This document contains derivation of formulas in paper: Estimating key traffic state parameters through parsimonious spatial queue models

# (1)

: cumulative arrival curve at time

: arrival rate function at time

# (2)

: cumulative departure curve at time

: capacity (or discharge rate), assumed to be a constant value

# (3)

: virtual queue length at time

: arrival rate function at time

: capacity (or discharge rate), assumed to be a constant value

# (4)

: virtual queue length at time

: the total delay between time and

: start time of congestion period

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# (5)

: start time of congestion period

# (6)

: time with maximum queue length

# (7)

: start time of congestion period

: time with maximum queue length

# (8)

: time with maximum queue length

: end time of congestion period

# (9)

: time with maximum arrival rate

# (10)

: start time of congestion period

: virtual queue length at time

# (11)

: end time of congestion period

: virtual queue length at time

# (12)

: discharge rate

: shape parameter

: start time of congestion duration

: time with longest queue

: a time instant without obvious physical meaning, is another root besides for and of the cubic net flow rate function ()

: time instant for vehicle entering the bottleneck (based on point queue model)

# (13)

# (14)

: oversaturation factor

: start time of congestion period

: end time of congestion period

: time with maximum queue length (minimum speed)

# (15)

Derivation of Eq. (15):

Let , then

and

The differential:

Remark:

(1) it is much more complicated than Newell’s quadratic model

(2) This is a 4-order polynomial with respect to time

(3) generally **cannot be observed or measured**, consequently, it should be removed in

It is observed that exists in the equation and no other terms include ,

implying that we can try to remove as a whole rather than purely remove .

We want to express through other parameters.

Mathematically, we want something like this:

We can resemble Newell’s deduction by observing

Remark: in LHS and RHS, terms and are common, for convenience, we include a new parameter ,

Note that should **NOT** equal zero

Recall that in Newell’s quadratic arrival rate model,

图表, 折线图

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(Newell’s quadratic arrival rate model)

图示

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# (16)

Derivation of Eq. (16)

Once we have the length of queue with respect to time , we are curious about the maximum value of this function, namely

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# (17)

Derivation of Eq (17)

图片包含 图示

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# (18)

Derivation of Eq.(18)

Note that

Let , then

and

The differential:

Function for calculating the integral:

Therefore…

# (19)

# (20)

Derivation of Eq. (20):

# (21)

Therefore,

# (22)

# (23)

# (24)

# (25)

# (26)

# (27)

# (28)

# (29)

# About :

If , then

If , then

Recall from the Newell’s quadratic polynomial arrival queue model, the and segments the whole peak hour into three intervals with equal length. In this case, we can compute =2/3. Note that in the cubic polynomial arrival queue model (Cheng et al. 2022), can **NOT** take the value of 2/3!

# About :

to simplify , arrange the terms containing together

Now let us observe the monotonical property in and

For any belongs to and , the following inequality always holds

Implying that

**is monotonically increasing in**

**is monotonically increasing in**

Discuss two cases,

# (1)

# (2)

# (30)

: start time of congestion period

: end time of congestion period

: the cumulative number of vehicles from to

Thus,

# (39)

# (40)

# (41)

# (42)

# (43)

# (44)

We need to calculate

Intermediate terms:

# (34)

# (35)

# (36)

Note that for the original paper missed a term: , which is a typo.

# (37)

# (38)