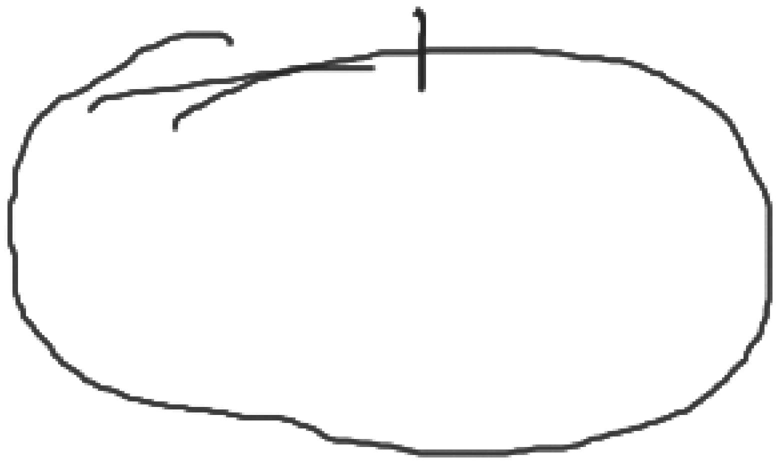
N cows



C - length 100

L - laps 2 $\rightarrow$ 200

20, $\underline{100}$, 70, 1

200/100 = 2 units of time

Sample output: $\boxed{4}$

100: 20, 1

70: 20, 1

$l(i)$ = number of laps done by cow i during the entire race $\in \mathbb{R}$

$$time_T = \frac{C \cdot L}{max(speed)}$$

$$l(i) = \frac{speed_i \cdot time_T}{C}$$

1 20 70 100

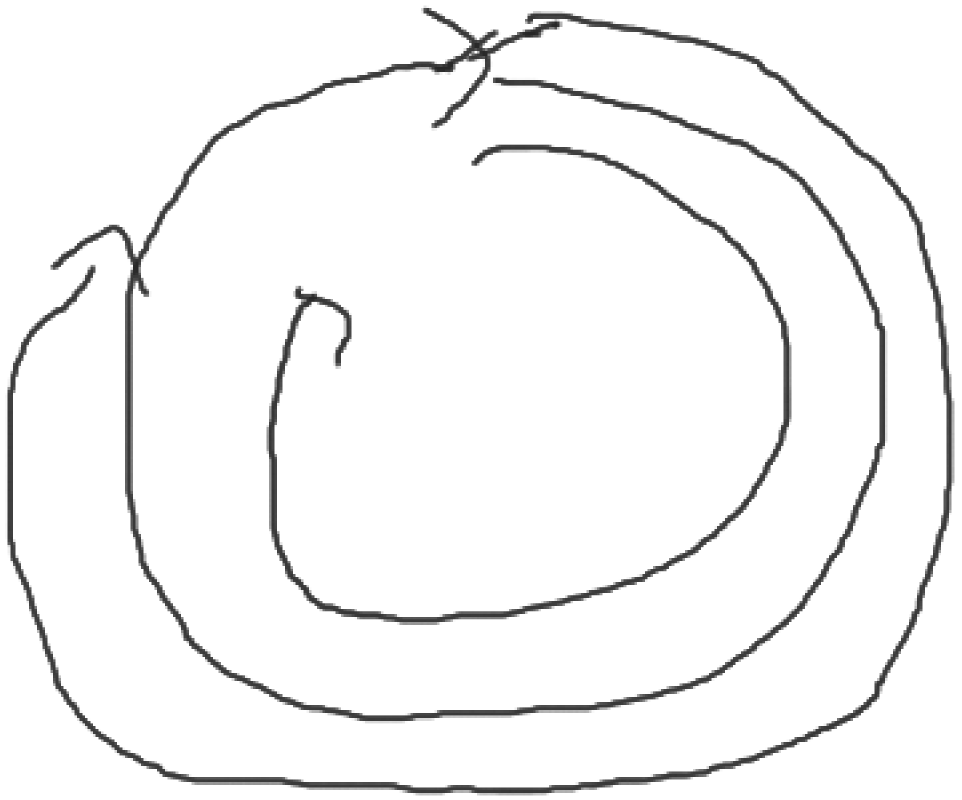
0.02 0.4 1.4 2

cows j , k

How many times does k overtake j

$$\text{floor}(\ell(k) - \ell(j))$$

25	50	100
1	2	4



1.8 0.8

$$\sum \left[\ell(b) - \ell(a) \right] \quad \text{for all } b > a$$

$$\sum \ell(b) - \ell(a) \quad \text{for all } b > a$$

$$[1 - (1)] + [2 \cdot 2 - (1+1)] + [4 \cdot 3 - (2+1+1)]$$

$$[1 \quad \underline{1} \quad 2 \quad 4]$$

$$[1 - (1)] + [(2-1) + (2-1)] + [(4-2) + (4-1) + (4-1)]$$

$$\sum \ell(b) - \ell(a) \quad \text{for } b > a$$

Naive: $O(N^2)$

$\ell(1) = 1 \qquad \ell(2) = 1 \qquad \ell(3) = 2 \qquad \ell(4) = 4$

$$\left[\ell(2) - \ell(1)\right] + \left[\left(\ell(3) - \ell(2)\right) + \left(\ell(3) - \ell(1)\right)\right]$$
$$+ \left[\left(\ell(4) - \ell(3)\right) + \ell(4) - \ell(2)\right) + \left(\ell(4) - \ell(1)\right)$$

$$= \underbrace{\left[\ell(2) - \cancel{\ell(1)}\right]}^{\delta_0} + \left[2 \cdot \ell(3) - \overbrace{\cancel{\left(\ell(2) + \ell(1)\right)}}^{\delta_1}\right] + \left[3\,\ell(4) - \overbrace{\cancel{\left(\ell(3) + \ell(2) + \ell(1)\right)}}^{\delta_2}\right]$$

$$\delta_i = \ell(i+1) + \delta_{i-1}$$

$$l = [1.2 \quad 1.3 \quad 2.2 \quad 4.4]$$

$$\lfloor l(2) - l(1) \rfloor = 0$$

$$\lfloor l(3) - l(2) \rfloor + \lfloor l(3) - l(1) \rfloor = 0 + 1 = 1$$

$$\lfloor l(4) - l(3) \rfloor + \ldots = 2 + 3 + 3 = \frac{8}{9}$$

$$l = [1.2 \quad 1.3 \quad 2.2 \quad 4.4] \qquad \sum \lfloor l(b) \rfloor - \lfloor l(a) \rfloor$$

$$\lfloor l(2) \rfloor - \lfloor l(1) \rfloor = 0$$

$$(\lfloor l(3) \rfloor - \lfloor l(2) \rfloor) + (\lfloor l(3) \rfloor - \lfloor l(1) \rfloor) = 1 + 1 = 2$$

$$(\lfloor l(4) \rfloor - \lfloor l(3) \rfloor) + \dots = 2 + 3 + 3 = \frac{8}{10}$$

1.3, 2.2 $\rightarrow$ problem: $\lfloor \ell(b) - \ell(a) \rfloor \neq$

$\lfloor \ell(b) \rfloor - \lfloor \ell(a) \rfloor$

$\{x\} = x - \lfloor x \rfloor$

$\{\ell(a)\} > \{\ell(b)\}$

$\{1.3\} = 0.3$

Binary-Index Tree / Fenwick Tree / BIT

$a = [\ 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\ ]$

Array	BIT	
$O(1)$	$O(\log N)$	update(i, val) — change the value at index i
$O(n)$	$O(\log N)$	query(i) = returns the sum of $\sum_{j=0}^{i} a[j]$

$l = [1.2 \quad 1.3 \quad 2.2 \quad 4.4]$

index

1: update(f(1.2), 1)

$O(N \log N)$

Runtime

BIT

$[\boxed{0 \quad 1} \quad 1 \quad 1]$

1	2	3	4
4.4	1.3	2.2	1.2

2: $\lfloor l(2) \rfloor - \lfloor l(1) \rfloor = 0$

errors: sum of all elements before 1.3 = 0, update(f(1.3), 1)

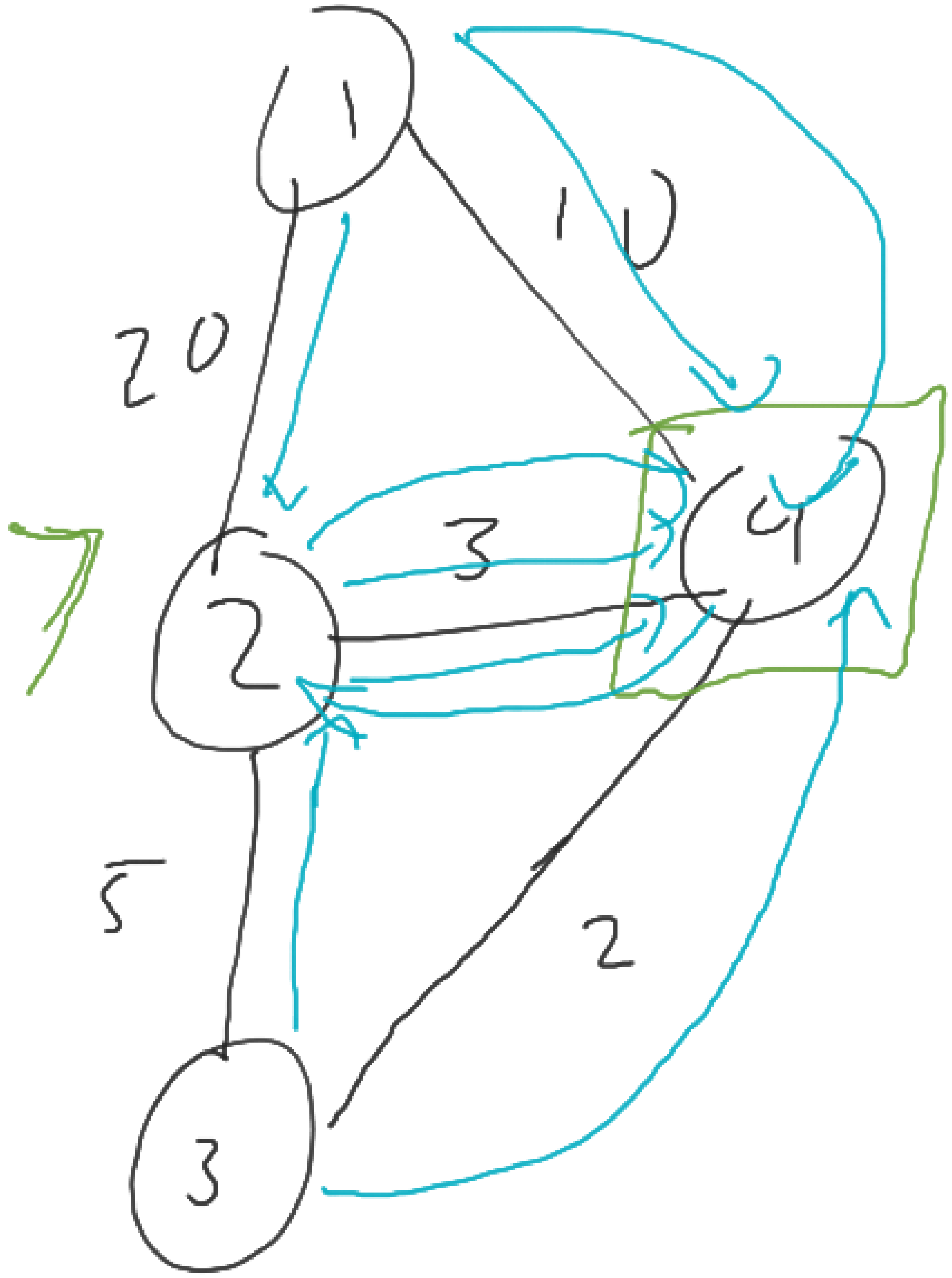
3. $(\lfloor l(3) \rfloor - \lfloor l(2) \rfloor) + (\lfloor l(3) \rfloor - \lfloor l(1) \rfloor) = 1 + 1 = 2$

errors: sum of all elements before 2.2 = 1, update(f(2.2), 1)

4. $\lfloor l(4) - \ldots \rfloor = 8$ errors = 0

$\sum \lfloor l(b) \rfloor - \lfloor l(c) \rfloor$ errors

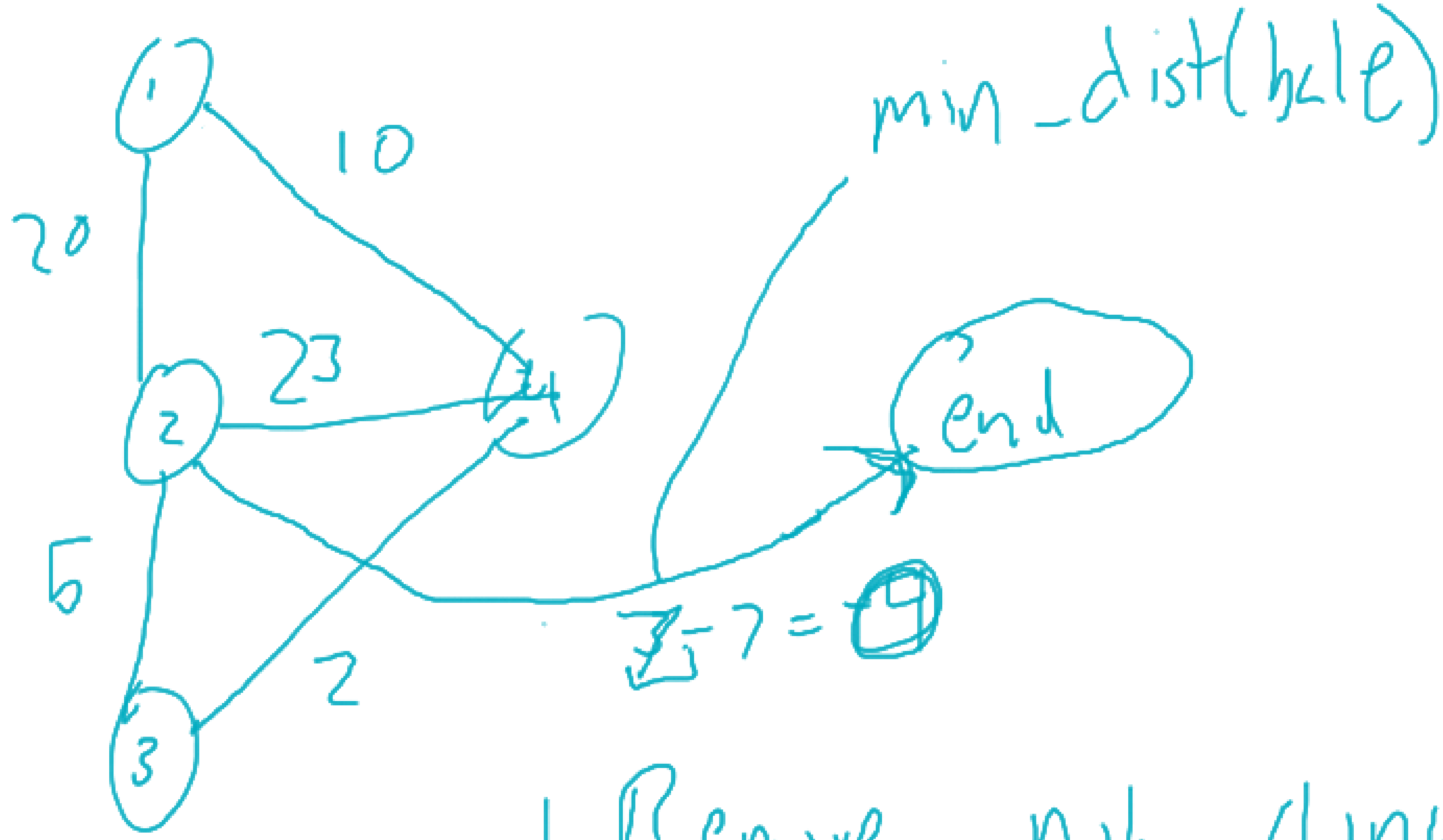
$8 + 2 - 1 = 9$



	Original	w/ [illegible]
1	10	~~23~~ 16
3	2	8

Dijkstras from node N

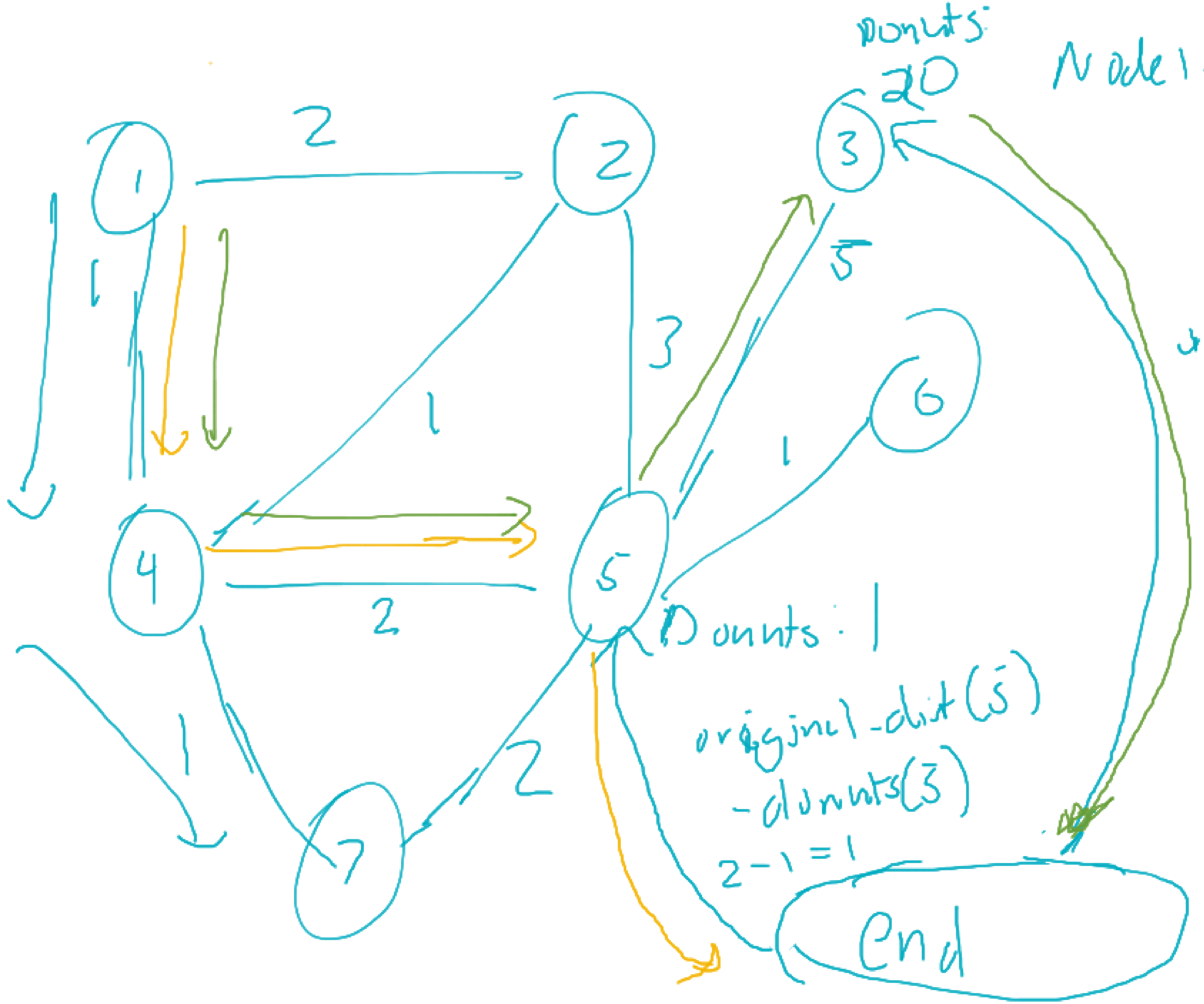
$O(M \log N)$



Goals:

1. Force the cow to actually get donuts
2. Reduce the path weight by donut yumminess

1. Remove not donut edges + add an edge to every donut



Node1: Original: 2
Donut: min(4, 5)

original_dist(3) - donuts(3)
7 - 20 = -13

```java
public class BIT {

    int[] tree;
    int N;

    public BIT(int N) {
        this.N = N;
        tree = new int[N+1];
    }

    public BIT(int N, int[] d){
        this.N = N;
        tree = new int[N+1];
        for (int i = 1; i < d.length; i++) {
            update(i,d[i]);
        }
    }

    public int query(int K) {
        int sum = 0;
        for (int i = K; i > 0; i -= (i & -i)) {
            sum += tree[i];
        }
        return sum;
    }

    public void update(int K, int val) {
        for (int i = K; i <= N; i += (i & -i)) {
            tree[i] += val;
        }
    }
}
```

J 30

D 10

$1 \rightarrow 1/2 \rightarrow 1/3 \rightarrow 1/4$

1000

	J 10	T 20	T 50
dist	10	15	25 $\longrightarrow$
time	10	20	50

$\text{dist} = 0 \rightarrow 10 \rightarrow 12.5$

$\text{time} = 0 \rightarrow 10 \rightarrow 15$

$\text{speed\_denom} = 1.0 \rightarrow 2.0 \rightarrow 3$

$T\ \overset{20}{30}$ $\quad \cancel{D10}$

$T\ \overset{10}{20}$ $\quad D\ \overset{90}{55}$

$\cancel{T15}^{\,5}$ $\quad D\ \overset{20}{20}$

$O(N^2)$

$O(N \log N)$

$$\begin{bmatrix} \cancel{T\ 15} \\ \cancel{T\ 20} \\ T\ 30 \end{bmatrix} \quad \begin{bmatrix} \cancel{D\ 10} \\ D\ 20 \\ D\ 55 \end{bmatrix}$$

dp[i] = min cost to protect cows 1...i
= f

f(i):
ans = ∞
for ℓ in range(1, M): $\leftarrow$ $O(N^2M)$
j := first cow covered if we end an umbrella of length ℓ at i
ans = min(ans, f(j-1) + c(ℓ))

$dp[i]$ = min cost of covering cows $1 \ldots i$

$O(N^2)$

$dp[i] = dp[i-1] + c[i]$

for every $k < i$

$dp[i] = \min(dp[i], \underbrace{dp[k]}_{\text{min cost of covering cows } 1 \ldots k} + \overbrace{c'[X_i - X_{k+1}]}^{\text{mincost of single umbrella covering cows } k+1 \ldots i})$

Larger umbrellas do not necessarily cost more than smaller umbrellas.

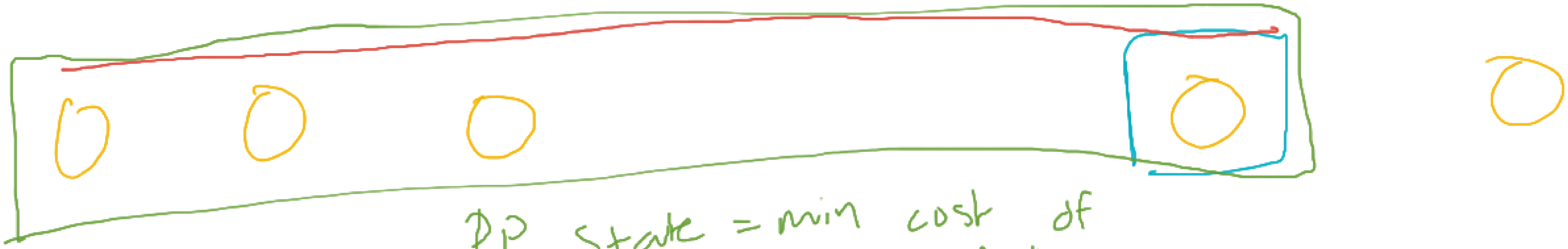
c

$2 \to 8$

$3 \to 7$

0 0

$$c'(l) = \min_{l' \geq l} c(l')$$



DP state = min cost of
covering all of these