Krylov and Matrix Balancing for fast Field of Value Type Inclusion Regions

Ian Zwaan

PhD student at TU/e

February 2, 2015

The field of values may be an excellent tool for generating a spectral inclusion region: it is easy to approximate numerically, and for many matrices this region fits relatively tightly around the eigenvalues. However, for some matrices the field of values may be a poor eigenvalue inclusion region: the numerical radius may be much larger than the spectral radius. We show that balancing the matrix may be helpful for generating a quality inclusion region based on the field of values. and introduce a new Krylov based balancing method. We believe that both the (sparse) balancing and the new "Krylov and balance" technique, combined with a projected field of values, render spectral inclusion regions that may be hard to beat in both quality and efficiency.

1. T. Betcke, *Optimal scaling of generalized and polynomial eigenvalue problems*, SIAM, J. Matrix Anal. Appl., 30 (2008), pp. 1320–1338.
2. T.-Y. Chen and J. W. Demmel, *Balancing sparse matrices for computing eigenvalues*, Linear Algebra and Its Applications, 309 (2000), pp. 261–287.
3. M. E. Hochstenbach, D. A. Singer, and P. F. Zachlin, *Numerical approximation of the field of values of the inverse of a large matrix*, Textos de Mathematica, 44, pp. 59–71.
4. ———, *Eigenvalue inclusion regions from inverses of shifted matrices*, Linear Algebra Appl., 429 (2008), pp. 2481–2496.
5. R. A. Horn and C. R. Johnson, *Matrix Analysis*, Cambridge University Press, Cambridge, Uk, 1985.
6. C. R. Johnson, *Numerical determination of the field of values of a general complex matrix*, SIAM J. Numer. Anal., 15 (1978), pp. 595–602.
7. T. A. Manteuffel and G. Starke, *On hubyrid iterative methods for nonsymmetric systems of linear equations*, Numer. Math., 73 (1996), pp. 489–506.
8. *The Matrix Market*, `http://math.nist.gov/MatrixMarket/`, a repository for test matrices.
9. G. W. Stewart, *A Krylov-Schur algorithm for large eigenproblems*, SIAM J. Matrix Anal. Appl., 23 (2001/02), pp. 601–614.
10. H. A. van der Vorst, *Iterative Krylov methods for Large Linear Systems*, vol. 13 of Cambridge Monographs on Applied and Computational Mathematics, Cambridge University Press, Cambridge, UK, 2003.

11. D. S. Watkins, *A case where balancing is harmful*, Electron. Trans. Numer. Anal., 23 (2006), pp. 1–4.