VITTORIO BARALDI

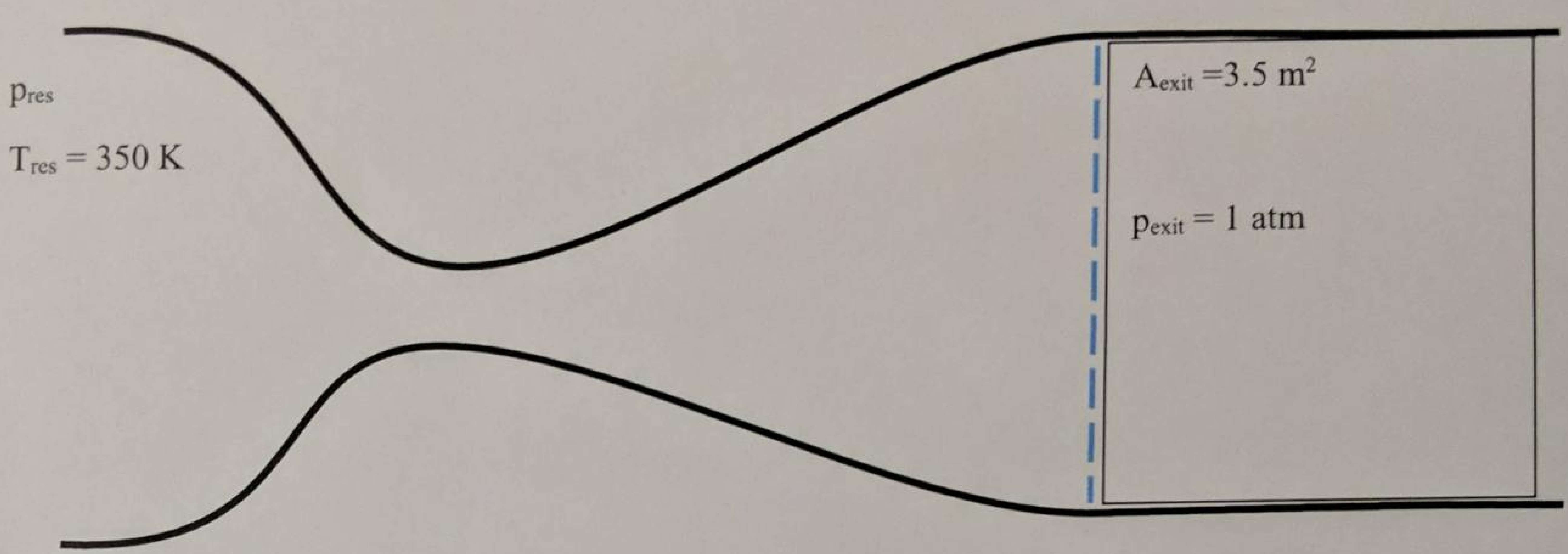
HOMEWORK 1

Due Tuesday Oct. 6 in class and Thursday Oct. 9 in class and

send by e-mail for the students who take the class on line

Problem 1 (10 points)

A converging diverging nozzle with $A_{throat} = 0.5\ m^2$ and $A_{exit} = 3.5\ m^2$ is used for a supersonic wind tunnel with test section pressure $p_{test} = p_{exit} = 1$ atm as shown in the figure below.



a) Fill up the following table for the operation conditions of the nozzle.

		Pres	M_{exit}
1	Choked subsonic	1.005 atm	0.085
2	Shock at A = 2.0 m^2	3.03 atm	0.235
3	Shock at the nozzle exit	5.59 atm	0.4496
4	Perfectly expanded flow	80 atm	3.54

$\dot{m} = 9458.19$ kg/s

For the cases 2 and 3, show all your work.

b) For the perfectly expanded flow case, calculate the mass flow rate through the test section.
c) For $p_{res} = 3.79$ atm a shock occurs in the nozzle find the location of the shock (A/A* or A) if it is given that $M_{exit} = 0.31$
d) Use the iterative method to find the location of the shock A/A* when $p_{res} = 3.79$ atm and the exit Mach number is not given.

(1)

$A_t = 0.5 m^2 = A^*$ $\quad A_e / A_t = 7$

$A_e = 3.5 m^2$

$T_{res} = 300 K$

$p_e = 1 atm$

$$\rightarrow \left(\frac{A}{A^*}\right)^2 = \frac{1}{M^2}\left[\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2} M^2\right)\right]^{(\gamma+1)(\gamma-1)}$$

$\rightarrow$ $M_e = 3.54$

chocked subsonic

From tables A.1 $\rightarrow$ $\boxed{M_e \cong 0.085}$

$P_{res} = p_e / 0.995$ (aerodynamic calculator)

$\rightarrow$ $\boxed{P_{res} = 1.005 \text{ atm}}$

Shock @ $A_1 = 2 m^2$

$\frac{A_1}{A_t} = 4$ $\rightarrow$ from appendix A $\rightarrow$ $M = 2.95$

From the normal shock relations, the mach number after the shock is $M_2 = 0.478$

For M_2, the value of A^* changes across the shock $\rightarrow$ $\frac{A_2}{A_2^*} = 1.4$ $\quad \frac{A_e}{A_2^*} = \frac{A_e}{A_t} \frac{A_t}{A_1} \frac{A_2}{A_2^*}$

Scanned by TapScanner

Since $A_1 = A_2 \Rightarrow \frac{A_e}{A_2^*} = 2.45$

So from tables $\rightarrow$ $\boxed{M_e = 0.235}$

So for $\begin{cases} \frac{A_e}{A_2^*} \rightarrow \frac{P_e}{P_{o2}} = 0.9623 \\ \text{shock} \rightarrow \frac{P_{o2}}{P_{o1}} = 0.3428 \end{cases}$

$P_{res} = P_{o1}$

$\rightarrow \frac{P_e}{P_{res}} = \frac{P_e}{P_{o2}} \cdot \frac{P_{o2}}{P_{o1}} = 0.3299$

$\rightarrow \boxed{P_{res} = 3.03 \text{ atm}}$

Shocks @ exit

$M_1 = M_e = 3.54$ $\qquad \frac{P_1}{P_{res}} = 0.01235$

After shock:

$M_e = M_2 = 0.4496 \rightarrow \boxed{M_e = 0.4496}$

$P_2/P_1 = 14.45$ $\qquad P_2 = P_e$

$$\rightarrow \frac{P_e}{P_{res}} = \frac{P_2}{P_1}\frac{P_1}{P_{res}} = 0.179 \rightarrow \boxed{P_{res} = 5.59 \text{ atm}}$$

perfectly exp.

$\boxed{M_e = 3.54}$ $\qquad P_{res} = P_e / 0.01235$

$\boxed{P_{res} = 80 \text{ atm}}$ $\qquad T_{o\,res} = 300\text{ K}$

b) $$\dot{m} = \frac{P_o A^*}{\sqrt{T_o}} \sqrt{\frac{\gamma}{R} \left(\frac{2}{\gamma+1}\right)^{(\gamma+1)(\gamma-1)}}$$

$$\rightarrow \dot{m} = \frac{8.106 \cdot 10^6 \cdot 0.5}{17.32} \sqrt{\left(\frac{1.4}{287} \cdot \left(\frac{2}{2.4}\right)^6\right)}$$

$\rightarrow \boxed{\dot{m} = 9458.19 \text{ kg/s}}$

c) knowing that $P_e = 1\,atm$

$P_{res} = 3.79\,atm$

$M_e = 0.31$

$\Rightarrow$ From Me $\rightarrow$ $\frac{P_e}{P_{oe}} = 0.9355$

$$\frac{P_e}{P_{res}} = \frac{P_e}{P_{oe}} \cdot \frac{P_{02}}{P_{01}} \Rightarrow \frac{P_{02}}{P_{01}} = 0{,}282$$

From this ratio, which represents the ~~static~~ total pressure change across the shock:

$\Rightarrow$ $M_1 = 3.18$ $M_2 = 0.466$

$\Rightarrow$ From A.1 for M_1 $\rightarrow$ $\frac{A}{A^*} = 5.1$ $A^* = 0.5$

$\Rightarrow$ Shock occurs at $\boxed{A = 2.55\,m^2}$

Scanned by TapScanner

AE 504 Compressible Flow

Table of Contents

Vittorio Baraldi

Homework 3

```
clear all
close all
clc
```

Ex. 1

Part c)

```
%initial data
Pres=3.79;          %reservoir pressure [atm]
Pe=1;               %exit pressure [atm]
At=0.5;             %throat area [m^2]
Ae=3.5;             %exit area [m^2]
Me=0.31;            %exit mach no.
g=1.4;              %heat air coefficient

exitratio=Ae/At;

syms asol msol
A=2;                %initial shock position guess
dA=.001;            %increment
options = optimoptions('fsolve','Display','none');
for i=1:1000
    %area-mach no. function
    fM=@(x) ((((2/(g+1))*(1+0.2*x^2))^6)/x^2)-(A/At)^2;
    %solving for Mach no. before shock
    M1=fsolve(fM,2,options);
    %mach no. after shock
    M2=sqrt(((g-1)*M1^2+2)/(2*g*M1^2-(g-1)));
    %total pressure ratio across shock
    Pratio=((((g+1)*M1^2)/((g-1)*M1^2+2))^(g/(g-1)))*...
        (((g+1)/(2*g*M1^2-(g-1)))^(1/(g-1)));
    %Area ratio based on M2
    fA=@(x) ((((2/(g+1))*(1+0.2*M2^2))^6)/M2^2)-x^2;
    Aratio=fsolve(fA,1.1,options);
    %finding the new area ratio between exit area and the sonic area
    %condition after shock
    Ae_A2t=exitratio*(At/A)*Aratio;
```

```
    %solving for the exit Mach no. given the area ratio above
    fM2=@(x) ((((2/(g+1))*(1+0.2*x^2))^6)/x^2)-Ae_A2t^2;
    Me=fsolve(fM2,0.9,options);
    %with isentropic relations, find the conditions at the exit
    [Pexit,~,~,~,~,~]=isentropicflow(Me,g);
    %Pfinal is the exit pressure, given the initial guess
    Pfinal=Pexit*Pratio*Pres;
    %difference between computed exit pressure and actual exit
 pressure
    diff=abs(Pfinal-Pe);
    %checking on the difference tolerance (1e-3)
    if diff<1e-3
        sol=A;
        break
    else
        %if condition is not satisfied, guess is incremented
        A=A+dA;
    end
end

fprintf('Shock occurs at A = %1.2f meters squared', sol)
```

```
Shock occurs at A = 2.50 meters squared
```

Published with MATLAB® R2020a

AE504 Adv. Compressible Flow

Isentropic Flow Relations

```
function
 [ratioP,ratioT,ratioR,ratioTs,ratioPs,ratioRs]=isentropicflow(M,g);
%{
Legend
ratioP  -- static pressure/total pressure
ratioT  -- static temperature/total temperature
ratioR  -- static density/total density
ratioTs -- sonic temperature/total temperature
ratioPs -- sonic pressure/total pressure
ratioRs -- sonic density/total density
M       -- Mach number
g       -- heat air coefficent
%}
for i=1:length(M)
    if M(i)<=0
        fprintf('Mach no. at position %i is either null or
 negative',i)
    else
        ratioP(i)=(1+((g-1)/2).*M(i)^2).^(-(g/(g-1)));
        ratioT(i)=ratioP(i)^((g-1)/g);
        ratioR(i)=ratioP(i)^(1/g);
        ratioTs(i)=2/(g+1);
        ratioPs(i)=ratioTs(i)^(g/(g-1));
        ratioRs(i)=ratioTs(i)^(1/(g-1));
    end
end
end
```

Published with MATLAB® R2020a