Mars Mission Simulation: Detailed Analysis

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Abstract

This document provides a detailed explanation of a MATLAB-based simulation for an interplanetary mission from Earth to Mars. The simulation covers key phases including Earth departure, Hohmann transfer, Mars orbit insertion, orbit propagation with J2 perturbations, atmospheric entry with drag, stable hovering using a PID controller, and a mission summary. Each section is explained with mathematical formulations, assumptions, and references to the code's functionality.

1 Introduction

The MATLAB code simulates a spacecraft's journey from a low Earth orbit (LEO) to a stable hover above the Martian surface. The simulation incorporates gravitational effects, atmospheric drag, and control systems to model a realistic mission profile. This document breaks down each section of the code, providing detailed explanations, mathematical derivations, and insights into the physical principles involved.

2 Constants

The simulation begins by defining fundamental constants used throughout the calculations. These include gravitational parameters, planetary radii, J2 perturbation coefficients, solar parameters, and vehicle properties.

- **Earth and Mars Gravitational Constants**: The gravitational parameter for Earth is $\mu_{\text{earth}} = 3.986004418 \times 10^5\,\text{km}^3/\text{s}^2$, and for Mars, $\mu_{\text{mars}} = 4.282837 \times 10^4\,\text{km}^3/\text{s}^2$. These values are used in orbital mechanics calculations.
- **Planetary Radii**: Earth's radius is $R_{\text{earth}} = 6378.1\,\text{km}$, and Mars' radius is $R_{\text{mars}} = 3389.5\,\text{km}$.
- **J2 Perturbations**: The J2 coefficients account for the oblateness of the planets, with $J2_{\text{earth}} = 1.08263 \times 10^{-3}$ and $J2_{\text{mars}} = 1.96045 \times 10^{-3}$.
- **Solar Parameters**: The astronomical unit (AU) is defined as $1.495978707 \times 10^8\,\text{km}$, and the Sun's gravitational parameter is $\mu_{\text{sun}} = 1.32712440018 \times 10^{11}\,\text{km}^3/\text{s}^2$.

- **Vehicle Properties**: The spacecraft has a mass of $1000\,\text{kg}$, a drag coefficient $C_d = 1.8$, and a reference area $A = 10\,\text{m}^2$.

These constants ensure consistency across all calculations and are critical for accurate modeling of orbital and atmospheric dynamics.

3 Earth Departure

The spacecraft begins in a low Earth orbit (LEO) at an altitude of 200 km, with a radius of $r_{\text{leo}} = R_{\text{earth}} + 200 = 6578.1\,\text{km}$. The circular orbit velocity is calculated as:

$$v_{\text{circ}} = \sqrt{\frac{\mu_{\text{earth}}}{r_{\text{leo}}}}$$

A characteristic energy $C3 = 12\,\text{km}^2/\text{s}^2$ defines the hyperbolic excess velocity $v_\infty = \sqrt{C3}$. The velocity for the trans-Mars injection (TMI) is:

$$v_{\text{hyper}} = \sqrt{v_\infty^2 + \frac{2\mu_{\text{earth}}}{r_{\text{leo}}}}$$

The required Δv for TMI is:

$$\Delta v_{\text{tmi}} = v_{\text{hyper}} - v_{\text{circ}}$$

The code computes $\Delta v_{\text{tmi}} \approx 3.65\,\text{km/s}$, which represents the velocity change needed to escape Earth's gravity and begin the interplanetary transfer.

4 Hohmann Transfer

The transfer from Earth to Mars follows a Hohmann elliptical orbit, which is the minimum-energy trajectory between two coplanar circular orbits. The semi-major axis of the transfer orbit is:

$$a_{\text{trans}} = \frac{r_1 + r_2}{2}$$

where $r_1 = 1\,\text{AU}$ (Earth's orbit) and $r_2 = 1.524\,\text{AU}$ (Mars' orbit). The transfer time is derived from Kepler's Third Law:

$$T_{\text{trans}} = \pi\sqrt{\frac{a_{\text{trans}}^3}{\mu_{\text{sun}}}}$$

The code calculates the transfer time as approximately 258.6 days, consistent with typical Hohmann transfer durations to Mars.

5 Mars Arrival and Mars Orbit Insertion (MOI)

Upon arrival at Mars, the spacecraft enters a hyperbolic trajectory with respect to Mars, characterized by the same $v_\infty = \sqrt{C3}$. The periapsis radius of the target orbit is $r_p = R_{\text{mars}} + 250 = 3639.5$ km, and the apoapsis radius is $r_a = R_{\text{mars}} + 10000 = 13389.5$ km. The semi-major axis of the elliptical orbit is:

$$a_m = \frac{r_p + r_a}{2}$$

The circular velocity at periapsis is:

$$v_{\text{circ,mars}} = \sqrt{\frac{\mu_{\text{mars}}}{r_p}}$$

The hyperbolic velocity at periapsis is:

$$v_{\text{hyper,mars}} = \sqrt{v_\infty^2 + \frac{2\mu_{\text{mars}}}{r_p}}$$

The velocity in the target elliptical orbit at periapsis is:

$$v_{\text{elliptic}} = \sqrt{2\left(\frac{\mu_{\text{mars}}}{r_p} - \frac{\mu_{\text{mars}}}{2a_m}\right)}$$

The Δv for Mars orbit insertion (MOI) is:

$$\Delta v_{\text{moi}} = v_{\text{hyper,mars}} - v_{\text{elliptic}}$$

The code computes $\Delta v_{\text{moi}} \approx 1.41$ km/s, which is the velocity change required to capture the spacecraft into the desired elliptical orbit around Mars.

6 Orbit Propagation with Mars J2

The spacecraft's orbit around Mars is propagated for 10 days, accounting for the J2 perturbation due to Mars' oblateness. The initial orbit is defined by orbital elements:

$$\text{oe0} = [a_m, e, i, \Omega, \omega, \theta] = [a_m, \frac{r_a - r_p}{r_a + r_p}, 30^\circ, 0, 0, 0]$$

These elements are converted to position and velocity vectors using the `oe2rv` function. The orbit is propagated using MATLAB's `ode45` solver with the `twobody_J2_mars` function, which includes the two-body acceleration and J2 perturbation:

$$\mathbf{a}_{\text{J2}} = 1.5J2\frac{\mu R^2}{r^4}\begin{bmatrix} \frac{x}{r}(5\frac{z^2}{r^2} - 1) \\ \frac{y}{r}(5\frac{z^2}{r^2} - 1) \\ \frac{z}{r}(5\frac{z^2}{r^2} - 3) \end{bmatrix}$$

The resulting trajectory is visualized in a 3D plot, showing the spacecraft's path in Mars' inertial frame.

7 Atmospheric Entry with Drag

The spacecraft enters Mars' atmosphere at an altitude of $h_{\text{entry}} = 125\,\text{km}$ with an entry velocity of $v_{\text{entry}} = 5.8\,\text{km/s}$ and a flight path angle of $\gamma = -12^\circ$. The initial conditions are:

$$\mathbf{y0}_{\text{entry}} = [x, z, v_x, v_z, h] = [0, R_{\text{mars}} + h_{\text{entry}}, v_{\text{entry}} \cos\gamma, -v_{\text{entry}} \sin\gamma, h_{\text{entry}}]$$

The dynamics are modeled using the `entry_with_drag` function, which includes gravitational acceleration and atmospheric drag:

$$\mathbf{a}_{\text{drag}} = -\frac{1}{2}\rho v^2 C_d \frac{A}{m} \frac{\mathbf{v}}{v}$$

The atmospheric density ρ is modeled using an exponential model based on altitude, similar to the Global Reference Atmospheric Model (GRAM). The trajectory is integrated over 300 seconds using `ode45` and plotted as downrange distance versus altitude.

8 Stable Hovering with PID Control

The spacecraft aims to maintain a stable hover at an altitude of $h_{\text{target}} = 10\,\text{km}$ using a PID controller. The Martian surface gravity at this altitude is:

$$g_{\text{mars}} = \frac{\mu_{\text{mars}}}{(R_{\text{mars}} + h_{\text{target}})^2}$$

The PID controller computes an acceleration command:

$$a_{\text{cmd}} = -(k_P e + k_I \int e\,dt + k_D \frac{de}{dt})$$

where $e = h - h_{\text{target}}$ is the altitude error, and the gains are $k_P = 0.5$, $k_I = 0.02$, and $k_D = 0.4$. The thrust is calculated as:

$$T = m(g_{\text{mars}} + a_{\text{cmd}})$$

with constraints to ensure non-negative thrust and a maximum of $2g_{\text{mars}}$. The dynamics are updated using a simple Euler integration over 120 seconds, and the altitude, velocity, and thrust are logged and plotted.

9 3D Animation of Atmospheric Entry

The atmospheric entry trajectory is visualized in 3D using the `comet3` function, showing the downrange (x), crossrange ($y = 0$), and altitude ($z - R_{\text{mars}}$) coordinates. A semi-transparent sphere representing Mars' surface is added for reference, with a radius of $R_{\text{mars}} = 3389.5\,\text{km}$.

10 Delta-V Budget and Mission Summary

The total Δv budget is the sum of the TMI and MOI maneuvers:

$$\Delta v_{\text{total}} = \Delta v_{\text{tmi}} + \Delta v_{\text{moi}}$$

The code reports:

- TMI Δv: 3.65 km/s
- Transfer Time: 258.6 days
- MOI Δv: 1.41 km/s
- Total Δv: 5.06 km/s
- Hover Altitude: 10 km
- Final Hover Error: Computed based on the final altitude

A success message is displayed if the final hover error is less than 0.01 km; otherwise, a warning is issued.

11 Exporting Figures

The simulation generates four figures (Mars J2 orbit, entry profile, 3D entry animation, and PID hover control), which are saved as PNG files with labels `Mars_J2_Orbit`, `Entry_Profile`, `Entry_3D`, and `Hover_PID`.

12 Conclusion

This simulation provides a comprehensive model of a Mars mission, from Earth departure to stable hovering. It incorporates realistic physical effects such as J2 perturbations and atmospheric drag, and employs a PID controller for precise altitude control. The results are visualized effectively, and the Δv budget provides a clear summary of the mission's propulsion requirements.