This document contains derivation of formulas in paper: A meso-to-macro cross-resolution performance approach for connecting polynomial arrival queue model to volume-delay function with inflow demand-to-capacity ratio

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: queued demand factor, which can convert to , and it represents the percentage of congested flows within the entire analysis period ()

Period volume : is the total lane-based volume loaded on a road link during an analysis period.

Inflow demand **/** congested flow : is the queued volume or queued demand which represents the total volume with travel speed under a specific cut-off speed ().

: cut-off speed, which can be used to distinguish “congested” vs. “uncongested” states of traffic bottleneck

About :

In Table 1, : lane-based ultimate hourly capacity

The hourly maximum flow rate per lane when the level of service is under E ( Branston, 1976 ; HCM, 2010 ) is used in this study as the “ultimate hourly capacity ”in .

# (24)

# (25a)

: peak period, i.e.,

: start time of congestion period

: end time of congestion period

: total in-flow demand during the entire congested period

: lane-based ultimate hourly capacity

: demand-to-capacity ratio

: the oversaturation-to-duration elasticity

: congestion duration constant in response to changes

# (25b)

: peak period, i.e.,

: start time of congestion period

: end time of congestion period

: total in-flow demand during the entire congested period

: lane-based ultimate hourly capacity

: demand-to-capacity ratio

: the oversaturation-to-duration elasticity

: congestion duration constant in response to changes

# (26a)

*:* queue discharge rate, assumed to be a constant value for each instance of a queue duration

: peak period, i.e.,

: start time of congestion period

: end time of congestion period

: total in-flow demand during the entire congested period

: lane-based ultimate hourly capacity

: demand-to-capacity ratio

: the oversaturation-to-duration elasticity

: congestion duration constant in response to changes

# (26b)

*:* queue discharge rate, assumed to be a constant value for each instance of a queue duration

: peak period, i.e.,

: start time of congestion period

: end time of congestion period

: total in-flow demand during the entire congested period

: lane-based ultimate hourly capacity

: demand-to-capacity ratio

: the oversaturation-to-duration elasticity

: congestion duration constant in response to changes

Comment: how to derive Eq. (26b)?

# (27a)

: magnitude of speed reduction

: cut-off speed, which can be used systematically to distinguish “congested” vs. “uncongested” states of traffic bottleneck

: lowest speed on a link

# (27b)

: magnitude of speed reduction

: peak period, i.e.,

: start time of congestion period

: end time of congestion period

: the duration-to-speed reduction elasticity factor

: an MSR reduction constant in response to changes of congestion duration

# (28a)

: the longest delay at compared with

: link length

: cut-off speed, which can be used systematically to distinguish “congested” vs. “uncongested” states of traffic bottleneck

: lowest speed on a link

: magnitude of speed reduction

: peak period, i.e.,

: start time of congestion period

: end time of congestion period

: the duration-to-speed reduction elasticity factor

: an MSR reduction constant in response to changes of congestion duration

# (28b)

: the longest delay at compared with

*:* queue discharge rate, assumed to be a constant value for each instance of a queue duration

: inflow curvature parameter used in polynomial form

: start time of congestion period

: end time of congestion period

: time index with maximum queue length

: peak period, i.e.,

# (29a)

: the longest delay at compared with

*:* queue discharge rate, assumed to be a constant value for each instance of a queue duration

: inflow curvature parameter used in polynomial form

: start time of congestion period

: end time of congestion period

: time index with maximum queue length

: peak period, i.e.,

# (29b)

: inflow curvature parameter used in polynomial form

*:* queue discharge rate, assumed to be a constant value for each instance of a queue duration

: link length

: cut-off speed, which can be used systematically to distinguish “congested” vs. “uncongested” states of traffic bottleneck

: an MSR reduction constant in response to changes of congestion duration

: the duration-to-speed reduction elasticity factor

: peak period, i.e.,

: start time of congestion period

: end time of congestion period

Comment: how to derive Eq. (29b)?

# (30a)

: inflow curvature parameter used in polynomial form

: lane-based ultimate hourly capacity

: link length

: total in-flow demand during the entire congested period

: demand-to-capacity ratio

: an MSR reduction constant in response to changes of congestion duration

: congestion duration constant in response to changes

: cut-off speed, which can be used systematically to distinguish “congested” vs. “uncongested” states of traffic bottleneck

: the duration-to-speed reduction elasticity factor

: the oversaturation-to-duration elasticity

: peak period, i.e.,

: start time of congestion period

: end time of congestion period

Comment: how to derive Eq. (30a)?

If , then we have

# (30b)

: inflow curvature parameter used in polynomial form

: lane-based ultimate hourly capacity

: link length

: total in-flow demand during the entire congested period

: demand-to-capacity ratio

: an MSR reduction constant in response to changes of congestion duration

: cut-off speed, which can be used systematically to distinguish “congested” vs. “uncongested” states of traffic bottleneck

: the duration-to-speed reduction elasticity factor

Comment: how to derive Eq. (30b)?

If , and , then we have

More comment:

If , and ,

# (31)

: time-dependent speed

: link length

: cut-off speed, which can be used systematically to distinguish “congested” vs. “uncongested” states of traffic bottleneck

: traffic delay departing at time

# (32)

: average delay during the whole peak period

: inflow curvature parameter used in polynomial form

*:* queue discharge rate, assumed to be a constant value for each instance of a queue duration

: peak period, i.e.,

: start time of congestion period

: end time of congestion period

: total in-flow demand during the entire congested period

# (33)

: average speed during the congestion duration

: link length

: cut-off speed, which can be used systematically to distinguish “congested” vs. “uncongested” states of traffic bottleneck

: average delay during the whole peak period

# (34)

