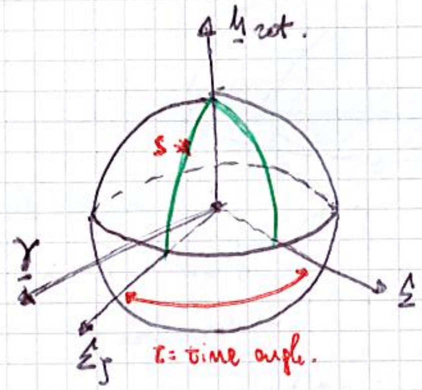
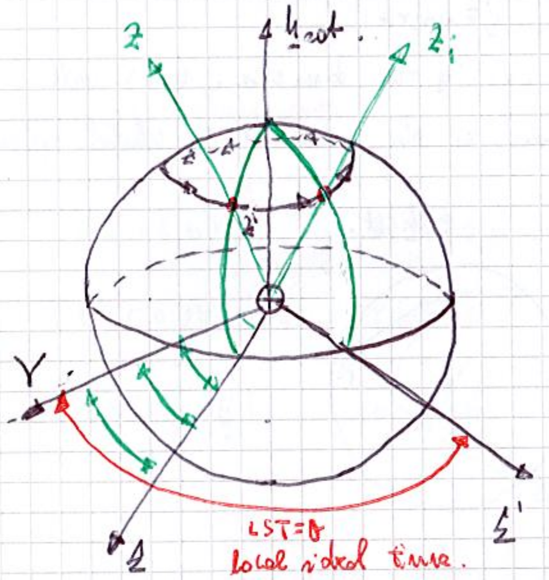
d) TIME MEASUREMENT,

Hp: Observed object are not moving (e.g. STARS)



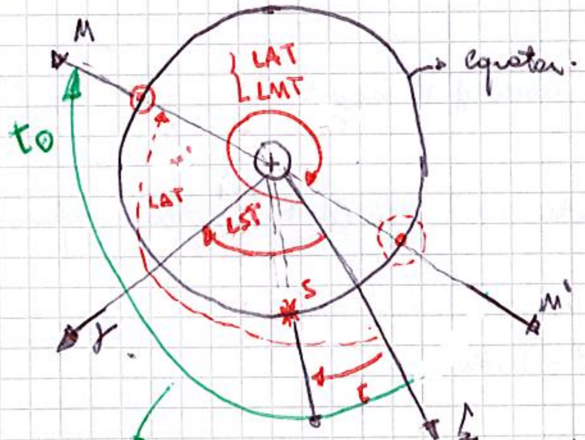
TIME ANGLE → angle between the meridian between the object observed and the meridian of the observer.



Time measure:

- t → ANGLE BETWEEN THE STAR PROJECTED ON EQUATOR AND THE ZENIT PROJECTED ON THE EQUATOR. ⇒ TIME ANGLE
- LST → ANGLE BETWEEN Υ AND THE PROJECTION OF THE ZENIT ON THE EQUATOR. ⇒ LOCAL SIDERAL TIME
- LMT' / LAT' → ANGLE BETWEEN ⊕-⊙ AND THE PROJECTION OF THE ZENIT ON THE EQUATOR. ⇒ LOCAL MEAN TIME
- LAT / LMT → ANGLE BETWEEN APPARENT POSITION OF SUN AND THE PROJECTION OF THE ZENIT ON THE EQUATOR. ⇒ LOCAL APPARENT TIME

Υ → FIXED AXIS ⇒ out of equinoxes conjunct earth-sun is different from Υ

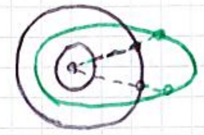


$\sqrt{\frac{\mu_\odot}{a^3_{\oplus\text{-}\odot}}}\, t_0 = E - e \sin E$

(latitude of the observer)

⇒ Projection of zenit axis on equator.)

Cause of elliptical orbit of earth around the sun. the velocity of the earth along the orbit is not always the same.



Definition of day:

* SIDERAL DAY: time interval between 2 consecutives passages of the meridian of the observer (Σ) on the equinoxes line (Υ).

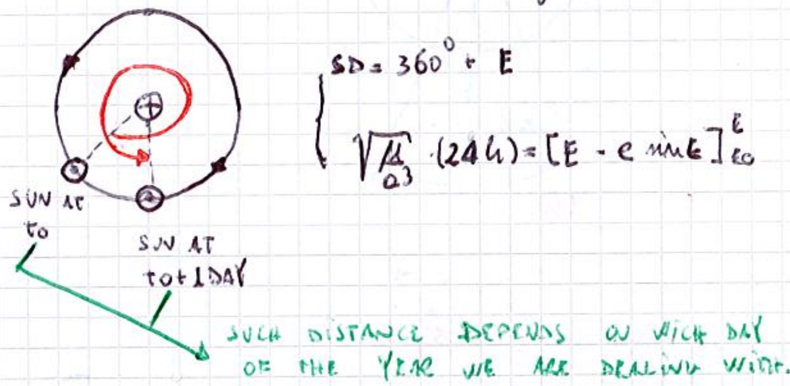
* SOLAR DAY: time interval between 2 consecutives passages of the meridian of the observer (Σ) on the conjunct earth-sun (⊕-⊙)

SIDERAL - DAY

$\boxed{360° = 23h\,56m. \quad \text{SIDERAL DAY}}$

SOLAR - DAY

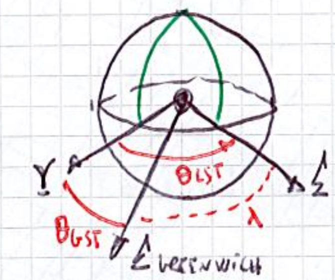
⇒ solar day > 360°



$$\begin{cases} SD = 360° + E \\ \sqrt{\frac{\mu}{a^3}}\,(24h) = [E - e \sin E]_{E_0}^{E} \end{cases}$$

⇓

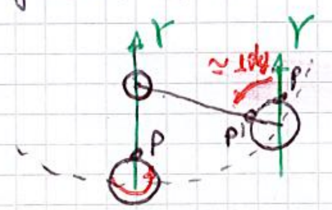
* LOCAL SIDERAL TIME: time angle between local meridian and the meridian passing through Υ



⇒ $\theta_{LST} = \lambda + \theta_{GST}$.

θ_{GST} → angle between greenwich meridian and Υ.

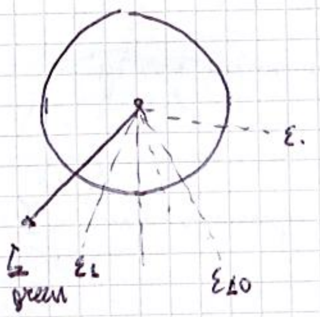
* LOCAL MEAN TIME: time angle between local meridian and conjunction ⊕-⊙



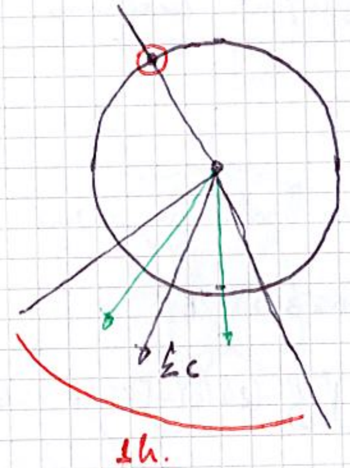
The last time measure we are interested in is the local civil-time.

FUSES: ARE DIGITALIZATION OF A DISCRETE QUANTITY TO VIZUAL REAL LIFE.

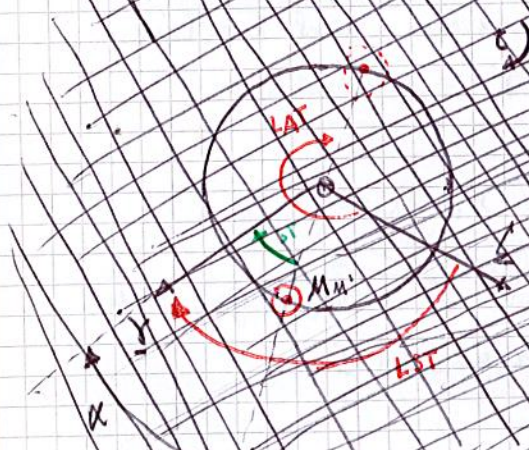
(IT). FUSI



⇒ if the median
is in the same SPICCHIO
We have to assume the
same hour..



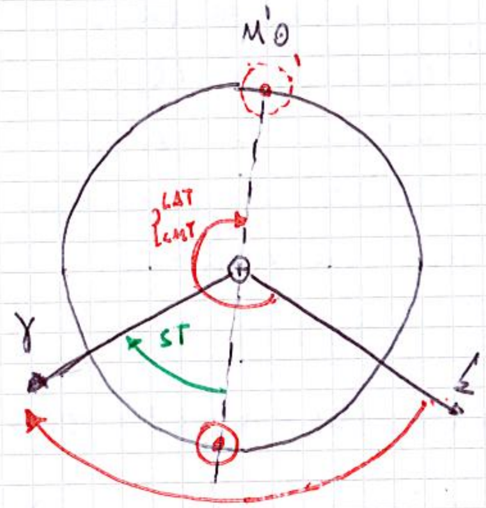
~~All time measures~~



~~$\left\{ \begin{array}{l} LST \Rightarrow \widehat{\mathcal{E}\gamma} \\ ST \Rightarrow \widehat{\odot\gamma} \end{array} \right.$~~ ~~$\left\{ \begin{array}{l} LMT \Rightarrow \widehat{\mathcal{E} M'_\odot} \\ LAT \Rightarrow \widehat{\mathcal{E} M_{M'}} \\ LCT \Rightarrow \widehat{\mathcal{E}_{cs} M_{M'}} \end{array} \right.$~~

~~$LST = \widehat{\mathcal{E}\odot} + \widehat{\odot\gamma} = LAT + ST$~~

⇒ ALL TIME MEASUREMENTS:



$(+) \left\{ \begin{array}{l} LST = \widehat{\mathcal{E}\gamma} \\ ST = \widehat{\odot\gamma} \end{array} \right.$

LAT of 1 DAY ≅ 12 h.

$(+) \left| \begin{array}{l} LMT = \widehat{\mathcal{E} M'_\odot} \\ LAT = \widehat{\mathcal{E} M_{M'}} \\ LCT = \widehat{\mathcal{E}_{cs} M_{M'}} \end{array} \right.$

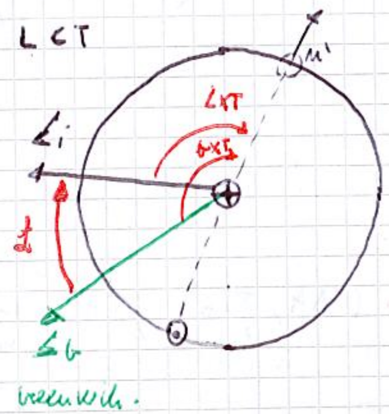
* $\left\{ \begin{array}{l} LMT \\ LAT \end{array} \right.$ ⇒ $LAT - E = LMT$ ⇒ $LAT - E = LMT$

$M - M_0 = (E - e \sin E)\Big|_{E_0}^{E}$

LAT → $\text{angle}_{LAT} = \sqrt{\frac{\mu_\odot}{a^3}} \cdot LAT$

LMT → $\sqrt{\frac{\mu_\odot}{a^3}} \cdot LMT = [\text{angle}_{LMT} - e \sin(\text{angle}_{LMT})]$

* LCT



⇒ $LCT = GMT \pm N \cdot 1^h$

N → number of the fuse.

⇒ $GXT = LXT + \lambda$ $\quad X = \left\{ \begin{array}{l} M \\ A \\ C \end{array} \right\}$ (= mean / apparent / civil)

⇓

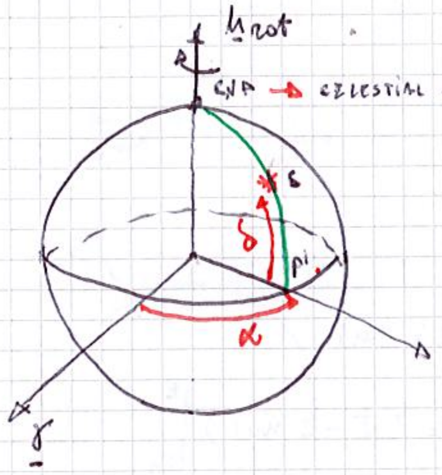
$GMT - \lambda = LMT.$

e) CHANGE IN REFERENCE SYSTEM.

considering the 3 systems seen right now:

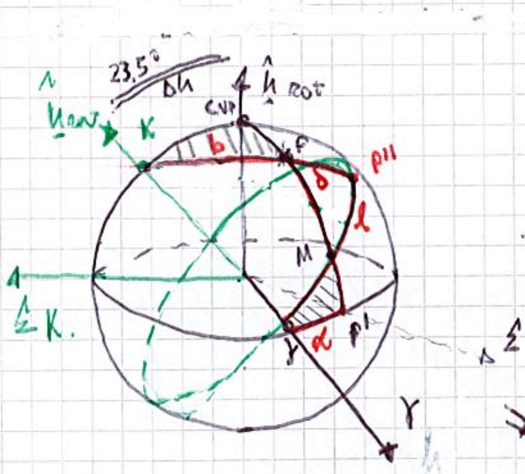
- EQUATORIAL ⇒ $(\alpha;\delta)$
- ECLIPTIC ⇒ $(l;\ b)$
- HORIZON (LOCAL) ⇒ $(Az;\ h)$

Using spherical geometry is possible to pass from one to another.

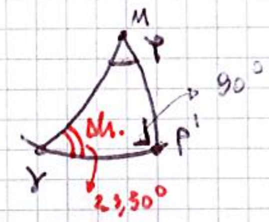


CNP ⇒ CELESTIAL NORTH POLE ⇒ intersection of $\hat{h}_{rot}$ with celestial sphere.

1) EQUATORIAL R.S. ⇄ ECLIPTIC R.S.

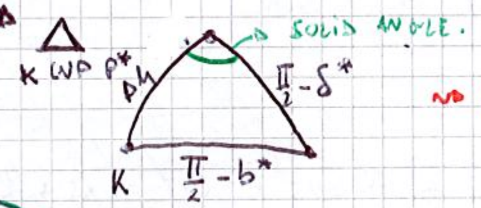


⇒ $\triangle P'M\Upsilon$

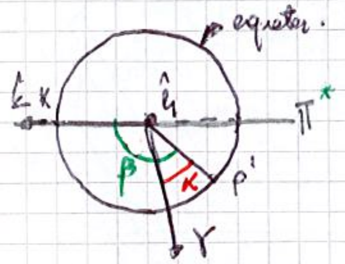


⇒ $(\Delta h;\ \frac{\pi}{2};\ \varphi$ ⇒ "surface angles")

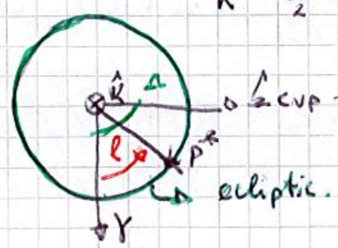
⇒ $\triangle$ K CNP P^*



⇒ $(\Delta h;\ \frac{\pi}{2}-b^*;\ \frac{\pi}{2}-\delta^*$ ⇒ "internal angles" ⇓ sides)



;



⇒ $\hat{\varepsilon}_{CNP}$ → projection of CNP on the ecliptic

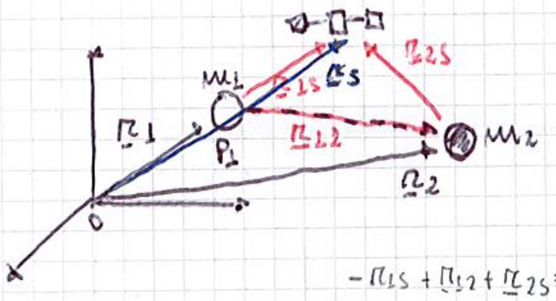
⇓ ; ⇓

$\beta = \frac{\pi}{2} + \alpha^*$; $\Omega = \frac{\pi}{2} - l$

↳ considering a plane ⊥ to $\underline{\Upsilon}$ (Π^*) ⇒ $(\hat{h}_{rot};\ \hat{h}_{rev}) \in \Pi^*$, since $\hat{\varepsilon}_K$ is the the projection of $\hat{h}_{rev}$ on such plane ⇒ must be orthogonal to $\underline{\Upsilon}$

10a) → "Perturbative effects"

1 → "Third-body perturbations (AND SOI COMPUTATION)"



$$\underline{r}_{1s} = \underline{r}_s - \underline{r}_1$$
$$\underline{r}_{2s} = \underline{r}_s - \underline{r}_2$$
↳ $\underline{r}_{12} = \underline{r}_2 - \underline{r}_1 = \underline{r}_{1s} - \underline{r}_{2s}$

$-\underline{r}_{1s} + \underline{r}_{12} + \underline{r}_{2s} = \underline{0}$ ⇒  ⇒ $\underline{r}_{1s} + \underline{r}_{12} = \underline{r}_{2s}$ ⇒ $\underline{r}_{12} = \underline{r}_{1s} - \underline{r}_{2s}$

Writing dynamic for the 3 bodies ⇒ WITH RESPECT TO "O" (INERTIAL R.F.)

(S) $m_s \dfrac{d^2\underline{r}_s}{dt^2} = -G\dfrac{m_s m_1}{r_{1s}^3}\underline{r}_{1s} - G\dfrac{m_s m_2}{r_{2s}^3}\underline{r}_{2s}$

(1) $m_1 \dfrac{d^2\underline{r}_1}{dt^2} = +G\dfrac{m_s m_1}{r_{1s}^3}\underline{r}_{1s} + G\dfrac{m_1 m_2}{r_{12}^3}\underline{r}_{12}$

(2) $m_2 \dfrac{d^2\underline{r}_2}{dt^2} = +G\dfrac{m_s m_2}{r_{2s}^3}\underline{r}_{2s} - G\dfrac{m_1 m_2}{r_{12}^3}\underline{r}_{12}$

↳ $\begin{cases}\underline{r}_s = \underline{r}_1 + \underline{r}_{1s}\\ \underline{r}_s = \underline{r}_2 + \underline{r}_{2s}\end{cases}$ AIM: Write dynamic of the system as a function of $(\underline{r}_{1s}, \underline{r}_{2s})$

S) $\dfrac{d^2\underline{r}_s}{dt^2} = \dfrac{d^2\underline{r}_1}{dt^2} + \dfrac{d^2\underline{r}_{1s}}{dt^2} = -\dfrac{G m_1}{r_{1s}^3}\underline{r}_{1s} - \dfrac{G m_2}{r_{2s}^3}\underline{r}_{2s}$

$$\Rightarrow \frac{d^2\underline{r}_{1s}}{dt^2} = -\frac{G m_1}{r_{1s}^3}\underline{r}_{1s} - \frac{G m_2}{r_{2s}^3}\underline{r}_{2s} - \underbrace{\frac{d^2\underline{r}_1}{dt^2}}_{(1)}$$
$$= -\frac{G m_1}{r_{1s}^3}\underline{r}_{1s} - \frac{G m_2}{r_{2s}^3}\underline{r}_{2s} - \frac{G m_s}{r_{1s}^3}\underline{r}_{1s} - \frac{G m_2}{r_{12}^3}\underline{r}_{12}$$
$$= -G\frac{(m_1+m_s)}{r_{1s}^3}\underline{r}_{1s} - \frac{G m_2}{r_{12}^3}\underline{r}_{12} - \frac{G m_2}{r_{2s}^3}\underline{r}_{2s}$$

$\underline{r}_{12} = \underline{r}_{1s} - \underline{r}_{2s}$

$$\Rightarrow \frac{d^2\underline{r}_{1s}}{dt^2} = -G\frac{(m_1+m_s)}{r_{1s}^3}\underline{r}_{1s} + G m_2\left[\frac{\underline{r}_{2s} - \underline{r}_{1s}}{r_{12}^3} - \frac{\underline{r}_{2s}}{r_{2s}^3}\right]$$

(S) $\dfrac{d^2\underline{r}_s}{dt^2} = \dfrac{d^2\underline{r}_{2s}}{dt^2} + \dfrac{d^2\underline{r}_2}{dt^2} = -\dfrac{G m_1}{r_{1s}^3}\underline{r}_{1s} - \dfrac{G m_2}{r_{2s}^3}\underline{r}_{2s}$

$$\Rightarrow \frac{d^2\underline{r}_{2s}}{dt^2} = -\frac{G m_1}{r_{1s}^3}\underline{r}_{1s} - \frac{G m_2}{r_{2s}^3}\underline{r}_{2s} - \underbrace{\frac{d^2\underline{r}_2}{dt^2}}_{(2)}$$
$$= -\frac{G m_1}{r_{1s}^3}\underline{r}_{1s} - \frac{G m_2}{r_{2s}^3}\underline{r}_{2s} - \frac{G m_s}{r_{2s}^3}\underline{r}_{2s} + \frac{G m_1}{r_{12}^3}\underline{r}_{12}$$

$\underline{r}_{12} = \underline{r}_{1s} - \underline{r}_{2s}$

$$= -G\frac{(m_2+m_s)}{r_{2s}^3}\underline{r}_{2s} + G m_1\left[\frac{\underline{r}_{1s} - \underline{r}_{2s}}{r_{12}^3} - \frac{\underline{r}_{1s}}{r_{1s}^3}\right]$$

So: The dynamic written in $\left(\dfrac{d^2\underline{r}_{1s}}{dt^2}; \dfrac{d^2\underline{r}_{2s}}{dt^2}\right)$ is:

$$\frac{d^2\underline{r}_{1s}}{dt^2} = -G\frac{m_1+m_s}{r_{1s}^3}\underline{r}_{1s} + G m_2\left(\frac{\underline{r}_{2s} - \underline{r}_{1s}}{r_{12}^3} - \frac{\underline{r}_{2s}}{r_{2s}^3}\right) \triangleq \underline{a}_{1s} + \underline{a}_{1d}(\underline{r}_{2s})$$
$$\frac{d^2\underline{r}_{2s}}{dt^2} = -G\frac{m_2+m_s}{r_{1s}^3}\underline{r}_{1s} + G m_1\left(\frac{\underline{r}_{1s} - \underline{r}_{2s}}{r_{12}^3} - \frac{\underline{r}_{1s}}{r_{2s}^3}\right) \triangleq \underline{a}_{2s} + \underline{a}_{2d}(\underline{r}_{1s})$$

→ Sphere of influence (SOI)

$\underline{a}_{1s}(\underline{r}_{1s}) = -\dfrac{G(m_1+m_s)}{r_{1s}^3}\underline{r}_{1s}$; $\underline{a}_{1d}(\underline{r}_{1s}, \underline{r}_{2s}) = G m_2\left(\dfrac{\underline{r}_{2s} - \underline{r}_{1s}}{r_{12}^3} - \dfrac{\underline{r}_{2s}}{r_{2s}^3}\right) \propto m_2$

$\underline{a}_{2s}(\underline{r}_{2s}) = -\dfrac{G(m_2+m_s)}{r_{2s}^3}\underline{r}_{2s}$; $\underline{a}_{2d}(\underline{r}_{1s}, \underline{r}_{2s}) = G m_1\left(\dfrac{\underline{r}_{1s} - \underline{r}_{2s}}{r_{12}^3} - \dfrac{\underline{r}_{1s}}{r_{1s}^3}\right) \propto m_1$

⇒ $\|\underline{r}_{soi}\|$ → centered in 1 $\|\underline{r}_{soi}\| = \|\underline{r}_{1s}\|$: $\dfrac{a_{1d}}{a_{1s}} = \dfrac{a_{2d}}{a_{2s}}$ (MODULUS EQUIVALENCE)
→ centered in 2 $\|\underline{r}_{soi}\| = \|\underline{r}_{2s}\|$

In computing radius of the sphere of influence:

$\dfrac{a_{1d}}{a_{1s}} = \dfrac{a_{2d}}{a_{2s}}$ → $a_{1d} \cdot a_{2s} = a_{2d} \cdot a_{1s}$ $\left\| \dfrac{G(m_2+m_s)}{r_{2s}^3}\right\| \|\underline{r}_{2s}\| = \dfrac{G(m_2+m_s)}{r_{2s}^2}$

$$G m_2 \left\| \frac{(\underline{r}_{2s} - \underline{r}_{1s})}{r_{12}^3} - \frac{\underline{r}_{2s}}{r_{2s}^3}\right\| + \frac{G(m_2+m_s)}{r_{2s}^2} = G m_1 \left\| \frac{(\underline{r}_{1s} - \underline{r}_{2s})}{r_{12}^3} - \frac{\underline{r}_{1s}}{r_{1s}^3}\right\| + G\frac{(m_1+m_s)}{r_{1s}^2}$$

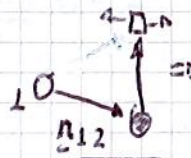
APPROXIMATION OF ALL ACCELERATIONS $a_{1s}, a_{2s}, a_{1d}, a_{2ds}$ SUPPOSING OF BEING REALLY CLOSE TO P1 (ML)

$\Rightarrow \delta \triangleq \frac{r_{1s}}{r_{12}} \ll 1$

→ $\|a_{1d}\|$

$$a_{1d} = \left\| \frac{\overbrace{r_{2s}-r_{1s}}^{=-r_{12}}}{r_{12}^3} - \frac{r_{2s}}{r_{2s}^3}\right\| = \sqrt{\left(-\frac{r_{12}}{r_{12}^3}-\frac{r_{2s}}{r_{2s}^3}\right)\cdot\left(-\frac{r_{12}}{r_{12}^3}-\frac{r_{2s}}{r_{2s}^3}\right)}$$

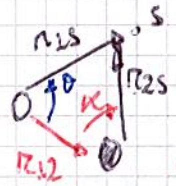
$$= \sqrt{\frac{r_{12}^2}{r_{12}^6}+\frac{r_{2s}^2}{r_{2s}^6}+2\frac{r_{12}\cdot r_{2s}}{r_{12}^3 r_{2s}^3}}\cdot G$$



$\Rightarrow$

$$= G\sqrt{\frac{1}{r_{12}^4}+\frac{1}{r_{2s}^4} - 2\frac{r_{21}\cdot r_{2s}}{r_{12}^3 r_{2s}^3}} = \sqrt{\frac{1}{r_{12}^4}+\frac{1}{r_{2s}^4}-\frac{2\cos\alpha}{r_{12}^2 r_{2s}^2}} \quad \text{(i)}$$

From the figure:



$\|r_{12}\| = r_{1s}\cos\theta + r_{2s}\cos\alpha \Rightarrow \cos\alpha = \frac{r_{12}-r_{1s}\cos\theta}{r_{2s}}$ (ii)

CARNOT'S THEOREM.

$$r_{2s}^2 = r_{1s}^2 + r_{12}^2 - 2 r_{1s} r_{12}\cos\theta.$$

$$\left(\frac{r_{2s}}{r_{12}}\right)^2 = \left(\frac{r_{1s}}{r_{12}}\right)^2 + 1 - 2\frac{r_{1s}}{r_{12}}\cos\theta.$$

$$\left(\frac{r_{2s}}{r_{12}}\right)^2 = \delta^2+1-2\delta\cos\theta. \quad \text{(iii)}$$

(i) $$a_{1d} = \sqrt{\frac{1}{r_{12}^4}+\frac{1}{r_{2s}^4}-\frac{2\cos\alpha}{r_{12}^2 r_{2s}^2}} = \frac{G}{r_{12}^2}\sqrt{1+\frac{1}{\left(\frac{r_{2s}}{r_{12}}\right)^4} - \frac{2\cos\alpha}{\left(\frac{r_{2s}}{r_{12}}\right)^2}}$$

(iii) $$= \frac{G}{r_{12}^2}\sqrt{1+\frac{1}{(\delta^2-2\delta\cos\theta+1)^2} - \frac{2\cos\alpha}{\delta^2-2\delta\cos\theta+1}}$$

(ii) $$= \frac{G}{r_{12}^2}\sqrt{1+\frac{1}{(\delta^2-2\delta\cos\theta+1)^2}-\frac{2(1-\delta\cos\theta)}{(\delta^2-2\delta\cos\theta+1)^{3/2}}}$$

$$\cos\alpha = \frac{r_{12}-r_{1s}\cos\theta}{r_{2s}} = \frac{1-r_{1s}/r_{12}\cos\theta}{r_{2s}/r_{12}}$$

$\Downarrow$

$$\cos\alpha = \frac{1-\delta\cos\theta}{(\delta^2+1-2\delta\cos\theta)^{1/2}}$$

→ Taylor envelope:

$$(1+x)^n = 1+nx+\frac{n(n-1)}{2}x^2+O(x^3)$$

→ to be arrested at the 2nd order.

Thanks to Taylor serie:

$$\underbrace{(\delta^2-2\delta\cos\theta+1)}_{x}^{-2} = 1-2(\delta^2-2\delta\cos\theta)+\frac{(-2)(-2-1)}{2}(\delta^2-2\delta\cos\theta)^2+O\ldots$$

$$(\delta^2-2\delta\cos\theta+1)^{-3/2} = 1-\frac{3}{2}(\delta^2-2\delta\cos\theta)+\frac{(-3/2)(-5/2)}{2}(\delta^2-2\delta\cos\theta)^2+\ldots$$

$$\Rightarrow -2(1-\delta\cos\theta)\left[1-\frac{3}{2}(\delta^2-2\delta\cos\theta)+\frac{15}{4}(\delta^4+4\delta^2\cos^2\theta-4\delta^3\cos\theta)\right]$$

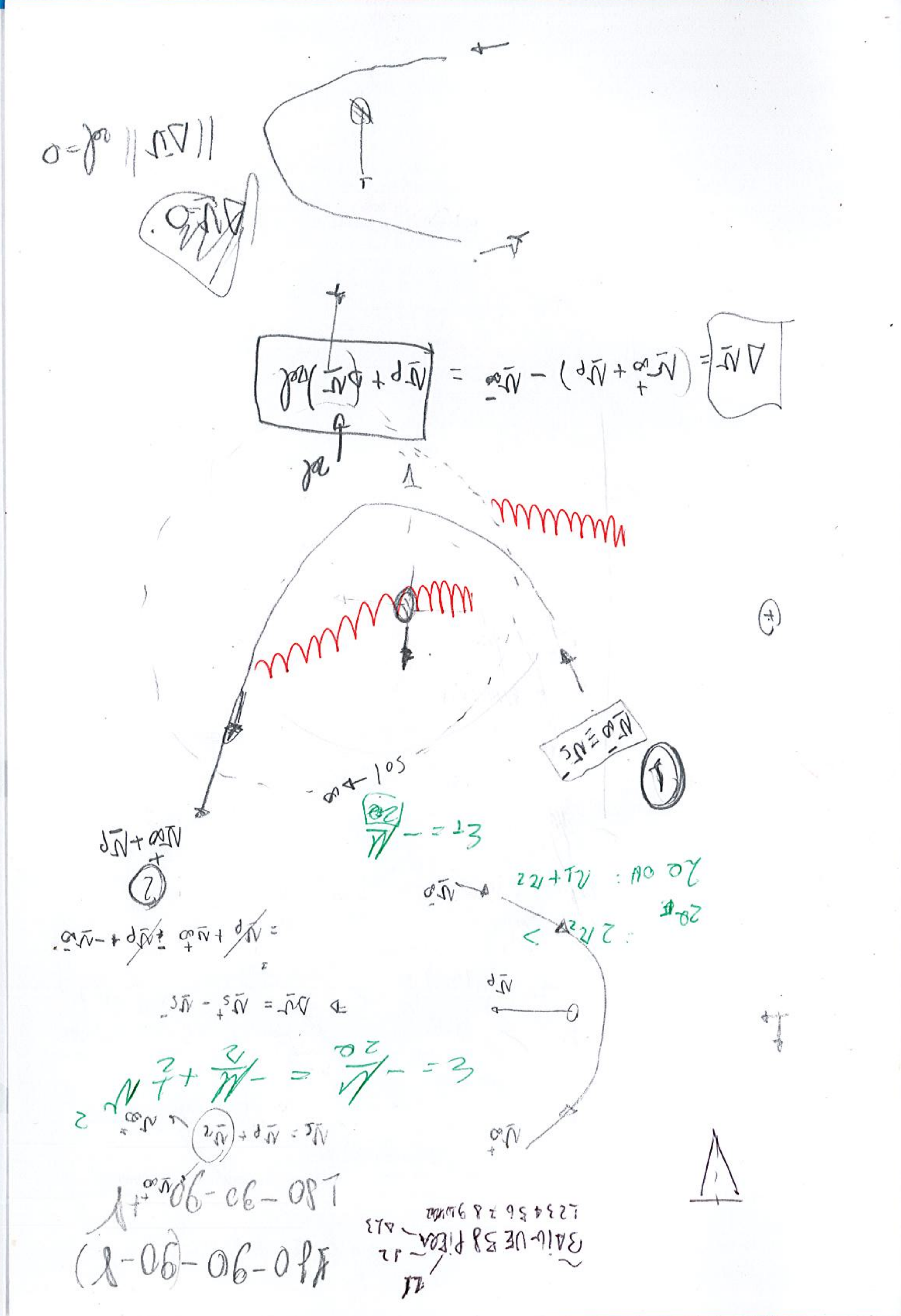
$$-2(1-\delta\cos\theta)\left[1-\frac{3}{2}\delta^2+3\delta\cos\theta+\underbrace{\frac{15}{4}\delta^4}_{\to 0}+15\delta^2\cos^2\theta-\underbrace{15\delta^3\cos\theta}_{\to 0}\right]$$

Approximation: Neglecting all terms in $(\delta^3;\delta^4)$ (AS TAYLOR SERIES HAS BEEN BLOCKED AT THE THIRD POWER)

$$-2(1-\delta\cos\theta)\left[1-\frac{3}{2}\delta^2+3\delta\cos\theta+15\delta^2\cos^2\theta\right]$$

$$-2\left[1-\frac{3}{2}\delta^2+3\delta\cos\theta+15\delta^2\cos^2\theta-\underbrace{15\delta^3\cos\theta}_{\to 0}-\delta\cos\theta-\underbrace{\frac{3}{2}\delta^3\cos\theta}_{\to 0}+3\delta^2\cos^2\theta-\underbrace{15\delta^3\cos^3\theta}_{\to 0}\right]$$

$$-2\left[1-\frac{3}{2}\delta^2+2\delta\cos\theta+18\delta^2\cos^2\theta\right.$$



Th* 7/02/2011

$\underline{r} = -11349{,}3378\,\underline{I} - 1160{,}6943\,\underline{J} - 399{,}7849\,\underline{K}$ [Km] → VENUS-CENTRIC R.F.

$\underline{v} = 4822{,}3911\,\underline{I} + 6460{,}3328\,\underline{J} - 40{,}19437\,\underline{K}$ [m/s]

$= 4{,}8226\,\underline{I} + 6{,}460\,\underline{J} - 0{,}0419437$ [Km/s]

$\mu_{\sigma^*} = \dfrac{m_\sigma}{m_{\sigma^*}}\,\mu_\sigma = 324860\ \mathrm{Km^3/s^2}$

a) ORBITAL PARAMETERS

$$\underline{h} = \underline{r} \times \underline{v} = \begin{vmatrix} \underline{I} & \underline{J} & \underline{K} \\ -11349{,}3378 & -1160{,}6943 & -399{,}7849 \\ 4{,}8226 & 6{,}460 & -4{,}0194 \times 10^{-2} \end{vmatrix}$$

$$= \underline{I}\,(-1160{,}6943 \cdot (-4{,}0194) + 399{,}7849 \cdot 6{,}460) -$$
$$- \underline{J}\,(-11349{,}3378 \cdot (-4{,}0194) + 4{,}8226 \cdot 399{,}7849) +$$
$$+ \underline{K}\,(-11349{,}3378 \cdot 6{,}460 + 1160{,}6943 \cdot 4{,}8226)$$

$\underline{h} = (2{,}6294\,\underline{I} - 2{,}3842\,\underline{J} - 6{,}7726) \times 10^9\ \mathrm{m^2/s}$

$= 2629{,}4\,\underline{I} - 2384{,}2\,\underline{J} - 6772{,}6\ \mathrm{Km^2/s}$

→ $\|\underline{h}\| = 7646{,}3236\ \mathrm{Km^2/s}$

- $\cos i = \dfrac{\underline{K} \cdot \underline{h}}{\|\underline{h}\|} = 0{,}8857$

  $i = 27{,}6622^\circ$

Patched conic Approach (LINKED FOR PROFESSOR LAVAGNA)

Hp:

(i) soi has ∞ radius when observed from the planet
(SOI OF THE PLANET)

(ii) SOI OF THE PLANET has 0 radius when observed from the sun.

(iii) Trajectories inside a soi are R2BP with the planet as main attractor.
Trajectories out of a soi are R2BP with the sun as main attractor

(iv) Planet orbits are circular and with $\begin{cases} \Omega = 0 \\ i = 0 \\ \omega = 0 \end{cases}$

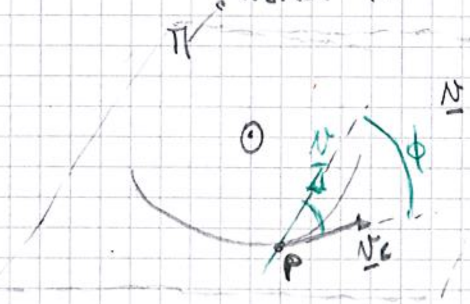
$\underline{v}_c \rightarrow$ circular velocity of the planet with respect to the sun.

$\underline{v} \rightarrow$ Heliocentric orbit velocity

$\Delta \underline{v} \rightarrow \Delta \underline{v} = \underline{v} - \underline{v}_c$.

$\phi \rightarrow$ flight-path angle of $\underline{v}$ (since orbits are circular $\phi : \underline{v} \hat{} \underline{v}_c$)

orbital plane of the plane



$\underline{v}$ can be any (each $\mathbb{R}^3$ vector)
anyway $\underline{v}_c$ and $\underline{v}$ always define a plane

Carnot's Theorem

$\Rightarrow$ at planet

① $\|\Delta \underline{v}_1\|^2 = \|\underline{v}_1\|^2 + \|\underline{v}_{c1}\|^2 - 2\,\|\underline{v}_{c1}\|\,\|\underline{v}_1\| \cos\phi_1$

② : $\|\Delta \underline{v}_2\|^2 = \|\underline{v}_2\|^2 + \|\underline{v}_{c2}\|^2 - 2\,\|\underline{v}_{c2}\|\,\|\underline{v}_2\| \cos\phi_2$

$\Rightarrow$ at the patch-point (P)

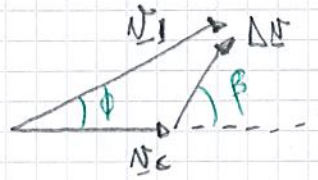
$\hookrightarrow$ Defining:

- $\underline{v}_{V/S} \rightarrow$ VEHICLE'S VELOCITY WITH RESPECT TO THE SUN
- $\underline{v}_{V/P} \rightarrow$ VEHICLE'S VELOCITY WITH RESPECT TO THE PLANET
- $\underline{v}_{P/S} \rightarrow$ PLANET'S VELOCITY WITH RESPECT TO THE SUN

~~[illegible]~~ $\underline{v}_1 \triangleq \underline{v}_{V/S1}$; $\underline{v}_{P/S} \triangleq \underline{v}_{S1}$;

$\Downarrow$

$$\underline{v}_\infty = \underline{v}_1 - \underline{v}_P \triangleq \Delta\underline{v}$$



$\Rightarrow$ • $v_c + \Delta v \cos\beta = v_1 \cos\phi$

(USING ONLY SCALAR QUANTITY)

$\downarrow$

• $v_1 \sin\phi = \Delta v \sin\beta$

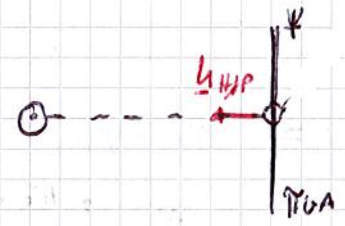
$\Rightarrow \dfrac{\Delta v \sin\beta}{\Delta v \cos\beta} = \tan\beta = \dfrac{v_1 \sin\phi}{v_1\cos\phi - v_c}$ $\Rightarrow$ $\beta \to$ angle between the velocity vector of the planet and the relative velocity of the satellite

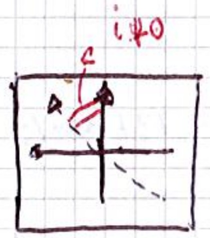
$$\tan\beta = \frac{v_1 \sin\phi}{v_1\cos\phi - v_c}$$; $\phi \to$ angle between $\underline{v}_p$ and $\underline{v}_{sc}$

G.A. → EXAMPLE 1

Soln ♆

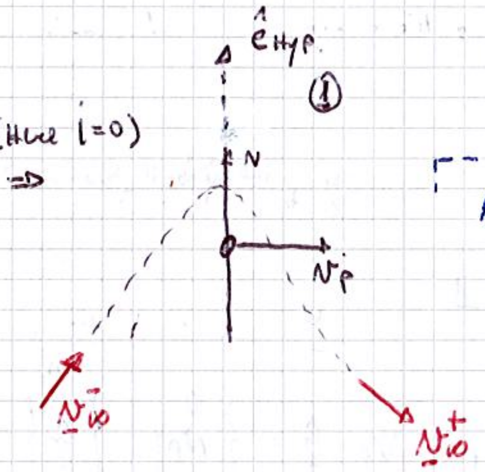
$\begin{cases} \hat{\underline{r}}_p = \text{North ♆} \Rightarrow \hat{\underline{e}}_{Hyp} = \hat{\underline{h}}_{rot} = \hat{h}_{rev} \\ \hat{\underline{h}}_{Hy} = \underline{r}_{\odot ♆} \end{cases}$





(Here $i = 0$)

$\Rightarrow$

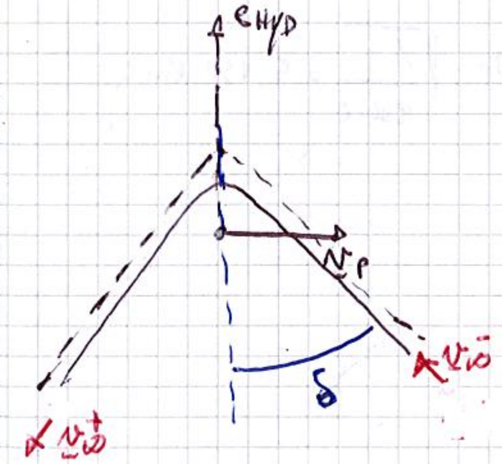


$\Delta\mathcal{E}^{GA} = 2 v_\infty \sin\delta \cos\sigma$

σ: $\widehat{\underline{v}_P\, \hat{\underline{e}}_{Hy}}$; δ: $\frac{1}{2}\, \widehat{\underline{v}_\infty^-\, \underline{v}_\infty^+}$

→ it's a choice (for the datas given) choice (1) instead of (2)

$\Rightarrow$ let's work on (2)



Datas:

$\mu_\odot = 1,327 \times 10^{20}\ m^3/s^2$

$e_{Hyp} = 2,4336$

$a_{Hyp} = -20649\ km$;

$\begin{cases} m_{♆} = 17,14\, m_\oplus \\ r_{♆-\odot} = 30,00\ A.U. \end{cases}$

$1\,A.U. = 149,6 \times 10^6\ Km$.

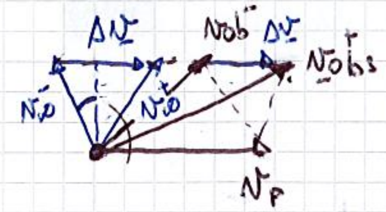
?? v_{max} ; $\|\Delta\underline{v}\|_{dos}$; $\underline{v}_{dos}^-$; $\underline{v}_{dos}^+$; $\Delta\xi \Rightarrow \widehat{\underline{v}_{dos}^-\, \underline{v}_{dos}^+}$, $a^+ \to$ SEMIAXIS OF THE RESULTING ORBIT AFTER G.A.

$\mu_{♆} \triangleq \mu$; $\mu = \dfrac{m_{♆}}{m_\oplus} \cdot \mu_\oplus = 6,832 \times 10^6\ Km^3/s$

$\dfrac{1}{e} = \sin\delta$ $\Rightarrow$ $\delta = \sin^{-1}\left(\dfrac{1}{e_{Hyp}}\right) = 24,00°$

$$v_\infty^- = v_\infty^+ = \sqrt{-\frac{\mu}{a_{Hyp}}} = 18,190\ Km/s$$

$\|\Delta \underline{v}_{obs}\| = \|\Delta \underline{v}_\infty\|$

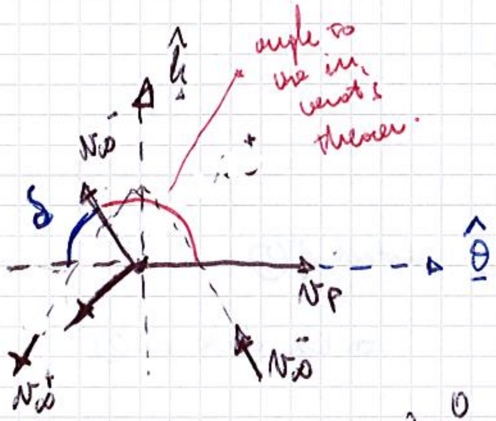


$$\begin{cases} v_{obs}^- = \underline{v}_\infty^- + \underline{v}_p \\ v_{obs}^+ = \underline{v}_\infty^+ + \underline{v}_p \end{cases} \Rightarrow \underline{v}_{obs}^+ - \underline{v}_{obs}^- = \underline{v}_\infty^+ - \underline{v}_\infty^- + \cancel{\underline{v}_p} - \cancel{\underline{v}_p} \Rightarrow \|\Delta \underline{v}_{obs}\| = \|\Delta \underline{v}_{rel}\|$$

$$\Delta v = 2 v_\infty \cdot \sin\delta = \frac{2 v_\infty}{e} \Rightarrow \|\Delta v_{obs}\| = \|\Delta v_{rel}\| = 14,797 \text{ km/s}$$

$$v_\infty^- = v_\infty^+ = v_\infty$$

$\Downarrow$



angle to the in. Carnot's theorem

$$\underline{v}_p = v_p \cdot \hat{\theta} = \begin{Bmatrix} 0 \\ v_p \\ 0 \end{Bmatrix} \quad \begin{Bmatrix} \hat{r} \\ \hat{\theta} \\ \hat{h} \end{Bmatrix}$$

$$v_\infty^- = \begin{Bmatrix} 0 \\ -v_\infty \cos\delta \\ v_\infty \sin\delta \end{Bmatrix}$$

$$v_{obs}^- = \begin{Bmatrix} 0 \\ -v_\infty \cos\delta + v_p \\ v_\infty \sin\delta \end{Bmatrix} \qquad v_p = \sqrt{\frac{\mu_\odot}{r_{\Psi-\odot}}} = 5,432 \text{ km/s}$$

$$v_{obs}^+ = \begin{Bmatrix} 0 \\ -v_\infty \cos\delta + v_p \\ -v_\infty \sin\delta \end{Bmatrix}$$

$$v_{obs}^- = \begin{Bmatrix} 0 \\ -11,18 \\ +7,398 \end{Bmatrix} \text{ km/s}, \qquad v_{obs}^+ = \begin{Bmatrix} 0 \\ -11,18 \\ -7,398 \end{Bmatrix} \qquad \|v_{obs}^+\| = \|v_{obs}^-\| = 13,39 \text{ km/s}$$

teacher: USED CARNOT'S THEOREM.

$$\|\underline{v}_{obs}^+\| = \sqrt{(v_\infty)^2 + (v_p)^2 - 2 v_p v_\infty \cos\delta} \qquad \Delta\xi = \cos^{-1}\left(\frac{v_{obs}^+ \cdot v_{obs}^-}{\|v_{obs}\|^2}\right)$$

$$\xi = -\frac{\mu_\odot}{2a^+} = \frac{1}{2}(v_{obs}^+)^2 - \frac{\mu_\odot}{r_{\Psi-\odot}} \Rightarrow a^+ = \frac{\mu_\odot}{\frac{1}{2}(v_{obs}^+)^2 - \frac{\mu_\odot}{r_{\Psi-\odot}}} = -7,37 \text{ A.U.}$$

NOT COPLANAR HOHMANN →