IDR-CGS-BiCGSTAB-IDR(s)
- a case of serendipity -

Peter Sonneveld [1]

In about 1976, I was preparing a renovation of the elementary course on numerical analysis in Delft University. In relation to the problem of solving a single nonlinear equation iteratively, I wondered whether the so-called 'secant method' could be generalized to systems of N nonlinear equations with N unknowns.

Before starting to read everything on a subject, I always try to think about it unbiased, and so I started with (probably) re-inventing the wheel. Had I seen the book by Ortega and Rheinboldt at that time, CGS, BiCGSTAB and IDR(s) probably wouldn't exist today. After a week of rather primitive numerical experiments, I decided that generalisations of the secant method to N dimensions were far too complicated for an elementary course. However, the experiments showed a surprising phenomenon, that appeared to be useful in the machinery of solving large sparse nonsymmetric *linear* systems.

The first application of this 'new wheel' was called IDR (Induced Dimension Reduction). Afterwards, CGS (Conjugate Gradients Squared) was developed as an 'improvement' of IDR, and also for other reasons. From then, starting with BiCGStab in cooperation with Henk van der Vorst, a lot of other methods of this kind were developed by many others. This went on until about 10 years ago.

In this short presentation I'll give a reconstruction of the strange history of these so-called 'Lanczos-type product methods'. It will be explained why this 'sleeping theory' woke up just after my retirement in 2006, resulting in a brand new family of methods: IDR(s). Since history is a continuing story, also some recently discovered interesting features of the IDR(s) methods are already part of it. Some will be mentioned in the lecture.

---

[1]Delft University of Technology, Delft Institute of Applied Mathematics, Mekelweg 4, 2628 CD, The Netherlands. E-mail: `P. Sonneveld, p.sonneveld@tudelft.nl`