Probabilistic bounds for the matrix condition number with extended Lanczos bidiagonalization

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Reliable estimates for the condition number of a large (sparse) matrix A are important in many applications. To get an approximation for the condition number $\kappa(A)$, an approximation for the smallest singular value is needed. Krylov subspaces are usually unsuitable for finding a good approximation to the smallest singular value. Therefore, we study extended Krylov subspaces which turn out to be ideal for the simultaneous approximation of both the smallest and largest singular value of a matrix. First, we develop a new extended Lanczos bidiagonalization method. With this method we obtain a lower bound for the condition number. Moreover, the method also yields probabilistic upper bounds for $\kappa(A)$. The user can select the probability with which the upper bound holds, as well as the ratio of the probabilistic upper bound and the lower bound.

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