Rational Least Squares Fitting using Krylov Spaces

Mario Berljafa* Stefan Güttel*

For given matrices $\{A, F\} \subset \mathbb{C}^{N\times N}$ and a vector $\mathbf{v} \in \mathbb{C}^N$, we consider the problem of finding a rational function $R_m^{\min}$ of type (m, m) such that

$$\|F\mathbf{v} - R_m(A)\mathbf{v}\|_2^2 \to \min,$$

and propose an iterative algorithm [1, 2] for its solution. At each iteration the algorithm constructs a rational Krylov space $\mathcal{Q}_{m+1}(A, \mathbf{v})$ and manipulates an associated Arnoldi decomposition to find better approximations to the poles of $R_m^{\min}$. In the special case when $A = \operatorname{diag}(\lambda_j)$ and $F = \operatorname{diag}(\psi_j)$ are diagonal we have a weighted rational least squares fitting problem $\sum_{j=1}^N |v_j|^2 \cdot |\psi_j - R_m(\lambda_j)|^2 \to \min$, and compare our method to the popular *vector fitting* [3].

References

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[3] B. Gustavsen and A. Semlyen, *Rational approximation of frequency domain responses by vector fitting*, IEEE Trans. Power Del., 14 (1999), pp. 1052–1061.

*School of Mathematics, The University of Manchester, Alan Turing Building, Oxford Road, M13 9PL Manchester, United Kingdom, `mario.berljafa@manchester.ac.uk`, `stefan.guettel@manchester.ac.uk`