20 Line Segments and Polygons

Intersection of Two Line-Segments

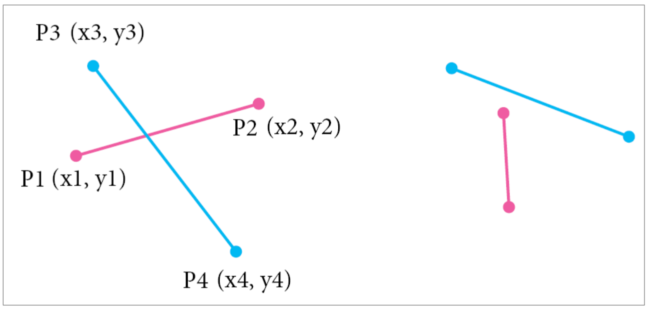
In geometry, a **line segment** is a part of a line that is bounded by two distinct end points, and contains every point on the line that is between its endpoints.

Let two line-segments are given. The points p1, p2 from the first line segment and p3, p4 from the second line segment. We have to check whether both line segments are intersecting or not.

We can say that both line segments are intersecting when these cases are satisfied:

- When (p1, p2, p3) and (p1, p2, p4) have a different orientation and
- (p3, p4, p1) and (p3, p4, p2) have a different orientation.

There is another condition is when two line segments are collinear.



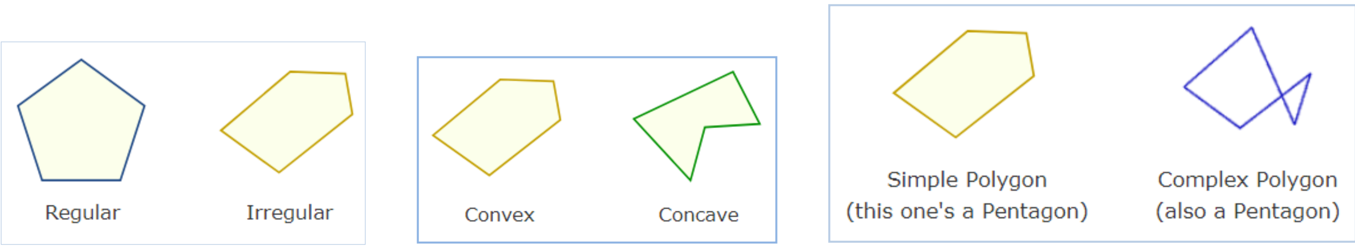
Exercise 1: Line segment and a point

Determine if a point is on a line segment.

Input: Two end points of the segment and the target point.

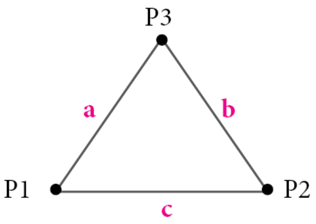
Polygons

Polygons are **2-dimensional** shapes. They are made of **straight lines**, and the shape is **closed** (all the lines connect up).

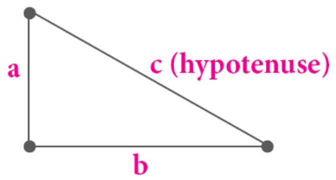


Triangle (Trigon)

- Any non-collinear three points determine a unique triangle.
- Sum of any 2 sides of a triangle must be greater than the measure of the third side

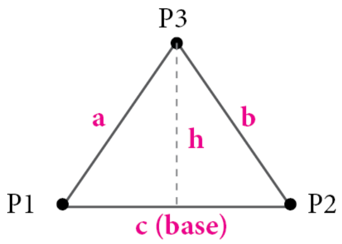


Pythagoras' Theorem



$c^2 = a^2 + b^2$

Area of a Triangle



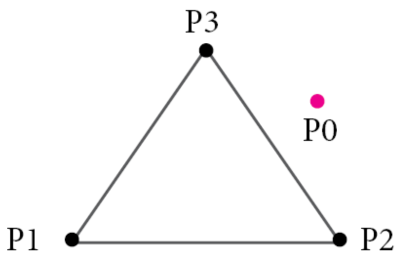
area = (base*h)/2

Heron's Formula:

area = sqrt(s*(s-a)*(s-b)*(s-c));
s = (a + b + c)/2

Point Location (Triangle)

Determine if point p0 is inside, outside, on a side or on a corner of the triangle.

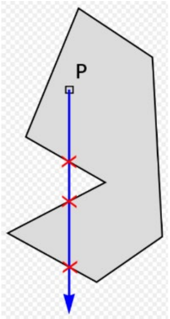


Solution 1: with the area of the triangle

Solution 2: with orientation of three points

Point Location (Polygon)

Point-in-polygon (**PIP**) problem: Determine if the point p0 is inside of a polygon.



Solution with Winding Number (Ray Casting) Algorithm

Winding number is defined by the number of times a curve travels counter clockwise around a point.

For any polygon, find all the edges of the polygon that cut through the line passing through the query point and is parallel to y_axis. For these edges check if the query point is on the left or right side of the edge when looking at all the edges in anticlockwise direction. Increase the value of winding number (wn) by one if query point is on the left side of an upward crossing and decrease the wn by one if query point is on the right side of a downward crossing. If the final winding number is non zero then the point lies inside the polygon.

Polygon Area

There are many ways to calculate the area of a polygon. Area of a well-known polygon like Triangles, Rectangles, Squares, Trapezoids, etc. can be calculated using simple mathematical formula.

If the vertices of a polygon are ordered in a clockwise or an anti-clockwise direction, then the area can be calculated using a **shoelace algorithm**.

Area of Polygon using Determinants

The area of a polygon with vertices

$$(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$$

listed counter-clockwise around the perimeter is given by

$$A = \frac{1}{2}\left(\begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & x_3 \\ y_2 & y_3 \end{vmatrix} + \ldots + \begin{vmatrix} x_n & x \\ y_n & y \end{vmatrix} \right)$$

Exercise 2: Set of unordered vertices of a convex polygon.

How can you calculate the area of a simple convex polygon when the vertices are given in a random order?

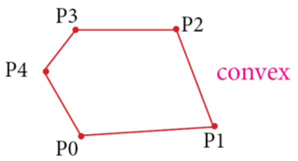
Convexity of a Polygon

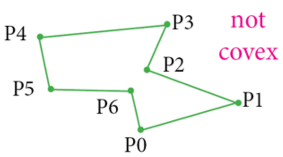
Determine if the given polygon is convex.

Input Format

Line 1: N

Line 2 to N+1: Each line denotes one corner points with two integers: x and y. Points are given in trigonometric order.





Diameter of a Convex Polygon

The diameter of a polygon is the largest distance between any pair of vertices. In other words, it is the length of the longest polygon diagonal (e.g., straight line segment joining two vertices). **Rotating calipers algorithm** can be used to find the algorithm of a polygon in **O(n)** time.

In computational geometry, the method of rotating calipers is an algorithm design technique that can be used to solve optimization problems including finding the width or diameter of a set of points.

The method is so named because the idea is analogous to rotating a spring-loaded vernier caliper around the outside of a convex polygon. Every time one blade of the caliper lies flat against an edge of the polygon, it forms an antipodal pair (opposite to each other) with the point or edge touching the opposite blade. The complete "rotation" of the caliper around the polygon detects all antipodal pairs.

Useful Links and References

https://en.wikipedia.org/wiki/Line_segment
https://towardsdatascience.com/is-the-point-inside-the-polygon-574b86472119
https://algorithmtutor.com/Computational-Geometry/Area-of-a-polygon-given-a-set-of-points/
https://mathworld.wolfram.com/PolygonDiameter.html
https://en.wikipedia.org/wiki/Rotating_calipers#cite_note-11