This document contains derivation of formulas in paper: An s-shaped three-parameter (S3) traffic stream model with consistent car following relationship.

Speed-density function:

# (1)

: density

: speed

: free flow speed

: critical density that can result in a maximum traffic flow rate

: positive parameter to be determined

: positive parameter to be determined

Flow-density function

# (2)

: density

: flow rate

: free flow speed

: critical density that can result in a maximum traffic flow rate

: positive parameter to be determined

: positive parameter to be determined

Comment: How to derive Eq. (2)?

Conservation law: , Speed-density function:,

# (3)

: density

: flow

: free flow speed

: critical density that can result in a maximum traffic flow rate

: positive parameter to be determined

: positive parameter to be determined

Comment: how to derive Eq. (3)?

Eq. (2):

Another important thing is that is equal to backward wave speed.

# (4)

: density

: flow

: free flow speed

: critical density that can result in a maximum traffic flow rate

: positive parameter to be determined

: positive parameter to be determined (now can be expressed by )

Comment: Eq. (4):

图表, 折线图

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This figure shows flow-density curve. The slope of tangent line at critical density is equal to zero.

How to derive Eq. (4)?

Then, we have:

# (5) and (6)

: density

: speed

: free flow speed

: critical density that can result in a maximum traffic flow rate

: positive parameter to be determined

: the maximum flow rate

How to derive Eq. (5) and (6)?

Then, we have

# (7)

: speed of critical density

: free flow speed

: critical density that can result in a maximum traffic flow rate

: positive parameter to be determined

Comment: About Eq. 7.

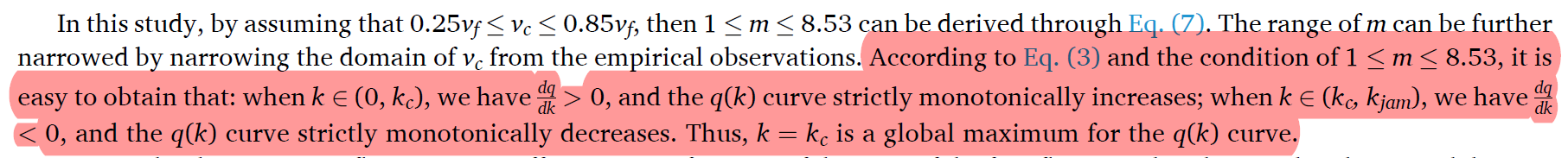
How to derive ?

More about Eq. 7:

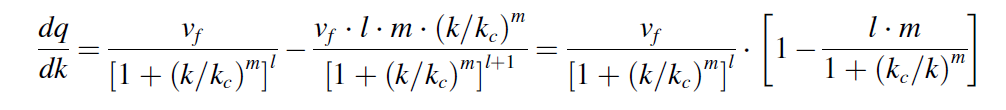
Assume that ,

文本, 信件

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About the above screenshot:



The sign of mainly depends on the sign of .

Depends on the relationship between 1 and

Depends on the relationship between 2 and

Depends on the relationship between 1 and

Remember that , we can know that when , leading to:

Continuous curve strictly monotonically increases in the interval of .

Similarly, when , curve strictly monotonically decreases.

# (8)

: flow

: density

: free flow speed

: critical density that can result in a maximum traffic flow rate

: positive parameter to be determined

How to derive Eq. (8)?

Eq. 3:

# (9)

: density at the inflection point

: critical density that can result in a maximum traffic flow rate

: positive parameter to be determined

Comment: how to derive Eq. (9)?

Remember that , then

Let:

Thus,

Remember that , we have , indicating .

When , , decreases ( increases), q-k curve is concave.

When , , increases ( decreases), q-k curve is convex.

图形用户界面, 文本, 应用程序

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图表, 折线图

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# (10)

: following vehicle index

: leading vehicle index

: acceleration of each vehicle at time

: reaction time of the following vehicle

: positive parameter to be determined

: space headway

: position of each vehicle at time , and

: difference in speed between the leading vehicle and the following vehicle

: speed of each vehicle at time

: parameter to be calibrated

: parameter to be calibrated

# (11)

: following vehicle index

: leading vehicle index

: acceleration of each vehicle at time

: reaction time of the following vehicle

: positive parameter to be determined

: space headway

: position of each vehicle at time , and

: difference in speed between the leading vehicle and the following vehicle

: speed of each vehicle at time

: free flow speed

: critical density that can result in a maximum traffic flow rate

# (12)

: following vehicle index

: leading vehicle index

: acceleration of each vehicle at time

: reaction time of the following vehicle

: positive parameter to be determined

: space headway

: position of each vehicle at time , and

: difference in speed between the leading vehicle and the following vehicle

: speed of each vehicle at time

: free flow speed

: critical density that can result in a maximum traffic flow rate

# (13)

: speed

: following vehicle index

: leading vehicle index

: positive parameter to be determined

: space headway

: position of each vehicle at time , and

: difference in speed between the leading vehicle and the following vehicle

: speed of each vehicle at time

: parameter to be calibrated

: parameter to be calibrated

# (14)

: speed

: positive parameter to be determined

: parameter to be calibrated

: parameter to be calibrated

: the instant of integration

# (15)

: speed

: density

: positive parameter to be determined

: free flow speed

: critical density that can result in a maximum traffic flow rate

: the instant of integration

# (16)

: number of observations

: observation index,

: th observation of speed

: th observation of density

: estimated speeds for the observations

: free flow speed (for calibration)

: critical density that can result in a maximum traffic flow rate (for calibration)

: maximum flow inertia coefficient (for calibration)

The objective of calibration is to minimize the sum of squared errors in the speed-density function .

# (17)

: number of observations

: observation index,

: th observation of speed

: th observation of density

: observed flows corresponding to density

: estimated speeds for the observations

: estimated flows corresponding to density

: free flow speed (for calibration)

: critical density that can result in a maximum traffic flow rate (for calibration)

: maximum flow inertia coefficient (for calibration)

: (for more about , please refer to original paper)

: Fitted variance in speed measurements

: Fitted variance in flow measurements

# (18)

Minimize the sum of squared errors in both speed and distance measures

: number of observations in the speed and space-headway measures

: time

: observed speed of the following vehicle at time

: estimated speed of the following vehicle at time

: observed space headway between the following vehicle and the leading vehicle

: estimated space headway between the following vehicle and the leading vehicle

: parameter to be calibrated

: parameter to be calibrated

# (19)

# (20)

# (21)

# (22)

: total number of observed follower-leader pairs

: total number of observations of speed and space-headway measures for the following vehicle

: index of follower-leader

: observed speed of the following vehicle at time

: estimated speed of the following vehicle at time

: observed space headway between the following vehicle and the leading vehicle

: estimated space headway between the following vehicle and the leading vehicle

# (23)

: observed value for observation

: predicted value for observation

# (24)

: speed

: density

: free flow speed

: critical density that can result in a maximum traffic flow rate