

# Vehicle motion control

Master Lab in Autonomous driving - Motion planning and control

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  - ③ Control design

# State-feedback control design

Recall that

$$\begin{bmatrix} \dot{e}_1 \\ \ddot{e}_1 \\ \dot{e}_2 \\ \ddot{e}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{2C_{af}+2C_{ar}}{mV_x} & \frac{2C_{af}+2C_{ar}}{m} & \frac{-2C_{af}l_f-2C_{ar}l_r}{mV_x} \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{2C_{af}l_f-2C_{ar}l_r}{I_zV_x} & \frac{2C_{af}l_f-2C_{ar}l_r}{I_z} & -\frac{2C_{af}l_f^2+2C_{ar}l_r^2}{I_zV_x} \end{bmatrix} \begin{bmatrix} e_1 \\ \dot{e}_1 \\ e_2 \\ \dot{e}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{2C_{af}}{m} \\ 0 \\ \frac{2l_fC_{af}}{I_z} \end{bmatrix} \delta$$

$$+ \begin{bmatrix} 0 \\ -\frac{2C_{af}l_f-2C_{ar}l_r}{mV_x} - V_x \\ 0 \\ -\frac{2C_{af}l_f^2+2C_{ar}l_r^2}{I_zV_x} \end{bmatrix} \dot{\psi}_{des}$$

By setting  $x = \begin{bmatrix} e_1 \\ \dot{e}_1 \\ e_2 \\ \dot{e}_2 \end{bmatrix}$ ,  $u = \delta$ ,  $d = \dot{\psi}_{des}$  the model can be compactly rewritten as

$$\dot{x} = A(V_x)x + B(V_x)u + B_d(V_x)d.$$

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If the system is controllable, then the eigenvalues of the closed-loop matrix  $A - BK$  can be arbitrarily placed ( $K = \text{place}(A, B, p)$ ).

**Exercise.** Is the stability of  $A - BK$  enough to achieve zero steady-state tracking error?

# State-feedback lateral control design. Exercise

Consider the road-aligned model with the following parameters

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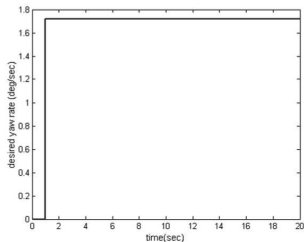
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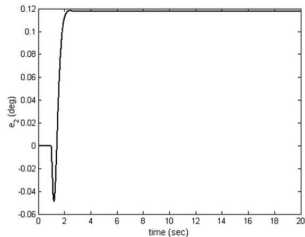
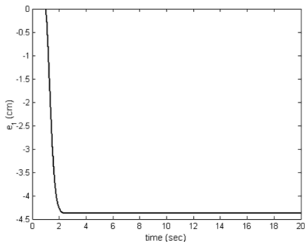
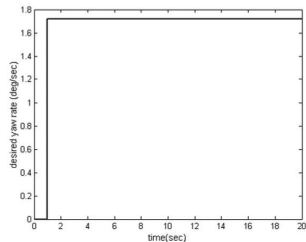
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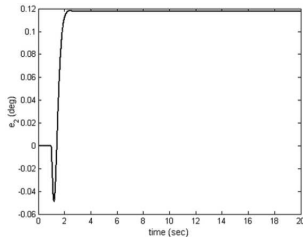
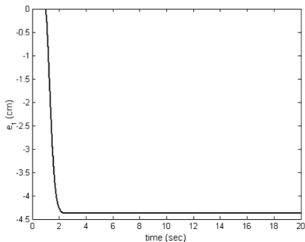
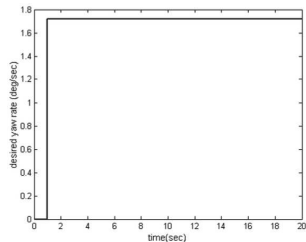
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Why aren't  $e_1$  and  $e_2$  not converging to 0?

# Steady-state errors

The presence of  $B_d \dot{\psi}_{des}$  does not allow  $e_1$  and  $e_2$  to converge to zero, when traveling on a curve, although  $A - BK$  is stable.



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At steady-state  $(A - BK)x_{ss} + B\delta_{ff} + B_d \frac{V_{des}}{R} = 0$ . By solving for  $x_{ss}$

$$x_{ss} = \begin{bmatrix} \frac{\delta_{ff}}{k_1} \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -\frac{1}{k_1} \frac{mV_x^2}{R(l_f + l_r)} \left[ \frac{l_r}{2C_{af}} - \frac{l_f}{2C_{ar}} + \frac{l_f}{2C_{ar}} k_3 \right] - \frac{1}{k_1 R} (l_f + l_r - l_r k_3) \\ 0 \\ \frac{1}{2RC_{ar}(l_f + l_r)} \left[ -2C_{ar}l_f l_r - 2C_{ar}l_r^2 + l_f mV_x^2 \right] \\ 0 \end{bmatrix}$$

While  $e_1$  can be made zero,  $e_2$  cannot.

# Non-zero orientation error

$e_1 = 0$  is achieved with

$$\delta_{ff} = \frac{mV_x^2}{RL} \left[ \frac{l_r}{2C_{\alpha f}} - \frac{l_f}{2C_{\alpha r}} + \frac{l_f}{2C_{\alpha r}} k_3 \right] + \frac{L}{R} - \frac{l_r}{R} k_3,$$

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Observe that, when  $e_1 = 0$ ,  $e_2$  can be made 0 as well at the speed

$$V_x = \sqrt{\frac{l_r}{ml_f} 2C_{\alpha r}(l_f + l_r)}.$$

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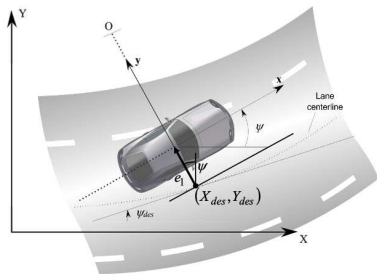
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that is, the orientation of the vehicle velocity vector in the global frame is the same as the path to follow.



# Effects of decoupled longitudinal and lateral control

Although the lateral and the longitudinal dynamics are coupled,

$$\dot{x} = A(V_x)x + B(V_x)u + B_d(V_x)d, \quad x = \begin{bmatrix} e_1 \\ \dot{e}_1 \\ e_2 \\ \dot{e}_2 \end{bmatrix}, \quad u = \delta, \quad d = \dot{\psi}_{des},$$

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So far, we learnt how to design  $\delta = -Kx + \delta_{ff}$ .

This leads to the CL matrix  $A(V_x) - B(V_x)K$ , which must be stable (eigenvalues with negative real part in CT, within the unitary circle in DT) *regardless the speed  $V_x$ .*

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Let  $V_x \in [V_{min}, V_{max}]$  and

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Wide speed intervals can be partitioned into smaller intervals and state-feedback gains designed (*speed-dependent gain-scheduling*).

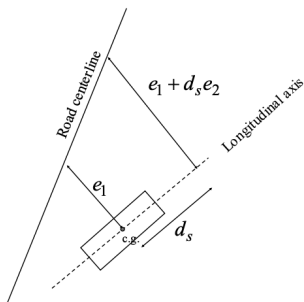
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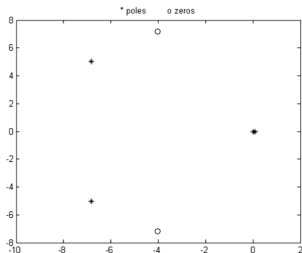
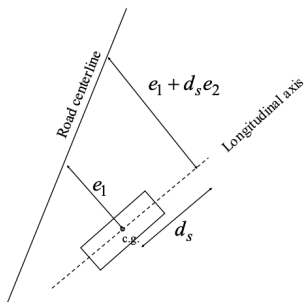
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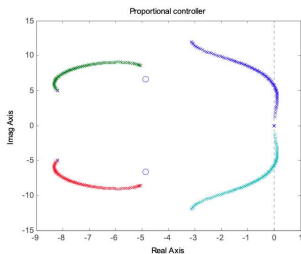
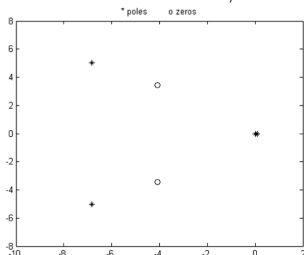
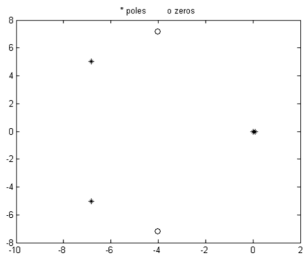
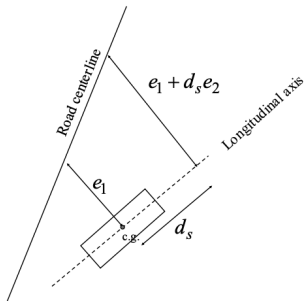
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If a proportional controller  $\delta = -Ky$  is used



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**Assumption.** A low level powertrain control system exists delivering a requested vehicle longitudinal acceleration  $a_{des}$  according to (first order dynamics)

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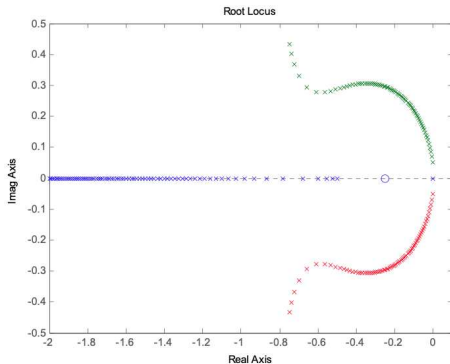
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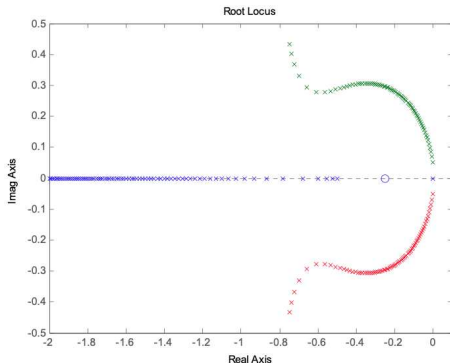
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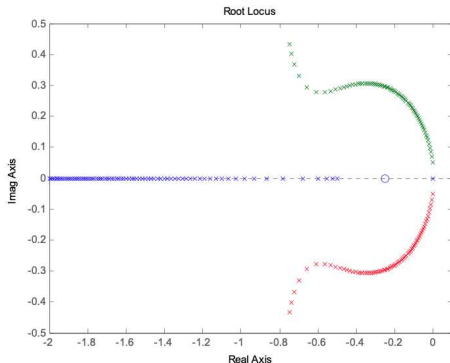
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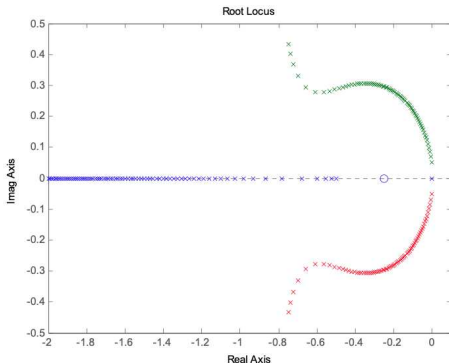
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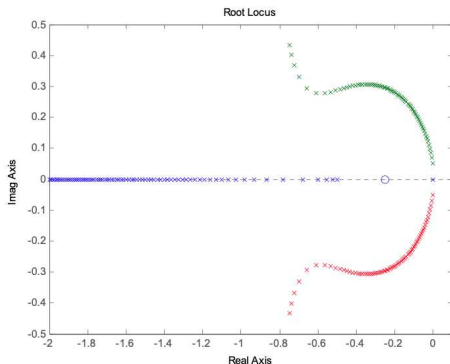
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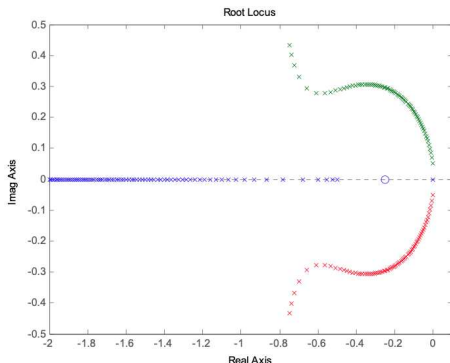
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- use sisotool, rlocus in Matlab to place the eigenvalues of the CL matrix as desired.

