My research on Krylov methods

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The field of values may be an excellent tool for generating a spectral inclusion region: it is easy to approximate numerically, and for many matrices this region fits relatively tightly around the eigenvalues. However, for some matrices the field of values may be a poor eigenvalue inclusion region: the numerical radius may be much larger than the spectral radius. We show that balancing the matrix may be helpful for generating a quality inclusion region based on the field of values. and introduce a new Krylov based balancing method. We believe that both the (sparse) balancing and the new "Krylov and balance" technique, combined with a projected field of values, render spectral inclusion regions that may be hard to beat in both quality and efficiency.

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