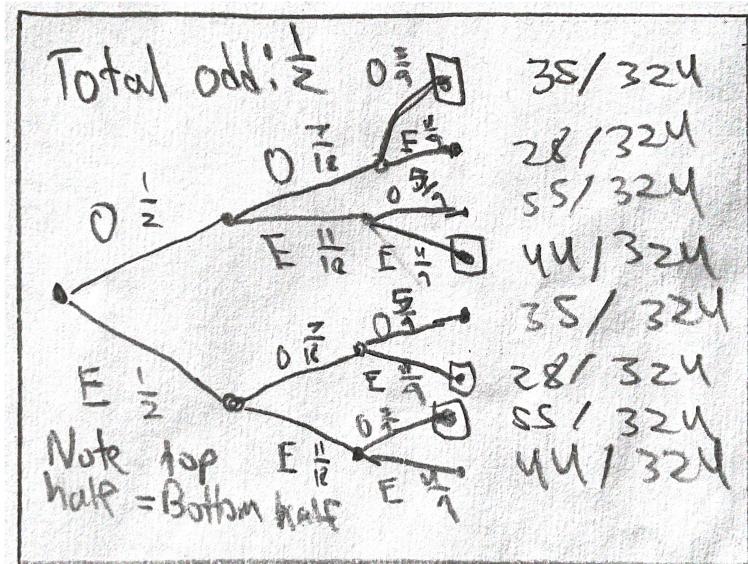


# Homework Assignment 9

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1.

a.  $\Omega = \{OOO, OOE, OEO, OEE, EOO, EOE, EEO, EEE\}$

b. Events of interest  $A = \{OOO, OEE, EOE, EEO\}$

c.  $Pr[OOO] = \frac{1}{2} \times \frac{7}{18} \times \frac{5}{9} = \frac{35}{324}$

$$Pr[OEE] = \frac{1}{2} \times \frac{11}{18} \times \frac{4}{9} = \frac{44}{324}$$

$$Pr[EOE] = \frac{1}{2} \times \frac{7}{18} \times \frac{4}{9} = \frac{28}{324}$$

$$Pr[EEO] = \frac{1}{2} \times \frac{11}{18} \times \frac{5}{9} = \frac{55}{324}$$

d.  $\frac{35}{324} + \frac{44}{324} + \frac{28}{324} + \frac{55}{324} = \frac{162}{324} = \frac{1}{2}$

2.

a.  $\frac{51! \times 2}{52!} = \frac{2}{52} = \frac{1}{26}$

b.  $\frac{\binom{6}{2}}{P(52,6)}$

c.  $\frac{3}{3^3} = \frac{1}{9}$

d.  $\frac{6 \times 6 \times 3}{6^3} = \frac{1}{2}$

e.  $1 - \frac{4^3}{6^3}$

f.  $n = 4$

3.

a.  $\frac{3}{5} \times \frac{7}{10} = \frac{21}{50}$

b.  $\frac{2}{5} \times \frac{3}{10} = \frac{3}{25}$

c.  $1 - \frac{3}{25} = \frac{22}{25}$

d.  $\frac{22}{25} - \frac{21}{50} = \frac{23}{50}$

e.  $2 \times \frac{1^5}{2} = \frac{1^4}{2} = \frac{1}{16}$

f.  $\frac{1^5}{2} = \frac{1}{32}$

g.  $\binom{5}{3} \times \frac{1^5}{2} = \frac{5}{16}$

h.  $\frac{1^2}{2} = \frac{1}{4}$

i.  $1 - \frac{1^5}{2} = \frac{31}{32}$

4. After picking any first sock, the second sock's pair probability is the same:  $\frac{1}{6-1} = \frac{1}{5}$

5.

a.  $\frac{\binom{10}{5}}{2^{10}}$

b.  $\frac{1}{2^3} = \frac{1}{8}$

c.  $\frac{1}{8} \times \frac{\binom{7}{2}}{2^7} = \frac{\binom{7}{2}}{2^{10}}$

d.  $\frac{\binom{10}{5}}{2^{10}} + \frac{1}{8} - \frac{\binom{7}{2}}{2^{10}}$

6.  $1 - \frac{P(300,20)}{300^{20}}$

7. One team:  $\binom{5}{3} \times \frac{1}{2}^6 = 10 \times \frac{1}{64} = \frac{5}{32}$ . Since either team can win:  $2 \times \frac{5}{32} = \frac{5}{16}$ .
8. With inclusion exclusion:  $p + p^2 - p^3$
9. The Vincents have 4 children, 3 of which are girls. Picking 2 girls probability:  $\frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$ .
10. Chance for 12:  $\binom{13}{1} \times \frac{1}{3}^{12} \times \frac{2}{3}$ . Change for 13:  $\frac{1}{3}^{13}$ .  
 Adding:  $\frac{1}{3}^{12} \times (13 \times \frac{2}{3} + \frac{1}{3}) = \frac{1}{3}^{12} \times \frac{27}{3} = \frac{1}{3}^{10}$ .
11. The probability is:  $(\frac{25000}{100000} + \frac{16666}{100000} + \frac{11111}{100000}) - (\frac{8333}{100000} + \frac{5555}{100000})$ . The multiples of 4,9, and 4,6,9 are the same:  $6 \mid 4 \times 9$ . The answer is:  $\frac{38889}{100000}$ .