Homework Assignment 5

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1.
   a. 60
   b. 12600
   c. True
   d. False
   e. False
   f. $2^n - 1$
   g. $8!$
   h. $(8!)^2$
   i. $\dfrac{(8!)^2}{3! \times 4!}$
   j. $(49 \times 50) \div 2 = 1225$
   k. $\binom{5}{2} \times \binom{6}{2} \times \binom{4}{3} = 10 \times 14 \times 4 = 560$
   l. $\dfrac{7!}{3! \times 2! \times 2!}$
   m. $6! \div 2!$
   n. $\dfrac{10!}{2!3!5!}$
   o. 36
   p. $\binom{7}{2} = 21$
2.
   a. $\binom{4}{1} \times \binom{6}{2} = 60$
   b. $\binom{4}{2} \times \binom{8}{1} = 48$
3. $\dfrac{9!}{2!2!3!}$ —> there is one way to ignore the last letter.
4. $3 \times \binom{12}{10}$
5. $(\binom{8}{5} \times 5!) \times (\binom{8}{4} \times 4!) \times 7!$

6. $\binom{4+95}{95} = \binom{99}{95}$: Each child gets at least $1M$ so $100M - 5M = 95M$.

7. $10! \div 5!$

8.

   a. $\binom{12}{6}$

   b. $\binom{12}{6} - \binom{7}{3} \times \binom{5}{3}$

9. $4 \times 3 \times \binom{11}{8}$

10. Let $x_i = 2k_i + 1$ where k_i is a non-negative integer, so $\sum_{i=0}^{4} (2k_i + 1) = 98$.

    Subtracting 4: $\sum_{i=0}^{4} 2k_i = 94$. Dividing by 2: $\sum_{i=0}^{4} k_i = 47$. $\binom{50}{47}$ combinations.

11. There are 2 ways to do this but I will give 1. First compare the first 3 oranges to the second 3. There are 2 cases:

    Case 1: The oranges have the same weight: Here, the lighter orange is from the 2 oranges left over. Compare those 2 oranges and the lighter orange is the one.

    Case 2: The lighter orange is one of the 3 oranges from the lighter group. Compare 2 of those 3 oranges. If they are the same weight the orange left out is the one, otherwise the lighter of the two oranges is the one.