

Rational Least Squares Fitting using Krylov Spaces

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For given matrices $\{A, F\} \subset \mathbb{C}^{N \times N}$ and a vector $\mathbf{v} \in \mathbb{C}^N$, we consider the problem of finding a rational function R_m^{\min} of type (m, m) such that

$$\|F\mathbf{v} - R_m(A)\mathbf{v}\|_2^2 \rightarrow \min,$$

and propose an iterative algorithm [1, 2] for its solution. At each iteration the algorithm constructs a rational Krylov space $\mathcal{Q}_{m+1}(A, \mathbf{v})$ and manipulates an associated Arnoldi decomposition to find better approximations to the poles of R_m^{\min} . In the special case when $A = \text{diag}(\lambda_j)$ and $F = \text{diag}(\psi_j)$ are diagonal we have a weighted rational least squares fitting problem $\sum_{j=1}^N |v_j|^2 \cdot |\psi_j - R_m(\lambda_j)|^2 \rightarrow \min$, and compare our method to the popular *vector fitting* [3].

References

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