Homework Assignment 2

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1.
   A.
      a. $\{-1,1\}$
      b. $\{1,2,3,4,5,6,7,8,9,10,11\}$
      c. $\{0,1,4,9,16,25,36,49,64,81\}$
      d. $\varnothing$
   B.
      a. $\{3x \mid x \in \mathbb{N} \land x \leq 4\}$
      b. $\{x-3 \mid x \in \mathbb{N} \land x \leq 6\}$
      c. $\{l \mid l \in letters \land l \leq p \land l \geq m\}$
   C.
      a. 1
      b. 1
      c. 2
      d. 3
   D.
      a. True
      b. True
      c. False
      d. True
      e. True
      f. False
   E. Power Set: $\{\varnothing, \{a\}, \{b\}, \{a,b\}\}$
      Cardinality: 4
   F. $A = \varnothing$ and $B = \{\varnothing\}$
      $(\varnothing \in \{\varnothing\}) \land (\varnothing \subseteq \{\varnothing\})$
2. $9 \times 8 \times 7 = 504$
   $9!/(9-k)!$
3. $26 \times 10 \times 60 \times 59 \times 58 \times 57 \times 56 \times 55 = 9{,}371{,}954{,}592{,}000$
4. $3^8 = 6561$
5. $8 \times 7 \times 6 = 336$
6. $x = 2, y = 2, z = 2$
   $x^{4116} + y^{4116} = z^{4117}$
   $2^{4116} + 2^{4116} = 2^{4117}$

$2 \times 2^{4116} = 2^{4117}$

$2^{4117} = 2^{4117}$

7. $26^{\lceil n/2 \rceil}$
8. $11 \times 16 \times 41 - 1 = 7215$
9. Number of even subsets is 2^{n-1}. For every element $x_1 \ldots x_{n-1}$ you can either include or not include it. Those 2 options for the $n-1$ elements is where 2^{n-1} comes from. The n^{th} element has no choice. If there is already an even number of elements in the subset, it is excluded, otherwise it is included.
10. $3! \times 10! \times 15! \times 4!$
11. Let B be the number of blue marbles and R be the number of red marbles. There are 3 combinations of picks:

    2 blues: $B = B - 2, R = R + 1$

    1 blue and 1 red: $R = R - 1$

    2 reds: $R = R - 1$

    Note that B starts out as 6695, and odd number. In all cases B either stays the same or decreases by 2, so B is always odd and can't become even. Therefore, B can never reach 0 and the last marble in the bag will be blue.
12. Yes, the value is at least $2/3$. Both 2^{4569} and 3^{2701} are integers so their difference will form an integer. $2^{4569} \neq 3^{2701}$ because they don't share any prime factors. The absolute value of a non-zero integer is greater than $2/3$.