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Mathematical foundation

1. Rank of matrix

Rank of matrix define argument of length square submatrix and row & coloums in a square matrix.

2. Show by matrix method

$2x+4y+z=5,\ x+y+z=6,\ 2x+3y+z=6$

$$A=\begin{bmatrix}2&4&1\\1&1&1\\2&3&1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=B\begin{bmatrix}5\\6\\6\end{bmatrix}$$

$$|A|=\begin{vmatrix}2&4&1\\1&1&1\\2&3&1\end{vmatrix}$$

$=2(1-3)-4(1-2)1(3-2)$

$=2(-2)-4(-1)+1(1)$

$=-4+4+1$

$=1$

$$\text{adj } A=\begin{bmatrix}2&4&1\\1&1&1\\2&3&1\end{bmatrix}$$

$A_{11}=\Big|\begin{matrix}1\\3\end{matrix}$

$A_{12}=\Big|\begin{matrix}1\\2\end{matrix}$

$A_{13}=\Big|$

$A_{21}=\Big|$

$A_{22}=$

$A_{23}=$

$=$

A

dation

submatrix

are

+ z = 6

$A_{11} = \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} = (1-3) = -2$

$A_{12} = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = (1-2) = 1$

$A_{13} = \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = (3-2) = 1$

$A_{21} = \begin{vmatrix} 4 & 1 \\ 3 & 1 \end{vmatrix} = (4-3) = 1$

$A_{22} = \begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix} = (2-2) = 0$

$A_{23} = \begin{vmatrix} 2 & 4 \\ 2 & 3 \end{vmatrix} = (6-8) = -2$

$A_{31} = \begin{vmatrix} 4 & 1 \\ 1 & 1 \end{vmatrix} = (4-1)$
$= 3$

$A_{32} = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = (2-1)$
$= 1$

$A_{33} = \begin{vmatrix} 2 & 4 \\ 1 & 1 \end{vmatrix} = (2-4)$
$= -2$

$$= \begin{bmatrix} -2 & 1 & 1 \\ -1 & 0 & 2 \\ 3 & -1 & -2 \end{bmatrix}^T$$

$$= \begin{bmatrix} -2 & 1 & 3 \\ -1 & 0 & -1 \\ 1 & 2 & -2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{1} \begin{bmatrix} -2 & -1 & 3 \\ 1 & 0 & -1 \\ 1 & 2 & -2 \end{bmatrix}$$

$$X = A^{-1} B$$

$$= \begin{bmatrix} -2 & -1 & 3 \\ 1 & 0 & -1 \\ 1 & 2 & -2 \end{bmatrix} \begin{bmatrix} 6 \\ 6 \\ 6 \end{bmatrix}$$

$$= \frac{1}{1}\begin{bmatrix} -10 & -6 & 18 \\ 5 & 0 & -6 \\ 5 & 12 & -12 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$\boxed{x = 2}$ $\boxed{y = -1}$ $\boxed{z = 5}$

3. If $A = 1/\sqrt{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ show that A is orthogonal

$A \times A^T = I$

Solu:

$A = 1/\sqrt{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

$A^T = 1/2\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$

$AA^T = 1/\sqrt{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} 1/\sqrt{2}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$

$= 1/2\begin{bmatrix} 1+1 & -1+1 \\ 1-1 & 1+1 \end{bmatrix}$

$= 1/2\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

given that matrix is orthogonal

4. If

4. If $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 & 0 \\ -1 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ prove that

Solu:

$AB \neq BA$

$$AB = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 & 0 \\ -1 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} (2-3+0) & (6+6+0) & (0+3+8) \\ (1-2+0) & (3+4+0) & (0+2+6) \\ (-1-1+0) & (-3+2+0) & (0+1+4) \end{bmatrix}$$

$$AB = \begin{bmatrix} -1 & 12 & 11 \\ -1 & 7 & 8 \\ -2 & -1 & 5 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 3 & 0 \\ -1 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ -1 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} (2+3+0) & (3+6+0) & (4+9+0) \\ (-2+2-1) & (-3+4+1) & (-4+6+2) \\ (0+0-2) & (0+0+2) & (0+0+4) \end{bmatrix}$$

$$BA = \begin{bmatrix} 5 & 9 & 13 \\ -1 & 2 & 4 \\ -2 & 2 & 4 \end{bmatrix}$$

$AB \neq BA$

$\therefore$ Hence proved.

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5. Find the adjoint of $\begin{bmatrix} 3 & 1 & 2 \\ 2 & 2 & 5 \\ 4 & 1 & 0 \end{bmatrix}$

Solu:

$A_{11} = +\begin{vmatrix} 2 & 5 \\ 1 & 0 \end{vmatrix} = (0-5) = -5$

$A_{12} = -\begin{vmatrix} 2 & 5 \\ 4 & 0 \end{vmatrix} = (0-20) = +20$

$A_{13} = +\begin{vmatrix} 2 & 2 \\ 4 & 1 \end{vmatrix} = (2-6) = -6$

$A_{21} = -\begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix} = (0-2) = -2$

$A_{22} = +\begin{vmatrix} 3 & 2 \\ 4 & 0 \end{vmatrix} = (0-8) = -8$

$A_{23} = -\begin{vmatrix} 3 & 1 \\ 4 & 1 \end{vmatrix} = (3-4) = 1$

$A_{31} = +\begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix} = (5-4) = 1$

$A_{32} = -\begin{vmatrix} 3 & 2 \\ 2 & 5 \end{vmatrix} = (15-4) = -11$

$A_{33} = +\begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix} = (6-2) = 4$

$$= \begin{bmatrix} -5 & +20 & -6 \\ -2 & -8 & 1 \\ 1 & -11 & 4 \end{bmatrix}$$

$$\text{adj} = \begin{bmatrix} -5 & -2 & 1 \\ 20 & -8 & -11 \\ -6 & 1 & 4 \end{bmatrix}$$