Homework Assignment 13

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1. The contrapositive of this statement is if G has an edge e in ≥ 2 cycles, G will hand an even cycles. Let e connect nodes u, v. With contradiction assume that G has no odd cycles. Then there are at least 3 ways to connect u, v: though e or ≥ 2 evenly sized paths. The evenly sized paths must intersect, otherwise there will be an even cycle though u, v. The distance the paths are separated must be both even or both odd, otherwise 1 other the paths will not be even. There are 2 paths connecting where the 2 evenly sized paths separate. Those 2 new paths make a cycle with even length (both even or both odd).
2. Pick an node in G to represent a node in T. From there, assign unused connected nodes in G to represent connected nodes in T. For any node in T, the node that correlates in G has enough connections for all the T node's connections. If n nodes have already been used, the G node has at least $m - n$ connections to unused nodes. The T node has at most $m - n$ children (m other nodes $-n$ used nodes).
3. Each non-leaf node takes what used to be 1 leaf and makes it i leaves. That means adding $2 ... k$ such transitions will result in $1 + ... + k - 1$ added leafs. Since the problem state that are $n - k + 1$ leafs, use algebra to solve for n: $1 + (1 + ... + k - 1) = n - k + 1$ or $1 + ... + k - 1 = n - k$ or $(k - 1)(k) \div 2 = n - k$ or $k(k - 1) = 2n - 2k$ or $k(k + 1) = 2n$, so $n = k(k + 1) \div 2$.
4. If G has no odd cycles, then $\chi(G) = 2$. All regions must be paths or cycles which are both 2 colorable. Otherwise if G has just 1 odd cycle, the cycle is 3 colorable and the remaining graph is 2 colorable, adding to a max of $\chi(G) = 5$. Otherwise, diversions from the odd cycles make even cycles which don't increase an odd cycles 3 colorability. That way the same technique can be applied to such graph leaving $\chi(G) \leq 5$.
5. Solve this problem for G with an induction on all H. The claim is H is $\leq d + 1$ colorable.
   BC: When there is 1 nodes in H, it is $\leq d + 1$ colorable because you only need 1 color.
   IH: Assume that when H's $n = k$ the claim holds.
   IS: Let v be the vertex being added to H. Because $\forall H, \delta(H) \leq d$, v can have at most d connections, and By the IH, H' (H without v) is $\leq d + 1$ colorable. There will be at least $(d + 1) - d = 1$ color left for v so H is $\leq d + 1$ colorable.
6. There are 3^{100} different colorings. That expands to about 5.1537752×10^{47}. That is about 5.1537752×10^{41} seconds or 8.5896253×10^{39} minutes or 8.5896253×10^{39} or 1.4316042×10^{38} hours or 5.9650176×10^{36} days.