Part I: Introduction

This section will introduce or review several key concepts to prepare for the rest of the report.

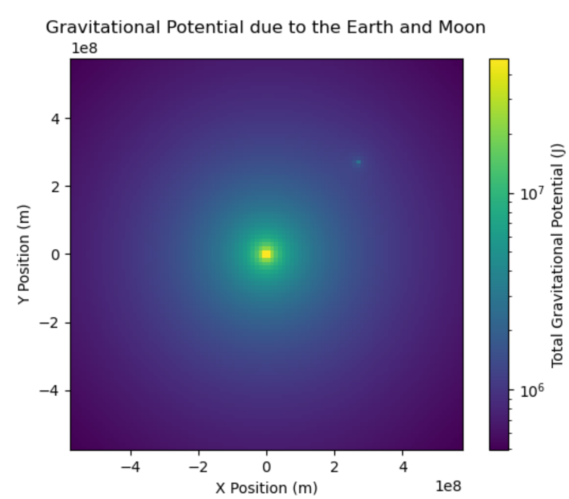
The **gravitational force** between two objects is the attraction between them due to their mass. This force is proportional to the product of the objects' masses, and inverse squarely proportional to the distance between them.

Energy is the ability of an object to do work. **Work** is force applied over a distance. The two kinds of energy we will be considering is **kinetic energy** and **gravitational potential energy.** Kinetic energy represents the speed at which an object is moving, while gravitational potential energy represents how much gravitational force can be applied to the object at maximum. Energy is a conserved quantity, so the sum of the kinetic and potential energy at the initial state (takeoff) is equal to the sum of the kinetic and potential energy at the final state. We can increase the initial state by increasing the velocity at takeoff in order to increase the final state, i.e. increasing the height the rocket gets to.

Part II: Gravitational Potential of Earth-Moon System

As mentioned previously, the final gravitational potential energy is related to the initial kinetic energy. As such, knowing the gravitational potential at the desired final location can tell us what the initial state must be.

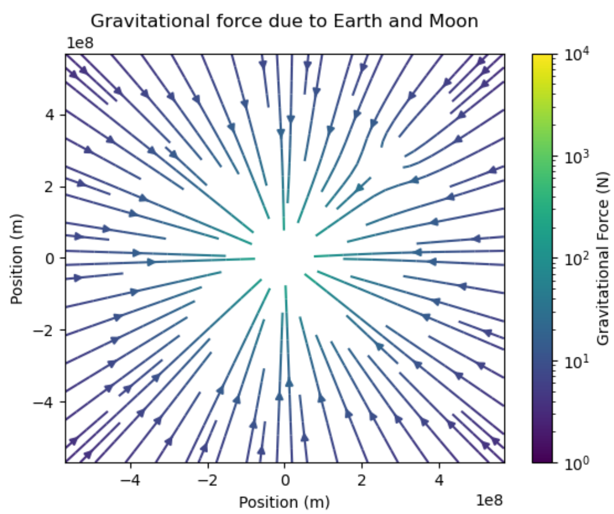
To calculate the gravitational potential of a fixed object, we created a function which takes in the object's mass and position and the testing position and returns the gravitational potential at the testing position. We then used this function to calculate the gravitational potential from the Earth at any point in the system, and to calculate the gravitational potential from the Moon at any point in the system. The graph below shows the sum of the potential energy due to the moon and the potential energy due to the Earth.



The colors of the graph represent the magnitude of the gravitational potential. The colors are on a logarithmic scale in order to represent these values better. As we can see, the potential spikes significantly around the moon and Earth. Most of the potential we see on the graph is due to the Earth, but the moon also has a significant impact on the area surrounding it. Therefore, we can see that the magnitude of the gravitational potential at the surface of the earth is much higher than magnitude of the gravitational potential far from the earth.

Part III: Gravitational Force of the Earth Moon System

Force is the product of the object's mass and its acceleration. The gravitational force is then important because it will oppose the acceleration caused by the rocket's engines. To show this, we modeled the force caused by the gravitational pull of the earth and moon below.



To produce this graph, we first defined a function that takes in the mass and position of two objects: the fixed large mass (Earth or Moon in this case) and the movable 'test' mass (or the Saturn V rocket). We then calculated two sets of data over all points, one for the earth and one for the moon. The final result is the vector summation of these forces.

In the graph, the arrows are pointing in the direction of the force. The color on the arrows is the magnitude of the force. Again, the magnitude is measured on a logarithmic scale to properly show the magnitude of the force. Both this graph and the one above only consider the moon at a fixed point, but more graphs can be created for the moon at different points.

Part IV: Projected Performance of the Saturn V Stage 1

The projected performance of the Saturn V rocket has been calculated using two measures, the burn time of the rocket and the projected altitude at that burn time.

The burn time of the rocket is the time it takes for the rocket to burn up all its fuel. This depends on the 'wet mass' of the rocket, which is the mass of the rocket with fuel, the 'dry mass' of the rocket, which is the mass of the rocket without the fuel, and the burn rate of the rocket. Based on these values, the projected burn time of the Saturn V Rocket is 157.69 seconds.

To calculate the projected altitude of the rocket, we first calculated the change in velocity over time. This relies on the lift given from the rockets engines and the gravitational force from the earth. From there, we integrated the equation from the liftoff time to the burn time (calculated in the last step) to find the final altitude. (Sidebar, but wouldn't integrating the change in velocity function give the final velocity, not the height?). Given this, the projected altitude at the burn time is 74.09km.

Part V: Discussion and Future Work

Several approximations have been made during these calculations. Most notably, we have neglected the drag force created from air resistance in our approximation of the final altitude. We have also approximated the gravitational force on the rocket to be 9.81 m/s^2, when it would change slightly as the rocket gets further from earth.

When comparing the calculations we found in this report to the tested measurements, we can see that our estimate for the burn time is slightly under the tested burn time, and our estimate for the final altitude is significantly above the tested altitude.

The burn time could be an underestimate for many reasons. For instance, in our calculations we account for a constant burn rate. In reality, this may fluctuate during the flight making the estimate slightly below what we would expect. As for the altitude, the main factor that makes our estimate inaccurate is the lack of drag force in our calculations. Drag force is the resistance caused by air as an object moves through it. This force would slow the rocket significantly on its way up and therefore results with a lower final altitude than estimated in this model.

To make our estimations more accurate in the future, I would recommend adding air resistance to our model of the altitude and adjusting the gravitational force relating to the height in our model of the velocity.