STALite/DTALite Users Guide

Working Document Version 1.0

Please feel free to send any questions, feedback, and corrections to Dr. Xuesong (Simon) Zhou (xzhou74@asu.edu) by adding comments in this document.

Table of Contents

1. Introduction

1.1. Motivation

Motivated by a wide range of transportation network analysis needs, static traffic assignment (STA) and dynamic traffic assignment (DTA) models have been increasingly recognized as a set of important tools for assessing operational performances of those applications at different spatial resolutions (e.g., network, corridor and individual segment levels) and across various analysis temporal regimes (e.g., peak hours, entire day and second-by-second). The mathematical modeling and related volume-delay functions are described in Appendix.

The advances of STA and DTA are built upon the capabilities of integrated flow assignment and simulation models in describing the formation, propagation, and dissipation of traffic congestion in a transportation network.

As a continuation of DTALite, the development of STALite (S stands for strategic or static assignment) is motivated by the following perspectives.

(1) Bridging the gap from macroscopic static assignment to mesoscopic dynamic assignment

Planning practitioners have recognized the full potential of DTA modeling methodologies that describe the propagation and dissipation of system congestion with time-dependent trip demands in a transportation network. In April 2009, the TRB Network Modeling Committee conducted a DTA user survey through the FHWA TMIP mail list, which identified the following top 5 technical barriers:

- DTA requires more data than are available or accessible to most users (47%)
- Setting up a DTA model consumed inordinate resource (44%)
- Cost/benefit of implementation is unclear (45%)
- DTA tools take too long to run (35%)
- The underlying modeling approaches are not transparent (35%)

The development goal of STALite aims to provide an integrated open-source package for strategic traffic analysis that includes both static traffic assignment and dynamic traffic simulation to reflect the impact of road capacity constraints. The underlying volume-delay models include BPR functions and its extension of BPR-X. Three traffic stream models, namely, point queue model, spatial queue model and simplified kinematic wave models, are embedded in the mesoscopic simulator to describe queueing behavior at bottlenecks with tight capacity constraints.

(2) Adopting open network standard of GMNS

General Travel Network Format Specification is a product of Zephyr Foundation, which aims to advance the field through flexible and efficient support, education, guidance, encouragement, and incubation. Further details can be found in https://zephyrtransport.org/projects/2-network-standard-and-tools/

(3) Integrated graphic user interface and analysis package

NeXTA (Network eXplorer for Traffic Analysis) is another open-source graphic user interface (GUI) for transportation network analysis, while the lower-case "*e*" stands for education with broader impacts. With both open-source traffic assignment/simulation engine (as a simple Windows console application) and graphic user interface, the software suite of STALite + NeXTA aims to

- provide an open-source code base to enable transportation researchers and software developers to expand its range of Strategic Traffic Assignment capabilities to various traffic management analysis applications.
- present results to other users by visualizing traffic flow dynamics and traveler route choice behavior in an integrated 2D environment.
- provide a free education tool for students to understand the complex decision-making process in transportation planning and optimization processes.

(4) parallel computing on shared memory multi-core computer

Emerging multi-core computer processor techniques are offering unprecedented available parallel computing power, on most of laptops and desktops currently available in the market. To exploit this paradigm change in computing, we will require a new software architecture and algorithm design so as to facilitate the most efficient use of emergent parallel hardware.

(5) Integrated signal timing optimization (to be added)

(6) Integrated OD demand estimation through path flow estimator (to be added)

The latest software release can be downloaded at our Github website. The source code can be downloaded at https://github.com/xzhou99/STALite. Table 1 illustrates the contents of different folders at Github https://github.com/xzhou99/STALite.

Table 1. contents of folders at Github.

Github Folder Name	Contents
Src	source code of STAlite
Release	a simple working dataset for console application STALite and visualizer of NeXTA.
Doc	user's guide and other documentations for STALite
Dataset	Five Sample datasets for STALite: 1. two_corridor 2. Braess's_paradox 3. Tempe ASU network 4. Sious_Falls 5. Chicago_sketch

The stable release and learning documents are located at https://github.com/xzhou99/stalite-dtalite_software_release.

1.2 System Architecture

The software architecture of STALite aims to integrate many rich modeling and visualization capabilities into an open-source traffic assignment model suitable for practical everyday use within the context of an entire large-scale metropolitan area network. Using a modularized design, the open-source suite of **simulation engine + visualization interface** can also serve future needs by enabling transportation researchers and software developers to continue to build upon and expand its range of capabilities. The **streamlined data flow** from static traffic assignment models can allow state DOTs and regional MPOs to rapidly apply the advanced STA/DTA methodology, and further examine the effectiveness of traffic mobility, reliability and safety improvement strategies, individually and in combination, for a large-scale regional network, a subarea or a corridor.

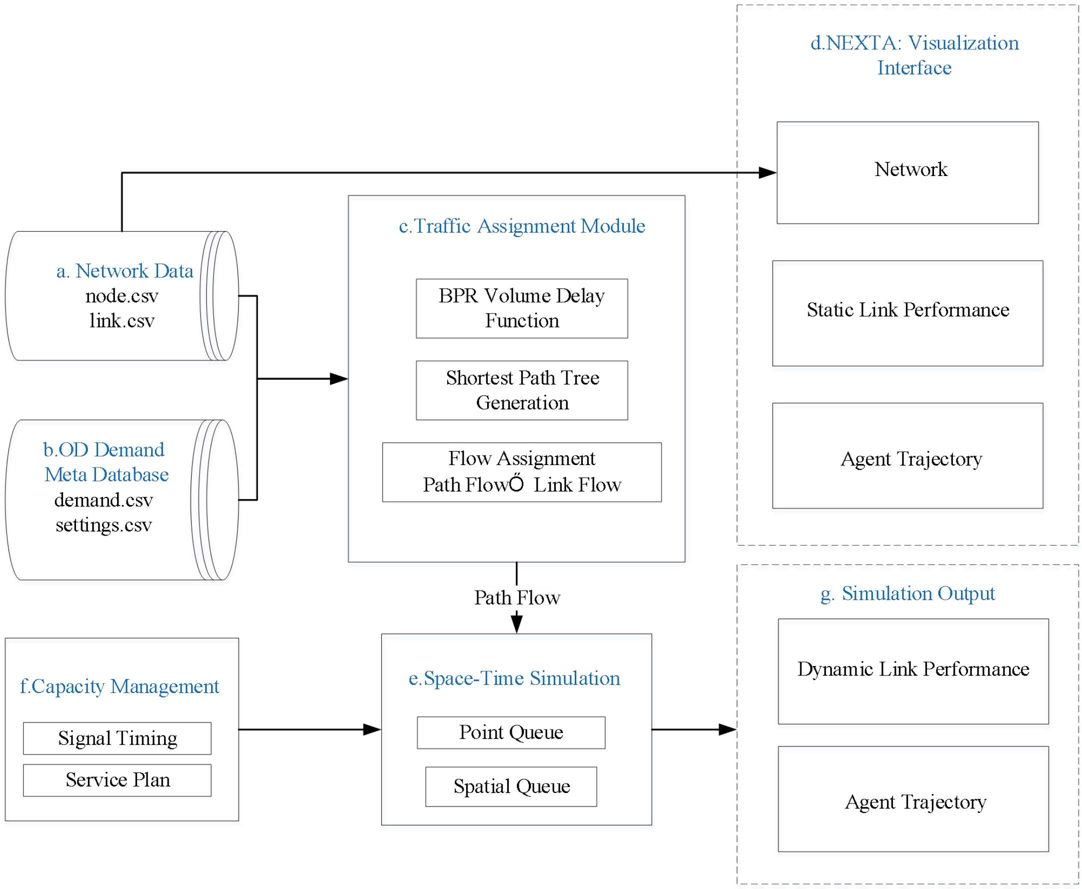


Figure 1.1 Software System Architecture

The components and different modules in the system are listed as following:

a. Network Data includes two essential files, node.csv and link.csv for the macroccopic network representation.

b. OD Demand Meta Database includes the setting.csv as the configuration file that describes information such as agent type, demand period, demand file list, which help users to represent the OD demand information for different user types at specific demand periods.

c. Traffic Assignment Module includes the key steps of the assignment, including the BPR Volume Delay Function, Shortest Path Tree Generation, and Flow Assignment, which generates the path flow and link flow according to the UE principle.

d. NEXTA: Visualization Interface Module is able to visualize the network and the output of traffic assignment, including Static Link Performance and Agent Trajectory.

e. Space-Time Simulation Module utilizes the path flow output of Traffic Assignment Module to perform Space-Time Simulation, while the underlying traffic flow models in the Space-Time Simulation Module are Point Queue (PQ) and Spatial Queue (SQ). A simplified kinematic wave (KW) model can be also used in an advanced mode, similar to DTALite.

f. Capacity Management aims to manage the static and time-dependent link capacity input for Space-Time Simulation, such as signal timing plans and multi-modal service plans.

g. Simulation Output Module covers the output file of Space-Time Simulation Module, including Dynamic Link Performance and Agent Trajectory in terms of link_performance.csv and agent.csv, which can be visualized in NeXTA.

Regarding parameters in settings.csv, Table 2 illustrates the differences between two key steps of Static Traffic Assignment and DTA + space-time simulation.

Table 2. The differences between Static Traffic Assignment and DTA+ space-time simulation

	Static Traffic Assignment	Dynamic Traffic Assignment + space-time simulation
Assignment_mode in settings.csv	UE	DTA
Travel time evaluation	BPR function with volume/capacity ratio (soft capacity constraints)	Space-time network based simulation with tight capacity constraints
Demand input	OD demand	OD demand or agent based
Output (1): link performance	Static Link performance VOC, volume, delay	Dynamic Link performance Queue, delay at time t
Output (2): path/agent data	Path flow for OD and k-paths, based on path pool based gradient projection methods	Individual agent trajectory with path sequence and time sequence

1.3. 5 steps of performing traffic analysis using CSV files

The specific instruction for the use of NeXTA and STALite is as follows:

Step 0: **[Download and locate the project folder]** Download and unzip the release software package from github. Locate STALite file folder with node.csv, link csv, demand.csv and settings.csv. Typically, copy STALite.exe and NeXTA.exe in the same folder for easy access.

Step 1: **[Check input files in Excel]** Open a file explorer, view or edit input files of node, link and demand csv files, in Excel or any text editor. Review and change the configuration in settings.csv in Excel.

Step 2: **[Visualize and validate network in NeXTA]** Click “NeXTA”—“File”—“Open Traffic Network Project” to choose the node.csv file in your network data set. Check the network connectivity through a simple path calculation by selecting one OD pair.

Step 3: **[Run STALite as a Windows console application]** Click on the executable of “STALite.exe” from a file explorer or run it from Windows command window, to perform traffic assignment and simulation. The output of this Windows console applications is displayed in screen and log file STALite_log.txt.

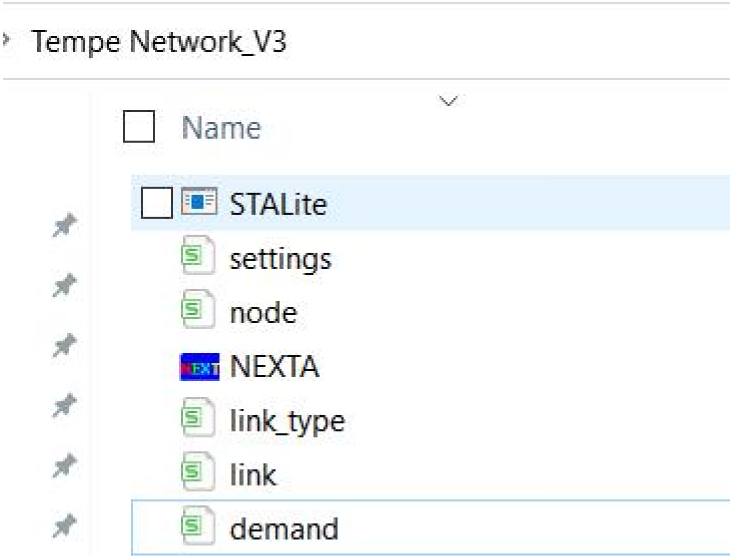
Step 4: [**Check output files in Excel**] After the completion of STALite, users can view the output link performance and agent files in Excel.

Step 5: [**Visualize output files in NeXTA**] For static traffic assignment, NeXTA is able to display view link travel time, speed and volume, as well as path display in the agent dialog. For dynamic assignment and simulation, one can use NeXTA to view time-dependent queue and density.

2. Getting Started from NeXTA graphical user interface and running STALite

Step 1: Download and locate the project folder, check CSV network files.

Locate the project folder of "Tempe network".



Check input files in Excel

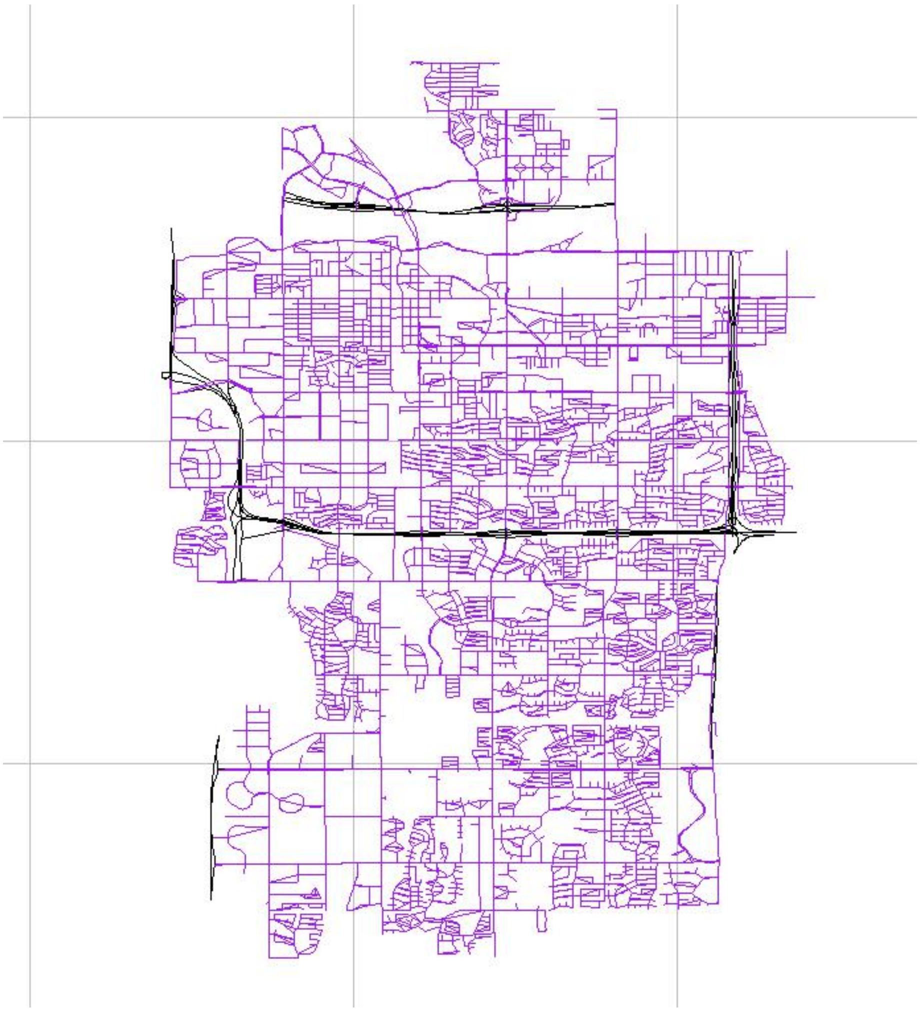
node.csv

name	node_id	zone_id	ctrl_type	node_type	activity_ty	is_bounda	x_coord	y_coord	geometry
	1	1			residential	0	-111.918	33.33409	POINT (-111.9176573 33.3340915)
	2	2			residential	0	-111.918	33.32785	POINT (-111.9180435 33.3278453)
	3	3		turning_cir	residential	0	-111.918	33.3284	POINT (-111.9181293 33.3283975)
	4	4			residential	0	-111.937	33.32473	POINT (-111.9370903 33.3247312)
	5	5			residential	0	-111.942	33.40662	POINT (-111.9419776 33.406618)
	6	6			residential	0	-111.94	33.33464	POINT (-111.9396516 33.3346409)
	7	7			residential	0	-111.965	33.42311	POINT (-111.9652898 33.4231118)
	8	8			residential	0	[illegible]	[illegible]	[illegible]

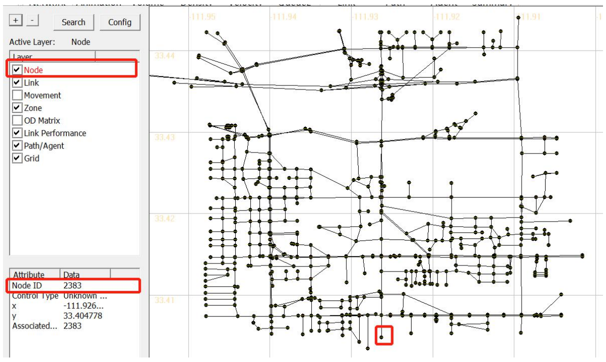
link.csv

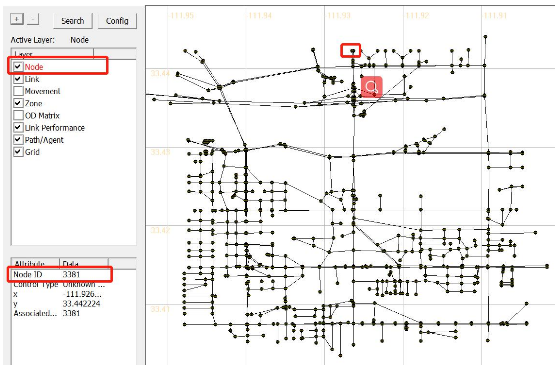
name	link_id	from_nod	to_node_i	dir_flag	length	lanes	free_spee	capacity	link_type_	link_type	geometry
East Calle	1a	1	4369	1	0.09217	1	29	699	residential	5	LINESTRING (-111.
East Calle	1b	4369	1	1	0.09217	1	29	699	residential	5	LINESTRING (-111.
East Calle	3a	1	4205	1	0.061443	1	29	699	residential	5	LINESTRING (-111.
East Calle	3b	4205	1	1	0.061443	1	29	699	residential	5	LINESTRING (-111.
East Ranch	5a	1	2524	1	0.146747	1	29	699	residential	5	LINESTRING (-111.
East Ranch	5b	2524	1	1	0.146747	1	29	699	residential	5	LINESTRING (-111.

Step 2: Visualize and validate network in NeXTA using shortest path finding

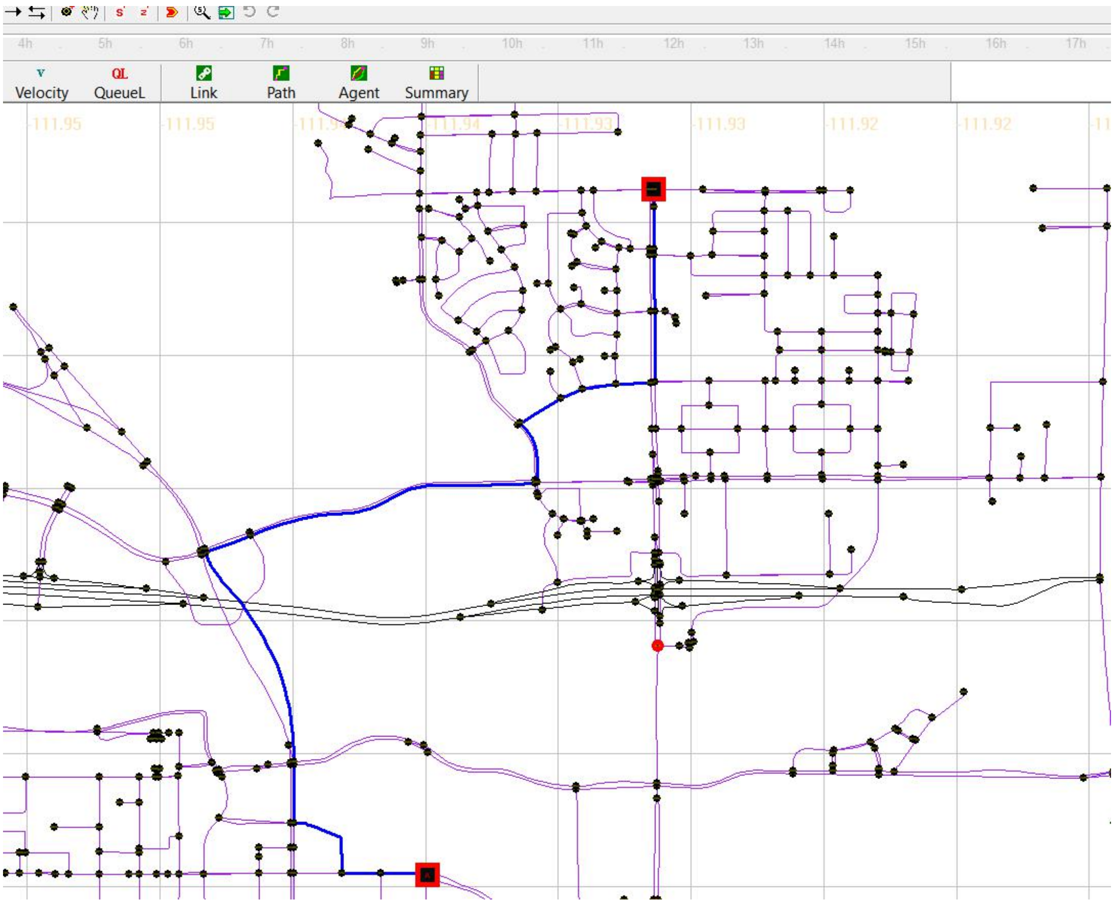


First, select the node layer in the left-hand-side GIS panel, we can use the mouse to select node 44, and node 86. Alternatively, one can use a keyboard shortcut of Control+f to search those nodes.

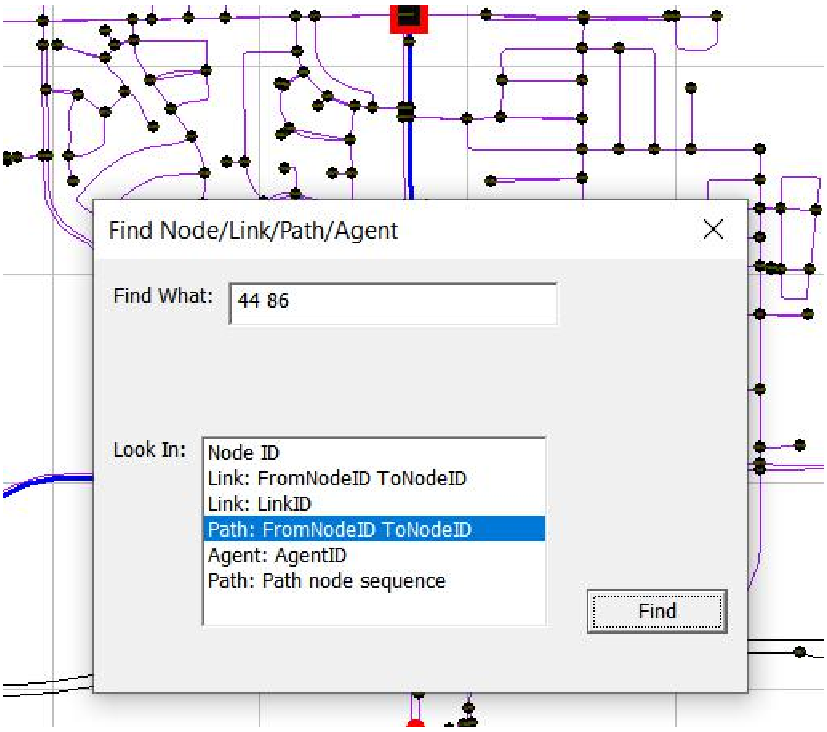




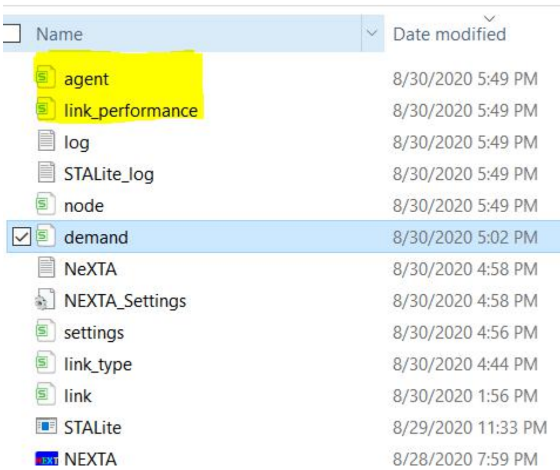
Go the path GIS layer, right click to check and confirm if this path is connected.



Alternatively, one can use a keyboard shortcut of Control+f to specify the origin and destination for the path.



Step 3. Run STALite as a Windows console application from File Explorer



The user now can check output files in Excel for the following two files:
link performance.csv
agent.csv

All STALite **data files** are in CSV format. The files for node, link and zone layers have geometric fields for importing from and exporting to GIS software.

3. Toy Examples for Computing Static User Equilibrium

3.1 Two-corridor example

This example uses a simple case with a single origin-to-destination pair and two paths p=1 for the primary path, p=2 for the alternative path, see in Figure 3.1 As each path has two links, path 1 has a free-flow travel time of 20 minutes, and path 2 has a free-flow travel time of 30 minutes.

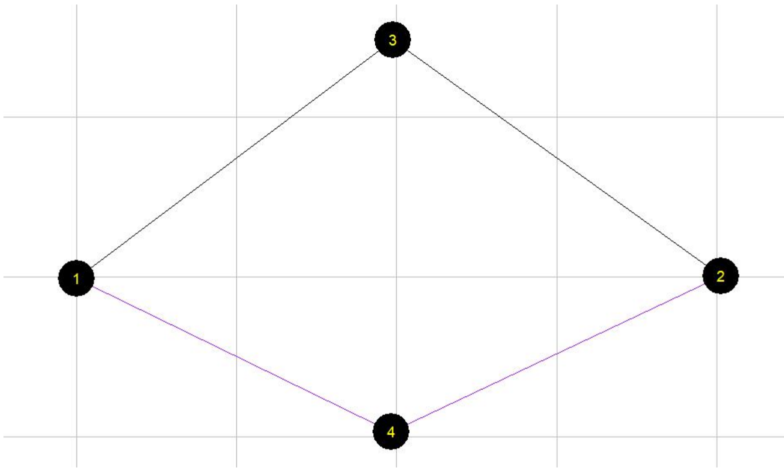


Figure 3.1 illustrative example of two-corridor network

For a given OD demand of 7,000 on this network, we can use the User Equilibrium method to perform traffic assignment. A graphic-based solution process can be described by Figure 3.2. As the path flow changes, the travel time on the two paths reaches the same equilibrium point, which satisfied the requirement of User Equilibrium. User equilibrium solution is reached when the freeway flow is 5400, and arterial flow as 7000-5400=1600, and this leads to the same travel time of 30 min on both routes.

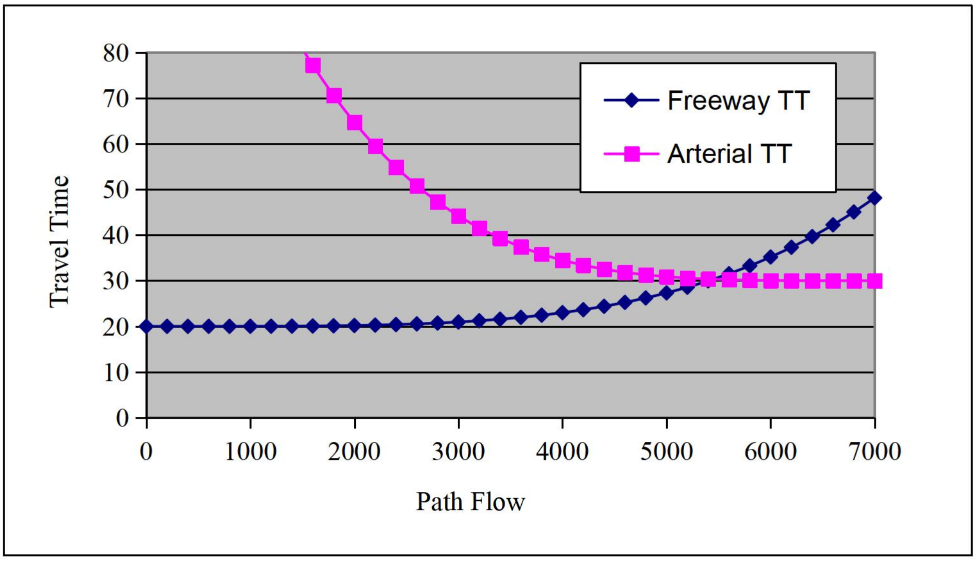


Figure 3.2 illustration of Equilibrium with X axis as freeway path flow.

The detailed parameters are in Table 3.1.

Table 3.1 parameters

Parameters	Value

Freeway flow travel time (min): Freeway:	20
Freeway flow travel time (min): Arterial:	30
Capacity (vehicles / hour): Freeway:	4000
Capacity (vehicles / hour):Arterial:	3000
Demand	7000
BPR alpha	0.15
BPR beta	4

The travel time function is

$$\text{Freeway\_TT} = \text{FFTT}[1 + 0.15(v/c)^4]$$

$$\text{Arterial \_TT} = \text{FFTT}[1 + 0.15((\text{demand}-v)/c)^4]$$

where:

TT = link travel time

FFTT= free-flow travel time of link

v = link flow

c = link capacity

3.2 Detailed data structure description

Generic network files used for STALite include files for three layers: physical layer, service layer and demand layer.

Table 3.1 File list for STALite

File type	Index: file name	Description
Input for physical layer	1a: *node.csv*	Define nodes in the network.
	1b.: *link.csv*	Define links in the network with essential attributes for assignment.
Input for demand layer	2: *demand.csv*	Define the demand of passengers on each OD pair, which could be extracted by *demand_file_list.csv*.
Input configuration file	3: *settings.csv*	Define basic setting for the Network, it contains five sections.
	Section of assignment	Set the number of iteration and the mode of assignment.
	Section of agent_type	Define attributes of each type of agent, including VOT (unit: dollar per hour) and PCE.
	Section of link_type	Define types of links in the network
	Section of demand_period	Define demand period, which could be extracted by

		demand_file_list
	Section of demand_file_list	Define demand type, period, and format type.
Input for service layer	4: *service_arc.csv*	Define space-time arcs for service based on the physical link with time window, time interval and travel time.
Output file	5a: *link_performance.csv*	Show the performance of each link, including the travel time, volume, and resource balance.
	5b: *agent.csv*	Show the results of the assignment, including the volume, toll, travel time and distance of each path of each agent, as well as the link sequence and time sequence.

The related files used in STALite are listed below.

(1) Prepare input data

- node.csv

Table 3.2 node.csv

node_id	zone_id	x_coord	y_coord
1	1	0.017882	-0.12518
2	2	40.25393	0.053648
3		19.77825	14.80687
4		19.68884	-9.69242

- link.csv

Table 3.3 link.csv

link_id	from_node_id	to_node_id	facility_type	dir_flag	length	lanes
1003	1	3	Freeway	1	10	1
3002	3	2	Freeway	1	10	1
1004	1	4	arterial	1	15	1
4002	4	2	arterial	1	15	1
capacity	**free_speed**	**link_type**	**VDF_fftt1**	**VDF_cap1**	**VDF_alpha1**	**VDF_beta1**
4000	60	1	20	4000	0.15	4
4000	60	1	0	4000	0.15	4
3000	60	2	30	3000	0.15	4
3000	60	2	30	3000	0.15	4

- demand.csv

Table 3.6 demand.csv

o_zone_id	d_zone_id	volume
1	2	7000

- settings.csv

There are different sections in the settings.csv file. And each section starts with the format of [section_name] along with the field names. There are five sections in the settings.csv, see in Table 3.4.

A	B	C	D	E	F	G
[assignment]		assignment_	number_of_	column_updating_iterations		
		ue	20	0		
[agent_type	agent_type	name		VOT	flow_type	PCE
	p	passenger		10	0	1
[link_type]	link_type	link_type_name		agent_type_	type_code	traffic_flow_
	1	Highway/Expressway			f	0
	2	Major arterial			a	0
[demand_pe	demand_pe	demand_period		time_period		
	1	AM		0700_0800		
[demand_fil	file_sequen	file_name		format_typ	demand_pe	agent_type
	1	demand.csv		column	AM	p

(2) Check output files

The files are the output of the previous input data.

- agent.csv

Table 3.10 agent.csv

agent_id	o_zone_id	d_zone_id	path_id	o_node_id	d_node_id
1	1	2	0	1	2
2	1	2	1	1	2
agent_type	**demand_period**	**agent_type**	**toll**	**travel_time**	**distance**
p	AM	p	0	30.3224	20
p	AM	p	0	30.3224	30
	node_sequence	**link_sequence**	**time_sequence**		
	1;3;2;	1003;3002;	0730:00;0800:19;0800:19;		
	1;4;2;	1004;4002;	0800:19;0830:38;0830:38;		

The volume in this file represents path volume, and the path is further represented in node_sequence.

For the above example (Table 3.10), when the assignment reaches equilibrium, there are two paths to choose. For path id 0, the origin zone id is 1 and destination zone id is 2, and the node sequence of this path is 1→ 3 → 2, and travel time on this path is 30.3224 minutes, and distance of this path is 20.

Table 3.11 link_performance.csv

link_id	from_node_id	to_node_id	time_period	volume	travel_time	speed	VOC	notes

1003	1	3	0700_0800	5447.848	30.322	39.575	1.362	period-based
3002	3	2	0700_0800	5447.848	0	0	1.362	period-based
1004	1	4	0700_0800	1552.149	30.322	59.362	0.517	period-based
4002	4	2	0700_0800	1552.149	0	0	0.517	period-based

From link_performance.csv, users are able to obtain the link volume, link travel_time, speed and VOC. For the above example (Table 3.11), the first link id is 1003, and the from-node of this link is 1, and the end-node of this link is 3. From 7:00 to 8:00, the volume on this link is 5447.848, and the travel time during this period is 30.322 (min), and the speed is 39.575 mile per hour, and the volume over capacity (VOC) is 1.362.

(3) Visualize the output in NEXTA

Open NEXTA, import the network, chose the time period that you set in demand_period.csv, and click volume, you can see the assignment outcome in Figure 3.3.

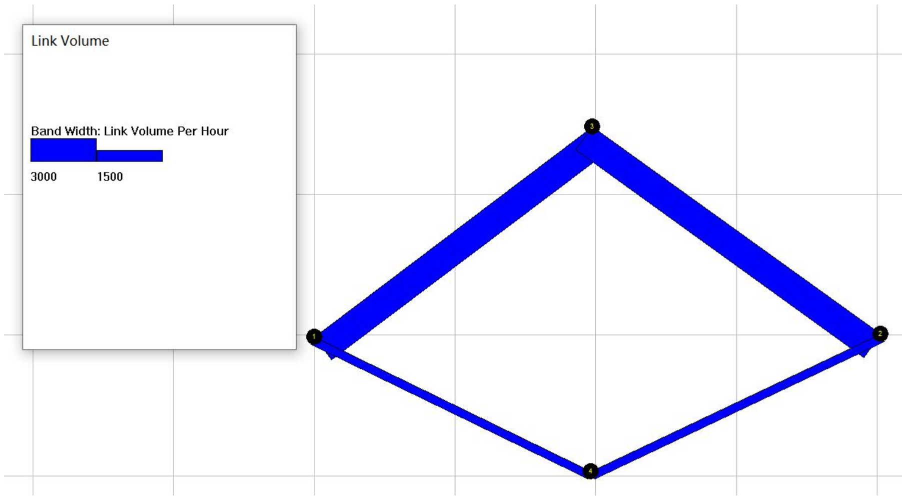


Figure 3.3 Link volume visualization

4. Detailed data structure descriptions

4.1 Input for network data

- The specific files for physical layer are *node.csv* and *link.csv*.
- Nodes in the physical network represent points of demand, including node_id, zone_id, and coordinates with an arbitrary coordinate system.
- A link is defined using upstream node and downstream node ids, with essential attributes such as length, free_speed, lanes, capacity, link_type, and coefficients of Volume Delay Function, typically required for static traffic assignment and mesoscopic traffic assignment.

File 1a: node.csv

Field Name	Description	Sample Value
name	Optional for visualization only	Main street @ Highland Dr.
node_id	Node identification number	1001
x_coord	Longitude or horizontal coordinate in any arbitrary geographic coordinate system.	100
y_coord	Latitude or vertical coordinate horizontal coordinate in any arbitrary geographic coordinate system	200
node_type	Optional text label for visualization and identifies of node	1
zone_id	Indication of node's physical location	1

File 1b: link.csv

Field Name	Description	Sample Values
name	Optional for visualization purposes	Main Street
link_id	Link identification number of the road	1003
from_node_id	Upstream node number of the link, must already be defined in *node.csv*	1
to_node_id	Downstream node number of the link, must already be defined in *node.csv*	3
link_type_name	Optional text label for visualization and data checking purposes	1
dir_flag	Indication of directions of the link (=0, bi-direction; =1, single direction)	1
length	The length of the link (between end nodes), measured in units of miles or km.	10
free_speed	Free-flow speed on defined link. Suggested Unit: mph or kmph	60
lanes	The number of lanes on the link	1

capacity	The number of vehicles per hour per lane	4000
link_type	Index of link type name	1
toll	Optional generalized toll cost of the link, which could also be the cost of fuel	0
VDF_cap1	Capacity used in the volume-delay function	4000
VDF_alpha1	Coefficient used in the volume-delay function	0.15
VDF_beta1	Coefficient used in the volume-delay function	4

4.2 Input for demand data

- The specific files for demand layer is *demand.csv*
- Travel demand is given by periods. Thus, one file defines total volume of demand and one file defines time periods.

File 2: demand.csv

Field Name	**Description**	**Sample Values**
o_zone_id	Origin zone number of the link, must already defined in *node.csv*	1
d_zone_id	Destination zone number of the link, must already defined in *node.csv*	2
demand	Travel demand	1500

4.3 Assignment and simulation configuration file

- The specific file for the configuration file is *settings.csv.*
- It defines basic setting for the Network, and contains five sections, namely assignment, agent_type, link_type, demand_period, demand_file_list.

File 3: settings.csv

Section Name	**Field Name**	**Description**	**Sample Values**
assignment	assignment_mode	Assignment_mode can be ue, dta or	1

		odme	
	number_of_iterations		40
	column_updating_iterations		40
agent_type	agent_type_id	Agent type identification number (type: integer)	1
	agent_type	Abbreviation of the agent type (type, string)	p
	name	Full name of the agent type	Passengers or vehicles
	VOT	Value of time for the agent	10
	PCE	Passenger Car Equivalent (PCE) of the agent	1
link_type	time_period	Time period expressed by HHMM_HHMM	0600_1200
demand_period	demand_period_id	Demand period identification number (type: integer)	1
	demand_period	Name of the demand period (type: string)	AM
	time_period	Time period expressed by HHMM_HHMM	0600_1200
demand_file_list	file_sequence_no	Sequence number of reading the files	1
	file_name	Name of the file to be read	demand
	format_type	column for three columns, agent_csv, routing policy	Column
	demand_period	Name of the demand period	AM
	agent_type	Abbreviation of the agent type	p

4.4 Input for signal timing and service layer

- The specific files for service layer is *service_arc.csv*.
- Service arcs are the physical link added with time dimension, which could describe the time periods of a certain link for open access.
- Agents providing transport service in the network must use certain services arcs, which are predetermined.

File 4: service_arc.csv

Field Name	**Description**	**Sample Values**
name	Optional name of the arc	vehicle arc
from_node_id	Upstream node number of the link, must already defined in *node.csv*	1
to_node_id	Downstream node number of the link, must already defined in *node.csv*	3
time_window	Time period when the link is open for service	0800_0900
time_interval	Time interval of each vehicle on the link	10
travel_time_delta	Free flow travel time on the link	25
capacity	Capacity for the services during this time period	1000

4.5 Output file

File 5a: link_performance.csv

Field Name	**Description**	**Sample Values**
link_id	Link identification number of the road	1
from_node_id	Upstream node number of the link, must already defined in input_node.csv	1
to_node_id	Downstream node number of the link, must already defined in input_node.csv	3
time_period	The simulation period of the agent HHMM format	0700_0800

volume	Link based flow volume for the defined period	5600
travel_time	Link travel_time in minute	15
speed	Average travel speed on the link	38
VOC	Volume /capacity ratio	0.4
notes	Some explanatory text	period-based

File 5b: agent.csv

Field Name	Description	Sample Value
agent_id	Node identification number	1
o_zone_id	Origin zone number of the agent	1
d_zone_id	Destination zone number of the agent	2
path_id	Path identification number	0
o_node_id	Origin node number of the agent	1
d_node_id	Destination node number of the agent	2
agent_type	Type of the agent	p
demand_period	Name of the demand period	AM
volume	Flow volume assigned on the agent	5600
toll	The amount of money/time that agent pays, unit: dollar	360
travel_time	The total time from the origin to the destination of the agent	31.5
distance	The total travel distance from the origin to the destination of the agent, unit: mile or km as users defined for length in link.csv	20
node_sequence	The number of nodes through which agents pass in turn	1;3;2;
link_sequence	The number of links through which agents pass in turn	1003;3002;

time_sequence	Time point through which agents pass in turn	0730:00;0745:45;0801:31;
time_decimal_sequence	The number of decimal times through which agents pass in turn	450.00;465.76;481.52;

Appendix: **From mathematical modeling to network-based assignment and simulation**

(1) Link volume-delay function in static traffic assignment

There are a number of key components for the static traffic assignment procedure.

- input trip table describes the flow per hour from each origin zone to each destination zone
- a traffic network consisting of nodes, links and link volume delay functions
- volume-delay function such as BPR **(Bureau of Public Roads)** relationship that shows increased link travel time as an increase in link volume

$$TT = FFTT[1 + 0.15(v/c)^4]$$

where:

TT = link travel time

FFTT= free-flow travel time of link

v = link flow

c = link capacity

[Remark: the link travel time function typically is only dependent on its own flow, while ignoring link volume on opposing or conflicting directions. The link capacity might not be a strict upper limit on flow, e.g. specified by highway capacity manual.]

As one of the simplest cases of behavior, User Equilibrium (UE) Principle assumes users are "greedy" and are familiar with the system. E*quilibrium requires iteration* to reach the following two principles:

- Principle A: No individual trip maker can reduce his path costs by switching routes.
- Principle B: All used routes between an O-D pair have equal and minimum costs

While all unused routes have greater or equal costs *(to the used path costs)*.

Wardrop (1952) proposed the user equilibrium and system optimal principles of route choice behavior in his seminal paper, and Beckman et al. (1956) formulated the static user equilibrium traffic assignment problem as an equivalent convex mathematical programming problem. Since their influential contributions, the development of the static network assignment formulations, algorithms and applications have made remarkable progress. The books by Sheffi (1985) and Patriksson (1994) provide the most comprehensive coverage on the static traffic assignment problem and its variants.

(2) General mathematical descriptions of traffic assignment

Traffic assignment loads an origin-destination (OD) trip matrix onto links of a traffic network, while satisfying a certain route choice behavioral model, e.g., deterministic user equilibrium. Traffic assignment is used to predict/estimate how trip-makers may shift to other routes or departure times in response to a number of strategies such as road pricing, incidents, road capacity improvement and traffic signal re-timing.

For example, tolling typically lead to traffic diversion on alternative routes and/or other transportation modes, and many traffic congestion mitigation strategies should be developed to improve the capacity to which the traffic may be diverted, for example, signal optimization, traveler information provision, and transit operation.

The common time periods include morning peak, afternoon peak and off-peak, and we can use the time of day factor to calculate the trip in the peak hour (e.g., morning peak may be 11% of daily traffic) from a 24 hour demand volume.

By using a simplified static traffic assignment formulation, the following mathematic description adopts the related sections in the paper titled "Equivalent Gap Function-Based Reformulation and Solution Algorithm for the Dynamic User Equilibrium Problem" by Lu, Mahmassani and Zhou in (2009). One can consider the extended DTA formulation by adding a time index dimension.

Consider a network G = (N, A), where N is a finite set of nodes and A is a finite set of directed links (i, j), $i \in N$ and $j \in N$. Associated with each link (i, j) is the link travel time $s_{ij}(t)$ required to traverse link (i, j) when departing at time interval $t \in S$ from node i. For simplicity and without loss of generality, $s_{ij}(t)$ is regarded as link travel time, though it can be generalized to include travel time, out-of-pocket cost and other travel impedances that may incur when traversing link (i, j) at time t. Travel time and cost are used interchangeably in this paper. Other important notation and variables are summarized below.

O	subset of origin nodes; $O \subseteq N$
D	subset of destination nodes; $D \subseteq N$.
T	set of departure time intervals.
o	subscript for an origin node, $o \in O$.
d	subscript for a destination node, $d \in D$.
$P(o,d)$	set of all feasible paths for a given triplet (o, d).
p	subscript for a path $p \in P(o,d)$.
q_{od}	number of trips departing from node o to node d.
r_{odp}	number of trips departing from o to d and assigned to path $p \in P(o,d)$.
r	path flow vector, $r = \{r_{odp}, \forall o \in O, d \in D, \text{and } p \in P(o,d)\}$.
$c_{odp}(r)$	path travel cost (or time) for the travelers departing from o to d and assigned to path $p \in P(o,d,\tau)$; $c_{odp}(r) = \sum_{(i,j)\in p} s_{ij}$, and is a function of the path flow vector r.
$c(r)$	vector of path travel costs; $c(r) = \{c_{odp}(r), \forall o \in O, d \in D, \text{and } p \in P(o,d)\}$.

The OD demand pattern for the entire planning horizon (i.e.,q_{od}, $\forall o$, d is assumed to be known *a priori*. The key behavioral assumption for the path choice decision is as follows: in a disutility-minimization framework, each trip-maker is rational and chooses a path that minimizes the travel cost. Specifically, for each trip-maker in(o,d), a path $p^* \in P(o,d)$ will be selected if and only if $c_{odp}(r) = min_{p \in P(o,d)}\, c_{odp}(r)$.

Given the assumptions above, the problem is to solve the UE traffic assignment problem, with a given OD demand, to obtain a path flow pattern satisfying the UE conditions. Specifically, the goal is to determine a UE path flow vector (routing policies) over a vehicular network for each OD pair and each departure time interval (i.e., $r^* \equiv \{r_{odp}*, \forall o, d, \text{and } p \in P(o,d)\}$.

By the above UE definition, all trips in a network are equilibrated in terms of actual experienced path costs, so it is necessary to determine the experienced path costs $c(r)$ for a given path flow vector r. To this end, a simulation-based dynamic traffic (or network loading) model is used to obtain the experienced path cost vector. It should be noted that the algorithm is independent of the specific dynamic traffic model selected; any (macroscopic, microscopic or mesoscopic) dynamic traffic model capable of capturing complex traffic flow dynamics, in particular the effect of physical queuing, as well as preventing violations of the first-in-first-out property, can be embedded into the proposed solution algorithm.

With the introduction of the gap function $Gap(r, \pi)$, the proposed nonlinear minimization problem (NMP) is presented as the following.

$$\underset{r,\pi}{Min} \sum_{o\in O} \sum_{d\in D} \sum_{\tau\in T} \sum_{p\in P(o,d)} r_{odp}[c_{odp}(r) - \pi_{od}] \tag{1}$$

$$\text{Subject to } \sum_{p\in P(o,d)} r_{odp} = q_{od},\ \forall o, d \tag{2}$$

$$c_{odp}(r) - \pi_{od} \geq 0,\ \forall o, d, \text{ and } p\in P(o, d) \tag{3}$$

$$r_{odp} \geq 0,\ \forall\ o, d, \text{ and } p\in P(o, d) \tag{4}$$

In the above NMP reformulation, both π and r are *independent* decision variables and hence the gap function is a function of both r and π (i.e., $Gap(r, \pi)$), where π and r are connected with each other through inequality constraint (3). $Gap(r, \pi)$ provides a measure of the violation of the UE conditions in terms of the difference between the total actual experienced path travel cost and the total shortest path cost evaluated at any given path flow pattern $r\in\Omega$. The difference vanishes when the path flow vector r^* satisfies the UE conditions. Thus, solving the UE problem can be viewed as a process of finding the path flow vector $r^*\in\Omega$ and π^* such that $Gap(r^*, \pi^*) = 0$.