Homework Assignment 3

Arjun Subramanian

1.
   a. $8!$
   b. $7! \times 2!$
   c. $2 \times (4!)^2$
   d. $4! \times 5!$
   e. $4! \times (2!)^4$
2. $26^4 - 25^4$
3. S_1, S_2, S_3 are the multiples of $2,3,5$ from $1...6000$. Number of unlucky number is $|S_1| + |S_2| + |S_3| - |S_1 \cap S_2| - |S_1 \cap S_3| - |S_2 \cap S_3| + |S_1 \cap S_2 \cap S - 3|$. This becomes $3000 + 2000 + 1200 - 1000 - 600 - 400 + 200 = 4400$. The number of lucky numbers is $6000 - 4400$, or 1600.
4.
   a. $P(n, k)$
   b. $\binom{n}{k}$
5.
   a. $\binom{15}{4}$
   b. $\sum_{n=3}^{15} \binom{15}{n}$
6. $\binom{25}{2}$
7. $2^{15} - \binom{15}{1}$
8. First, divide the players into 2 groups by $\binom{2n}{n}$. Now we need to pair each player from the first group with a player from the second group. There are $P(n, n)$ or $n!$ ways to do this. Due the to multiplication rule: $\binom{2n}{n} \times n! = \frac{(2n)!}{n! \times n!} \times n! = \frac{(2n)!}{n!}$. Every match is being counted twice because the players are ordered. After pairing players, the result must be divided by 2^n for every $\times 2$ per n matches. The expression will become $\frac{(2n)!}{2^n \times n!}$ which is the same expression that the question states.

9.
   a. 9×8^6
   b. $\binom{10}{2} \times (2^7 - 1) + 10$
10. Since Alice doesn't know the color of her hat, Bob and I can't both have white hats. Otherwise, the white hats would have run out and Alice would have known she has a red hat. Since Bob, doesn't know the color of his hat, I can't have a white hat. Otherwise, Bob would have known that both him and I can't have white hats, so his would have been red. In conclusion, since I don't have a white hat, I must have a red hat.
11. 10 numbers are GREEN, only squares have an odd number of factors.
12. Take 1,2,...,8 marbles from the 1^{st},2^{nd}, . . . ,8^{th} boxes respectively, and weight them together. If all the boxes had 12 unit Brazilian marbles there would be $(\sum_{x=1}^{8} x = 36) * 12 = 432$ units on the scale. Since that is not the case and Indian marbles weigh 1 unit less then Brazilian marbles, the difference between 432 and the weight the scale reads will be the box number in which the Indian marbles are located.