# Homework Assignment 11

**Arjun Subramanian**

1.
   a. $Pr[A \mid W]$ or $\dfrac{Pr[A \cap W]}{Pr[W]}$ or $\dfrac{Pr[A]}{1/11 \times (0/10 + 1/10...)}$ or $\dfrac{1/11}{1/11 \times 55/10}$ or $\dfrac{2}{11}$.
   b. It's the same until: $\dfrac{Pr[A]}{1/11 \times (0/10^k + 1/10^k \dots)}$ or $\dfrac{1}{0/10^k + 1/10^k \dots}$ or $\dfrac{10^k}{\sum_{i=0}^{10} i^k}$.
2.
   a. The events are independent because $Pr[A \cap B] = 1/36$ and $Pr[A] \times Pr[B] = 1/6 \times 1/6 = 1/36$.
   b. The events are independent. $Pr[A \cap C] = 1/36$. There are $6$ options that satisfy $C$, $Pr[C] = 6/36 = 1/6$. So $Pr[A] \times Pr[C] = 1/6 \times 1/6 = 1/36$.
   c. The events are independent for similar reasons as part b. $Pr[B \cap C] = 1/36$ and $Pr[B] \times Pr[C] = 1/6 \times 1/6 = 1/36$.
   d. The $3$ events are not independent. $Pr[A \cap B \cap C] = 1/36$ because the rolls are $1$ and $6$, they add to $7$. $Pr[A] \times Pr[B] \times Pr[C] = 1/6 \times 1/6 \times 1/6 = 1/216$.
   e. The events aren't independent because if the max is $2$ the min can't be $5$. Otherwise $5$ would be the max and $2$ the min. That means $D \cap E = \varnothing$ or $Pr[D \cap E] = 0$. Both $D$ and $E$ are possible, $Pr[D] \times Pr[E] = 1/12 \times 1/12 = 1/144$.
3. Part a has $p$ probability. Part b has probability $p^2 + 2p(1-p)(1/2)$ or $p^2 + p(1-p)$ or $p^2 + p - p^2$ or $p$. Both strategies have the same probability.
4. Let $R$ be the event of the games working out as they did, $W$ the event that I am slightly worse than my opponent, $E$ the event that my opponent an I are equal, and $B$ the event that I am slightly better than my opponent. That answer is $Pr[W \mid R] = \dfrac{Pr[W \cap R]}{Pr[R]}$.

   Expanding: $\dfrac{Pr[W] \times Pr[R \mid W]}{Pr[W] \times Pr[R \mid W] + Pr[E] \times Pr[R \mid E] + Pr[B] \times Pr[R \mid B]}$.

   $Pr[W], Pr[E], Pr[B]$ are all $1/3$. Replacing:

   $\dfrac{1/3 \times Pr[R \mid W]}{1/3 \times (Pr[R \mid W] + Pr[R \mid E] + Pr[R \mid B])}$ or $\dfrac{Pr[R \mid W]}{Pr[R \mid W] + Pr[R \mid E] + Pr[R \mid B]}$.

   $Pr[R \mid W] = 3/5 \times 2/5 \times 3/5 \times 3/5$, $Pr[R \mid E] = 1/2^4$,

   $Pr[R \mid B] = 2/5 \times 3/5 \times 2/5 \times 2/5$. Substituting: $\dfrac{54/625}{54/625 + 1/16 + 24/625}$ or

   $\dfrac{54}{54 + 625/16 + 24}$ or $\dfrac{864}{864 + 625 + 384}$ or $\dfrac{864}{1873}$ or $\approx 46\,\%$.

5. The one person committee has probability $p$ of making the correct decision. The committee of three has probability $p^2(1/2) + 2p(1-p)(1/2) + p^2(1/2)$ or $p^2 + p - p^2$ or $p$ of making the correct decision. Both committee have the same probability.
6. 
   a. The events are independent, $Pr[A \cap B] = 1/16 = 1/4 \times 1/4 = Pr[A] \times Pr[B]$.
   b. The events are independent, $Pr[A \cap B] = 1/16 = 1/4 \times 4/16 = Pr[A] \times Pr[B]$.
   c. The events are independent, $Pr[A \cap B] = 1/16 \neq 3/16 \times 9/16 = Pr[A] \times Pr[B]$.
7. Let $V$ be the event of picking a vowel, $E$ the event that the man is English, and $A$ the event that the man is American. The answer is $Pr[E \mid V]$ or $\dfrac{Pr[E \cap V]}{Pr[V \cap E] + Pr[V \cap A]}$ or $\dfrac{Pr[E] \times Pr[V \mid E]}{Pr[E] \times Pr[V \mid E] + Pr[A] \times Pr[V \mid A]}$. Substituting calculated values, $\dfrac{2/5 \times 3/6}{2/5 \times 3/6 + 3/5 \times 2/5}$ or $\dfrac{1/5}{1/5 + 6/25}$ or $\dfrac{5}{5+6}$ or $\dfrac{5}{11}$ or $45\,\%$
8. There are $16!$ ways everyone can park. To satisfy the condition the $3$ of us can park in $2\binom{16}{3}$ ways. That is $\dfrac{2\binom{16}{3} \times 13!}{16!}$ $(13!$ for others) or $\dfrac{2 \times 560}{16 \times 15 \times 14}$ or $\dfrac{1120}{3360}$ or $\dfrac{1}{3}$.
9. Let $W_A$ be the event that a white ball is selected from $A$, $W_B$ the event that a white ball is selected from $B$, $W_C$ the event that a white ball is selected from $C$, and $W_2$ the event that $2$ white are selected. The question is asking $Pr[W_A \mid W_2]$ or $\dfrac{Pr[W_A \cap W_2]}{Pr[W_2]}$ or $\dfrac{Pr[W_A] \times (Pr[W_B \cap \overline{W_C}] + Pr[\overline{W_B} \cap W_C])}{Pr[W_A \cap W_B \cap \overline{W_C}] + Pr[W_A \cap \overline{W_B} \cap W_C] + Pr[\overline{W_A} \cap W_B \cap W_C]}$. Given probabilities $Pr[W_A] = 2/6 = 1/3$, $Pr[W_B] = 8/12 = 2/3$ and $Pr[W_C] = 1/4$: $\dfrac{1/3 \times (1/2 + 1/12)}{1/6 + 1/36 + 1/9}$ or $\dfrac{1/3 \times 7/12}{6/36 + 1/36 + 4/36}$ or $\dfrac{7/36}{11/36}$ or $\dfrac{7}{11}$.
10. Let $W$ be the event that the worlds series is $6$ games, $R$ the event that the Royals win the world series, and $C$ the event that the Cubs win the world series. The question is asking $Pr[R \mid W]$ or $\dfrac{Pr[R \cap W]}{Pr[W]}$ or $\dfrac{Pr[R \cap W]}{Pr[W \cap R] + Pr[W \cap C]}$. Substituting with calculations: $\dfrac{\binom{10}{5}\frac{2}{3}^4\frac{1}{3}^2}{\binom{10}{5}\frac{2}{3}^4\frac{1}{3}^2 + \binom{10}{5}\frac{1}{3}^4\frac{2}{3}^2}$ or $\dfrac{\frac{160}{729}}{\frac{160}{729} + \frac{40}{729}}$ or $\dfrac{160}{200}$ or $\dfrac{4}{5}$.

11. Let $H$ be the event that $14$ heads are flipped, and $T$ the event that no $2$ consecutive coin flips are tails. That question is asking $Pr[T \mid H]$ or $\frac{Pr[T \cap H]}{Pr[H]}$. $Pr[H]$ is $\binom{20}{14}\frac{1}{2}^{14}\frac{1}{2}^{6}$.

If $T$ then the arrangement must at least be $T, H, T, H, T, H, T, H, T, H, T$. There are $9$ more heads that have to be inserted in $7$ spots $Pr[T \cap H] = \binom{15}{9}\frac{1}{2}^{20}$. Substituting:

$$\frac{\binom{15}{9}\frac{1}{2}^{20}}{\binom{20}{14}\frac{1}{2}^{14}\frac{1}{2}^{6}} \text{ or } \frac{\binom{15}{9}}{\binom{20}{14}} \text{ or } \frac{5005}{38760} \text{ or } \frac{1001}{7752}.$$