Recent progresses in Krylov subspace methods for solving complex symmetric linear systems

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Complex symmetric linear systems (CSLSs) with the following form

$$A\boldsymbol{x} = \boldsymbol{b}, \quad A \neq A^H, \text{ but } A = A^T \in \mathbb{C}^{n\times n}, \quad \boldsymbol{b} \in \mathbb{C}^n$$

arise in many important applications such as numerical computations in quantum chemistry, eddy current problems, modeling the waveguide discontinuities and electromagnetic simulations. Hence, there is a strong need for the fast solutions of complex symmetric linear systems. During the past few years, a variety of specified Krylov subspace methods (KSMs) for solving such systems are proposed and used, such as COCG, COCR, QMR-SYM and BiCGCR methods.

In this talk, I will mainly revisit and focus on SCBiCG, which is also known as one of methods for solving such linear system. SCBiCG can be derived by substituting a matrix polynomial, which is expressed by the complex conjugate coefficient matrix and initial residual vector, to the initial shadow residual of the BiCG algorithm. Moreover, we clarify that SCBiCG can be transformed to some methods which have been previously proposed. Besides, in our talk we will prove that the preconditioned BiCGCR is mathematically equivalent to preconditioned COCR in detail, and then give an overview of the recent progress in other KSMs with suitable preconditioning techniques for solving CSLSs. Finally, numerical experiments involving many electromagnetic model problems are employed to investigate the convergence behaviors of these solvers, and then some remarks on future research of this topic will be also summarized.

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