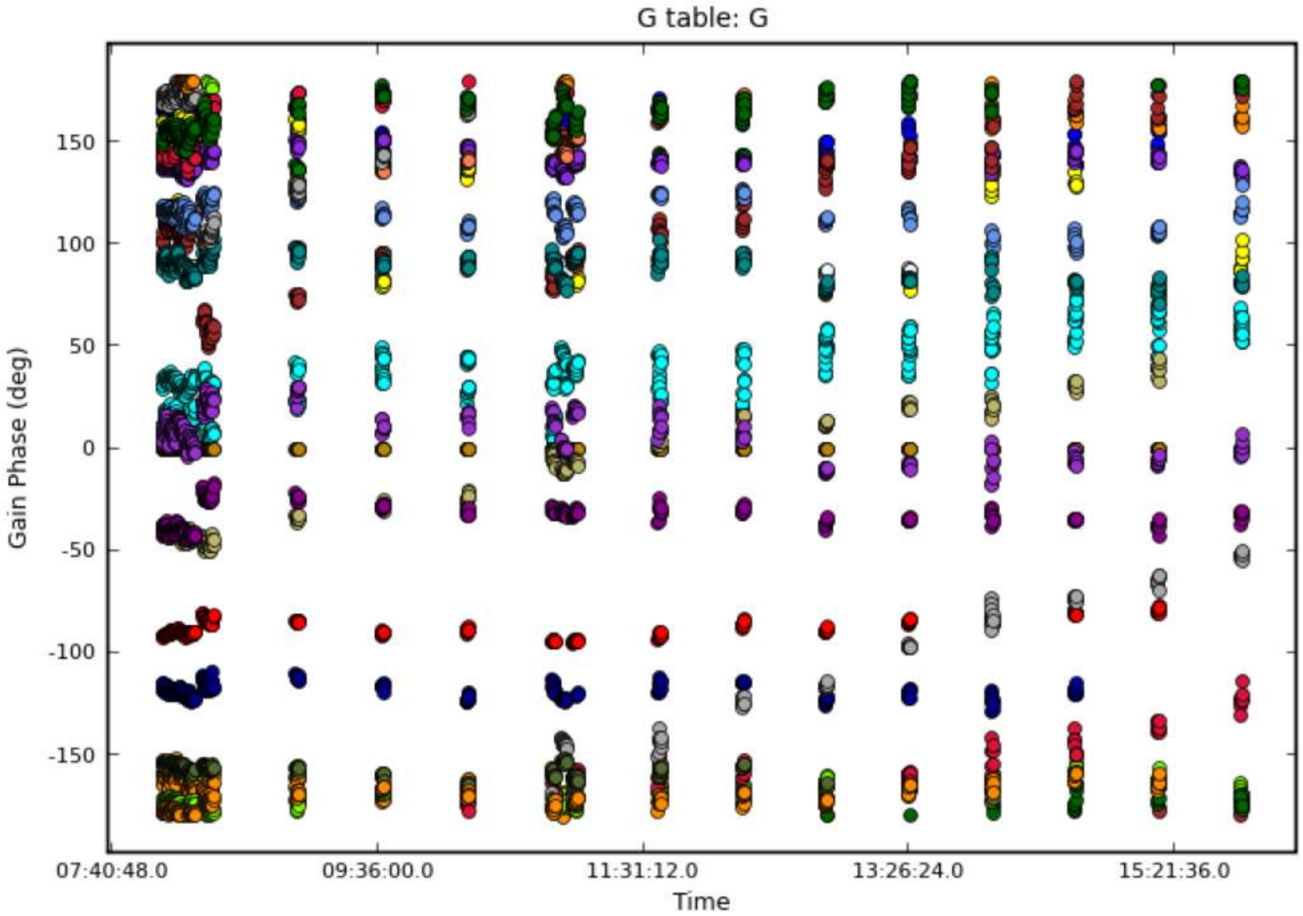
- Bootstrap flux density scale by enforcing gain amplitude consistency over all calibrators:

$$\left\langle \frac{|G_i|}{|G_i(fd\ cal)|} \right\rangle_{time,antennas} = 1.0$$

- Correct data (interpolate, as needed):

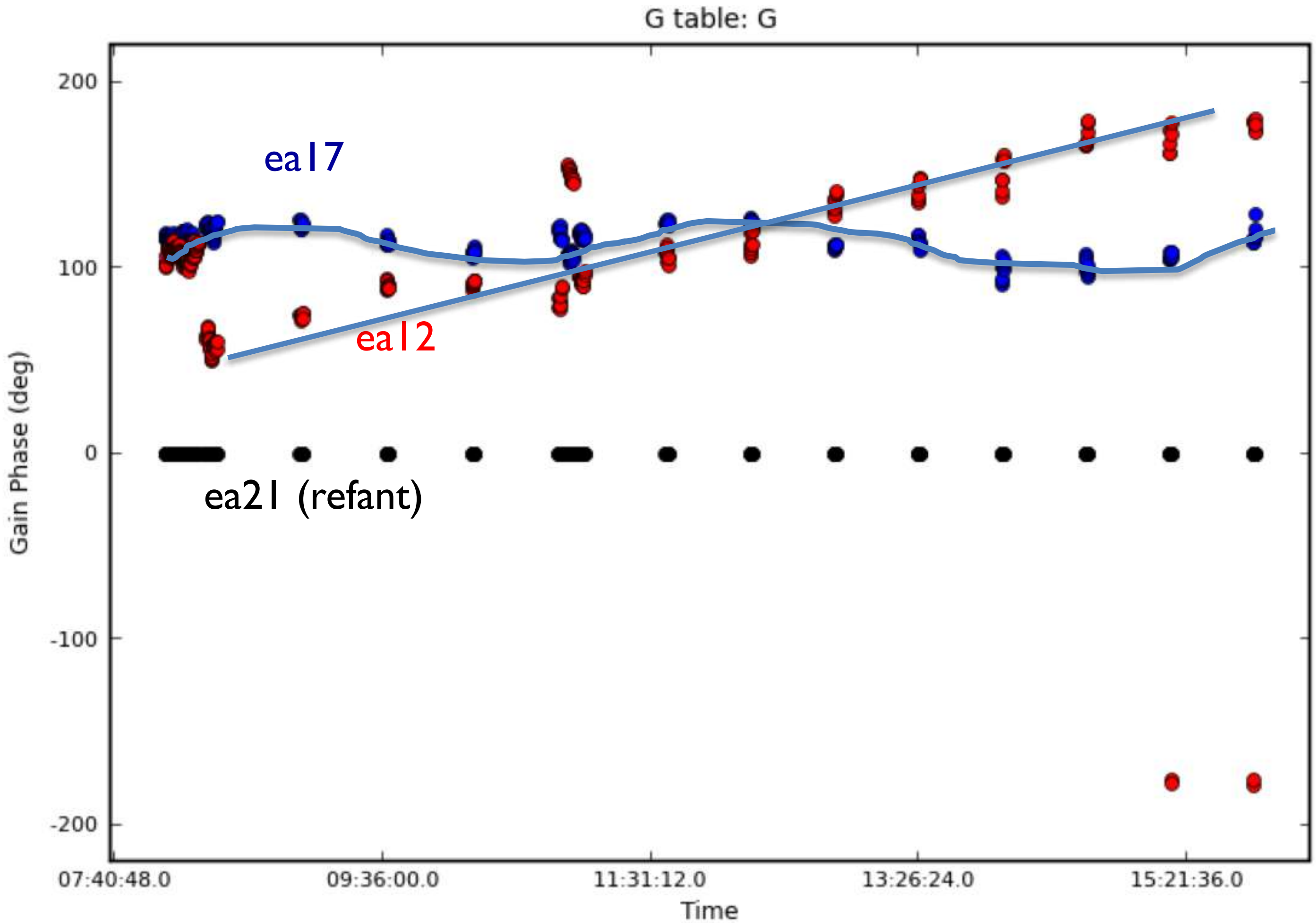
$$V_{ij}^{cor} = G_i^{-1} G_j^{*-1} \; V_{ij}^{obs}$$

The Antenna-based Calibration Solution

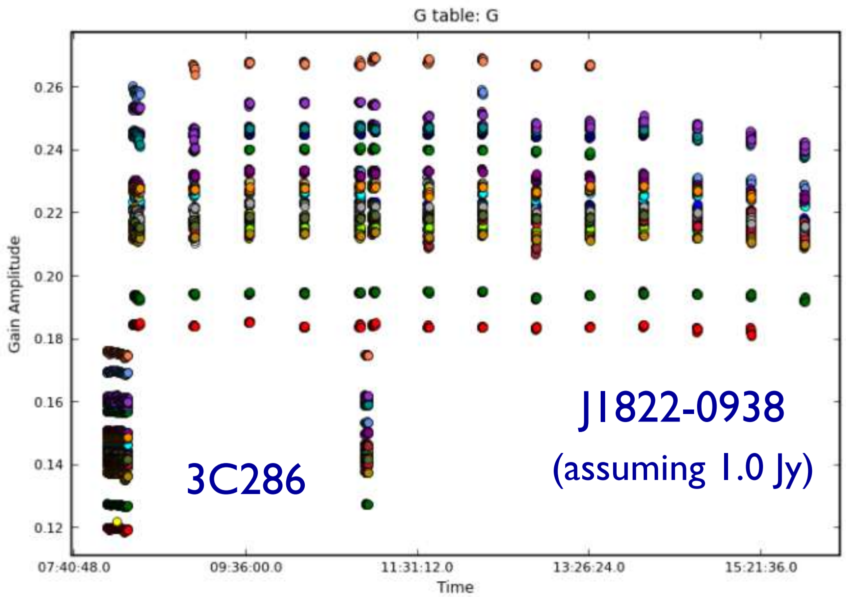


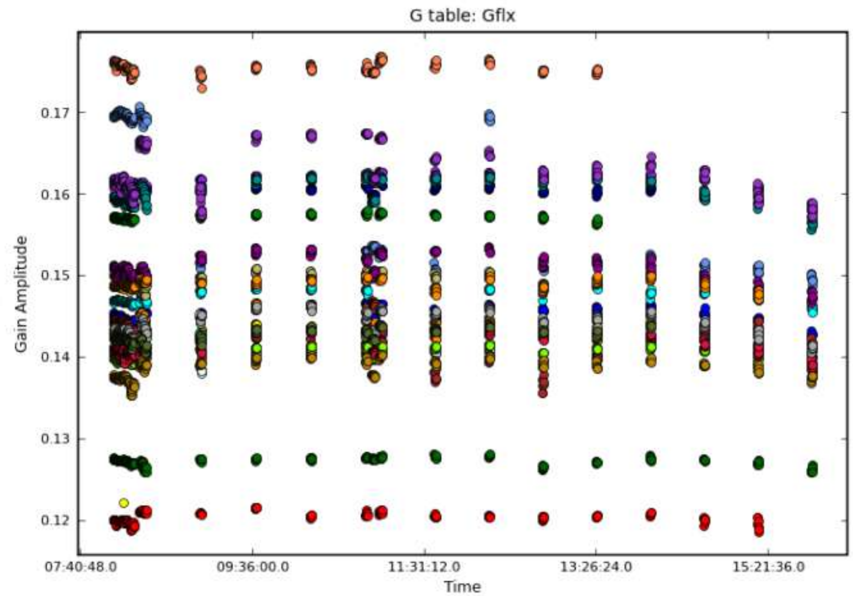
- Reference antenna: ea21 (phase = 0)

The Antenna-based Calibration Solution



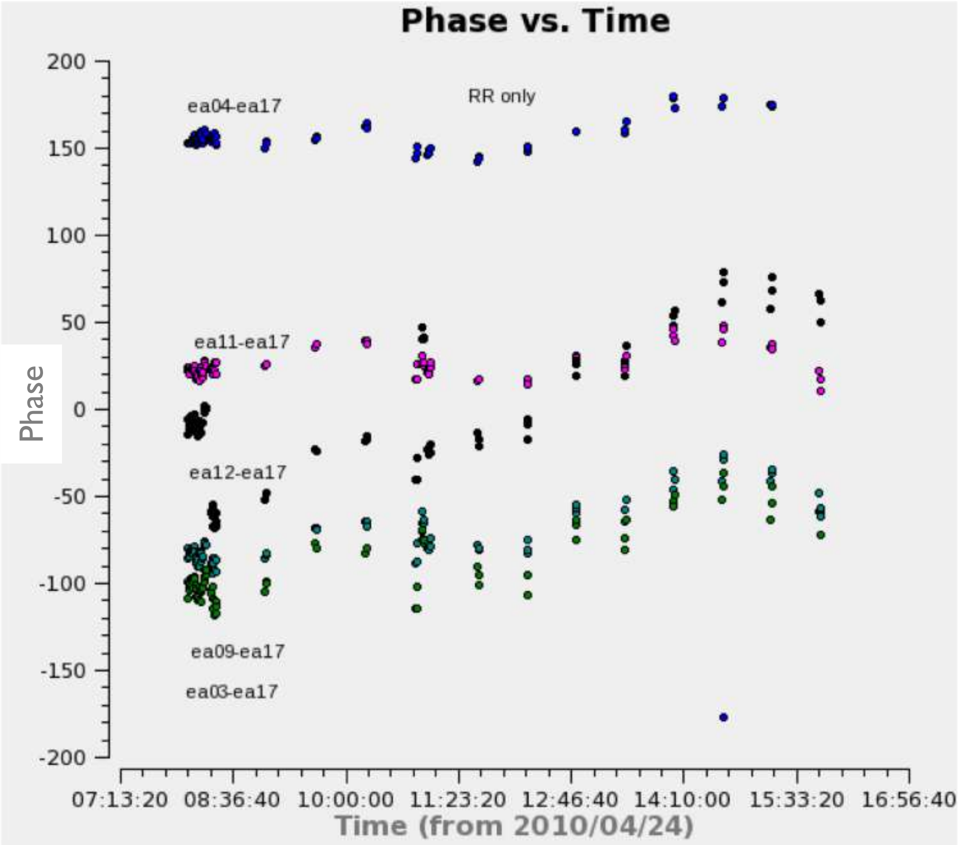
Flux Density Bootstrapping



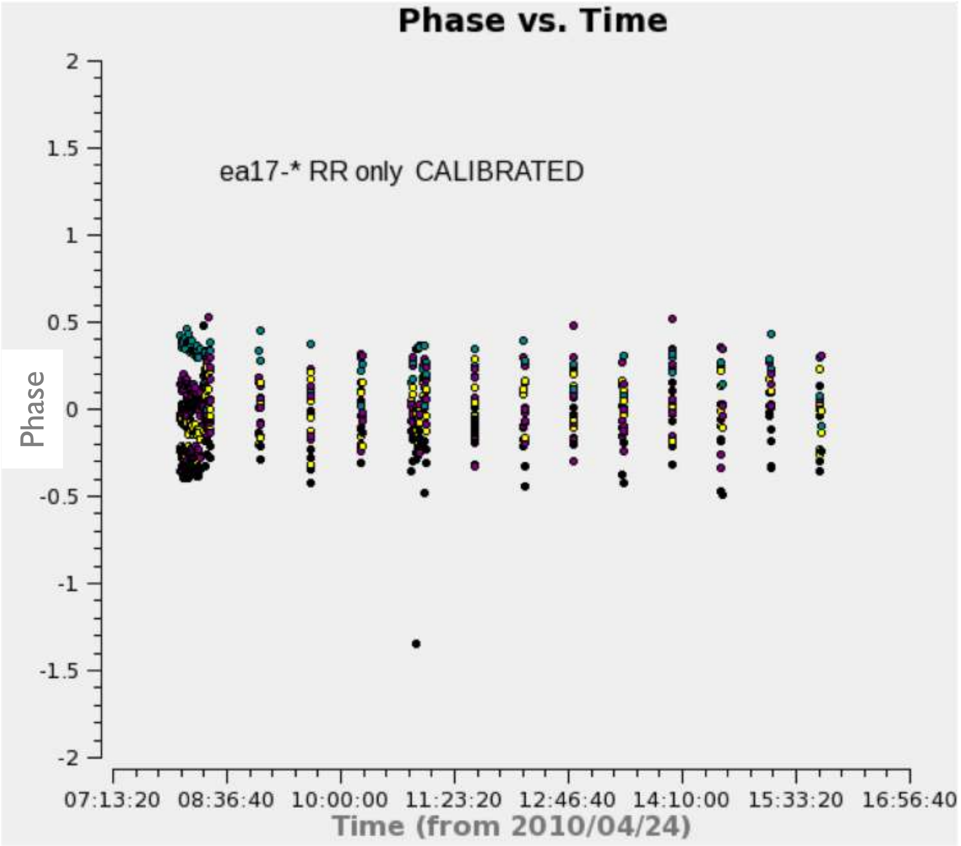


- 3C286's gains have correct scale $\left(\sim\sqrt{\mathrm{Jy}^{-1}}\right)$
- Thus, J1822-0938 is 2.32 Jy (not 1.0 Jy, as assumed)

Effect of Antenna-based Calibration: Phase (before)

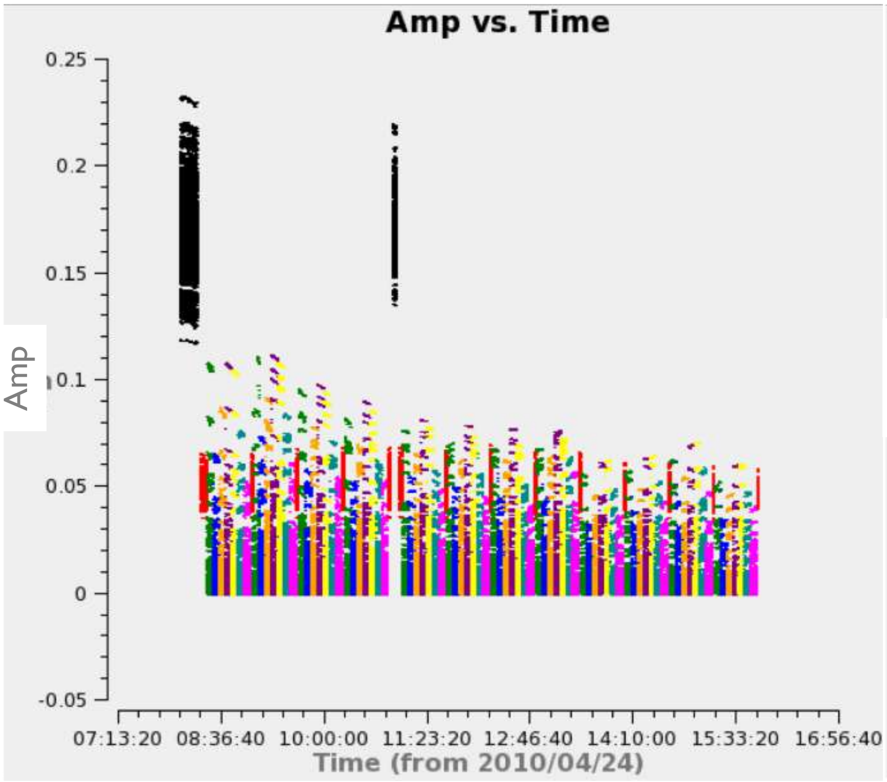


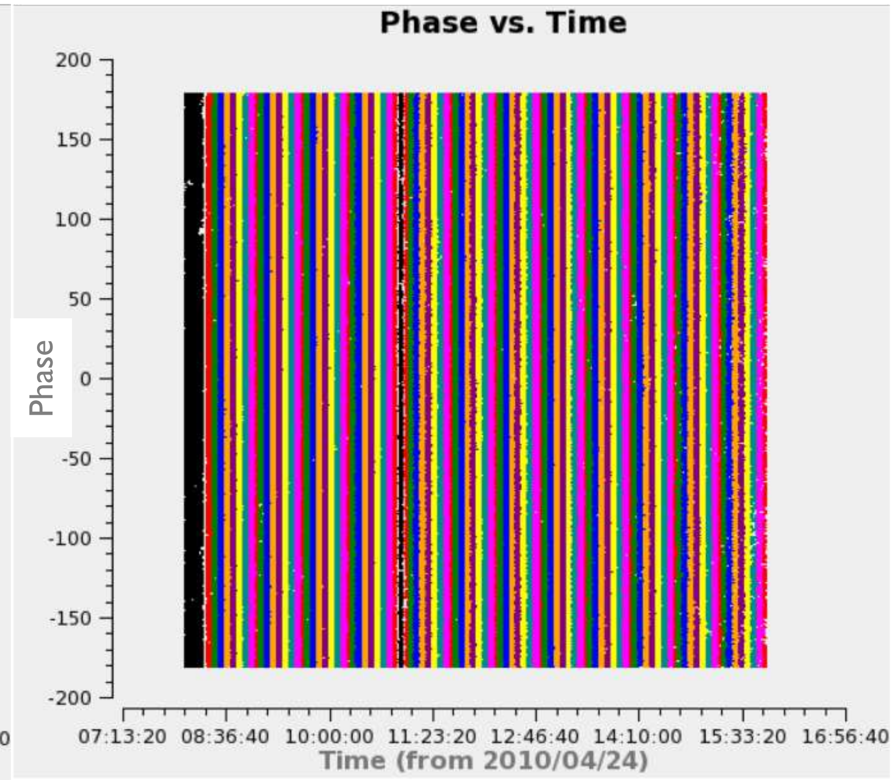
Effect of Antenna-based Calibration Phase (after)



Effect of Antenna-based Calibration

Field colors

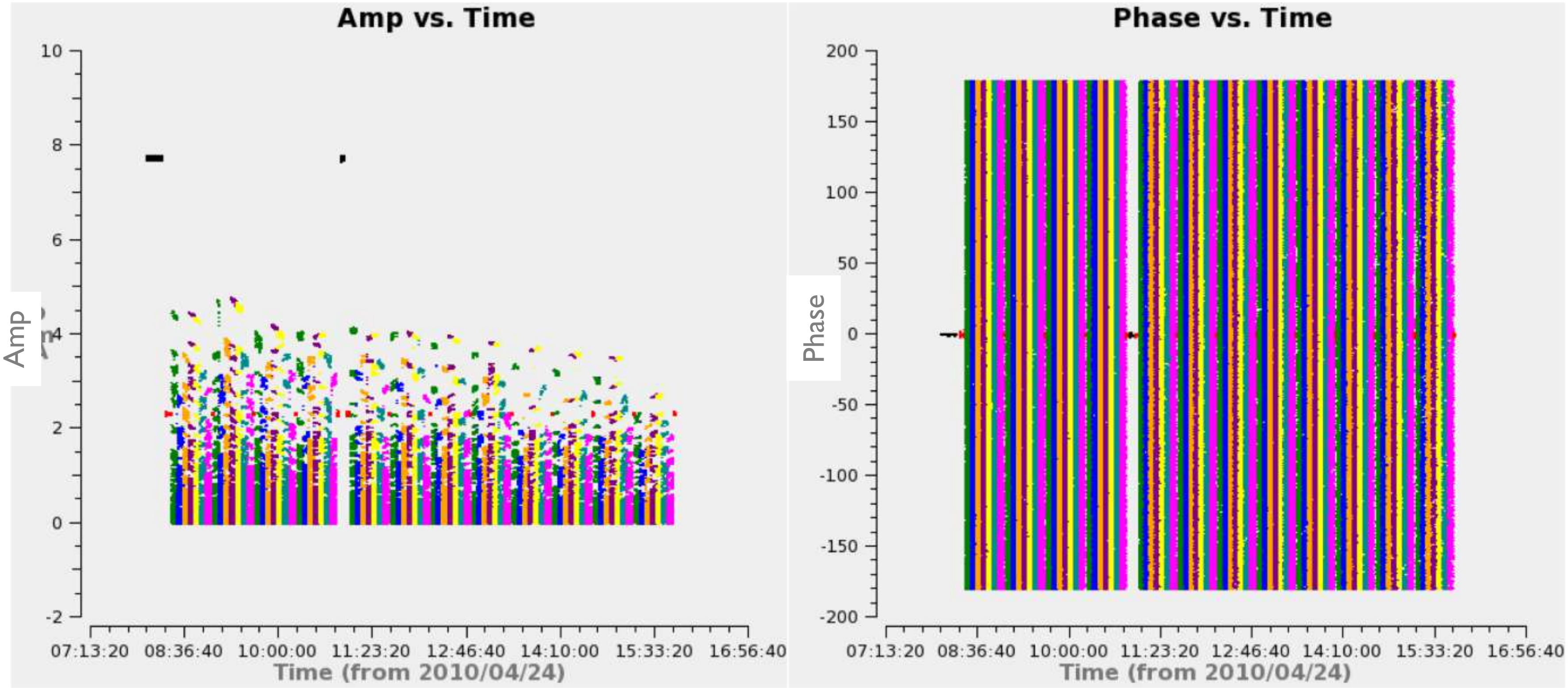




UNCALIBRATED

Effect of Antenna-based Calibration

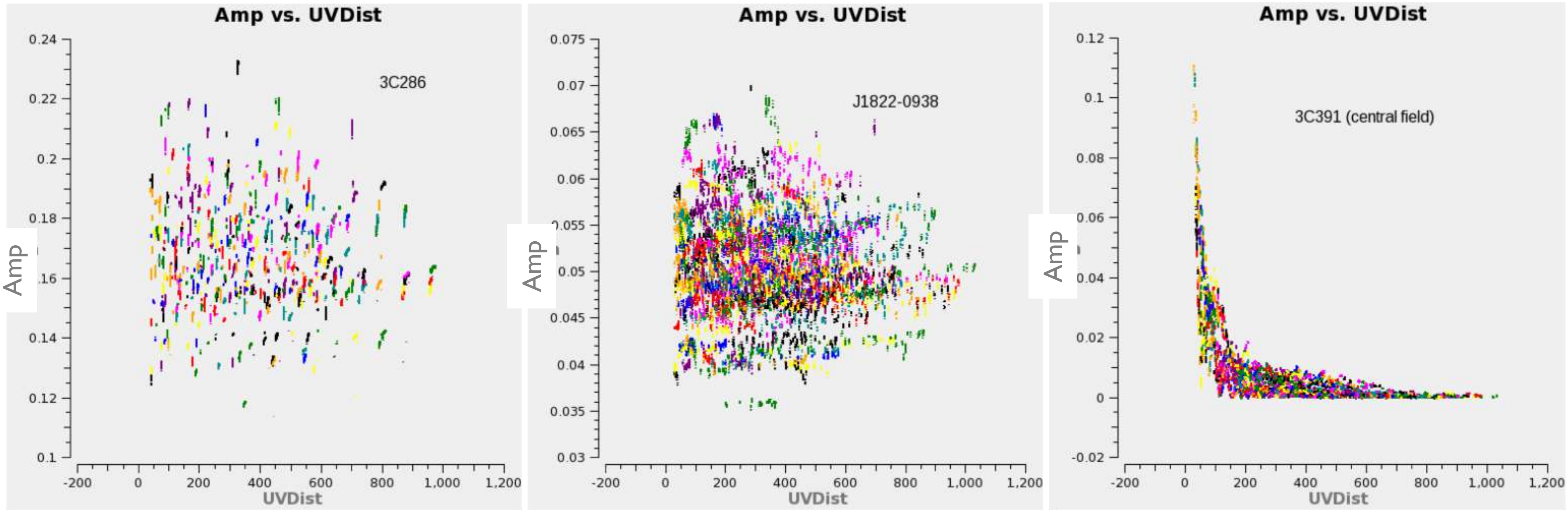
Field colors



CALIBRATED

Effect of Antenna-based Calibration

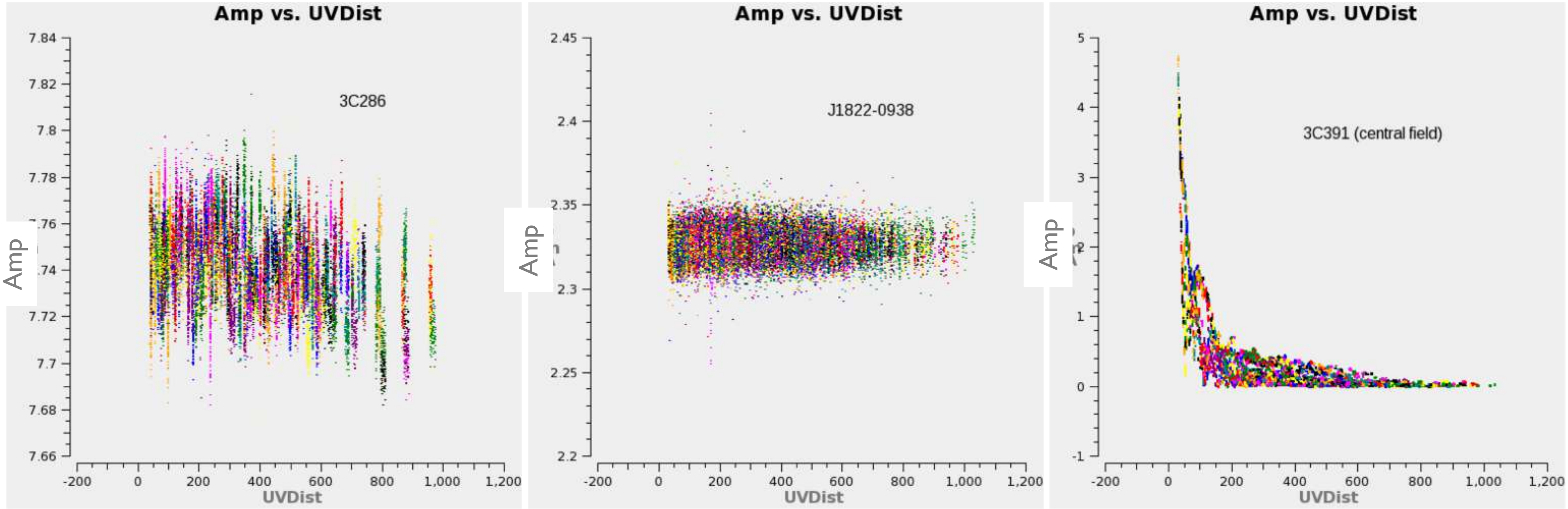
Baseline colors



UNCALIBRATED

Effect of Antenna-based Calibration

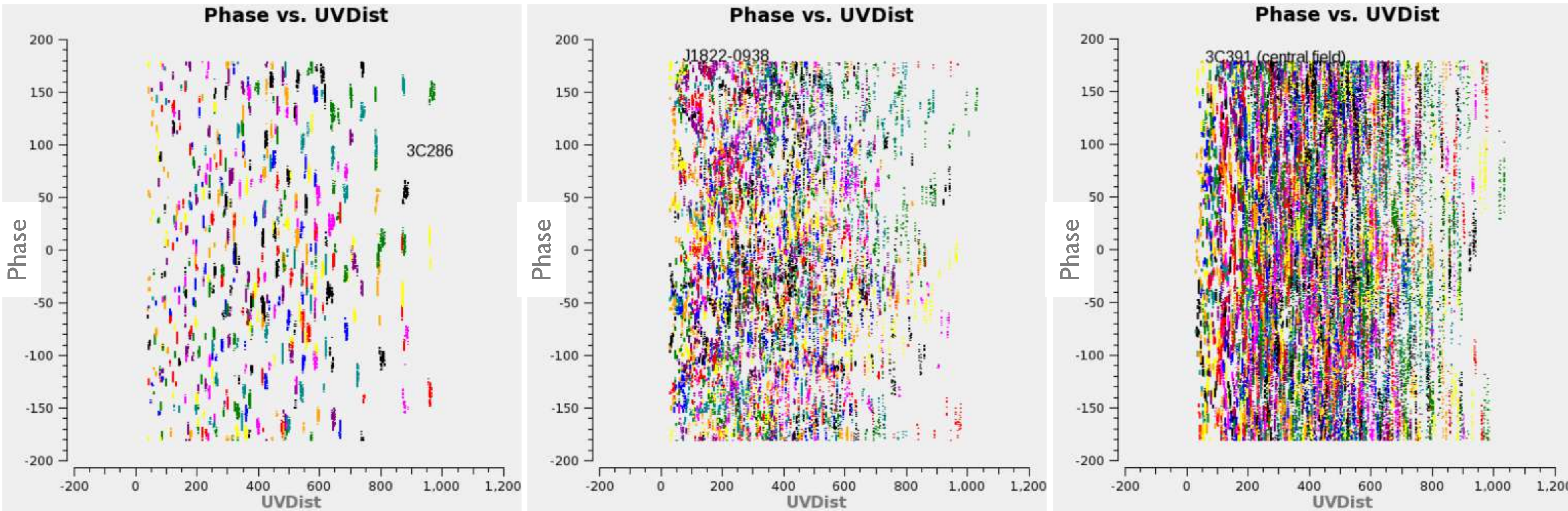
Baseline colors



CALIBRATED

Effect of Antenna-based Calibration

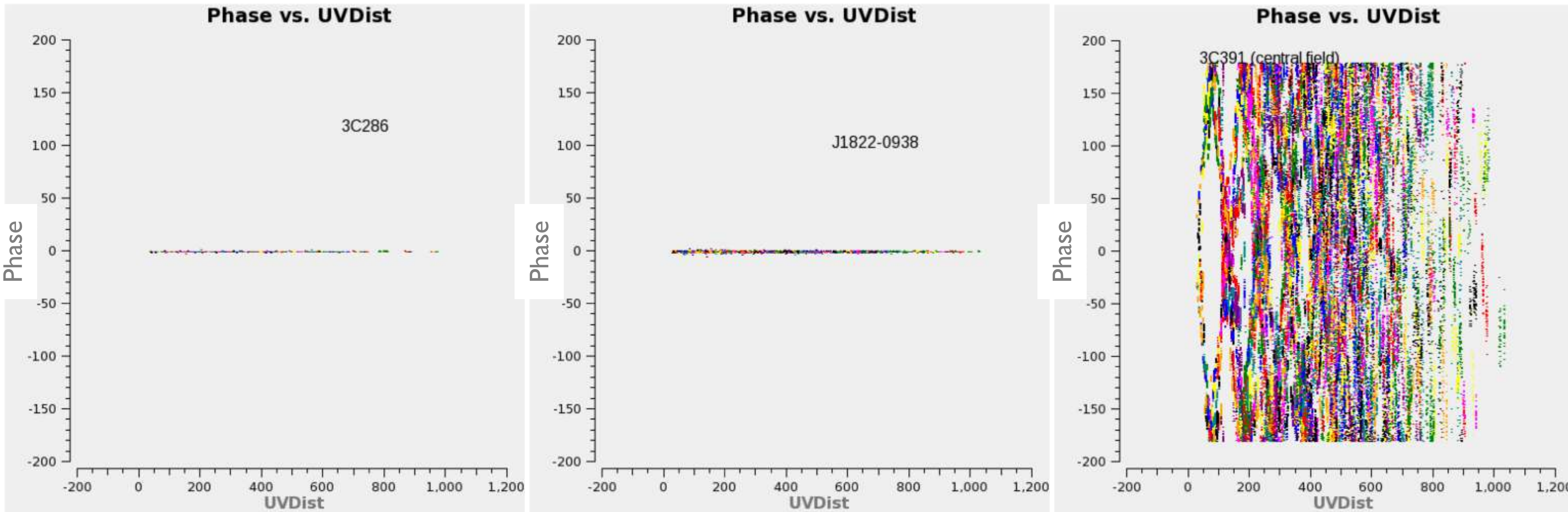
Baseline colors



UNCALIBRATED

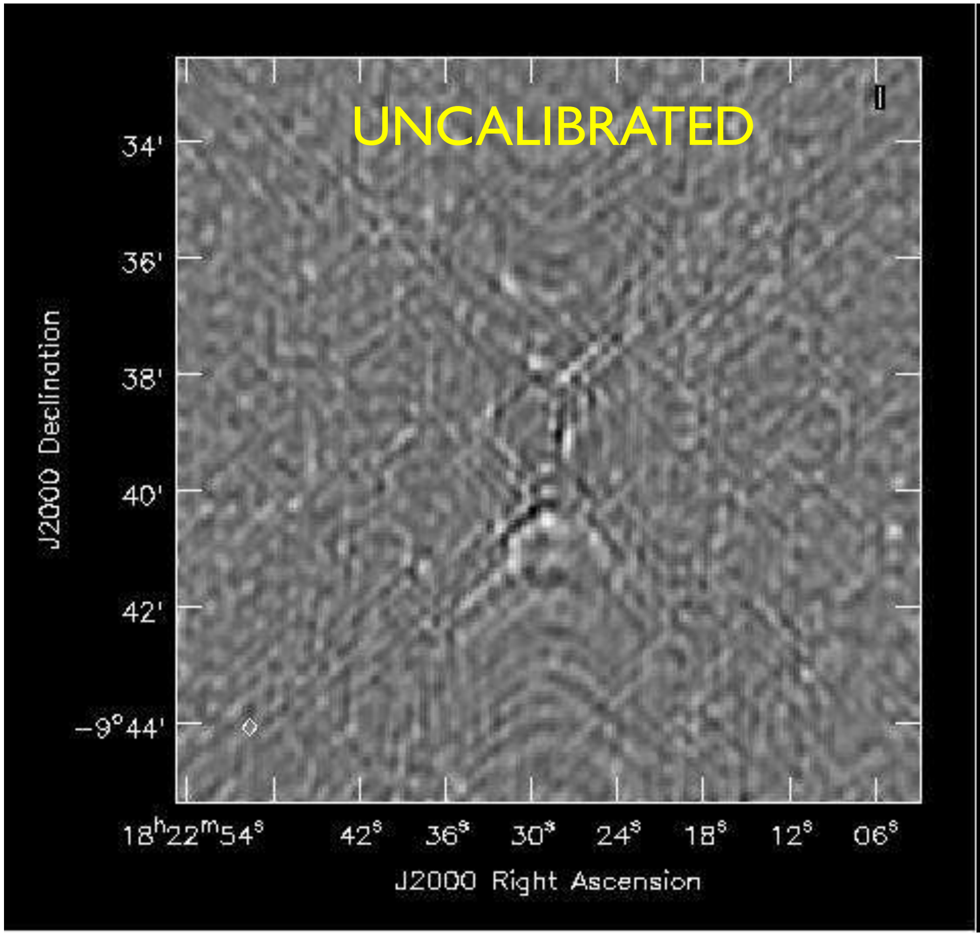
Effect of Antenna-based Calibration

Baseline colors

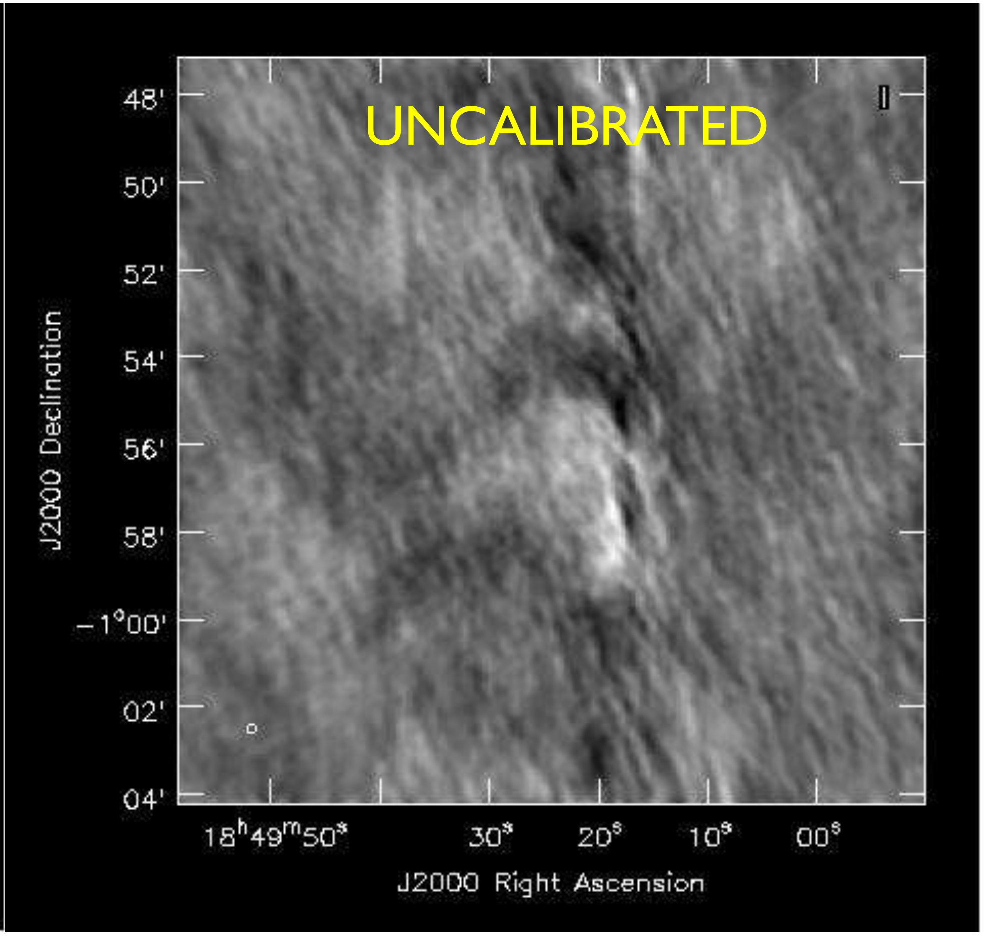


CALIBRATED

Calibration Effect on Imaging

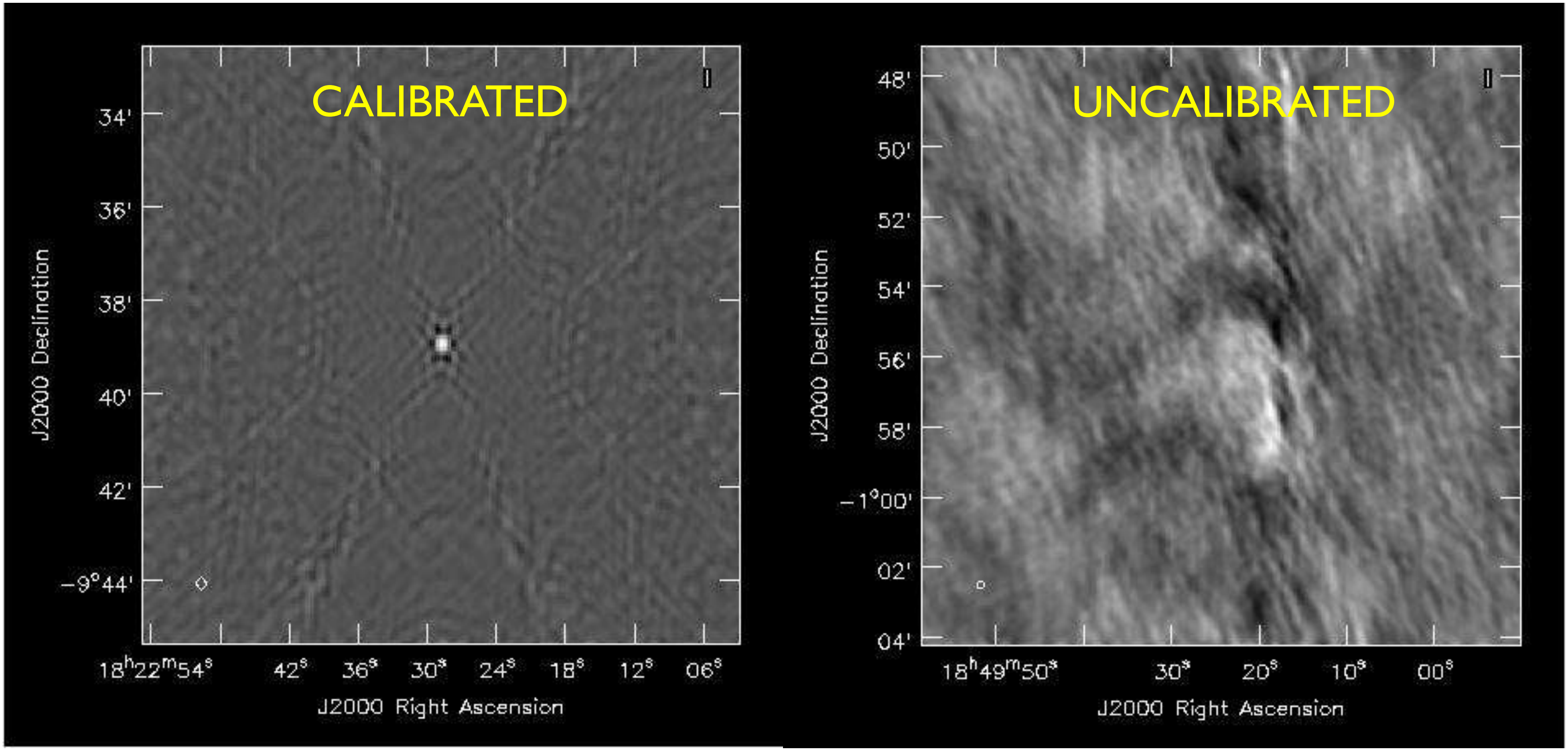


J1822-0938
(calibrator)



3C391
(science)

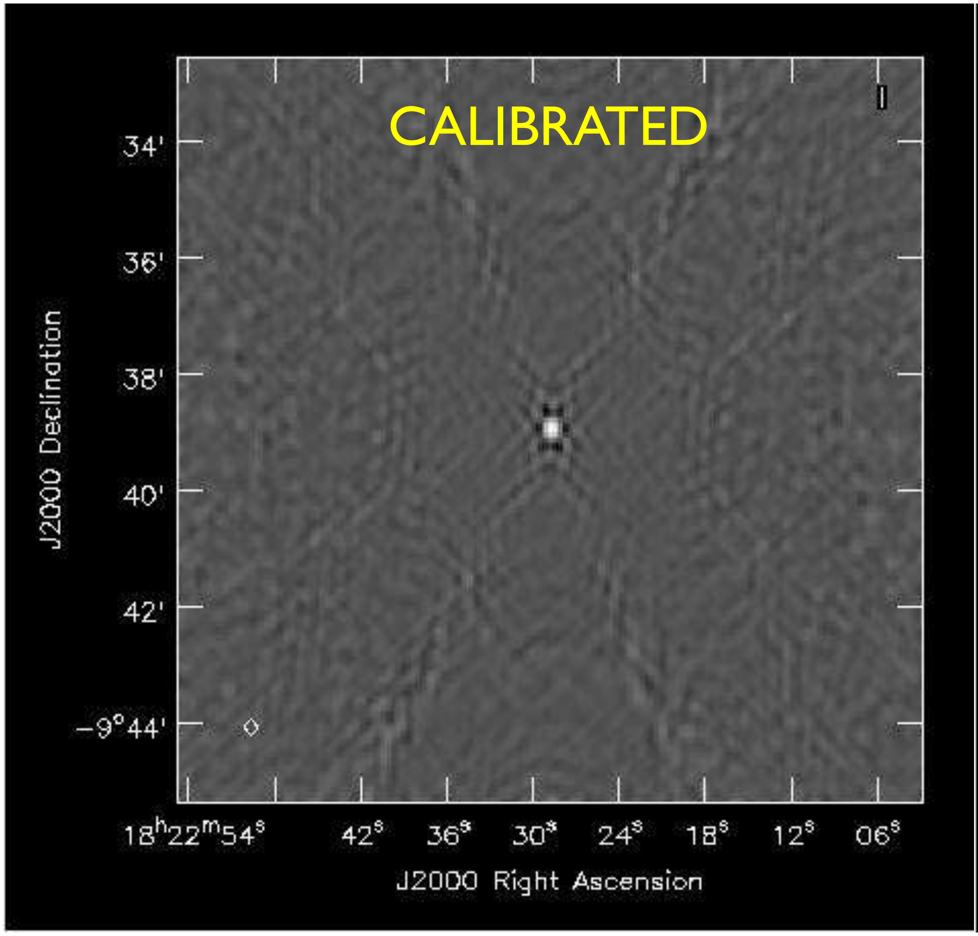
Calibration Effect on Imaging



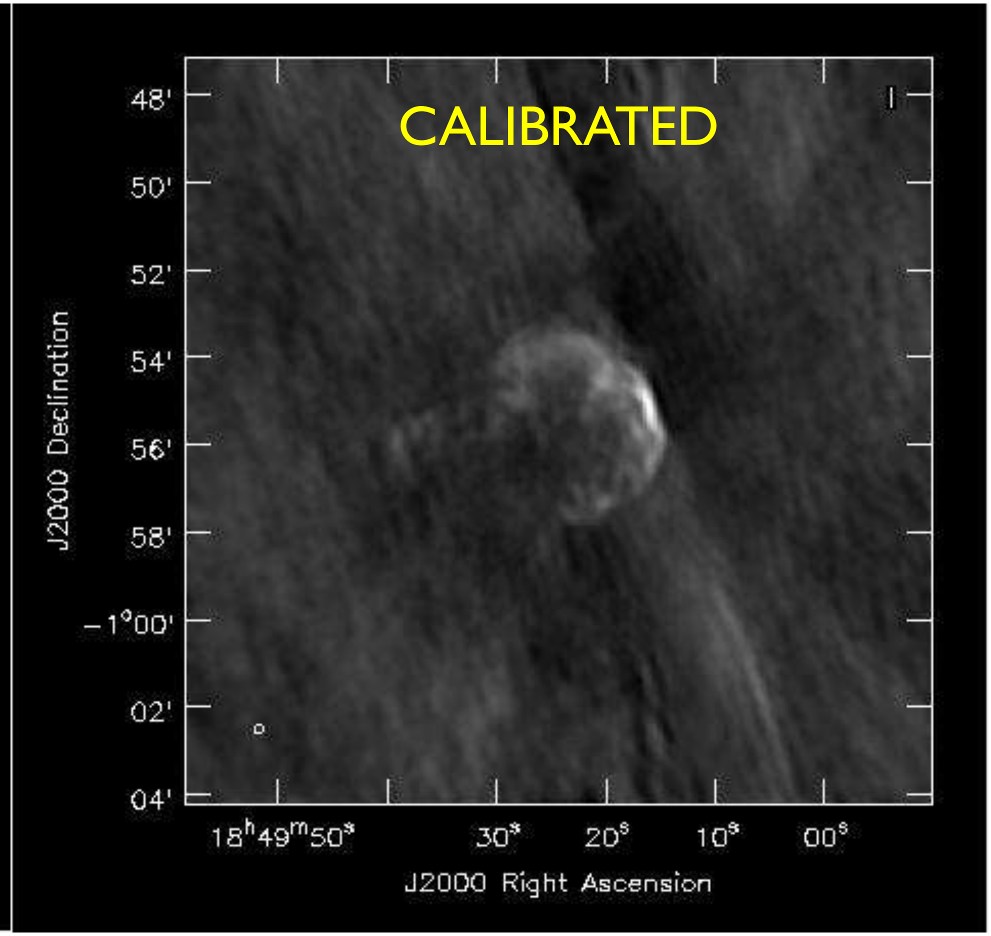
J1822-0938
(calibrator)

3C391
(science)

Calibration Effect on Imaging

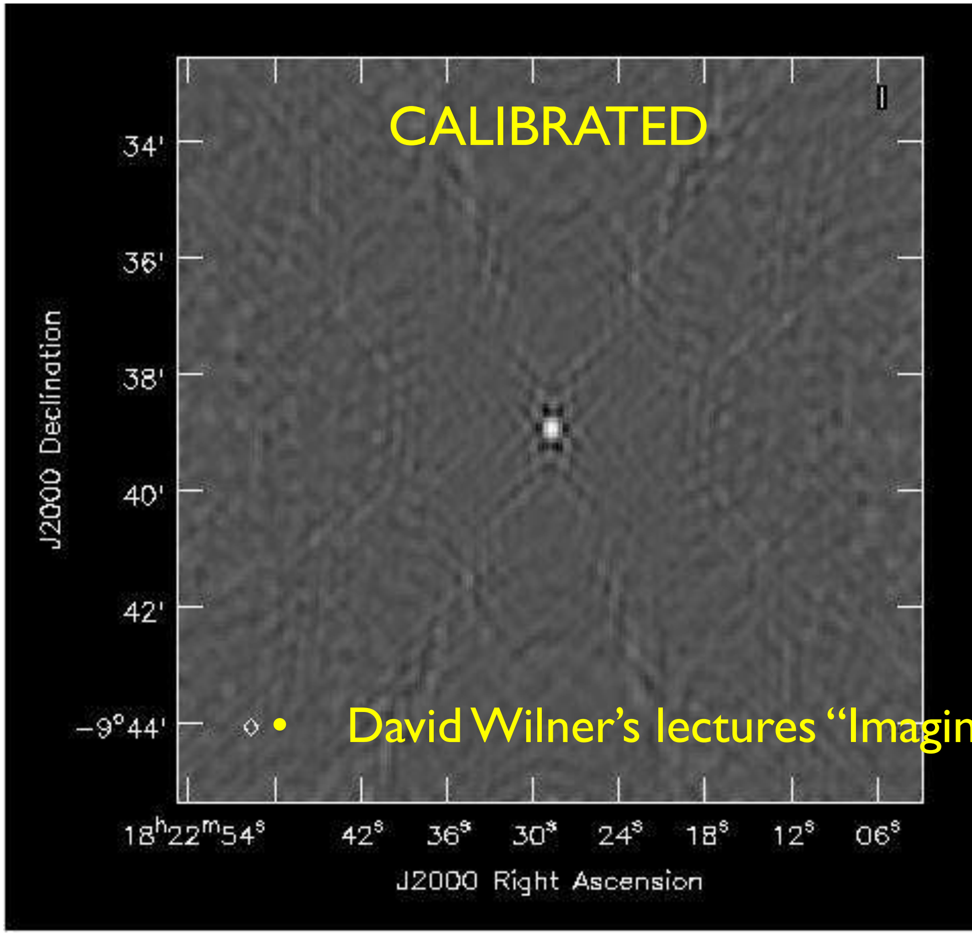


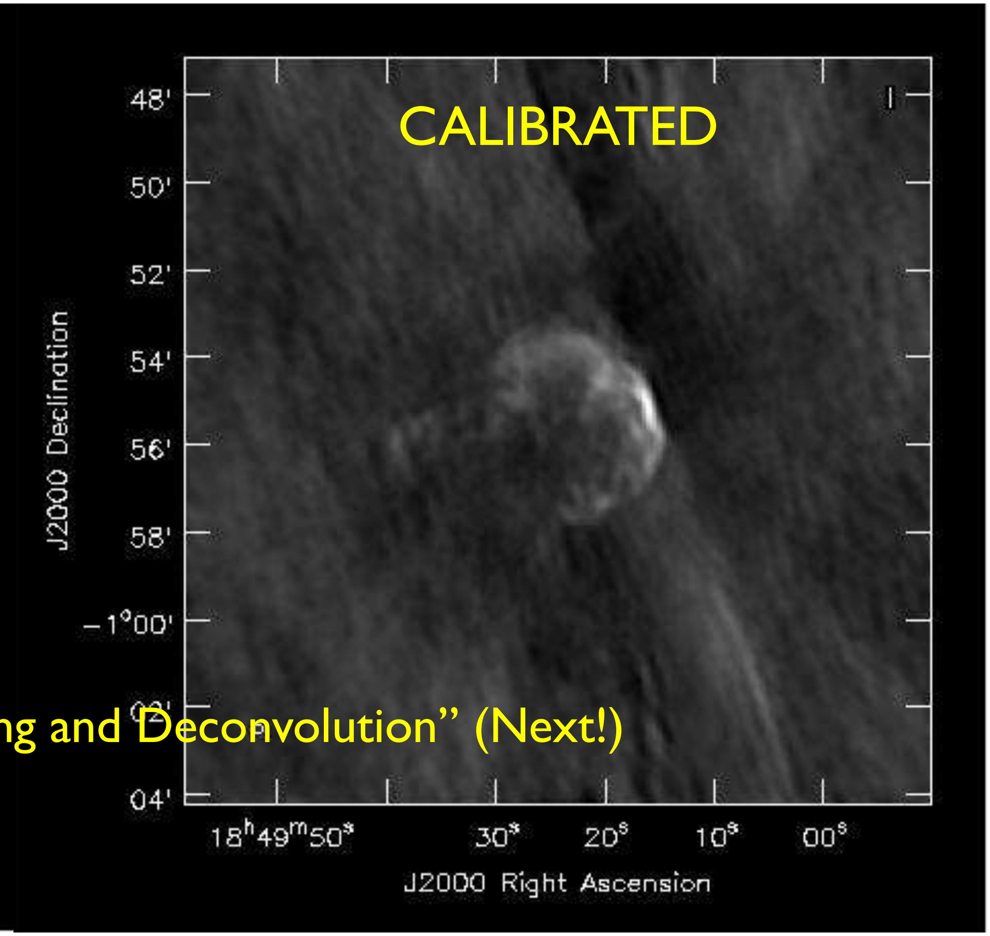
J1822-0938
(calibrator)



3C391
(science)

Calibration Effect on Imaging





- David Wilner's lectures "Imaging and Deconvolution" (Next!)

J1822-0938
(calibrator)

3C391
(science)

Evaluating Calibration Performance

- Are solutions ~continuous?
  - Noise-like solutions are just that—noise (beware: calibration of pure noise generates a spurious point source)
  - Discontinuities may indicate instrumental glitches (interpolate with care)
  - Any additional editing required?
    - Provisional calibration can make bad data easier to see
  - Evidence of unsampled variation?
    - Flag uncalibrateable data
    - (Consider faster cadence next time!)
- Are calibrator data fully described by antenna-based effects?
  - Phase and amplitude *closure errors* are the baseline-based residuals
  - Are calibrators sufficiently point-like? If not, self-calibrate: model calibrator visibilities (by imaging, deconvolving and transforming) and re-solve for calibration; iterate to isolate source structure from calibration
    - Crystal Brogan's lectures: "Advanced Calibration" (this afternoon)
- Greg Taylor's lecture: "Error Recognition" (Tuesday)

Summary of Scalar Example

- Dominant calibration effects are ***antenna-based***
  - Minimizes degrees of freedom
  - More precise
  - Preserves closure
  - Permits higher dynamic range *safely*!

- Point-like calibrators effective
- Flux density bootstrapping

- Deconvolution necessary ("Imaging")

Generalizations and Specializations

- Full-polarization Matrix Formalism
- Calibration Effects Factorization
- Calibration Heuristics and ‘Bootstrapping’

Full-Polarization Formalism (Matrices!)

- Need dual-polarization basis (*p*,*q*) to fully sample the incoming EM wave front, where *p*,*q* = *R*,*L* (circular basis) or *p*,*q* = *X*,*Y* (linear basis):

$$\vec{I}_{circ} = \overleftrightarrow{S}_{circ}\vec{I}_{Stokes}$$

$$\begin{pmatrix} RR \\ RL \\ LR \\ LL \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & i & 0 \\ 0 & 1 & -i & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} I+V \\ Q+iU \\ Q-iU \\ I-V \end{pmatrix}$$

$$\vec{I}_{lin} = \overleftrightarrow{S}_{lin}\vec{I}_{Stokes}$$

$$\begin{pmatrix} XX \\ XY \\ YX \\ YY \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & i \\ 0 & 0 & 1 & -i \\ 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} I+Q \\ U+iV \\ U-iV \\ I-Q \end{pmatrix}$$

- Stokes Parameters:

  I = Total Intensity; Q,U = Linear Polarization; V = Circular Polarization

- Devices can be built to sample these circular (R,L) or linear (X,Y) basis states in the signal domain (Stokes Vector is defined in "power" domain)
- Some components of J_i involve mixing of basis states, so dual-polarization matrix description desirable or even required for proper calibration

Full-Polarization Formalism: Signal Domain

- Substitute:

$$s_i \rightarrow \vec{s}_i = \begin{pmatrix} s^p \\ s^q \end{pmatrix}_i, \qquad J_i \rightarrow \overleftrightarrow{J}_i = \begin{pmatrix} J^{p\rightarrow p} & J^{q\rightarrow p} \\ J^{p\rightarrow q} & J^{q\rightarrow q} \end{pmatrix}_i$$

- The *Jones matrix* thus corrupts the vector wavefront signal as follows:

$$\vec{s}_i' = \overleftrightarrow{J}_i \vec{s}_i$$

$$\begin{pmatrix} s'^p \\ s'^q \end{pmatrix}_i = \begin{pmatrix} J^{p\rightarrow p} & J^{q\rightarrow p} \\ J^{p\rightarrow q} & J^{q\rightarrow q} \end{pmatrix}_i \begin{pmatrix} s^p \\ s^q \end{pmatrix}_i$$

$$= \begin{pmatrix} J^{p\rightarrow p} s^p + J^{q\rightarrow p} s^q \\ J^{p\rightarrow q} s^p + J^{q\rightarrow q} s^q \end{pmatrix}_i$$

Full-Polarization Formalism: Correlation - I

- Four correlations are possible from two polarizations. The *coherency matrix* represents correlation in the matrix formalism:

$$\overleftrightarrow{V}_{ij}^{true} = \left\langle \vec{s}_i \cdot \vec{s}_j^{\,*+} \right\rangle = \left\langle \begin{pmatrix} s^p \\ s^q \end{pmatrix}_i \cdot \begin{pmatrix} s^{p*} & s^{q*} \end{pmatrix}_j \right\rangle = \begin{pmatrix} \left\langle s_i^p \cdot s_j^{p*} \right\rangle & \left\langle s_i^p \cdot s_j^{q*} \right\rangle \\ \left\langle s_i^q \cdot s_j^{p*} \right\rangle & \left\langle s_i^q \cdot s_j^{q*} \right\rangle \end{pmatrix}$$

- Observed visibilities:

$$\overleftrightarrow{V}_{ij}^{obs} = \left\langle \vec{s}_i' \cdot \vec{s}_j'^{\,*} \right\rangle = \left\langle \left(\overleftrightarrow{J}_i \vec{s}_i \right) \cdot \left(\overleftrightarrow{J}_j^* \vec{s}_j^{\,*} \right)^+ \right\rangle = \overleftrightarrow{J}_i \left\langle \vec{s}_i \cdot \vec{s}_j^{\,*+} \right\rangle \overleftrightarrow{J}_j^{*+} = \overleftrightarrow{J}_i \overleftrightarrow{V}_{ij}^{true} \overleftrightarrow{J}_j^{*+}$$

Full-Polarization Formalism: Correlation - II

- And finally, ***for fun***, expand the correlation of corrupted signals:

$$\overleftrightarrow{V}_{ij}^{obs} = \overleftrightarrow{J}_i \left\langle \vec{s}_i \cdot \vec{s}_j^{\,*+} \right\rangle \overleftrightarrow{J}_j^{*+}$$

$$= \begin{pmatrix} \begin{matrix} J_i^{p\to p} J_j^{*p\to p} \left\langle s_i^p \cdot s_j^{*p} \right\rangle + J_i^{p\to p} J_j^{*q\to p} \left\langle s_i^p \cdot s_j^{*q} \right\rangle + \\ J_i^{q\to p} J_j^{*p\to p} \left\langle s_i^q \cdot s_j^{*p} \right\rangle + J_i^{q\to p} J_j^{*q\to p} \left\langle s_i^q \cdot s_j^{*q} \right\rangle \end{matrix} & \begin{matrix} J_i^{p\to p} J_j^{*p\to q} \left\langle s_i^p \cdot s_j^{*p} \right\rangle + J_i^{p\to p} J_j^{*q\to q} \left\langle s_i^p \cdot s_j^{*q} \right\rangle + \\ J_i^{q\to p} J_j^{*p\to q} \left\langle s_i^q \cdot s_j^{*p} \right\rangle + J_i^{q\to p} J_j^{*q\to q} \left\langle s_i^q \cdot s_j^{*q} \right\rangle \end{matrix} \\ \\ \begin{matrix} J_i^{p\to q} J_j^{*p\to p} \left\langle s_i^p \cdot s_j^{*p} \right\rangle + J_i^{p\to q} J_j^{*q\to p} \left\langle s_i^p \cdot s_j^{*q} \right\rangle + \\ J_i^{q\to q} J_j^{*p\to p} \left\langle s_i^q \cdot s_j^{*p} \right\rangle + J_i^{q\to q} J_j^{*q\to p} \left\langle s_i^q \cdot s_j^{*q} \right\rangle \end{matrix} & \begin{matrix} J_i^{p\to q} J_j^{*p\to q} \left\langle s_i^p \cdot s_j^{*p} \right\rangle + J_i^{p\to q} J_j^{*q\to q} \left\langle s_i^p \cdot s_j^{*q} \right\rangle + \\ J_i^{q\to q} J_j^{*p\to q} \left\langle s_i^q \cdot s_j^{*p} \right\rangle + J_i^{q\to q} J_j^{*q\to q} \left\langle s_i^q \cdot s_j^{*q} \right\rangle \end{matrix} \end{pmatrix}$$

- UGLY, but we rarely, if ever, need to worry about algebraic detail at this level---just let this occur "inside" the matrix formalism, and work (think) with the matrix short-hand notation
- Synthesis instrument design driven by minimizing off-diagonal terms in J_i

The Matrix Measurement Equation

- We can now write down the Measurement Equation in matrix notation:

$$\vec{V}_{ij}^{obs} = \int_{sky} \left(\overleftrightarrow{J}_i \overleftrightarrow{I}_c(l,m) \overleftrightarrow{J}_j^{*+} \right) e^{-i2\pi\left(u_{ij}l + v_{ij}m\right)} \, dl \, dm$$

  - $I_c(l,m)$ is the 2x2 matrix of Stokes parameter combinations corresponding to the coherency matrix of correlations (basis-dependent)
    - Circular basis: $I_c = \begin{pmatrix} RR & RL \\ LR & LL \end{pmatrix} = \begin{pmatrix} I+V & Q+iU \\ Q-iU & I-V \end{pmatrix}$
    - Linear basis: $I_c = \begin{pmatrix} XX & XY \\ YX & YY \end{pmatrix} = \begin{pmatrix} I+Q & U+iV \\ U-iV & I-Q \end{pmatrix}$

- …and consider how the J_i are products of many effects…

A Dictionary of Calibration Components

- J_i contains many components, in principle:
  - F = ionospheric effects
  - T = tropospheric effects
  - P = parallactic angle
  - X = linear polarization position angle
  - E = antenna voltage pattern, gaincurve
  - D = polarization leakage
  - G = electronic gain
  - B = bandpass response
  - K = geometry
  - M, A = baseline-based corrections

$$\overleftrightarrow{J}_i = \overleftrightarrow{K}_i \overleftrightarrow{B}_i \overleftrightarrow{G}_i \overleftrightarrow{D}_i \overleftrightarrow{E}_i \overleftrightarrow{X}_i \overleftrightarrow{P}_i \overleftrightarrow{T}_i \overleftrightarrow{F}_i$$

- Order of terms ~follows signal path (right to left)
- Each term has matrix form of J_i with terms embodying its particular algebra (on- vs. off-diagonal terms, etc.)
- Direction-dependent terms must stay inside FT integral
- 'Full' calibration is traditionally a bootstrapping process wherein relevant terms (usually a minority of above list) are considered in decreasing order of dominance, relying on approximate separability

Ionospheric Effects, *F*

$$\overleftrightarrow{F}^{RL} = e^{i\Delta\phi}\begin{pmatrix} e^{-i\varepsilon} & 0 \\ 0 & e^{i\varepsilon} \end{pmatrix}; \ \overleftrightarrow{F}^{XY} = e^{i\Delta\phi}\begin{pmatrix} \cos\varepsilon & \sin\varepsilon \\ -\sin\varepsilon & \cos\varepsilon \end{pmatrix}$$

- The ionosphere introduces a dispersive path-length offset: $$\Delta\phi \propto \frac{\int n_e \, dl}{\nu}$$
  - More important at lower frequencies (<5 GHz)
  - Varies more at solar maximum and at sunrise/sunset, when ionosphere is most active and variable
  - Direction-dependent within wide field-of-view
- The ionosphere is *birefringent*: Faraday rotation: $$\varepsilon \propto \frac{\int B_{\parallel} \, n_e \, dl}{\nu^2}$$
  - as high as 20 rad/m^2 during periods of high solar activity will rotate linear polarization position angle by $\varepsilon = 50$ degrees at 1.4 GHz
  - Varies over the array, and with time as line-of-sight magnetic field and electron density vary, violating the usual assumption of stability in position angle calibration

– Frank Schinzel's lecture: "Polarization" (Friday)

– Tracy Clark's lecture: "Low Frequency Interferometry" (Friday)

Tropospheric Effects, *T*

$$\overset{\leftrightarrow}{T} = \begin{pmatrix} t & 0 \\ 0 & t \end{pmatrix} = t \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

- The troposphere causes polarization-independent amplitude and phase effects due to emission/opacity and refraction, respectively
  - Up to 2.3m excess path length at zenith compared to vacuum
  - Higher noise contribution, less signal transmission: Lower SNR
  - Most important at ν > 15 GHz where water vapor and oxygen absorb/emit, and where path length errors are a larger fraction (or multiple!) of the wavelength
  - Zenith-angle-dependent (more troposphere path nearer horizon)
  - Clouds, weather = variability in phase and opacity; may vary across array
  - Water vapor radiometry (estimate phase from power measurements)
  - Phase transfer from low to high frequencies (delay calibration)

- ALMA!
  - Crystal Brogan's lectures: "Advanced Calibration" (today)

Parallactic Angle, *P*

$$\overleftrightarrow{P}^{RL} = \begin{pmatrix} e^{-i\chi} & 0 \\ 0 & e^{i\chi} \end{pmatrix}; \ \overleftrightarrow{P}^{XY} = \begin{pmatrix} \cos\chi & \sin\chi \\ -\sin\chi & \cos\chi \end{pmatrix}$$

- Changing orientation of sky in telescope's field of view
  - Constant for equatorial telescopes
  - Varies for alt-az-mounted telescopes:

$$\chi(t) = \arctan\left(\frac{\cos l \sin h(t)}{\sin l \cos\delta - \cos l \sin\delta \cos h(t)}\right)$$

$$l = \text{latitude},\ h(t) = \text{hour angle},\ \delta = \text{declination}$$

  - Rotates the position angle of linearly polarized radiation
  - Analytically known, and its variation provides leverage for determining polarization-dependent effects

- Frank Schinzel's lecture: "Polarization" (Friday)

Linear Polarization Position Angle, *X*

$$\overleftrightarrow{X}^{RL} = \begin{pmatrix} e^{-i\Delta\chi} & 0 \\ 0 & e^{i\Delta\chi} \end{pmatrix}; \;\; \overleftrightarrow{X}^{XY} = \begin{pmatrix} \cos\Delta\chi & \sin\Delta\chi \\ -\sin\Delta\chi & \cos\Delta\chi \end{pmatrix}$$

- Configuration of optics and electronics (and refant) causes a net linear polarization position angle offset
- Can be treated as an offset to the parallactic angle, P
- Calibrated by registration with a strongly polarized source with known polarization position angle (e.g., flux density calibrators)
- For circular feeds, this is a phase difference between the R and L polarizations, which is frequency-dependent (a R-L phase bandpass)
- For linear feeds, this is the orientation of the dipoles (in the frame of the telescope) projected onto sky coordinates

- Frank Schinzel's lecture: "Polarization" (Friday)

Antenna Voltage Pattern, *E*

$$\overleftrightarrow{E}^{pq} = \begin{pmatrix} E^p(l,m) & 0 \\ 0 & E^q(l,m) \end{pmatrix}$$

- Antennas of all designs have direction-dependent gain within field-of-view
  - Important when region of interest on sky comparable to or larger than λ/D radians
  - Important at lower frequencies where radio source surface density is greater and wide-field imaging techniques required
  - Beam squint: E^R and E^L offset, yielding spurious Stokes V polarization
  - Sky rotates within field-of-view for alt-az antennas, so off-axis sources move through the pattern
  - Direction dependence of polarization leakage (*D*) may be included in *E* (off-diagonal terms then non-zero)
- Shape and efficiency of the voltage pattern may change with zenith angle: 'gain curve'

  – Brian Mason's lecture: "Mosaicking" (Friday)

  – Urvashi Rao Venkata's lecture: "Widefield Imaging" (Monday)

Polarization Leakage, *D*

$$\overleftrightarrow{D} = \begin{pmatrix} 1 & d^p \\ d^q & 1 \end{pmatrix}$$

- Antenna & polarizer are not ideal, so orthogonal polarizations not perfectly isolated
  - Well-designed feeds have *d* ~ a few percent or less
  - A geometric property of the optics design, so frequency-dependent
  - For *R,L* systems, total-intensity imaging affected as *~dQ, dU,* so only important at high dynamic range (*Q,U,d each ~few* %, typically)
  - For *R,L* systems, linear polarization imaging affected as *~dI,* so almost always important
  - For small arrays (no *differential* parallactic angle coverage), only relative D solution is possible from standard linearized solution, so parallel-hands cannot be corrected absolutely (closure errors)
- Best calibrator: Strong, point-like, observed over large range of parallactic angle (to separate source polarization from *D*)
- Frank Schinzel's lecture: "Polarization" (Friday)

“Electronic” Gain, *G*

$$\overleftrightarrow{G}^{pq} = \begin{pmatrix} g^p & 0 \\ 0 & g^q \end{pmatrix}$$

- Catch-all for most amplitude and phase effects introduced by antenna electronics and other generic effects
    - Most commonly treated calibration component
    - Dominates other effects for most standard observations
    - Includes scaling from engineering (correlation coefficient) to radio astronomy units (Jy), by scaling solution amplitudes according to observations of a flux density calibrator
    - Includes any internal system monitoring, like EVLA switched power calibration
    - Often also includes tropospheric and (on-axis) ionospheric effects which are typically difficult to separate uniquely from the electronic response
    - Excludes frequency dependent effects (see *B*)
- Best calibrator: strong, point-like, near science target; observed often enough to track expected variations
    - – Also observe a flux density standard

Bandpass Response, *B*

$$\overleftrightarrow{B}^{pq} = \begin{pmatrix} b^p(\nu) & 0 \\ 0 & b^q(\nu) \end{pmatrix}$$

- *G*-like component describing frequency-dependence of antenna electronics, etc.
    - Filters used to select frequency passband not square
    - Optical and electronic reflections introduce ripples across band
    - Often assumed time-independent, but not necessarily so
    - Typically (but not necessarily) normalized
    - ALMA Tsys is a "bandpass" (freq-dependent calibration to K)
- Best calibrator: strong, point-like; observed long enough to get sufficient *per-channel* SNR, and often enough to track variations

- Ylva Pihlstrom's lecture: "Spectral Line Data Analysis" (Friday)

Geometry, *K*

$$\overleftrightarrow{K}^{pq} = \begin{pmatrix} k^p & 0 \\ 0 & k^q \end{pmatrix}$$

- Must get geometry right for Synthesis Fourier Transform relation to work in real time
    - Antenna positions (geodesy)
    - Source directions (time-dependent in topocenter!) (astrometry)
    - Clocks
    - Electronic path-lengths introduce delays (polarization, spw differences)
    - Longer baselines generally have larger relative geometry errors, especially if clocks are independent (VLBI)
    - Importance scales with frequency *and* bandwidth
- *K* is a clock- & geometry-parameterized version of *G*
    - All-sky observations used to isolate geometry parameters

  – Adam Deller's lecture: "Very Long Baseline Interferometry" (Friday)

Non-closing Effects: *M*, *A*

- Baseline-based errors which do not factor into antenna-based components
  - Digital correlators designed to limit such effects to well-understood and ***uniform*** (not dependent on baseline) scaling laws (absorbed in *f.d.* calibration)
  - Simple noise (additive)
  - Averaging in time and frequency over variation in antenna-based effects and visibilities (practical instruments are finite!)
  - Instrumental polarization effects in parallel hands (not properly factored)
  - Correlated "noise" (e.g., RFI)
- Difficult to distinguish from source structure (visibility) effects
  - Geodesy and astrometry observers consider determination of radio source structure—a baseline-based effect—as a required *calibration* if antenna positions are to be determined accurately
- Separate factors for each element of the coherency matrix; *M* multiplies, *A* adds

Solving the Measurement Equation

- Formally, solving for any antenna-based visibility calibration component is always the same general non-linear fitting problem:

$$\vec{V}_{ij}^{corrected\cdot obs} = \overleftrightarrow{J}_i \vec{V}_{ij}^{corrupted\cdot mod} \overleftrightarrow{J}_j^{*+}$$

  - Generalization of scalar non-linear LS approach
  - Observed and Model visibilities are corrected/corrupted by available prior calibration solutions/information downstream and upstream of the solved-for component, respectively
  - Resulting solution used as prior in subsequent solves, as necessary
  - Each solution is relative to priors and assumed source model
  - Iterate sequences, as needed → generalized self-calibration
- Viability and accuracy of the overall calibration depends on isolation of different effects using *proper calibration observations*, and *appropriate solving strategies (heuristics)*

Measurement Equation Heuristics

- When considering which effects are relevant to a particular observation, and how to sequence calibration determination, it is convenient to express the Measurement Equation in a "Heuristic Operator" notation:

$$V^{obs} = M\ B\ G\ D\ E\ X\ P\ T\ F\ V^{true} + A$$

- Rigorous notation, antenna-basedness, etc., suppressed
- **Usually, only a subset of terms are considered, though highest-dynamic range observations may require more**
- An expression of a "Calibration Model"
  - Order is important (handled in software)
  - Solve for terms in decreasing order of dominance, iterate to isolate
  - NB: Non-trivial direction-dependent solutions involve convolutional treatment of the visibilities, and is coupled to the imaging and deconvolution process---see advanced imaging lectures…)

Decoupling Calibration Effects

- All calibration terms are a function of prior information!
- Multiplicative gain (G) term will soak up many different effects; known priors should be compensated for *explicitly*, especially when direction-dependent differences (e.g., between calibrator and target) will limit the accuracy of calibration transfer:
  - Zenith angle-dependent atmospheric opacity, phase (T,F)
  - Zenith angle-dependent gain curve (E)
  - Antenna position errors (K)
- Early calibration solves (e.g., G) are always subject to more subtle, uncorrected effects
  - Instrumental polarization (D), which introduces gain calibration errors and causes apparent closure errors in *parallel-hand* correlations
  - When possible, iterate and alternate solves to decouple effects…

Calibration Heuristics – Spectral Line

Total Intensity Spectral Line (K=antenna positions, B=bandpass, G=gain, E=gaincurve, T=opacity):

$$V^{obs} = K\ B\ G\ E\ T\ V^{true}$$

1. Preliminary Gain solve on B-calibrator:

   $$(K' V^{obs}) = \underline{G_B}\ (E\ T\ V^{mod})$$

2. Bandpass Solve (using G_B) on B-calibrator (then discard G_B):

   $$(K' V^{obs}) = \underline{B}\ (G_B\ E\ T\ V^{mod})$$

3. Time-dependent Gain solve (using inverse of B) on all calibrators:

   $$(B'\ K'\ V^{obs}) = \underline{G}\ (E\ T\ V^{mod})$$

4. Flux Density scaling:

   $G \rightarrow G_f$ (enforce gain consistency)

5. Correct with inverted (primes) solutions:

   $$V^{cor} = T'\ E'\ G_f'\ B'\ K'\ V^{obs}$$

6. Image!

Calibration Heuristics – Polarimetry

Polarimetry (B=bandpass, G=gain, D=instr. poln, X=pos. ang., P=parallactic ang.):

$$V^{obs} = B\,G\,D\,X\,P\,V^{true}$$

1. Preliminary Gain solve on B-calibrator:

   $$V^{obs} = \underline{G_B}\,V^{mod}$$

2. Bandpass (B) Solve (using G_B) on B-calibrator (then discard G_B):

   $$V^{obs} = \underline{B}\,(G_B\,V^{mod})$$

3. Gain (G) solve (using parallactic angle *P*, inverse of *B*) on calibrators:

   $$(B'\,V^{obs}) = \underline{G}\,(P V^{mod})$$

4. Instrumental Polarization (*D*) solve (using *P*, inverse of *G*,*B*) on instrumental polarization calibrator:

   $$(G'\,B'\,V^{obs}) = \underline{D}\,(P\,V^{mod})$$

NRAO

Calibration Heuristics – Polarimetry

5. Polarization position angle solve (using *P*, inverse of *D*,*G*,*B*) on position angle calibrator:

   $(D' \, G' \, B' \, V^{obs}) = \underline{X} \, (P \, V^{mod})$

6. Flux Density scaling:

   $G \rightarrow G_f$ (enforce gain consistency)

7. Correct with inverted solutions:

   $V^{cor} = P' \, X' \, D' \, G_f' \, B' \, V^{obs}$

8. Image!

- To use external priors, e.g., *T* (opacity), *K* (ant. position errors), *E* (gaincurve), revise step 3 above as:

3. $(B' \, K' \, V^{obs}) = G \, (E \, P \, T \, V^{mod})$

– and carry *T*, *K*, and *E* forward along with *G* to subsequent steps

Modern Calibration Challenges

- Gain calibration optimizations
  - 'Delay-aware' gain (self-) calibration: Troposphere and Ionosphere introduce time-variable phase effects easily parameterized as functions of frequency
  - Inter-band gain transfer (high-frequency ALMA)
  - Water Vapor Radiometry
- Polarization calibration optimizations
  - Frequency-dependent Instrumental Polarization √
  - High Dynamic Range (I, Q, U, & V))
  - More robust gain refant algorithms
  - Routine Full Polarization Treatments
- Voltage pattern for wide fields of view, mosaicking
  - Frequency-dependent voltage pattern
  - Wide-field accuracy (sidelobes, rotation)
  - Instrumental polarization (incl. frequency-dependence)
- RFI mitigation
- Pipelines/Science Ready Data Products (SRDP)
  - Generalized Heuristics vs. observational flexibility…
  - Modern instruments' sensitivity to more subtle effects…

- **Increasing sensitivity: Can implied dynamic range be reached by our calibration and imaging techniques?**

Summary

- Determining calibration is as important as determining source structure—can't have one without the other
- Data examination and editing an important part of calibration
- Calibration dominated by antenna-based effects
  - permits efficient, accurate and scientifically defensible separation of calibration from astronomical information (satisfies closure)
- Full calibration formalism algebra-rich, but is *modular*
- Calibration an iterative process, improving various components in turn, as needed
- Point sources are the best calibrators
- Observe calibrators according requirements of calibration components