Homework Assignment 14

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1. $E[X] = \sum_{i=1}^{k} i \times 1/k$ or $E[X] = 1/k \times k(k+1) \div 2$ or $E[X] = (k+1) \div 2$.
2. $E[X] = \sum_{i=1}^{6} i^2 \times 1/6$ or $1/6 \times (1 + 4 + 9 + 16 + 25 + 36)$ or $\$91/6$ or about $\$15.17$.
3. $E[R] = \frac{1}{6^6} \times \sum_{i=1}^{6} i \times (i^6 - (i-1)^6)$ or $\frac{1}{6^6} \times (6 \times 6^6 - 5^6 - 4^6 - 3^6 - 2^6 - 1^6)$.

   Calculating each term gives $E[R] = \frac{259421}{46656}$ or about 5.56.
4.
   a. $1/2$
   b. $\$8$
   c. $E[S]$ is $1/2 \times 4 + 1/2 \times 16$ or $\$10$. I should switch, the expected value $\$10 > \$8$.
   d. Switch. Since 7 is not even, it can't be the doubled box. Other box has $2 \times 7 = \$14$.
5. Let X_i be the random value of a pair having people with opposite sex. That way $X = \sum_{i=1}^{14} X_i$

   so $E[X] = \sum_{i=1}^{14} E[X_i]$. Since every X_i has to same probability of $2(8/15 \times 7/14)$,

   $E[X_i] = 8/15$. Substituting $E[X] = 14 \times 8/15$ or $112/15$ about 7.47 pairs.
6. Let X_i the the random variable that the string "math" appears at index i. $E[X_i] = (1/26)^4$.

   That means that X (total times) is $\sum_{i=1}^{1000000-4} X_i$, so $E[X] = (1000000 - 4)(1/26)^4$.

   Simplifying gives $E[X] = 999996/456976$ or $249999/114244$ about 2.19 times.
7. Let X_i be a random variable that a pair is male-female. $E[X_i] = 2 \times n/2n \times n/(2n-1)$ or $n/(2n-1)$. There are n pairs so, $E[X]$ (number of male-female) is $n^2/(2n-1)$.
8. Let X_i be if the the i^{th} number is underlined. That means

   $E[X_i] = \frac{1}{n}(\frac{n-1}{n-1} + \frac{n-2}{n-1} \ldots \frac{0}{n-1})$ or $\frac{1}{n(n-1)}(n - 1 + n - 2 + \ldots + 0)$ or

   $\frac{1}{n(n-1)} \times \frac{n(n-1)}{2}$ or $\frac{1}{2}$. There are $n-1$ such number so $E[X] = \frac{n-1}{2}$.

9.

a. $1 - (1/2)^3 = 7/8$

b. Since $E[X_i] = 7/8$, $E[X] = 7 \times 7/8 = 49/8$ or 6.125.

c. If there is no way to make > 6 or 7 propositions true then $E[X] \leq 6$. Since the expected number of propositions is > 6 the must be at least 1 way to make all propositions true. Letting all x_i be true is an example of this.