# Wide Bandwidth Imaging



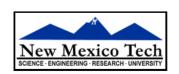
14th NRAO Synthesis Imaging Workshop

13 - 20 May, 2014, Socorro, NM

Urvashi Rau National Radio Astronomy Observatory









### Why do we need wide bandwidths?

Broad-band receivers => Increased 'instantaneous' imaging sensitivity

Continuum sensitivity : 
$$\sigma_{cont} = \frac{\sigma_{chan}}{\sqrt{(N_{chan})}} \propto \frac{T_{sys}}{\sqrt{N_{ant}(N_{ant}-1) \ \delta \tau \delta \nu}}$$

50 MHz → 2 GHz => Theoretical improvement : 
$$\sqrt{\frac{2GHz}{50 MHz}} \approx 6$$
 times.

In practice, effective broadband sensitivity for imaging depends on bandpass shape, data weights, and regions of the spectrum flagged due to RFI. For VLA L-band, we typically use 70% of the band.

### Why do we need wide bandwidths?

Broad-band receivers => Increased 'instantaneous' imaging sensitivity

Continuum sensitivity : 
$$\sigma_{cont} = \frac{\sigma_{chan}}{\sqrt{(N_{chan})}} \propto \frac{T_{sys}}{\sqrt{N_{ant}(N_{ant}-1) \ \delta \tau \delta \nu}}$$

50 MHz → 2 GHz => Theoretical improvement : 
$$\sqrt{\frac{2GHz}{50MHz}}$$
 ≈ 6 times.

In practice, effective broadband sensitivity for imaging depends on bandpass shape, data weights, and regions of the spectrum flagged due to RFI. For VLA L-band, we typically use 70% of the band.

#### Some bandwidth jargon.....

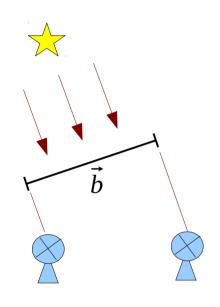
Frequency Range :	${f v}_{min}$ , ${f v}_{max}$	(1 – 2 GHz)	(4 – 8 GHz)	(8 - 12 GHz)
Bandwidth :	${f v}_{max} - {f v}_{min}$	1 GHz	4 GHz	4 GHz

Bandwidth Ratio: 
$$v_{max}$$
:  $v_{min}$  2:1 2:1 1.5:1

Fractional Bandwidth : 
$$(v_{max} - v_{min})/v_{mid}$$
 66% 66% 40%

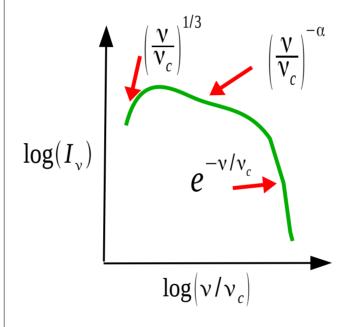
# The instrument and the sky change with frequency...

#### **UV-coverage**



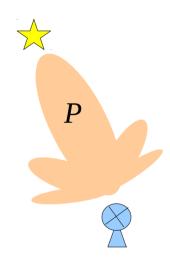
$$S(u,v)_{v} = \frac{\vec{b}}{\lambda} = \frac{\vec{b}v}{c}$$

#### Sky Brightness



$$I(\mathbf{v})$$

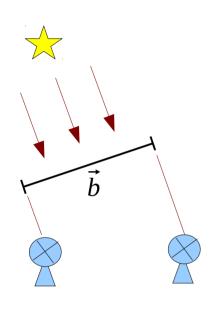
#### **Primary Beam**



$$HPBW_{v} = \frac{\lambda}{D} = \frac{c}{vD}$$

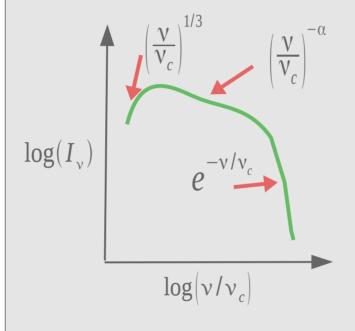
# The instrument and the sky change with frequency...

#### **UV-coverage**



$$S(u,v)_{v} = \frac{\vec{b}}{\lambda} = \frac{\vec{b}v}{c}$$

#### Sky Brightness

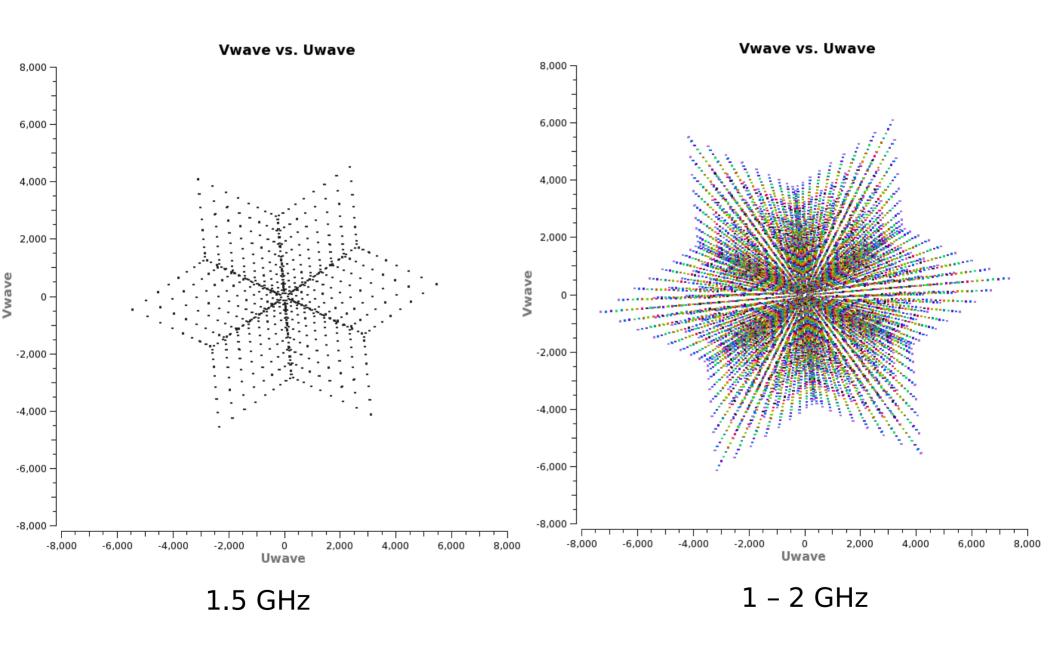


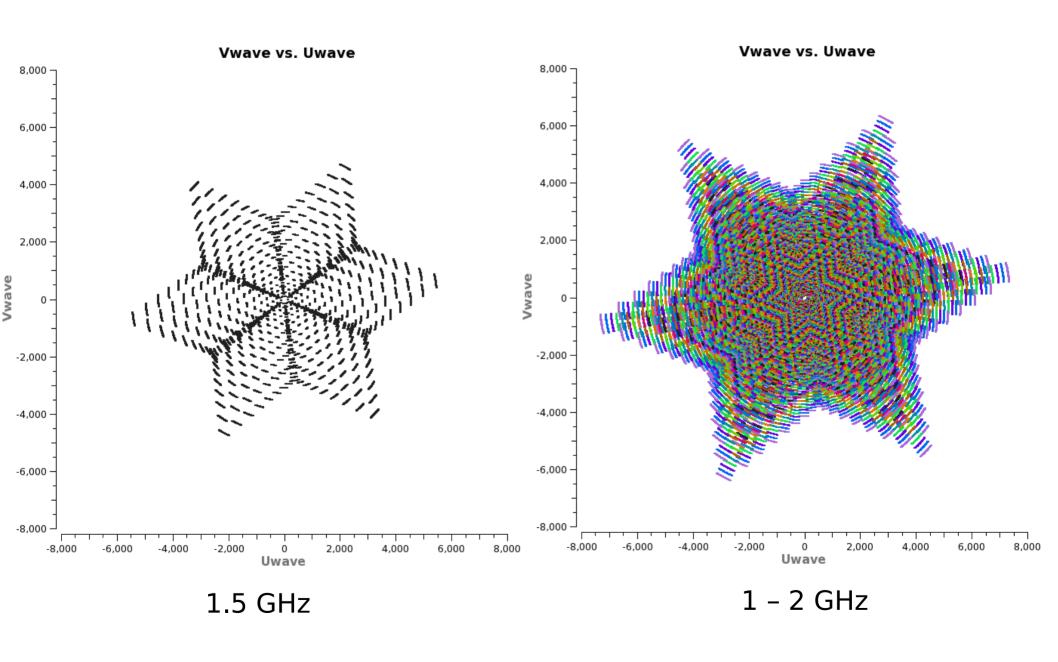
$$I(\mathbf{v})$$

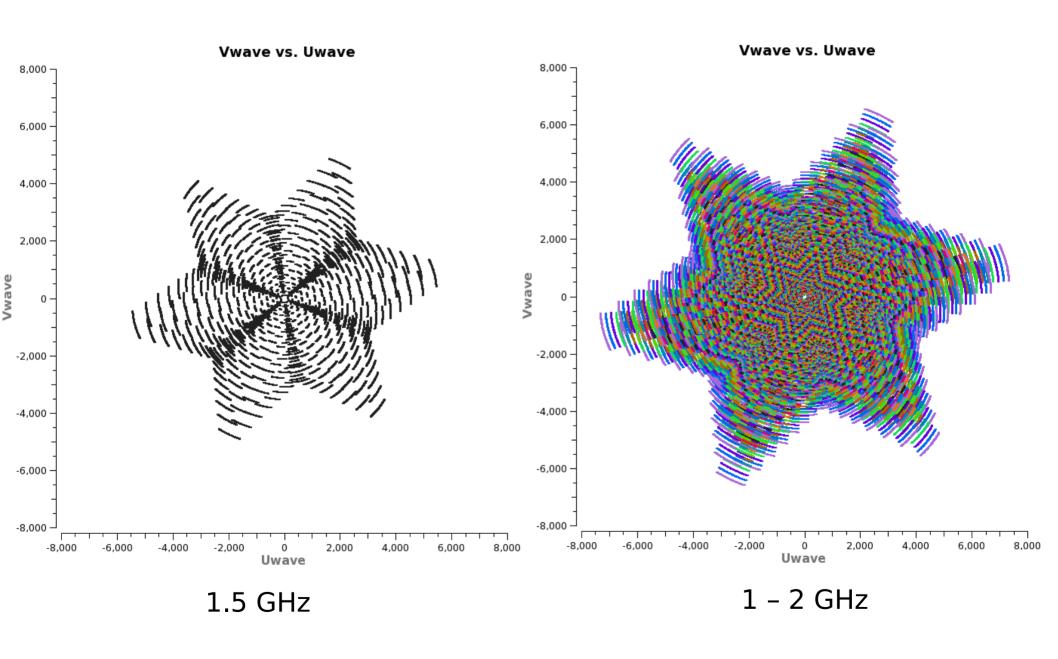
#### Primary Beam

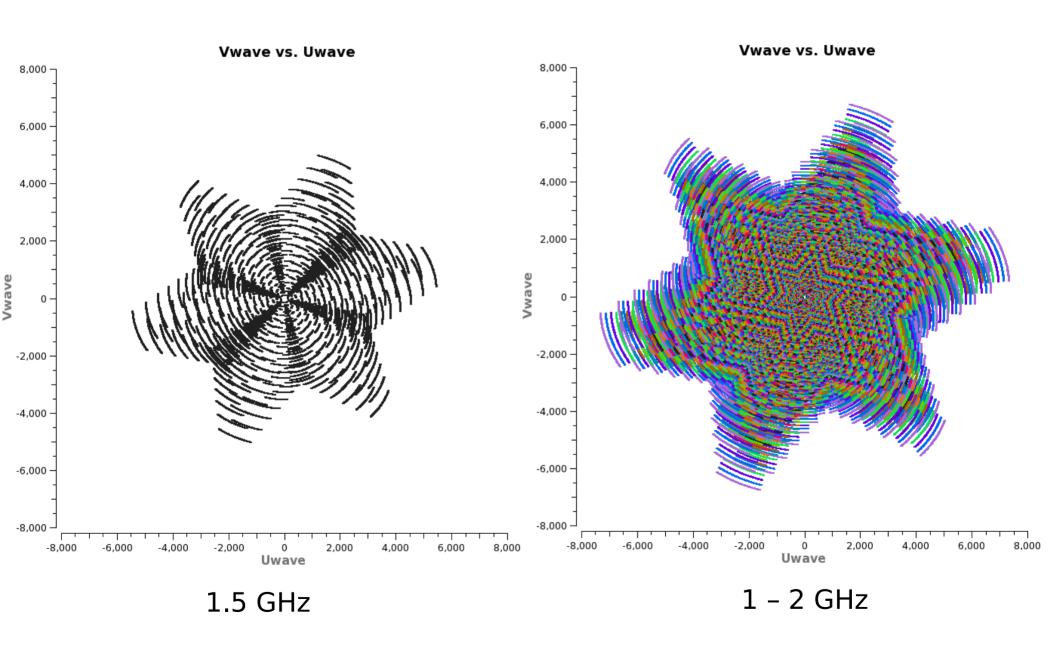


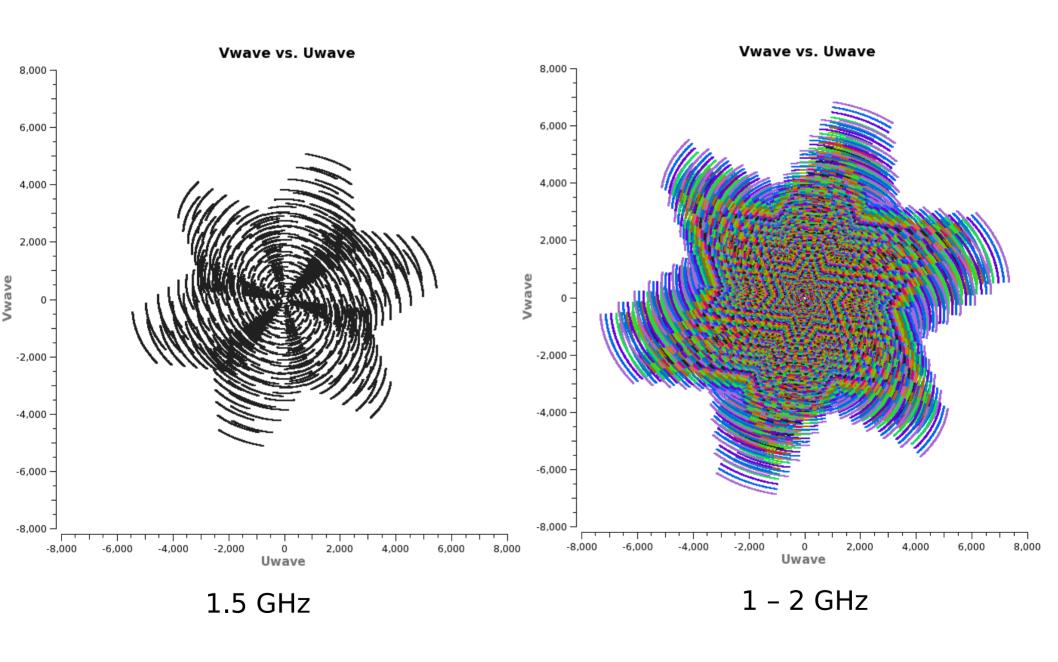
$$HPBW_{v} = \frac{\lambda}{D} = \frac{c}{vD}$$

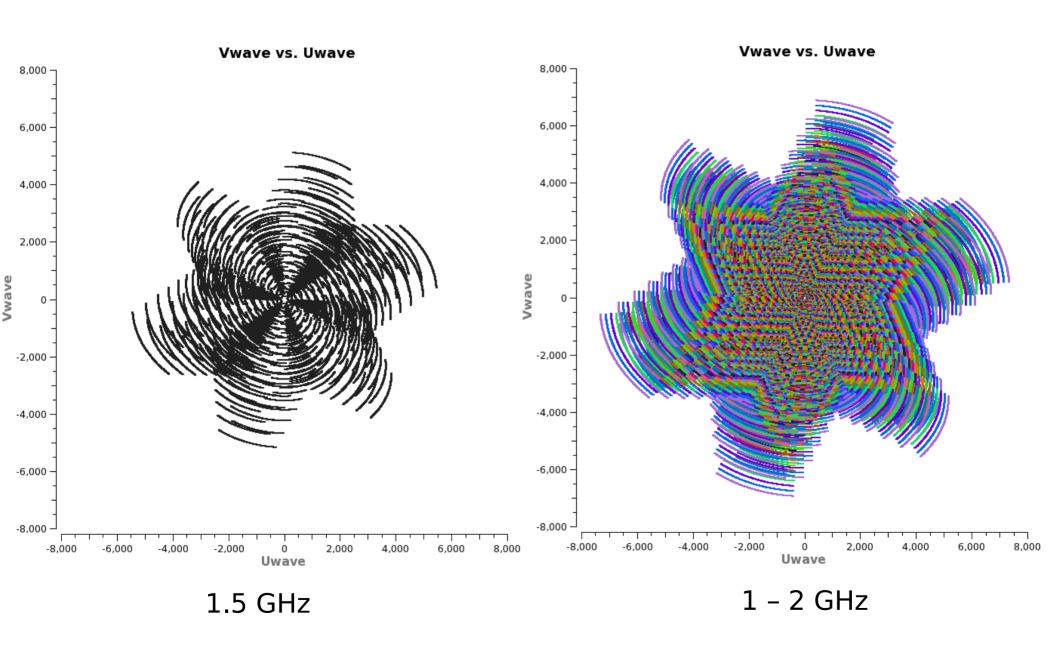


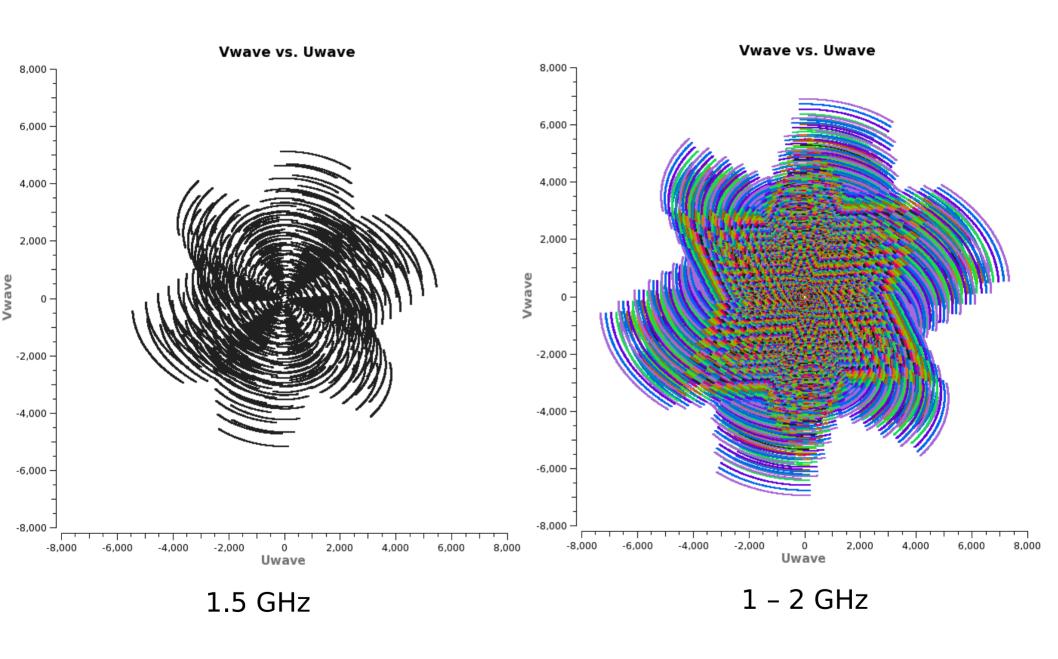


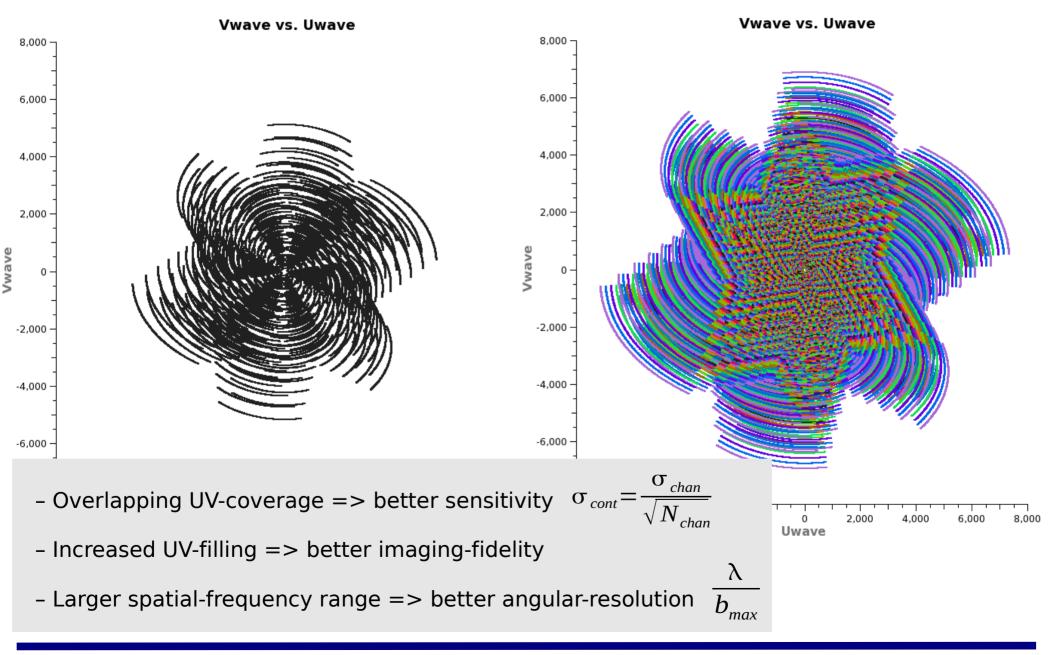






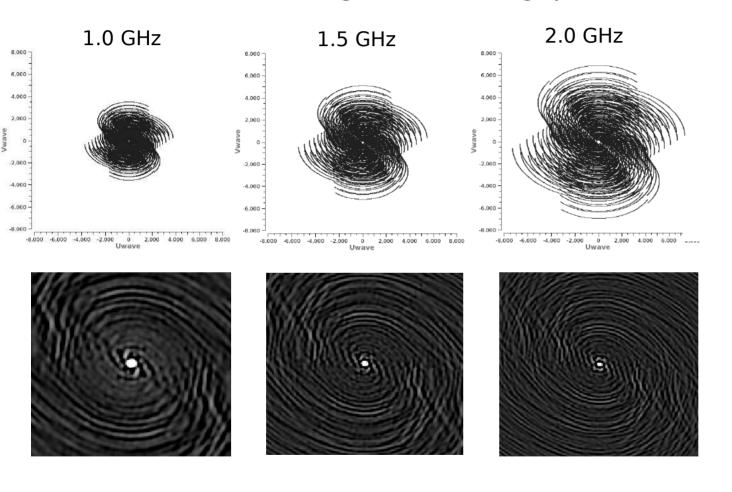


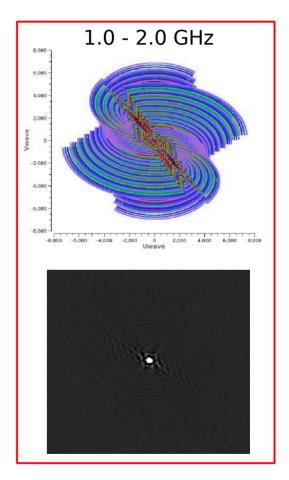




### Imaging Properties change with frequency

- Angular-resolution increases at higher frequencies
- Sensitivity to large scales decreases at higher frequencies
- Wideband UV-coverage has fewer gaps => lower Psf sidelobe levels

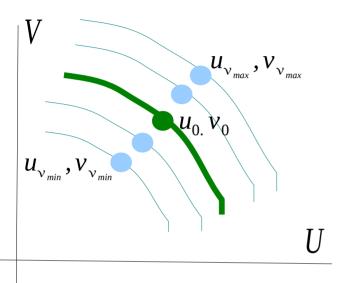




Measure visibilities in frequency 'channels' and place them at their correct locations on the UV-plane.

# Bandwidth smearing (chromatic aberration)

Suppose the entire receiver bandwidth was measured in one channel  $\, \, \nu_{0} \,$ 



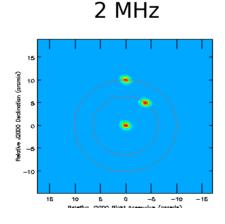
$$V(u_{\scriptscriptstyle 
m v})$$
 is mistakenly mapped to  $\; rac{{f v}_0}{{f v}} u_{\scriptscriptstyle 
m v} \;$ 

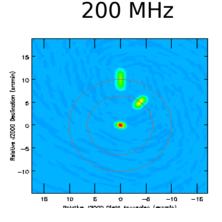
Similarity theorem of Fourier-transforms :

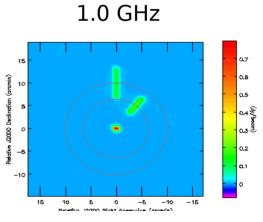
Radial shift in source position with frequency. => Radial smearing of the sky brightness

Excessive channel averaging during post-processing has a similar effect.

Bandwidth smearing limit for HPBW field-of-view :  $\delta v < 1$ 



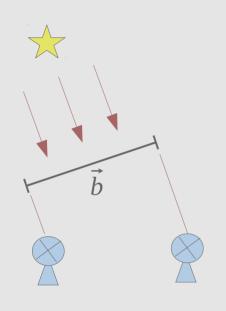




Bandwidth Smearing limits at L-Band (1.4 GHz), 33 MHz (VLA D-config), 10 MHz (VLA C-config), 3 MHz (VLA B-config), 1 MHz (VLA A-config)

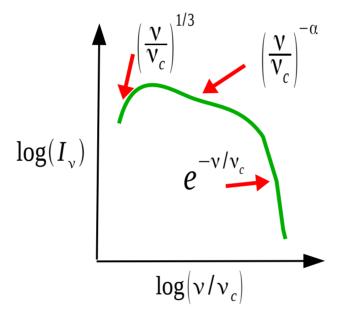
# The instrument and the sky change with frequency...

#### **UV-coverage**



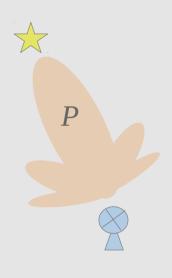
$$S(u,v)_{v} = \frac{\vec{b}}{\lambda} = \frac{\vec{b}v}{c}$$

#### Sky Brightness



$$I(\mathbf{v})$$

#### **Primary Beam**



$$HPBW_{v} = \frac{\lambda}{D} = \frac{c}{vD}$$

### **Imaging Equations**

#### Narrow Band / Flat spectrum sky

$$I^{obs} = I^{sky} * PSF$$

$$I_{wb}^{obs} \approx I^{sky} * \left[ \sum_{v} PSF_{v} \right]$$

#### Image reconstruction

- = deconvolution : remove the effect of the instrument's response to a flat spectrum point source.
- = non-linear fitting of a narrow-band model of the sky to the data

(Ref: Imaging and Deconvolution lecture)

# Wide Band Sky with spectral structure

$$I_{wb}^{obs} = \sum_{v} \left[ I_{v}^{sky} * PSF_{v} \right]$$

Wideband Image reconstruction

= Treat each frequency separately
(Ref : Spectral Line Analysis lecture)

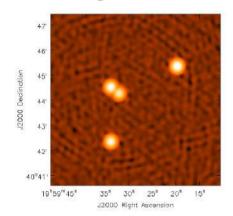
(or)

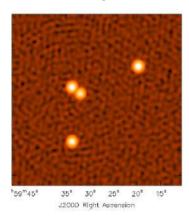
- = joint deconvolution : remove the effect of the instruments response to a point source with spectral features
- = non-linear fitting of a wide-band model of the sky to the data

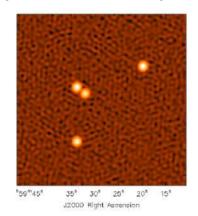
# Single-channel vs MFS imaging – Angular Resolution

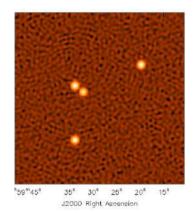
3 flat-spectrum sources + 1 steep-spectrum source (1-2 GHz)

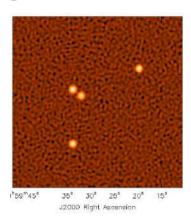
Images made at multiple frequencies (Spectral Cube / Image Cube)



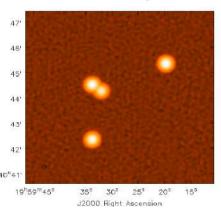




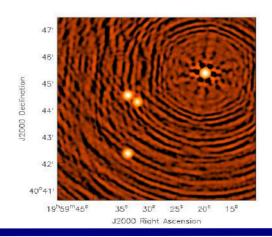




Combine single-frequency images (after smoothing)

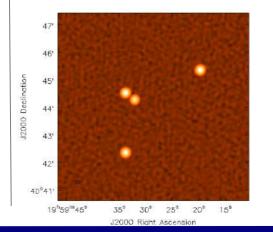


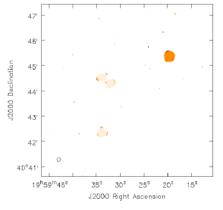
Do MFS using all data, but ignore spectra



Do MFS using all data

- + Model and fit for spectra too
- = Intensity and Spectral-Index





# Algorithm: Multi-Term MFS (with multi-scale)

Sky Model: Collection of multi-scale flux components whose amplitudes follow a Taylor polynomial in frequency

Reconstruction Algorithm: Linear least squares + deconvolution

Data Products : Taylor-Coefficient images  $I_{0,I_1}^m I_{1,I_2}^m$ ...

that represent the sky spectrum 
$$I_{v}^{sky} = \sum_{t} I_{t} \left( \frac{v - v_{0}}{v_{0}} \right)^{t}$$

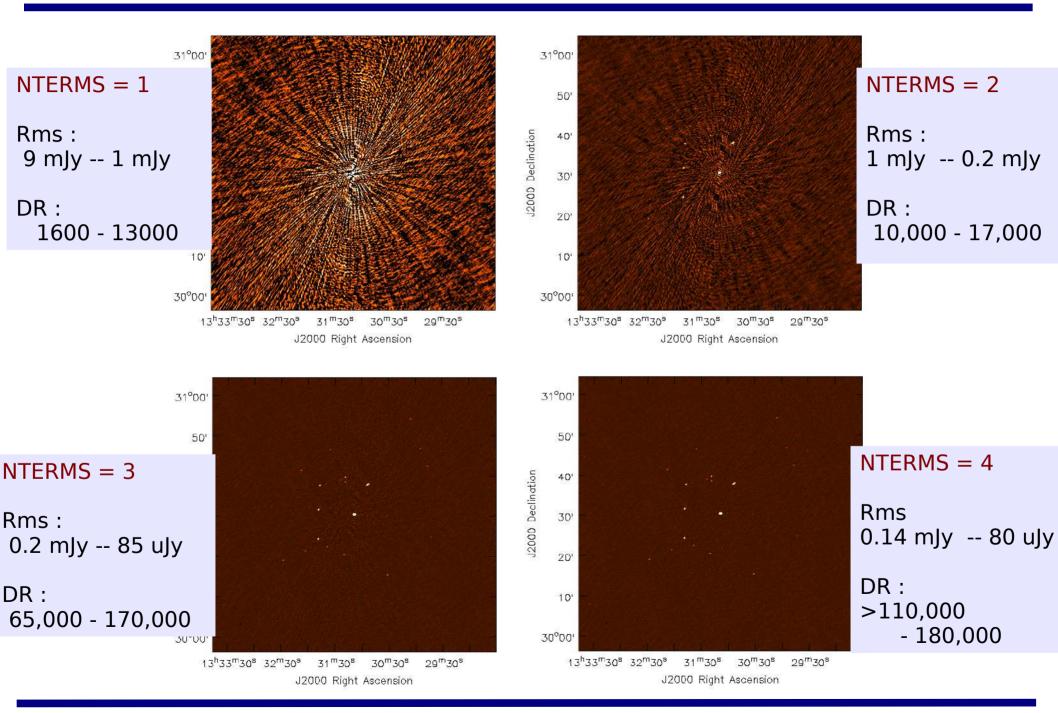
Interpretation:

As a power-law (spectral index and curvature)

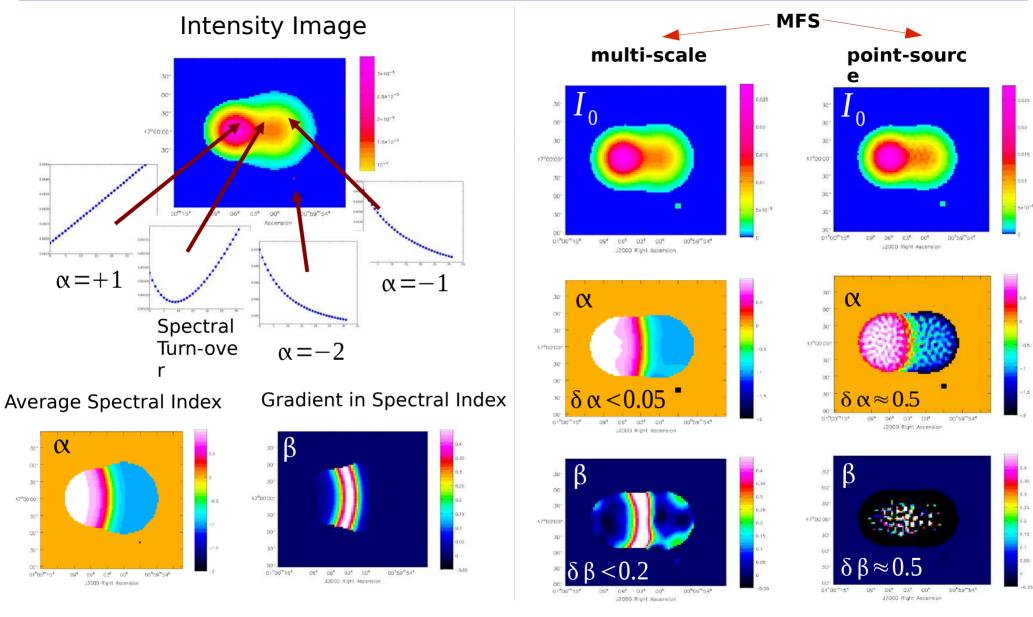
$$I_{\nu} = I_{\nu_{0}} \left(\frac{\nu}{\nu_{0}}\right)^{\alpha + \beta \log(\nu/\nu_{0})} \qquad I_{0}^{m} = I_{\nu_{0}} \qquad I_{1}^{m} = I_{\nu_{0}} \alpha \qquad I_{2}^{m} = I_{\nu_{0}} \left(\frac{\alpha(\alpha - 1)}{2} + \beta\right)$$

Sault &Wieringa, 1994 Rau &Cornwell, 2011

# Dynamic-range with MS-MFS: 3C286 example: Nt=1,2,3,4

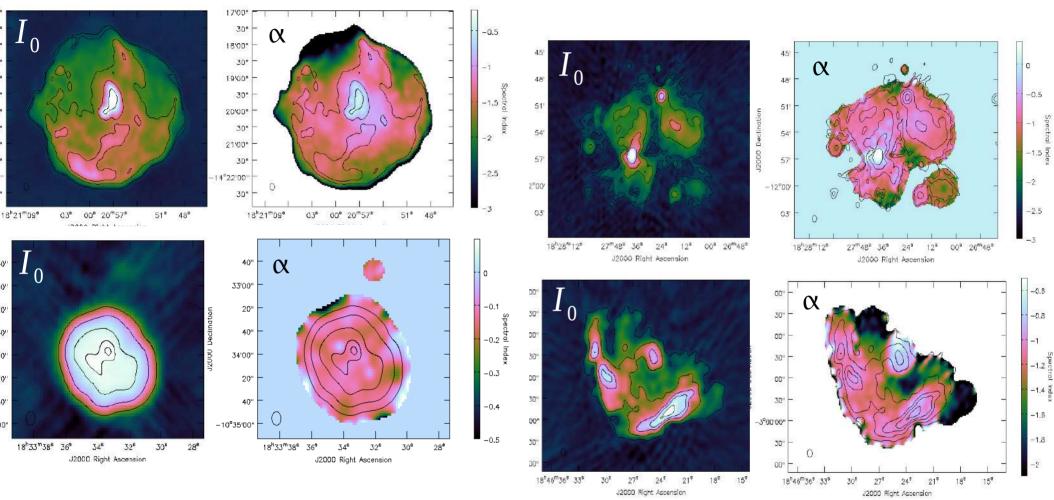


# Example of wideband-imaging on extended-emission



Spectral-index error is dominated by 'division between noisy images'a multi-scale model gives better spectral index and curvature maps

#### Supernova Remnants at L and C Band [Bhatnagar et al, 2011]

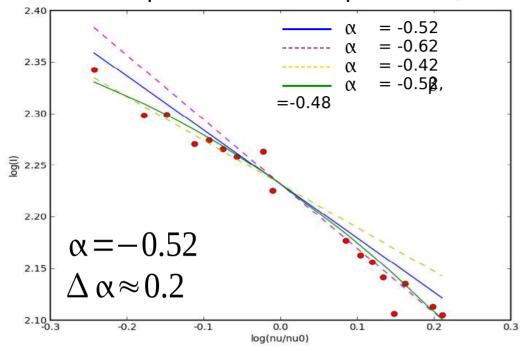


These examples used nterms=2, and about 5 scales.

- => Within 1-2 Ghz and 4-8 GHz, spectral-index error is < 0.2 for SNR>100.
- => Dynamic-range limit of few x 1000 ---> residuals are artifact-dominated

# **Spectral Curvature**

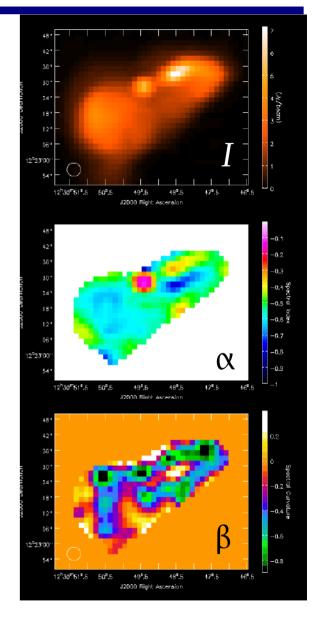
Data: 10 VLA snapshots at 16 frequencies (1.2 - 2.1 GHz)



From existing P-band (327 MHz), L-band(1.42 GHz) and C-band (5.0 GHz) images of the core/jet

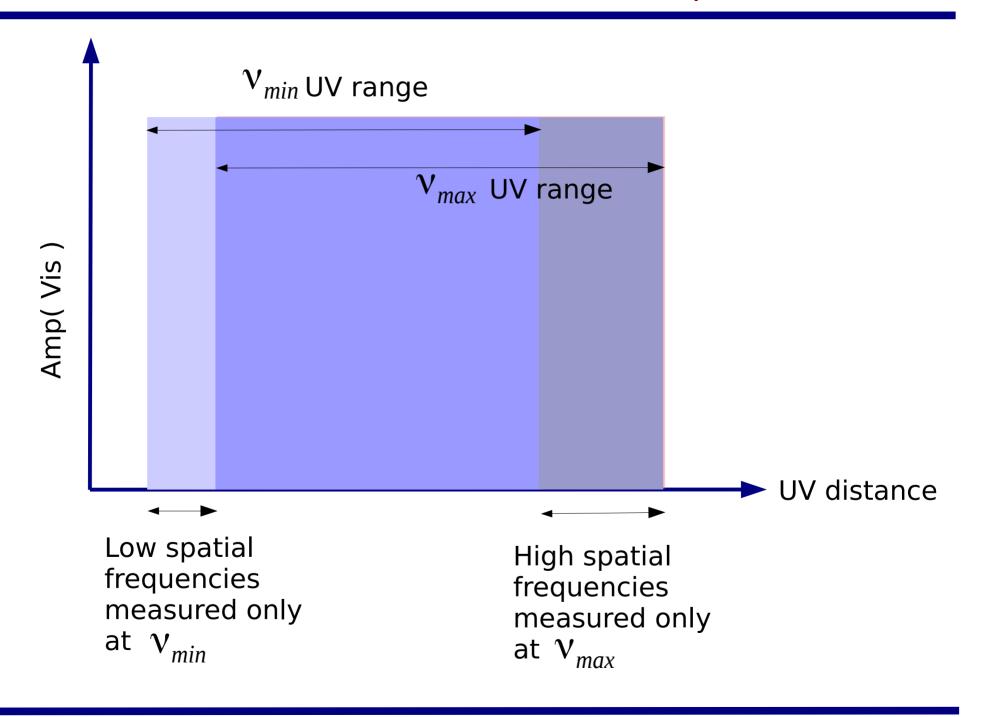
P-L spectral index :  $-0.36 \sim -0.45$ 

L-C spectral index :  $-0.5 \sim -0.7$ 

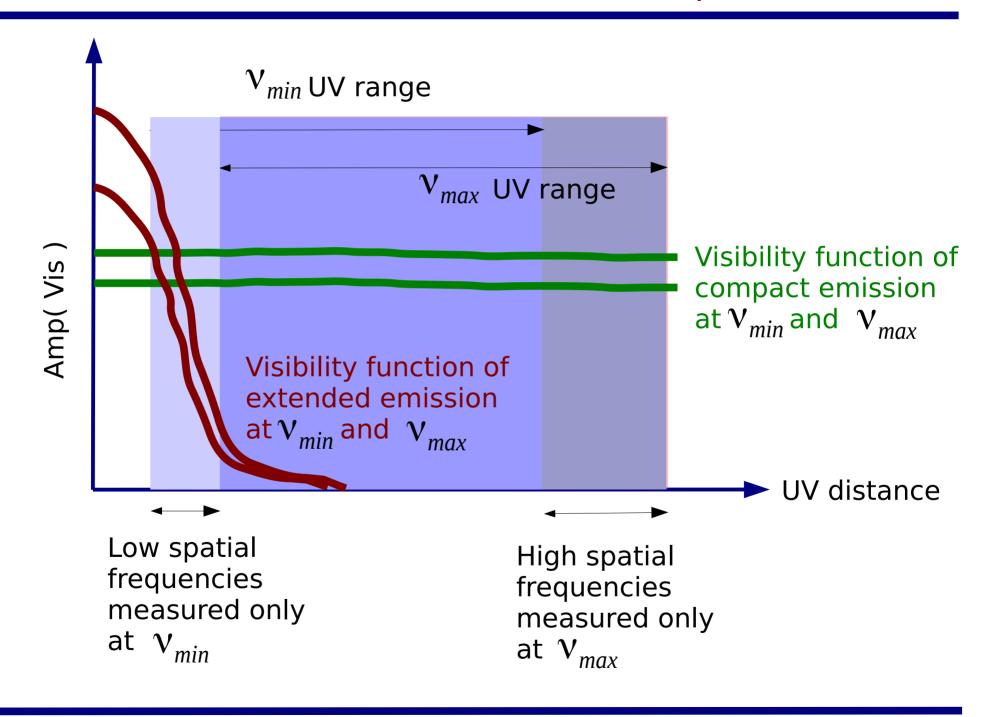


=> Need SNR > 100 to fit spectral index variation  $\sim$  0.2 (at the 1-sigma level ... ) => Be very careful about interpreting  $\beta$ 

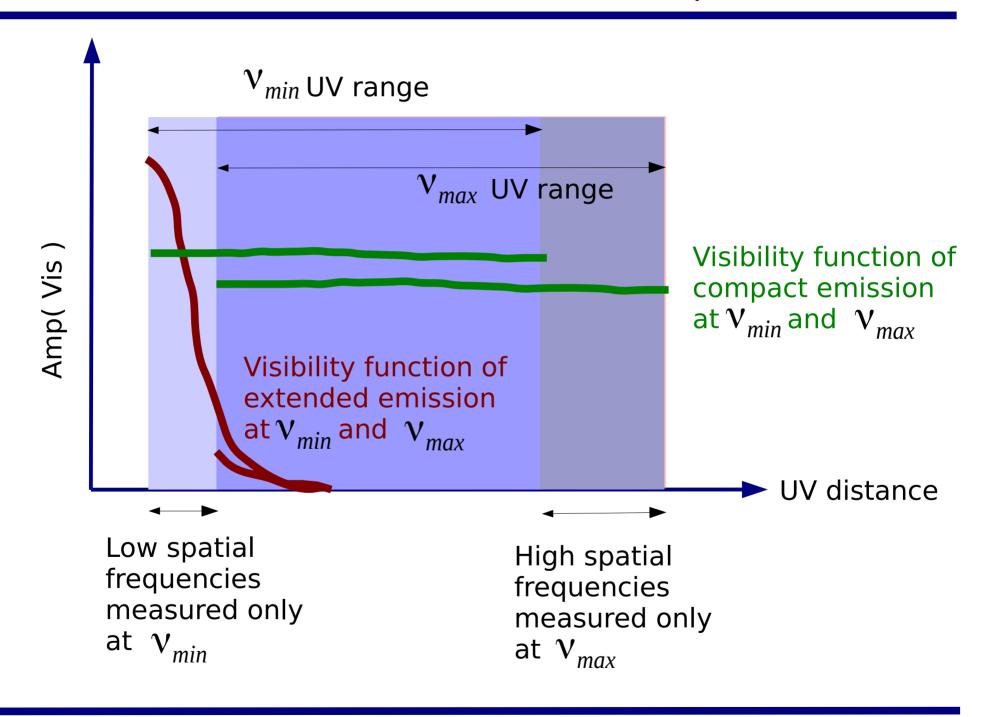
# For which scales can we reconstruct the spectrum?



### For which scales can we reconstruct the spectrum?

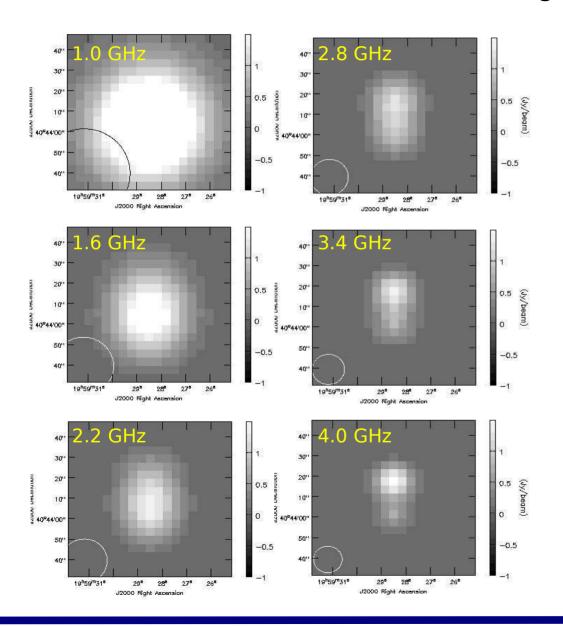


### For which scales can we reconstruct the spectrum?

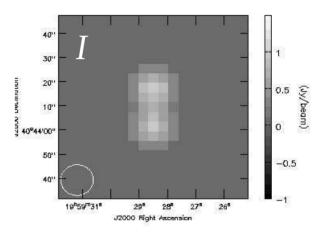


# Moderately Resolved Sources + High SNR

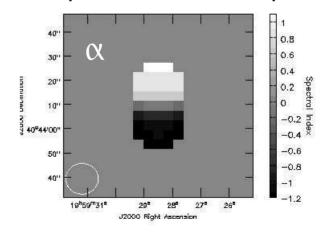
Can reconstruct the spectrum at the angular resolution of the highest frequency (only high SNR)



#### Restored Intensity image

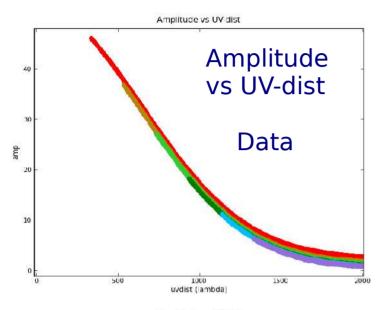


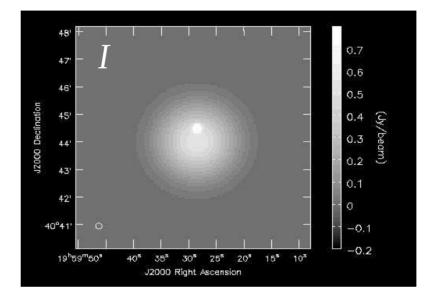
#### Spectral Index map



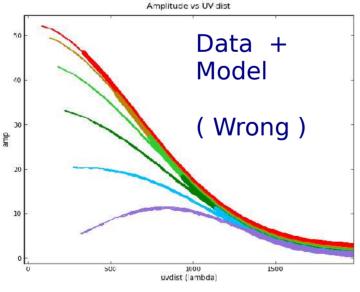
# Very large spatial scales – Unconstrained spectrum

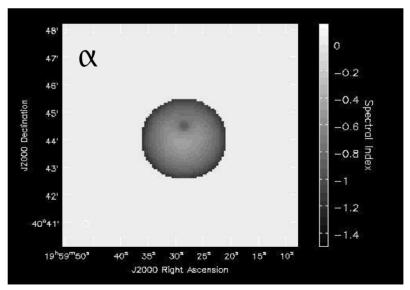
The spectrum at the largest spatial scales is NOT constrained by the data





True sky has one steep spectrum point, and a flat-spectrum extended emission



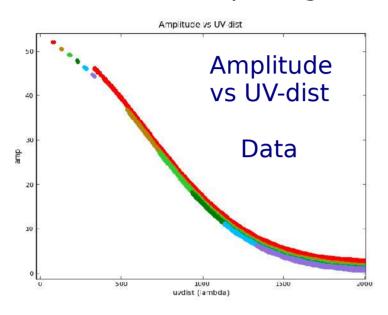


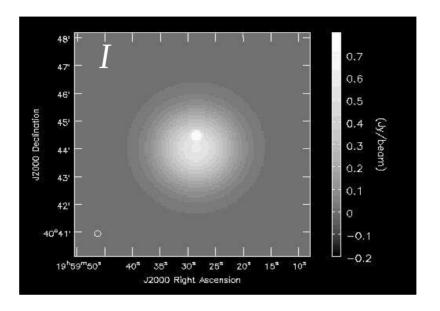
No short spacings to constrain the spectra

=> False steep spectrum reconstruction

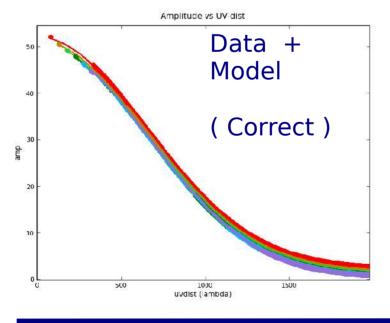
### Very large spatial scales – Need additional information

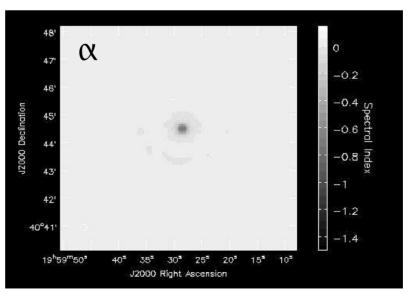
External short-spacing constraints (visibility data, or starting image model)





True sky has one steep spectrum point, and a flat-spectrum extended emission



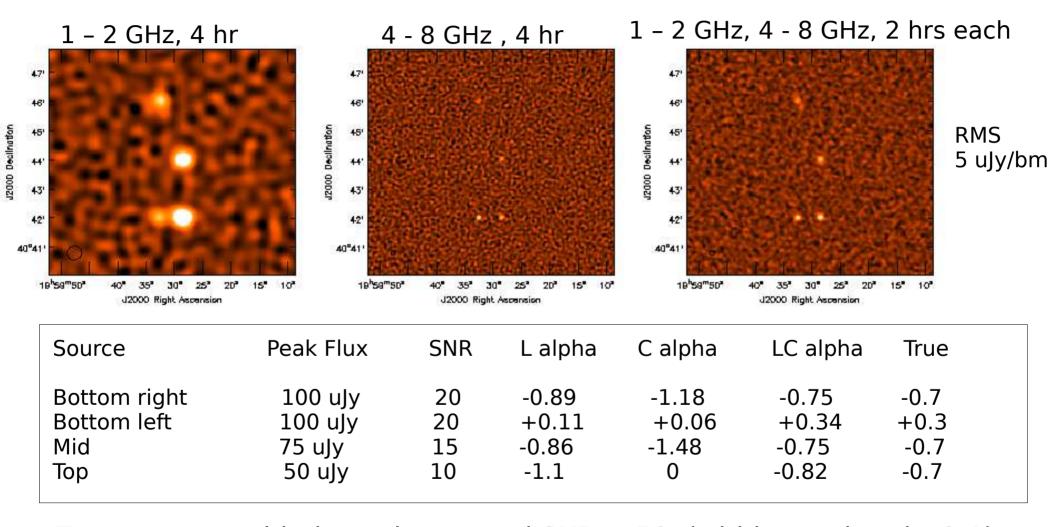


With short spacing info,

Correct reconstruction of a flat spectrum

# Spectral Index Accuracy (for low signal-to-noise)

#### Accuracy of the spectral-fit increases with larger bandwidth-ratio

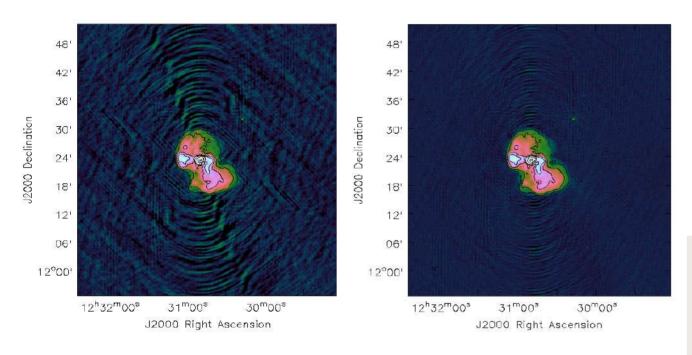


To trust spectral-index values, need SNR > 50 (within one band – 2:1) For SNR < 50 need larger bandwidth-ratio.

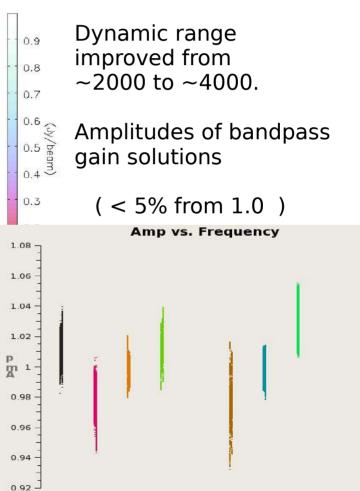
### Wide-band Self-Calibration (for HDR imaging)

- -- First, get a wide-band sky model.
- -- Follow with 'bandpass' calibration
- -- Check amplitude solutions carefully before applying them.

( easy to impose an artificial spectrum on your data )



In these VLA data (of M87), each SPW had been calibrated, imaged, and phase self-cal'd separately, prior to joint MFS imaging and wide-band self-cal to smooth out the spectrum.

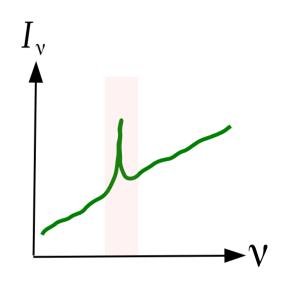


1.6

# Using Wide-Band Models for other processing....

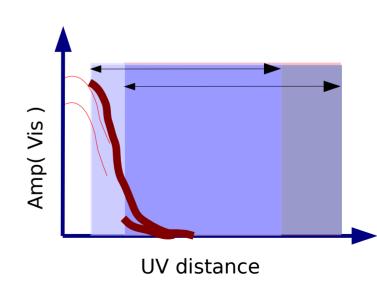
#### (1) Continuum Subtraction

- De-select frequency channels with spectral-lines
- Make a wide-band image model
- Predict model-visibilities over all channels
- Subtract these model visibilities from the data



#### (2) Combining with single-dish data

- Make Taylor-coefficient maps from multi-frequency single-dish images
- Use as a starting model in the MT-MFS interferometric reconstruction



### Wide-Bandwidths and Polarization / Faraday-Rotation

#### Stokes Q,U,V can also change with frequency

- If the expected variation  $< \sim 1\%$  of the peak, MFS (nt=1) will suffice
- If not, it is safest to make a Cube (as the spectra may not smooth)

#### **Faraday Rotation-Measure Synthesis**

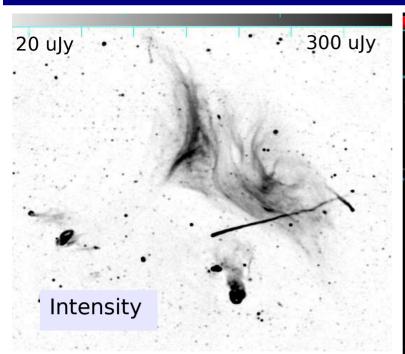
Images of polarized surface-brightness at various Faraday-depths :  $F(\phi)$ 

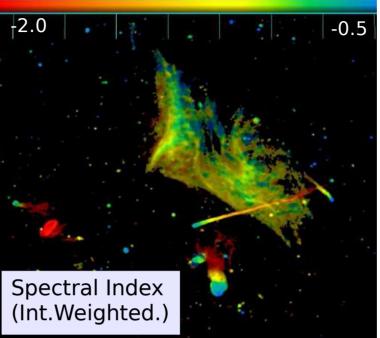
- -P = Q + i U: Make spectral cubes for Q and U separately, and calculate P
- For each pixel in the P-cube, solve  $P(\chi^2) = \int F(\varphi) e^{2\pi i \varphi \chi^2} d\varphi$  for  $F(\varphi)$

This calculation is currently done post-deconvolution, but it could be folded into the image reconstruction framework.

(Ref : Polarization in Interferometry" lecture)

### Wideband VLA imaging of Abell 2256 [Owen et al, 2014]





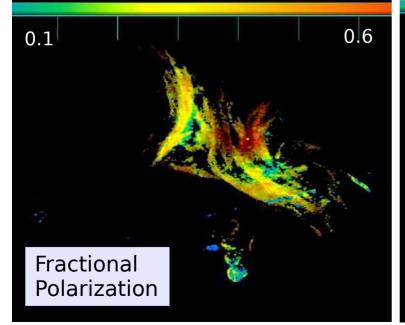
VLA A,B,C,D at L-Band (1-2 GHz)

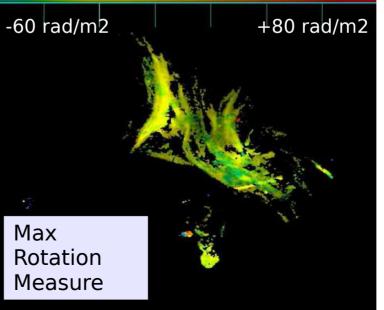
VLA A, at S&C bands(2-4, 4-6, 6-8 GHz)

Calibration and Auto-flagging in AIPS.

Intensity and Spectral index Imaging in CASA. (with Pbcor only post-deconv.)

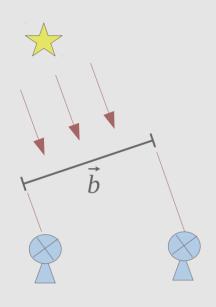
Polarization and Rotation Measure Imaging in AIPS.





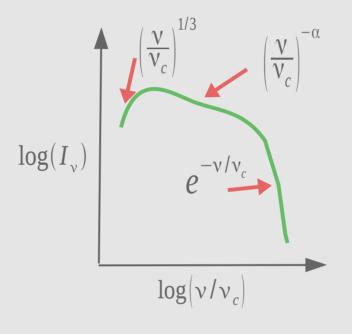
# The instrument and the sky change with frequency...

#### **UV-coverage**



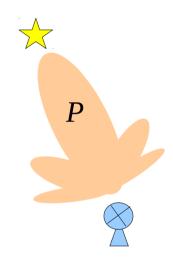
$$S(u,v)_{v} = \frac{\vec{b}}{\lambda} = \frac{\vec{b}v}{c}$$

#### Sky Brightness



$$I(\mathbf{v})$$

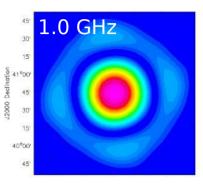
#### **Primary Beam**

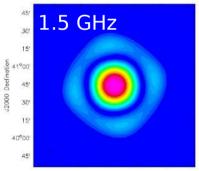


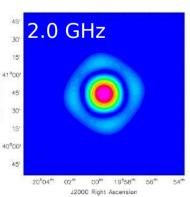
$$HPBW_{v} = \frac{\lambda}{D} = \frac{c}{vD}$$

# Wide-Band Wide-Field Imaging: Primary Beams

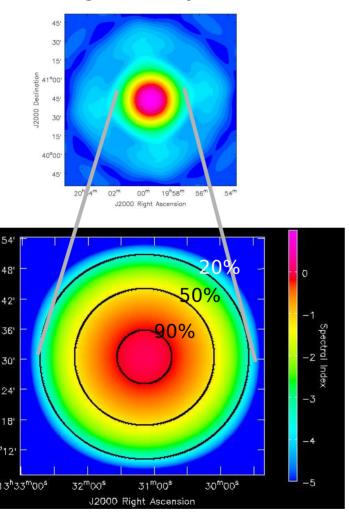
#### **VLA PBs**







Average Primary Beam



Spectral Index of PB

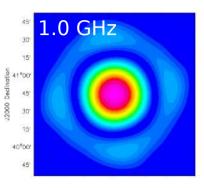
MFS : artificial 'spectral index' away from the center

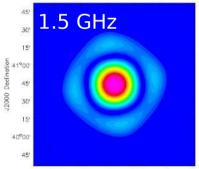
For VLA L-Band (1-2 GHz)

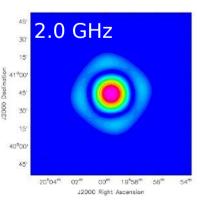
- About -0.4 at the PB=0.8 (6 arcmin from the center)
- About -1.4 at the HPBW(15 arcmin from the center)

## Wide-Band Wide-Field Imaging: Primary Beams

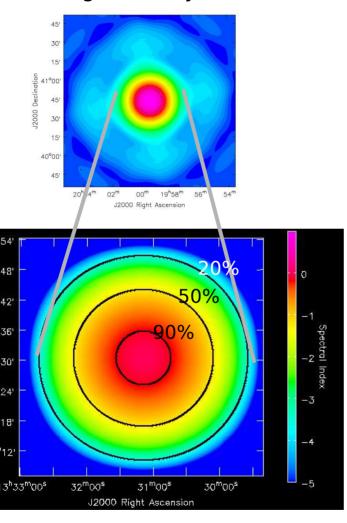
#### **VLA PBs**







Average Primary Beam



Spectral Index of PB

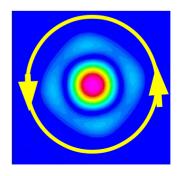
MFS : artificial 'spectral index' away from the center

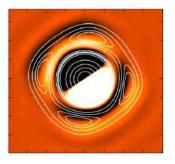
For VLA L-Band (1-2 GHz)

- About -0.4 at the PB=0.8 (6 arcmin from the center)
- About -1.4 at the HPBW(15 arcmin from the center)

### Primary beams also

- rotate with time
- have polarization structure (beam squint, etc...)





(Ref: Wide-Field Imaging – Full Beams lecture)

# Wide-Band Primary Beam Correction





-- Divide the output image at each frequency by  $P(\mathbf{v})$ 

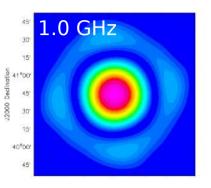
### Multi-Term MFS Imaging

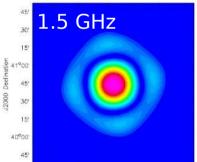
- -- Taylor coefficients represent  $I(\mathbf{v})P(\mathbf{v})$
- -- Polynomial division by PB Taylor coefficients

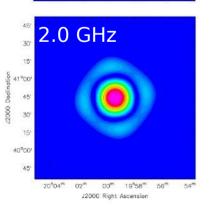
$$\frac{(I_{0,}^{m}I_{1,}^{m}I_{2,...}^{m})}{(P_{0,}P_{1,}P_{2,...})} = (I_{0,}^{sky}I_{1,}^{sky}I_{2}^{sky}...)$$

### Wideband A-Projection

- -- Remove P(v) during gridding (before model fitting)
- -- Also handles PB rotation/squint
- -- Output spectral index image represents only the sky







# Imaging Options: MT-MFS [y/n], A-Projection [y/n]

#### **MT-MFS**

Multi-term MFS (wideband) Imaging
+
Absorb PB spectrum into sky model
+
Post-deconvolution Wideband PBcor
for intensity and alpha

Sault & Wieringa 1994, Rau & Cornwell, 2011

### MT-MFS + WB-A-Projection

Multi-term MFS with wideband A-Projection to remove PB spectrum during gridding

Minor cycle sees only sky spectrum

Post-deconvolution PBcor of intensity only.

Bhatnagar, Rau, Golap, 2013

### **Cube**

Per channel Hogbom/Clark/CS Clean
+
Per channel post-deconvolution Pbcor
+
Smooth to lowest resolution
+

Fit spectrum per pixel, collapse chans

Hogbom 1974, Clark 1980, Schwab & Cotton 1983, Schwarz, 1978

### **Cube + A-Projection**

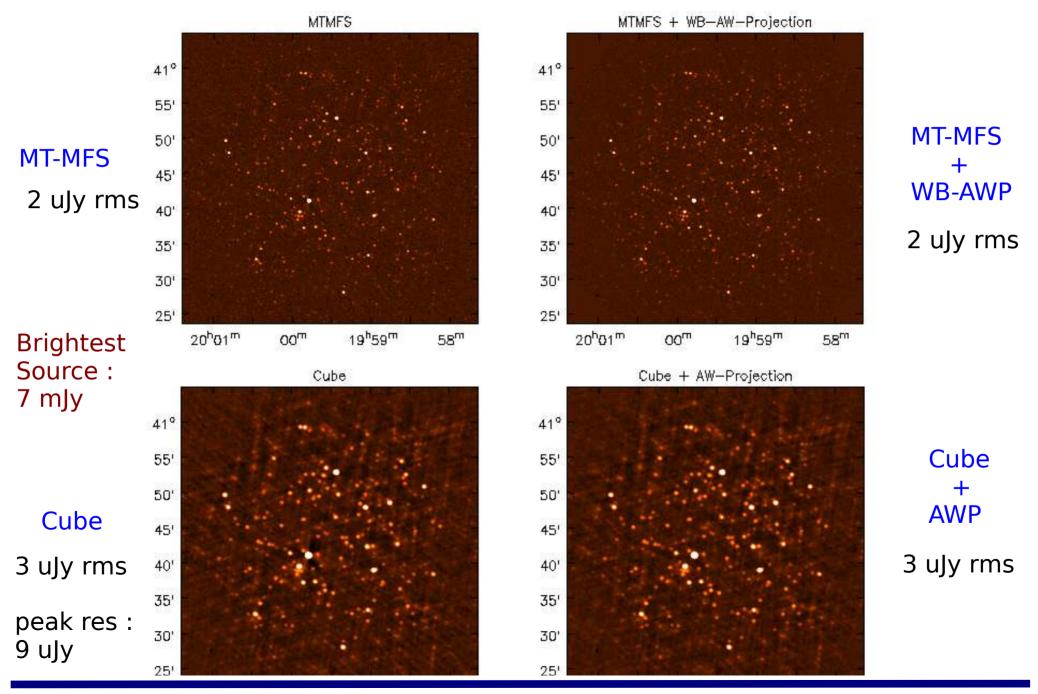
Same as Cube,

 with narrow-band A-Projection per channel

( A-Projection : Construct gridding convolution operators from antenna aperture illumination models. Removes beam squint and accounts for aperture rotation )

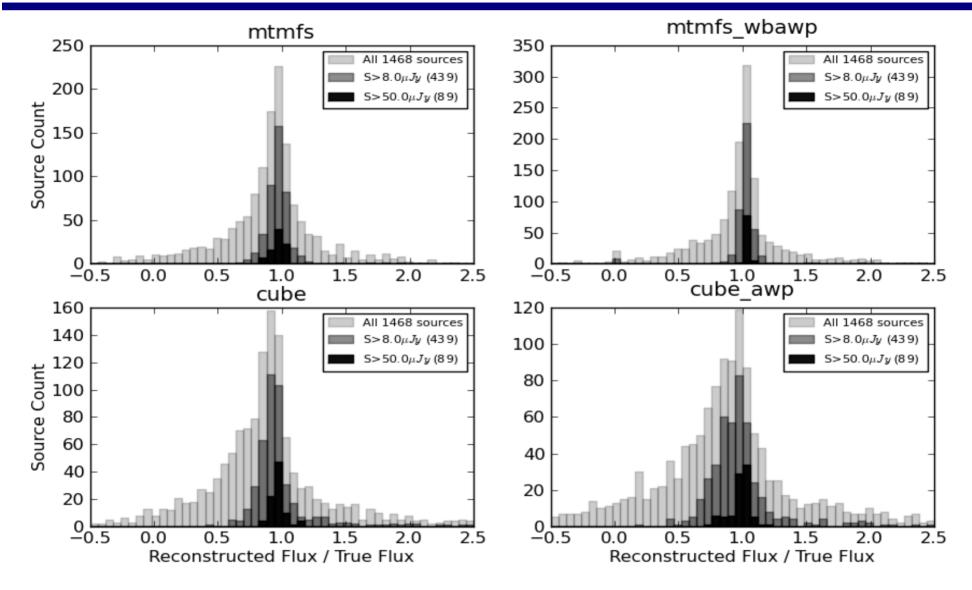
Bhatnagar, Cornwell, Golap, Uson, 2004

# Low dynamic range test (< 10<sup>4</sup>) – compare four methods



14<sup>th</sup> NRAO Synthesis Imaging Workshop, 19 May 2014

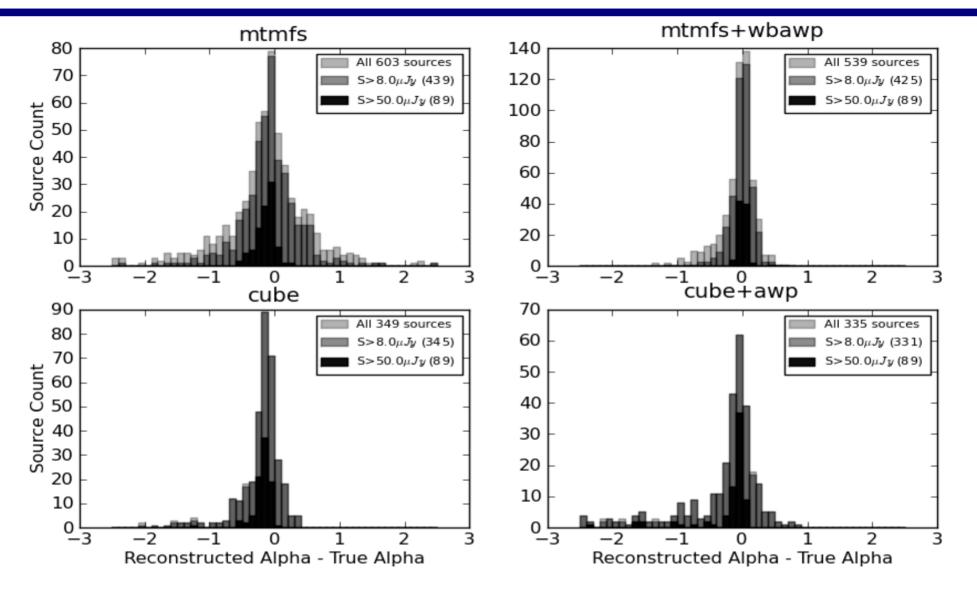
# Histogram of Reconstructed / True Intensity



=> Brighter sources and MFS methods are more accurate

( Different shades in the plots indicate different source intensity ranges )

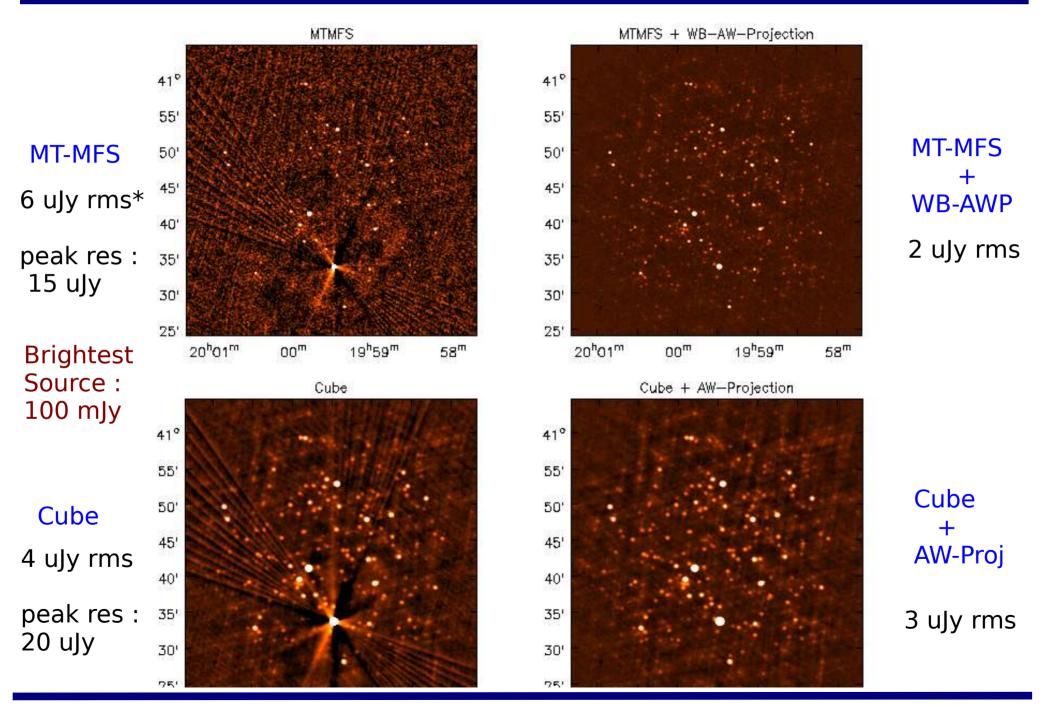
# Histogram of Reconstructed – True Spectral Index



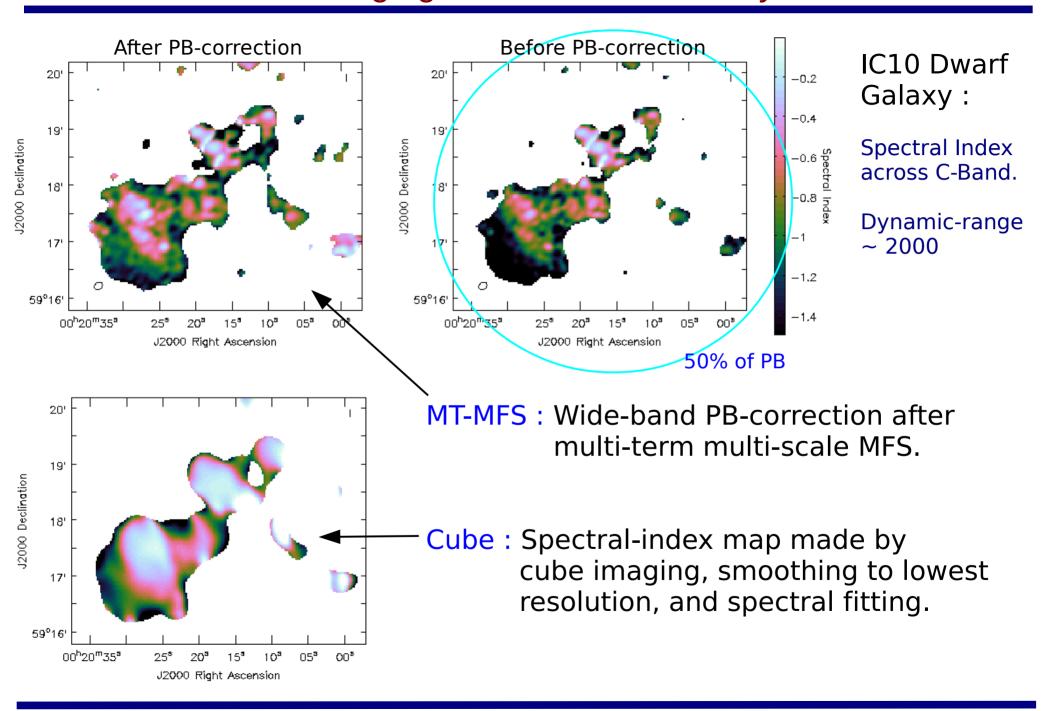
=> Spectral index accuracy degrades faster than intensity...

( Different algorithms produced different #s of usable spectral indices )

# High dynamic range test (>10<sup>4</sup>) - compare four methods

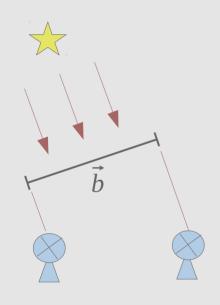


### Wideband VLA imaging of IC10 Dwarf Galaxy [Heesen et al, 2011]



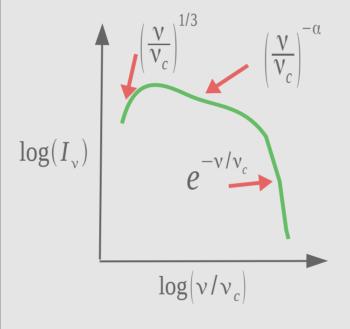
# The instrument and the sky change with frequency...

### **UV-coverage**



$$S(u,v)_{v} = \frac{\vec{b}}{\lambda} = \frac{\vec{b}v}{c}$$

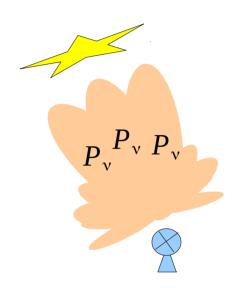
### Sky Brightness



$$I(\mathbf{v})$$

### **Primary Beams**

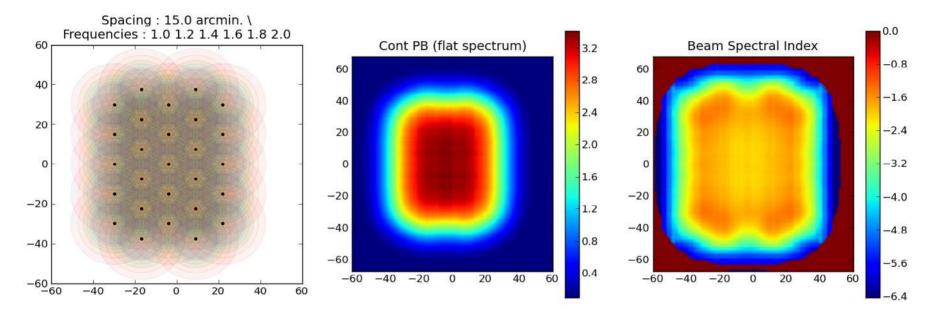
( Mosaic )



$$HPBW_{v} = \frac{\lambda}{D} = \frac{c}{vD}$$

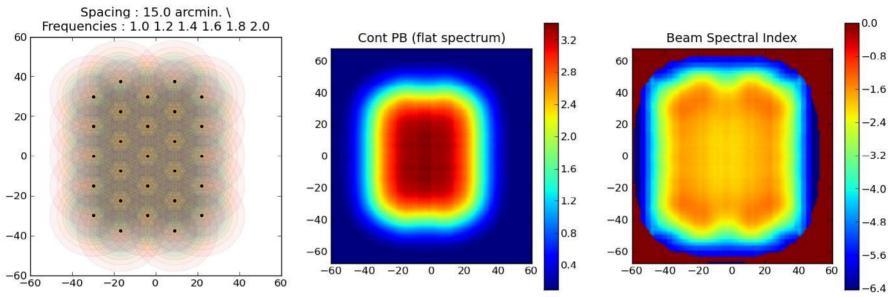
## Wide-Band Wide-Field Imaging: Mosaics

The mosaic primary beam has an artificial spectral index all over the FOV



## Wide-Band Wide-Field Imaging: Mosaics

The mosaic primary beam has an artificial spectral index all over the FOV



#### Algorithms:

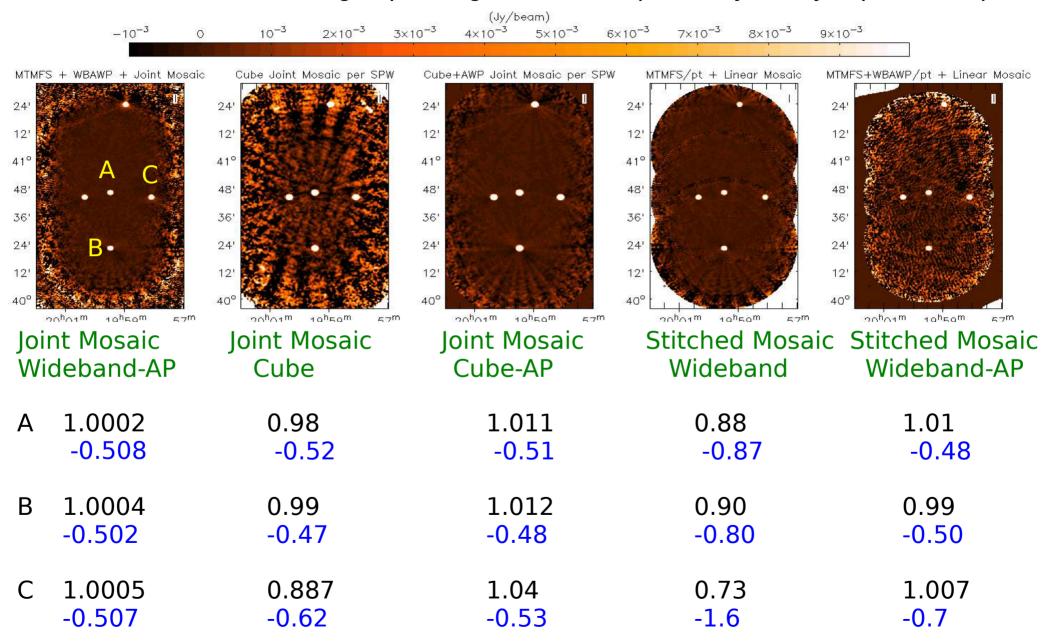
- Deconvolve Pointings separately or together (Stitched vs Joint Mosaic)
  - Impacts image fidelity, especially of common sources.

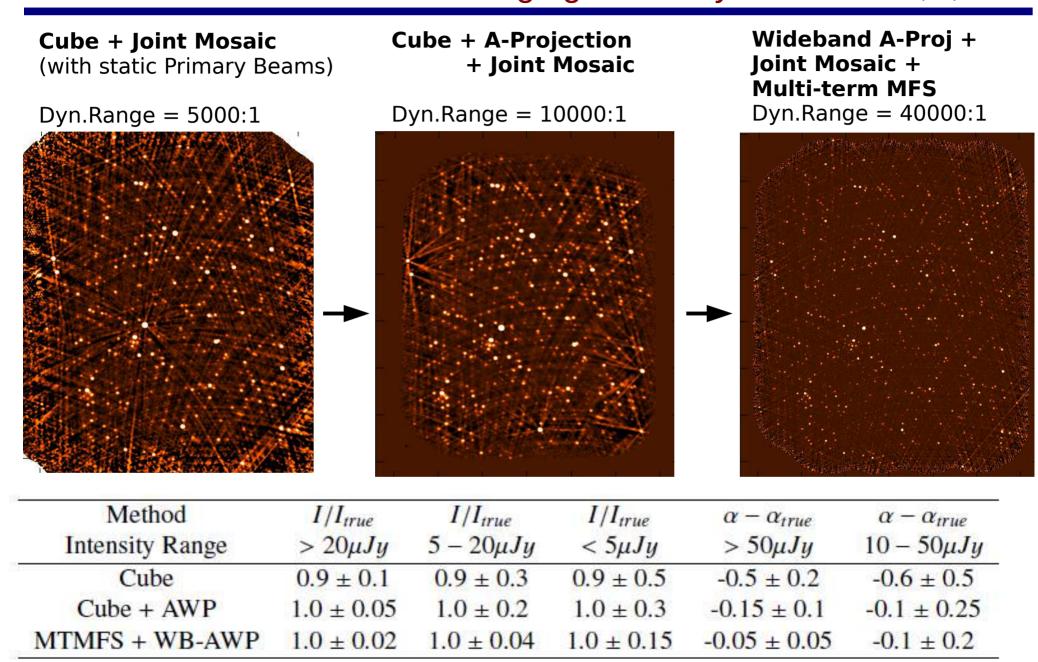
(Ref: Wide-Field Imaging – Mosaicing lecture)

- Deconvolve Channels separately or together ( Cube vs MFS )
  - Impacts imaging fidelity and sensitivity, dynamic range
- Use A-Projection or not ( Accurate vs Approximate PB correction )
  - Impacts dynamic range and spectral index accuracy

## Comparison of several wideband mosaic methods

Dataset: L-Band D-config, 3 pointings, 5 sources (intensity = 1 Jy, alpha = -0.5)

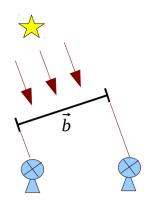




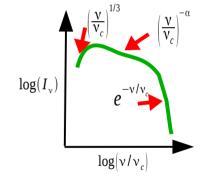
So far, none of our methods produced accurate spectral indices below 10 micro Jy.

# Wide-Band (wide-field) Imaging - Summary

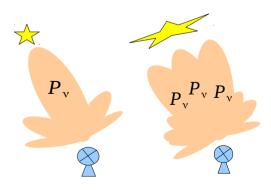
- UV coverage changes with frequency
  - -- Avoid bandwidth-smearing
  - -- Use multi-frequency-synthesis
    - -- to increase the uv-coverage and image-fidelity
    - -- to make images at high angular-resolution



- Sky brightness changes with frequency
  - -- reconstruct intensity and spectrum together (MT-MFS)
  - -- (or) make a Cube of images



- Instrumental primary beam changes with frequency
  - -- divide PB-spectrum from observed sky-spectrum.
  - -- apply wide-field imaging techniques to eliminate the PB frequency dependence during imaging.
  - -- Stitched vs Joint mosaics

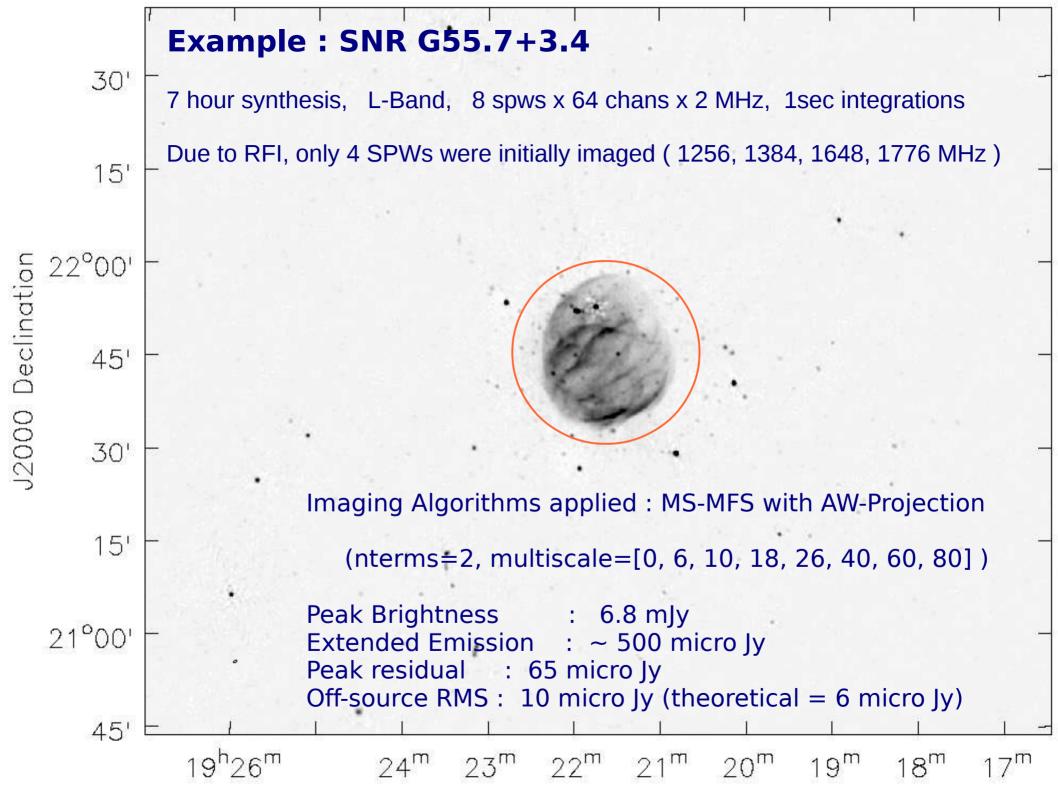


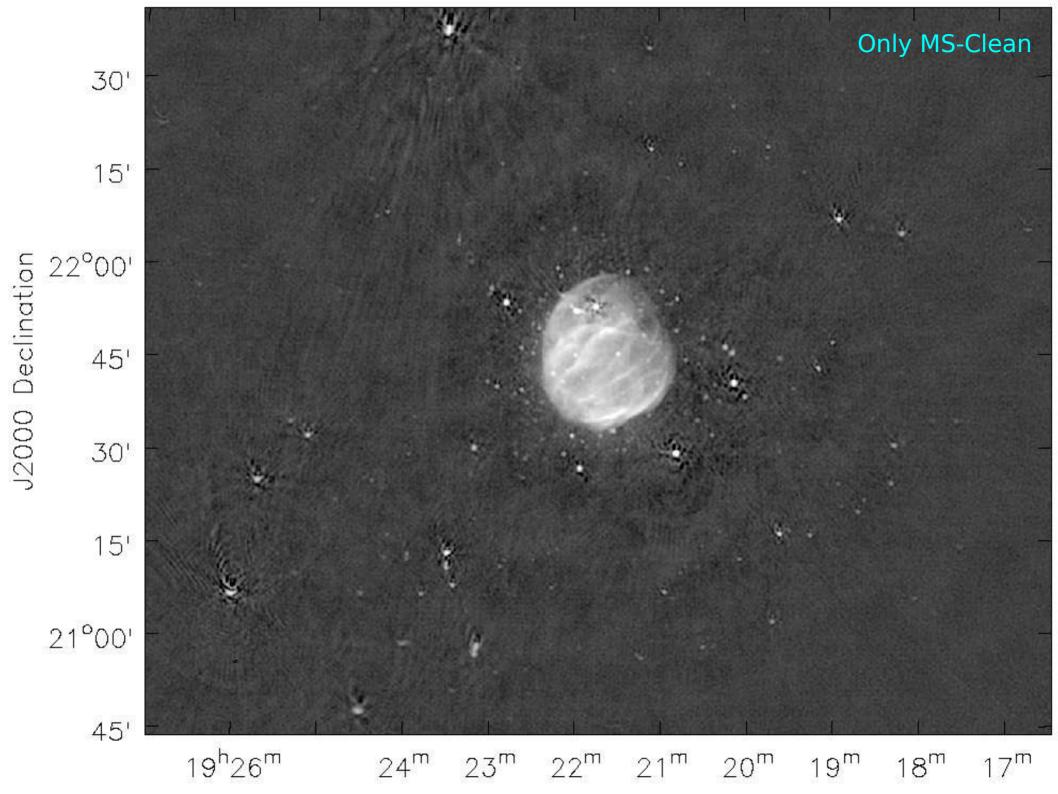
# Wide Band (wide field) Imaging – some guidelines

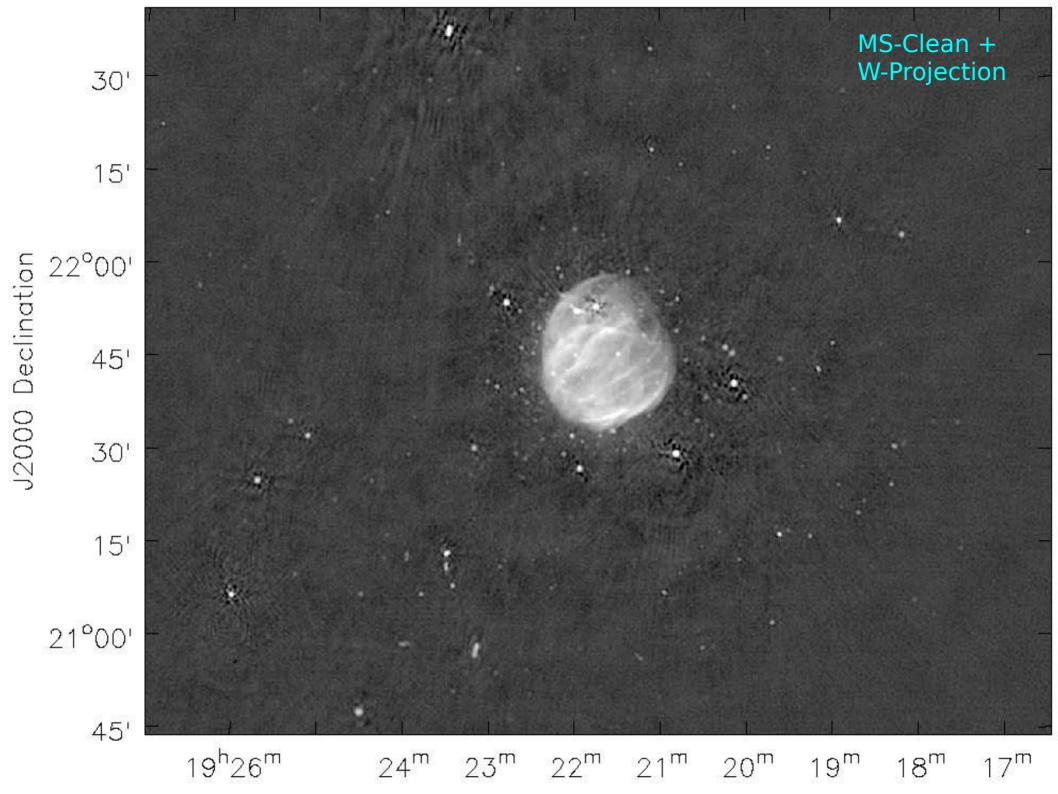
- → MFS has better imaging fidelity, resolution and sensitivity than Cube
- For 2:1 bandwidth, the dynamic range limit with standard MFS (no spectral model) is few 100 to 1000 for a spectral index of -1.0
- For point sources,

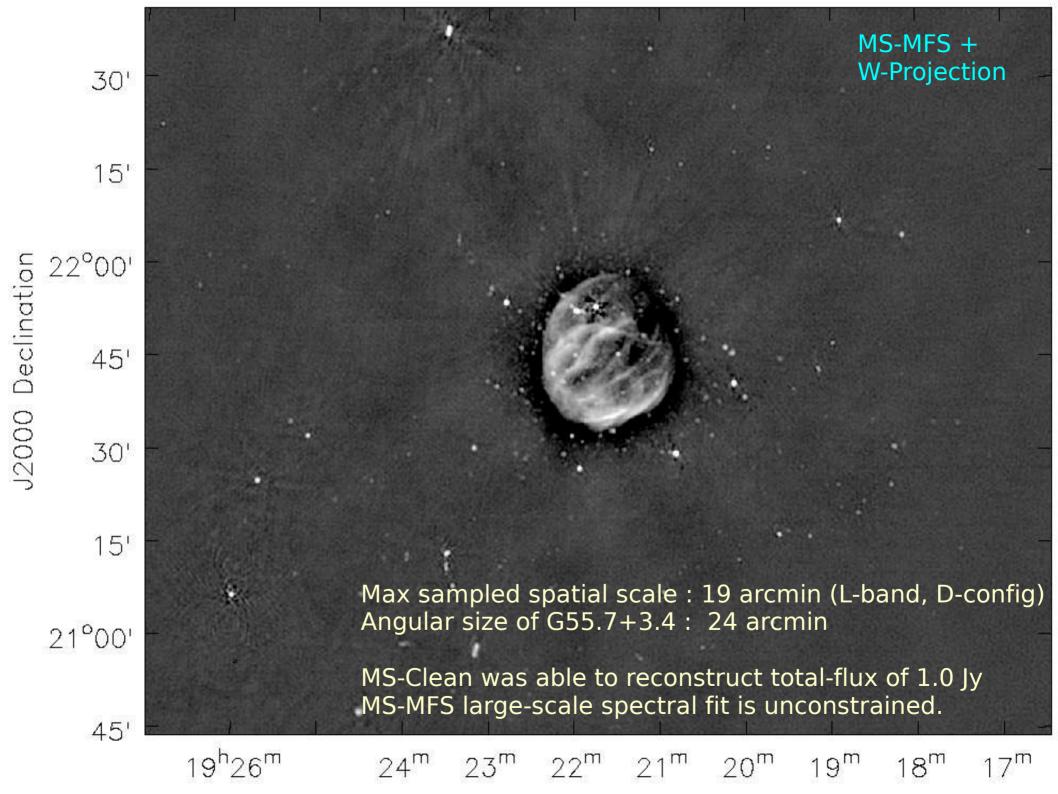
  MT-MFS spectral index errors < 0.1 for SNR > 50 ( 2:1 bwr )

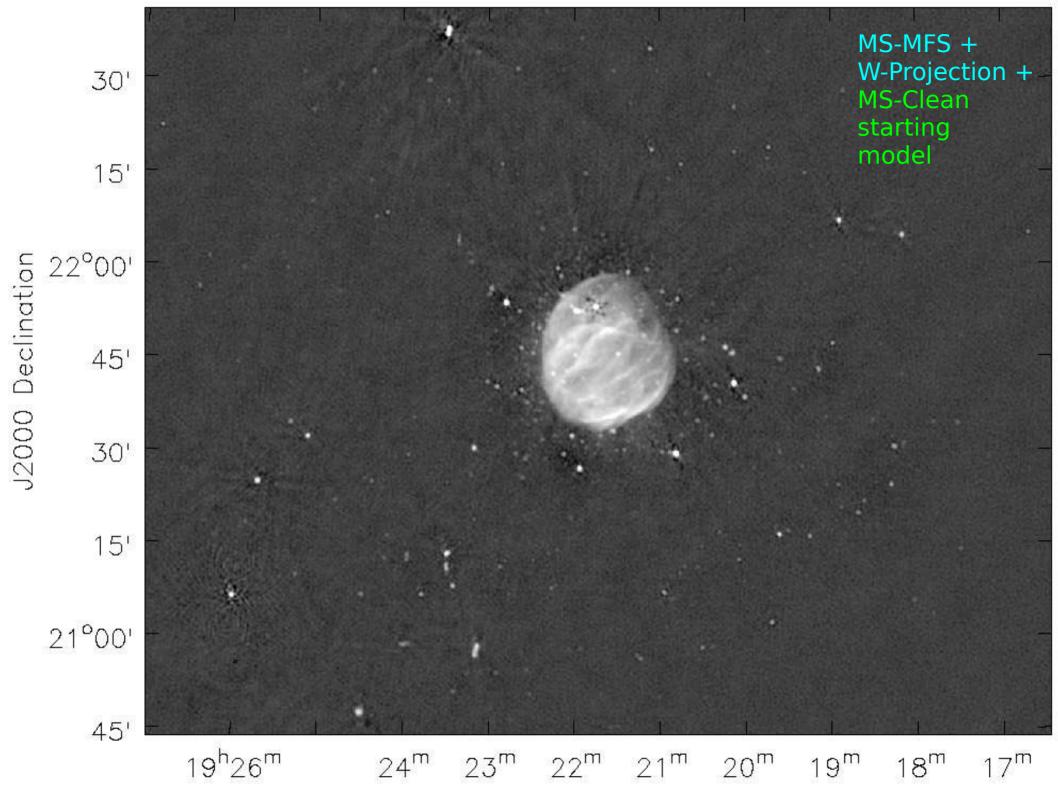
  for SNR > 10 ( 4:1 bwr )
- → For extended emission MT(MS)-MFS spectral index errors < 0.2 for SNR > 100
- For 2:1 bwr, the PB's artificial spectral index at the HPBW is -1.4
- → VLA beam squint and rotation effects appear at the few x 10<sup>4</sup> DR.
- → Joint mosaics have better imaging fidelity than stitched mosaics.
- The current most practical approach to wideband mosaicing is cube joint mosaicing using A-Projection (accuracy vs cost vs software)





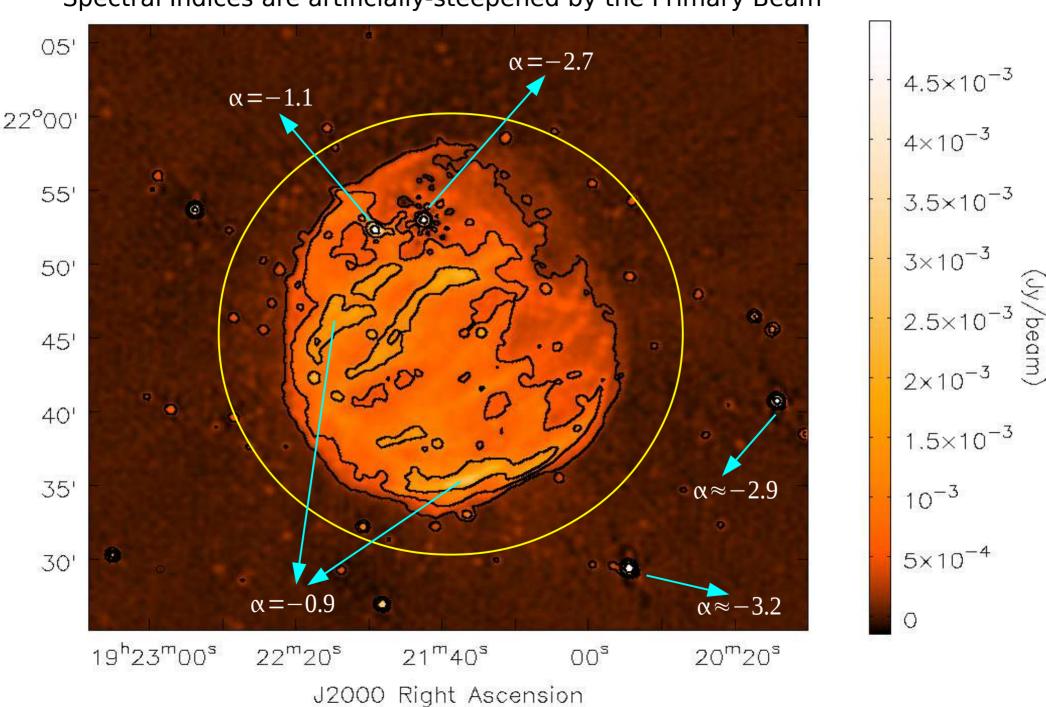






# G55.7+3.4 : Supernova-Remnant + Pulsar

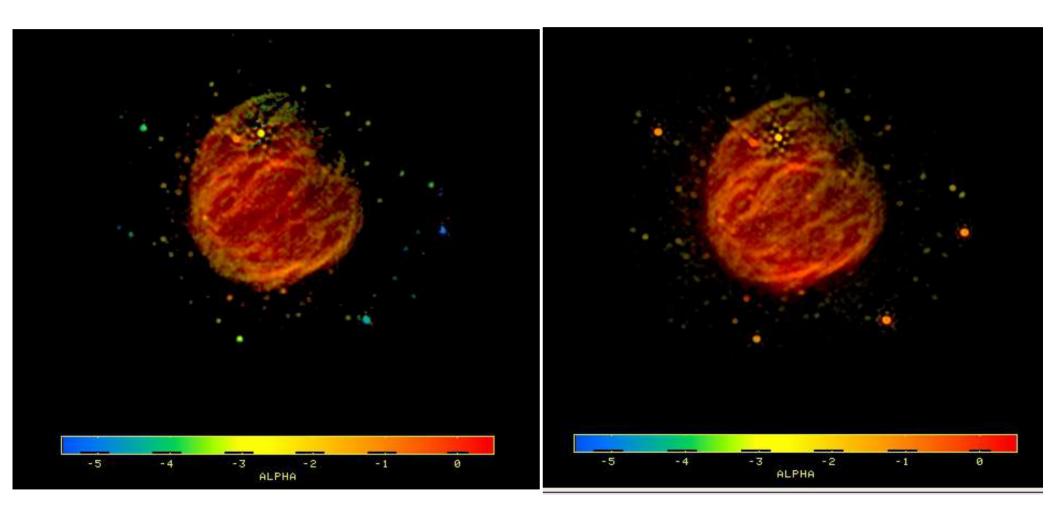
Spectral Indices are artificially-steepened by the Primary Beam



# Spectral Indices before and after WB-A-Projection

Without PB correction
Outer sources are artificially steep

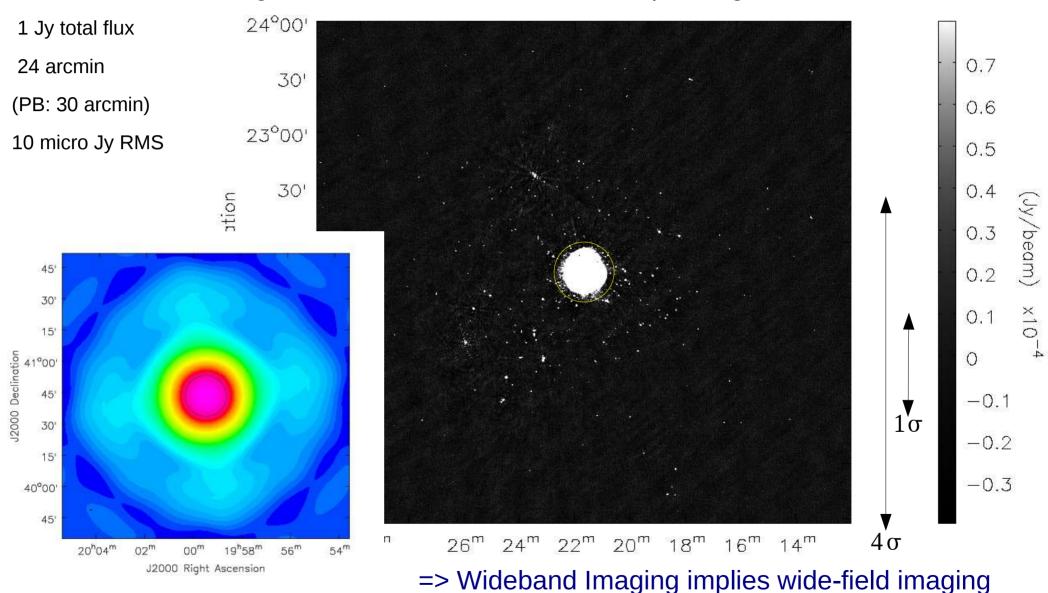
With PB correction (via WB-AWP)
Outer sources have correct spectra



Intensity-weighted spectral index maps (color = spectral index from -5.0 to +0.2)

# Wide-field sensitivity because of wide-bandwidths

G55.7+3.4: 4 x 4 degree field-of-view from one EVLA pointing



## **Summary**

Broad-band receivers provide increased instantaneous sensitivity

Cube-imaging will suffice for a quick-look, and bright simple targets

For deep imaging, do wideband MFS (intensity and spectrum)

Apply appropriate wideband primary beam correction

Choose your algorithms based on desired accuracy and computing cost

Pay attention to the **many** sources of error in this whole process.

New astrophysics made possible by new instruments!

→ High dynamic range, wideband, full-polarization, mosaic imaging --> An ACTIVE area of research for VLA and other new telescopes