Polarimetry II

Astrophysical applications and advanced techniques

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26 September 2017

Background image credit: Mulcahy, Beck & Heald (2017, A&A, 600, 6)

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Topics

- Astrophysical motivation: Generation of polarization and propagation effects
  - Synchrotron
  - Faraday rotation
  - Zeeman splitting

- Widefield polarimetry
  - Antenna beam response
  - A projection

- Advanced analysis techniques
  - Rotation measure synthesis / deconvolution
  - Faraday tomography
  - QU fitting & Faraday dispersion modeling

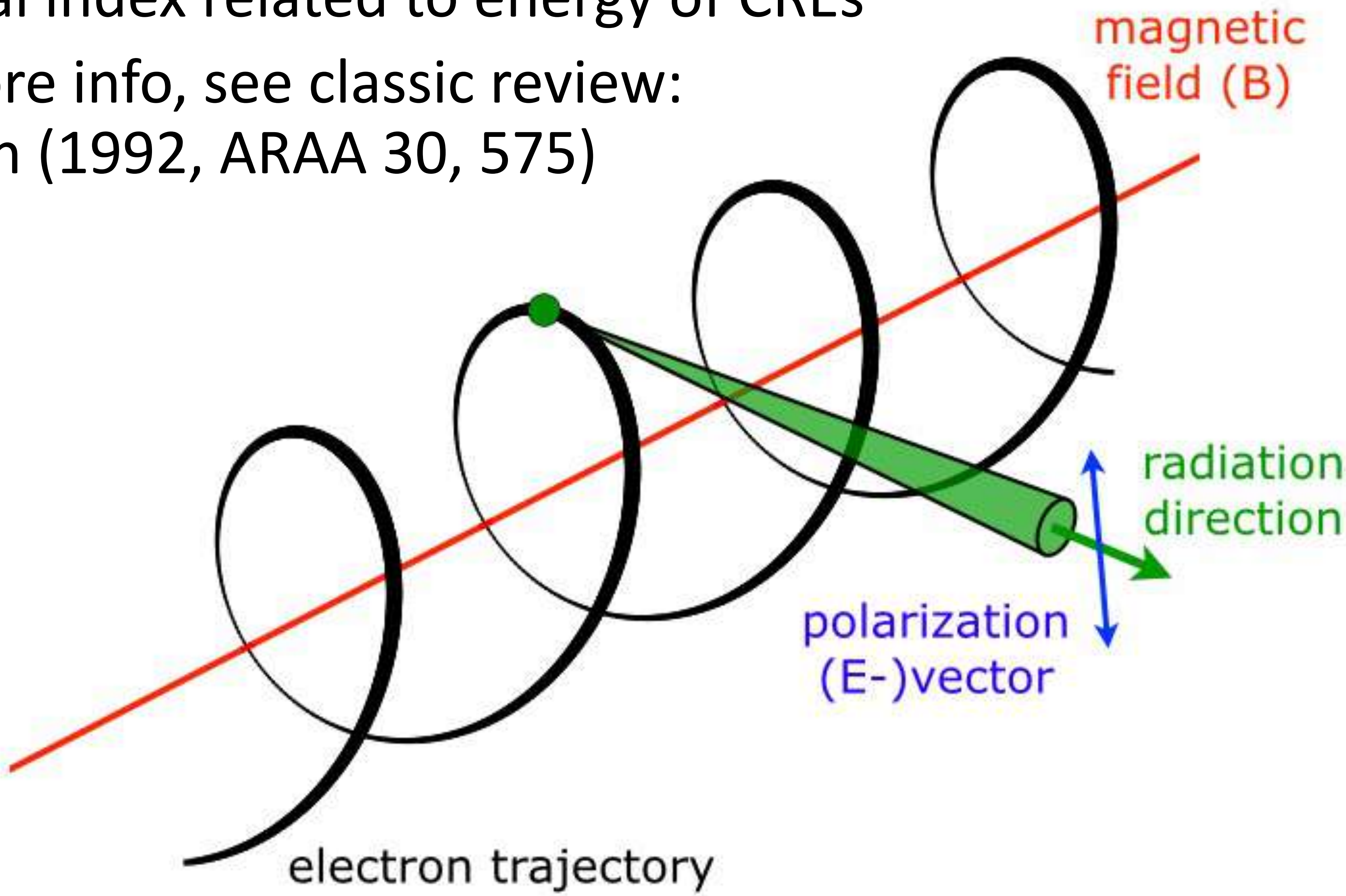
Astrophysical motivation

… but why?

- Polarisation is an excellent probe of magnetic fields in the Universe

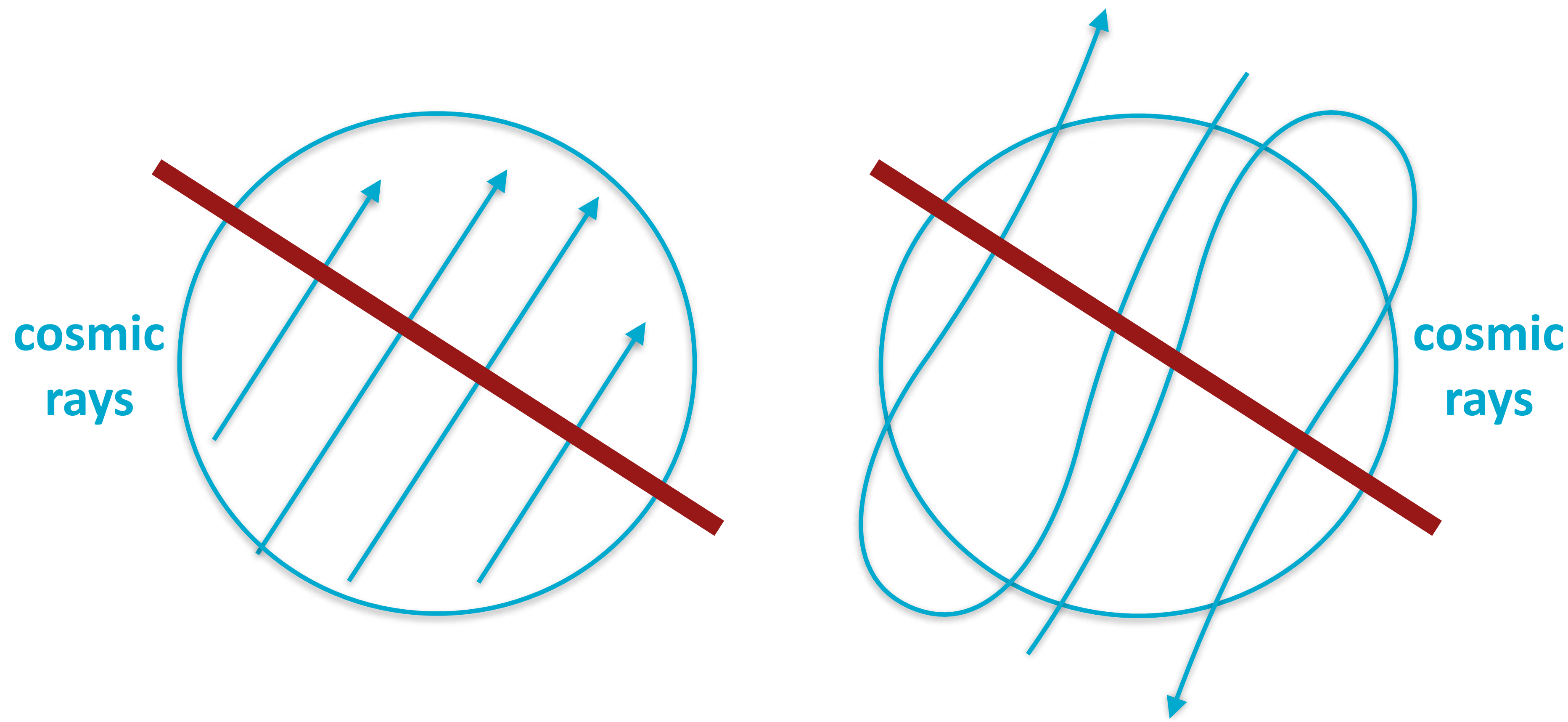
Synchrotron radiation

- Probes number density of CREs and strength of magnetic field in the plane of the sky
- Spectral index related to energy of CREs
- For more info, see classic review: Condon (1992, ARAA 30, 575)



Synchrotron polarisation

- Linear polarisation from ordered fields in plane of the sky (max fractional polarization typically ~0.7-0.8)
- Can be regular/coherent or anisotropic/tangled



Faraday rotation

- Birefringence of magnetised and ionised medium leads to an effect called "Faraday rotation"

Rotation of linear polarization angle is proportional to λ^2:

$$\chi = \chi_0 + \mathrm{RM} \times \lambda^2$$

Proportionality constant traces conditions in intervening medium:

$$\mathrm{RM} \propto \int n_e \vec{B} \cdot d\vec{l}$$

Cooper & Price (1962, Nature)

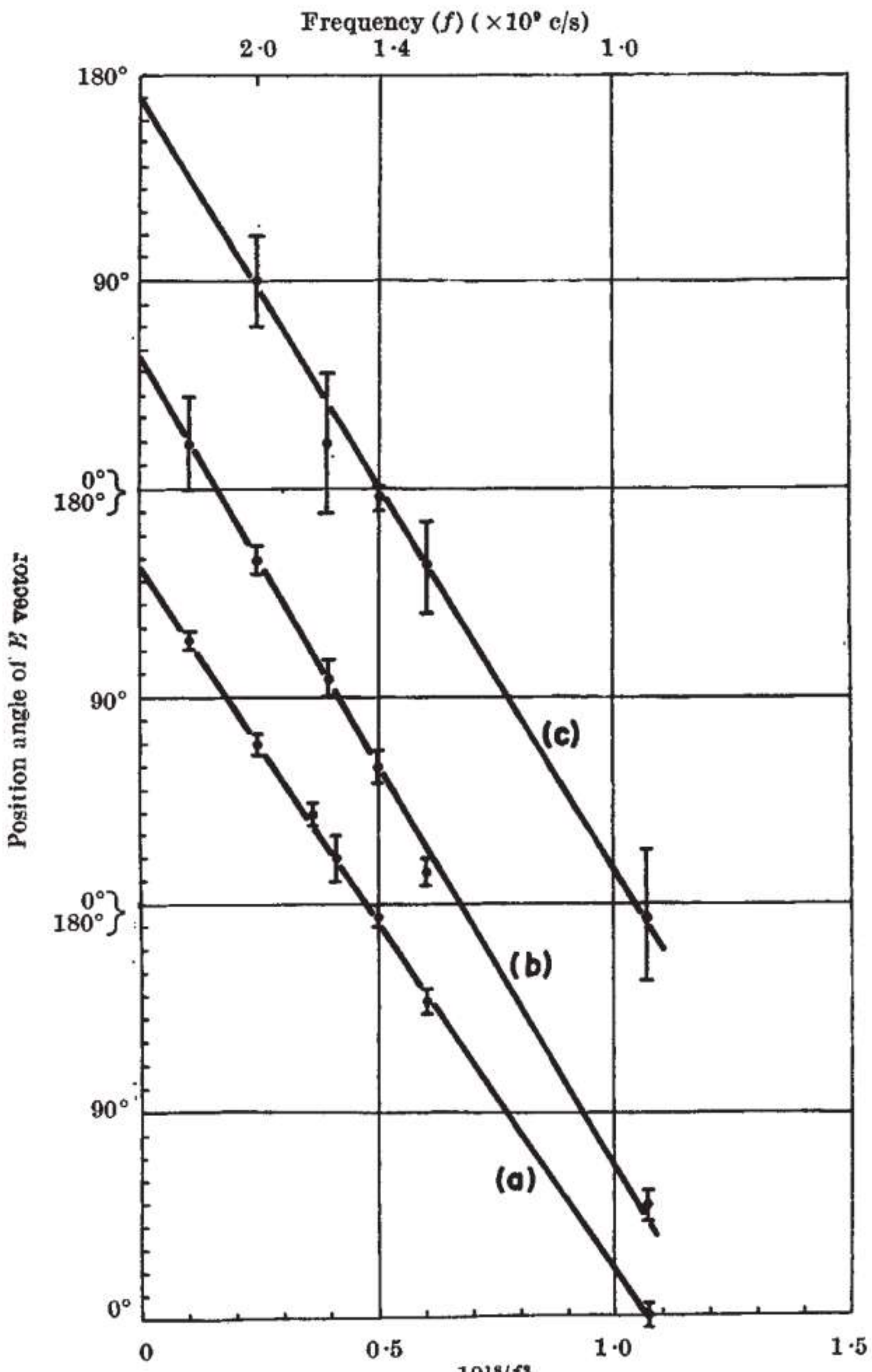
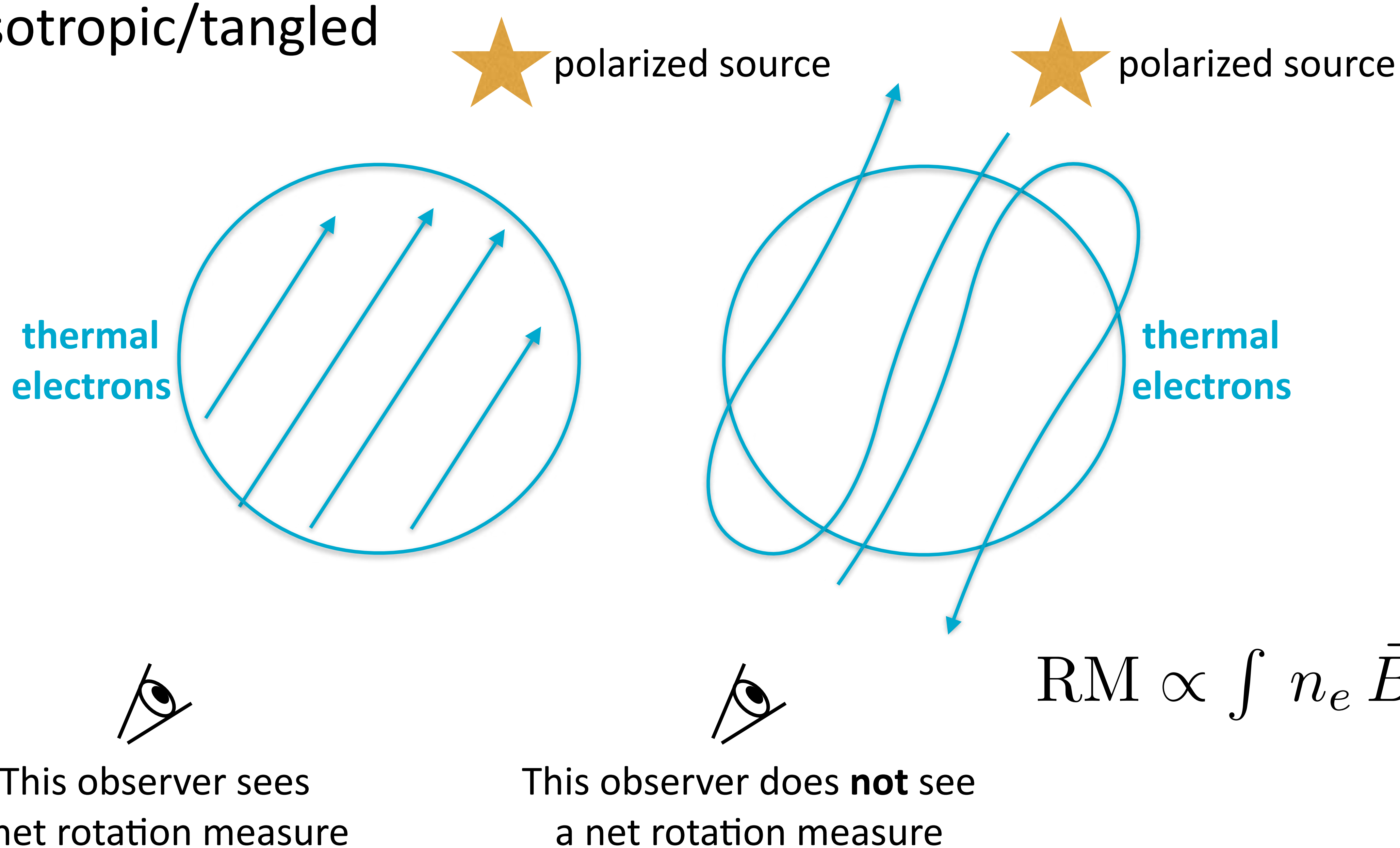


Fig. 1. Frequency dependence of the polarization position angle at three points in Centaurus *A*. *f*, Frequency in c/s. *a*, R.A. 13h 23m 15s; declination −42° 48·5′. *b*, R.A. 13h 24m 00s; declination −42° 30′. *c*, R.A. 13h 20m 30s; declination −44° 25′. Curve *c* has been arbitrarily displaced upwards by 180° relative to *a* and *b*

Faraday rotation

- Proportional to component of magnetic field along the line of sight
- Highly sensitive to whether field is regular/coherent or anisotropic/tangled



$$\mathrm{RM} \propto \int n_e \vec{B} \cdot d\vec{l}$$

Faraday Rotation

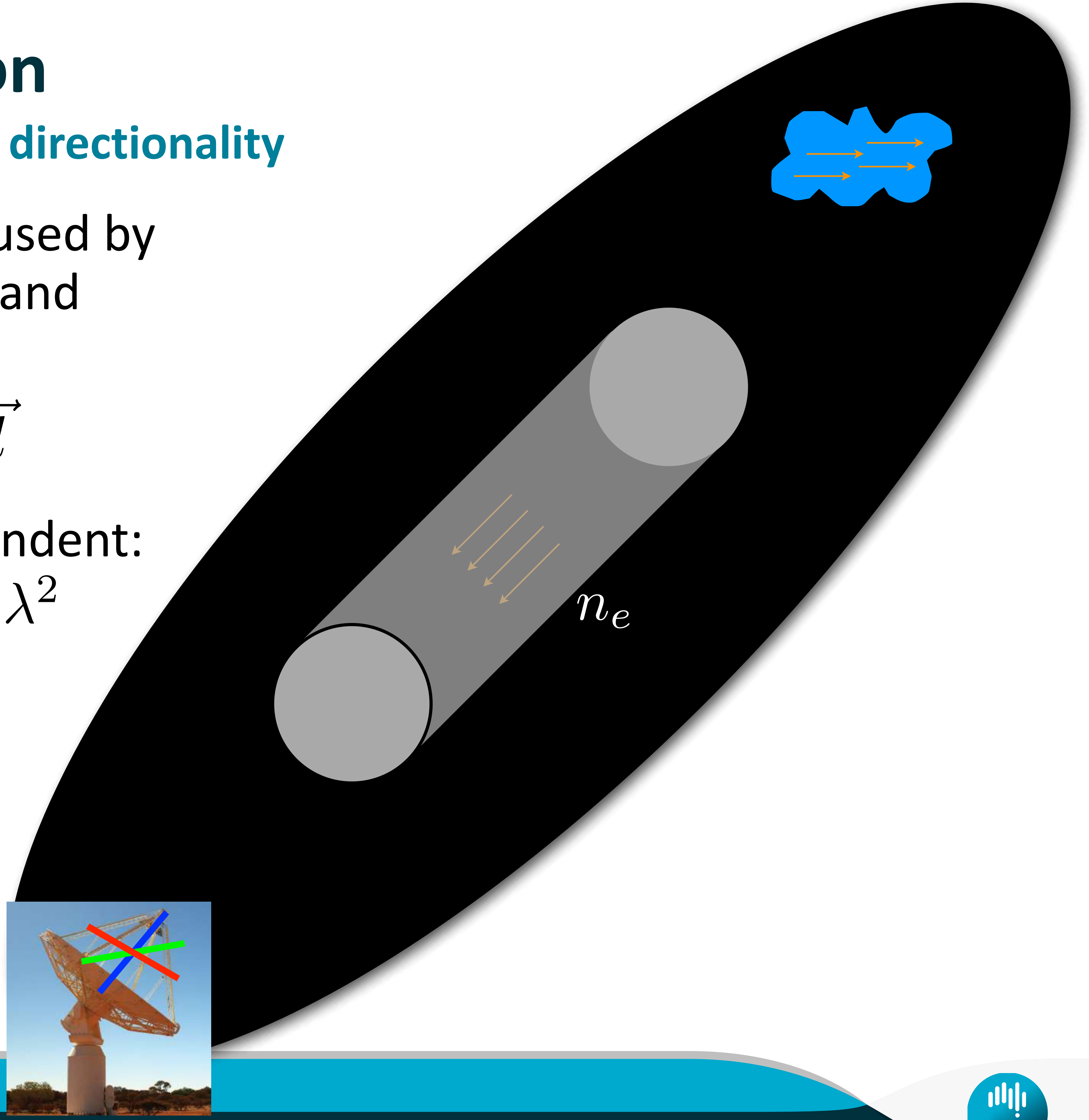
Probing magnetic field directionality

- Faraday rotation caused by LOS magnetic field, and thermal electrons:

$$\mathrm{RM} \propto \int n_e \, \vec{B} \cdot d\vec{l}$$

- It is frequency dependent:

$$\chi = \chi_0 + \mathrm{RM} \times \lambda^2$$



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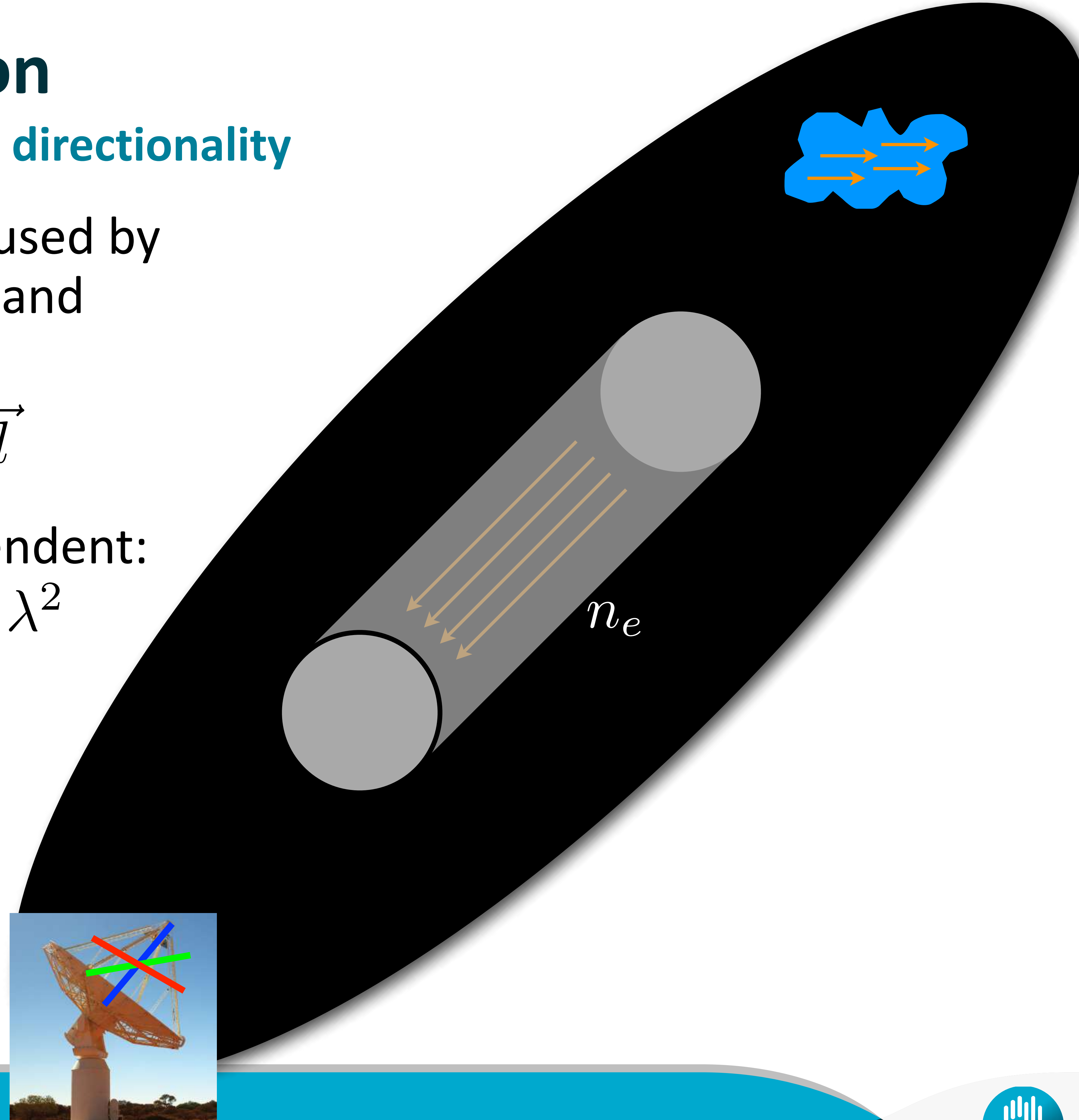
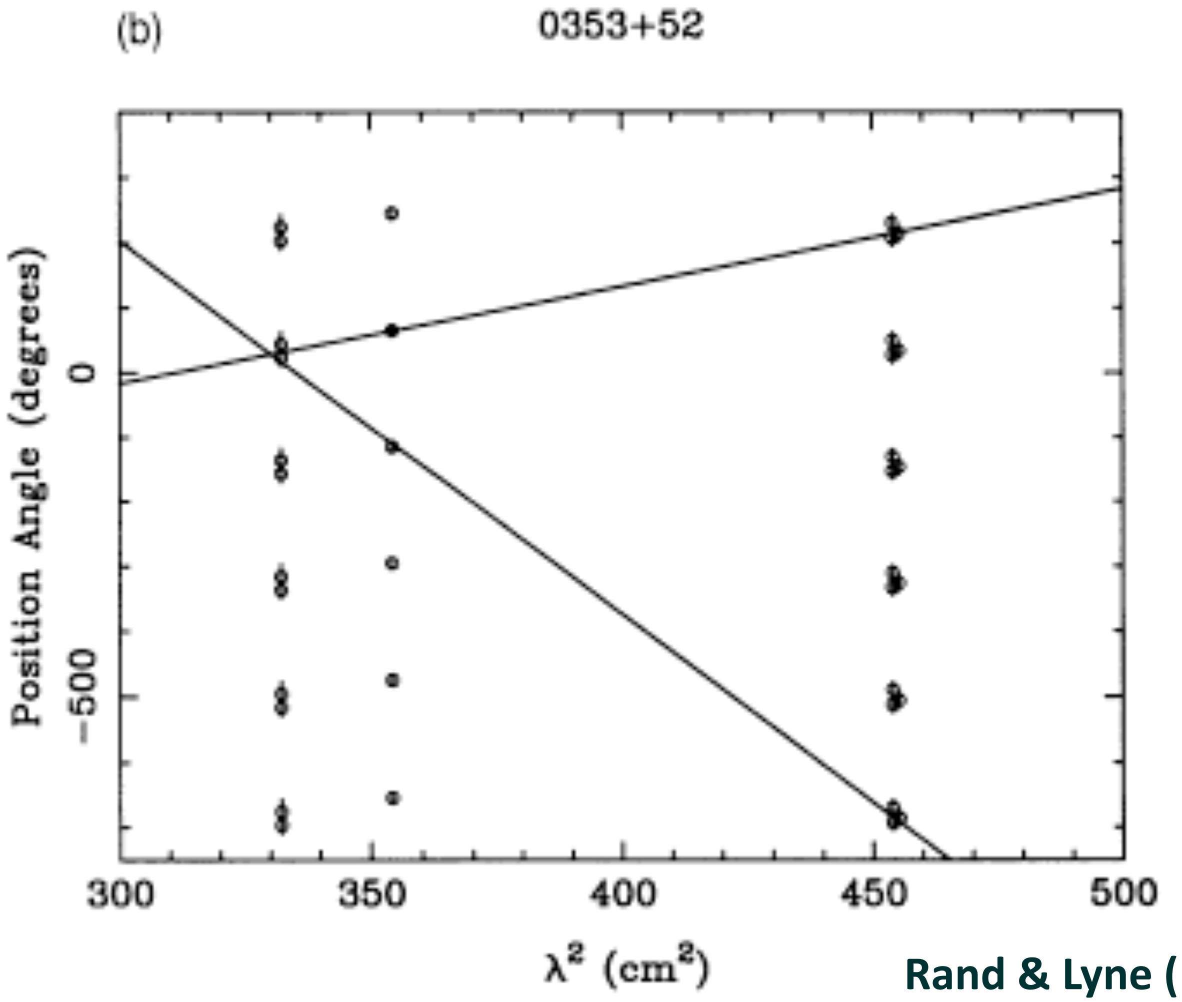


Faraday rotation: the nπ ambiguity

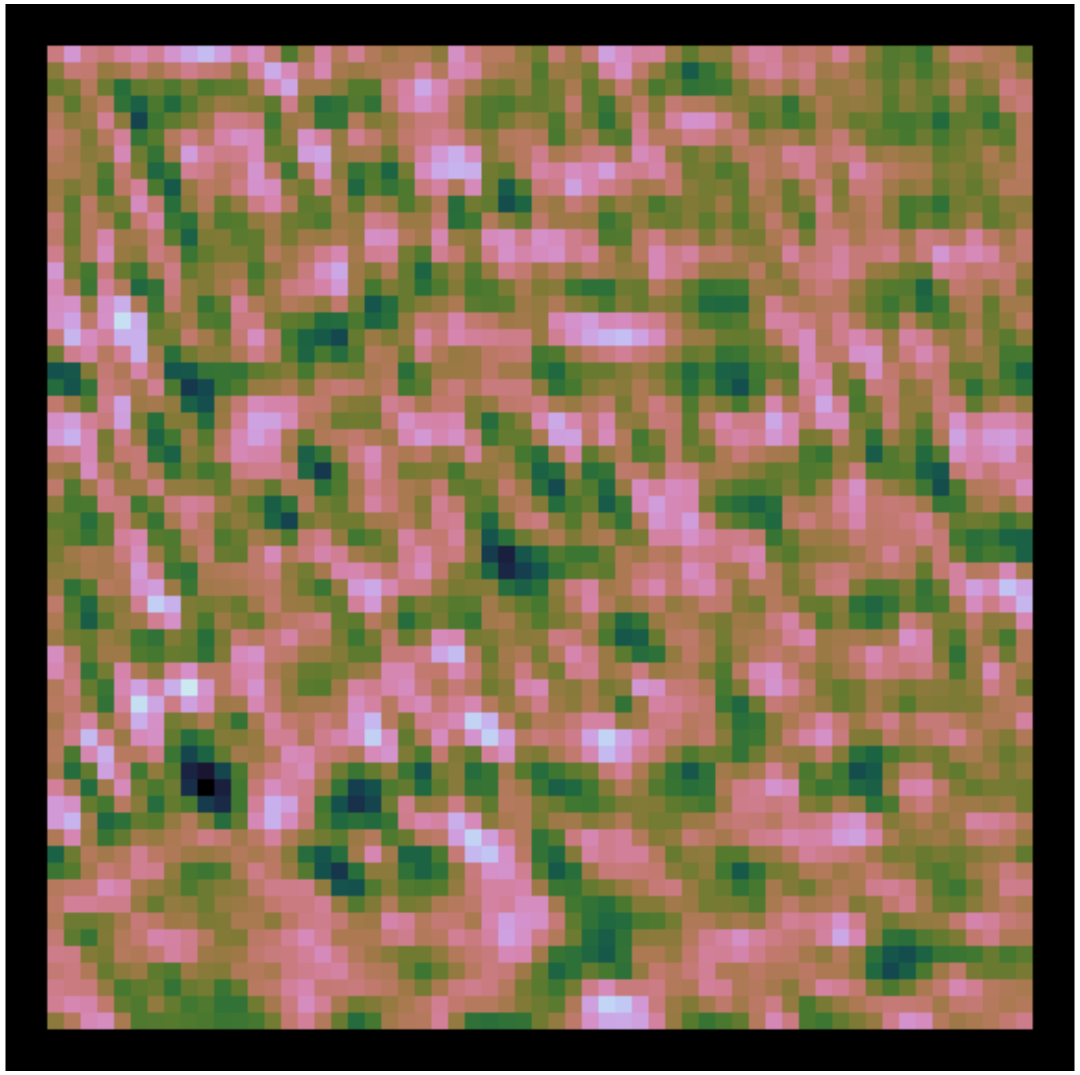
- The ambiguity in polarization angle can lead to errors in Faraday rotation measure (both magnitude and sign!)
  - Critical for interpretation of magnetic field directionality
  - One of the reasons that RM Synthesis was developed - discussed in the Advanced Techniques section of this lecture.



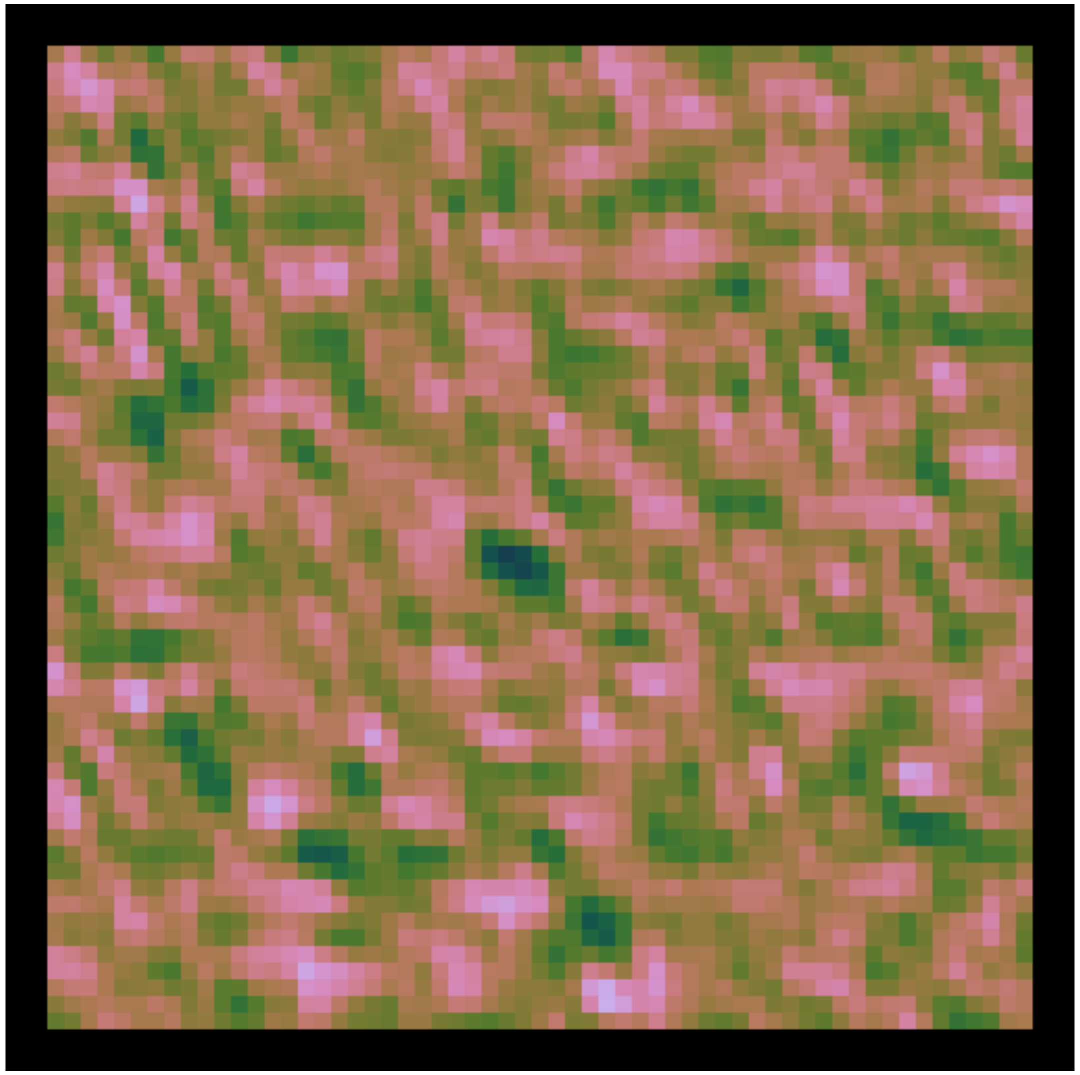
Rand & Lyne (1994)

Faraday rotation: What it looks like

- LOFAR observation (120-170 MHz, steps of 50 kHz)
- Stokes Q and U, one frame per channel
- Polarized pulsar with RM=-64 rad/m^2 (more about this later)



Stokes Q



Stokes U

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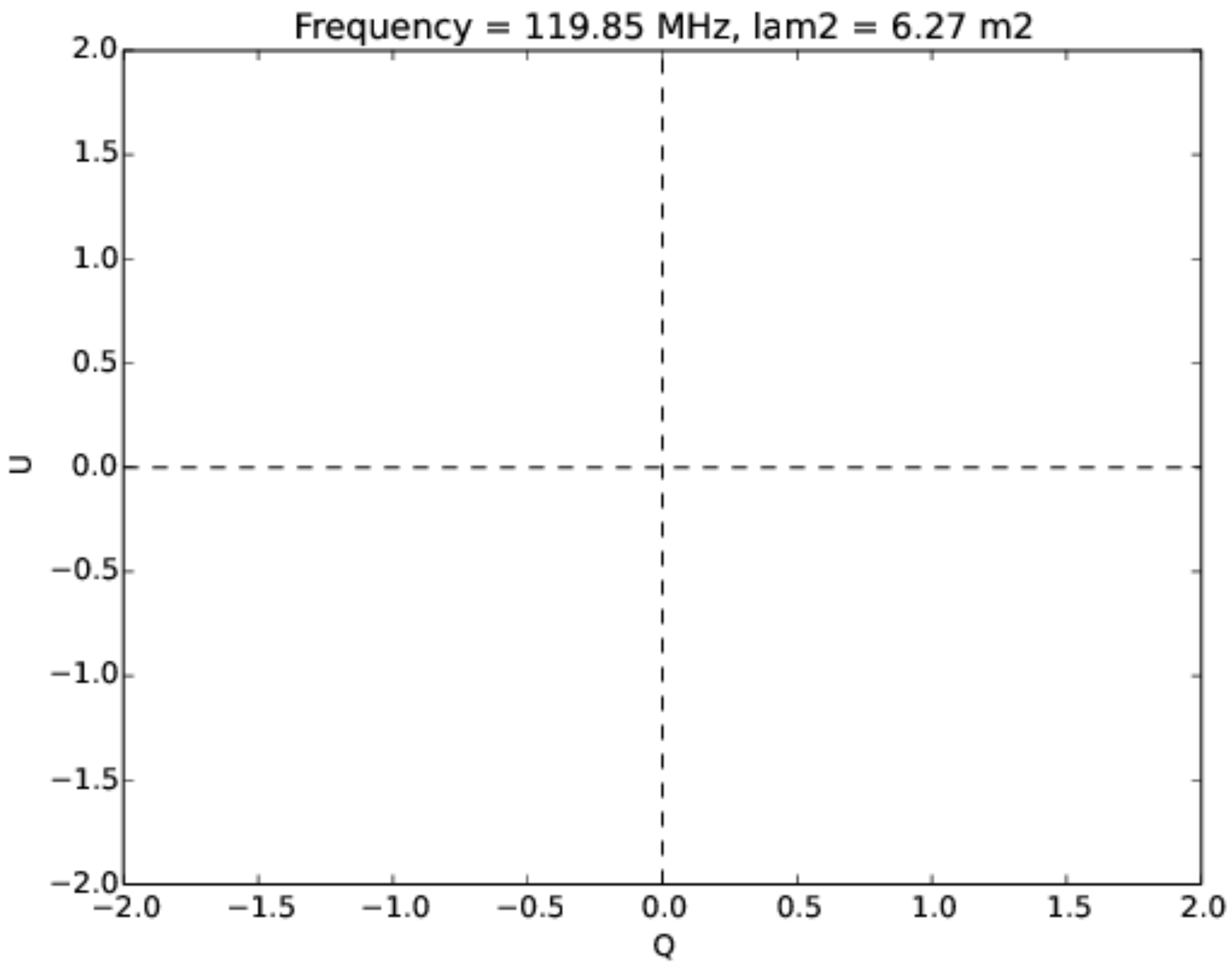

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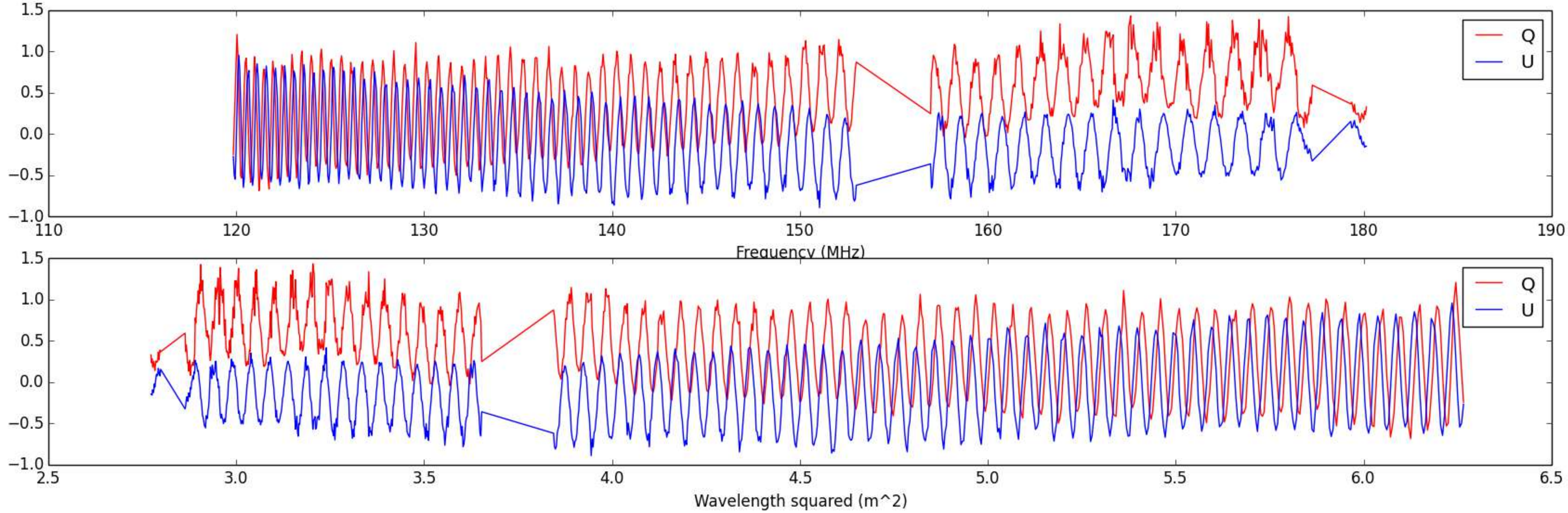
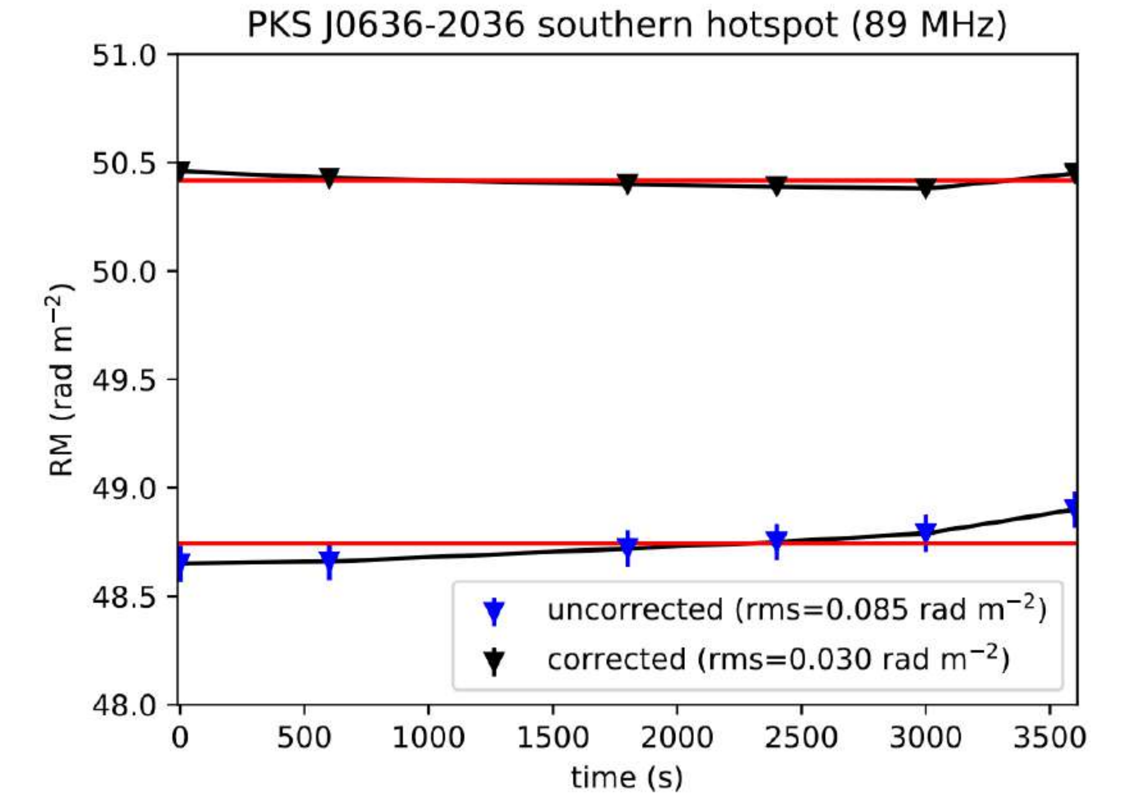


Ionospheric Faraday rotation

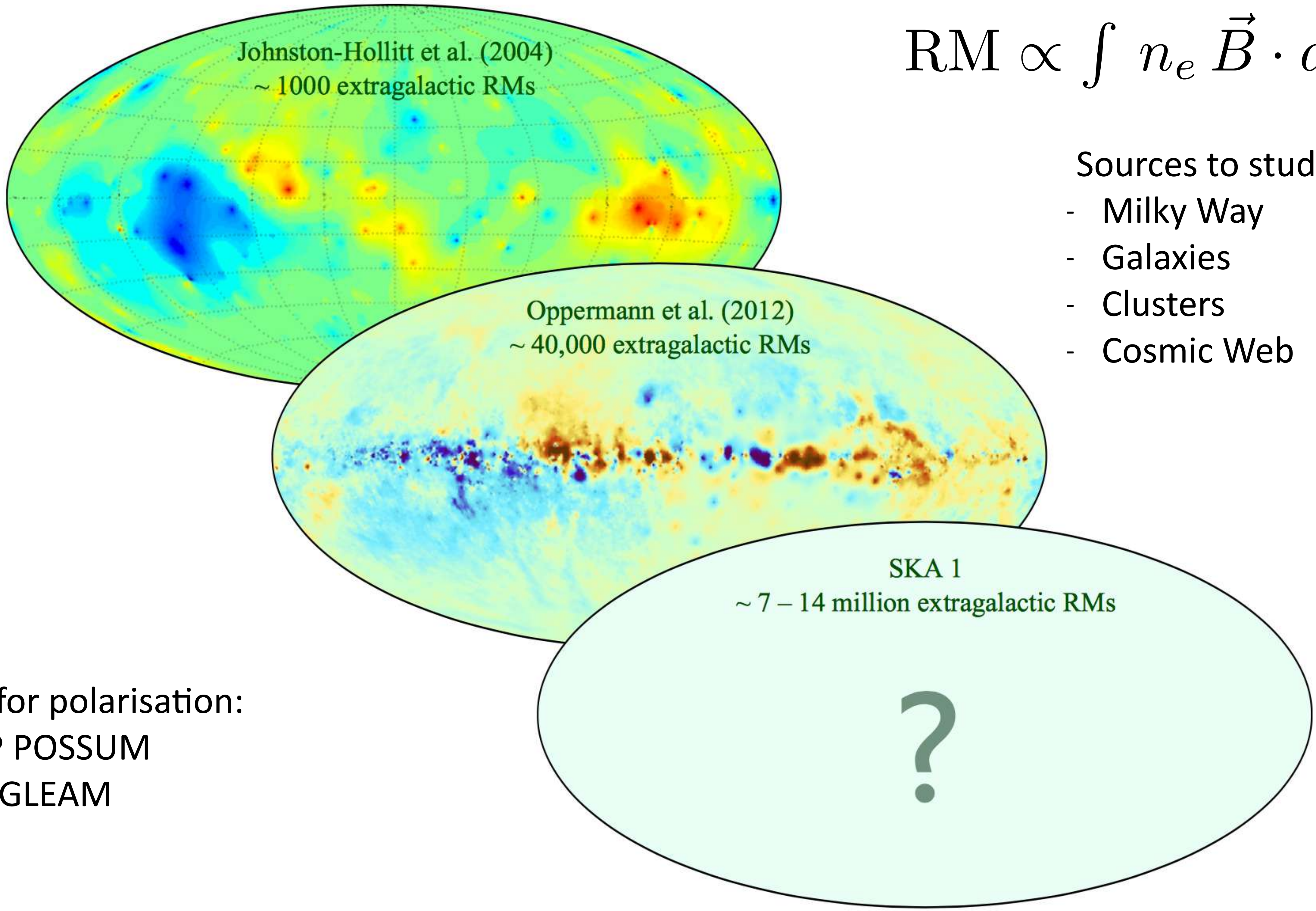
- Ionosphere is also a magnetised plasma, and contributes up to ~1-2 rad/m^2 of Faraday rotation depending on look direction and space weather conditions -- this is highly time variable.
- Several recent publications on this topic in the literature specifically for low frequency observations. See e.g. Emil Lenc's excellent overview of MWA polarization techniques and applications: Lenc et al (2017, PASA, in press; arxiv:1708.05799)
  - ionFR (Sotomayor-Beltran et al 2013, A&A 552, 58)
  - RMextract (M. Mevius; http://github.com/maaijke/RMextract)



Lenc et al (2017)

RM Grid: Fundamental tool for the future

$$\mathrm{RM} \propto \int n_e \vec{B} \cdot d\vec{l}$$



Sources to study:
- Milky Way
- Galaxies
- Clusters
- Cosmic Web

Surveys for polarisation:
- ASKAP POSSUM
- MWA GLEAM
- SKA ...

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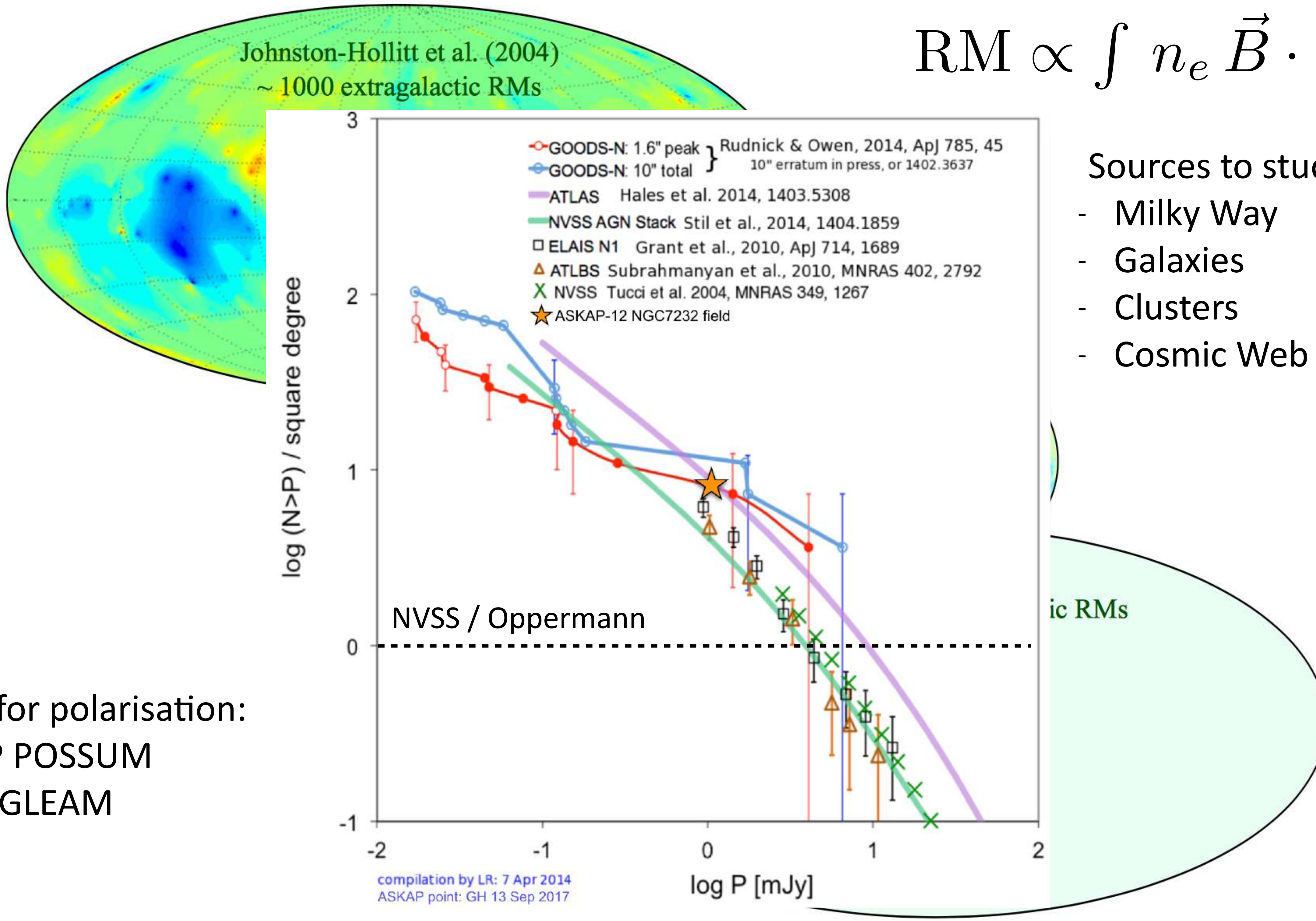

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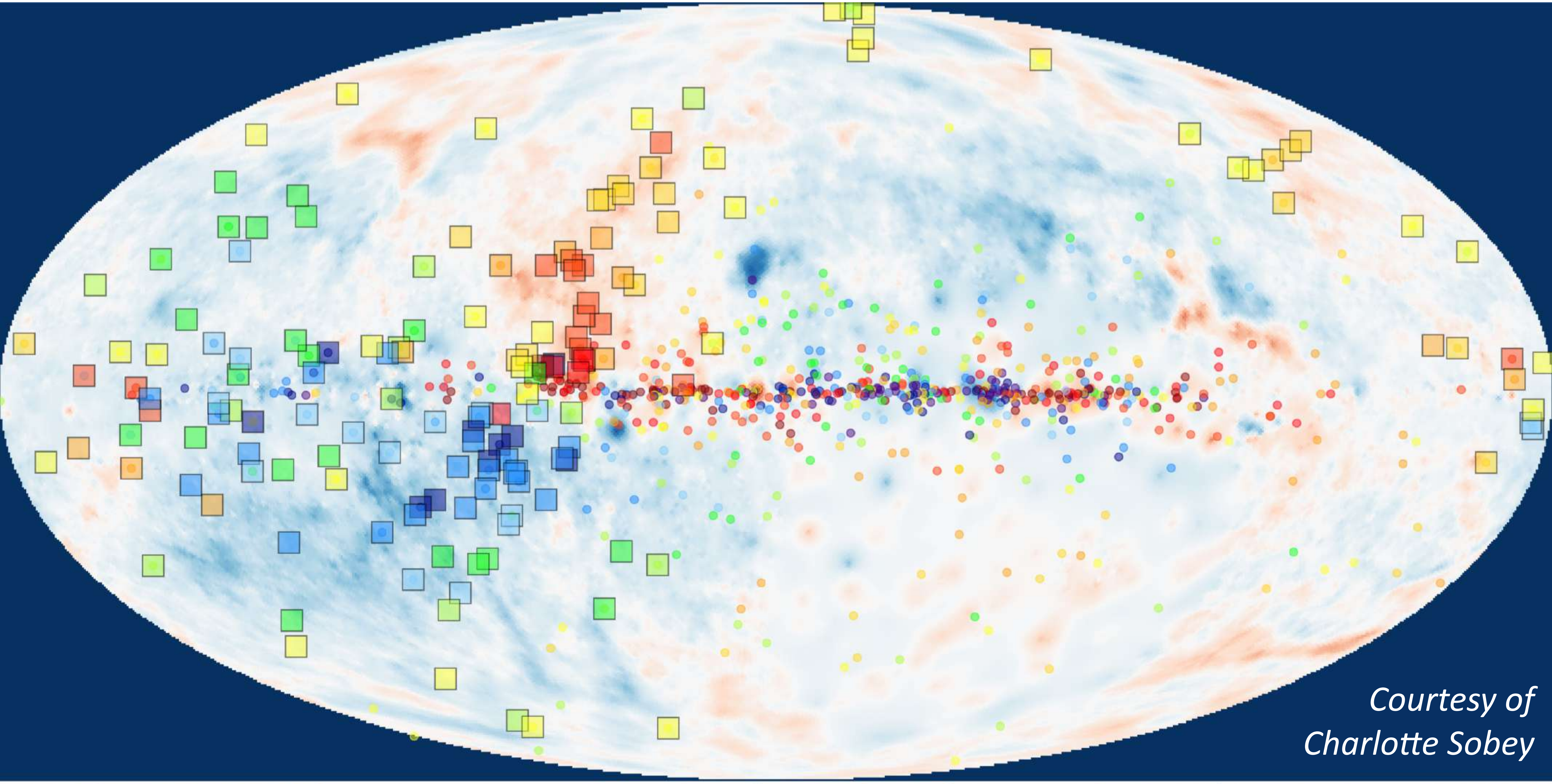
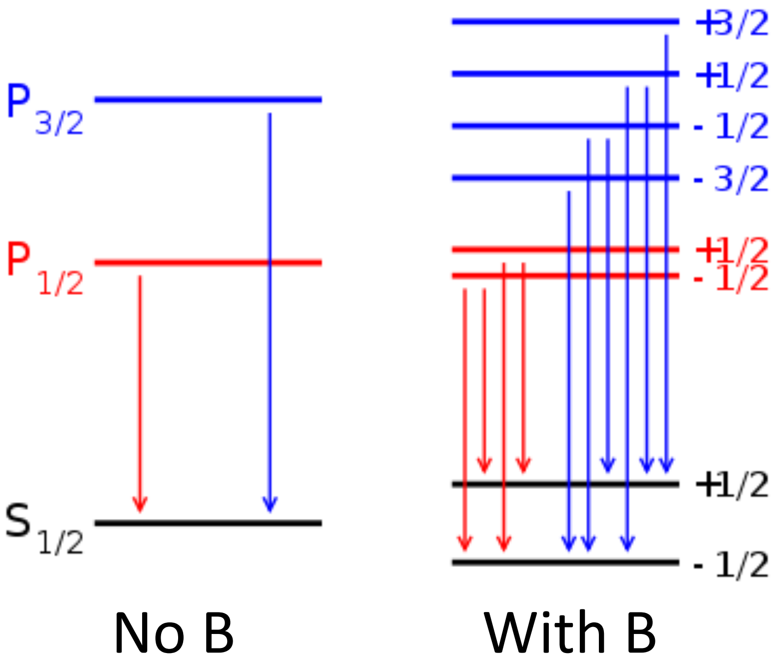
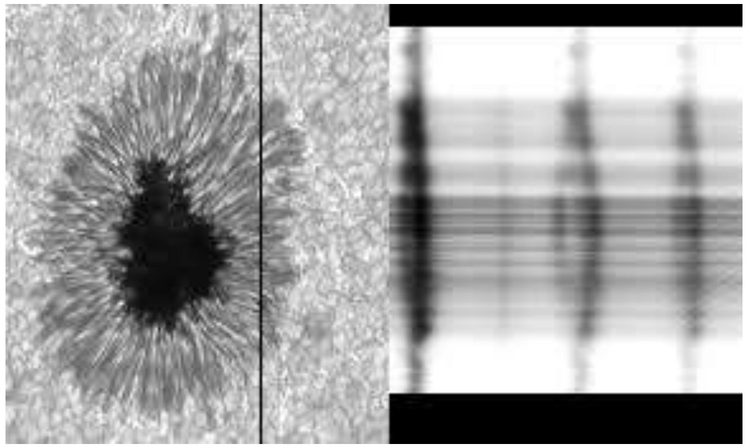


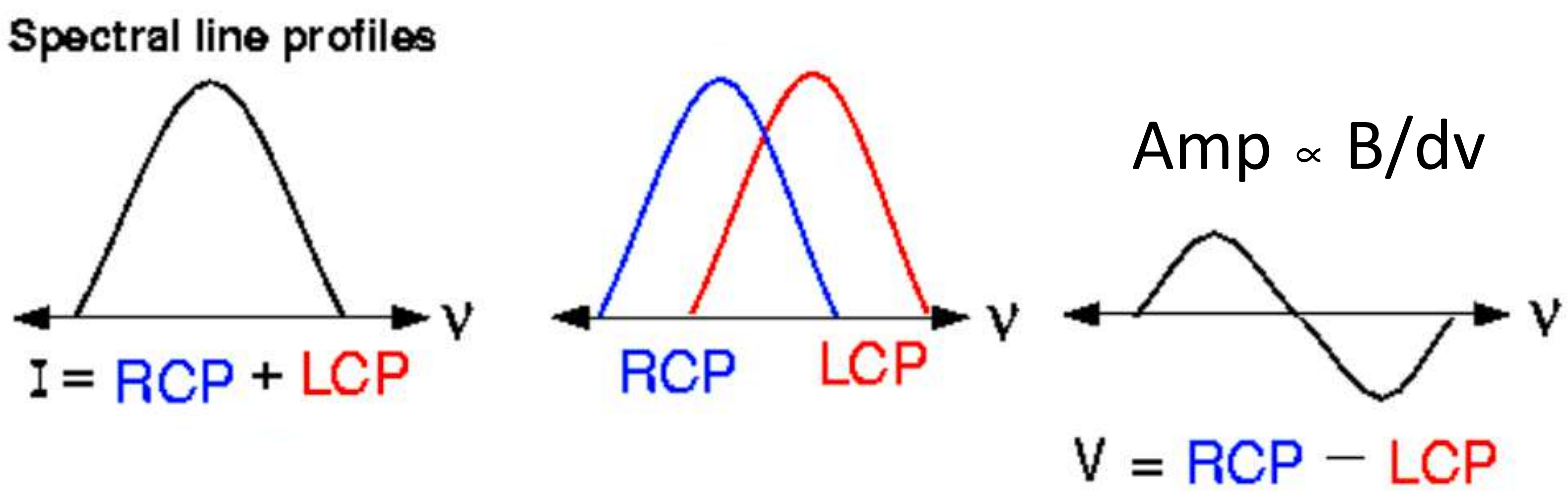
- LOFAR HBA RMs (Sobey et al. in prep., ~200 squares)
- Current pulsar RM catalogue (Manchester et al. 2005, 680 circles)
- Extragalactic sources (Oppermann et al. 2014, background)

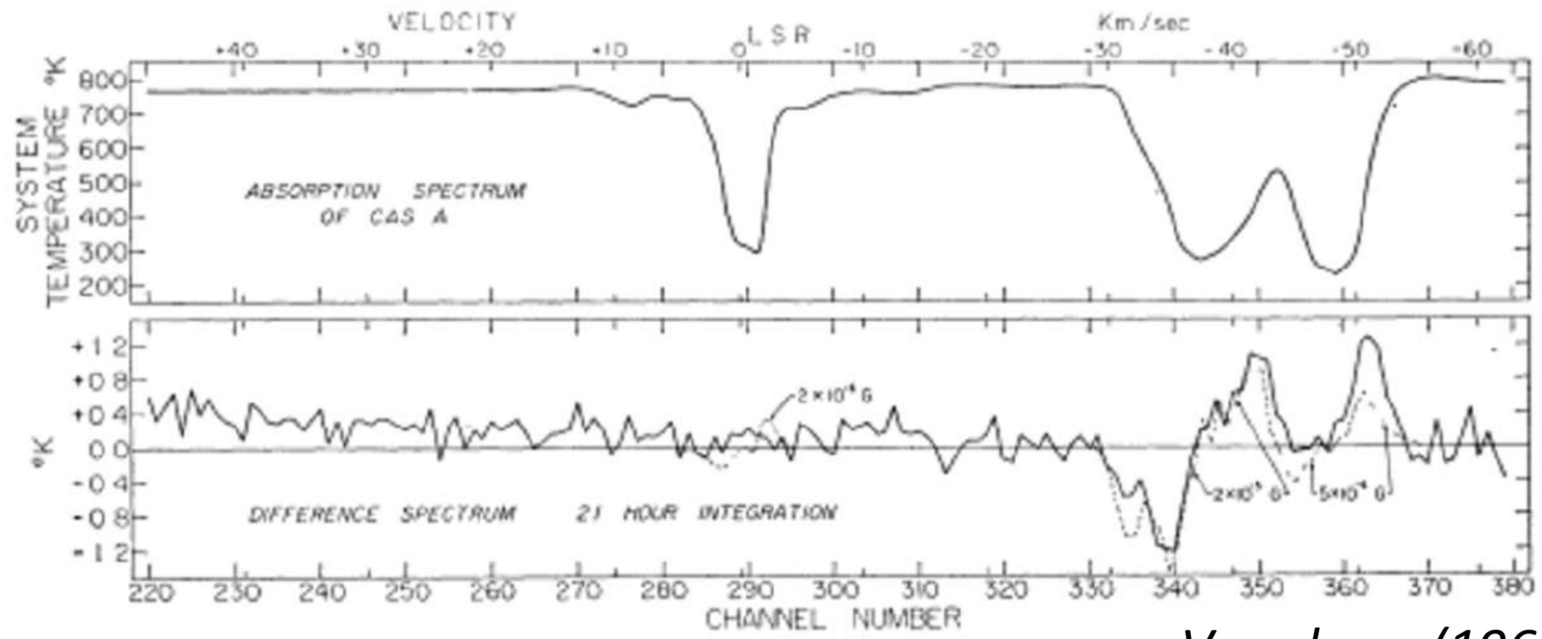
Circular polarisation: Zeeman effect

- Atoms/molecules with net magnetic moment have split energy levels (and thus transitions) in presence of magnetic field
- If magnetic field parallel to LoS, circularly polarised components









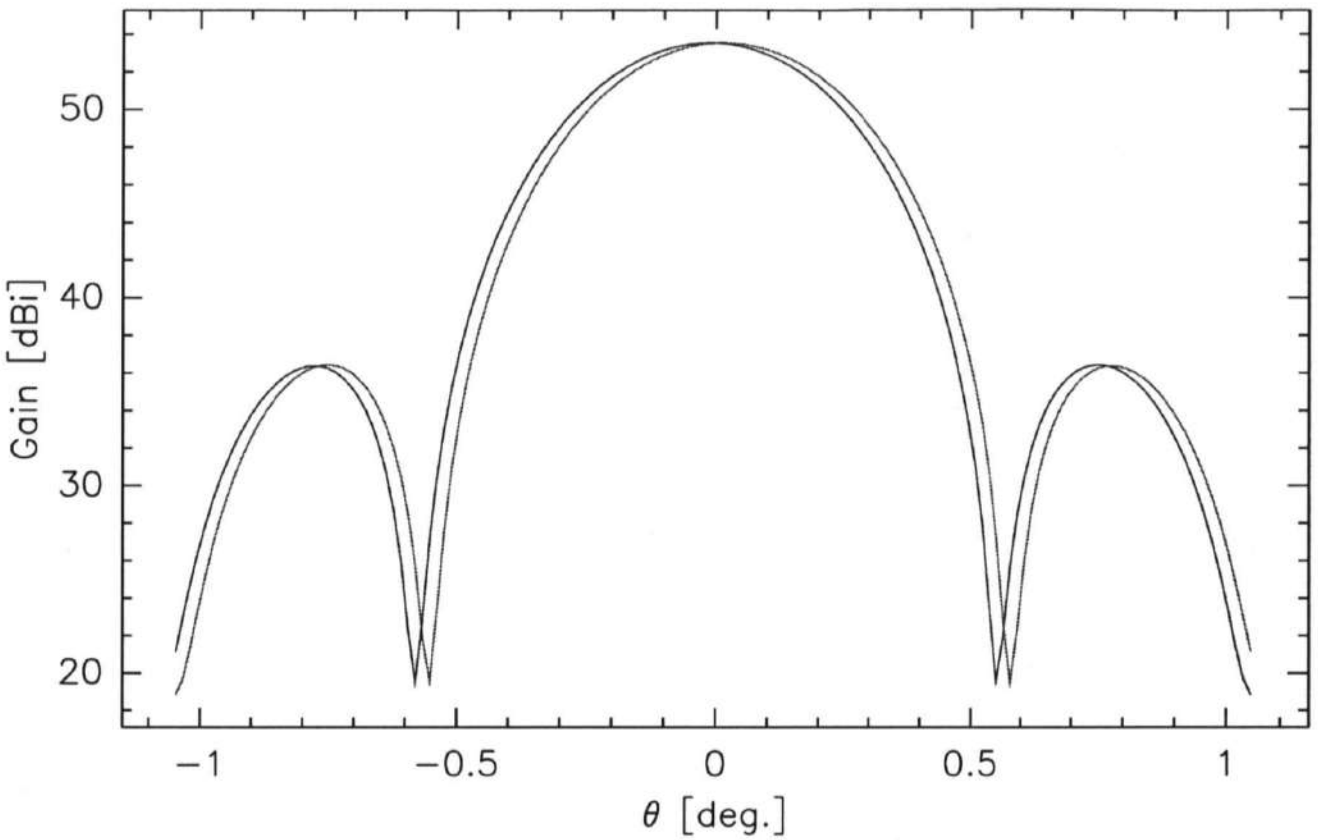
Verschuur (1969)

Widefield polarimetry

Widefield polarimetry with SPF antennas

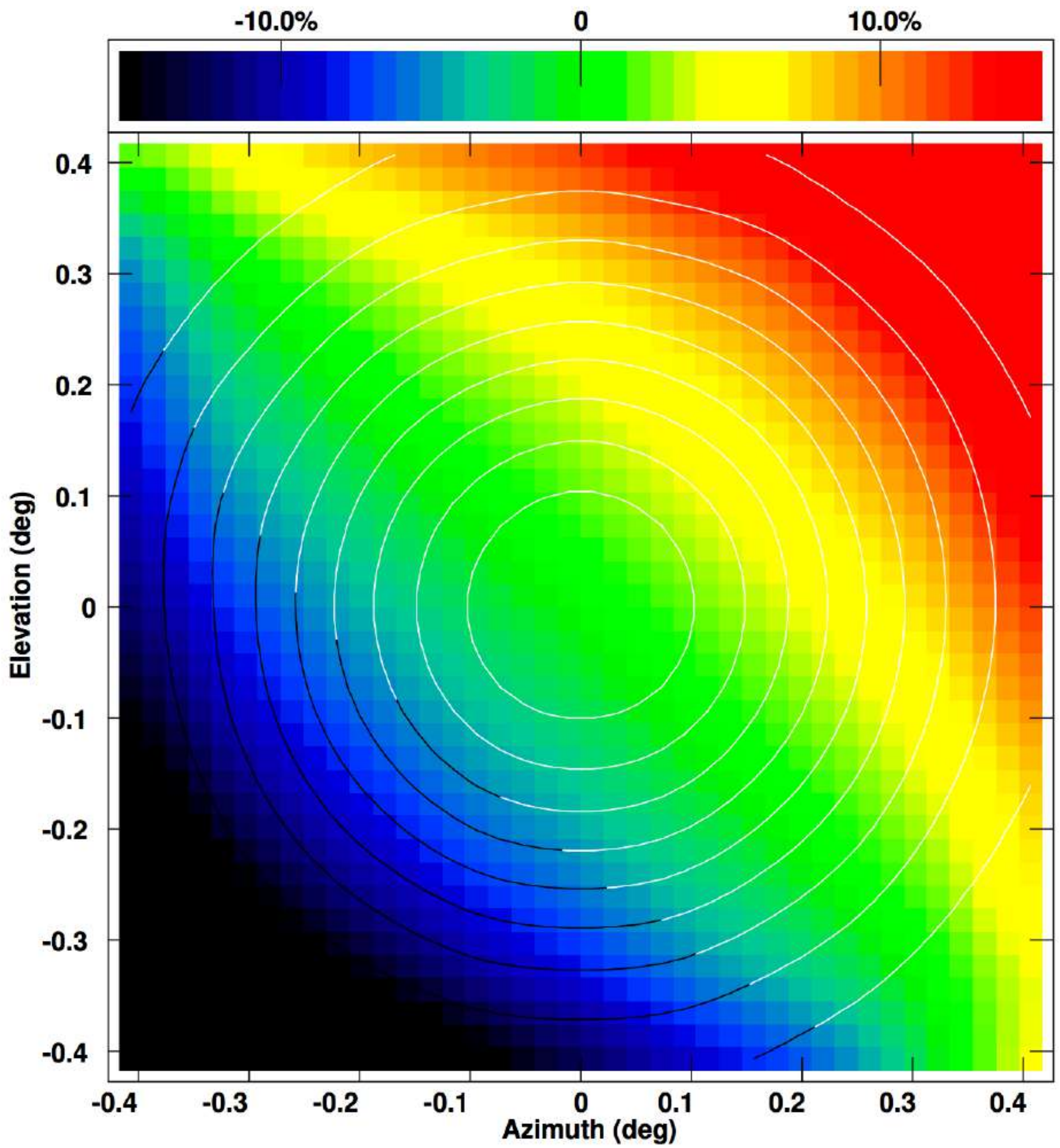
- Widefield response of antenna to the different feeds leads to a complicated pattern in polarization quantities

The VLA's L and R polarizations effectively point in different directions:



Resulting (idealized) Stokes V response ➡

EVLA Memo 58: Brisken



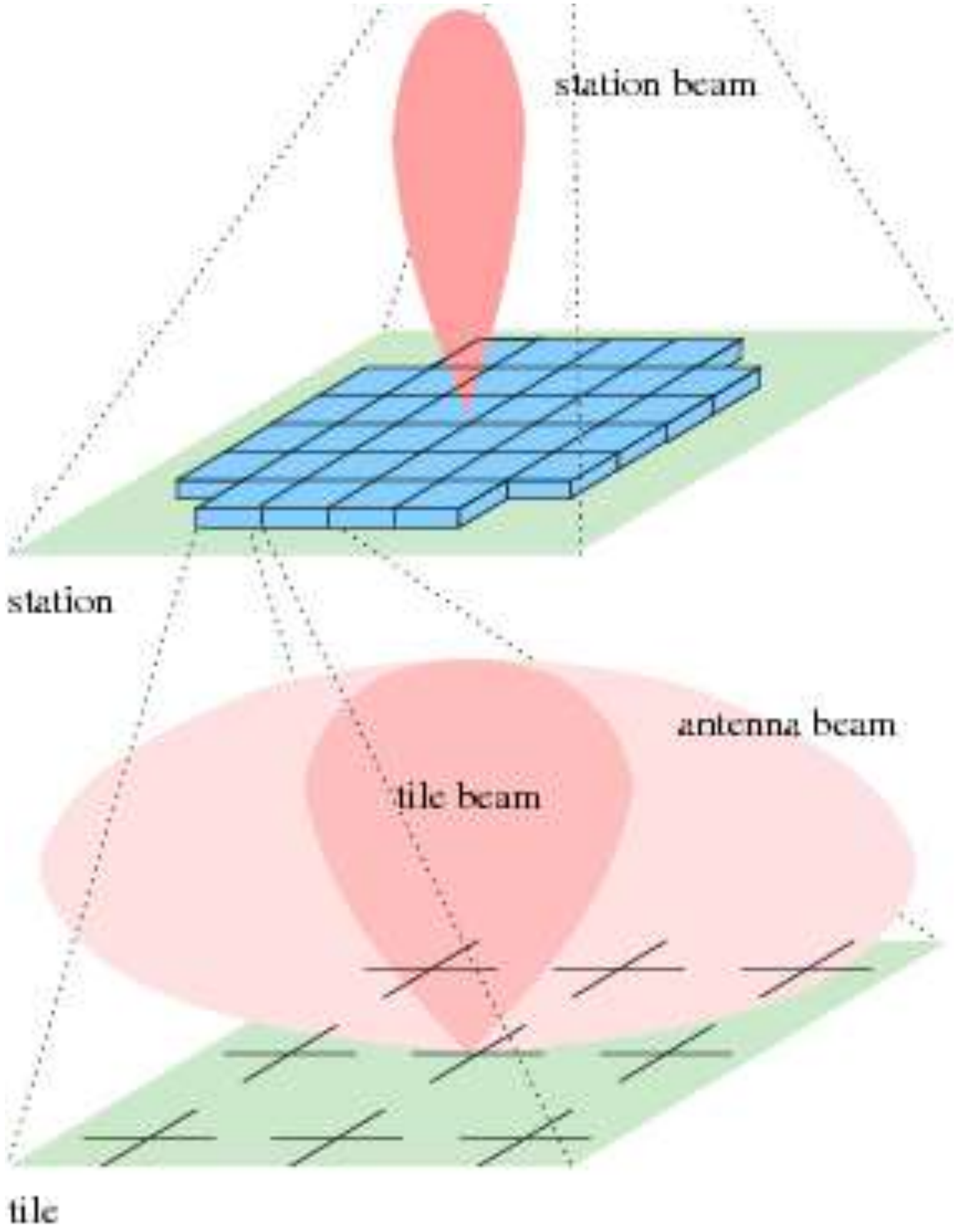
EVLA Memo 113: Cotton & Uson

Widefield polarimetry with SPF antennas

- For ATCA, from the User Guide:
  - On-axis instrumental polarisation is typically below 2-3%, reducing to ≲0.1% after calibration
  - Off-axis polarisation increases roughly as the square of the distance from the pointing centre at least up to the half-power point
    - At 16, 6 and 3cm, the instrumental polarisation is about 3%, 1.6% and 3% of the apparent total intensity at the half power point, respectively
    - At 16 and 6cm, almost purely linear instrumental polarization; at 3cm the circularly polarised component is <1% at the half-power point
    - Alt-az mount ➡ off-axis response varies with parallactic angle
    - `miriad` task `offpol` can be used to simulate off-axis polarimetric response of a long synthesis observation. Mosaicing smears out the off-axis response still further, by as much as an order of magnitude.

Widefield polarimetry for aperture arrays

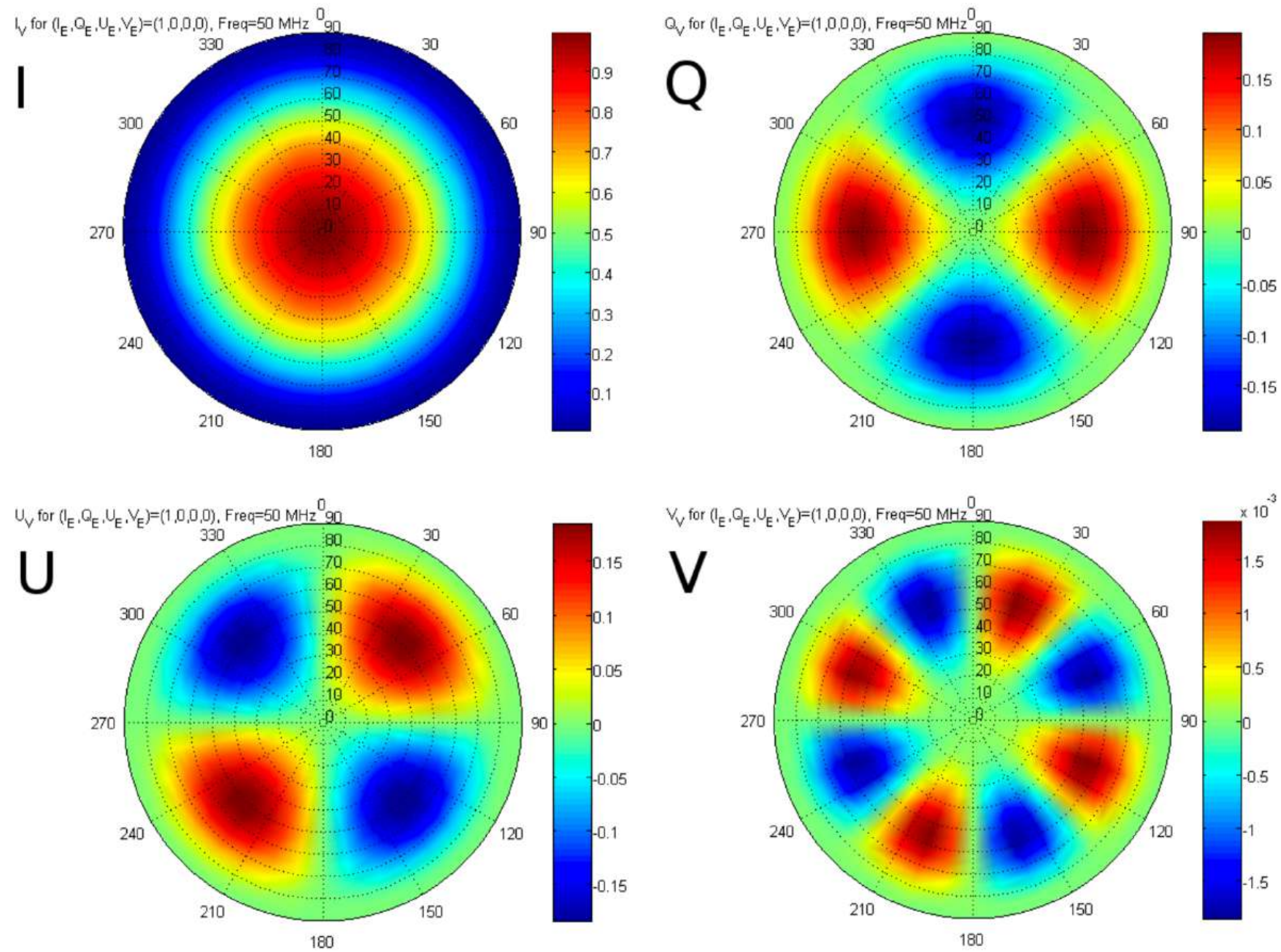
- Aperture arrays have fixed dipoles (with fixed response pattern on the sky), combined in one or more beamforming stages (analogue and/or digital)
  - Like ASKAP PAF, but fixed on ground



Widefield polarimetry for aperture arrays

- In principle, large leakage signals are expected to result if direction-, time-, and frequency-dependent beam corrections are not applied to the data

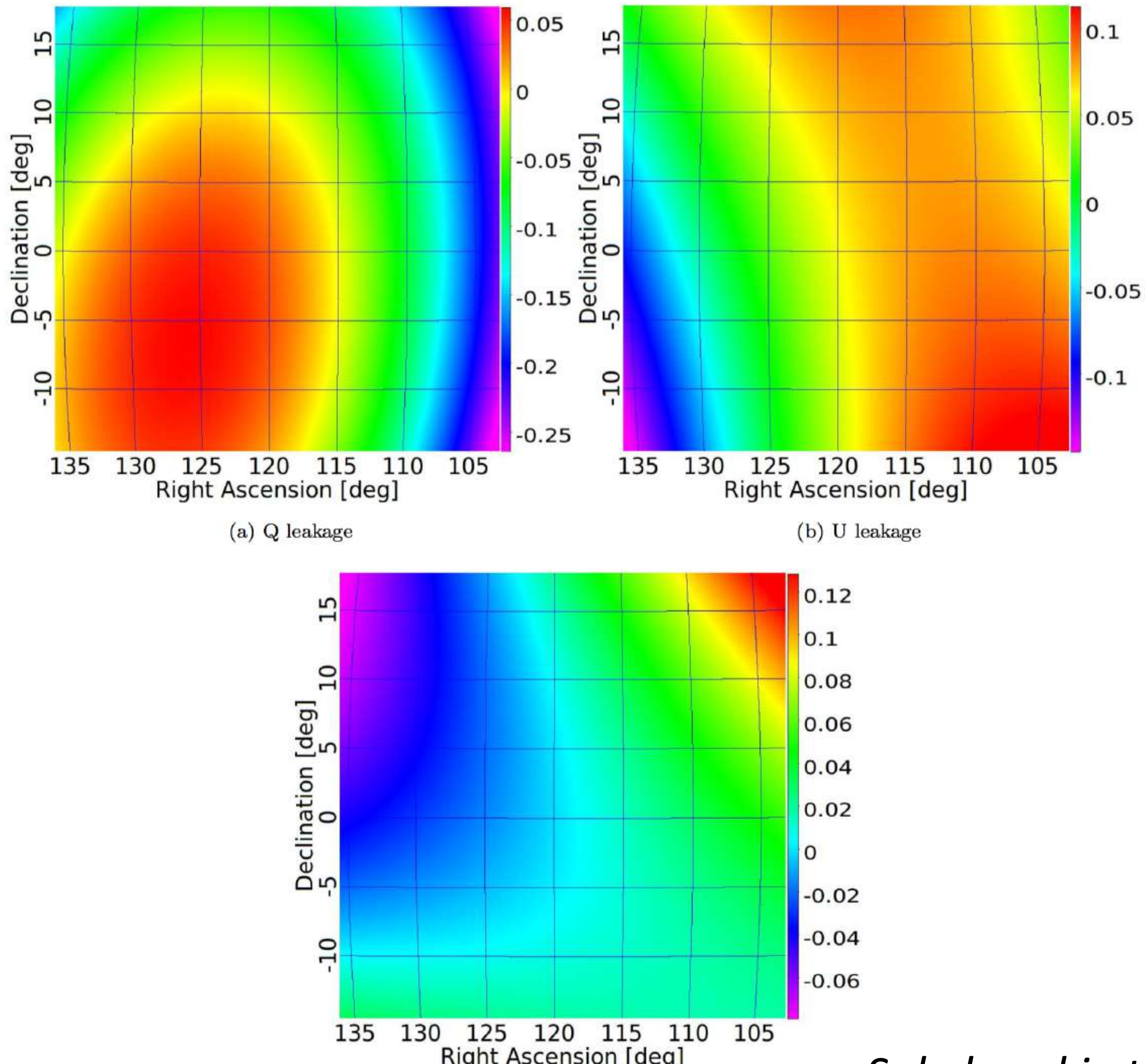
LBA element beam model (50 MHz)



Brentjens

Widefield polarimetry for aperture arrays

- In practice, leakage signals ~10s of percent of Stokes I are seen e.g. MWA:



(a) Q leakage

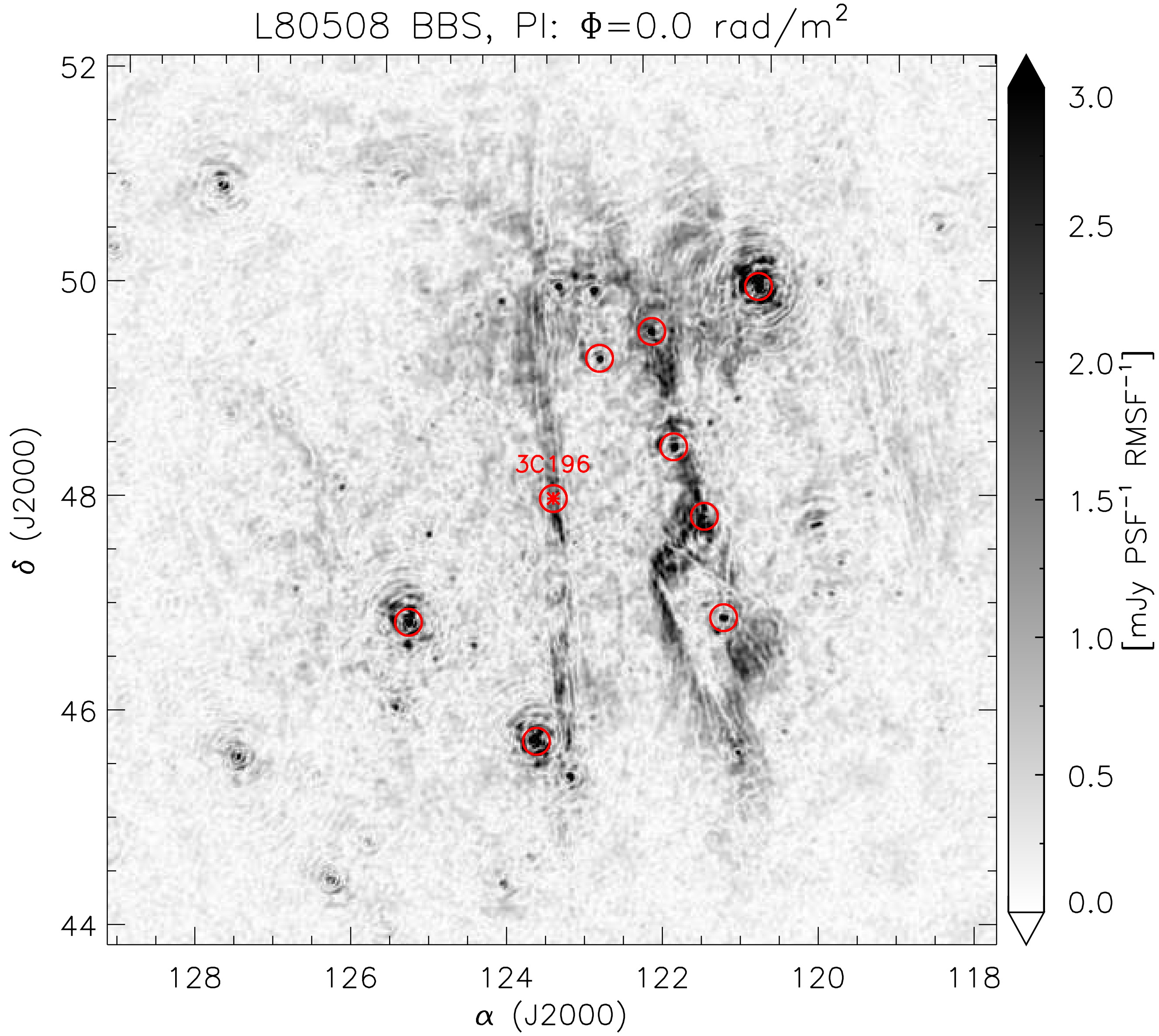
(b) U leakage

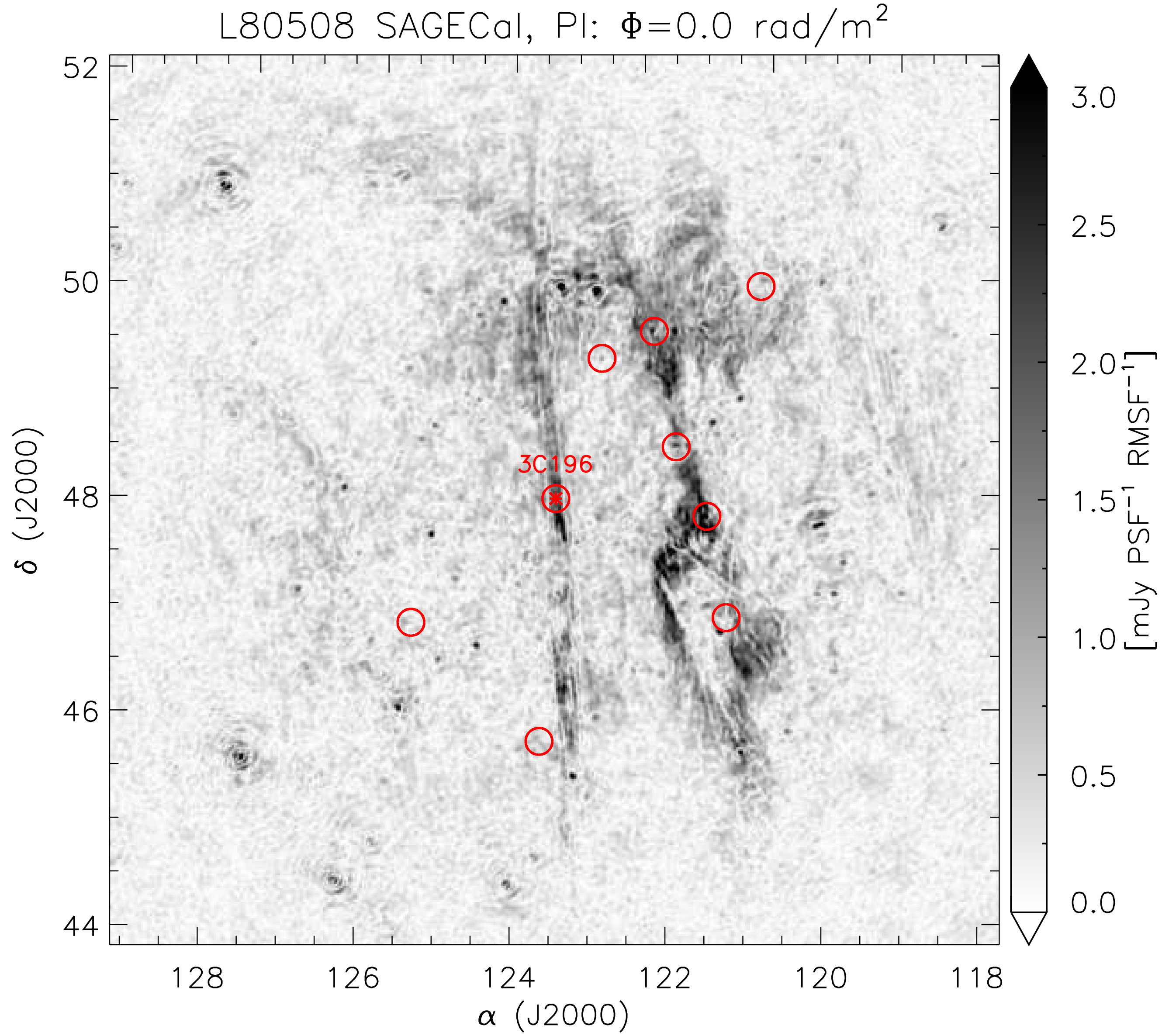
(c) V leakage

Sokolowski et al (submitted)

"Peeling" as a key step for polarization

- Removal of instrumentally polarized source using direction-dependent calibration mitigates artificial diffuse polarization:





Jelic et al (2015)

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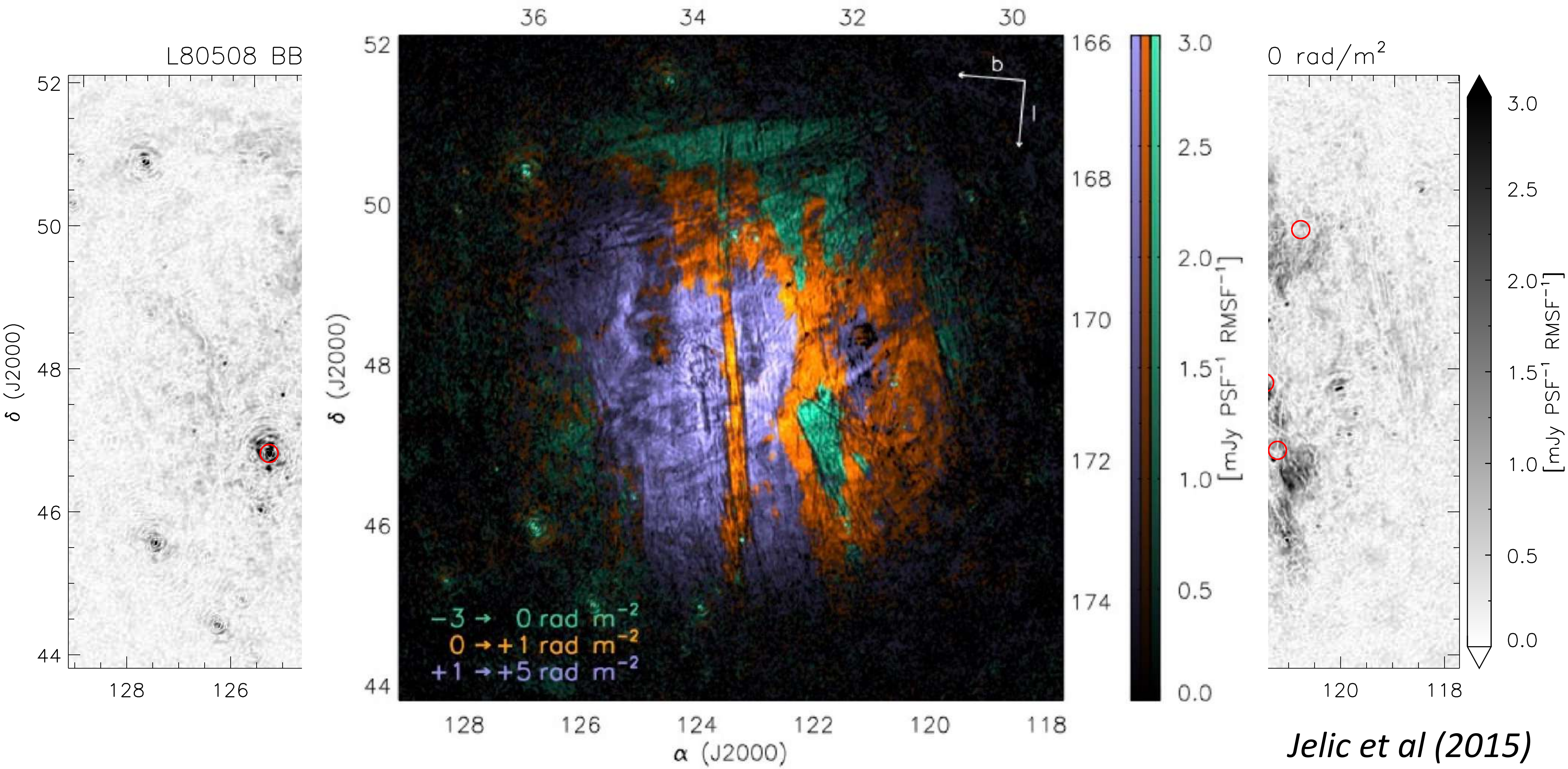
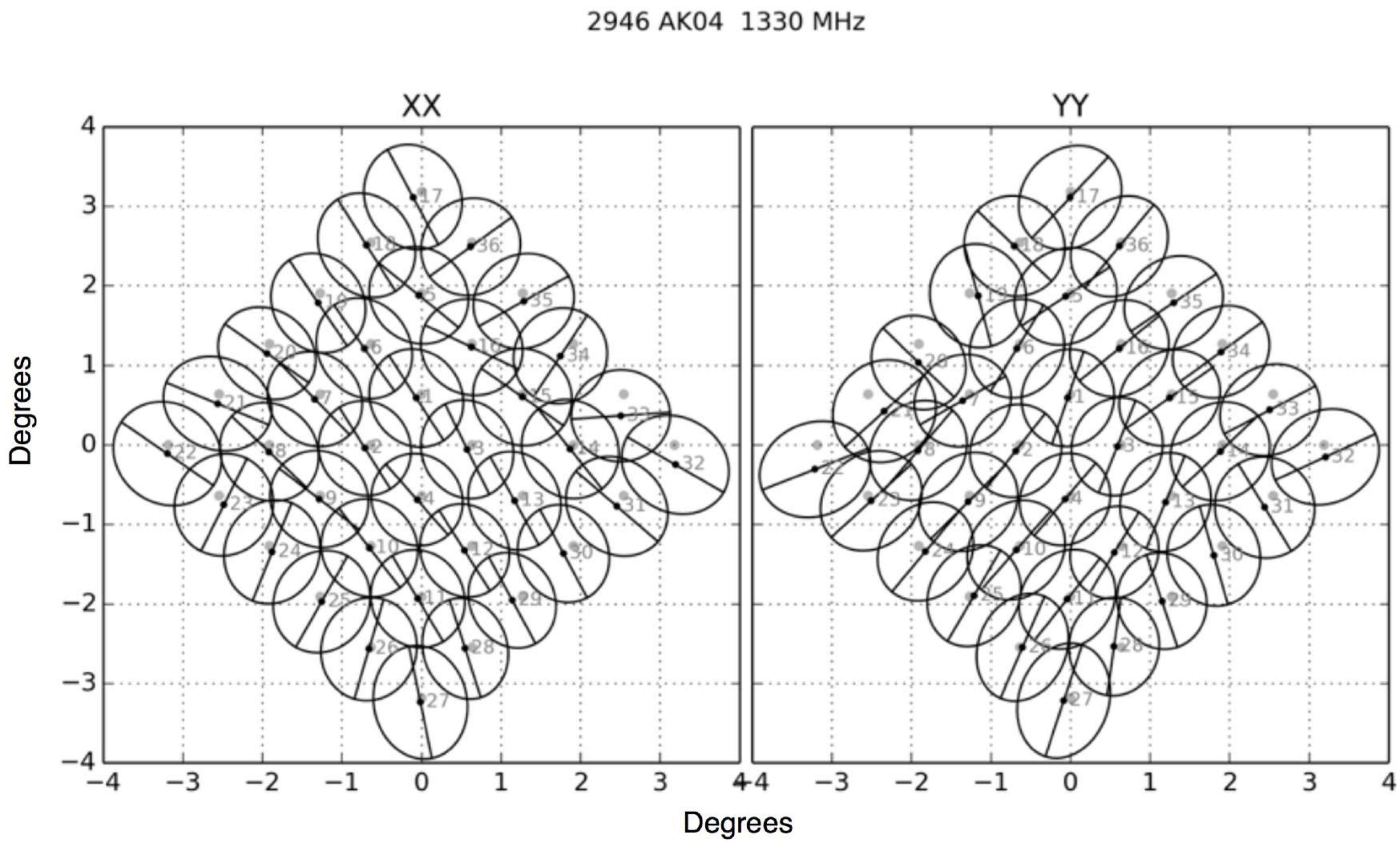


Jelic et al (2015)

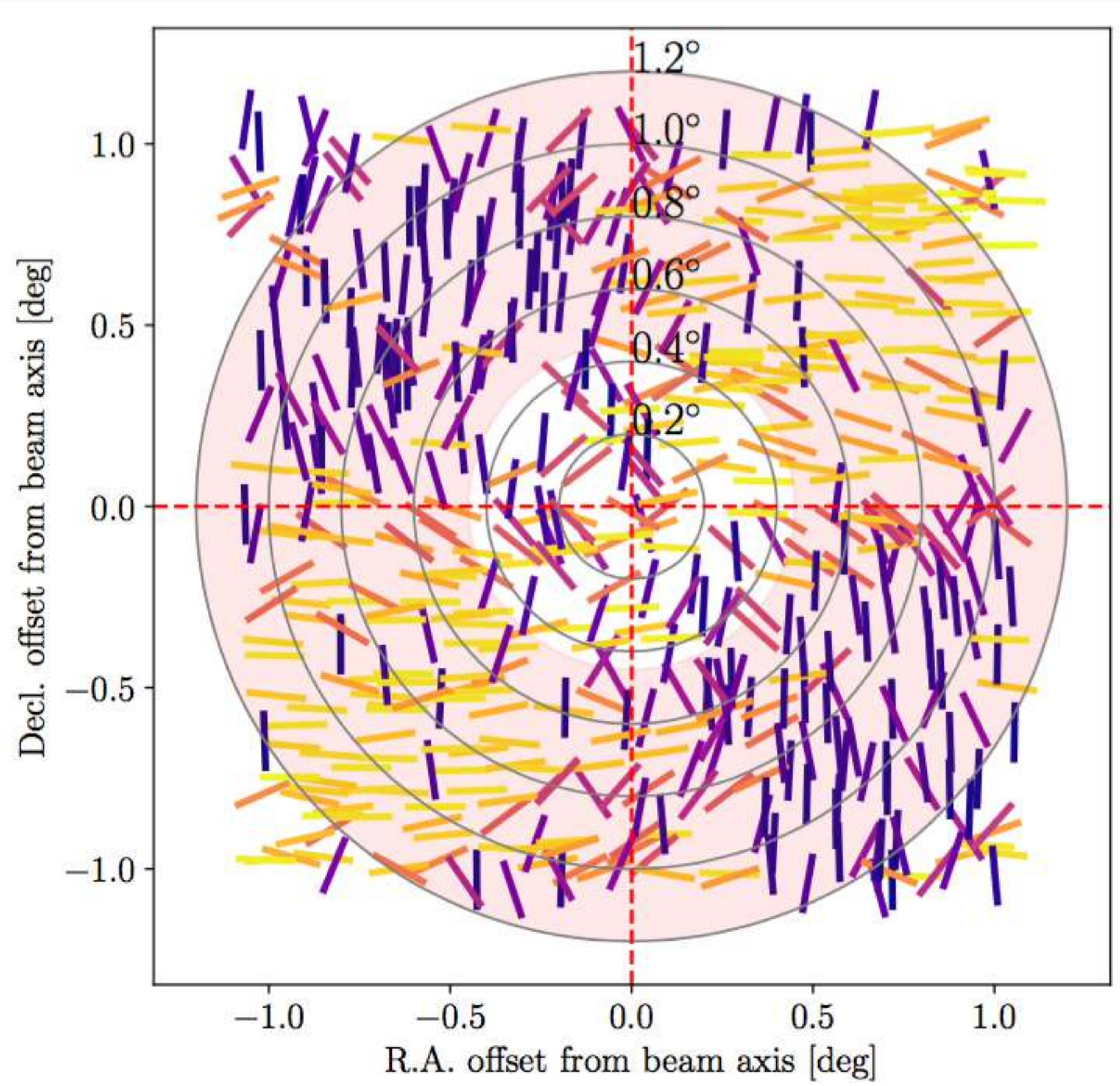
ASKAP beam shapes

- Plots courtesy of Dave McConnell:



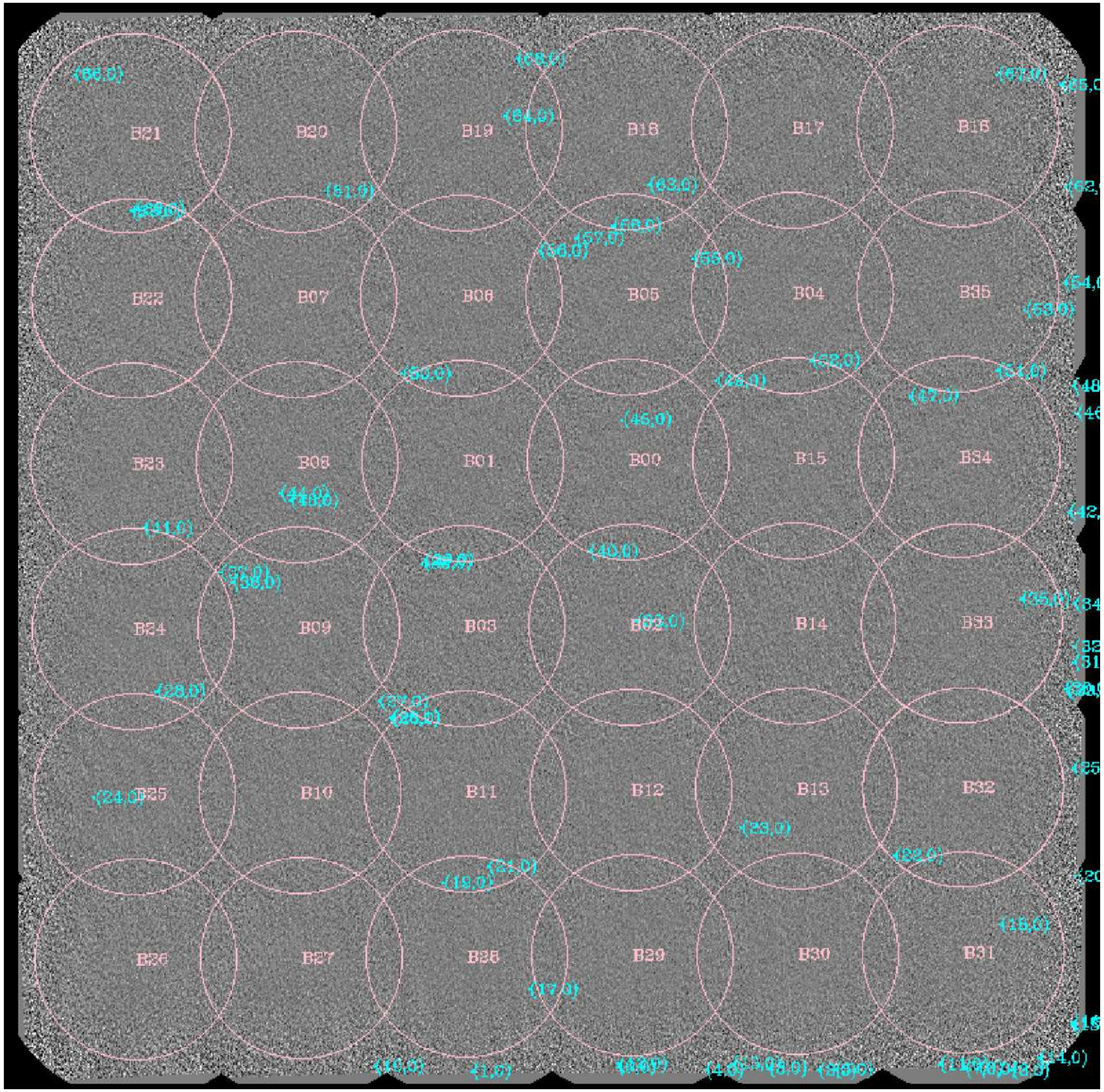
ASKAP instrumental polarization: linear

- Instrumental polarization in Q,U from several beams overlapped



Craig Anderson

ASKAP polarimetric mosaic: Stokes V



GH

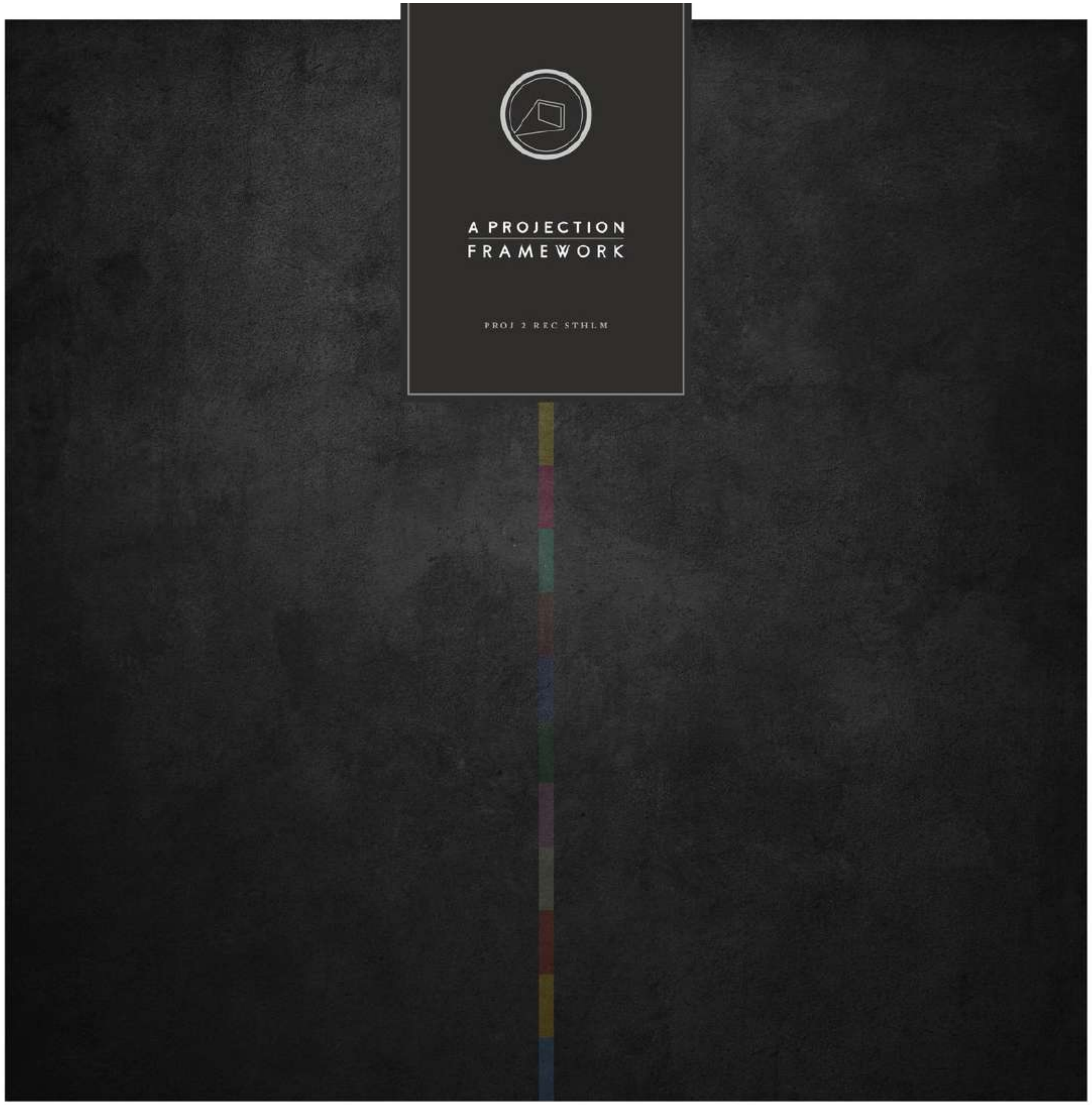
A-projection

- Postpunk band from Stockholm, Sweden (latest album cover: a nice clean image free of leakage?)

- ... is ***also*** a sophisticated technique to take into account the full-polarization antenna response during the imaging process

  Bhatnagar et al (2008)
  (See Tim Cornwell's lecture)

$$V_{ij}^{\mathrm{Obs}} = \mathsf{M}_{ij} \int \mathsf{M}_{ij}^{\mathsf{Sky}}(\boldsymbol{s}) I(\boldsymbol{s}) \mathrm{e}^{2\pi\iota \boldsymbol{s}\cdot\boldsymbol{b}_{ij}} \mathrm{d}\boldsymbol{s}$$



Widefield polarimetry: key concepts

- Antenna response to X,Y (or R,L) is different, and this leads to complicated polarimetric response across the beam
- "Leakage" between Stokes parameters is the empirical result
- Aperture arrays and PAFs require thinking about a ***hierarchical*** beam pattern (e.g. MWA: dipole and tile beam in combination)
- Leakage can be mitigated in several ways, e.g.:
  - Removal of bright sources (does not fix leakage for other sources)
  - Application of corrections in the image plane (for identical beams)
  - Application of per-antenna response as a Jones matrix in the measurement equation (see A-projection for details)

Advanced techniques

Rotation measure synthesis

- Rotation measure synthesis is a simple technique to search for periodic signal in the complex Q,U polarization vector, as a function of the primary observational dimension, λ^2
- This is achieved with a direct Fourier transform
  - Conjugate coordinate is "Faraday depth" (equivalent to RM, for a simple source)
- Can be thought of as a brute-force search for the RM that maximises the polarized signal
- Benefits:
  - Avoids $n\pi$ ambiguity described earlier (makes use of full frequency information available with modern spectrometers)
  - Allows separation of multiple sources with different RMs within the synthesized beam (including instrumental polarization at RM ~0 rad/m^2)
  - Permits use of full broadband sensitivity, without risking bandwidth depolarization that would result from frequency averaging -- this is very important for sources with large RM values

RM Synthesis: basic equations

- Introduced by Burn (1966), further developed in full detail by Brentjens & de Bruyn (2005)

$$RM \rightarrow \phi = 0.81 \int_{\mathrm{source}}^{\mathrm{observer}} n_e \vec{B} \cdot d\vec{l}$$

Faraday depth

- The polarization properties of a source at a particular Faraday depth and observing wavelength:

$$\mathbf{P} = |P|\, e^{2i\chi} = |P|\, e^{2i(\chi_0 + \phi\lambda^2)} = \underbrace{|P|\, e^{2i\chi_0}}_{\text{Intrinsic polarization}}\ \underbrace{e^{2i\phi\lambda^2}}_{\text{Faraday rotation}}$$

- Defining "Faraday dispersion function" and summing over Faraday depths:

$$\mathbf{P}(\lambda^2) = \int_{-\infty}^{+\infty} \underline{\mathbf{F}(\phi)}\, e^{2i\phi\lambda^2}\, d\phi$$

RM Synthesis: basic equations

- We note that the previous equation is like a Fourier transform, and since we know & love Fourier transforms we know how to invert it:

$$\mathbf{P}(\lambda^2) = \int_{-\infty}^{+\infty} \mathbf{F}(\phi)\, e^{2i\phi\lambda^2}\, d\phi$$

$$\mathbf{F}(\phi) = \boxed{\int_{-\infty}^{+\infty}} \mathbf{P}(\lambda^2)\, e^{-2i\phi\lambda^2}\, d\lambda^2$$

Physics

Observed polarization

RM Synthesis: basic equations

- The fact that we don't observe all frequencies, and in particular that we can't observe negative values of λ^2, means that we must work instead with a discrete (rather than continuous) form of the RM Synthesis equation:

$$\mathbf{F}(\phi) = \frac{1}{N}\sum_{i=1}^{N}\mathbf{P}_i\, e^{-2i\phi\lambda_i^2}$$

- Two additional tweaks are made in practice: (i) addition of "shift" term in the exponential to slow down rotation of polarization vector in **F**(φ) [this is purely cosmetic]; and (ii) the application of per-channel weights (can be done similar to imaging):

$$\boxed{\mathbf{F}(\phi) = K\sum_{i=1}^{N} w_i\,\mathbf{P}_i\, e^{-2i\phi(\lambda_i^2-\lambda_0^2)}}$$

RM Synthesis: basic equations

- Observational consequences: bandwidth and channelisation

$$\delta\phi \approx \frac{2\sqrt{3}}{\Delta\lambda^2}$$

More bandwidth ➡ better RM precision

$$\phi_{\max} \approx \frac{\sqrt{3}}{\delta\lambda^2}$$

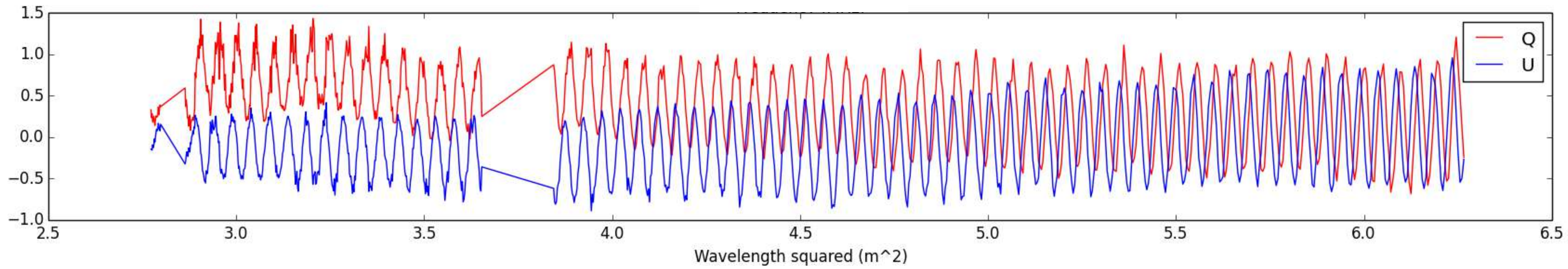
Finer channels ➡ recover higher RMs

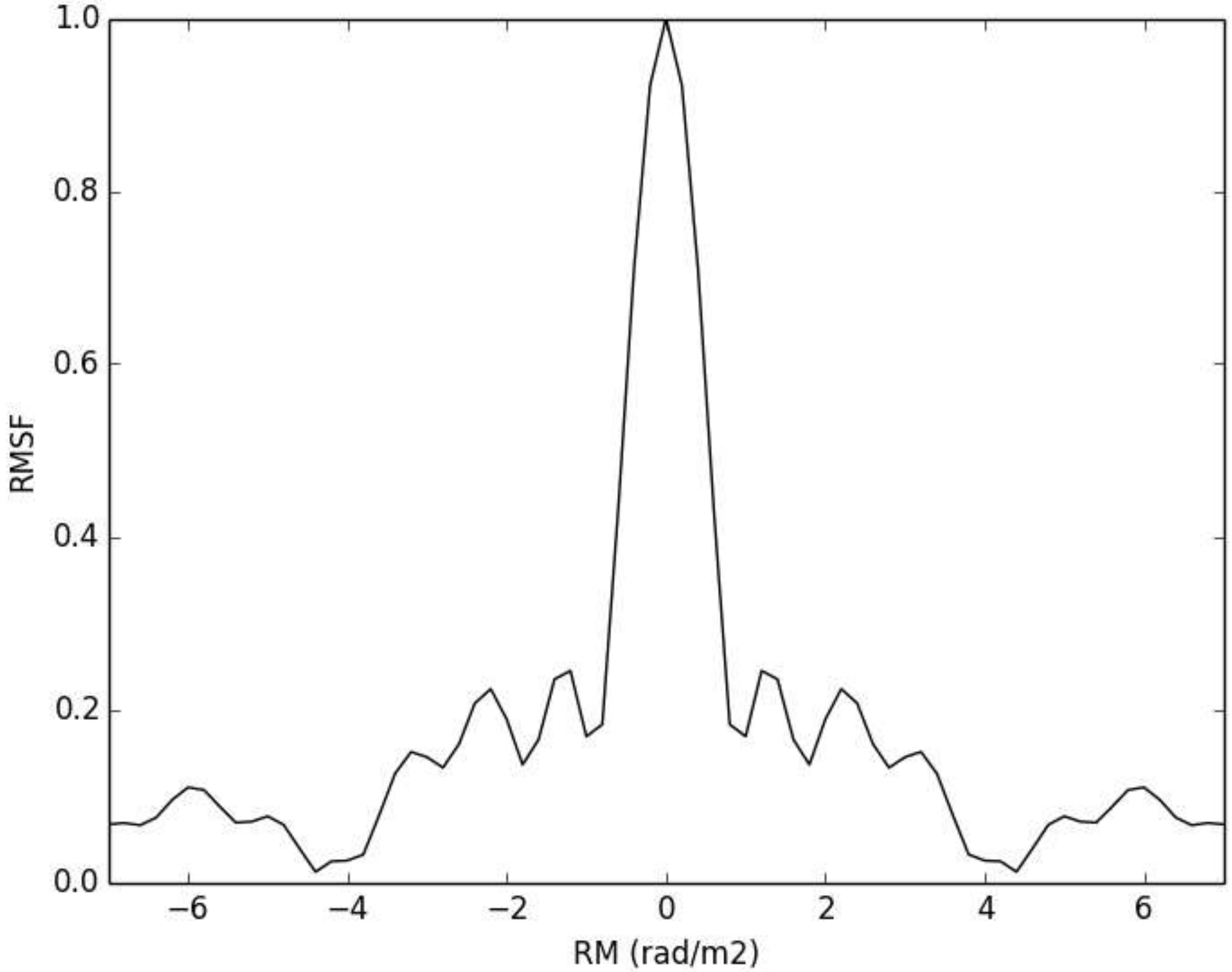
- Pros and cons of high vs low frequency:

	"High" frequency (>700 MHz)	"Low" frequency (<350 MHz)
😎	Recovery of complicated emission	Excellent precision on RMs
💩	Poor precision on RMs	Loss of all but simplest emission

Rotation measure synthesis

- Recall previous example: polarized pulsar with RM=-64 rad/m^2





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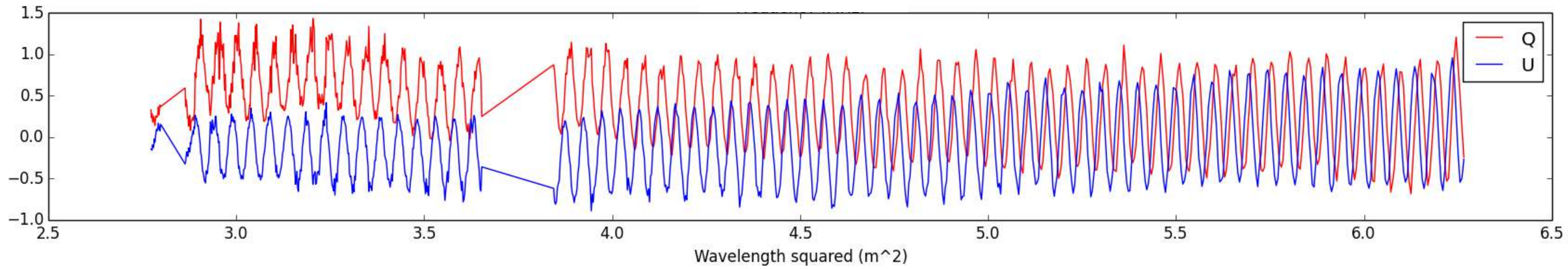
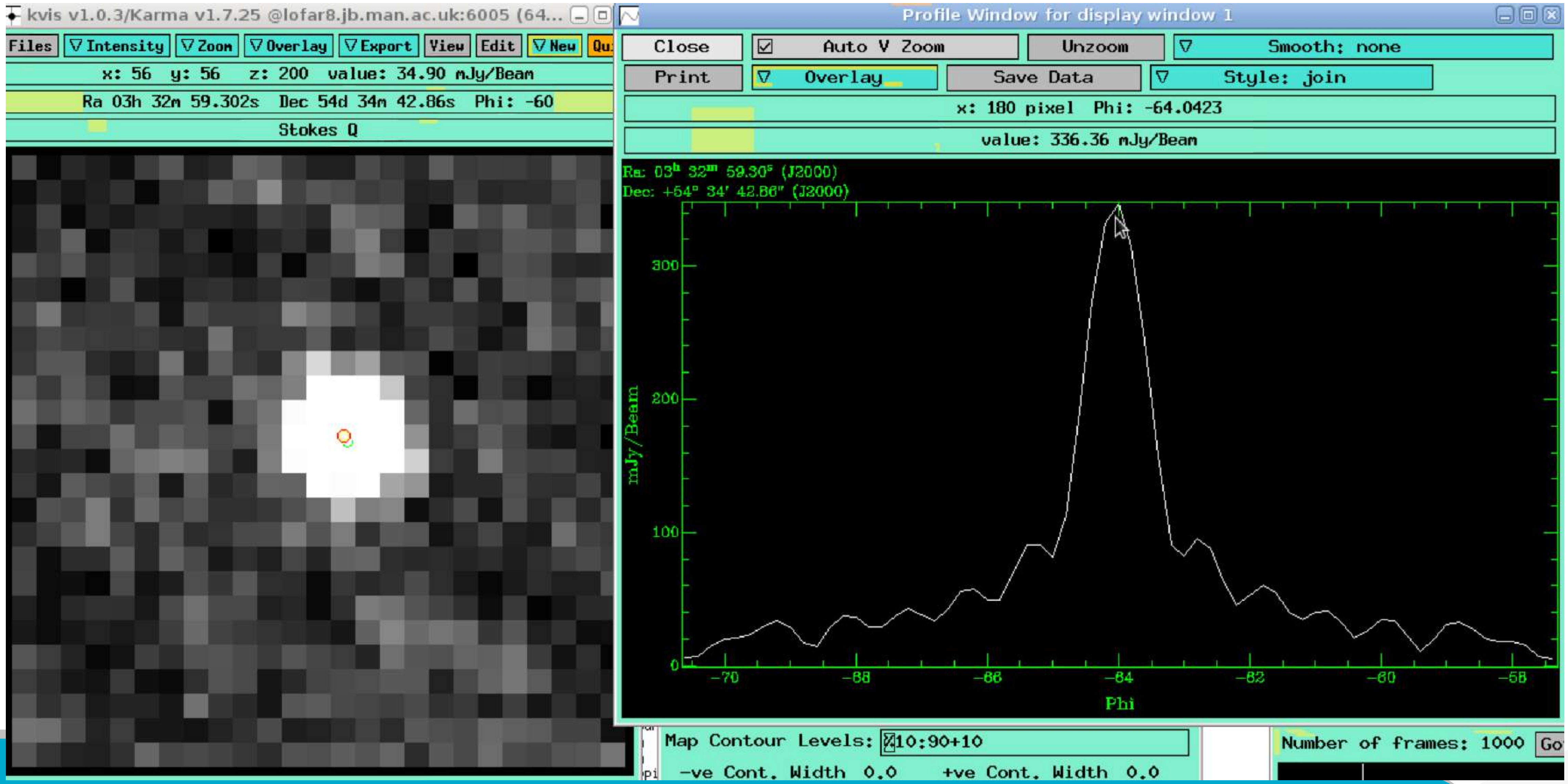




RM Deconvolution (RM CLEAN)

- Mathematically, the sparse frequency coverage leads to a convolution of the Faraday dispersion function with the so-called Rotation Measure Spread Function (RMSF), similar to the "dirty beam" in synthesis imaging:

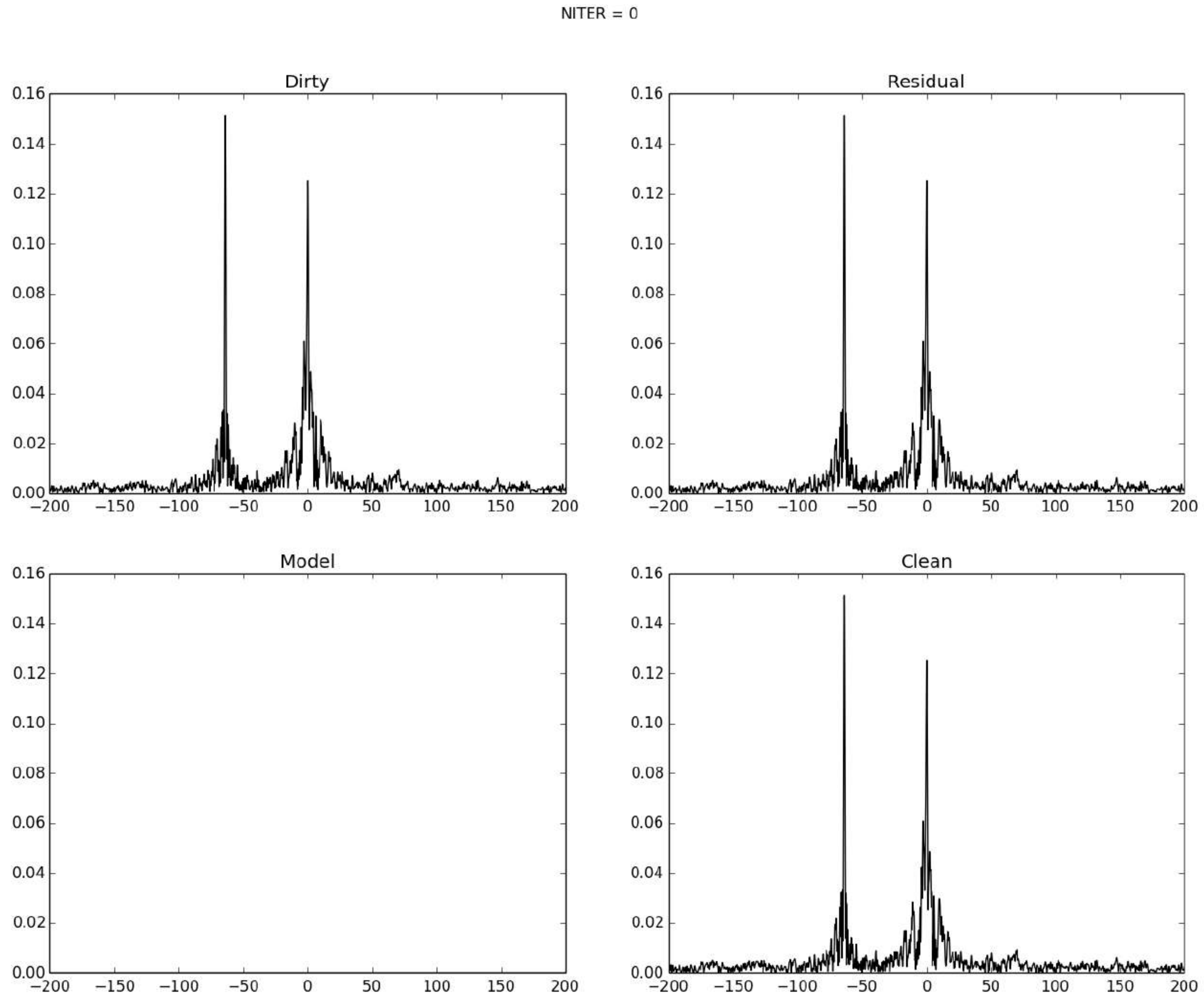
$$\mathbf{R}(\phi) \equiv \frac{1}{N} \sum_{i=1}^{N} e^{-2i\phi(\lambda_i^2 - \lambda_0^2)}$$

$$\tilde{\mathbf{F}}(\phi) = \mathbf{F}(\phi) \star \mathbf{R}(\phi)$$

- Thus, the RMSF can be deconvolved from the recovered Faraday dispersion function leading to a better estimate of the intrinsic Faraday dispersion function
- Tasks to do this are available (`RMCLEAN` in `miriad`, for example)

RM Deconvolution (RM CLEAN): example

- Polarized pulsar displayed previously:
  (NB: polarized intensity shown, but RMCLEAN operates on Q,U)



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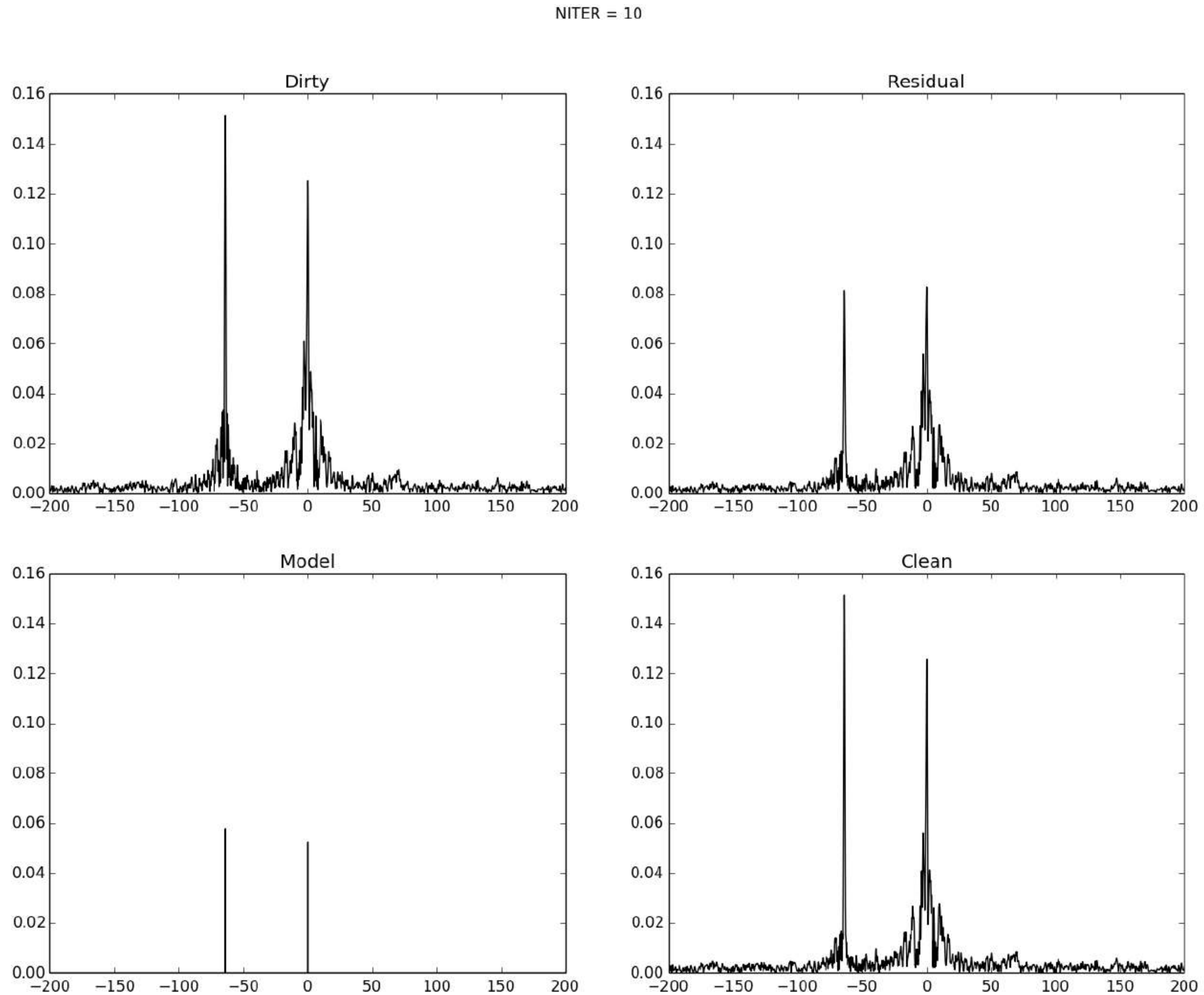

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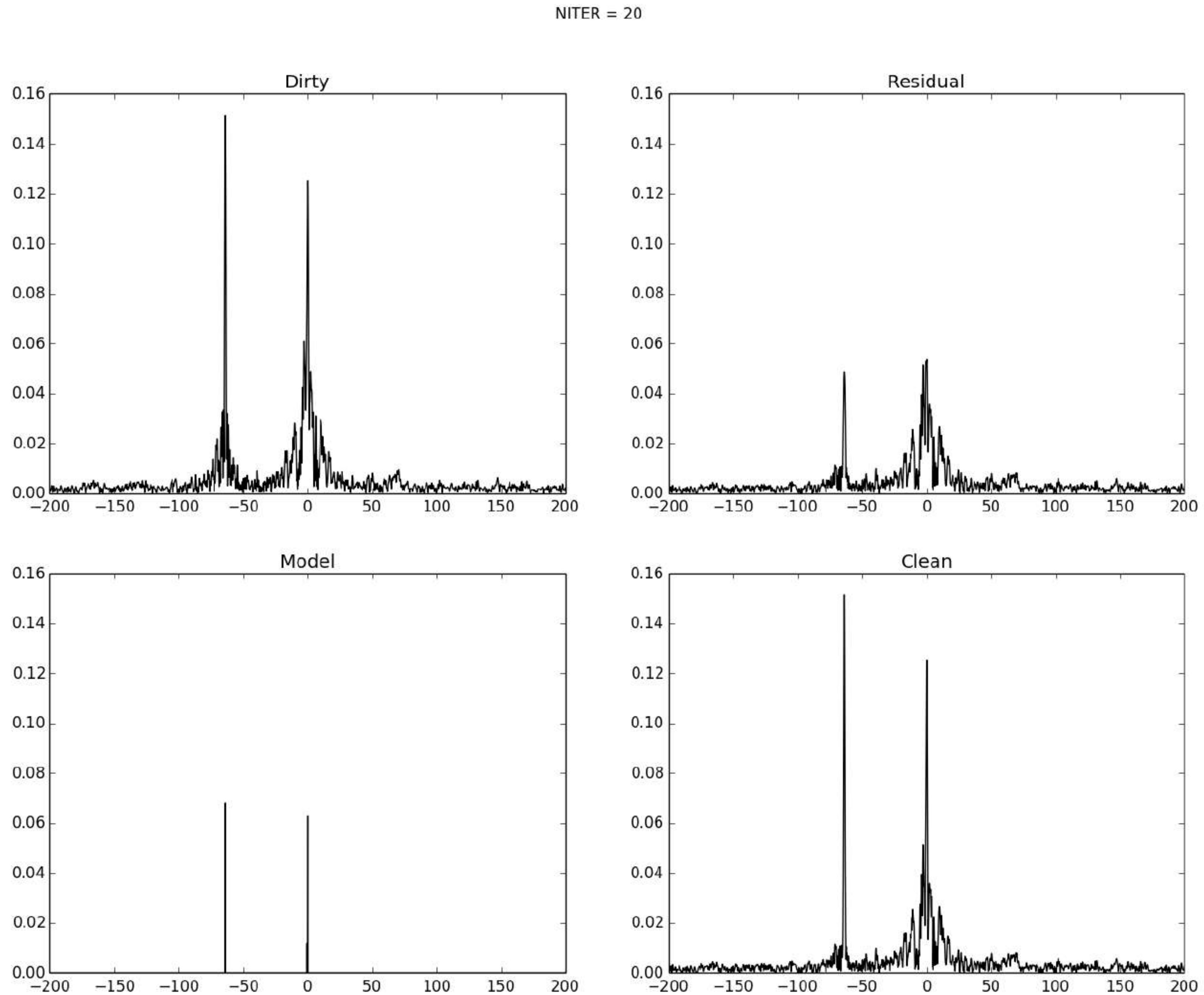

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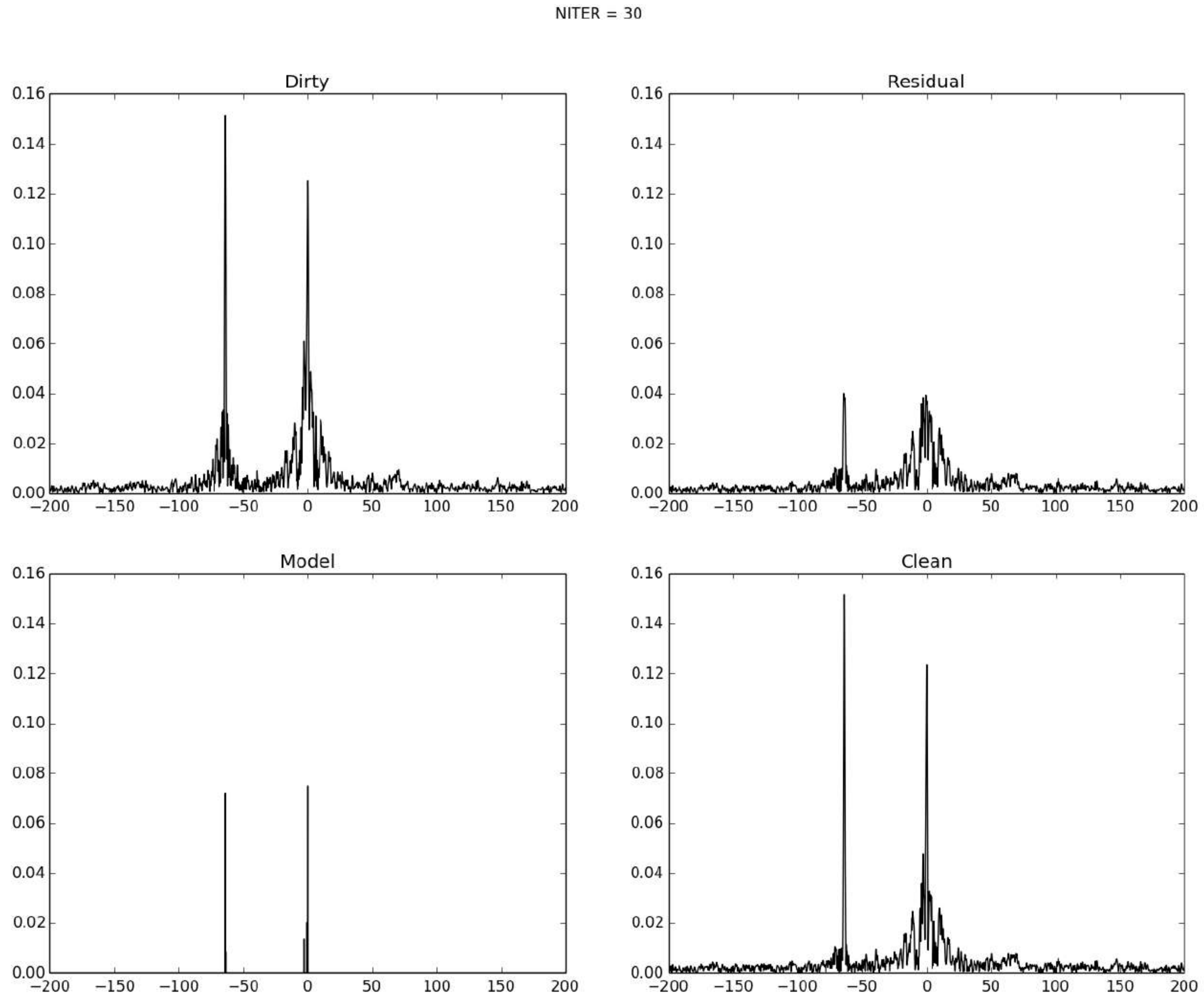

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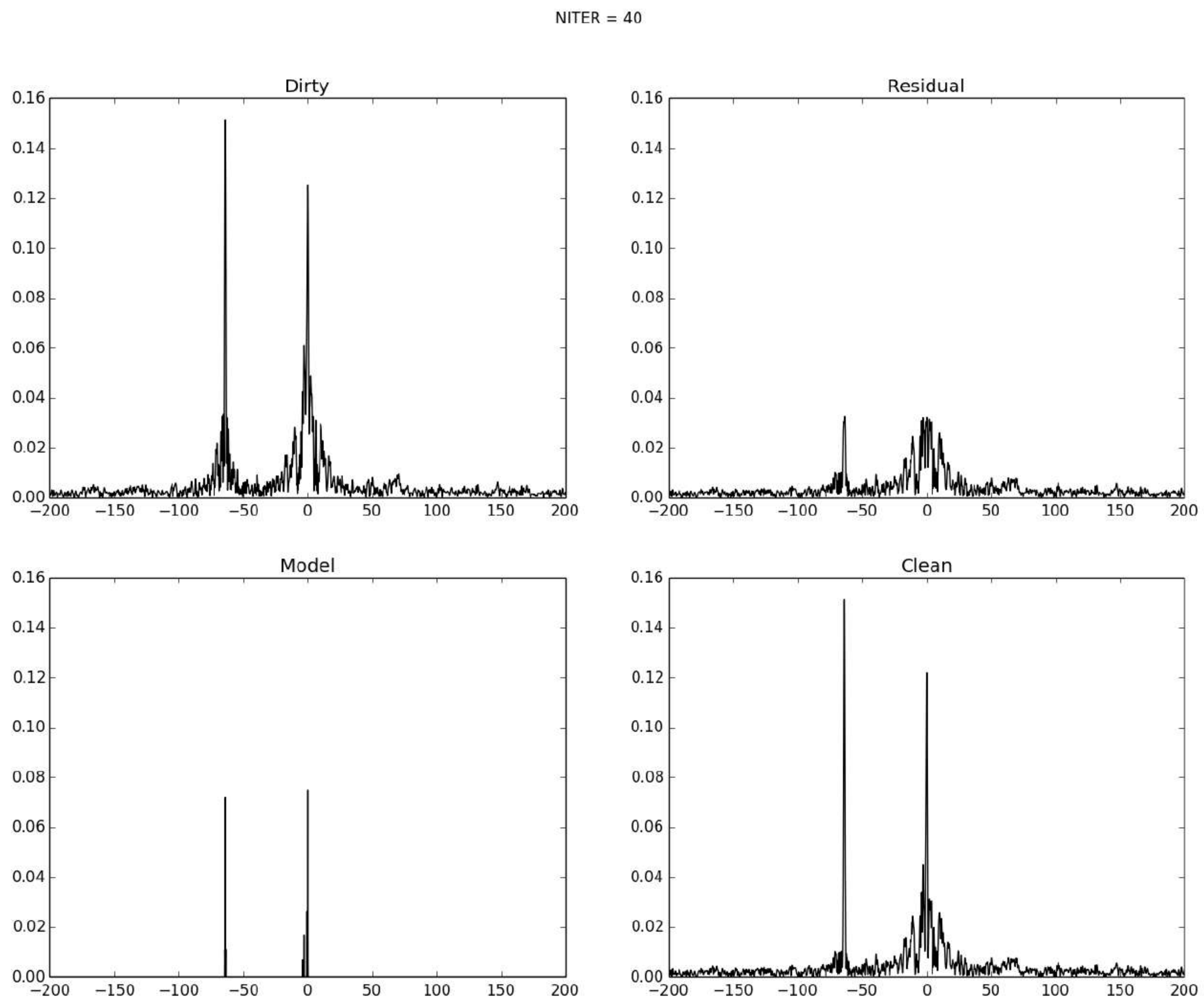

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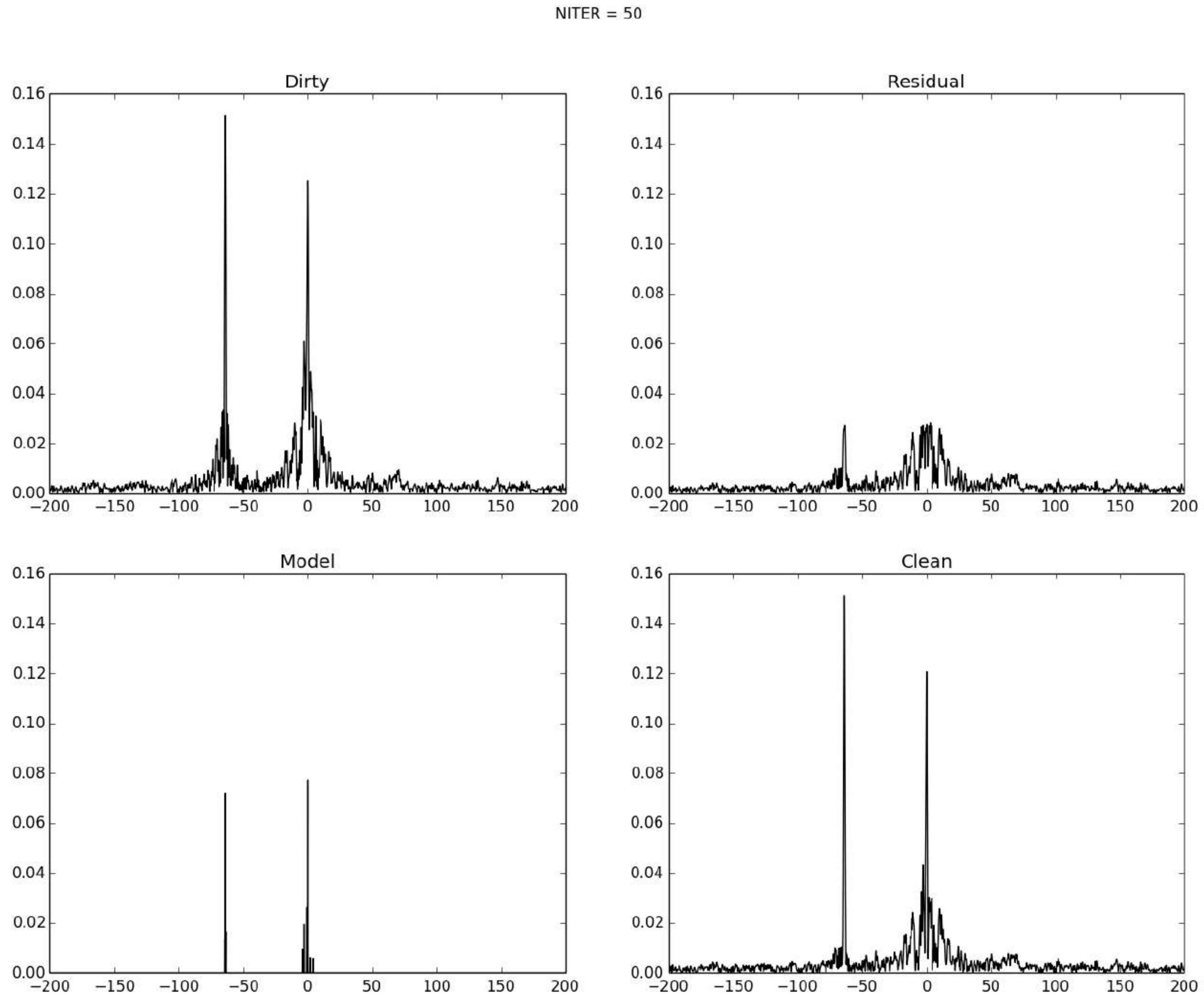
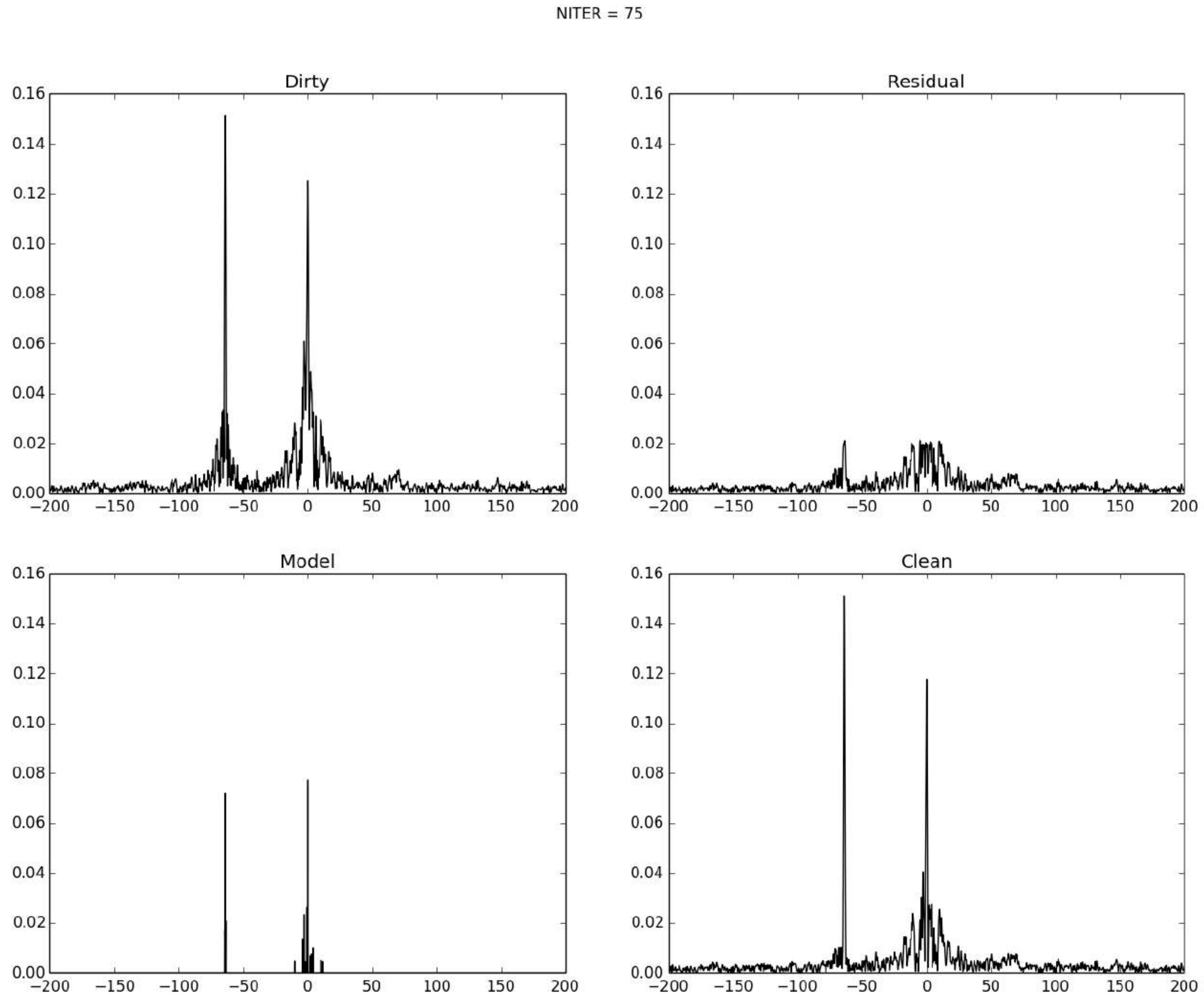


RM Deconvolution (RM CLEAN): example

- Polarized pulsar displayed previously:
  (NB: polarized intensity shown, but RMCLEAN operates on Q,U)



RM Deconvolution (RM CLEAN): example

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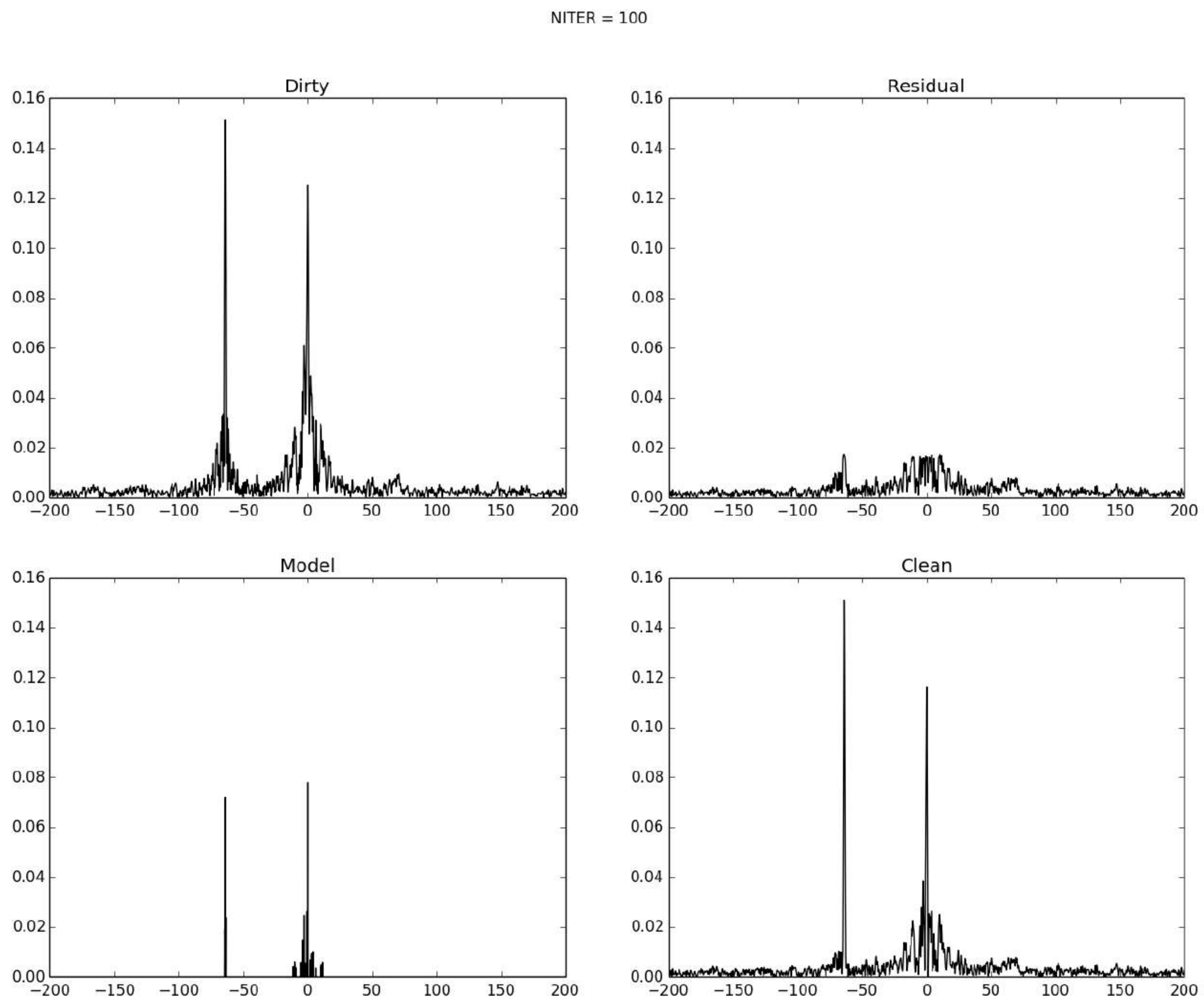
  (NB: polarized intensity shown, but RMCLEAN operates on Q,U)



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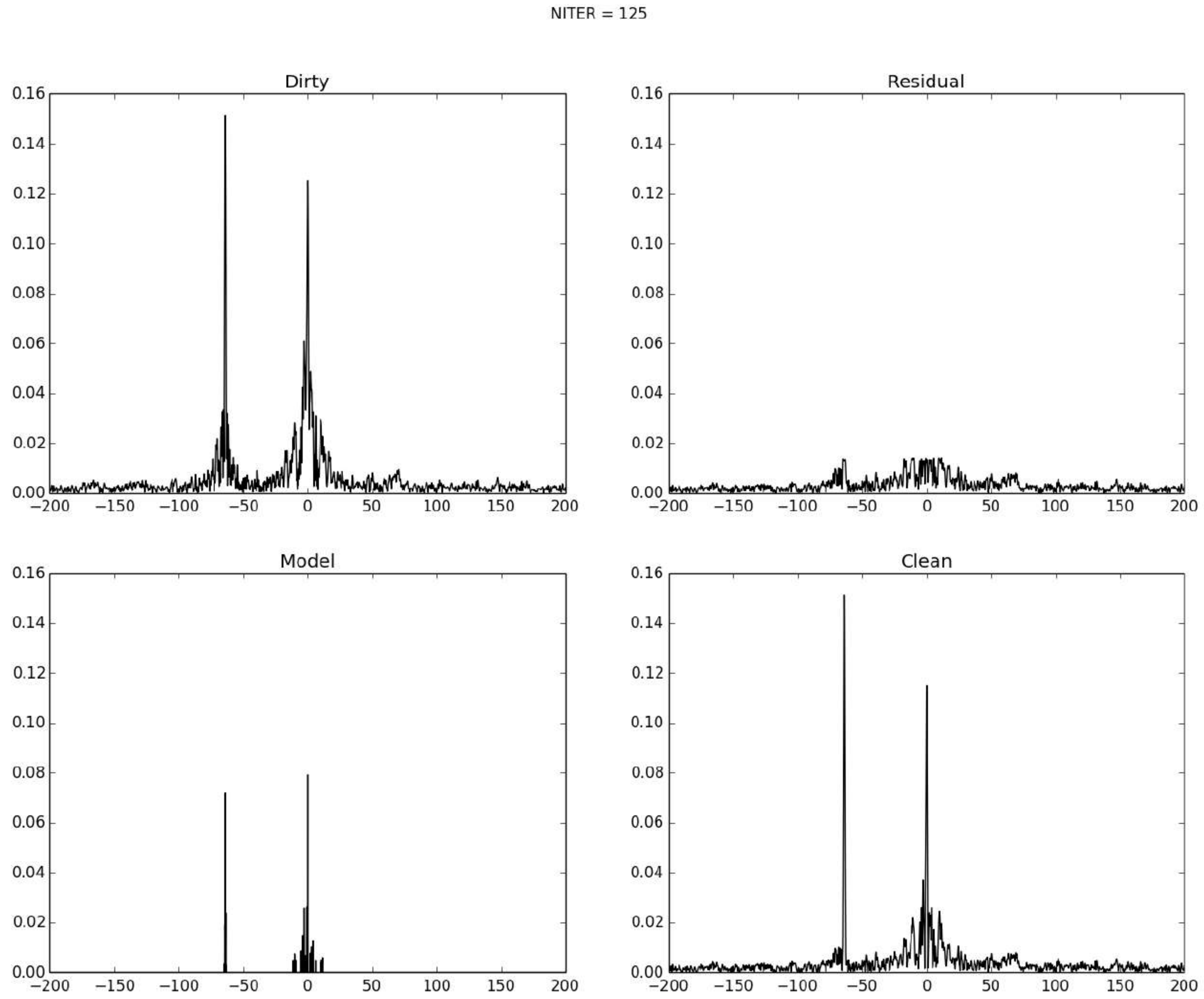

RM Deconvolution (RM CLEAN): example

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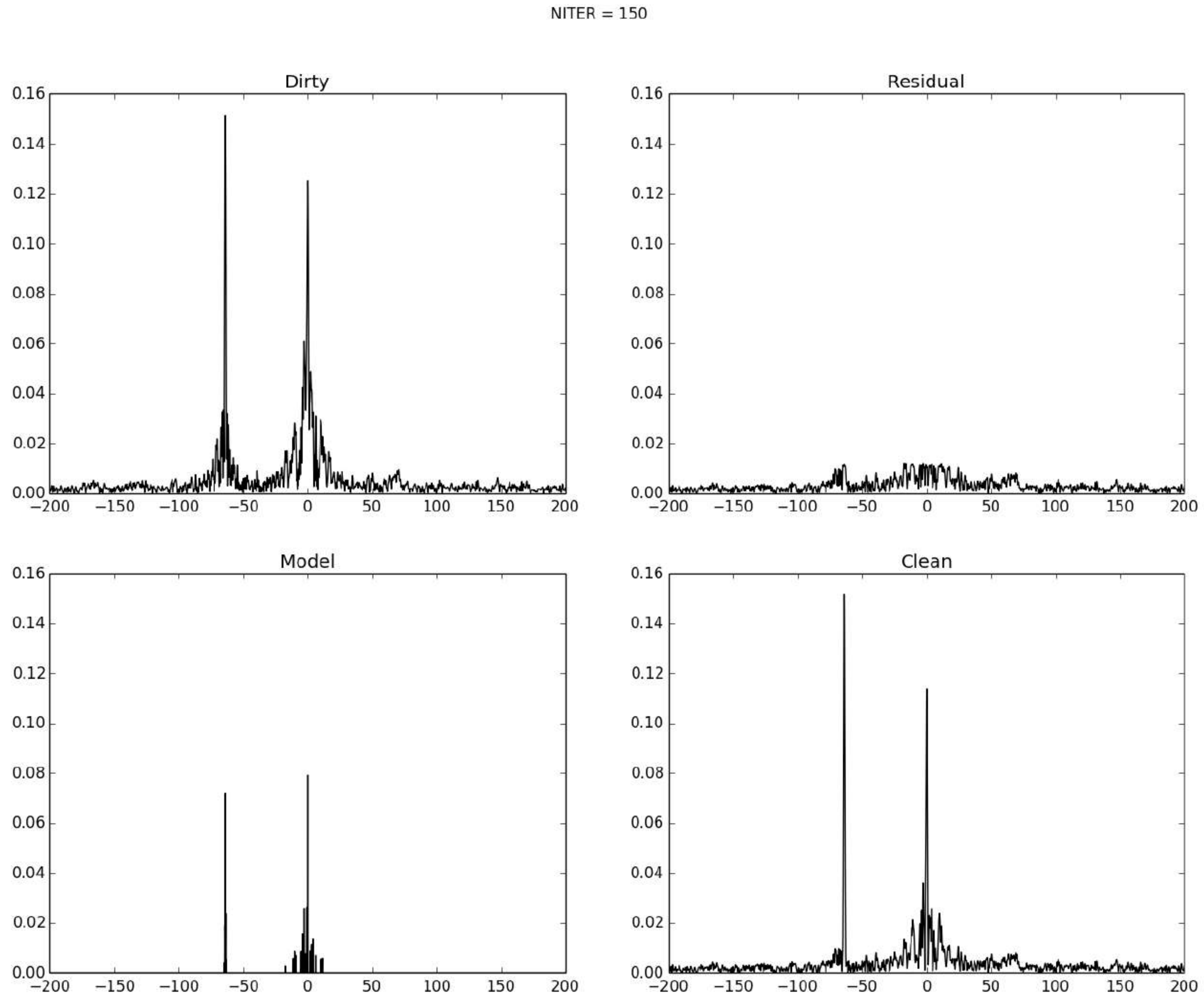

RM Deconvolution (RM CLEAN): example

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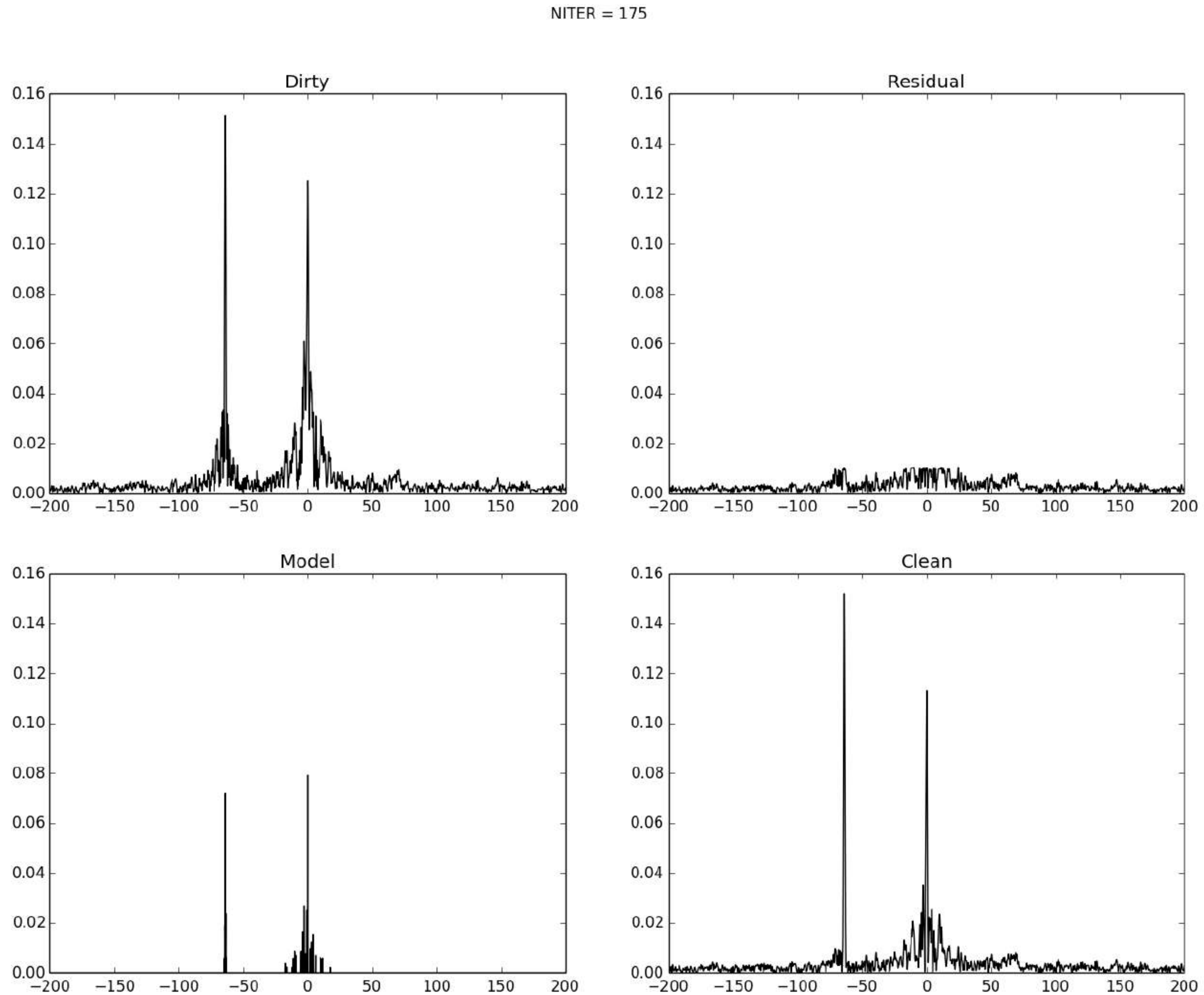

RM Deconvolution (RM CLEAN): example

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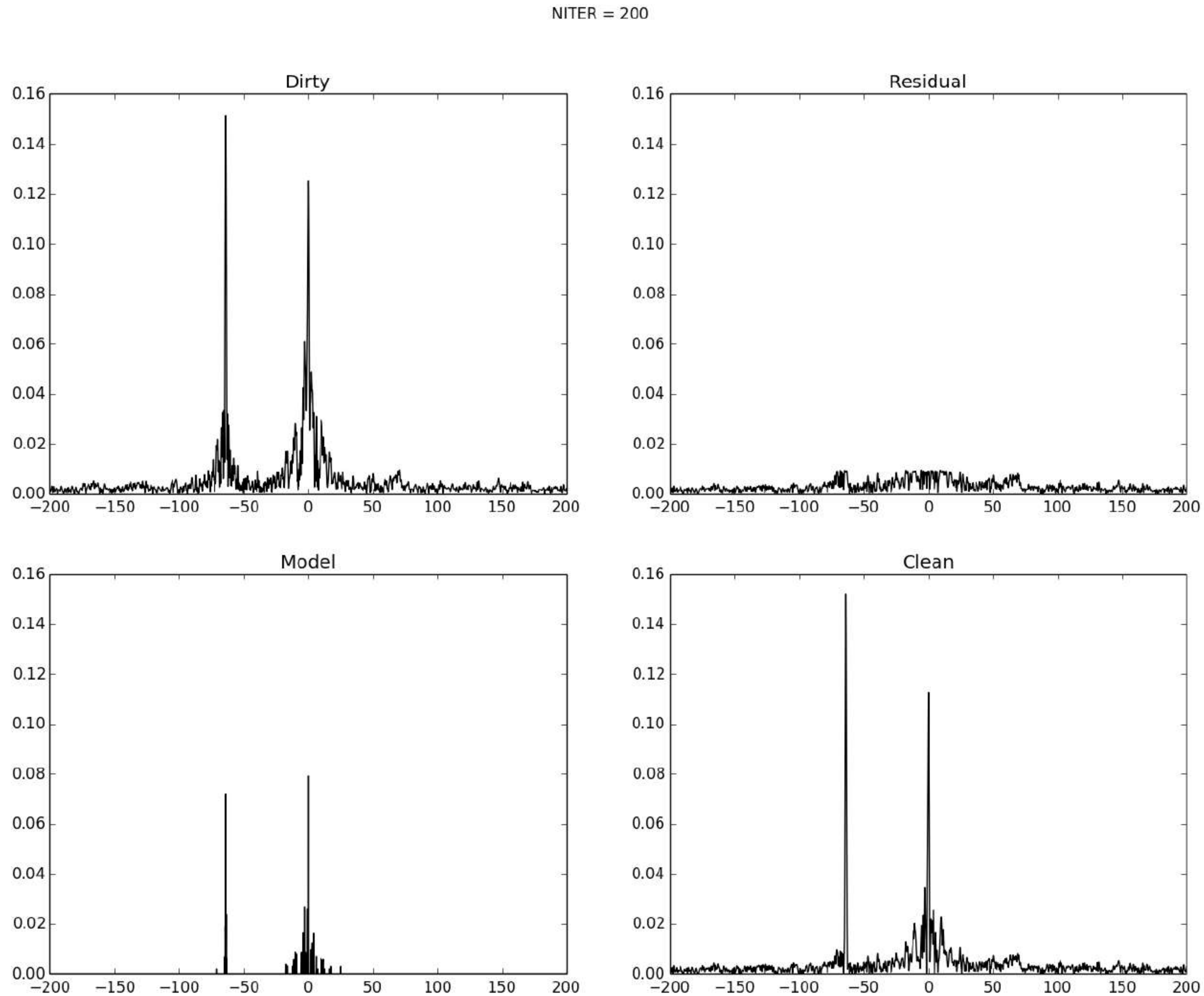

RM Deconvolution (RM CLEAN): example

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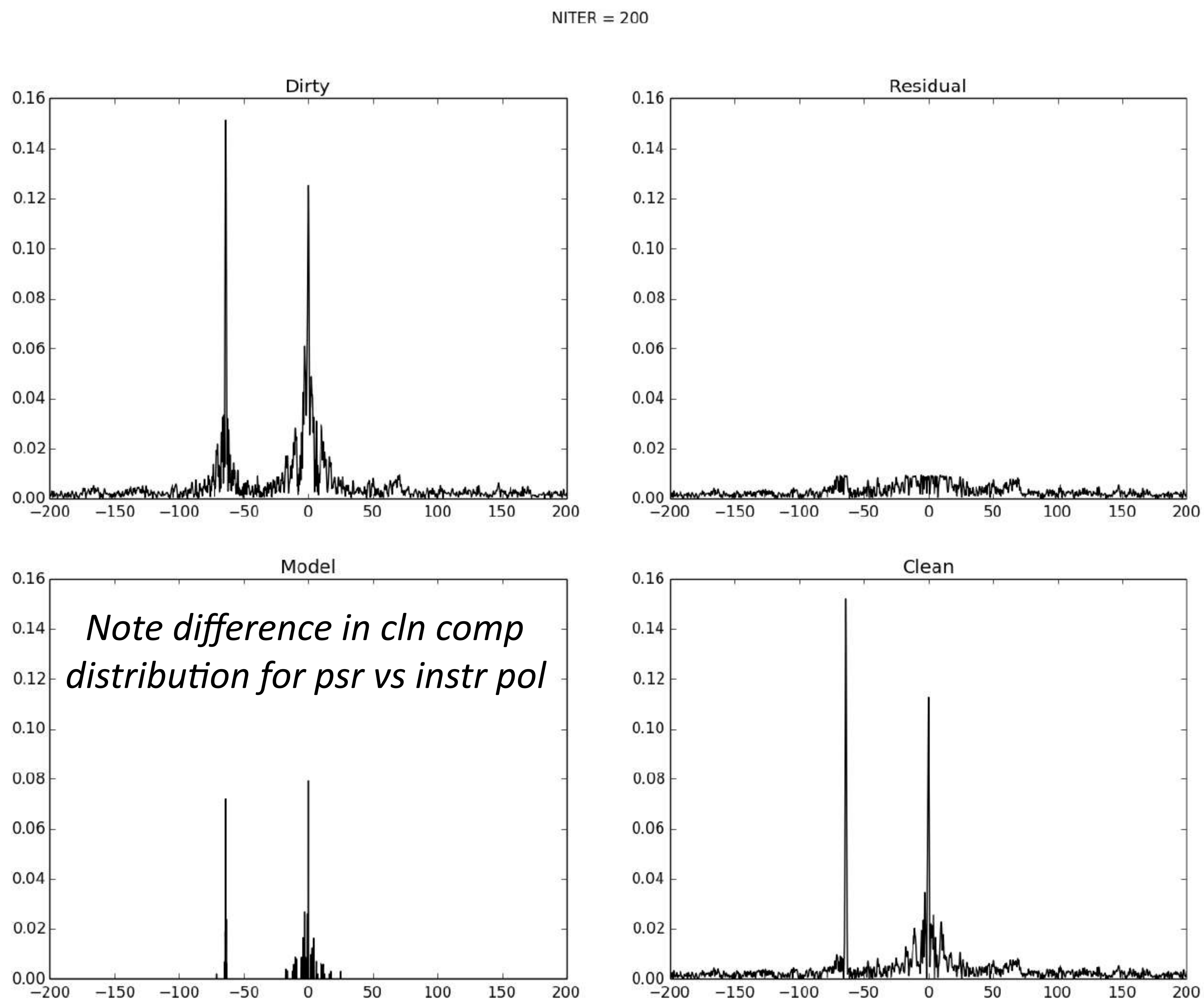
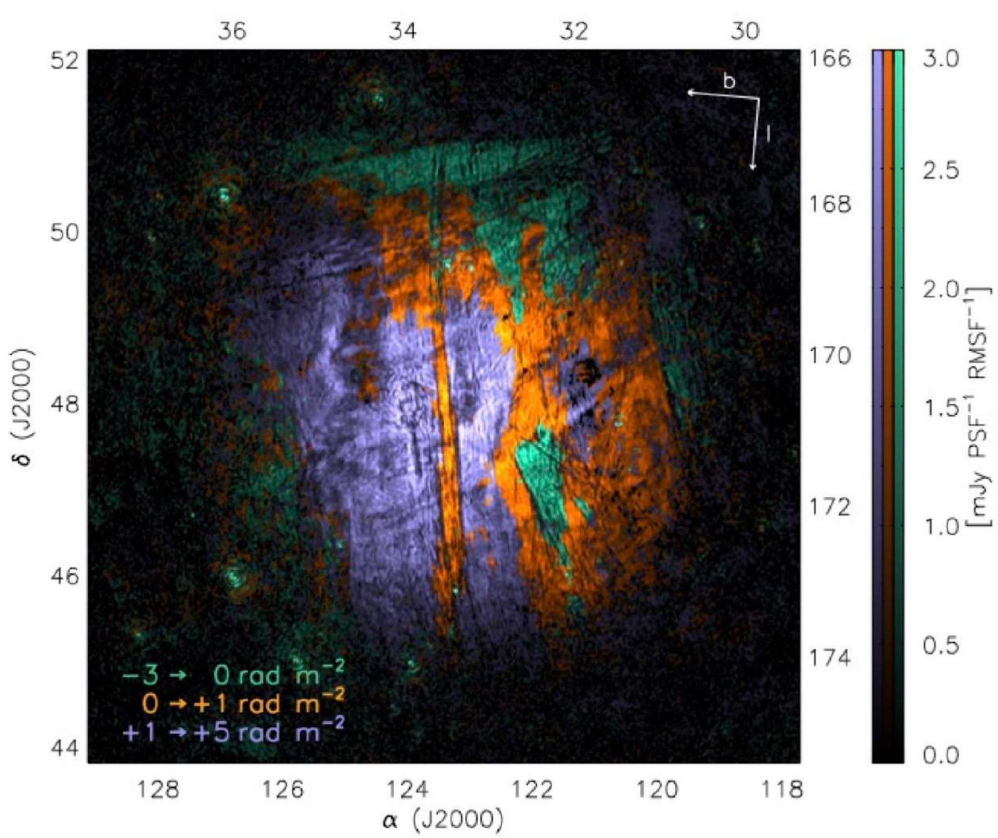
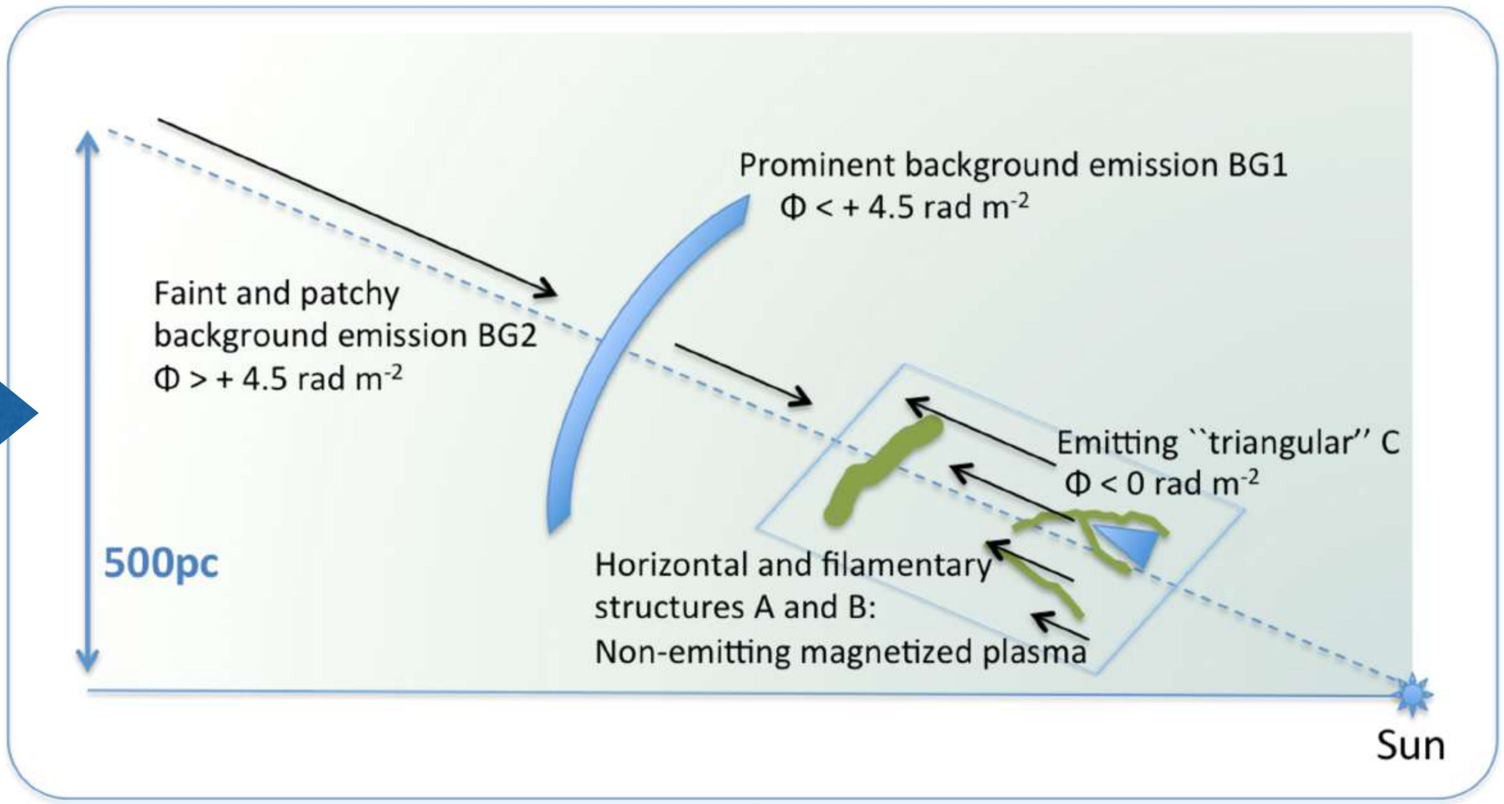


Faraday tomography

- Faraday depth does not directly translate to physical depth
- Other considerations, such as angular/physical scale and cumulative effect of multiple Faraday rotating layers, are required
- This translation to a 3-D physical picture is ***Faraday tomography***





Jelic et al (2015)

QU fitting & Faraday dispersion modeling

- In practice, limitations of RM Synthesis and RM-CLEAN lead us to consider a complementary approach in some cases: namely, model-fitting in the observational (λ^2) domain
- Principles of Fourier transforms as described by Max and Mark apply here, too: e.g. large structures are small in the conjugate domain, and vice-versa; sharp edges in one domain correspond to power on a broad range of scales in the conjugate domain
- For broadband polarimetry, we are probing sources with polarized emission that is present at a range of Faraday depth, e.g.:
  - turbulent magnetic fields and/or thermal electron density
  - ordered magnetic field embedded in a thermal plasma with cosmic rays (mixed synchrotron-emitting and Faraday-rotating)
- Example: "Exclamation mark" source observed with 3 broadband telescopes: ATCA, ASKAP/BETA, and MWA

QU fitting & Faraday dispersion modeling

- Two components favoured with properties consistent with high-resolution imaging

Two “RM component” models

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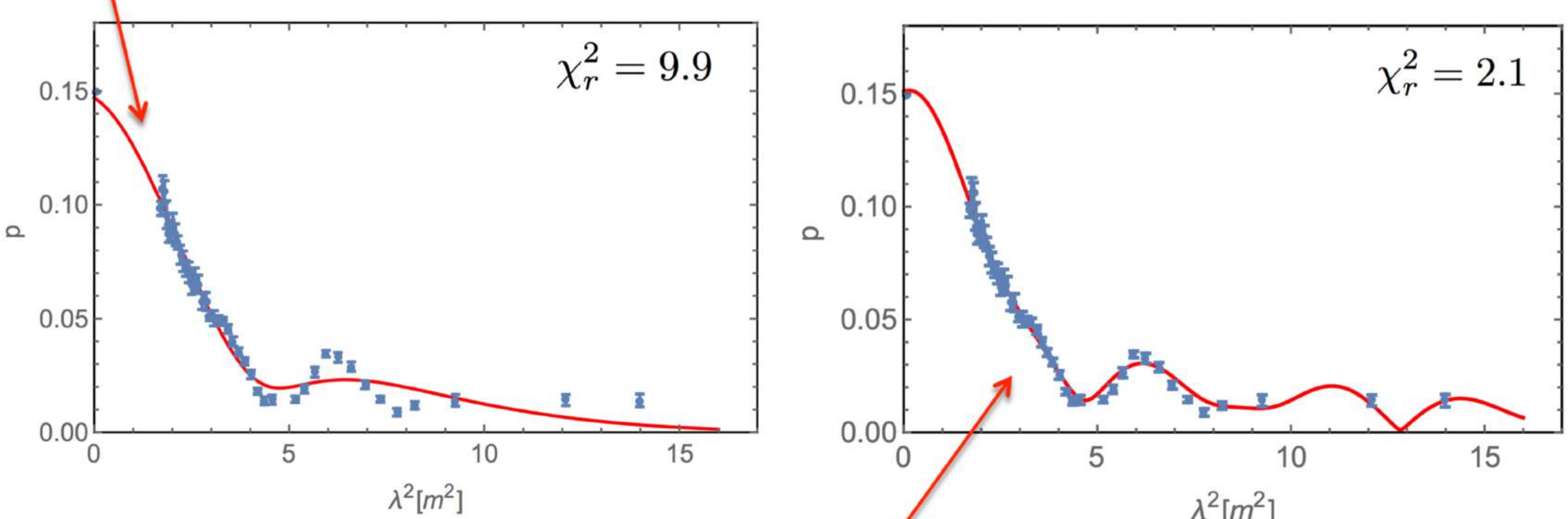
$$P = p_{01}\, e^{2i(\chi_{01}+\mathrm{RM}_1\lambda^2)} e^{-2\sigma^2_{\mathrm{RM}_1}\lambda^4} + p_{02}\, e^{2i(\chi_{02}+\mathrm{RM}_2\lambda^2)} e^{-2\sigma^2_{\mathrm{RM}_2}\lambda^4}$$



Slide courtesy of Shane O'Sullivan

$$P = p_{01}\, e^{2i(\chi_{01}+\mathrm{RM}_1\lambda^2)} \frac{\sin \Delta\mathrm{RM}_1\lambda^2}{\Delta\mathrm{RM}_1\lambda^2} + p_{02}\, e^{2i(\chi_{02}+\mathrm{RM}_2\lambda^2)} \frac{\sin \Delta\mathrm{RM}_2\lambda^2}{\Delta\mathrm{RM}_2\lambda^2}$$

O'Sullivan et al (2017, in prep)

Questions?