LQR Optimal Control and Kalman Filter Implementation for Satellite Relative Motion Manuevers

Alex Perez, A01686764, Chao Yuan, A01963548

I. Introduction

Even though the inertial equations for satellite motion are nonlinear, the relative motion equations that describe the position of one spacecraft relative to another are linear equations. These linear equations, together with weighting matrices can formulate a manuevering optimal control problem using a Linear Quadratic Regulator (LQR). The controller takes as a basic input, a target position and velocity relative to a specified satellite. The controller would then calculate the optimal trajectory to go from the initial position and velocity to the target position and velocity. The cost function of the LQR system is the energy of the manuever and for the specific case of spacecraft motion, it is the Δv of each manuever and the final states of the chaser spacecraft. The manuevers will employ continuous control throughout the trajectory. Most spacecraft use impulsive manuevers but for the sake of the project, we will keep the control continuous.

The solution to this problem can be very useful for autonomous satellite flight. Autonomous satellites can find the optimal path and manuevers to inspect or dock with other satellites or resident space objects. The conservation of fuel is especially important for small satellites like cubesats or other crafts with tight fuel tolerances. These optimal control tools can then be used in larger spacecraft and onboard manned missions as a useful tool. Some work has been done with optimal control for a station-keeping trajectory [1] and control of satellite constellations and formation flying [2], [3]. We intend to further this research with unique trajectories and manuevers. We will also formulate this problem in the continuous time domain with throttable engines.

The objective is to build an optimal controller that can be used for given satellite relative motion trajectories using a LQR system. The goal is to be able to give the optimal controller arbitrary initial conditions and target position and velocity terms and then have the controller compute an optimal path for the spacecraft in a local-vertical local horizontal (LVLH) frame. Another objective is to understand the constraints of the system and find how robust our optimal control algorithms are for many different initial conditions and rendezvous, tracking and circumnavigation missions.

II. Theory

We will make use of the Clohessy-Wiltshire equations [4] as our linear system. These equations are given as:

$$\ddot{x} = 3n^2x + 2n\dot{y}$$

$$\ddot{y} = -2n\dot{x}$$

$$\ddot{z} = -n^2z$$

where the x axis is in the radial direction, the y axis is in the along-track or downrange direction and the z axis is in the cross-track direction as shown in Figure 1.

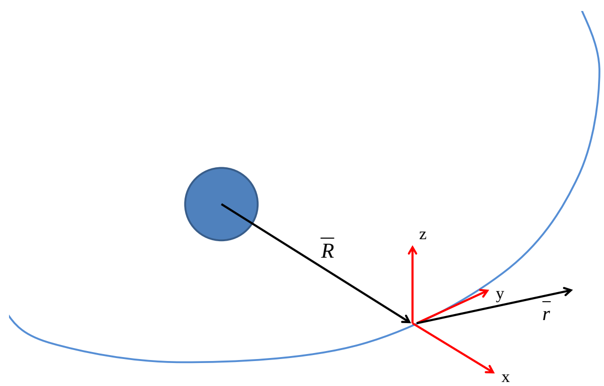

Fig. 1. Local-Vertical Local-Horizontal (LVLH) Frame

From these equations we can get linear-time-invariant solutions where we can relate the states from one time to another through a state transition matrix of the form

$$x_{k+1} = \Phi(t, t_0)x_k$$

where

$$\Phi = \begin{bmatrix} \Phi_{rr} & \Phi_{rv} \\ \Phi_{vr} & \Phi_{vv} \end{bmatrix}$$

and

$$\Phi_{rr} = \begin{bmatrix} 4-3\cos nt & 0 & 0 \\ 6(\sin nt - nt) & 1 & 0 \\ 0 & 0 & \cos nt \end{bmatrix}$$

$$\Phi_{rv} = \begin{bmatrix} \frac{1}{n}\sin nt & \frac{2}{n}(1-\cos nt) & 0 \\ -\frac{2}{n}(1-\cos nt) & \frac{1}{n}(4\sin nt - 3nt) & 0 \\ 0 & 0 & \frac{1}{n}\sin nt \end{bmatrix}$$

$$\Phi_{vr} = \begin{bmatrix} 3n\sin nt & 0 & 0 \\ -6n(1-\cos nt) & 0 & 0 \\ 0 & 0 & -n\sin nt \end{bmatrix}$$

$$\Phi_{vv} = \begin{bmatrix} \cos nt & 2\sin nt & 0 \\ -2\sin nt & 4\cos nt - 3 & 0 \\ 0 & 0 & \cos nt \end{bmatrix}$$

For the purpose of this problem we will choose a constant time step dt so the state transition matrix Φ becomes a constant matrix A so ($\Phi = A$) and the matrix that multiplies the control can be written as B where

$$B = \begin{bmatrix} \Phi_{rv} \\ \Phi_{vv} \end{bmatrix}$$

These matrices make up our discrete linear time-invariant system

$$x_{k+1} = Ax_k + Bu_k$$

A. *Initial Relative Orbit Determination (IROD) with a Camera Offset*

Recent work with Dr. David Geller has proven that the once unobservable problem of taking angle-only measurements has become observable if you take into account a camera offset from the center of mass of the chaser vehicle. When the camera is located at the center of mass, the linearized dynamics from the HCW equations do not allow the chaser to take angle-only measurements and recover relative position information. With a camera offset in the crosstrack direction, the dynamics change just enough for the camera relative to the center of mass of the chaser vehicle that a simple algorithm can be used to take angle measurements and then recover the initial relative position and velocity states. Three observations (six angle measurements) are initially needed for the problem to be observable. The first measurement equation can be written in this form

$$d(0)^{LVLH} = -\bar{R}_0 + k_0 i_{los}(0)$$

where $d(0)^{LVLH}$ is the camera offset in the LVLH frame, k_0 is a scale factor that is also determined in the IROD process, and $i_{los}(0)$ is the line-of-sight unit vector at time $t = 0$. The second and third measurement equations are given as

$$d(1)^{LVLH} = -\Phi_{rr}(1)\bar{R}_0 - \Phi_{rv}(1)\bar{V}_0 + k_1 i_{los}(1)$$

$$d(2)^{LVLH} = -\Phi_{rr}(2)\bar{R}_0 - \Phi_{rv}(2)\bar{V}_0 + k_2 i_{los}(2)$$

where Φ_{rr} and Φ_{rv} are block matrices that are part of the dynamics matrix, A in this way $A = \begin{bmatrix} \Phi_{rr} & \Phi_{rv} \\ \Phi_{vr} & \Phi_{vv} \end{bmatrix}$. We can combine these three measurement equations into a linear matrix equation in this way

$$\bar{x} = \begin{bmatrix} \bar{R}_0 \\ \bar{V}_0 \\ k_0 \\ k_1 \\ k_2 \end{bmatrix}$$

$$M = \begin{bmatrix} -I & 0 & i_{los}(0) & 0 & 0 \\ -\Phi_{rr}(1) & -\Phi_{rv}(1) & 0 & i_{los}(1) & 0 \\ -\Phi_{rr}(2) & -\Phi_{rv}(2) & 0 & 0 & i_{los}(2) \end{bmatrix}$$

$$\bar{d} = \begin{bmatrix} d(0)^{LVLH} \\ d(1)^{LVLH} \\ d(2)^{LVLH} \end{bmatrix}$$

And we have the the linear matriv equation

$$M\bar{x} = \bar{d}$$

which can be easily inverted to recover the initial position and velocity of the chaser vehicle.

$$\hat{x} = M^{-1}\bar{d}$$

B. *LQR Optimal Control Derivation*

The state of the system will include the position and velcity of the chaser spacecraft and the position of the target spacecraft is fixed in the LVLH frame. The cost function for this LQR infinite-horizon problem can be written as

$$J_{LQR} = \int_0^\infty \left(x^T Q x + u^T R u\right) dt$$

where we will be focused on reducing the energy of the control manuevers and matching the final states of the target vehicle for rendezvous missions or matching given final states. Part of the problem will be chosing the appropriate weighting matrices for different mission designs. Once we have chosen acceptable weighting matrices, we can use the development of linear quadratic regulators to find a gains for our optimal controller. This includes solving the algebraic Riccati equation for our given linear system

$$A^T P + PA - PBR^{-1}B^T P + Q = 0$$

and then using this result to calculate an optimal control.

$$u = -Kx$$

where $K = R^{-1}B^T P$.

We chose the Q and R matrices to reduce the settling time to get to the target vehicle but also to reduce the amount of fuel spent in the process. We found that we could choose Q and R values that would shoot toward the target, get there quickly but spend a lot of fuel. Some other values for Q and R would cause the chaser to oscillate around the target for a long time and never quite zone in on the desired position vector. These Q and R equations have to be chosen carefully and often by trial and error.

C. Kalman Filter Equations

The Kalman filter processes measurements and uses the measurements to estimate the state of the chaser spacecraft. Without the influence of measurements, the Kalman filter continues to propagate the states and the state covariance matrix in an intelligent way. The measurements provide information about the state of the chaser and the Kalman filter updates the estimate. The main equation for a generic discrete-time Kalman filter are given below.

	Governing Equations
Model	$x_{k+1} = Ax_k + Bu_k + w_k \quad w_k \sim N(0, Q_k)$
Model Measurement	$\tilde{y} = Cx_k + v_k \quad v_k \sim N(0, R_k)$
Kalman Gain	$K_k = P_k^- C_k^T \left[C_k P_k^- C_k^T + R_k\right]^{-1}$
Update State	$\hat{x} = \hat{x} + K_k \left[\tilde{y} - C_k \hat{x}\right]$
Update Covarianve	$P_k^+ = \left[I - K_k C_k\right] P_k^-$
Propagate State	$\hat{x} = A\hat{x} + Bu$
Propagate Covariance	$P_{k+1}^- = AP_k^+ A + Q_k$

TABLE I
KALMAN FILTER EQUATIONS

For our simulation we assume that we are getting LIDAR (Laser Imaging, Detecting and Ranging) measurements. This means that we are getting angle and range information about our current relative position. Thes measurements, with some noise added, are used the the Kalman Filter to update the state estimate and state covariance matrices.

III. RESULTS

We used the IROD scheme, the LQR optimal control and Kalman Filter on a number of initial conditions and trajectories. The case we will present here has the initial relative conditions

$$\bar{x}_0 = \begin{bmatrix} 100\,m \\ 1\,m \\ 1\,m \\ -0.0016\,m/s \\ 0.001\,m/s \\ 0.001\,m/s \end{bmatrix}$$

And other paramters include the camera measurement noise $\sigma^2_{camera} = 10^{-5}$, the LIDAR measurement error $v_k = 10^{-3}$ and the process noise $w_k = 10^{-5}$. These are reasonable values for the three different sources of error in this problem. After applying LQR optimal control and getting measurements from the Kalman Filter, the optimal trajectory is shown in Figure 2. The optimal control profile for all three LVLH directions is shown in Figure 3 and the state covariance estimate over time is shown in Figure 4.

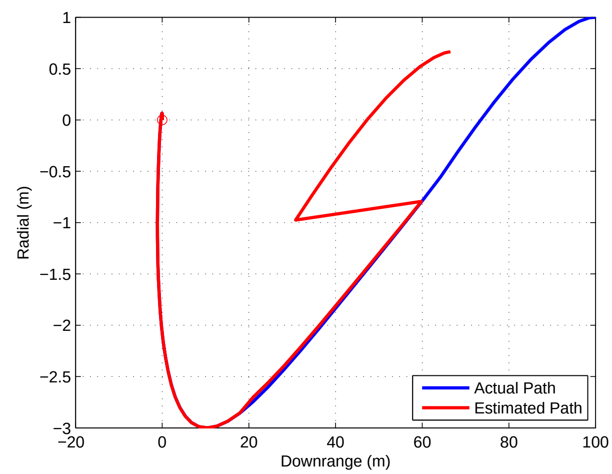

Fig. 2. Local-Vertical Local-Horizontal (LVLH) Frame

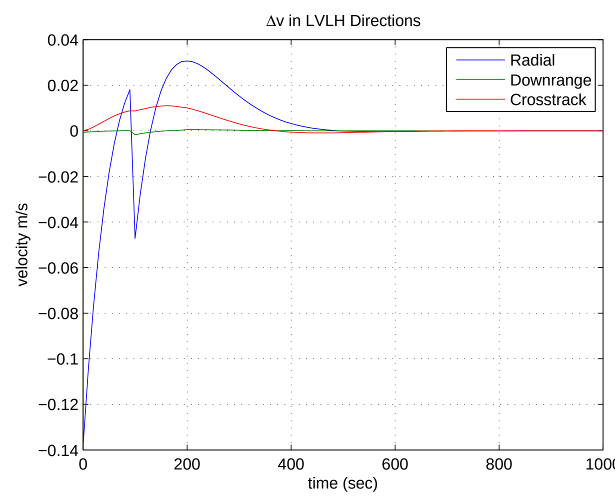

Fig. 3. Local-Vertical Local-Horizontal (LVLH) Frame

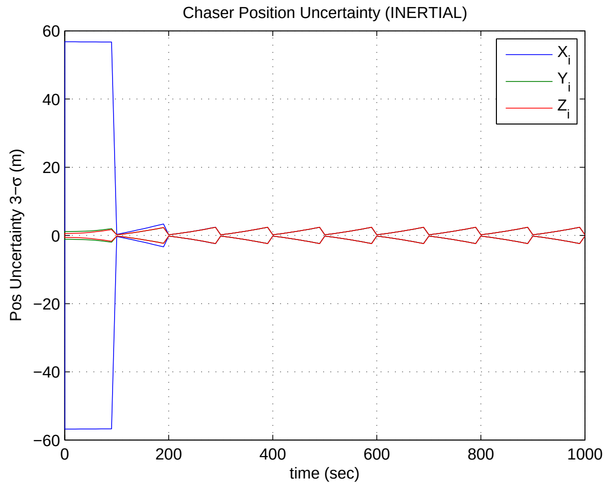

Fig. 4. Local-Vertical Local-Horizontal (LVLH) Frame

Figure 3 shows that the initial estimate of the position of the chaser spacecraft from the IROD scheme is a little incorrect but after the first LIDAR measurement the actual path of the spacecraft and the estimated path meet sync up and stay together throughout the remainder of the trajectory. This tells us that the Kalman filter is working and that the estimation models match well with the actual models of the system even with noise introduced.

IV. Conclusion

The three stages of this project, the IROD scheme, the LQR optimal control and the Kalman filter all work well together to simulate a hypothetical rendezvous mission given that our initial states are unknown and there are measurement and process noise included in the problem. The LQR control works well alongside the Kalman Filter and is a great tool to generate possible optimal trajectories. Future work to adjust the control gains should be pursued to get the desired amount of fuel usage and time to arrive at the target vehicle. Futher cases with varying initial conditions, noise levels and other parameter constraints should be considered to make this research more global and impactful.

References

[1] David C. Redding, Neil J. Adams, and Edward T. Kubiak. Linear-quadratic stationkeeping for the sts orbiter. *Journal of Guidance*, 12(2):248–255, January 1988.

[2] Yuri Ulybyshev. Long-term formation keeping of satellite constellation using linear-quadratic controller. *Journal of Guidanc*, 21(1):109–115, January-February 1998.

[3] Vikram Kapila, Andrew G. Sparks, James M. Buffington, and Qiguo Yan. Spacecraft formation flying: Dynamics and control. *Journal of Guidance, Control, and Dynamics*, 23(3):561–564, 1999.

[4] W.H. Clohessy and R.S. Wiltshire. Terminal guidance system for satellite rendezvous. *Journal of the Aerospace Sciences*, 27(9):653–658, September 1960.