Orbital Mechanic's project report

Transfer from Ganymede to Europa with gravity assist around Io: optimium transfer and perturbation analysis

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1 Initial data's acquistion

The state vector (position and velocity with respect to Jupiter) of the the Jovian moons has been obtained thanks to Horizons software (available on the internet at: http://ssd.jpl.nasa.gov/?horizons) that provides real positions and velocities for planets and moons of the Solar system.

Ephemeris of Ganymede, Io, Europa and Jupiter have been aquired from Horizons 's database across the whole launch window with a time step of two hours (7200s)[1]. Data had been printed in four different text files containing the state of the three Jovian moons and Jupiter itself

- `data_Ganymede.txt` file contains position and velocity of Ganymede with respect to Jupiter for the whole launch window : form 00:00 (UT) 01/jan/2018 to 01/may/2024 with a time step of 2 hours
- `data_Europa.txt` file contains position and velocity of Europa with respect to Jupiter for the whole launch window : from 00:00 (UT) 01/jan/2018 to 01/may/2024 with a time step step of 2 hours
- `data_Io.txt` file contains position and velocity of Io with respect to Jupiter for the whole launch window : form 00:00 (UT) 01/jan/2018 to 01/may/2024 with the a time step of 2 hours
- `data_Jupiter.txt` file contains position and velocity of Jupiter with respect to the Sun for the whole launch window : from 00:00 (UT) 01/jan/2018 to 01/may/2024 with a time step of 2hours

Thank to a properly implemented function (`generation_file_data.m`) the four text data files just presented have been modified and saved into a format more suitable with the Matlab® workspace: `HORIZON_data.mat`, which contains in a single file the position and velocity vectors of all four bodies. Whereas possibile the data relative to Jupiter, Ganymede, Europa and Io have been taken directly from the acquired values (all data multiples of two hours from the initial date, as it has been done in the scripts `project_OPTIMUM_research.m` and `project_OPTIMUM_plots.m`), otherwise a linear interpolation between the two closest acquired data have been performed thank to the implemented function `interp_data.m`.

[1]The choice of a two hours time step is given by what it seemed to be a reasonable step to appreciate a diffence from the previous configuration without being too significant and risk to lose valuable solutions.

The necessity of using real ephemeris instead of unperturbed orbits can be easily shown through a qualitative comparison between the orbits of Ganymede, Io and Europa computed integrating a common restricted two body problem[2] ($\ddot{\mathbf{r}} = -\frac{\mu_{Jupiter}}{r^3}\mathbf{r}$) and the orbits plotted with the real ephemeris acquired from Horizons throughout the entire launch window. Differences immediatly appears significant cause of high perturbative effects affecting Ganymede, Io and Europa's orbits as shown in Fig1 in comparison with Fig2.

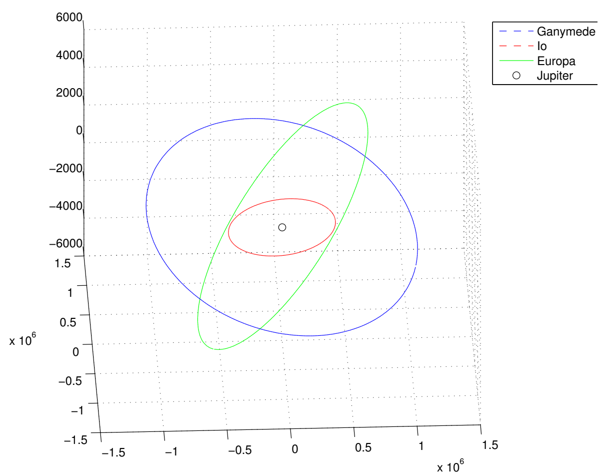


Figure 1: Unperturbed orbits of Ganymede, Europa, Io.

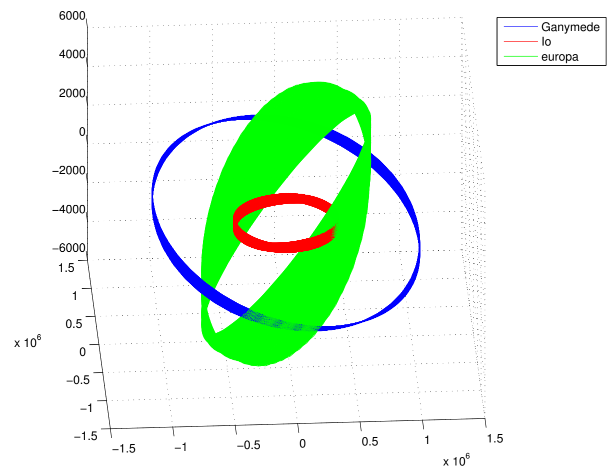


Figure 2: Real positions of Ganymede, Europa, Io across the launch window.

The presented plots are to be considered only qualitatively. I,J,K axes are unscaled in order to better appreciate the differences between the three Jovian moons orbits.
Hereafter a scaled plot to get a better feeling of the effective order of magnitude of the orbits with respect to Jupiter true size.

[2]Using initial conditions always got from Horizons at the beginning of the launch window (01/jan/2018)

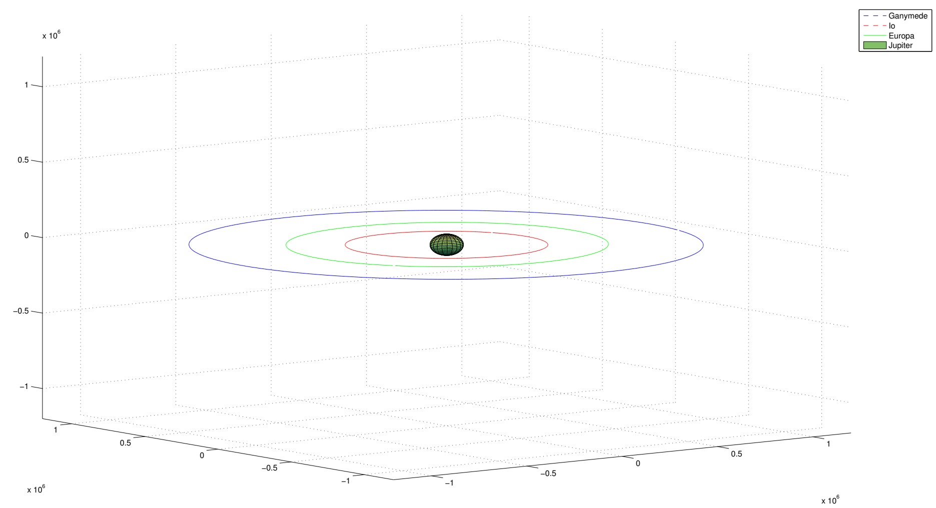


Figure 3: Scaled orbits of Ganymede, Europa, Io.

It's important to observe also that Horizons takes into account effects due to Einstein's relativity. In fact, when Horizons provides the state vectors of an astronomic body, it considers the relative time measured from a coordinate reference frame centered on the desidered body as origin with its own relative velocity with respect to the Sun [3].
In our particular case the time step computed by the software for the three Jovian moons considers the relative time due to Jupiter's velocity with respect to the Sun, therefore the same time interval computed on Earth, according to Einstein's theory, would be different because of the different velocity of the two planets. Anyways, thank to Lorentz's transformation, it's possible to show that time dilatation due to the different circular velocity of Earth and Jupiter is completely negligible. In fact:

$$\Delta t_{Earth} = \gamma \cdot \Delta t_{Jupiter}$$

Where γ represents Lorentz's factor depending on the difference in velocity between Earth and Jupiter ($u = ||\mathbf{v}_{Earth} - \mathbf{v}_{Jupiter}||$) and on the speed of

[3] See Horizons' documentation available at:http://ssd.jpl.nasa.gov/?horizons_doc

light (c) through the relation:

$$\gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$$

Since circular velocities for Earth and Jupiter are:

$$v_{Earth} = \sqrt{\frac{\mu_{Sun}}{1A.U.}} = 29.783 Km/s$$

,

$$v_{Jupiter} = \sqrt{\frac{\mu_{Sun}}{5.2A.U.}} = 13.056 Km/s$$

then:

$$\gamma = 1.00000056 \Longrightarrow \Delta t_{Earth} \sim \Delta t_{Jupiter}$$

Finally we have demonstrated that we are allowed to assume null the time difference between earthly time, Jovian time and the time evaluated on all the other astronomic bodies considered in this project. In other words, under this assumption , all "temporal" quantities presented in this report (times of flight, departure dates etc..) might be considered indipendent from the chosen reference system in witch time is measured.
All dates will be reported in the form : hh:mm:ss (UT) gg/month/year
(e.g 12:15 (UT) 13/apr/2014 [4]).

[4]Dates refer to the coordinate universal time "UTC" coincident with the "GMT" (Greenwich mean time)

2 Orbital transfer

The assigned project covers the trajectory design for a transfer from Ganymede to Europa, including a powered gravity assist performed around Io, for a launch window extending from 2018 to 2023.
To design the "Jupiter-centric" transfers among the three moons, two different Lambert's problems has been solved by using the provided Matlab® function `lambert_mick.m`. Thanks to Lambert's theorem the conical arc connecting initial and final position is univocaly defined, once time of flight is assigned, allowing us to compute initial and final velocity vectors.
At a later stage, once incoming and outcoming velocity vectors of each different Jovian moons are known, a linked-conics approach has been used to design both the gravity assist around Io and the incoming maneouver required to reach the assigned operational orbit around Europa.
Finally has been chosen, among the several different possible trajectories, the one which minimize the overall ΔV cost.

2.1 Description of the maneuver

2.1.1 Starting maneuver

As requested, the velocity of the spacraft has been retained null with respect to the starting moon (Ganymede) . Under this assumpition the first impulse cost has been computed as:

impulse1
departure impulse: $\Delta v_1 = ||\mathbf{v}_{start1} - \mathbf{v}_{Ga}||$

where :

- $\mathbf{v}_{Ga}$ represents the velocity of Ganymede at the departure time instant t_0
- $\mathbf{v}_{start1}$ represents the velocity of the spacecraft resulting from the solution of the first Lambert's problem connecting Ganymede to Io in a given time of flight (tof_1).

2.1.2 Powered gravity assist

Incoming and outcoming absolute velocities are known from the computation of the two "Jupiter-centric" transfer orbits.[5]

- $\mathbf{v}_{abs_i} \triangleq \mathbf{v}_{Lamb_1 f}$ represents the velocity of the spacecraft, resulting from the solution of the first Lambert's problem, evalueted at the end of the conical arc connecting Ganymede to Io in the time interval tof_1

- $\mathbf{v}_{abs_o} \triangleq \mathbf{v}_{Lamb_2 i}$ represents the velocity of the spacecraft, resulting from the solution of the second Lambert's problem, evalueted at the beginning of the conical arc connecting Io to Europa in the time interval tof_2

Therefore it's possible to compute the relative infinitum velocities at the edge of Io's sphere of influence by subtracting the moon velocity (evaluated at $t_1 = t_0 + tof_1$, when the spacecraft reaches the moon) to the previously defined absolute velocities[6].

$$\mathbf{v}_{rel_i} = \mathbf{v}_{abs_i} - \mathbf{v}_{Io}(t_1) \triangleq \mathbf{v}_{\infty}^{-}$$
$$\mathbf{v}_{rel_o} = \mathbf{v}_{abs_o} - \mathbf{v}_{Io}(t_1) \triangleq \mathbf{v}_{\infty}^{+}$$

Since the two infinitum velocities are univocally defined (once set t_0, tof_1 and tof_2), and generally different in modulus and direction a powered gravity assist maneouver is required to connect the two hyperbola branches. We designed such maneuver in order to have both hyperbola arcs on the same plane with the same pericentric radius so that we might operate the impulse required by the maneuver at the pericenter.
It's important to observe that, once the relative incoming and outcoming velocities are established, the pericentric radius shared by the two resultant hyperbolas is automatically determined.
In fact after having determined the angle included between $\mathbf{v}_{\infty}^{-}$ and $\mathbf{v}_{\infty}^{+}$ (δ) through the relation:

$$\delta = \arccos\left(\frac{\mathbf{v}_{\infty}^{-} \cdot \mathbf{v}_{\infty}^{+}}{||\mathbf{v}_{\infty}^{-}|| \cdot ||\mathbf{v}_{\infty}^{-}||}\right)$$

[5]Obtained as solution of two different Lambert's problem.

[6]Since linked-conics model has been chosen, the time required to the spacraft to cross Io's sphere of influence is retained infinitesimal, reason why both initial and final relative velocites are subtracted by Io's velocity at $t_1 = t_0 + tof_1$

it's possible to see that, due to hyperbolic geometry, the following five relations must be simultaneously satisfied:

1)$e_1 = 1 + \frac{r_p {v_\infty^-}^2}{\mu_{Io}}$

2)$e_2 = 1 + \frac{r_p {v_\infty^+}^2}{\mu_{Io}}$

3)$\delta_1 = \arcsin(\frac{1}{e_1})$

4)$\delta_2 = \arcsin(\frac{1}{e_2})$

5)$\delta = \delta_1 + \delta_2$

Where:

- $\delta 1$ and $\delta 2$ represents respectively the angles included between the axis normal to the eccentricity vector (common to both hyperbolas) and $\mathbf{v}_\infty^-$ and $\mathbf{v}_\infty^+$
- e_1 and e_2 represents respectively the eccentricity of the first and second hyperbola.

The solution with respect to r_p of this non-linear system is unique and, once r_p is known, the geometry of the two hyperbola is fully defined.
After the evaluation of the pericentric radius a check over the obtained value has been done in order to avoid those trajectories which would cause the satellite to impact on Io's soil. Such a control has been performed verifying that r_p was greater than Io's radius plus a safety margin.
Finally the impulse cost required to pass from the first hyperbolic branch to the second can be computed as:

impulse 2
powered gravity impulse : $\Delta v_2 = |v_{p_2} - v_{p_1}|$

where :

- $\mathbf{v}_{p_1}$ represents the pericentric velocity of the incoming hyperbola arc.
- $\mathbf{v}_{p_2}$ represents the pericentric velocity of the outcoming hyperbola arc.

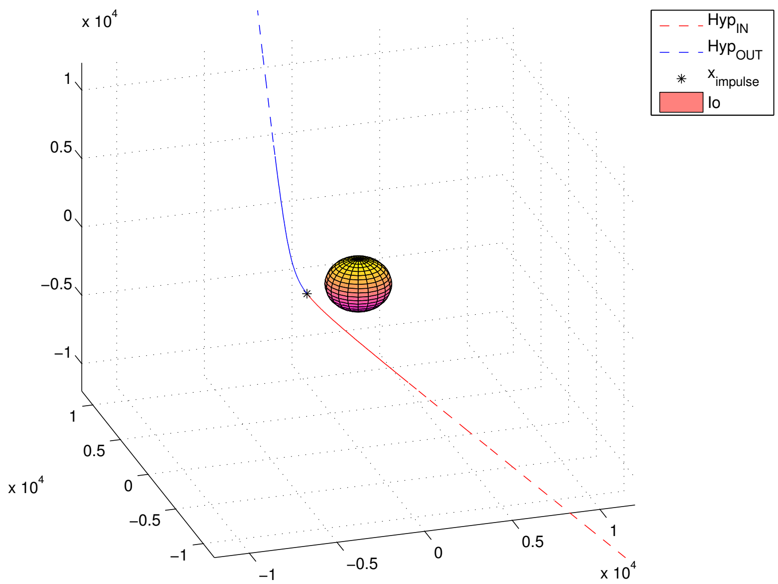


Figure 4: Representation of a generic powered gravity assist maneuver around Io

2.1.3 Incoming maneuver

The general idea for the incoming maneuver is to operate a circularization of the incoming hyperpola on Europa at the pericenter, proceding with a simple change of plane from the obtained circular orbit (same inclination of the hyperbola) to an idential circular orbit with an inclination equal to the one of the operational orbit and finally make an in-plane maneuver to reach to the final orbit.
The reason for such a choice is due to the simplicity of the maneuver: two in-plane maneuvers and a change of plane (change of inclination solo). In fact we did not want to deal with a single impulse maneuver in order to avoid the use of spherical triangles and the errors that would have derived from such approximation[7]. On the other hand we might have opted for a

[7]The spherical triangle formulas are valid strictly for circular orbits, therefore the farther your orbits are to be circular, the greater is the error of such approximation (from

two impulses manuever: an in-plane one from the incoming hyperbola to the circular orbit and afterward a change of plane form the circular orbit directly to the operational one. In this case the error due to the use of spherical triangles would have probably be negligible since operating between a circular orbit and an elliptic one with a value of eccentricity near to zero ($e_f = 0.0232$), but still in order to operate a simpler and more rigorous choice we decided to perform the three impulses maneuver previously described.
In order to design the entire incoming maneuver two other assumptions have been made:

1. The infinitum velocity vector has been placed[8] in such a way that the nodal axis of the hyperbola was coincident with the nodal axis of the operational orbit, and the same nodal axis has been kept even for the two circular orbit in order to operate a simple change of inclination at the nodal axis.

2. The pericenter radius of the incoming hyperbola has been chosen equal to the one of the final orbit, as well as it has been done for the two circular orbit in order to guarantee the intersections incoming hyperbola-first circular orbit and second circular orbit-operational orbit at the pericenter.

The assigned operational orbit is characterized by the data:

Pericenter eigth [Km]	Apocenter eight [Km]	i[deg]	Ω [deg]	ω [deg]
120	200	78	250	80

The equivalent orbital parameters are:

semiaxis a_f [Km]	e_f	i_f[deg]	Ω_f [deg]	ω_f [deg]
17205	0.0232	78	250	80

Incoming absolute velocitiy of the spacecraft into Eupropa's sphere of influence is known from the computation of the second "Jupiter-centric" transfer orbit [9], therefore it's possible to compute the infinitum velocity vector at

hyperbolic to elliptic errors might be significant).

[8] In linked-conics approach the position of the infinitum velocity vector at the edge of the sphere of influence is unspecified, therefore it might be placed where more convenient.

[9] Obtained as solution of the second Lambert's problem.

the Europa's sphere of influence considering Europa's velocity at the time instant $t_2 = t_0 + tof_1 + tof_2$ when the spacecraft reaches the Jovian moon:

$$\mathbf{v}_{rel_i} = \mathbf{v}_{abs_i} - \mathbf{v}_{Eu}(t_2) \triangleq \mathbf{v}_\infty^-$$

Once the incoming hyperbola is defined, we procede by evaluating the two circular orbits:

1. $circ_1$ of radius r_{p_f}, coplanar to the incoming hyperbola and intercepting the hyperbola at the pericenter.

2. $circ_2$ of radius r_{p_f}, coplanar to the operationa orbit and intercepting the operational orbit at the pericenter.

These circular arcs meet where the orbital plane of the hyperpola intercepts the operational orbit's plane; such axis obviously coincides with the nodal axis common to the incoming hyperbola and the operational orbit as it was meant by design.

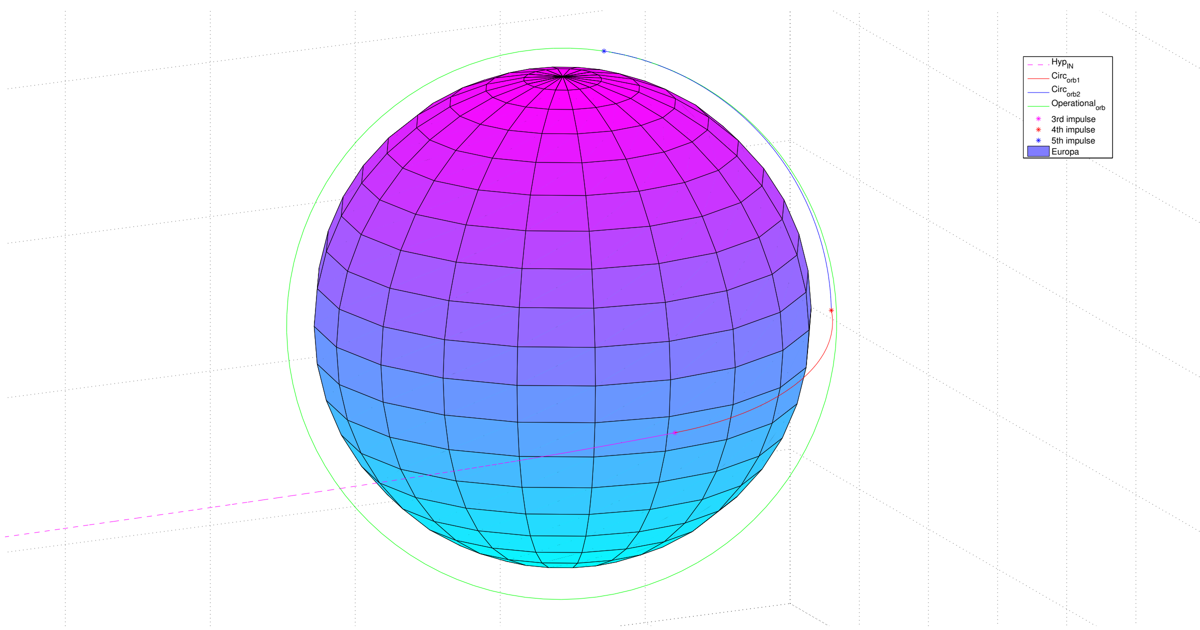


Figure 5: General incoming maneuverr

Finally when all the transfer orbits are defined it is possible to evaluate the cost of the three impulses required by the maneuver as it follows:

impulse3
braking impulse : $\Delta v_3 = |v_{p_{hyp}} - v_{circ_1}|$

impulse4
plane-change impulse : $\Delta v_4 = |2 \cdot v_{circ_1} \cdot \sin(\frac{i_f - i_{hyp}}{2})|$

impulse5
final impulse : $\Delta v_5 = |v_{p_f} - v_{circ_1}|$

2.2 Criteria for the optimum research

In order to find the optimal transfer trajectories (the ones capable to minimize the overall ΔV cost), without modifying the design of the incoming maneuver and of the powered gravity assist, a minimization with respect to the three following degrees of freedom has been performed[10]:

1. **time start - t_0 :** departure time expressed in seconds starting from the date 00:00 (UT) 01/jan/2018 to 00:00(UT) 01/jan/2024 with a time step of 2 hours(7200s).

2. **time of flight 1 - tof_1 :** time of flight expressed in seconds of the first Lambert transfer from Ganymede to the bound of Io's sphere of influence. Set of values tested starting from $57600s$(16 hours) to $345600s$(4 days) with a time step of 2 hours(7200s).

3. **time of flight 2 - tof_2:** time of flight of the second Lambert transfer from Io to the bound of Europa's sphere of influence. Set of values tested starting from $57600s$(16 hours) to $345600s$(4 days) with a time step of 2 hours(7200s).

The necessity of testing the departure date across the whole launch window is due to perturbative effects affecting Ganymede, Io and Europa's orbit. In fact, if unperturbed orbits are considered, the same configuration (in the meaning of reciprocal position of the Jovian moons) repeats around each 21 days[11] and therefore the research of the optimum condition could have been reduced at that time frame.
The minium value chosen to test the times of flight ($57600s$) is the lowest one, empirically found after running several tests, capable to solve both Lambert's problem with a closed conic which was multiple of the time step[12].
The maximum value selected to test tof_1 and tof_2 ($345600s$) has been chosen after having observed that further increasing of the times of flight would have produced none substantial decreasing in the global cost.
Results of the optimization are the following:

[10] Optimization procedure is performed into Matlab®'s file: `project_OPTIMIZER.m`

[11] Approximate period of Ganymede, Europa and Io is respectively of 1, 3 and 7 days.The least common multiple between the orbital periods of the three moons guarantees the repetition of the configuration

[12] The necessity of using a time step of 2 hours is given by the fact that the data acquired from the Horizon software have that same time step; this allows us to recall directly the needed data without undergoing an interpolation process.

optimal t_0[s]	**departure date**	**optimal tof_1[s]**	**optimal tof_1[s]**
106941600 s	18:00 (UT) 21/may/2021	180000s	165600s

To show graphically that the best possible values of tof_1 and tof_2 were chosen, a pork-chop diagram is provided in Fig.6. Such pork chop diagram has been obtained testing all the values of tof_1 and tof_2 contained in the previously described set: [57600÷345600]s and maintainig the departure date settled on the 20:00 (UT) 12/sept/2022.

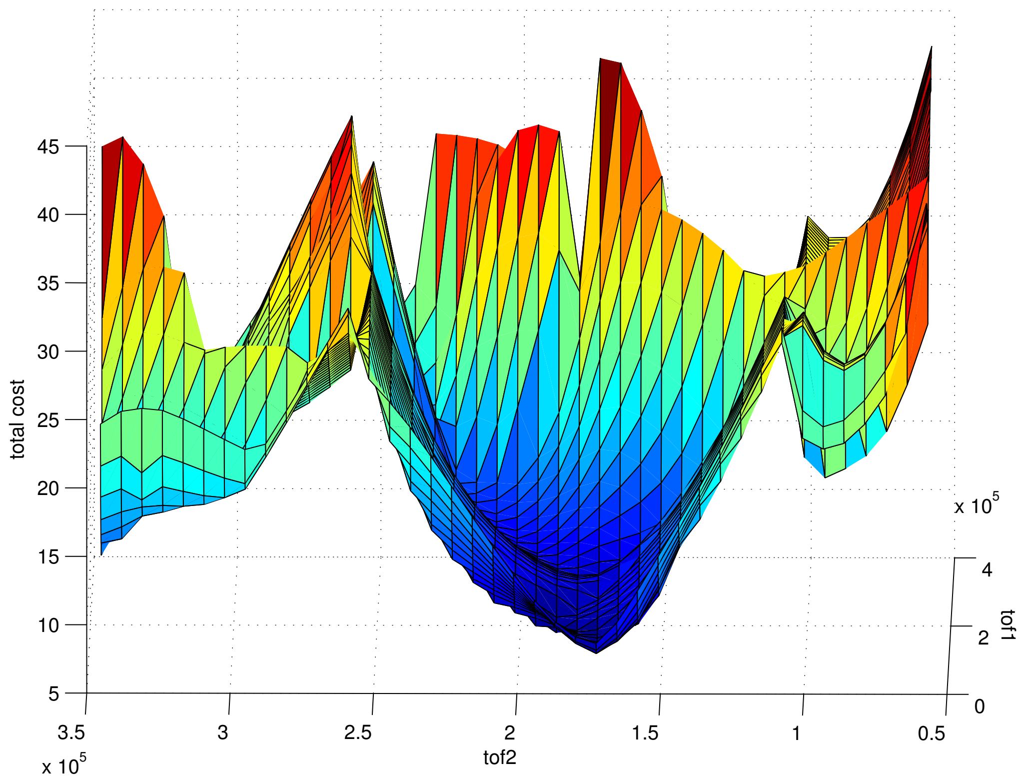


Figure 6: Pork-chop diagram considering all the solution got through the optimization procedure

Unfortunately the most part of the solutions computed impacts on Io's soil and therefore have to be excluded. In other words, a really small part of the values of tof_1 combined with tof_2 generates two Labert's trajectories capabable to perform a powered gravity assist avoiding an impact on Io's soil (as shown in Fig.7).

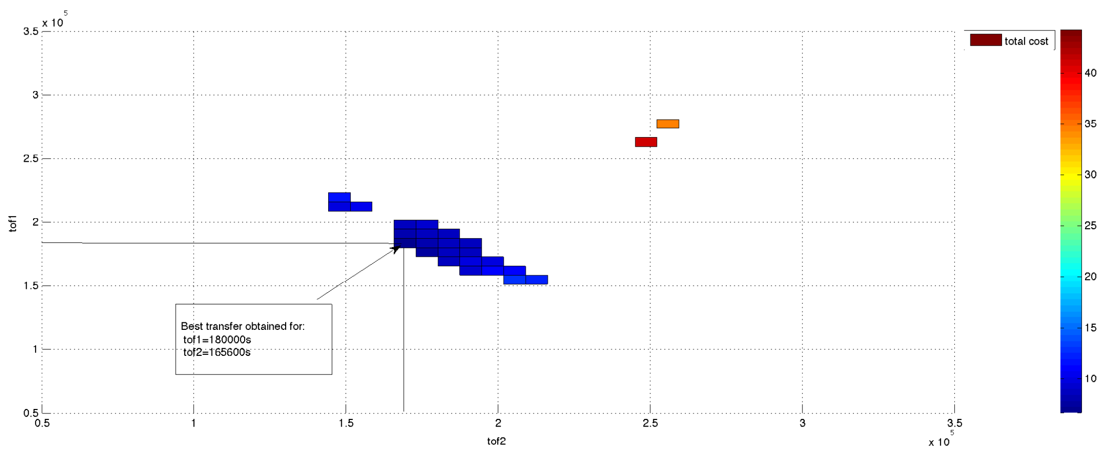


Figure 7: Pork-chop diagram considering only solutions that don't impact on Io's soil

2.3 Optimal transfer

Results hereunder presented[13] refer to the best transfer obtained through the minimization cost procedure previously explained ($t_0 = 106941600s$, $tof_1 = 180000s$ and $tof_2 = 165000s$).

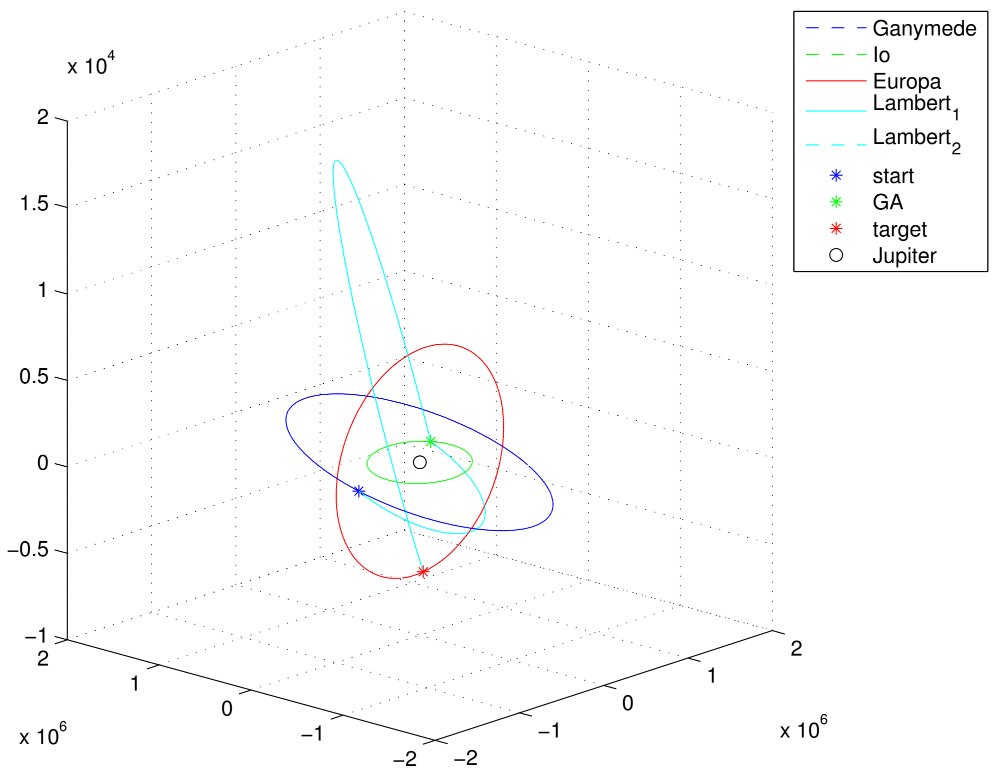


Figure 8: Best "Jupiter-centric" transfers obtained through the optimization process illlustrated in paragraph 1.3. (scale on vertical axis has been decreased in order to appriciate the plane change)

[13] Optimal transfer is provided into Matlab® script: `project_OPTIMUM_PLOT.m`.

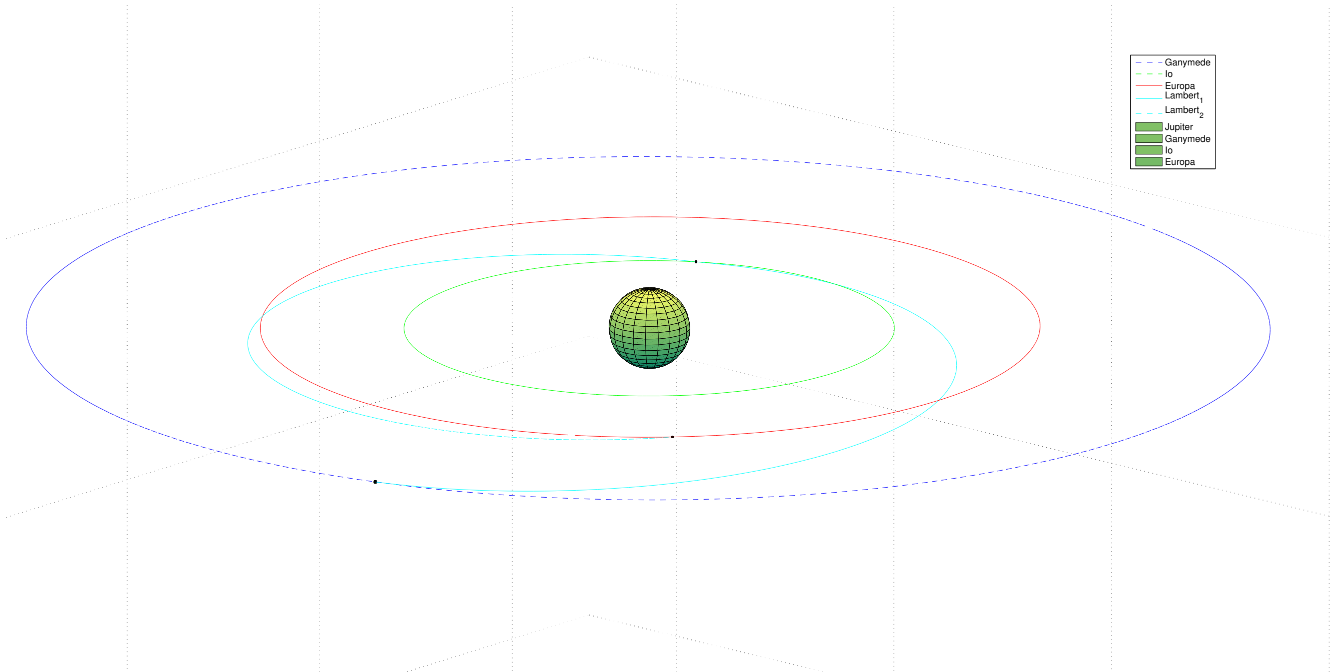


Figure 9: Best “Jupiter-centric” transfers obtained through the optimization process. (scale of all axes has been kept equal to appriciate true proportions)

1 First Lambert arc

The spacecraft will depart from Ganymede on date: **18:00 (UT) 21/may/2021**. At this time instant the first impulse must be given to the spacecraft:

impulse1
departure impulse: $\Delta v_1 = 2.7530 Km/s$

The first Lambert arc is characterized by the following orbital parameters:

semiaxis a_1 [Km]	e_1	i_1[deg]	Ω_1 [deg]	ω_1 [deg]	θ_{start1}	θ_{end1}
743778.9	0.4431	0.246	20.25	352.98	182.17	17.27(377.27)

2 Powered gravity assist

Io will be reached on date **20:00 (UT) 23/may/2021**.
At this time instant a second impulse must be given to the spacraft [14]:

impulse 2
powered gravity impulse : $\Delta v_2 = 0.3982 Km/s$

Gravity assist contribution[15] : Δv=1.0920

Incoming hyperbola's branch si characterized by the following orbital parameters:

semiaxis $a_{in.}$ [Km]	$e_{in.}$	$i_{in.}$[deg]	$\Omega_{in.}$ [deg]	$\omega_{in.}$ [deg]
-407.2501	5.6957	159.57	91.41	274.82

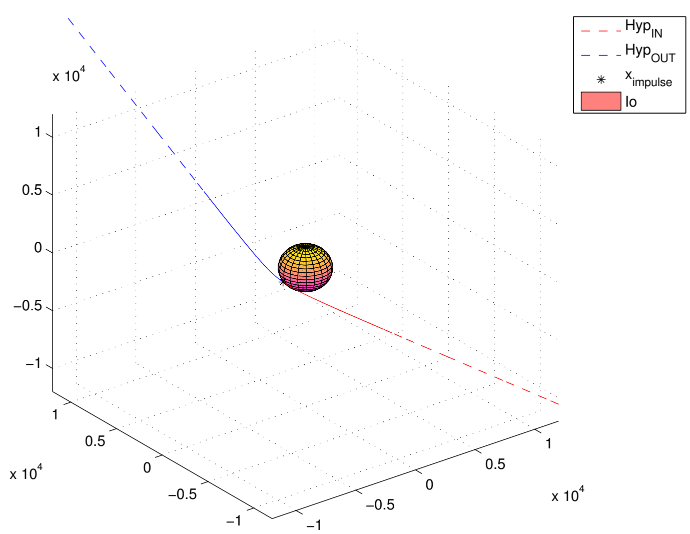


Figure 10: Best powered gravity assist maneouver

[14]Linked-conics approach suppose null time to cross the SOI of the planet
[15]It's the contribution given to the whole maneuver by the presence of the attractor $(\Delta v_\infty - \Delta v_{poweredGA})$

While the outcoming hyperbola's branch is characterized by the following orbital parameters:

semiaxis $a_{out.}$ [Km]	e_{out}	$i_{out.}$[deg]	$\Omega_{out.}$ [deg]	$\omega_{out.}$ [deg]
-534.2901	4.5792	159.57	91.41	274.82

After few easy calculation it's immediate to see that the two hyperbolas share the same pericenter radius equal to: $||\mathbf{r_p}|| = 1912.30Km$ [16].

3 Second Lambert arc

Second "Jupiter-centric" transfer is characterized by the follwing orbital parameters:

semiaxis a_2 [Km]	e_2	i_2[deg]	Ω_2 [deg]	ω_2 [deg]	θ_{start2}	θ_{end2}
582086.2	0.3058	1.529	28.86	328.58	33.06	227.47

Thank to the properly designed powered gravity assist maneuver the spacecraft will automatically enter this trajectory once it has left Io's sphere of influence with no need for other impulses to be performed.

4 Incoming maneuver

Europa will be reached on date **18:00 (UT) 25/may/2021**.
The impulse to pass from the incoming hyperbola to the first auxiliary circumference ("braking-impulse") must be given at the this time instant.

It's important to remark the fact that the optimization procedure operates with incoming hyperbolas which tend to be the closest possible to the solution coplanar to the operational orbit in order to reduce the entire maneuver to a single in-plane impulse from the hyperbola directly to the final orbit. In fact, by computation, Δi=0.02 [deg] for the optimal solution found.

The orbital parameters characterizing the incoming hyperbola *hyp* are:

semiaxis a_{hyp} [Km]	e_{hyp}	i_{hyp}[deg]	Ω_{hyp} [deg]	ω_{hyp} [deg]
-212.12	8.9224	77.98	250.00	268.65

[16] Io radius $R_{Io} = 1821.3Km$

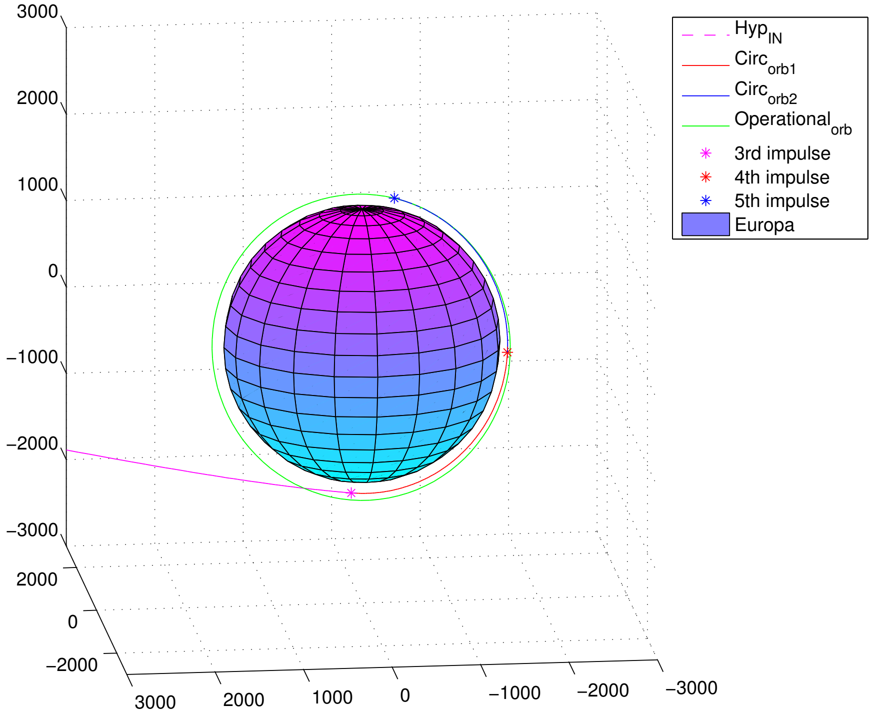


Figure 11: Best overall incoming maneuver

The orbital parameters characterizing the first auxiliary circumference $circ_1$ are:

semiaxis a_{circ_1} [Km]	e_{circ_1}	i_{circ_1}[deg]	Ω_{circ_1} [deg]	ω_{circ_1} [deg]	$\theta_{start_{circ1}}$	$\theta_{end_{circ1}}$
1680.50	0	77.98	250.00	0	0	91.35

impulse3
breaking-impulse : $\Delta v_3 = 2.962 Km/s$

This first auxiliary circular orbit will cross the nodal axis and meet the second one at the time instant **18:32:25 (UT) 25/may/2021**, at this moment the plane-change impulse will be performed.

Orbital parameters of the second circular conic are:

semiaxis a_{circ_2} [Km]	e_{circ_2}	i_{circ_2}[deg]	Ω_{circ_2} [deg]	ω_{circ_2} [deg]	$\theta_{start_{circ2}}$[deg]	$\theta_{end_{circ2}}$ [deg]
1680.50	0	78	250	0	91.35	171.35

impulse4
plane-change impulse : $\Delta v_4 = 0.001 Km/s$

It's now possible to compute the cost of immission into final orbit:

impulse5
final impulse: $\Delta v_5 = 0.01592 Km/s$

Finally the cost of the overall transfer from Ganymede to Europa performing the powered gravity assist will be:

OVERALL
MISSION COST: $\Delta v_{tot} = \Delta v_1 + \Delta v_2 + \Delta v_3 + \Delta v_4 + \Delta v_5 = 6.1297 Km/s$

Final orbit will be reached at pericenter at the time instant :
19:01:48 (UT) 25/may/2021 .

3 Perturbation analysis

A perturbation analysis is hereafter provided showing how the non uniformity of Europa's mass, the solar radiation pressure[17] and the presence of a third body (Jupiter) affect the keplerian parameters of the assigned operational orbit around Europa once reached[18].
As a reminder, the initial keplerian parameters of the (un-perturbed) operational orbit and the date of arrival are reported:

19:01:48 (UT) 25/may/2021

a_i [Km]	e_i	i_i[deg]	Ω_i [deg]	ω_i [deg]
1720.5	0.0232	78	250	80

Physical quantities necessary to compute the perturbation analysis, are the following:

- $J_{2_{Eu}} = 435.5 \times 10^{-6}$coefficient of Europa's first zonal harmonic
- $\varphi = 50.57 frac W/m^2$ solar flux

While provided data of the spacecraft are the following:

- $A_{spacecraft} = 3m^2$ surface of the spacecraft
- $m_{spacecraft} = 780Kg$ mass of the spacecraft
- $\epsilon = 0.34$ reflectivity of the spaceraft's surface

Perturbations have been computed using a VOP [19] approach performed through a numerical integration thanks to Matlab®'s algorithm `ode113`. In order to ensure an acceptable convergence of the method really low tolerances have been set in performing time integration[20].
As requested a differentiated analysis between short and long period has been performed:

[17]A double eclipse factor has been considered: one for Europa with respect to Jupiter and another for the spacecraft with respect to Europa.
[18]Since Europa has no significant presence of a gaseous atmosphere, perturbative effects due to drag and other aerodynamical forces have been completly neglected.
[19]Variations of orbital parameters
[20]Tolerences set through Matlab®'s command: `odeset('RelTol',1e-12,'AbsTol',1e-24)`.

1. **short period analysis:** perturbative effects have been evaluated for the duration of a period of the operational orbit ($\mathbf{T_{op} = 7942.6s}$).

2. **long period analysis:** perturbative effects have been evaluated from the arrival date to either the next solar year or the impact date depending on which of the two occurs first.

In order to better appreciate the mean behavior of each perturbative effect (for long period analysis), a filtering process has been performed. Such result has been obtained computing the mean value of each orbital parameter over 50 periods of the operational orbit[21].

The architecture of the Matlab®'s scripts necessary to compute the perturbation analysis is the following:

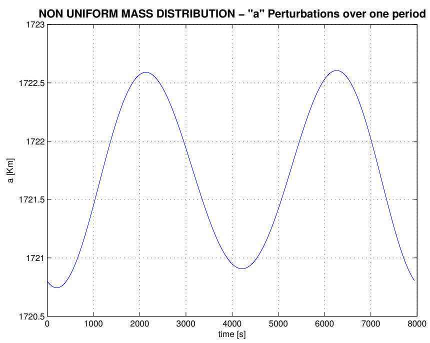
perturbative effect	**main script**	**recalled function**
non uniformity of mass	`perturbation_NON_UNIF_MASS.m`	`kp_mass.m`
third body presence	`perturbation_THIRD_BODY.m`	`kp_3rdb.m`
solar radiation pressure	`perturbation_SOL_RAD_PRES.m`	`kp_solar_wind.m`
global effect	`perturbation_TOTAL.m`	`kp_total.m`

[21] Because of the shortness of T_{op} the simple use of it would have caused to obtain a filter almost equal to the function itself. After running several tests $50T_{op}$ seemed to us the best choice to get the main behavior of the analized perturbation.

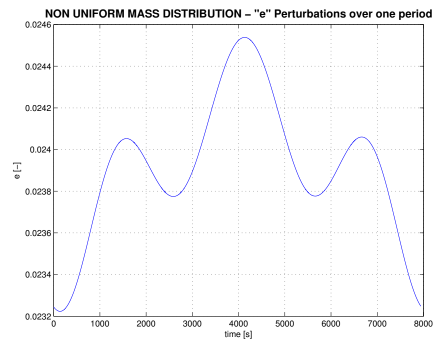
3.1 Short period analysis

3.1.1 Perturbation due to non uniformity of mass (Europa)

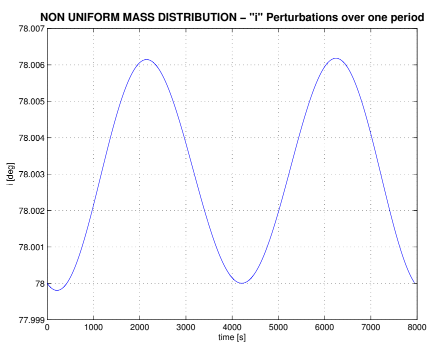
a [Km]



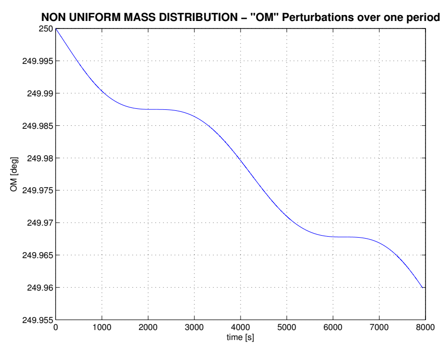
e



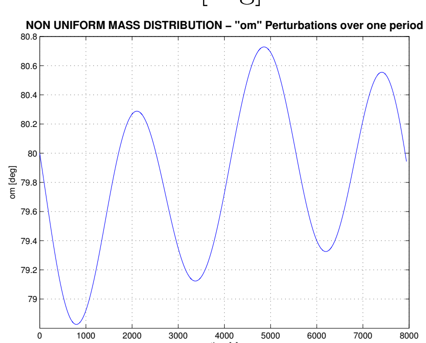
i[deg]



Ω [deg]

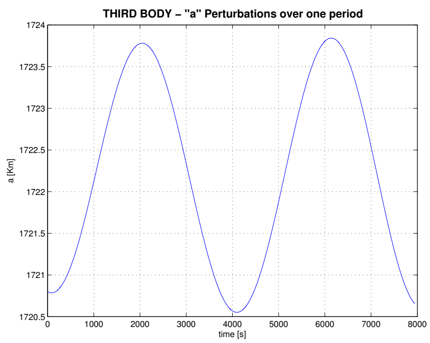


ω [deg]

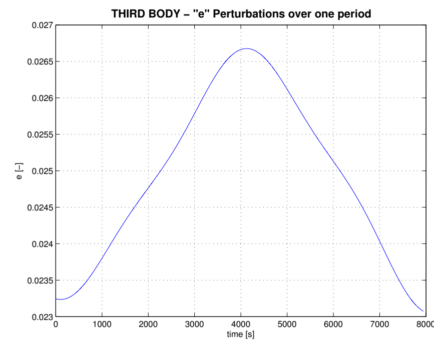


3.1.2 Perturbation due to third body presence (Jupiter)

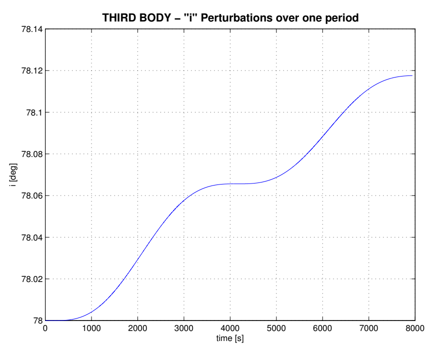
a [Km]



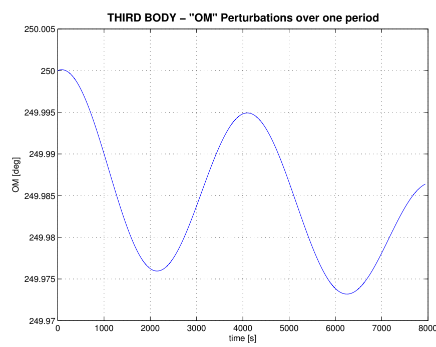
e



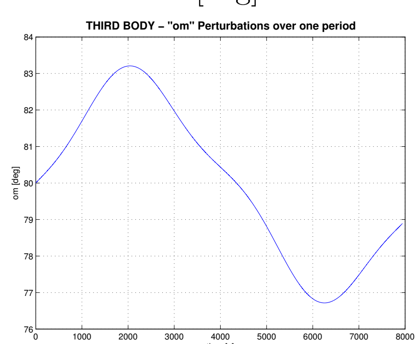
i[deg]



Ω [deg]

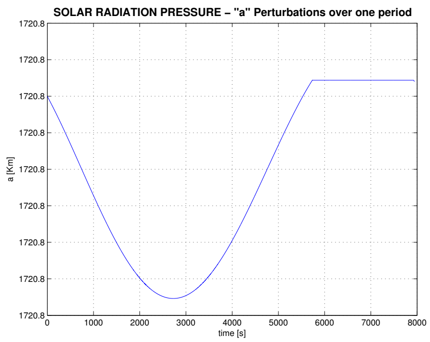


ω [deg]

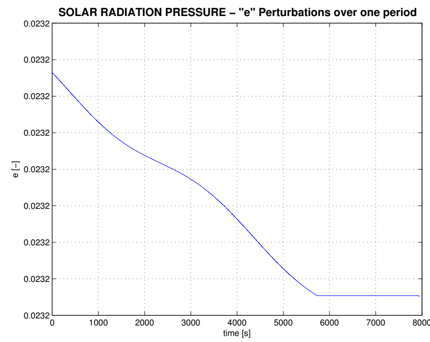


3.1.3 Perturbation due to solar radiation pressure

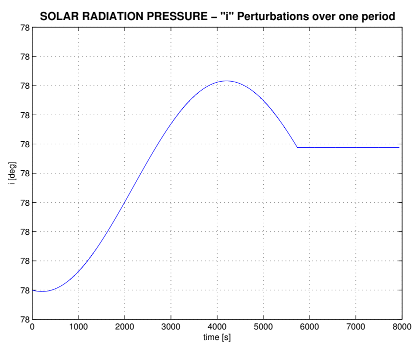
a [Km]



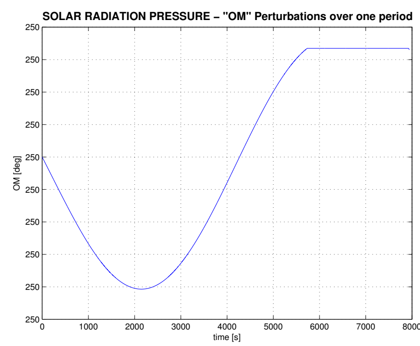
e



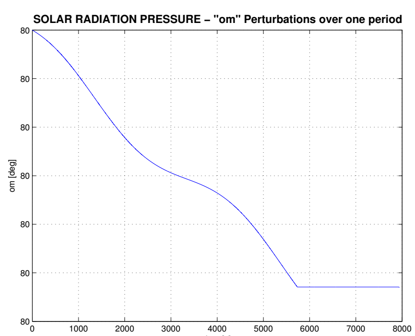
i[deg]



Ω [deg]

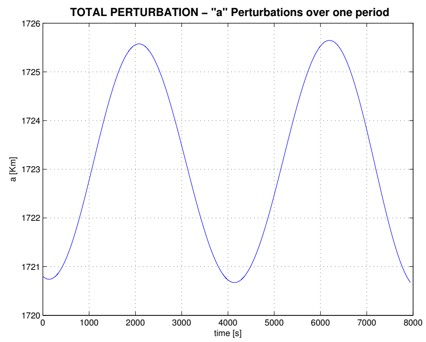


ω [deg]

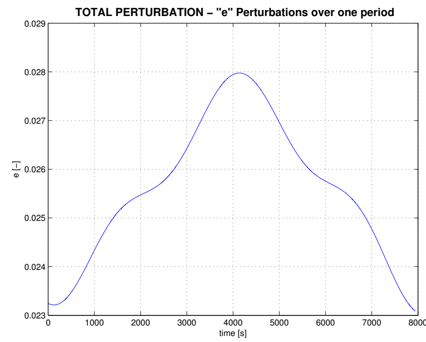


3.1.4 Global perturbative effect

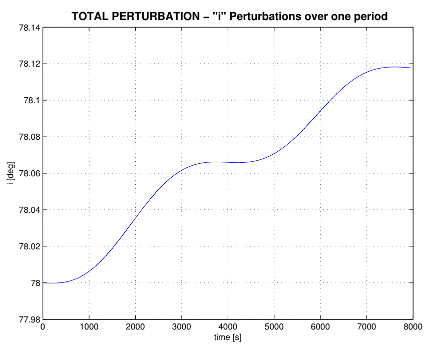
a [Km]



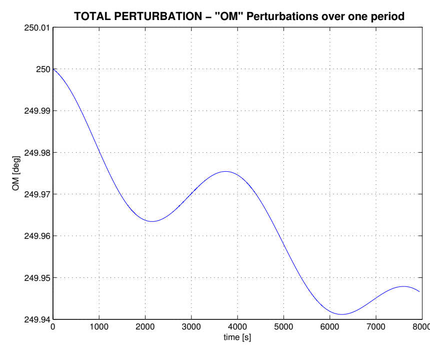
e



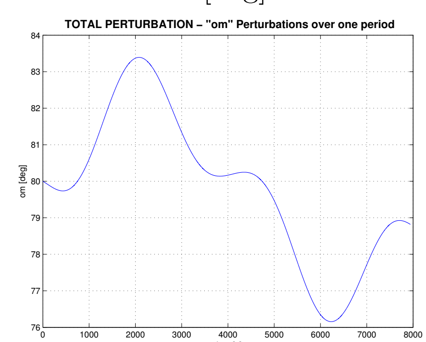
i[deg]



Ω [deg]



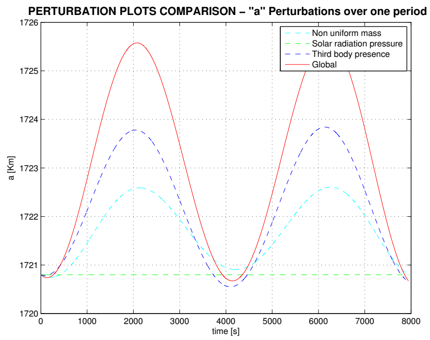
ω [deg]



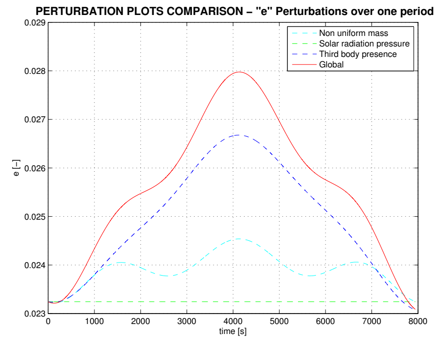
In order to better appreciate the influence of each perturbative effect on the global result, graphs plotting all the contributes are hereafter shown[22].

[22]Only short period graphs are presented because same trend would have been appreciated on long period and the computational cost would have been much higher.

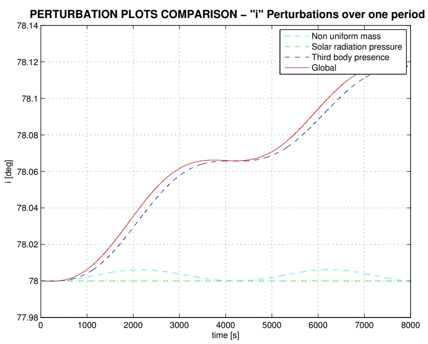
a [Km]



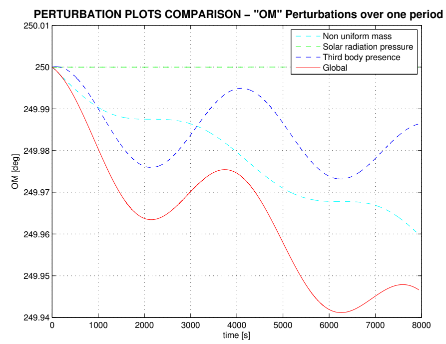
e



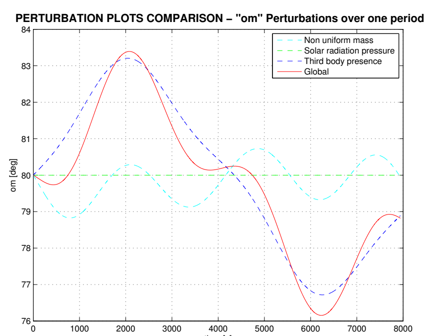
i[deg]



Ω [deg]



ω [deg]



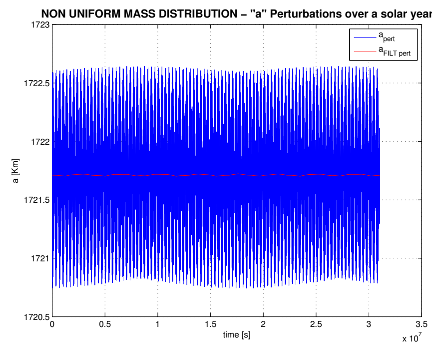
It's now evident that the dominant perturbation affecting the spacecraft orbit it's the presence of a third body, as expectable considering Jupiter's high mass and the small distance of Europa from Jupiter.

Perturbative effects comparison is provided into Matlab® script: `pert_PLOTS_COMPARISON.m`

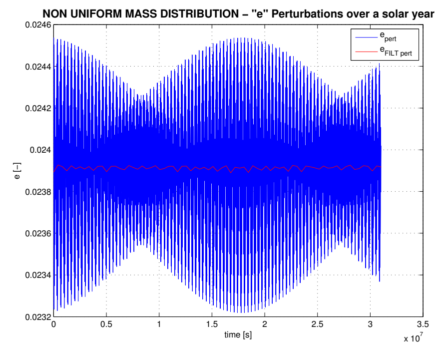
3.2 Long period analysis

3.2.1 Perturbation due to non uniformity of mass (Europa)

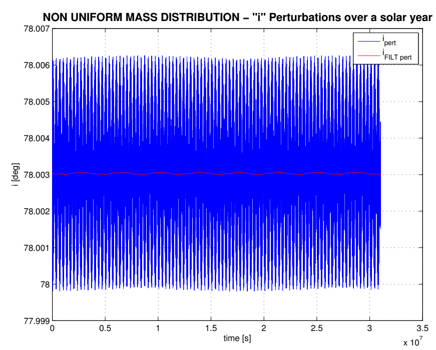
a [Km]



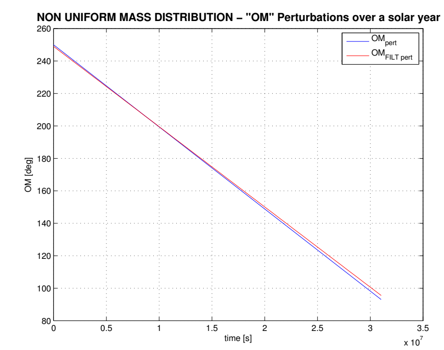
e



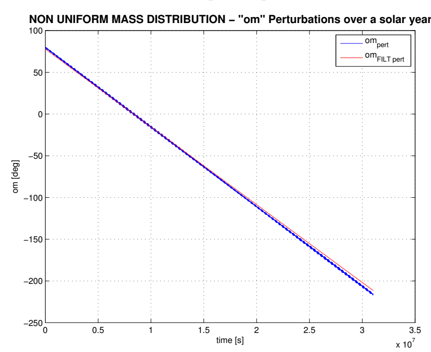
i[deg]



Ω [deg]

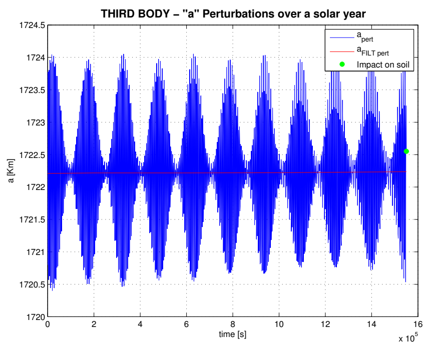


ω [deg]

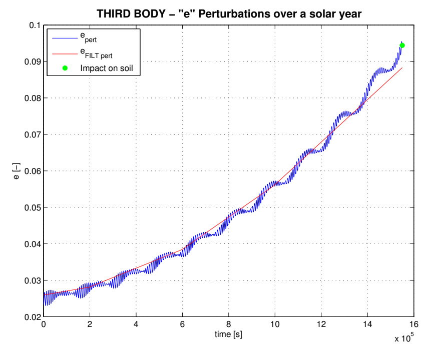


3.2.2 Perturbation due to third body presence (Jupiter)

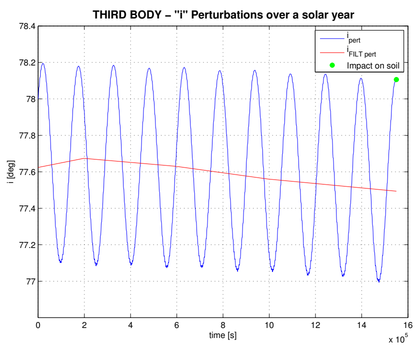
a [Km]



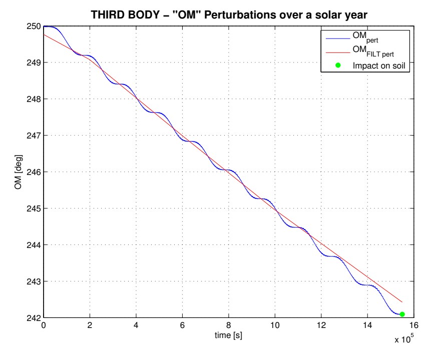
e



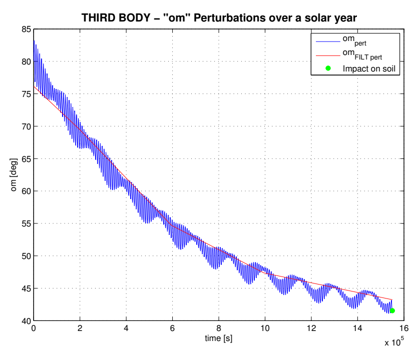
i[deg]



Ω [deg]



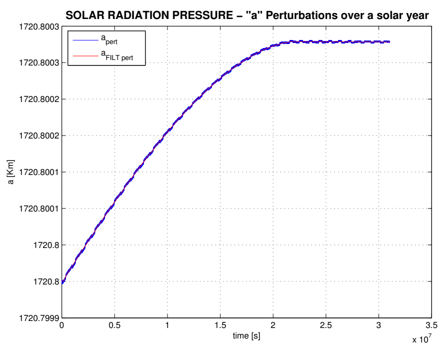
ω [deg]



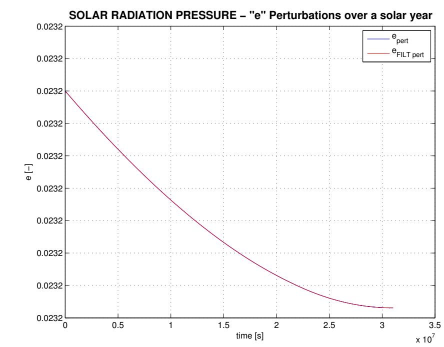

```
Impact on Europa's soil after:
17 days
22 hours
21 minutes
45 seconds
```

3.2.3 Perturbation due to solar radiation pressure

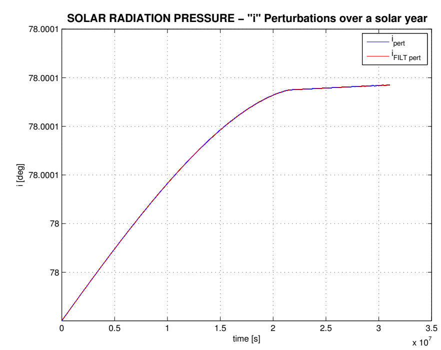
a [Km]



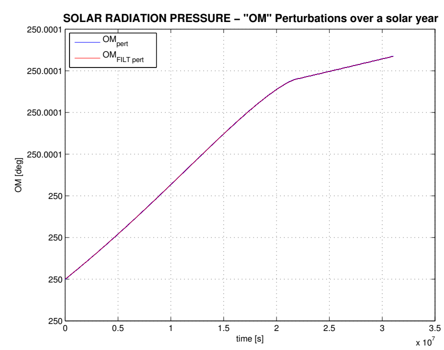
e



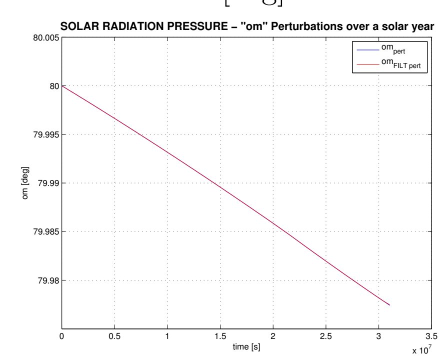
i[deg]



Ω [deg]

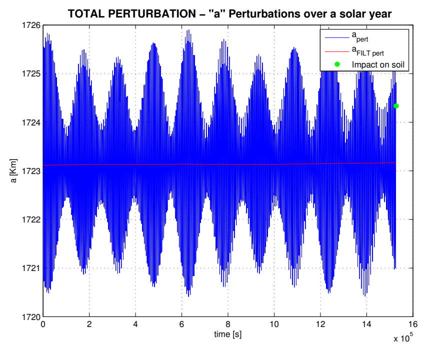


ω [deg]

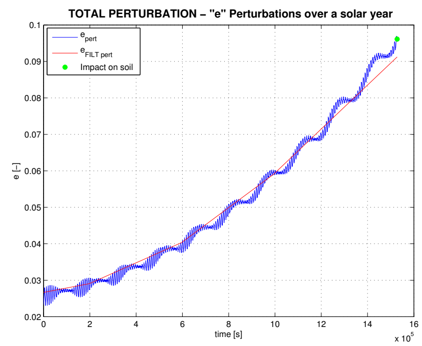


3.2.4 Global perturbative effect

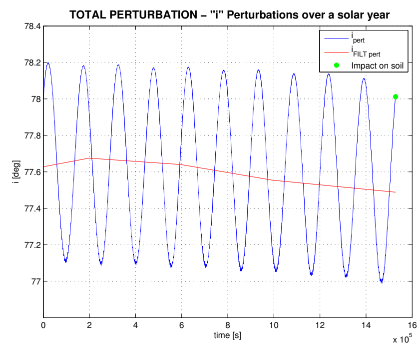
a [Km]



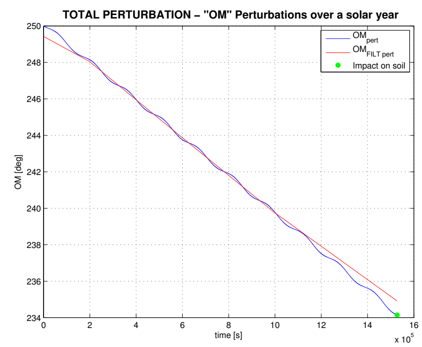
e



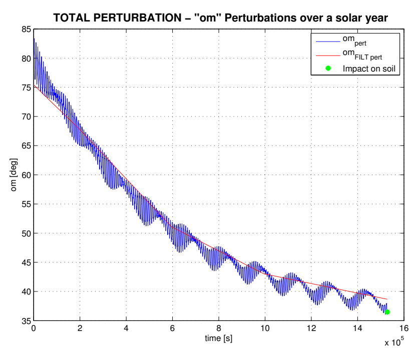
i[deg]



Ω [deg]



ω [deg]



`Impact on Europa's soil after:`

`17 days`
`16 hours`
`12 minutes`
`17 seconds`

3.3 Operational orbit after perturbation analysis

If no control system is provided on board, after about 17 days, the spacecraft will impact on Europa's soil.
Throughout this time frame the inital operational orbit:

a_i [Km]	e_i	i_i[deg]	Ω_i [deg]	ω_i [deg]
1720.5	0.0232	78	250	80

will be perturbed into:

a_f [Km]	e_f	i_f[deg]	Ω_f [deg]	ω_f [deg]
1724.30	0.0961	78.01	234.14	36.48

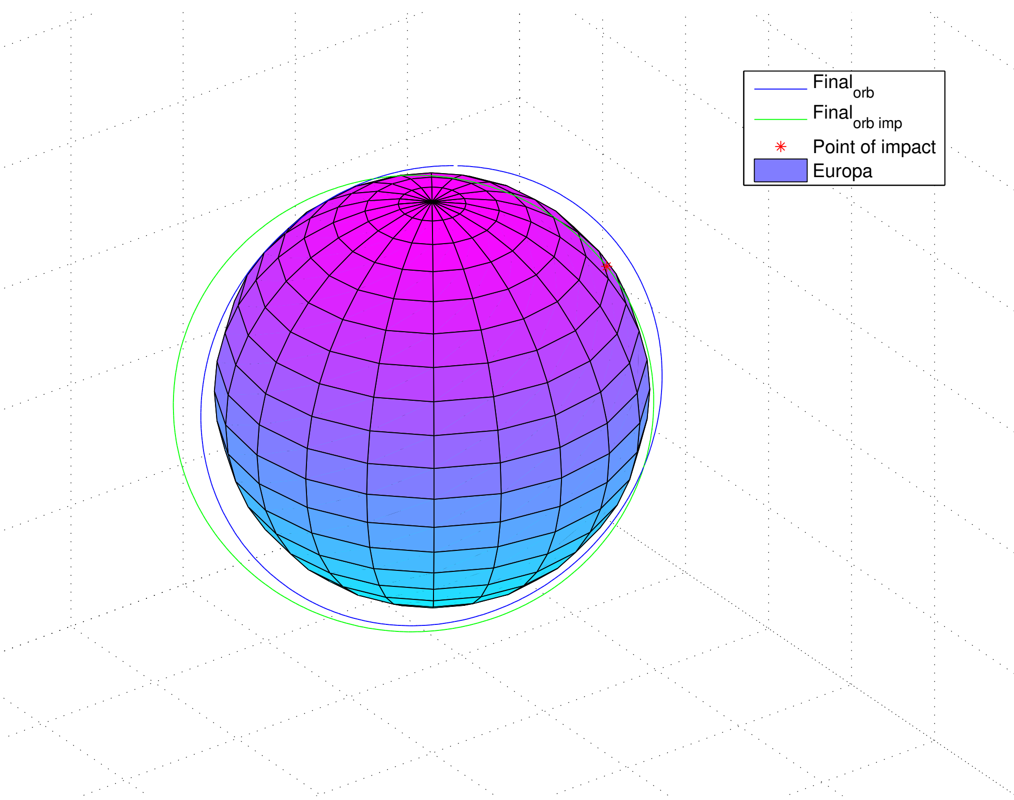
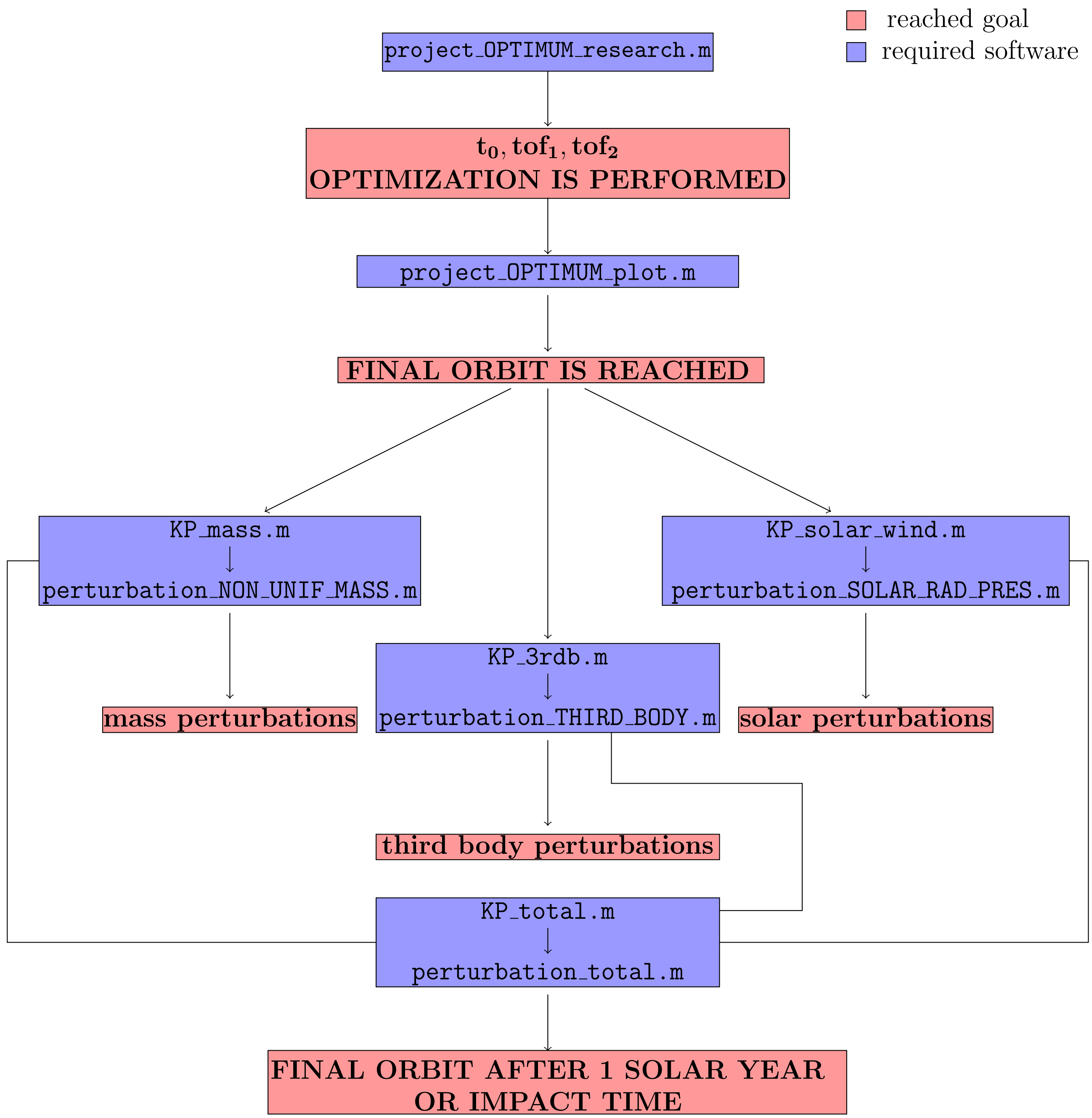


Figure 12: Spacecraft's orbit change due to overall perturbative effects

4 Flow diagram



5 Faced problems

Even though hints of several different encountered problems might be present in the previous part, in the following sections all faced problems are going to be listed and individually treated.

5.1 Data acquisition

At first the problem of data acquisition consisted in finding either the state vectors for the three Jovian moons or the three position vectors plus the mean orbital parameters values at the beginning of our launch window[23]. With these informations we would have been able to compute state vectors of each moon at any time, hence proceding with all the necessary calculations. Afterward, as described in section 1, we noticed that the variations of the orbital parameters were not negligible and therefore we proceded acquiring a set of data sufficient to cover properly the entire launch window.

5.2 Defining orbits orietation

In the defining process of the orbits orientation we often encountered throughout calculations a lack of data, therefore we frequently solved it thanks to the geometry of the problem.
Oftentimes to get the $\hat{\mathbf{h}}$[24] we exploited the fact that the result of a vectorial multiplication is a vector perpendicular to the plane containing the previous two vectors, hence $\hat{\mathbf{h}} = \frac{\mathbf{v}_a \times \mathbf{v}_b}{||\mathbf{v}_a \times \mathbf{v}_b||}$ would be a unitary vector perpendicular to the plane including $\mathbf{v}_a$ and $\mathbf{v}_b$.

5.3 Computational problems

Amoung the encountered problems a couple of them were strictly of computational nature.

[23]The given function `uplanet_mick.m` to generate the initial data for the project did not comprehend our assigned astronomic bodies.

[24]Directional unitary vector of angular momentum.

5.3.1 `ode113` convergency

When integrating over long time periods the propagation of approximation errors starts to be significant[25], therefore it's needed to be set really low tolerance's values in order to guarantee the convergency even though it means a sensible increase of the computational cost.

5.3.2 Resolution of non linear equation

Solving the non-linear system (presented in section 2.1.2) with respect to δ_1, the resulting non-linear function shows a really flat slope in the proximity of the zero[26] as shown in Fig.13.

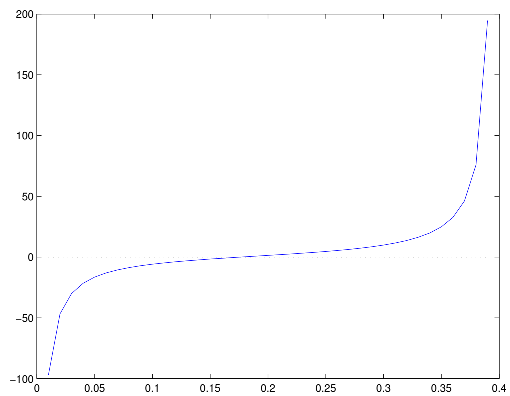


Figure 13: Function of the non-linear resulting equation for the evaluation of δ_1

In this scenario the use of a Newton method for the evaluation of the solution is not recommended therefore a bisection algorithm has been chosen.

[25] As described in section 3

[26] local value of the derivate near to zero

6 References

- Horizon software: http://ssd.jpl.nasa.gov/?horizons)
- Text book: Giovanni Mengali, Alessandro A. Quarta, Fondamenti di meccanica del volo spaziale, Pisa university press 2013.