CLASS 5

Work on any problems we discussed and feel free to ask me anything!

Break 1:	10:30 AM - 10:45 AM
Lunch:	12:15 PM - 1:30 PM
Break 2:	3:00 PM - 3:15 PM
End:	4:45 PM

AlphaStar
Academy

1. DFS w/ pruning
2. Sorting on K, DSU w/ keeping track of sizes
3. Inclusion-Exclusion
4. Pick a start point + boundary tracing (think about it as if you were in a dark maze)
5. DP O(N^2) - DP[i][j?] = i books used, j - maybe shelves used.
6. Eulers Tour OR maintain stacks of the last time we saw a certain type of cow in the DFS (think about LCA)
7. Think about what it means to be balanced (prefix sums with +1 for an open and -1 for a closed), DP[i][Holsteins Open]~~[Guernseys Opens~~] = i is the index of the string that's input

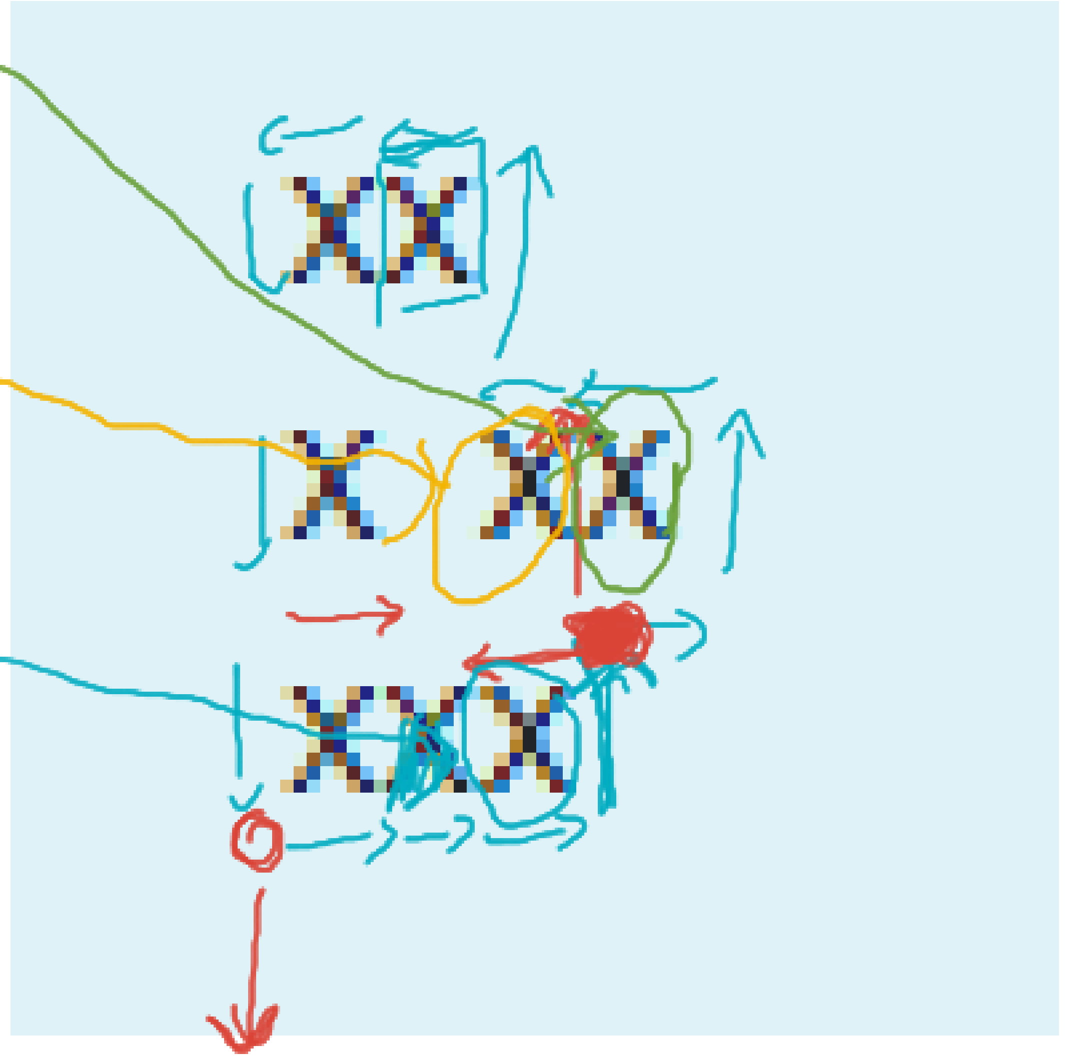
Before Lunch

#4 Perimeter

1 If you can turn right, then do it

2. If you can go forward, then do it

3. If you can turn left, then do it



O(N log N)

If you can turn right, then do it

If you can turn left, then do it ↑ If there is an X that allows you turn left while clinging onto it

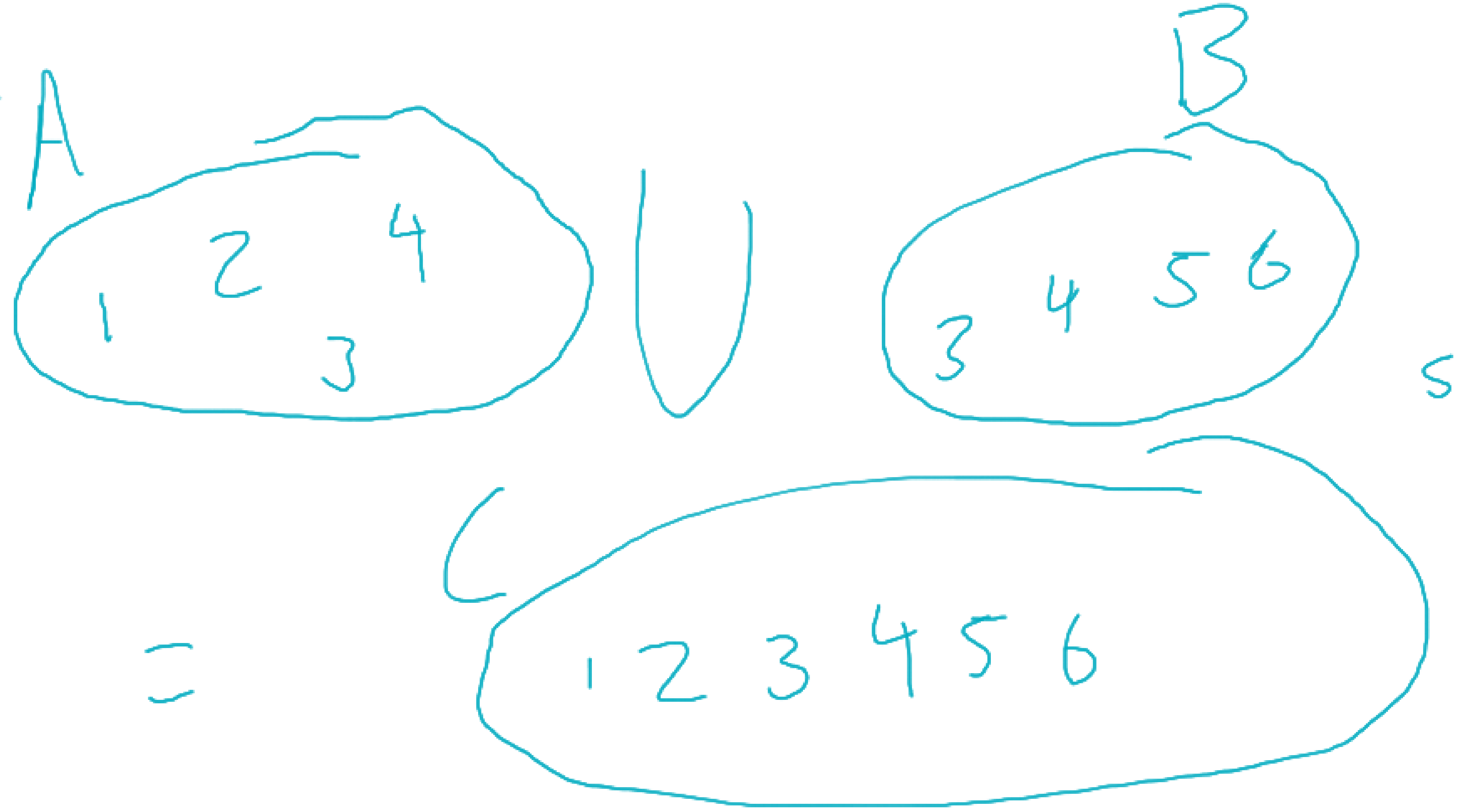
If you can go forward, then do it

Data Structure to have efficient

-contains → O(log n
set

#3 Cowpatibility

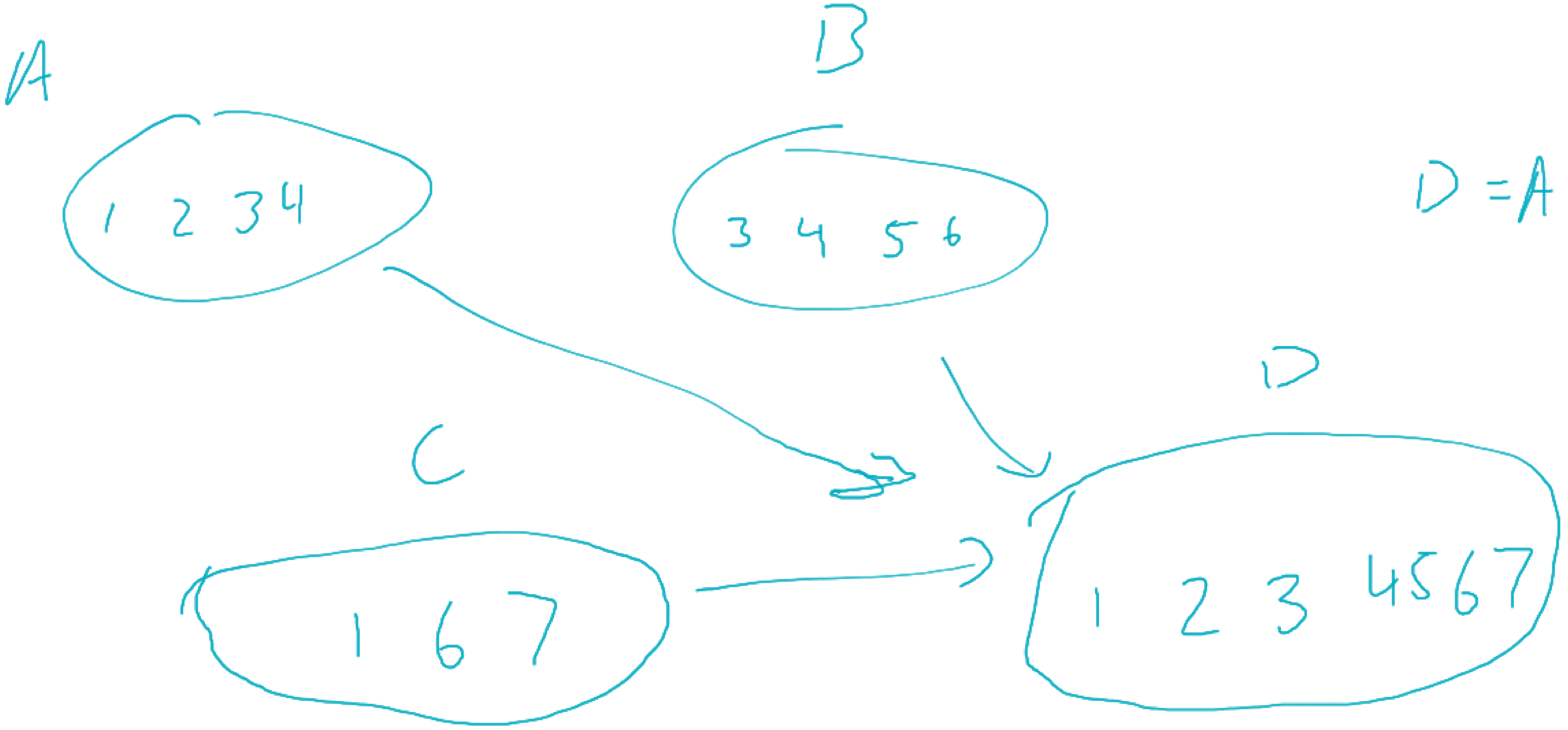
PIE - Inclusion-Exclusion



size(C)

= size(A) + size(B)

- size(A ∩ B)



$$D = A \cup B \cup C$$

$$A + B + C - A \cap B - B \cap C + A \cap B \cap C$$

$$- A \cap C$$

$A \quad B \quad C \quad D \quad E$

$$\binom{5}{1} \qquad \binom{5}{2}$$

$$A + B + C + D + E - (A \cap B + A \cap C + A \cap D + A \cap E + B \cap C + B \cap D \ldots)$$

$$\binom{5}{3}$$

$$+ (A \cap B \cap C + A \cap B \cap D + \ldots)$$

$$\binom{5}{4}$$

$$- (A \cap B \cap C \cap D + \ldots)$$

$$\binom{5}{5}$$

$$+ (A \cap B \cap C \cap D \cap E)$$

$\binom{5}{i}$ $i:0..5$

$2^5 - 1 = 31$

$$A = \left[\begin{array}{cccccccccccc} 2 & 2 & 2 & 1 & 1 & 1 & 1 & 2 & 1 & 2 & & \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & & \\ & & & & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & \cdots & & 50 & 60 & 70 & 80 & 90 \end{array}\right]$$

A[i] = number of cows which like this flavor

1. $(1 \quad 2 \quad 3 \quad 4 \quad 5) \Rightarrow A[1] + A[2] + A[3] \ldots \to 0$

2. $(1 \quad 2 \quad 3 \quad 10 \quad 8) \to A[1] + A[2] + A[3] + A[10] + A[8]$

$\to 3$

3. $(10 \quad 9 \quad 8 \quad 7 \quad 6) \to \sum A[i] \to 2$

$$\frac{4(4-1)}{2} = 6$$

4 0

A | | | | |

1 2 3 4 5 6 7 8 9 10 50 60 70 80 90

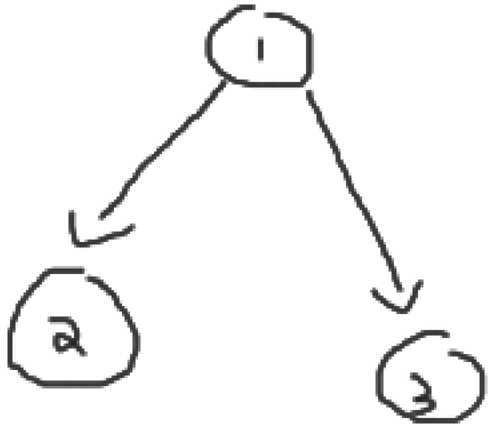
| | | 0

1 ∩ 2 1 ∩ 3 1 ∩ 4 . . . 1 ∩ 6

| |

1 ∩ 2 ∩ 3 1 ∩ 2 ∩ 4

⋮



1. (1 2 3 4 5)

PIE

$$(A[1] + A[2] + \cdots) - (A[1 \cap 2] + A[1 \cap 3] + \cdots) + (A[1 \cap 2 \cap 3] + \cdots)$$

↳ 0

2. (1 2 3 10 8)

↳ 1

3. → 1 4. → 0

#5 Bookshelf

N < 2000

W(i) - width of book i

H(i) - height of book i

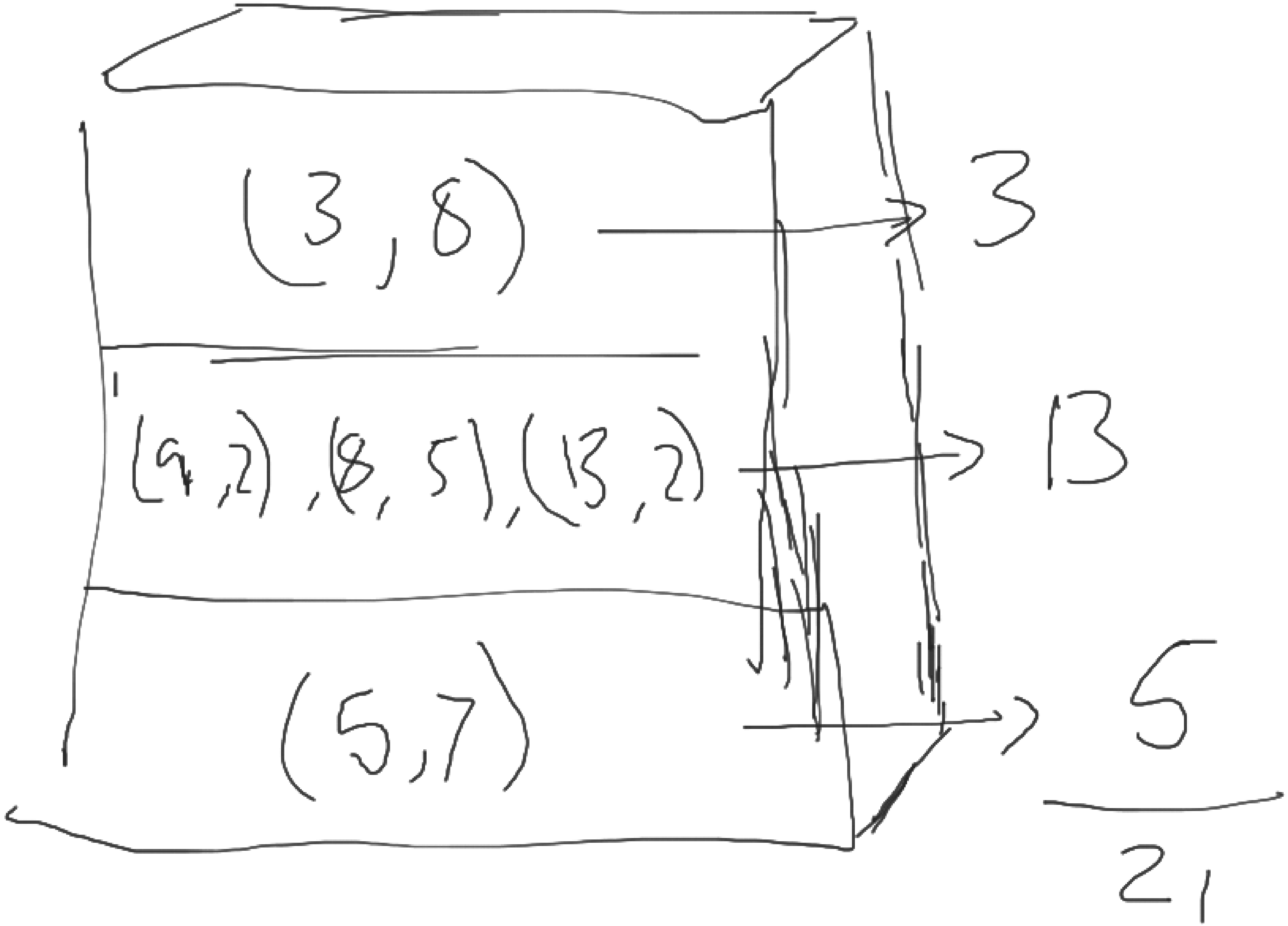
Shelf has total width <= L

Books have to be added in order?

minimize total height of the bookshelf where the height of each shelf max height of books in that shelf

$N = 5 \quad L = 10$

5	7
9	2
8	5
13	2
3	8
↑	↑
H(i)	ω(i)



$DP[i]$ = the minimum height of fitting books $1 \dots i$ in a bookshelf

$DP[1] = 5$

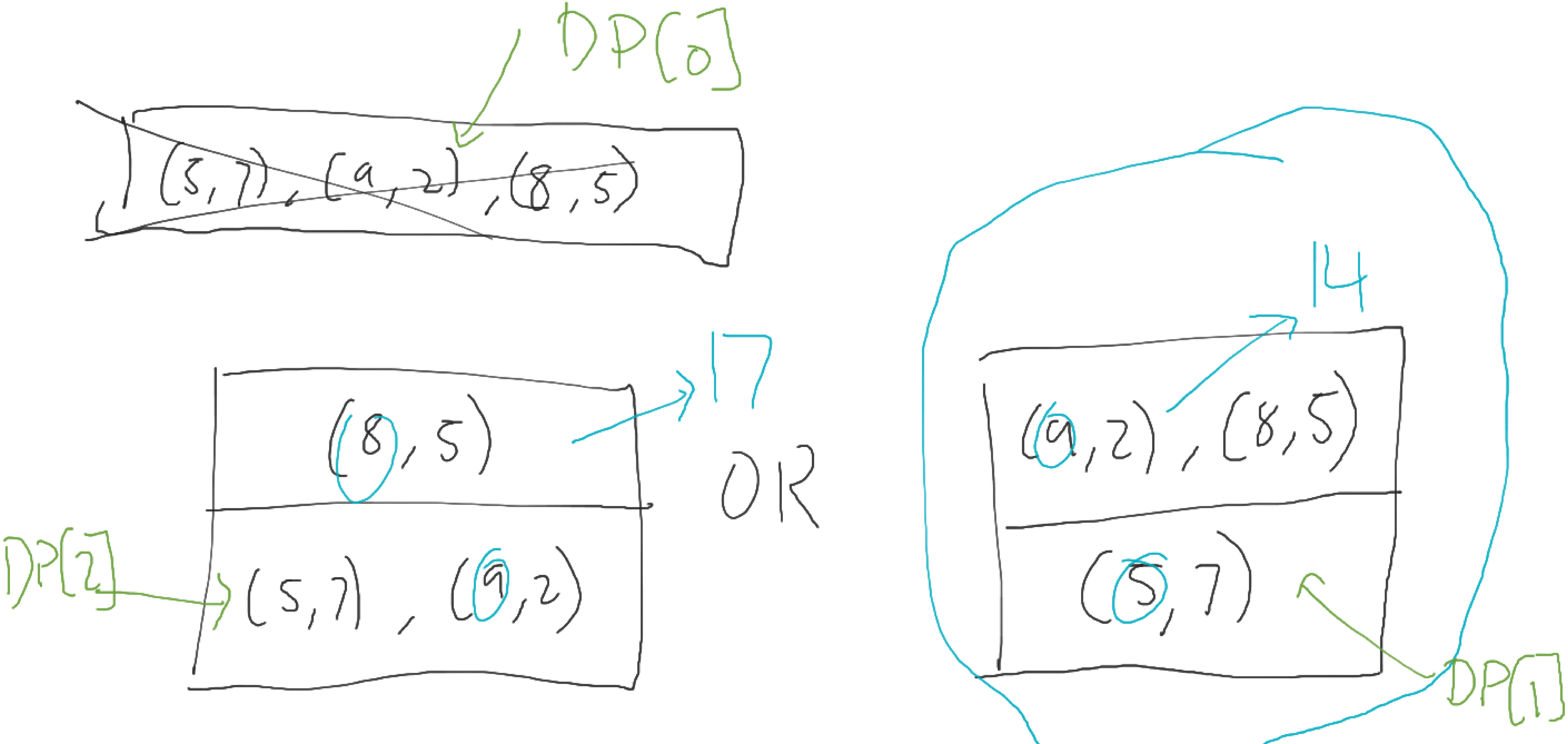
$DP[2] = 9$

$DP[3] = 14$

$DP[4] = 18$

$$\boxed{(5,7),\ (9,2)} \longrightarrow \textcircled{9}$$

DP [3]



$DP[i] \longleftarrow DP[k]$ for a $k<i$

(insert books from $k+1 \ldots i$ into a single shelf (if valid)

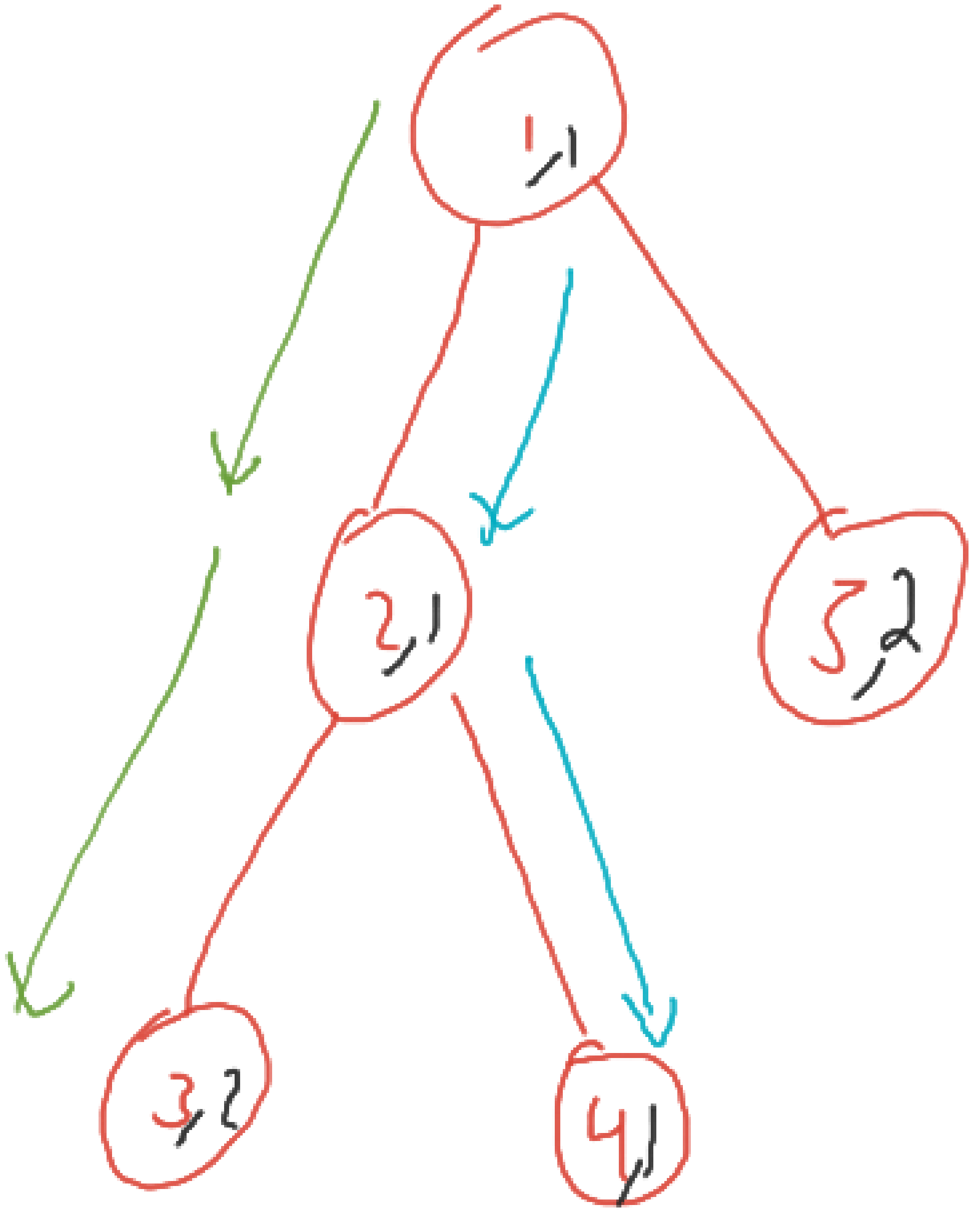
$$DP[k]+\max(h(k+1 \ldots i)) \text{ if } \text{sum}(w(k+1 \ldots i))<=L$$

instead of $k : 0 \rightarrow i-1$

$k : i-1 \longrightarrow 0$

$\text{maxHeight} = \max(\text{maxHeight}, h(k))$ might need to be k+1

$\hookrightarrow := \max(h(k..i))$

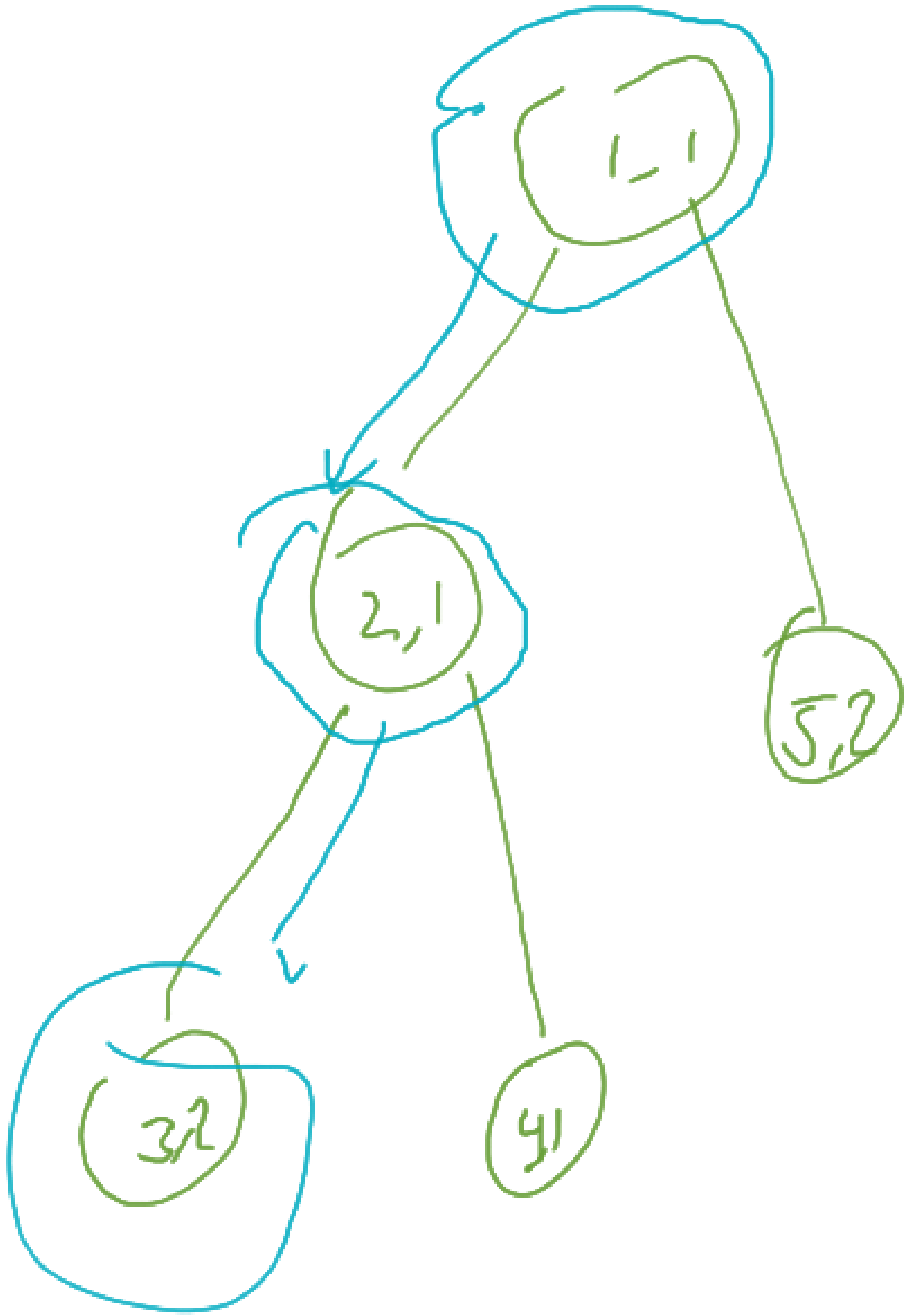


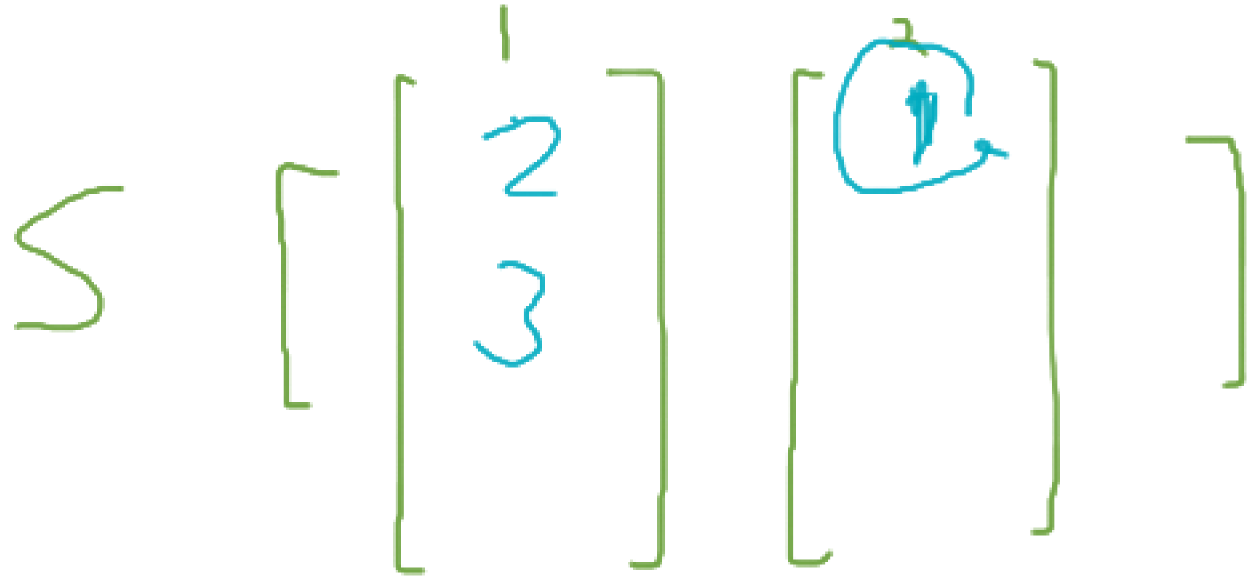
$1 \to 4$ $\boxed{1}$ ✓

$1 \to 4$ $\boxed{2}$ ✗

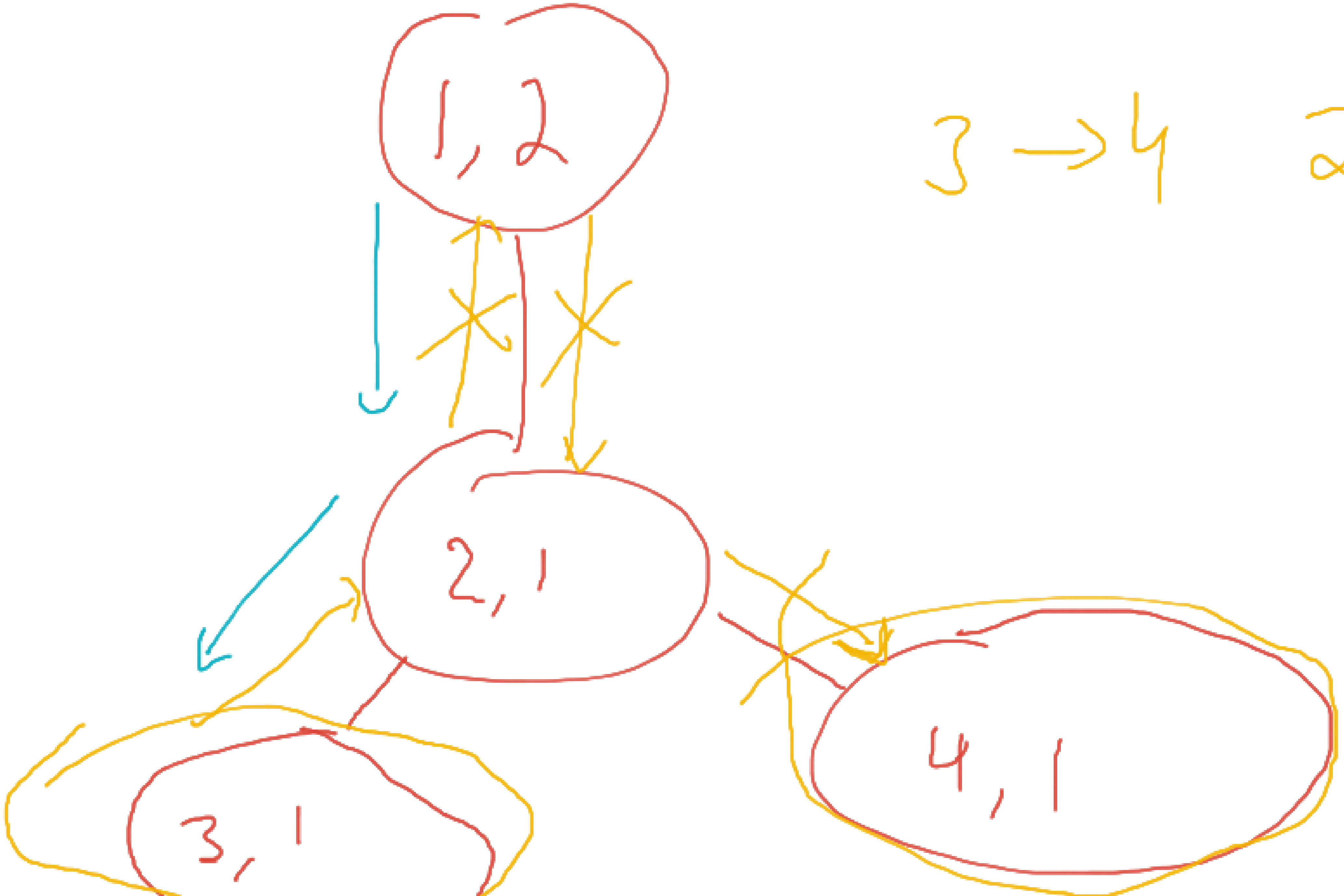
$1 \to 3$ $\boxed{2}$ ✓

Brute Force: $O(NM)$



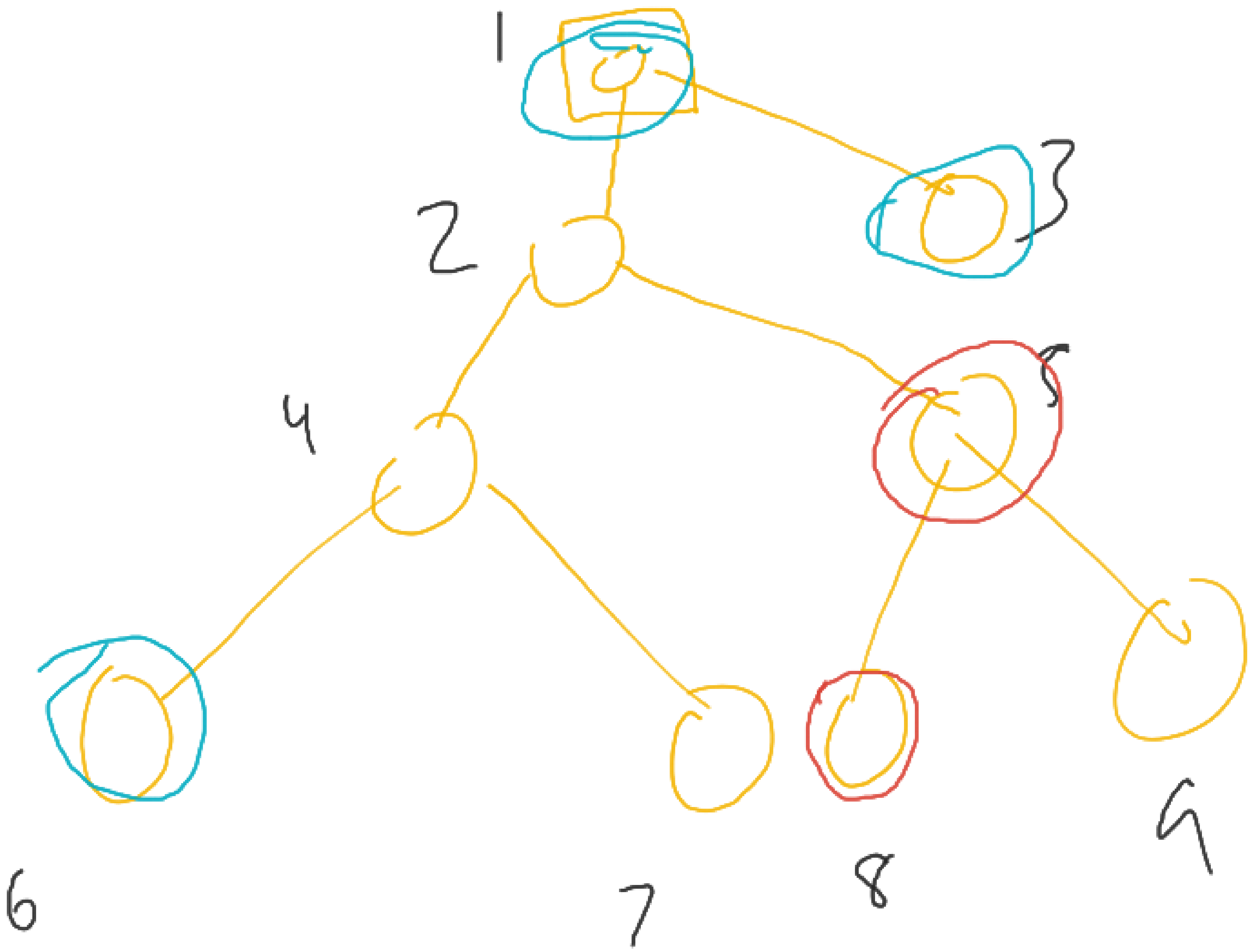


3 → 4 2



Aside LCA

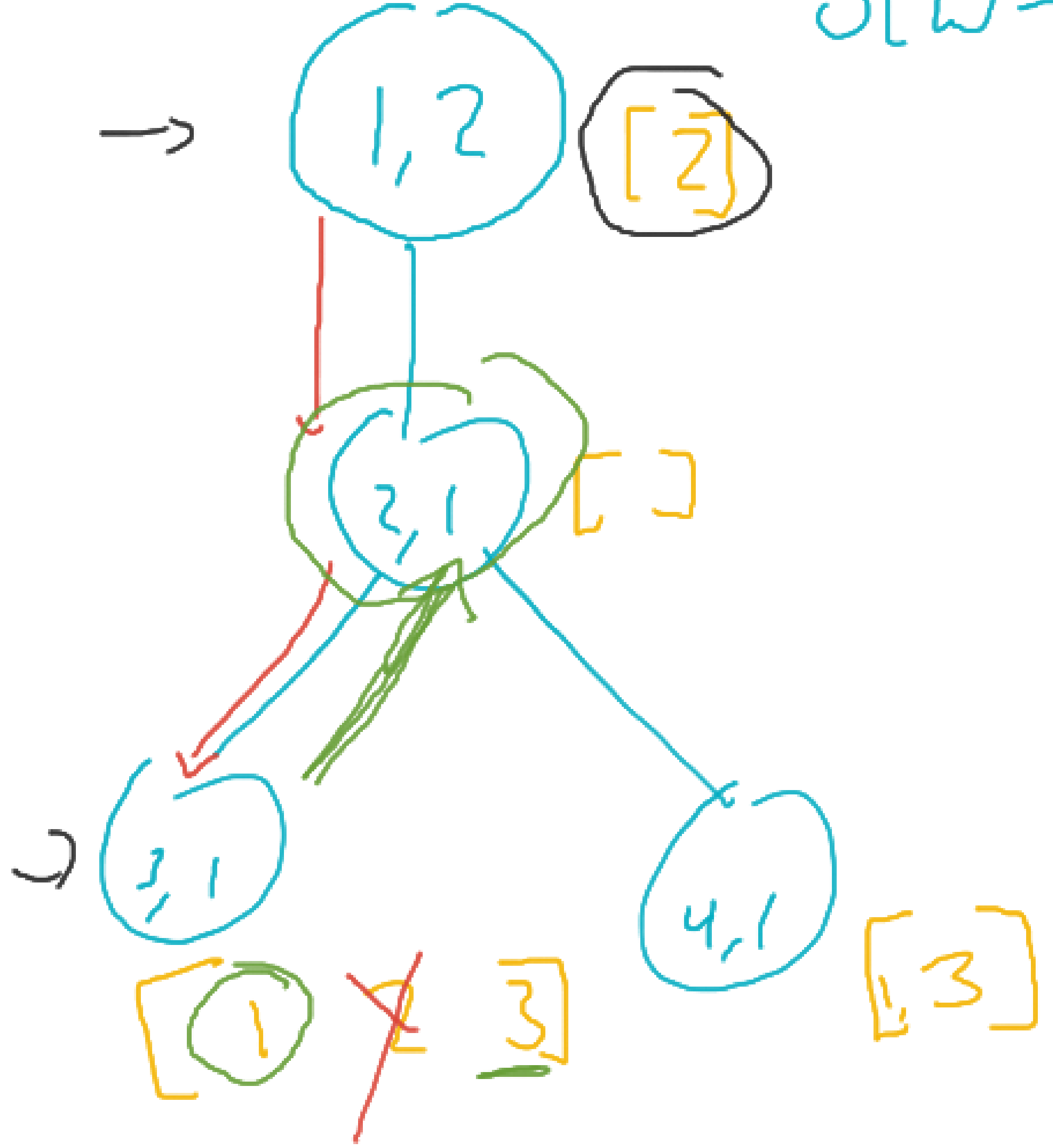
Lowest Common Ancestor

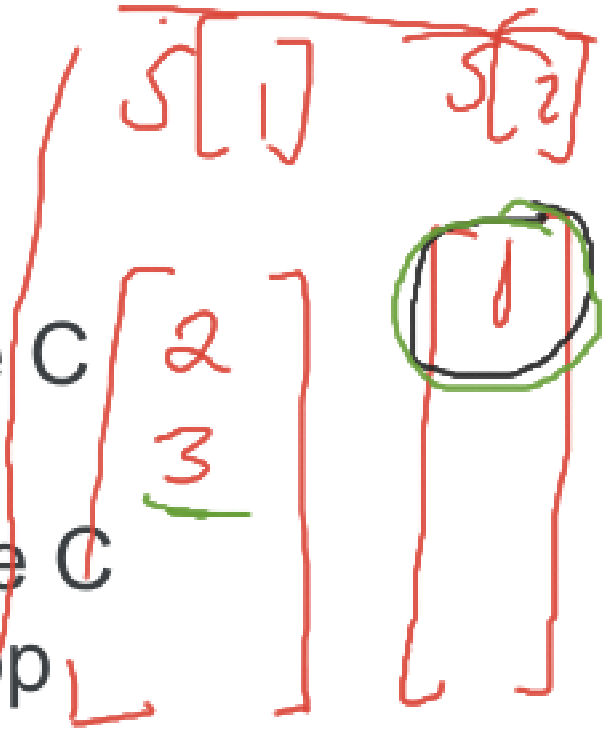


Sparse Tables

every ancestor of LCA(A,B) is a common ancestor of A and B

S[i] = stack of times we've seen cows with milk type i





Given a query of a path from A to B with milk type C

CLAIM: If there does exist any cows with milk type C on the path from A to B, one of them will be the top of stack S[C] at position A or B in the DFS

At Position A, the top of S[C] represents the lowest ancestor of A which has milk type C

S[C].top() is on the path from A to B if and only if LCA(A,B) = LCA(S[C].top, B)

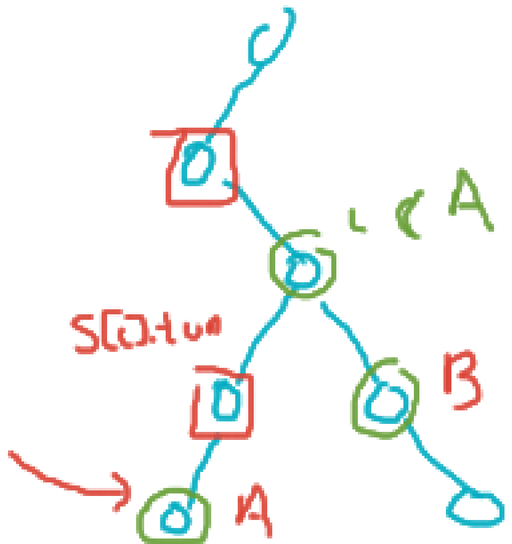
Congruently, it is below LCA(A,B)

O(1)

1. 3 → 4, 2 ?? ✗

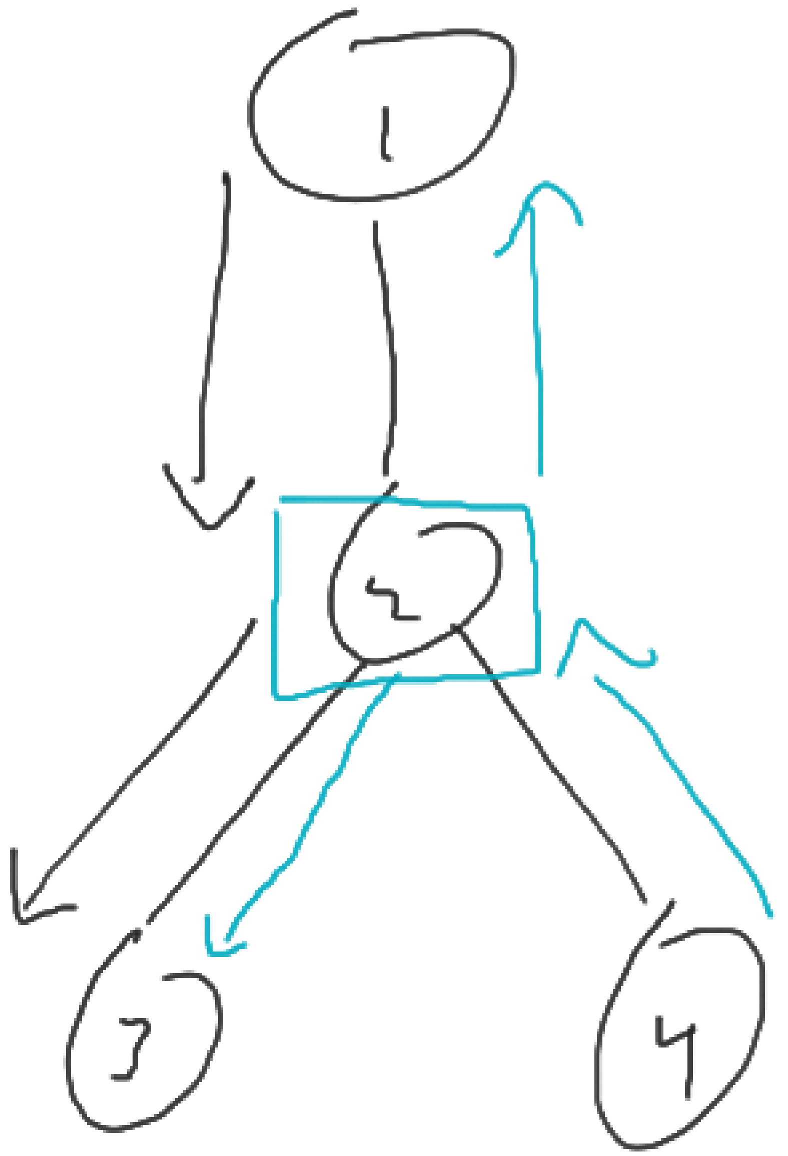
2. 3 → 1, 2 ✓

3. 4 → 3, 1 ✓



$S[C]_T$

is 1 above the LCA(3, 4)

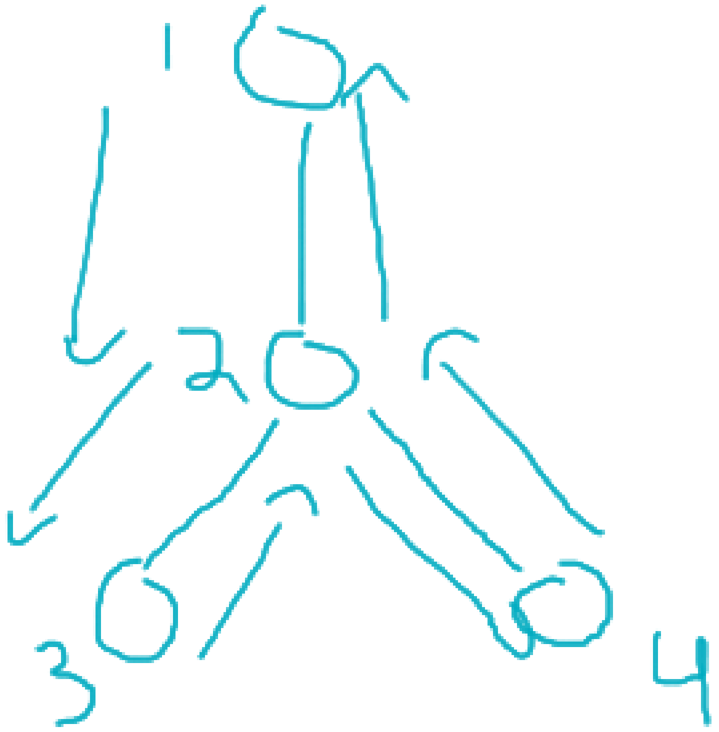


We know S[C].T is an ancestor of A. Hence, if it is also an ancestor of B, then it is either the LCA or above the LCA

Black Box O(1) - Compute if something is an ancestor of A or not

-> to check if LCA, check if next node is also ancestor of B

How to check if X is an ancestor of Y in a tree



0	1	2	3	4	5	6
1	2	3	2	4	2	1

	first	last
1	0	6
2	1	5
3	2	2
4	4	4

if interval completely contained, then it is an ancestor