User's Manual for `multihop_3d.py`

3D HF Propagation Simulator with O/X Mode Splitting

Version 1.0

Generated with Claude Code

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Contents

1 Introduction

`multihop_3d.py` is a Python-based simulator for high-frequency (HF) radio wave propagation through the Earth's ionosphere. It models the splitting of radio waves into ordinary (O) and extraordinary (X) magnetoionic modes, computing their different propagation paths, arrival angles, and polarization states.

1.1 Key Features

- **3D Ray Tracing**: Full three-dimensional treatment of ray paths
- **O/X Mode Splitting**: Separate tracking of ordinary and extraordinary waves
- **Elevation Splitting**: Different reflection heights for O and X modes
- **Azimuth Splitting**: Lateral deflection due to magnetized plasma
- **Multihop Propagation**: Support for 1, 2, 3, or more ionospheric hops
- **Faraday Rotation**: Computation of polarization rotation along path
- **Comprehensive Output**: Text summaries, diagnostic plots, and data files

1.2 Applications

- HF direction-finding system analysis
- Interferometric HF array design
- Polarimetric receiver calibration
- Ionospheric research and modeling
- Amateur radio propagation prediction
- Time signal reception analysis (WWV, CHU, etc.)

2 Installation and Requirements

2.1 Dependencies

The simulator requires Python 3.7+ and the following packages:

```
pip install numpy matplotlib scipy
```

Optional dependencies for enhanced functionality:

```
pip install requests # For fetching space weather data pip install ppigrf # For IGRF
magnetic field model
```

2.2 Files

The simulator consists of a single Python file:

- `multihop_3d.py` — Main simulator script

No additional configuration files are required. The script is self-contained and can be run directly.

3 Quick Start

3.1 Basic Usage

Run the simulator with default parameters (WWV to OVRO path):

```
python3 multihop_3d.py
```

This simulates propagation from WWV (Fort Collins, CO) to Owens Valley Radio Observatory at the default frequencies (5, 10, 15, 20, 25 MHz).

3.2 Custom Path

Specify transmitter and receiver locations:

```
python3 multihop_3d.py -tx-lat 45.29 -tx-lon -75.75 \
-rx-lat 37.23 -rx-lon -118.28 \
-frequencies 3.33 7.85 14.67 \
-output chu_ovro
```

3.3 Multihop Propagation

For longer paths requiring multiple ionospheric reflections:

```
python3 multihop_3d.py -hops 2 -frequencies 5 10 15 -output my_2hop
```

4 Command-Line Reference

4.1 Synopsis

```
python3 multihop_3d.py [OPTIONS]
```

4.2 Options

Table 1: Command-line options for `multihop_3d.py`

Option	Default	Description
`-tx-lat`	40.68	Transmitter latitude (degrees N)
`-tx-lon`	−105.04	Transmitter longitude (degrees E)
`-rx-lat`	37.23	Receiver latitude (degrees N)
`-rx-lon`	−118.28	Receiver longitude (degrees E)
`-frequencies`, `-f`	5 10 15 20 25	Frequencies to simulate (MHz)
`-hops`, `-n`	1	Number of ionospheric hops
`-date`	now	Date/time in ISO format
`-output`, `-o`	hf_3d	Output file prefix
`-no-plots`	False	Skip generating plots
`-help`, `-h`		Show help message

4.3 Coordinate Convention

- **Latitude**: Positive = North, Negative = South
- **Longitude**: Positive = East, Negative = West
- Example: New York City is approximately 40.71°N, −74.01°E

5 Output Files

The simulator generates 7 output files for each run:

5.1 Text Summary

`<prefix>_summary.txt` — Human-readable summary containing:

- Path geometry (coordinates, distance, bearing)
- Simulation date/time
- Propagation conditions including Maximum Usable Frequency (MUF)
- Results table with O/X reflection heights, angular splitting, and MUF status
- Notes on interpretation

The results table includes a `Status` column indicating whether each frequency is above or below the MUF:

- `OK` — Frequency is below MUF; normal ionospheric reflection expected
- `ABOVE_MUF` — Frequency exceeds MUF; wave may penetrate ionosphere

Note: The MUF calculation uses the per-hop distance. For multi-hop paths, the shorter individual hop distances result in lower MUF values (because the rays are more vertical).

5.2 Polarization Summary

The summary file includes a **RECEIVED POLARIZATION** section showing differential O/X losses and resulting polarization state:

```
RECEIVED POLARIZATION (due to differential O/X absorption)
-------------------------------------------------------------
    Freq   Loss_O   Loss_X    DLoss    LCP/RCP   L/R (dB)    Axial    Ellip     %LCP     %RCP
   (MHz)     (dB)     (dB)     (dB)      ratio               ratio    (deg)
-------------------------------------------------------------
    2.50    425.6    583.5    157.9      >1000       >100    1.000    -45.0    100.0      0.0
    5.00    231.9    246.0     14.1      25.75      +14.1    0.671    -33.9     96.3      3.7
   10.00    156.6    160.0      3.4       2.16       +3.4    0.191    -10.8     68.4     31.6
```

Key columns:

- **Loss_O, Loss_X**: Total path loss for O and X modes (dB)
- **ΔLoss**: Differential loss ($\text{Loss}_X - \text{Loss}_O$); positive = X-mode absorbed more
- **LCP/RCP ratio**: Power ratio of left to right circular polarization at receiver
- **L/R (dB)**: Same ratio in decibels
- **Axial ratio**: Minor/major axis of polarization ellipse (0 = linear, 1 = circular)
- **Ellip**: Ellipticity angle χ ($-45°$ = pure LCP, $0°$ = linear, $+45°$ = pure RCP)
- **%LCP, %RCP**: Percentage of power in each circular component

Practical interpretation: When LCP/RCP ratio is >10, an LCP antenna provides >10 dB SNR advantage over RCP. This is common for daytime low-frequency paths in the Northern Hemisphere.

5.3 Diagnostic Plots

The simulator generates 6 diagnostic plot files, each showing different aspects of the propagation analysis:

Table 2: Output plot files

File	Contents
`_overview.png`	4-panel overview: O-mode ray paths, O vs X comparison, Faraday rotation vs frequency, path loss vs frequency. Shows MUF in title.
`_polarization.png`	6-panel polarization analysis: ellipses for each frequency, Stokes parameters (I,Q,U,V), ellipse parameters (ψ, χ), O/X reflection heights.
`_ray_details.png`	6-panel ionospheric parameters along ray path: electron density N_e, magnetic field B, plasma/gyro frequencies, O and X refractive indices (with evanescent regions shaded).
`_3d_rays.png`	4-panel 3D visualization: O and X ray paths in 3D space, O-X phase difference vs frequency, arrival angle of arrival (elevation and azimuth).
`_rx_polarization.png`	Receiver polarization state showing the polarization ellipse at the receiver for each frequency.
`_angular_splitting.png`	4-panel O/X splitting analysis: O and X elevation angles, elevation splitting, O and X azimuths, azimuth splitting. Shows MUF line.

5.3.1 Overview Plot

The overview plot (Figure 1) provides a quick summary of propagation characteristics:

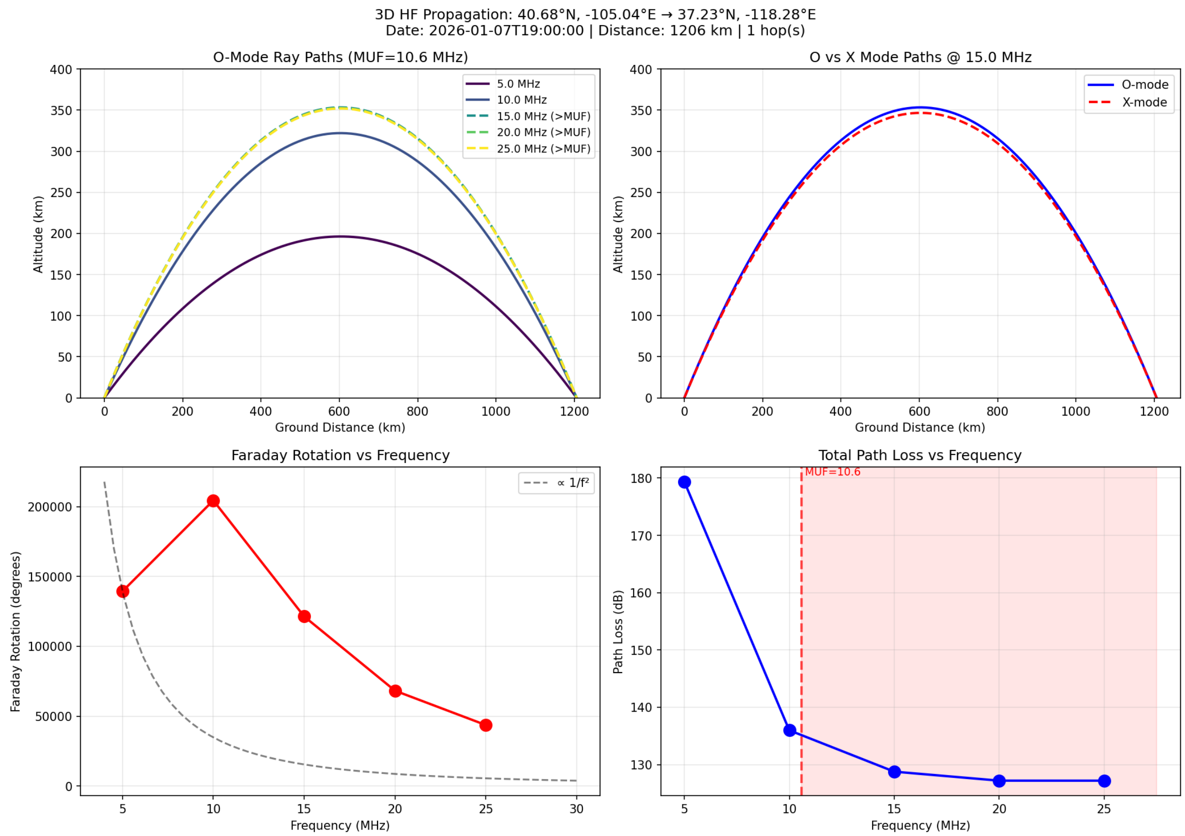

Figure 1: Example overview plot showing ray paths (upper left), O vs X mode comparison (upper right), Faraday rotation (lower left), and path loss (lower right). The MUF is shown in the ray paths title, and frequencies above MUF use dashed lines.

5.3.2 Polarization Plot

The polarization plot (Figure 2) shows the polarization state of the received signal:

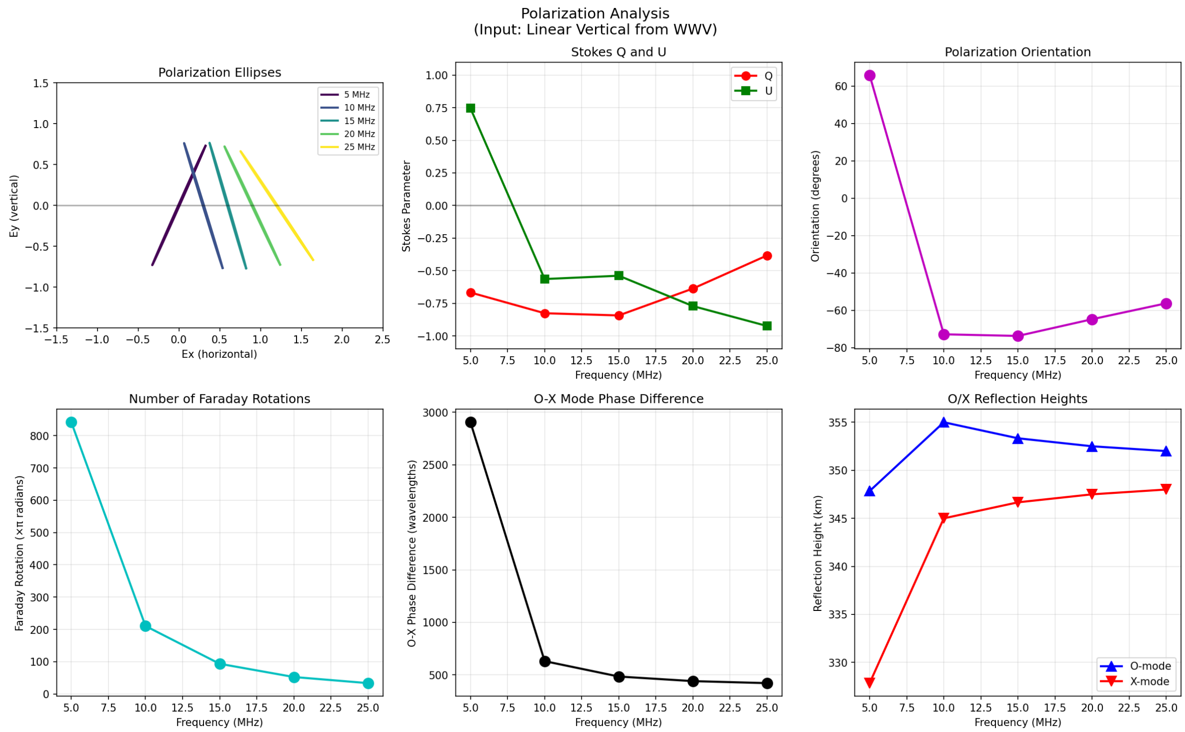

Figure 2: Polarization analysis showing ellipses for each frequency, Stokes parameters, ellipse orientation (ψ) and ellipticity (χ), and O/X reflection height comparison.

5.3.3 Ray Details Plot

The ray details plot (Figure 3) shows ionospheric parameters along the ray path:

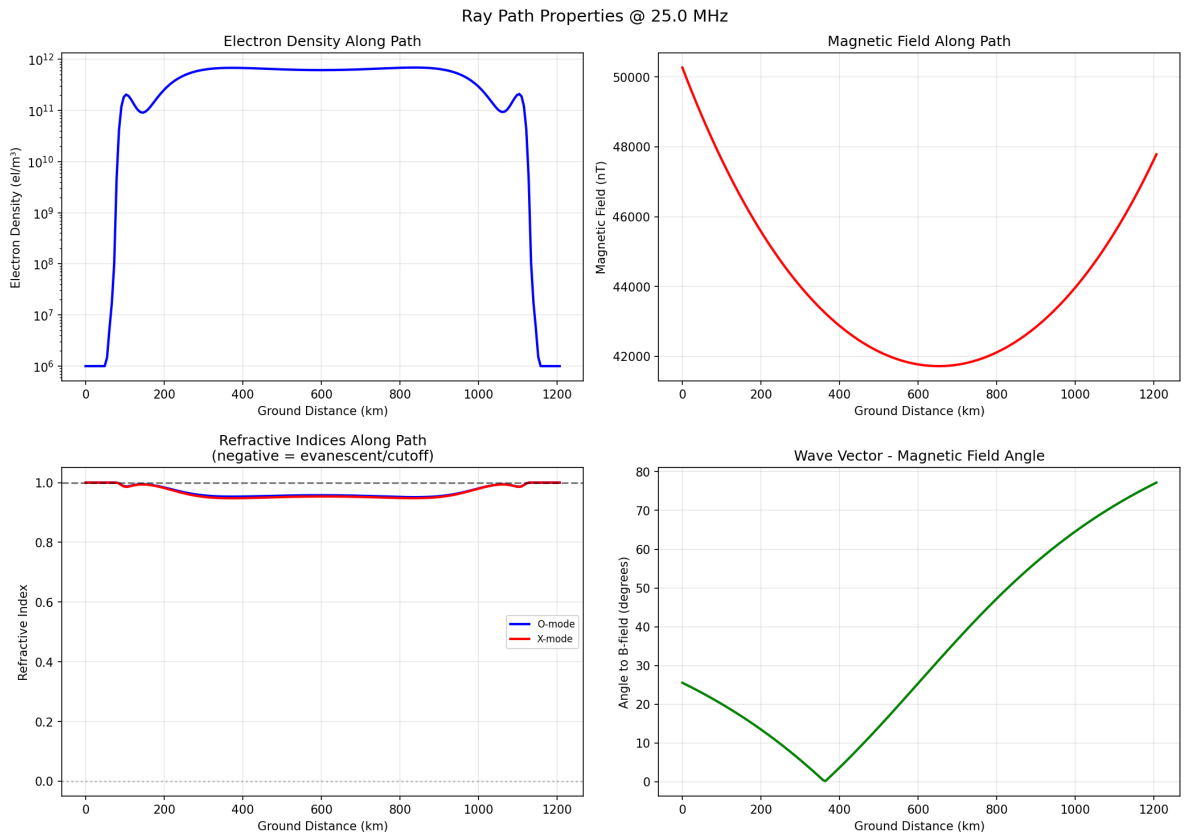

Figure 3: Ray details showing electron density profile, magnetic field strength, plasma and gyro frequencies, and O/X refractive indices along the path. Red shading indicates evanescent (cutoff) regions.

5.3.4 3D Rays Plot

The 3D rays plot (Figure 4) provides spatial visualization:

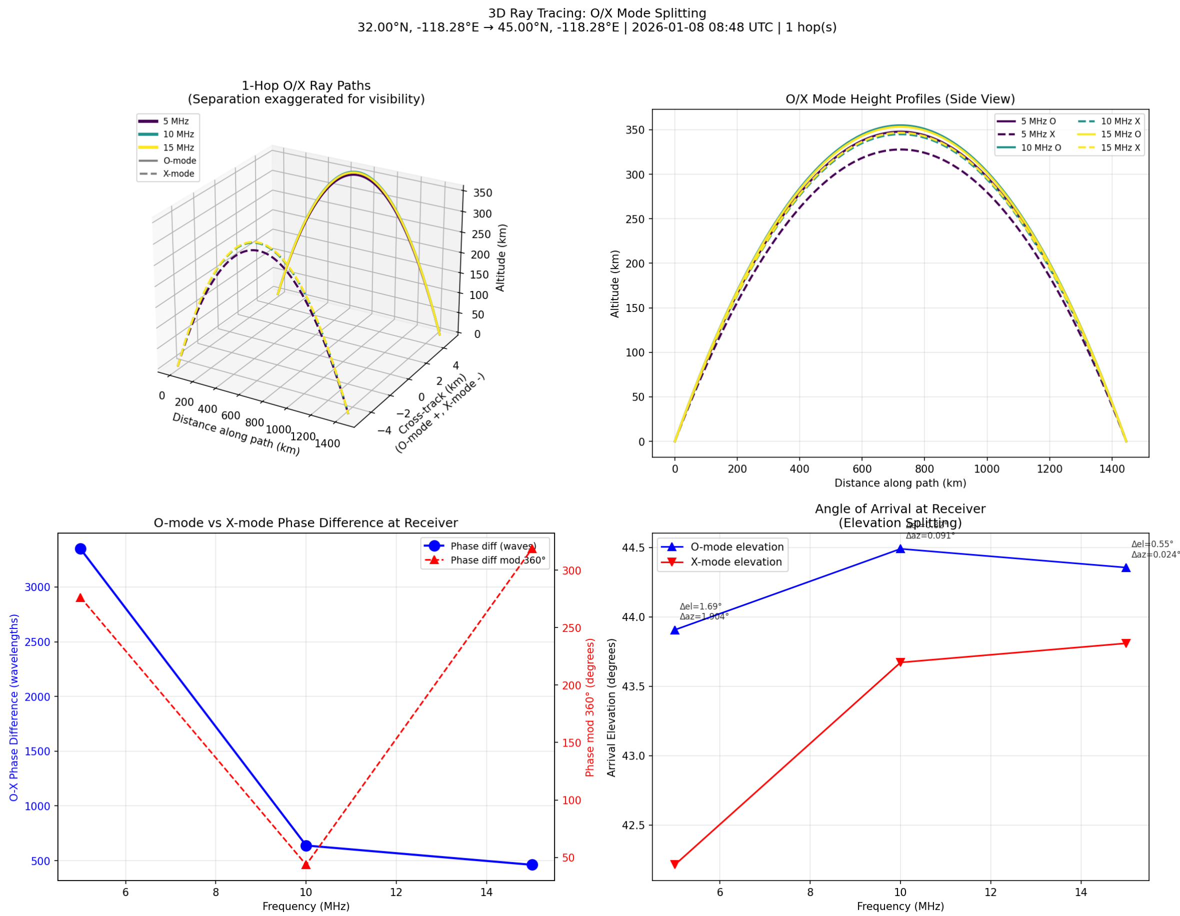

Figure 4: 3D visualization of O-mode (blue) and X-mode (red) ray paths, along with phase difference and arrival angles.

5.3.5 Receiver Polarization Plot

The receiver polarization plot (Figure 5) shows the polarization ellipse at the receiver:

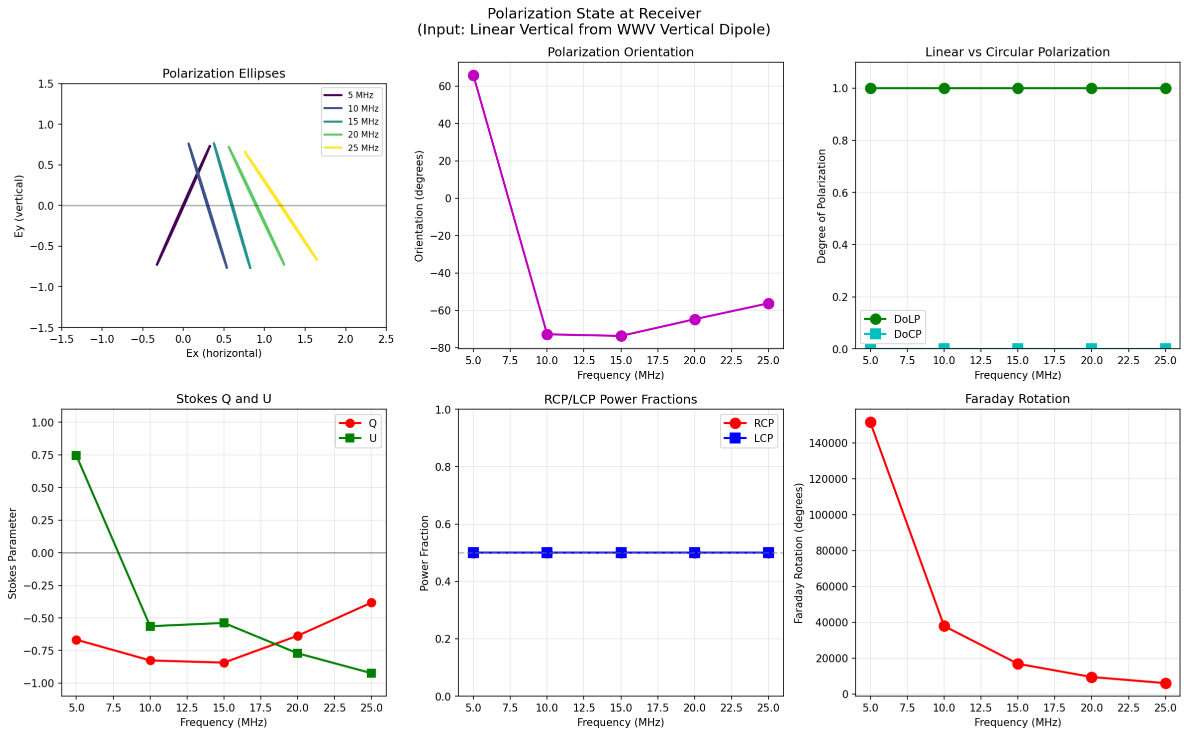

Figure 5: Receiver polarization state showing the polarization ellipse for each frequency after Faraday rotation.

5.3.6 Angular Splitting Plot

The angular splitting plot (Figure 6) shows O/X mode angular separation:

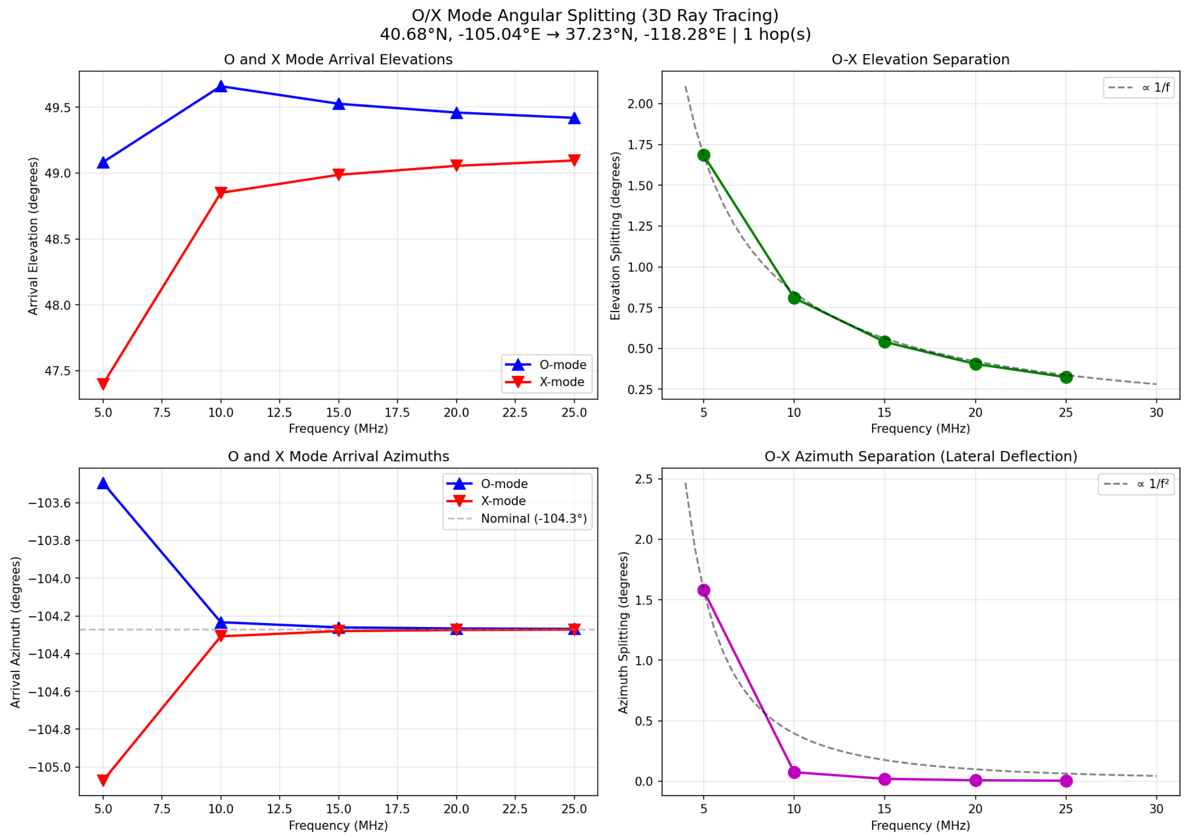

Figure 6: O/X mode angular splitting showing elevation comparison, elevation splitting, azimuth comparison, and azimuth splitting. The red dashed line indicates the MUF.

6 Physical Background

6.1 Magnetoionic Theory

When a radio wave enters the ionosphere, it splits into two characteristic modes due to the presence of free electrons and the Earth's magnetic field:

- **Ordinary (O) mode**: Refractive index closer to 1; penetrates deeper before reflecting; left-hand circular polarization (in Northern Hemisphere)
- **Extraordinary (X) mode**: Lower refractive index; reflects at lower altitude; right-hand circular polarization

The refractive indices are given by the Appleton-Hartree equation:

$$n^2 = 1 - \frac{X}{1 - \frac{Y_T^2}{2(1-X)} \pm \sqrt{\frac{Y_T^4}{4(1-X)^2} + Y_L^2}} \tag{1}$$

where:

$$X = \left(\frac{f_p}{f}\right)^2 \quad \text{(plasma parameter)} \tag{2}$$

$$Y = \frac{f_g}{f} \quad \text{(gyro parameter)} \tag{3}$$

$$Y_L = Y \cos\theta \quad \text{(longitudinal component)} \tag{4}$$

$$Y_T = Y \sin\theta \quad \text{(transverse component)} \tag{5}$$

and f_p is the plasma frequency, f_g is the electron gyrofrequency, and θ is the angle between the wave vector and magnetic field.

6.2 Elevation Splitting

Because $n_O > n_X$ (O-mode refractive index is closer to 1), the O-mode wave penetrates higher into the ionosphere before reflecting. The height difference is approximately:

$$\Delta h \approx \frac{5 \times 10^{10}}{f} \text{ meters} \tag{6}$$

This results in different arrival elevation angles at the receiver.

6.3 Azimuth Splitting

The O and X modes experience different lateral deflections perpendicular to both the density gradient and the magnetic field direction. This causes the modes to arrive from slightly different azimuths.

6.4 Faraday Rotation and Output Polarization

6.4.1 Physical Mechanism

When linearly polarized radiation enters the ionosphere, it splits into two characteristic modes:

- **O-mode** (ordinary): Becomes left-hand circular polarization (LCP) in Northern Hemisphere
- **X-mode** (extraordinary): Becomes right-hand circular polarization (RCP) in Northern Hemisphere

For linear input, the power splits **equally** between O and X modes (50% each) *at the transmitter.*

6.4.2 Phase Difference and Faraday Rotation

The O and X modes propagate with different phase velocities. The phase difference accumulated over the path is:

$$\delta = \phi_O - \phi_X = 2\Omega \tag{7}$$

where Ω is the Faraday rotation angle:

$$\Omega = \frac{2.365 \times 10^4}{f^2} \int N_e B_\parallel \, ds \quad \text{(radians)} \tag{8}$$

with N_e = electron density (el/m^3), $B_\parallel$ = magnetic field component along ray (T), and f = frequency (Hz).

6.4.3 Differential O/X Losses

The O and X modes experience **different losses** during propagation:

1. **Path length difference**: O-mode reflects at higher altitude (larger refractive index $n_O \approx 1$), resulting in a slightly longer path than X-mode.

2. **D-region absorption**: The absorption coefficient differs:

$$\kappa_O \propto \frac{f^2}{f^2 - f_H^2} \tag{9}$$

$$\kappa_X \propto \frac{f^2}{f^2 + f_H f \cos\theta} \tag{10}$$

   where $f_H \approx 1.4$ MHz is the electron gyrofrequency. X-mode typically absorbs 10–30% more than O-mode.

3. **Collisional damping**: Slightly different damping rates in the magnetized plasma.

6.4.4 Output Polarization: Elliptical Due to Differential Loss

At the receiver, the O and X modes have *unequal amplitudes* A_O and A_X:

$$\mathbf{E}_O = \frac{A_O}{\sqrt{2}}(\hat{x} + i\hat{y})e^{i\phi_O} \quad \text{(LCP)} \tag{11}$$

$$\mathbf{E}_X = \frac{A_X}{\sqrt{2}}(\hat{x} - i\hat{y})e^{i\phi_X} \quad \text{(RCP)} \tag{12}$$

The Stokes parameters for the recombined wave are:

$$I = A_O^2 + A_X^2 \tag{13}$$

$$Q = 2A_O A_X \cos(\delta) \tag{14}$$

$$U = 2A_O A_X \sin(\delta) \tag{15}$$

$$V = A_X^2 - A_O^2 \quad \text{(non-zero when } A_O \neq A_X) \tag{16}$$

Key result: Because X-mode typically suffers more loss than O-mode ($A_X < A_O$), the output has:

- $V < 0$: Net LCP component (excess O-mode power)
- Ellipticity angle: $\chi = \frac{1}{2}\arcsin(V/I)$
- The polarization is *slightly elliptical*, not purely linear

Normalized (to $I = 1$):

$$Q = \frac{2A_O A_X}{A_O^2 + A_X^2}\cos(\delta) \tag{17}$$

$$U = \frac{2A_O A_X}{A_O^2 + A_X^2}\sin(\delta) \tag{18}$$

$$V = \frac{A_X^2 - A_O^2}{A_O^2 + A_X^2} \tag{19}$$

6.4.5 Practical Implications

- At HF, δ is typically thousands to millions of radians
- The effective polarization angle $\theta = \frac{1}{2}\arctan(U/Q) \mod 180°$ appears "random"
- Small changes in electron density cause large polarization angle changes ("polarization fading")
- The ellipticity is small (typically $|\chi| < 5°$) but non-zero due to differential absorption
- Daytime paths show larger differential loss (more D-region absorption) than nighttime
- Lower frequencies have larger differential loss (absorption $\propto 1/f^{1.98}$)
- To isolate individual modes, use circular polarization feeds (RCP selects X-mode, LCP selects O-mode)

6.5 Maximum Usable Frequency (MUF)

The Maximum Usable Frequency is the highest frequency that will be reflected by the ionosphere for a given path geometry. Above the MUF, radio waves penetrate through the ionosphere rather than being reflected.

The MUF is calculated as:

$$\text{MUF} = f_0F_2 \cdot \sec(\phi) \tag{20}$$

where f_0F_2 is the critical frequency of the F2 layer and ϕ is the angle of incidence at the ionosphere. For oblique paths:

$$\sec(\phi) \approx \sqrt{1 + \left(\frac{D}{4h}\right)^2} \tag{21}$$

where D is the hop distance and h is the reflection height (300 km).

Key points:

- MUF varies with solar activity (higher SSN $\rightarrow$ higher MUF)
- MUF is higher during daytime than nighttime
- Longer hop distances have higher MUF (more oblique incidence)
- Multi-hop paths use per-hop distance for MUF calculation
- Frequencies above MUF are marked `ABOVE_MUF` in output

The simulator displays MUF in the title of the ray paths plot and adds a red dashed vertical line at the MUF frequency in frequency-dependent plots.

7 Worked Examples

7.1 Example 1: Short Path (354 km)

Scenario: Los Angeles to Owens Valley Radio Observatory

```
python3 multihop_3d.py -tx-lat 34.05 -tx-lon -118.24 \
-rx-lat 37.23 -rx-lon -118.28 \
-frequencies 5 10 15 \
-output example_short
```

Results:

Freq (MHz)	Distance	O Elev	X Elev	ΔElev	ΔAz
5	354 km	75.74°	74.91°	0.83°	0.47°
10	354 km	76.02°	75.63°	0.39°	0.02°
15	354 km	75.95°	75.69°	0.26°	0.01°

Key observations:

- Short paths have high elevation angles ($> 75°$)
- O/X splitting is modest ($< 1°$) but measurable
- Splitting decreases with increasing frequency

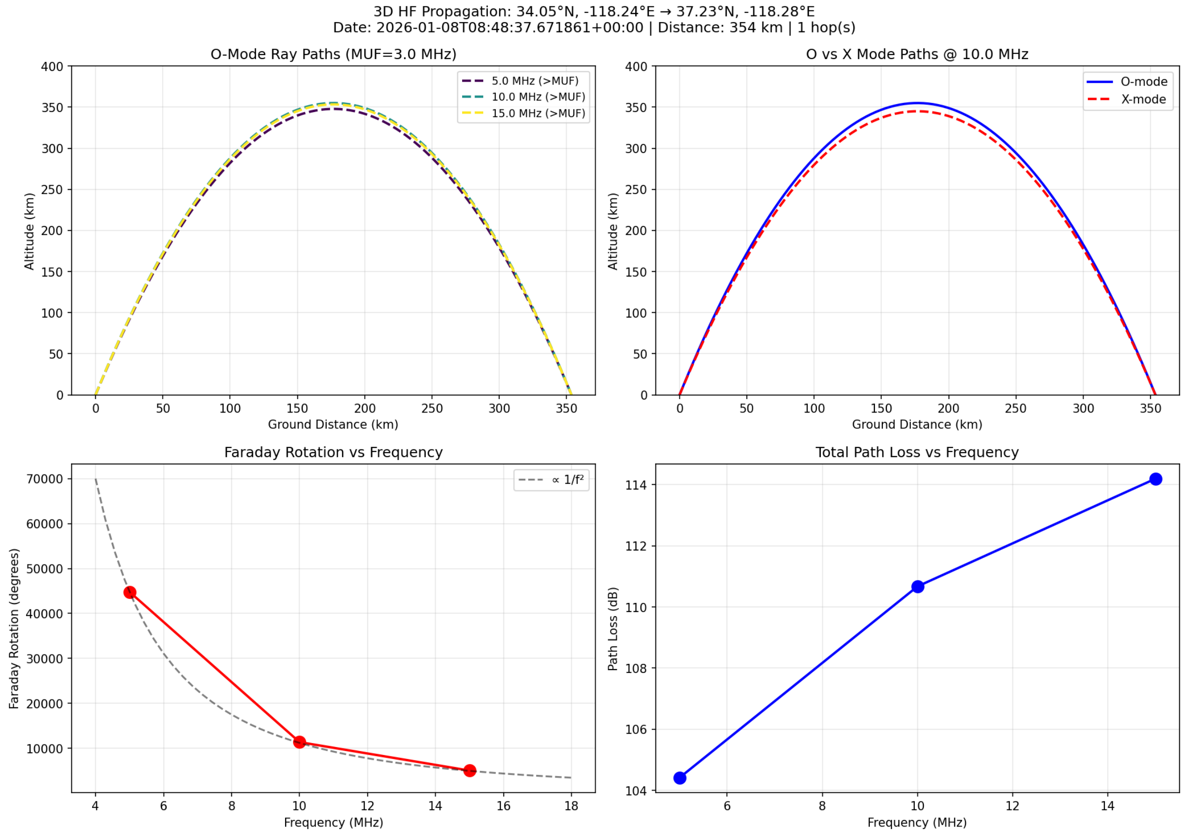

Figure 7: Short path (354 km) simulation results showing steep ray paths.

7.2 Example 2: Medium Path — WWV to OVRO (1206 km)

Scenario: Standard WWV reception at Owens Valley

```
python3 multihop_3d.py -frequencies 5 10 15 20 25 -output wwv_ovro
```

Results:

Freq (MHz)	h_O (km)	h_X (km)	O Elev (°)	X Elev (°)	ΔElev (°)	ΔAz (°)
5	347.8	327.8	49.08	47.40	1.69	1.58
10	355.0	345.0	49.66	48.85	0.81	0.08
15	353.3	346.7	49.53	48.99	0.54	0.02
20	352.5	347.5	49.46	49.05	0.41	0.01
25	352.0	348.0	49.42	49.10	0.32	0.004

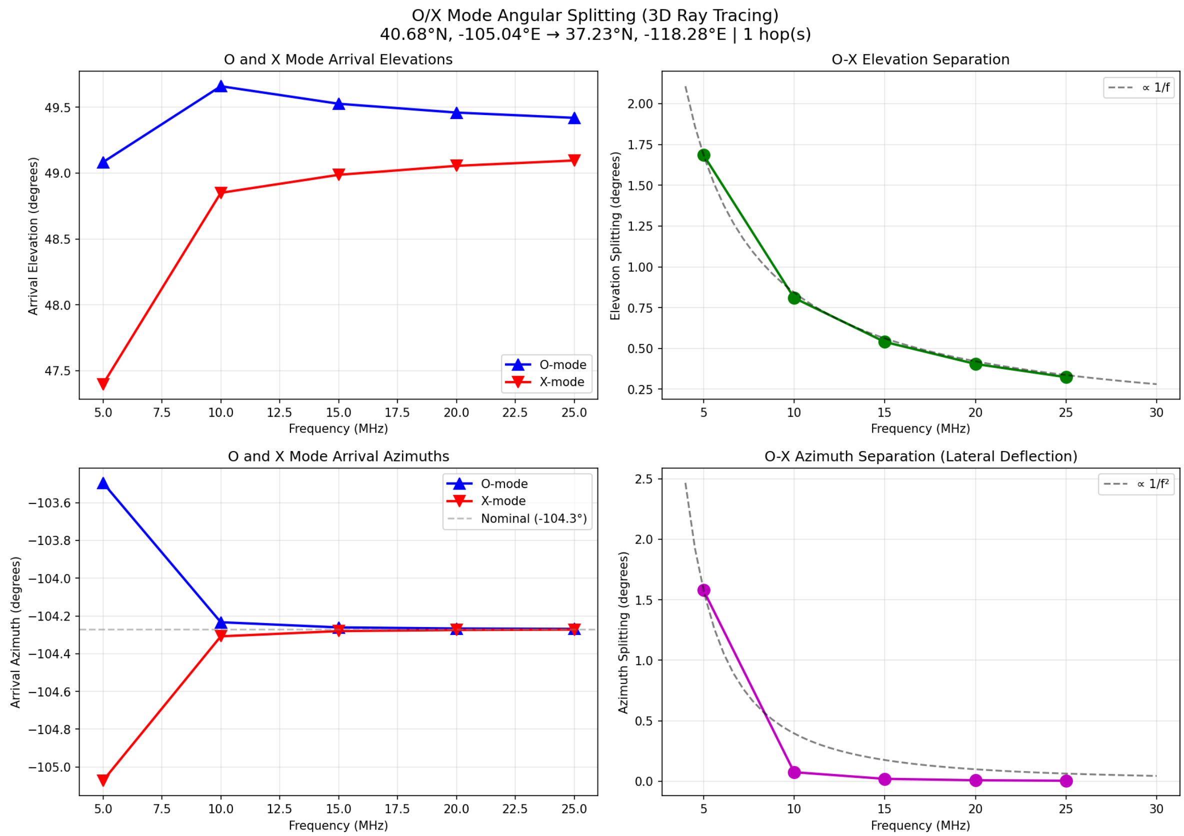

Figure 8: WWV to OVRO O/X angular splitting analysis.

7.3 Example 3: North-South Path (1446 km)

Scenario: Purely meridional path to compare with east-west propagation

```
python3 multihop_3d.py -tx-lat 32.0 -tx-lon -118.28 \
-rx-lat 45.0 -rx-lon -118.28 \
-frequencies 5 10 15 \
-output example_northsouth
```

Results:

Freq (MHz)	Bearing	O Elev	X Elev	ΔElev	ΔAz
5	0.0° (N)	43.91°	42.21°	1.69°	1.90°
10	0.0° (N)	44.49°	43.67°	0.82°	0.09°
15	0.0° (N)	44.35°	43.81°	0.55°	0.02°

Key observation: North-south paths show similar elevation splitting but different azimuth splitting patterns due to the magnetic field geometry.

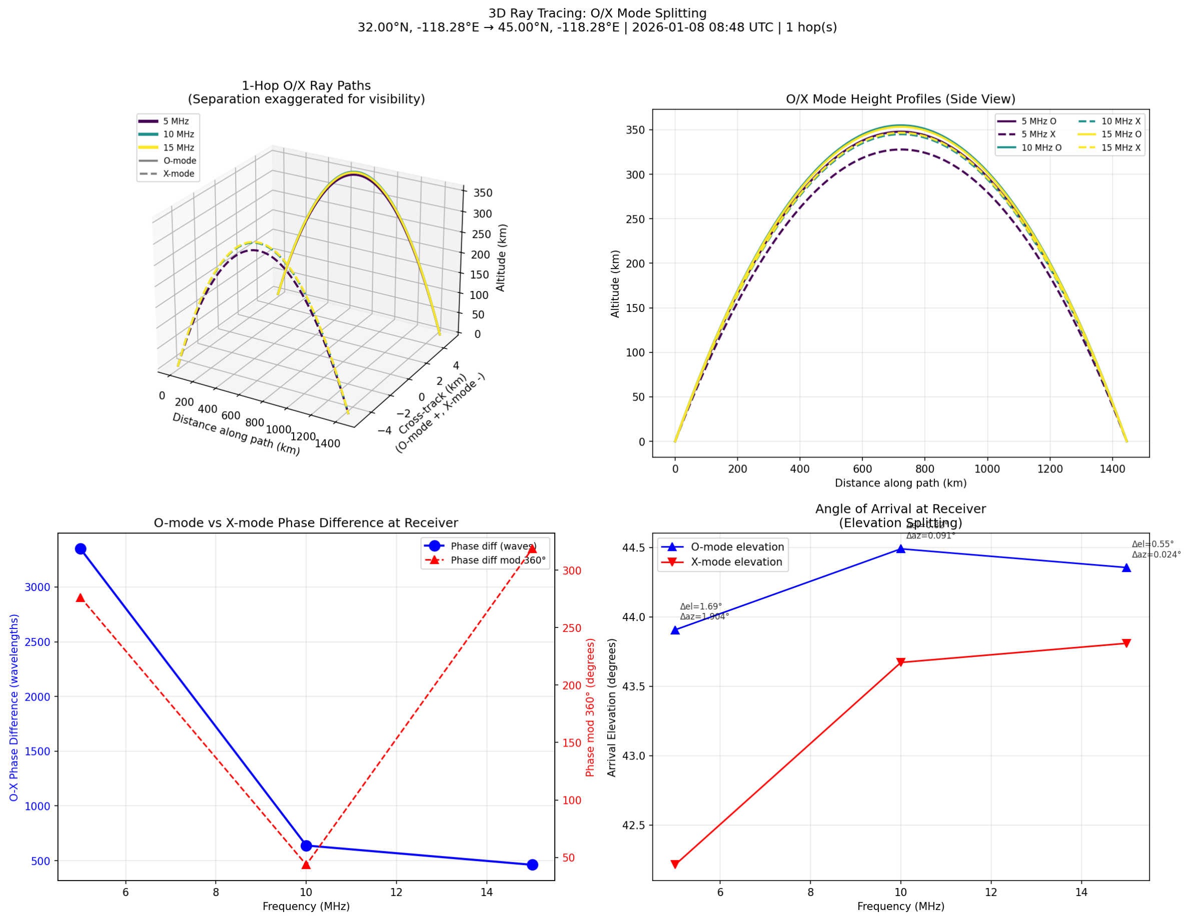

Figure 9: North-south path (1446 km) 3D ray visualization.

7.4 Example 4: Long Path with Multiple Hops — CHU to OVRO (3621 km)

Scenario: Canadian time signal station CHU received at Owens Valley

For a 3621 km path, single-hop propagation requires very low elevation angles. Compare 1, 2, and 3 hop modes:

```
# 1-hop (marginal) python3 multihop_3d.py -tx-lat 45.29 -tx-lon -75.75 \
-rx-lat 37.23 -rx-lon -118.28 \
-frequencies 3.33 7.85 14.67 -hops 1 \
-output chu_ovro_1hop
# 2-hop (recommended) python3 multihop_3d.py -tx-lat 45.29 -tx-lon -75.75 \
-rx-lat 37.23 -rx-lon -118.28 \
-frequencies 3.33 7.85 14.67 -hops 2 \
-output chu_ovro_2hop
# 3-hop python3 multihop_3d.py -tx-lat 45.29 -tx-lon -75.75 \
-rx-lat 37.23 -rx-lon -118.28 \
-frequencies 3.33 7.85 14.67 -hops 3 \
-output chu_ovro_3hop
```

Comparison at 7.85 MHz:

Hops	Hop Dist (km)	Elevation (°)	ΔElev (°)	ΔAz (°)	Faraday (°)	Loss (dB)
1	3621	20.2	0.71	2.42	644,594	129.8
2	1811	36.3	1.05	1.04	519,460	138.4
3	1207	47.8	1.09	1.75	446,284	147.2

Key observations:

- Single-hop over 3621 km has low elevation (20°) — marginal propagation
- 2-hop gives moderate elevation (36°) with reasonable loss
- 3-hop has high elevation (48°) but +18 dB additional loss
- Elevation splitting *increases* with more hops
- 2-hop is the best compromise for this path

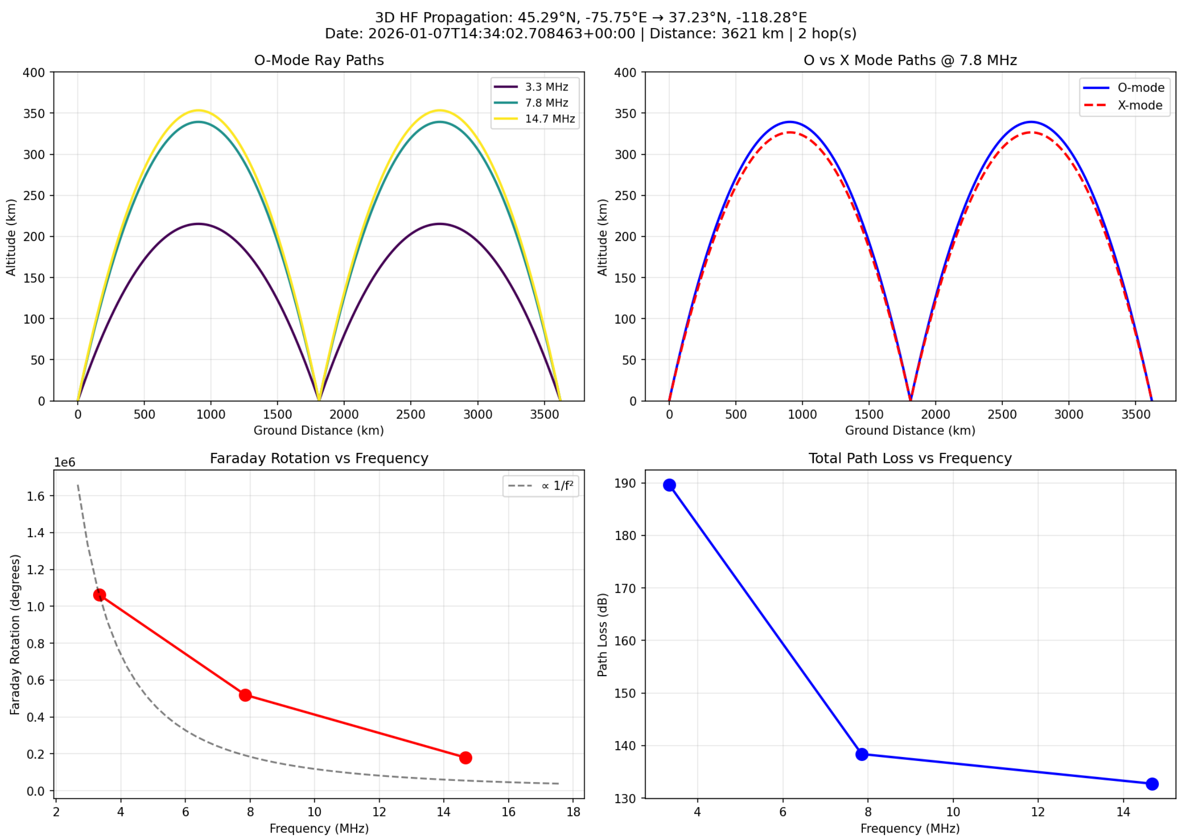

Figure 10: CHU to OVRO 2-hop propagation overview.

7.5 Example 5: Day vs Night Comparison

Scenario: Compare daytime and nighttime ionospheric conditions

7.5.1 Nighttime (06:00 UTC)

```
python3 multihop_3d.py -date 2026-01-07T06:00:00+00:00 \
-frequencies 5 10 15 \
-output example_night
```

At 06:00 UTC (11 PM local time in Colorado), the ionosphere is in nighttime conditions with reduced electron density. The MUF is low (typically 3–5 MHz).

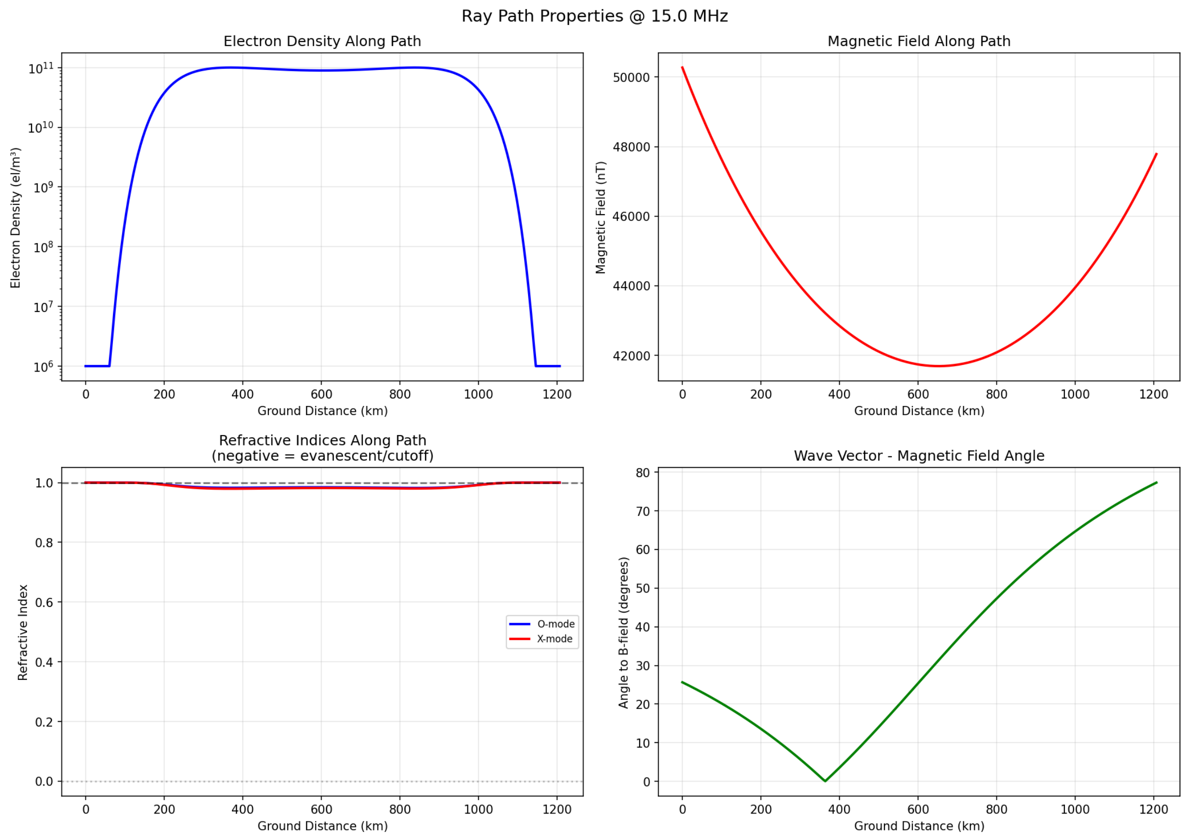

Figure 11: Nighttime propagation showing reduced electron density along path. Note the lower peak N_e and lower plasma frequency compared to daytime.

7.5.2 Daytime (19:00 UTC)

```
python3 multihop_3d.py -date 2026-01-07T19:00:00+00:00 \
-frequencies 5 10 15 \
-output example_day
```

At 19:00 UTC (noon local time in Colorado), the ionosphere has maximum electron density due to solar illumination. The MUF is higher (typically 10–15 MHz for this path).

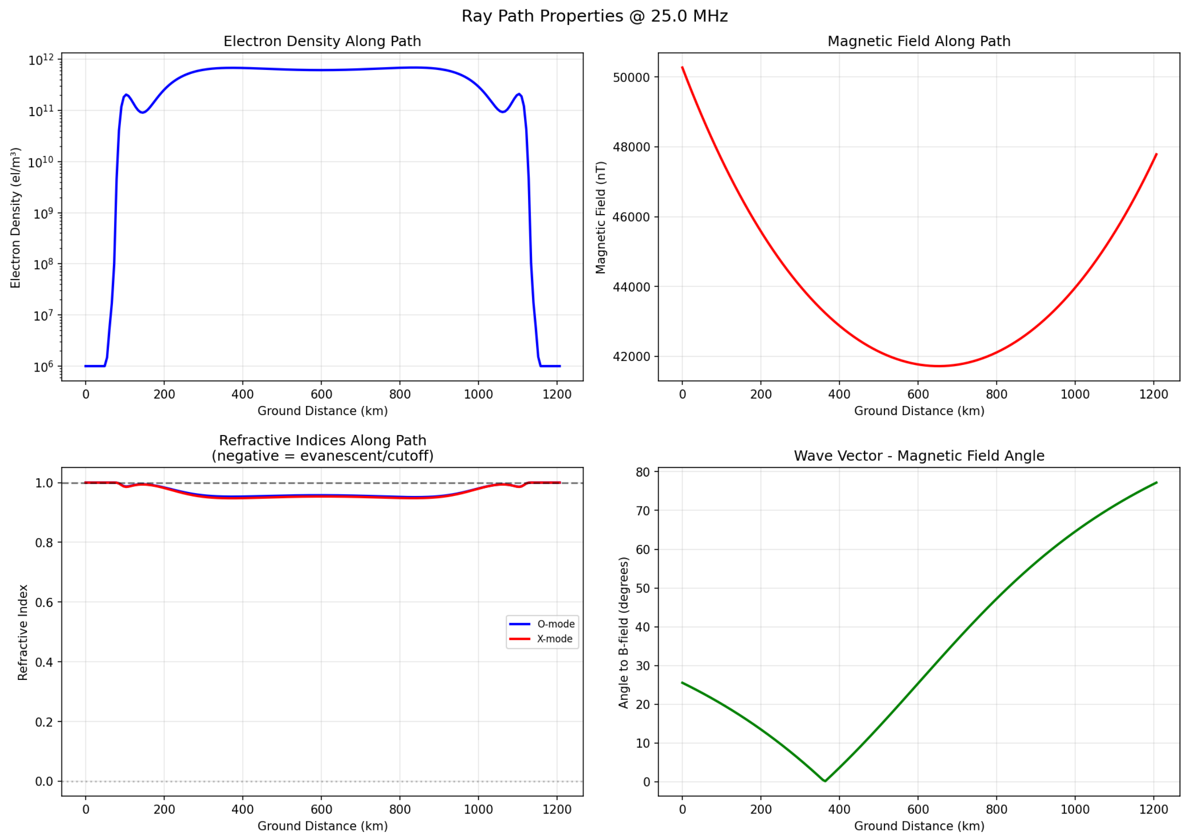

Figure 12: Daytime propagation showing enhanced electron density along path. Note the higher peak N_e and higher plasma frequency compared to nighttime.

7.5.3 Key Differences

Table 3: Day vs Night comparison for WWV to OVRO path

Parameter	Nighttime (06:00 UTC)	Daytime (19:00 UTC)
MUF	∼4 MHz	∼11 MHz
Peak N_e	Lower	Higher
Faraday rotation	Lower	Higher
D-layer absorption	None	Present
5 MHz status	ABOVE_MUF	OK
10 MHz status	ABOVE_MUF	OK
15 MHz status	ABOVE_MUF	ABOVE_MUF

7.6 Example 6: Polarization Effects — WWV to Boston (2816 km)

Scenario: Demonstrate dramatic differential O/X absorption on a daytime 2-hop path

This example shows why antenna polarization selection matters enormously for HF reception.

```
# Daytime (19:00 UTC = 2 PM Eastern) python3 multihop_3d.py -tx-lat 40.68 -tx-lon
-105.04 \
-rx-lat 42.36 -rx-lon -71.06 \
-frequencies 2.5 5 10 15 20 25 \
-hops 2 -date 2026-01-07T19:00:00 \
-output wwv_boston_day
# Nighttime (06:00 UTC = 1 AM Eastern) python3 multihop_3d.py -tx-lat 40.68 -tx-lon
-105.04 \
-rx-lat 42.36 -rx-lon -71.06 \
-frequencies 2.5 5 10 15 20 25 \
-hops 2 -date 2026-01-07T06:00:00 \
-output wwv_boston_night
```

7.6.1 Daytime Polarization Results

Table 4: WWV to Boston daytime polarization (19:00 UTC)

Freq (MHz)	Loss_O (dB)	Loss_X (dB)	ΔLoss (dB)	LCP/RCP (dB)	Ellip (deg)	Recommendation
2.5	426	584	158	>100	−45	**LCP essential**
5.0	232	246	14	+14	−34	**LCP strongly preferred**
10.0	157	160	3.4	+3.4	−11	LCP preferred
15.0	142	144	1.6	+1.6	−5	LCP slight advantage
20.0	138	139	0.9	+0.9	−3	LCP marginal advantage
25.0	137	137	0.6	+0.6	−2	Either acceptable

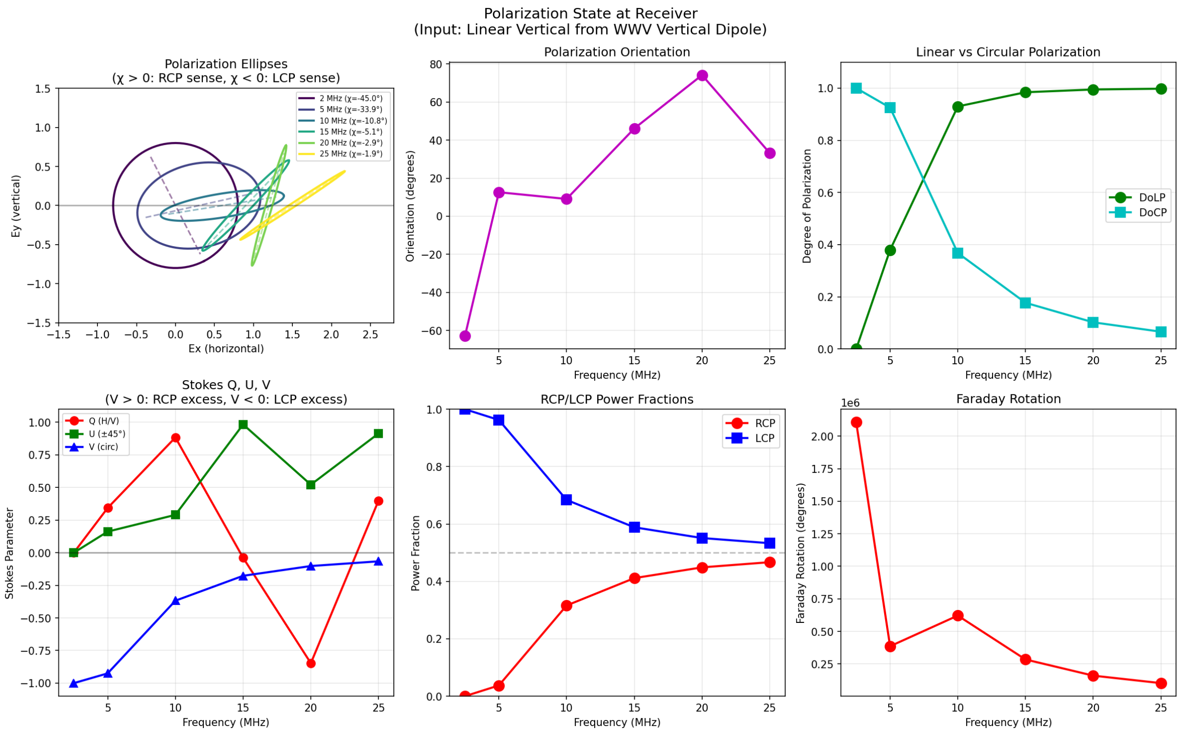

Figure 13: WWV to Boston daytime received polarization. At 2.5 MHz, the signal is nearly pure LCP due to complete X-mode absorption. Higher frequencies show progressively more linear polarization.

Key insight: At 2.5 MHz during daytime, the X-mode (RCP) is effectively *completely absorbed*. An RCP antenna would receive *nothing*, while an LCP antenna captures the full O-mode signal. This 158 dB differential represents the difference between signal and no signal.

7.6.2 Nighttime Polarization Results

Table 5: WWV to Boston nighttime polarization (06:00 UTC)

Freq (MHz)	Loss_O (dB)	Loss_X (dB)	ΔLoss (dB)	LCP/RCP (dB)	Ellip (deg)	Recommendation
2.5	110	110	−0.2	−0.2	+0.7	Either acceptable
5.0	117	117	−0.1	−0.1	+0.4	Either acceptable
10.0	123	123	−0.1	−0.1	+0.2	Either acceptable

At night, D-region absorption is essentially zero (the D-layer disappears without solar illumination). Both O and X modes arrive with nearly equal amplitude, resulting in linear polarization. Antenna handedness does not matter for SNR.

7.7 Example 7: CHU to OVRO (3621 km)

Scenario: Canadian time station CHU frequencies on a long transcontinental path

```
# Daytime python3 multihop_3d.py -tx-lat 45.2917 -tx-lon -75.7533 \
-rx-lat 37.23 -rx-lon -118.28 \
-frequencies 3.33 7.85 14.67 \
-hops 2 -date 2026-01-07T19:00:00 \
-output chu_ovro_day
# Nighttime python3 multihop_3d.py -tx-lat 45.2917 -tx-lon -75.7533 \
-rx-lat 37.23 -rx-lon -118.28 \
-frequencies 3.33 7.85 14.67 \
-hops 2 -date 2026-01-07T06:00:00 \
-output chu_ovro_night
```

Table 6: CHU to OVRO daytime polarization (19:00 UTC)

Freq (MHz)	Loss_O (dB)	Loss_X (dB)	ΔLoss (dB)	LCP/RCP (dB)	Ellip (deg)	Assessment
3.33	339	404	65	>60	−45	Pure LCP
7.85	177	182	5.4	+5.4	−17	Strong LCP
14.67	145	147	1.7	+1.7	−5.5	Moderate LCP

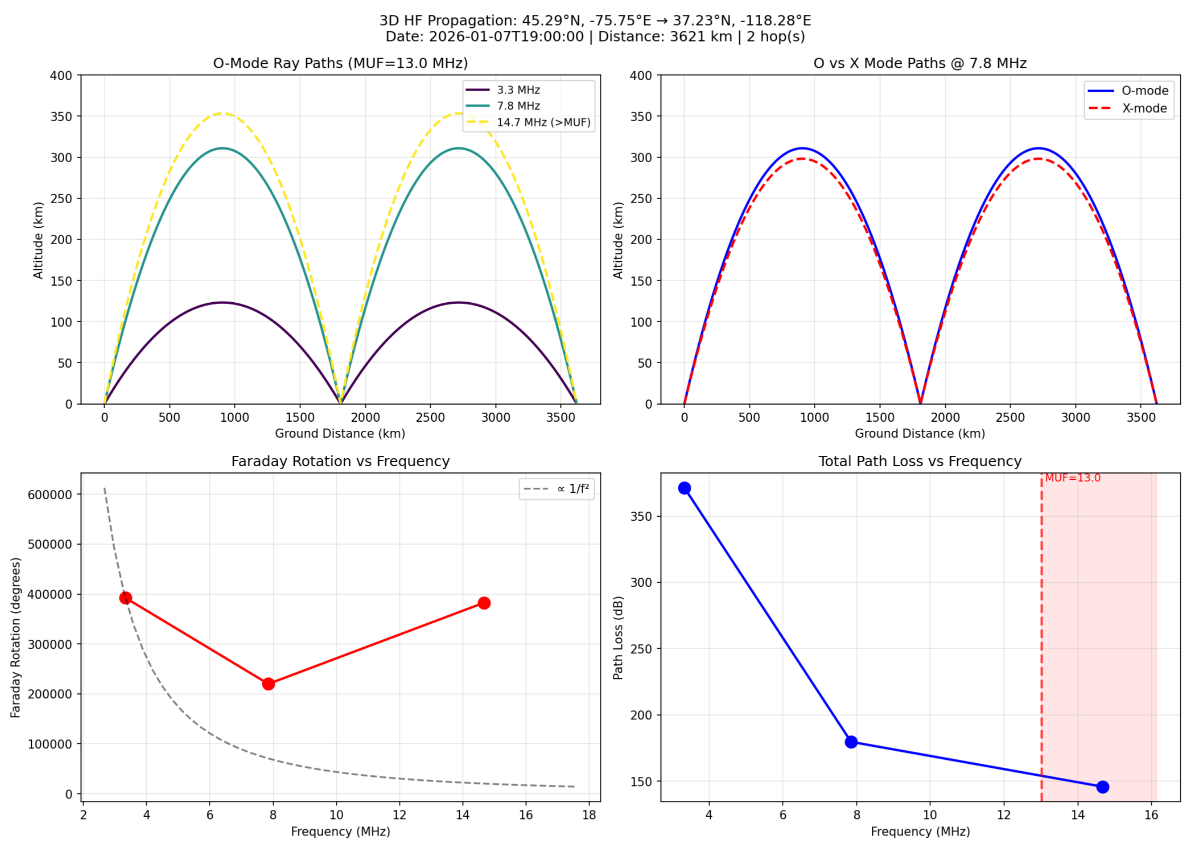

Figure 14: CHU to OVRO daytime propagation overview. The 2-hop path geometry results in moderate elevation angles (∼35–38°).

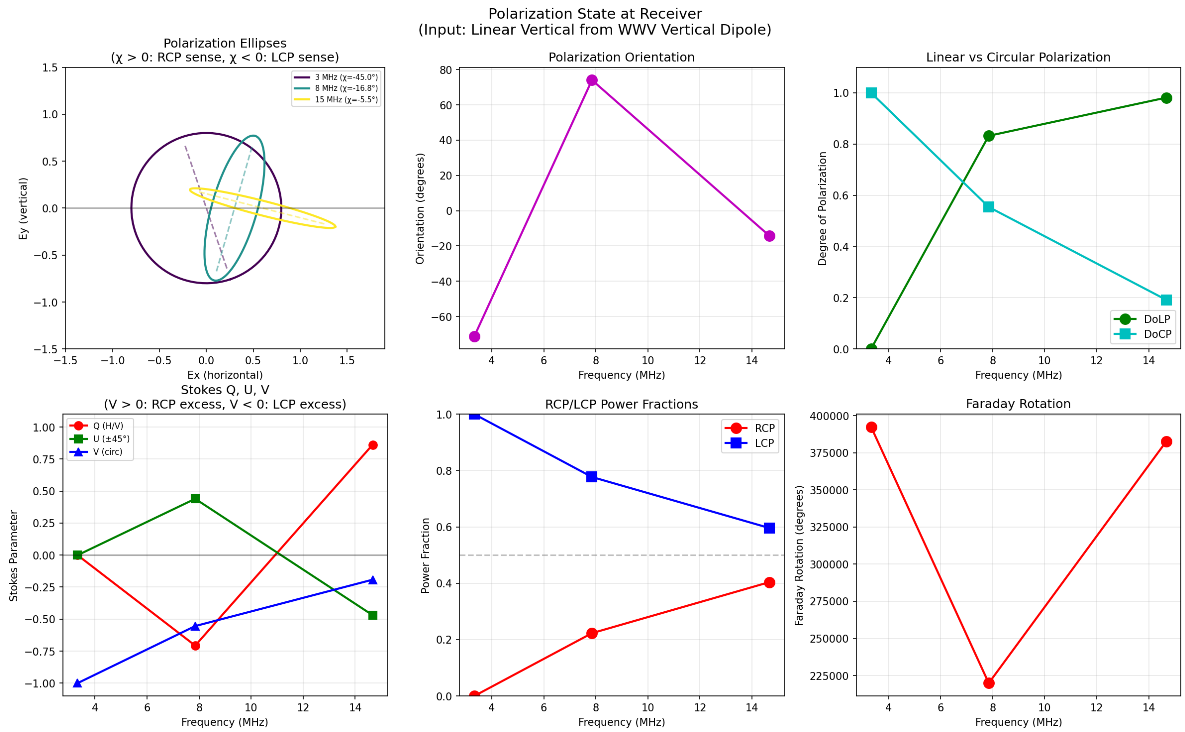

Figure 15: CHU to OVRO daytime received polarization. Note the nearly pure LCP at 3.33 MHz due to 65 dB differential absorption.

8 Interpreting Results

8.1 Elevation Splitting

The elevation splitting $\Delta\theta_{el}$ between O and X modes follows an approximate $1/f$ scaling:

$$\Delta\theta_{el} \propto \frac{1}{f} \tag{22}$$

Typical values:

- 5 MHz: 1–2°
- 10 MHz: 0.5–1°
- 15 MHz: 0.3–0.6°
- 25 MHz: 0.2–0.4°

8.2 Azimuth Splitting

The azimuth splitting $\Delta\theta_{az}$ follows an approximate $1/f^2$ scaling and depends strongly on path geometry relative to the magnetic field:

$$\Delta\theta_{az} \propto \frac{1}{f^2} \tag{23}$$

East-west paths tend to show larger azimuth splitting than north-south paths due to the orientation of the Earth's magnetic field.

8.3 Faraday Rotation

Faraday rotation is typically very large at HF frequencies:

- 5 MHz: $> 100{,}000°$ (hundreds of full rotations)
- 10 MHz: $\sim 25{,}000°$
- 15 MHz: $\sim 10{,}000°$
- 25 MHz: $\sim 4{,}000°$

This means linear polarization is effectively randomized at HF. Circular polarization (RCP or LCP) should be used to select specific modes.

8.4 Reflection Heights

Typical reflection heights:

- E-layer: 100–120 km (lower frequencies, daytime)
- F2-layer: 250–350 km (higher frequencies, primary for HF)
- O-mode reflects 10–30 km higher than X-mode

8.5 Path Loss

Total path loss includes:

1. Free-space spreading loss ($\propto f^2 d^2$)
2. D-region absorption (daytime, $\propto 1/f^2$)
3. Additional loss per hop (∼6–10 dB)

9 Understanding the Output Plots

9.1 Overview Plot (`_overview.png`)

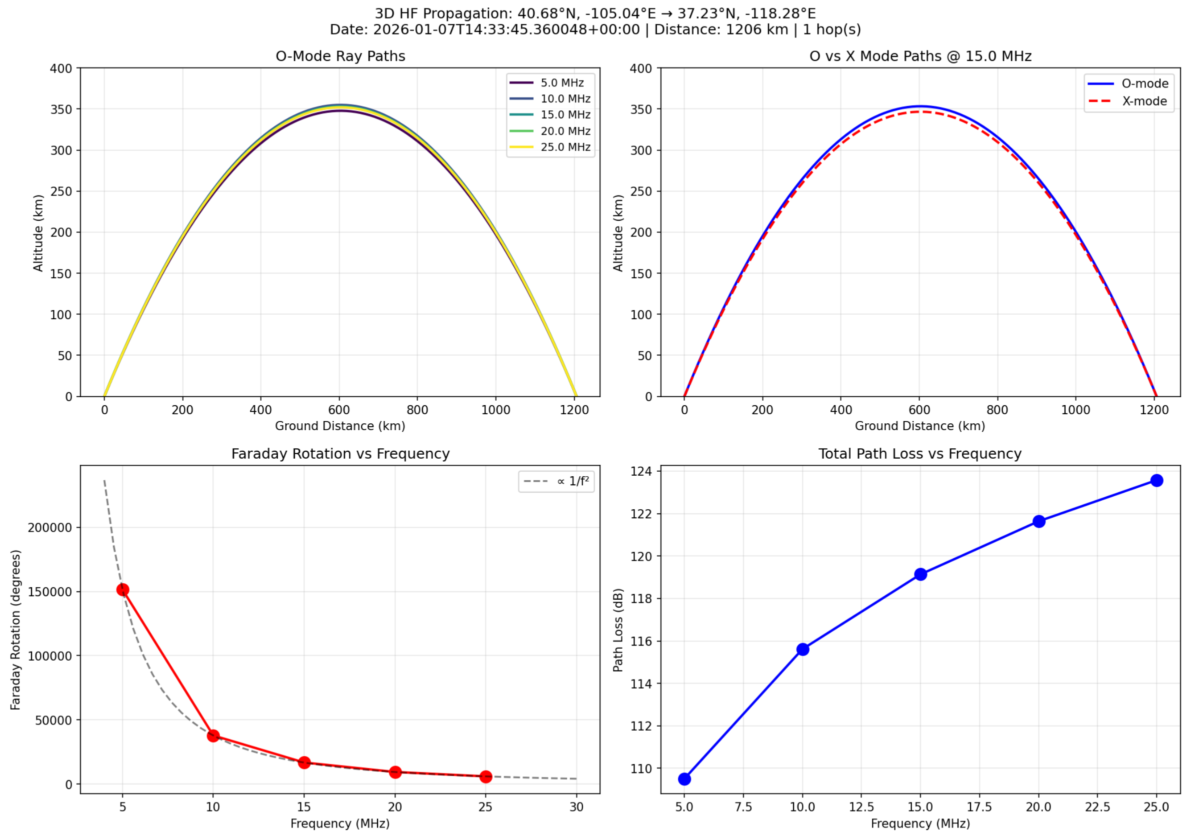

Figure 16: Annotated overview plot.

1. **Top-left**: O-mode ray paths for all frequencies, showing altitude vs. ground distance
2. **Top-right**: O vs X mode comparison for middle frequency
3. **Bottom-left**: Faraday rotation vs frequency with $1/f^2$ reference line
4. **Bottom-right**: Total path loss vs frequency

9.2 Angular Splitting Plot (`_angular_splitting.png`)

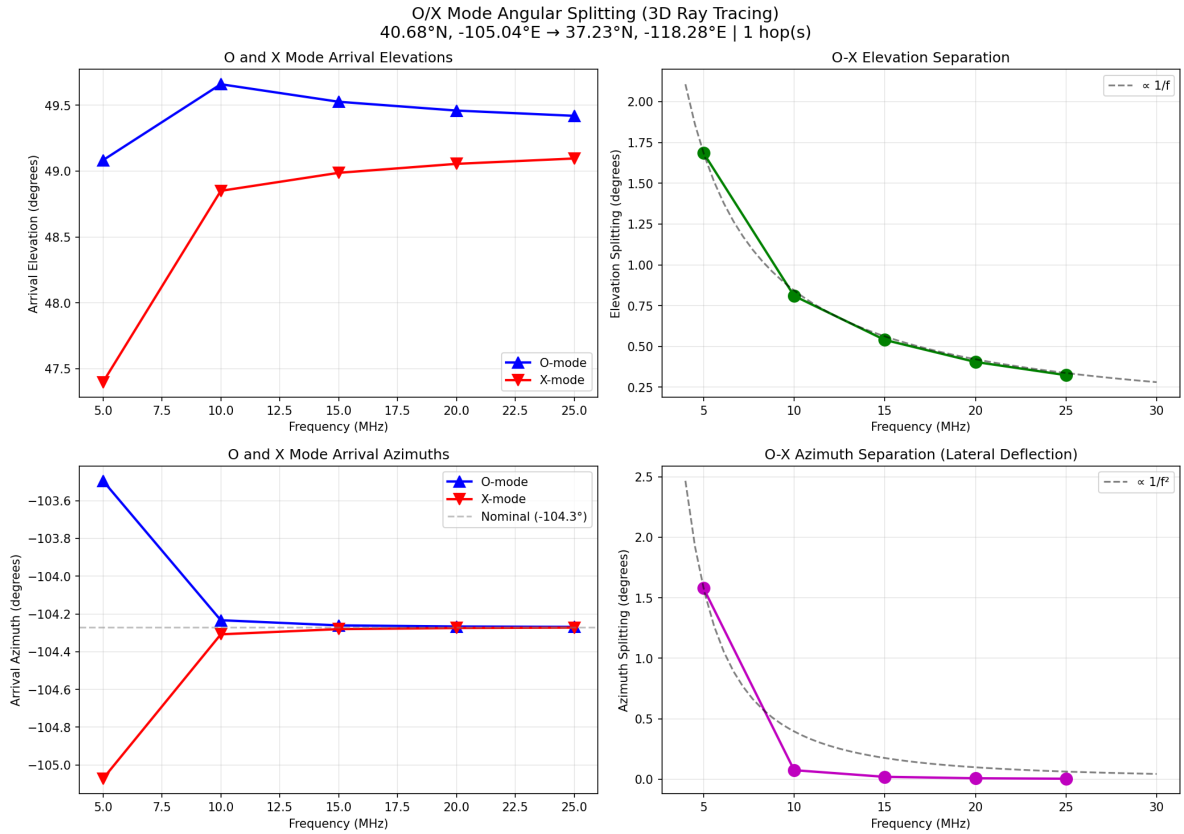

Figure 17: Annotated angular splitting plot.

1. **Top-left**: O and X mode arrival elevations
2. **Top-right**: Elevation splitting with $1/f$ reference
3. **Bottom-left**: O and X mode arrival azimuths
4. **Bottom-right**: Azimuth splitting with $1/f^2$ reference

9.3 3D Rays Plot (`_3d_rays.png`)

1. **Top-left**: 3D view of ray paths (O solid, X dashed)
2. **Top-right**: Side view (altitude profile)
3. **Bottom-left**: O-X phase difference
4. **Bottom-right**: Angle of arrival with splitting annotations

9.4 Ray Details Plot (`_ray_details.png`)

Shows physical quantities along the ray path:

1. Electron density N_e (el/m^3)

2. Magnetic field strength B (nT)
3. Refractive indices n_O and n_X (negative = evanescent)
4. Wave vector - magnetic field angle

9.5 Polarization Plots

The polarization analysis is split across three complementary plots, which are of particular interest for interferometric and polarimetric applications.

9.5.1 Main Polarization Plot (`_polarization.png`)

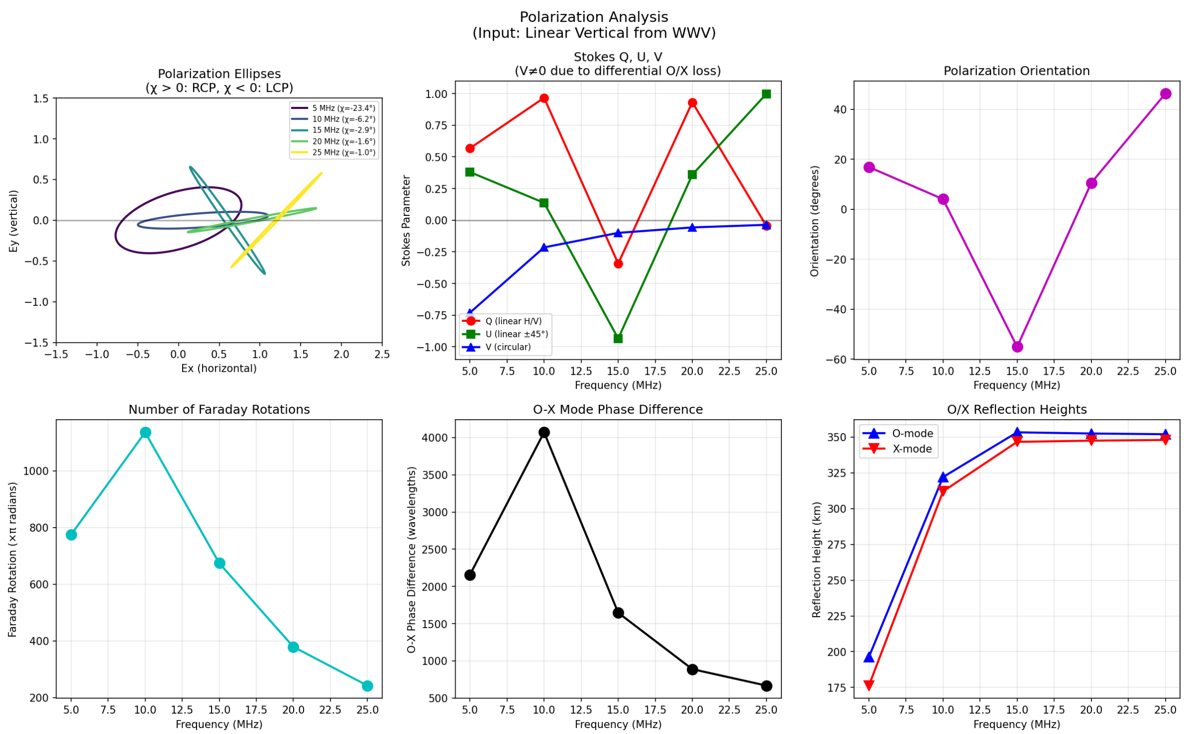

Figure 18: Polarization analysis plot showing frequency-dependent polarization state.

This 6-panel plot shows:

1. **Polarization ellipses**: Visual representation of polarization state for each frequency
2. **Stokes parameters (I,Q,U,V)**: Quantitative polarization description
3. **Orientation angle** ψ: Ellipse tilt from horizontal
4. **Ellipticity** χ: Shape of polarization ellipse ($\pm 45°$ = circular)
5. **O/X reflection heights**: Comparison of mode reflection altitudes
6. **Additional diagnostics**: Path geometry and summary

9.5.2 Receiver Polarization Plot (`_rx_polarization.png`)

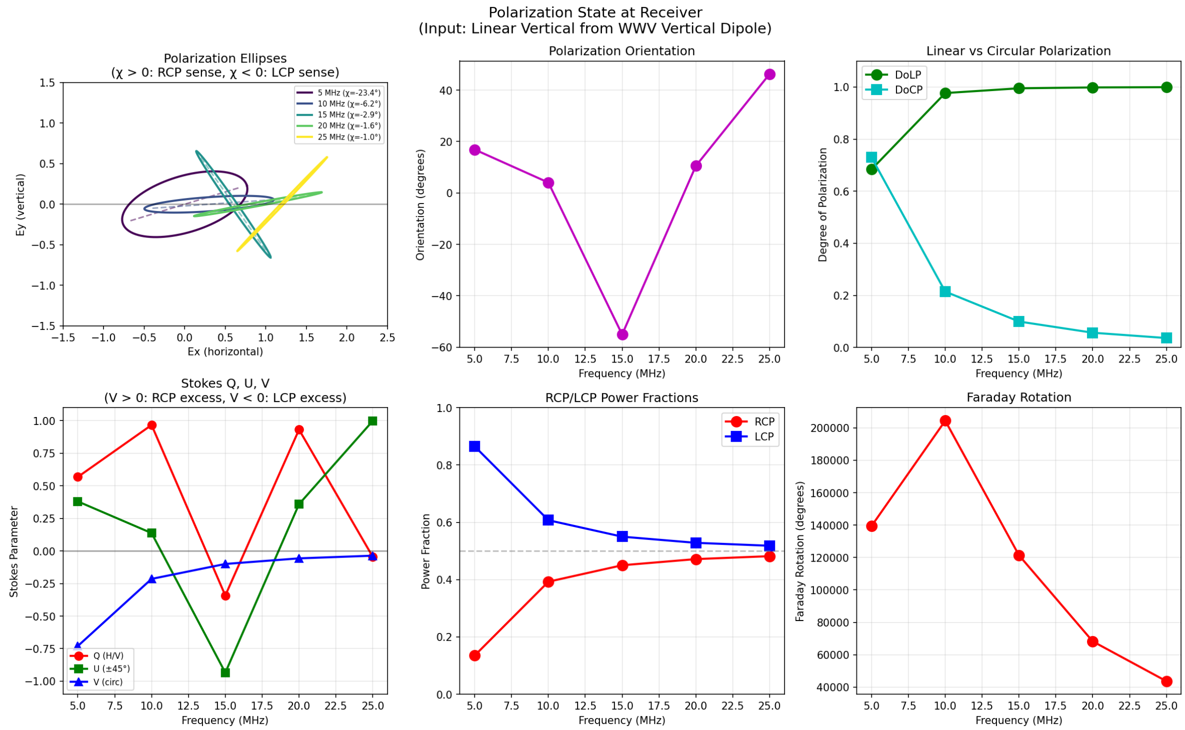

Figure 19: Receiver polarization state showing ellipses after Faraday rotation.

This plot shows the actual polarization ellipse at the receiver for each frequency. Key features:

- Ellipse orientation after Faraday rotation
- Degree of Linear Polarization (DoLP)
- Degree of Circular Polarization (DoCP)
- Axial ratio for each frequency

10 Best Practices

10.1 Choosing Number of Hops

Table 7: Recommended number of hops by path length

Path Length	Recommended Hops
<500 km	1
500–1500 km	1
1500–3000 km	1–2
3000–4500 km	2
>4500 km	2–3

Rule of thumb: Maximum single-hop distance is approximately 3500–4000 km with F2-layer reflection.

10.2 Frequency Selection

- Always check the Maximum Usable Frequency (MUF) in output
- Frequencies above MUF may not propagate reliably
- Lower frequencies have larger O/X splitting but more absorption
- 10–15 MHz is often optimal for daytime propagation

10.3 Time Considerations

- Use `-date` to simulate specific times
- Daytime: Higher electron densities, more D-region absorption
- Nighttime: Lower densities, less absorption, F-layer only
- Sunrise/sunset: Rapid ionospheric changes

11 Troubleshooting

11.1 Common Issues

"ABOVE_MUF" status for all frequencies

The ionospheric critical frequency is too low to support propagation at the requested frequencies. Try:

- Lower frequencies
- Different time of day (daytime has higher MUF)
- More hops (shorter hop distance = higher elevation angle)

Very low elevation angles ($< 10°$)

Path is too long for single-hop. Increase `-hops`.

X-mode refractive index shows zero/negative

This is physically correct — the X-mode is evanescent (cutoff) in regions of high electron density. The simulation handles this appropriately.

Missing optional dependencies

If `requests` is not installed, space weather data defaults to moderate solar activity (F10.7=150). If `ppigrf` is not installed, a tilted dipole magnetic field model is used instead of IGRF.

11.2 Performance Tips

- Use `-no-plots` for faster execution when only text output is needed
- Reduce number of frequencies for quick tests
- The simulation is CPU-bound; no GPU acceleration is used

12 Physical Model Details

12.1 Ionosphere Model

The ionosphere is modeled using Chapman layers:

$$N_e(h) = N_{max} \exp\left[\frac{1}{2}\left(1 - z - e^{-z}\right)\right] \tag{24}$$

where $z = (h - h_{max})/H$ and H is the scale height.
Two layers are included:

- **E-layer**: $h_{max} = 110$ km, $H = 10$ km
- **F2-layer**: $h_{max} = 300$ km, $H = 70$ km

The F2-layer peak density scales with solar activity:

$$N_{max,F2} \propto F10.7^{0.8} \tag{25}$$

12.2 Magnetic Field Model

Without IGRF, a tilted dipole model is used:

- Dipole moment: $M = 7.94 \times 10^{22}$ A · m^2
- Magnetic pole: 80.5°N, 72.6°W

12.3 Ray Path Model

Parabolic approximation for each hop:

$$h(t) = 4h_{reflect} \cdot t(1 - t), \quad t \in [0, 1] \tag{26}$$

This gives arrival elevation:

$$\theta_{el} = \arctan\left(\frac{4h_{reflect}}{d_{hop}}\right) \tag{27}$$

13 References

1. Davies, K. (1990). *Ionospheric Radio.* Peter Peregrinus Ltd.
2. Budden, K.G. (1985). *The Propagation of Radio Waves.* Cambridge University Press.
3. Ratcliffe, J.A. (1959). *The Magneto-Ionic Theory and its Applications to the Ionosphere.* Cambridge University Press.
4. ITU-R P.531: Ionospheric propagation data and prediction methods required for the design of satellite services and systems.

14 Version History

v1.0 (January 2026) Initial release with O/X elevation and azimuth splitting, multihop support, comprehensive output plots.

v1.1 (January 2026) Added differential O/X absorption, received polarization state calculation, LCP/RCP power ratios, and polarization summary in output files.

15 Quick Reference: Antenna Selection

Based on simulation results, here are practical guidelines for antenna polarization selection:

Table 8: Recommended antenna polarization for HF reception

Location	Time	Frequency	Recommendation
Northern Hemisphere	Day	<5 MHz	**LCP required**
Northern Hemisphere	Day	5–10 MHz	LCP strongly preferred
Northern Hemisphere	Day	10–20 MHz	LCP preferred
Northern Hemisphere	Day	>20 MHz	LCP slight advantage
Northern Hemisphere	Night	Any	Either acceptable
Southern Hemisphere	Day	<5 MHz	**RCP required**
Southern Hemisphere	Day	5–10 MHz	RCP strongly preferred
Southern Hemisphere	Day	10–20 MHz	RCP preferred
Southern Hemisphere	Day	>20 MHz	RCP slight advantage
Southern Hemisphere	Night	Any	Either acceptable

Summary: During daytime, use LCP in Northern Hemisphere, RCP in Southern Hemisphere. At night, polarization selection doesn't affect SNR.

A Sample Output: WWV to OVRO Summary File

The following is a complete example of the text summary file produced by `multihop_3d.py` for the WWV to Owens Valley path at 5, 10, 15, 20, and 25 MHz with single-hop propagation.

```
================================================================================
3D HF PROPAGATION SIMULATION - SUMMARY
================================================================================

SIMULATION VERSION: 3D Ray Tracing with O/X splitting (elevation and azimuth)

PATH GEOMETRY
----------------------------------------
Transmitter: 40.6800°N, -105.0400°E
Receiver:    37.2300°N, -118.2800°E
Distance:    1206.0 km
Number of hops: 1
Bearing:     -104.3°

DATE/TIME
```

```
----------------------------------------
Simulation: 2026-01-08T08:46:50.149985+00:00

PROPAGATION CONDITIONS
----------------------------------------
Maximum Usable Frequency (MUF): 4.0 MHz
(Frequencies above MUF will penetrate the ionosphere)

RESULTS BY FREQUENCY
----------------------------------------------------------------------------------------------------------------------
    Freq      h_O      h_X     O Elev     X Elev    Elv Spl       O Az       X Az     Az Spl      Status
   (MHz)     (km)     (km)      (deg)      (deg)      (deg)      (deg)      (deg)      (deg)
----------------------------------------------------------------------------------------------------------------------
     5.0    347.8    327.8      49.08      47.40     1.6853     -103.5     -105.1     1.6261   ABOVE_MUF
    10.0    355.0    345.0      49.66      48.85     0.8095     -104.2     -104.3     0.0769   ABOVE_MUF
    15.0    353.3    346.7      49.53      48.99     0.5397     -104.3     -104.3     0.0202   ABOVE_MUF
    20.0    352.5    347.5      49.46      49.05     0.4047     -104.3     -104.3     0.0082   ABOVE_MUF
    25.0    352.0    348.0      49.42      49.10     0.3238     -104.3     -104.3     0.0041   ABOVE_MUF

====================================================================================================
NOTES ON 3D RAY TRACING
====================================================================================================

1. ELEVATION SPLITTING: O-mode penetrates higher into the ionosphere before
   reflecting (because n_O is closer to 1). X-mode reflects at lower altitude.
   This results in different arrival elevation angles.

2. LATERAL (AZIMUTHAL) SPLITTING: The O and X modes experience different
   lateral deflections due to the magnetized ionosphere. The deflection is
   perpendicular to both the density gradient and the magnetic field.

3. MULTIHOP: For paths requiring multiple hops, the code divides the path
   into equal segments. Each hop adds 180 deg phase shift at ground reflection.
   Faraday rotation and absorption accumulate across all hops.

4. The O/X splitting is typically small (< 1 degree) but can be significant
   for precise direction-finding or interferometric applications.
```

Interpreting This Output

Key observations from this example:

- The path is 1206 km from WWV (Fort Collins, CO) to OVRO (Owens Valley, CA)
- Bearing of $-104.3°$ indicates a west-southwest propagation direction
- **MUF = 4.0 MHz**: This simulation was run at night (08:46 UTC = 01:46 local), resulting in low ionospheric electron density and hence low MUF
- All frequencies show `ABOVE_MUF` status, indicating they would penetrate the ionosphere under these conditions
- O-mode reflects 10–20 km higher than X-mode (height difference decreases with frequency)
- Elevation splitting ranges from 1.69° at 5 MHz to 0.32° at 25 MHz

- Azimuth splitting is most pronounced at 5 MHz (1.63°) and negligible above 10 MHz
- The $1/f$ scaling of elevation splitting is evident in the data

Note: In practice, propagation above MUF may still occur via scatter modes or sporadic-E, but the simulator assumes classical F-layer reflection.