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THE POSSIBILITY OF NEW FERMIONS WITH $\Delta I = 0$ *MASS**

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In the Glashow–Weinberg–Salam model the fermions have $\Delta I = \frac{1}{2}$ masses from the breaking of the weak SU(2) gauge symmetry. In many enlarged models, such as those from grand unified and/or supersymmetric theories, there are additional fermions with undetermined $\Delta I = 0$ (SU(2) invariant) masses. We study these new fermions. They induce flavour changing neutral currents. We show that the mixing angles of $\Delta I = 0$ fermions of mass order M with normal $\Delta I = \frac{1}{2}$ fermions of mass order $m \ll M$ are order η or η^2, where $\eta = m/M$. If $M > 150\,\delta$, δ being a model-dependent mixing parameter of order a normal fermion mass, the amplitudes of all FCNC processes are below the experimental sizes and limits. Thus for $\delta \leqslant 0.1$ GeV, M can be as low as 20 GeV, close to the present experimental lower bound. δ is fixed, and less than 0.1 GeV for all relevant cases, if we assume the mass hierarchy of the known fermions is not the result of a particular choice of $\Delta I = 0$ mass parameters. If produced $\Delta I = 0$ mass fermions will be noticeable by the mass degeneracy within their isospin multiplets. There will be an enhanced ratio of neutral to charged decays over the normal fermions. Standard GUT predictions are changed little.

1. Introduction

Fermions can be classified by their quantum numbers under the standard model $SU(3)_C \times SU(2)_L \times U(1)_Y$ [1]. These quantum numbers are of two kinds: those belonging to $SU(3)_C \times U(1)_Q$ which are therefore conserved (C) and those belonging to $SU(2)_L \times U(1)_Y$ which are non-conserved (NC). We call the known fermions (u, d, e, ν_e; c, s, μ, ν_μ; . . .) *normal*, the fermions with standard C-quantum numbers but with different NC-quantum numbers *pseudoexotic* and the fermions with some non-standard C-quantum numbers *exotic*. In general pseudoexotic fermions will mix with normal ones after the breaking of $SU(2)_L \times U(1)_Y$.

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The closeness of the measured parameter $\rho = (M_W/M_Z)\cos\theta_w$ to 1 establishes that the normal fermions get their mass, when the standard model spontaneously breaks to $SU(3)_C \times U(1)_Q$, primarily from the coupling $\bar{f}_L f_R$ to a Higgs doublet ($I = \frac{1}{2}$). They have a $\Delta I = \frac{1}{2}$ mass (the left-handed fields are doublets and the right-handed fields singlets). Can the standard model tolerate the addition of new fermions with $SU(2)_L \times U(1)_Y$ invariant ($\Delta I = 0$) masses? Does experiment constrain the mass (M) and other properties of such fermions? If the mixing angles between different fermions were Cabibbo-like, for example, as commonly assumed, the contribution of these new fermions to flavour changing neutral current (FCNC) processes would in general exceed the experimental bounds [2, 3]. The angles, however, are not Cabibbo-like. We shall prove that the mixing angles between $\Delta I = 0$ and $\Delta I = \frac{1}{2}$ fermions are order η or η^2, where $\eta \sim m/M$ ($m_{\Delta I=1/2} \ll M_{\Delta I=0}$). Rare processes are suppressed and heavy $\Delta I = 0$ fermions decouple [4]. $\Delta I = 0$ mass fermions can thus be very light, perhaps as low as 20 GeV, depending on the structure of the normal fermion part of the theory (see sect. 2). Such fermions would have striking signatures. Masses within isospin multiplets would be nearly degenerate and the ratio of neutral to charged decays would be enhanced over that for normal fermions. Their presence is compatible with the predictions of grand unified theories (GUTS).

Before starting our analysis we note some models with $\Delta I = 0$ fermions. Grand unified theories (GUTS) based on SO(10) [5] and E_6 [6], GUTS derived from $N = 8$ supergravity and supersymmetric unified models all contain $\Delta I = 0$ fermions with masses originating from the extra mass scales. For example, consider the model proposed by Ellis, Gaillard and Zumino (EGZ) [7]. Its SU(5) content is

$$\text{F:}\quad \underset{\Delta I = 1/2}{\frac{3(\bar{5}+10)}{\text{normal}}} + \frac{9(1)+3(\bar{5}+5)+9(10+\overline{10})}{\text{pseudoexotic}} + \underset{\Delta I = 0}{\frac{4(24)+(45+\overline{45})}{\text{pseudoexotic and exotic}}}\,. \tag{1.1}$$

In GUTS $\Delta I = 0$ fermions are usually assumed to be very heavy. We emphasize, however, that their masses are a priori arbitrary. Examples are known of natural GUTS where these $\Delta I = 0$ fermions acquire masses approximately ten orders of magnitude smaller than the grand unification mass [8, 9]. In sect. 4 we discuss another example based on the EGZ model.

In supersymmetric models [10] the partners of the usual Higgs and gauge bosons can acquire $\Delta I = 0$ masses and there can also be genuine $\Delta I = 0$ supermultiplets. Our analysis may then be relevant. The situation, however, is very model dependent; in particular it is crucial to know whether supersymmetry is broken at high or low energy. Non-renormalization theorems allow one to arrange some particles to have light mass and/or very small mixing angles; thus our assumptions need not apply*.

* In supersymmetric models with supersymmetry broken at low energy the supersymmetric partners of the known particles do not have masses larger than the scale of breaking. In any model in which their mass is low their phenomenological effects should be carefully studied.

Our results apply if the effective theory has $\Delta I = 0$ fermions much heavier than normal fermions and each class of normal fermions gets mass from only one light Higgs scalar multiplet*.

In sect. 2 we give a theorem on the mixing angles between normal and $\Delta I = 0$ fermions. The details are given in the appendices. We also discuss the characteristics of these new fermions. Sect. 3 discusses the experimental constraints on $\Delta I = 0$ fermions from standard phenomenology, GUTS and cosmology. In sect. 4 we give an example of a model with low-mass $\Delta I = 0$ fermions and sect. 5 contains final remarks and conclusions.

This paper is one of two on $\Delta I = 0$ fermions. In the other [4] we evaluate, to one loop, the rate for the process $\mu \to e\gamma$ in a simple model, and discuss how the $\Delta I = 0$ fermions decouple.

2. Mixing angles and main characteristics of $\Delta I = 0$ fermions

In this section we elaborate on the crucial point of our analysis, the values of the mixing angles, and study the main characteristics of $\Delta I = 0$ fermions. Our starting point is normal fermions with left-handed (LH) doublets and right-handed (RH) singlets. They acquire mass when the standard model (GWS) is broken. $\Delta I = 0$ fermions have LH and RH parts in the same type of multiplet. Their mass comes from a GWS-invariant term and is thus arbitrary, but apparently heavier than normal masses.

We must consider two kinds of vertices: fermion–fermion–gauge boson (FFG) and fermion–fermion–Higgs boson (FFH). We begin with the gauge mixing angles. They are obtained by diagonalizing a general mass matrix and rewriting the FFG vertices determined by the fermion content in terms of mass eigenstates. The values of the mass matrix entries are guided by experiment. $\Delta I = 0$ entries come from GWS-invariant mass terms which we take of order a large mass M. $\Delta I = \frac{1}{2}$ entries come from the Yukawa terms when GWS is broken and are of order a normal mass m which is much less than M. We assume all $\Delta I \geqslant 1$ entries are negligible as evidenced by the ratio of the charged to neutral current strengths $\rho \sim 1$ and the smallness of neutrino masses [11]. The details of the diagonalization and the mixing matrices are given in appendix A where we prove the theorem below.

Before GWS is broken all the mixing angles are zero, and weak current and mass eigenstates coincide. When GWS breaks we obtain mixing angles as an expansion in the mass ratio $\eta = m/M$. Because $\eta \ll 1$, the mass eigenstates will coincide with the current eigenstates to order 0 in η. In this sense we will speak about mass eigenstates having a well-defined isospin. Gauge bosons do not change the fermion helicity. Thus the FFG vertices will involve only LH (RH) fermions.

* In particular this means that light scalars couple with typically small Yukawa couplings and only one neutral light scalar, that giving mass, couples to each class of normal fermions.

2.1. FFG MIXING ANGLE THEOREM

Theorem: LH(RH) fermions whose isospin quantum numbers differ by $\frac{1}{2}$ will mix with an angle $\eta \sim m/M$. Different LH(RH) fermions with the same isospin will mix with angles of order η^2 (except normal fermions in the charged current which mix with Cabibbo angles) and LH(RH) fermions differing in isospin by 1 or more will have mixing smaller than η^2.

Thus normal fermions mix with $\Delta I = 0$ fermions and $\Delta I = 0$ among themselves with angles η or η^2 depending on the isospin of the new $\Delta I = 0$ fermions, and normal fermions mix among themselves in the neutral current with angles η^2. The mixing angles quoted above are upper bounds; they may be smaller or zero for particular sets of fermions (see tables).

An exception to this theorem appears when two heavy $\Delta I = 0$ fermions with the same C-quantum numbers are nearly degenerate in mass. This is particularly relevant when such fermions have different NC-quantum numbers. To zeroth order in η the mixing angle in this case is maximal (45°). Other fermions mix with the degenerate multiplets with the maximum possible mixing angle (e.g. a doublet will mix with such a maximal mixture of a degenerate doublet and a singlet with an angle order η). All the low-energy consequences of the theorem, however, still hold. In particular, the net effect of interchanging these heavy fermions in low-energy processes is the same. Leading-order mixing effectively cancels. Such exceptions are not detectable in low-energy experiments. We will not discuss any further this case, except to note that one must treat carefully the expressions containing differences of large masses in the denominator (see appendices). When these masses are degenerate the corresponding mixing angles diverge and a more delicate analysis is necessary, the conclusion being that just presented.

As an example of theorem 2.1 consider the vertices involved in a typical lepton-number changing process such as $\mu \to e\gamma$ via gauge boson exchange. The fermion

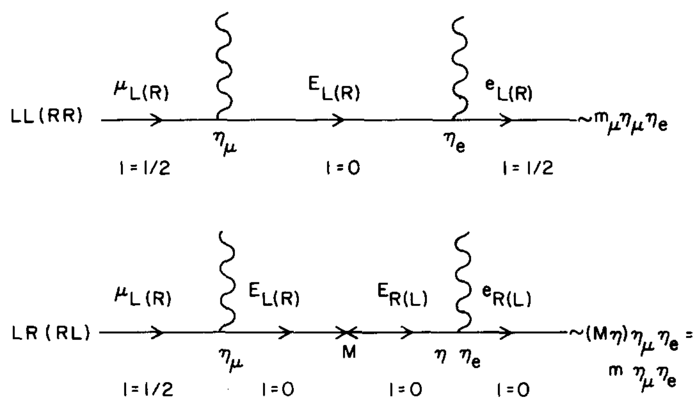

Fig. 1. Fermion line for one-loop $\mu \to e$ transitions. The m_μ in the LL(RR) diagram comes from the external momentum of the μ line.

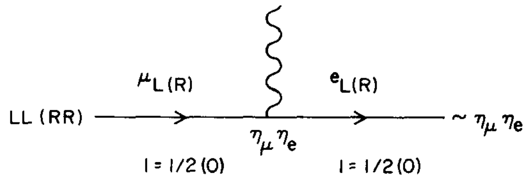

Fig. 2. Tree-level $\mu \to e$ transition.

lines for the one-loop diagrams arising from $\Delta I = 0$ lepton-E exchange are shown in fig. 1. Fig. 2 shows the tree level mixings which are of the same order since two $\Delta I = \frac{1}{2}$ vertices are comparable in mixing to one $\Delta I = 0$ vertex.

Let us emphasize the phenomenological consequences of these results. The mixing angles go to zero with the scale of GWS breaking, as expected. The explicit functional dependencies, however, are non-trivial. For contrast, imagine all the mixing angles were $\sqrt{m/M}$ instead of the values we have quoted. FCNC would then almost certainly forbid $\Delta I = 0$ mass fermions. Consider the case of $\mu \to e\gamma$ [4]. Typical diagrams where this process is mediated by exchange of $\Delta I = 0$ mass fermions are shown in fig. 3. The diagrams with a mass insertion on the internal fermion line (LR) give a contribution proportional to M up to the mixing angles. If these angles were $\sim\sqrt{m/M}$ the net contribution would be $\sim M(\sqrt{m/M})^2$, i.e. independent of M, and with the absence of the GIM mechanism [12] $\Delta I = 0$ mass fermions would, in general, be forbidden. We have found that the mixing-angle behavior is significantly different. In the example of an intermediate lepton with $I = 0$ one vertex has $\Delta I = \frac{1}{2}$ ($\mu_L \to E_L$) and the other has $\Delta I = 0$ ($E_R \to e_R$). Therefore the net mixing-angle suppression is $(m/M)(m/M)^2 \sim (m/M)^3$ as opposed to $(\sqrt{m/M})^2$. The phenomenological consequences are then very different. $\Delta I = 0$ fermions induce FCNC, on which there are strong experimental bounds, at the tree level. These processes have $\Delta I = 0$ vertices (see fig. 2) and thus according to our theorem have mixing-angle suppression order $(m/M)^2$ (the consequent limits on M would be very different for less mixing-angle suppression). In summary any normal $\to$ normal transition forbidden in the absence of $\Delta I = 0$ fermions is suppressed by a factor η^2; heavy $\Delta I = 0$ fermions decouple.

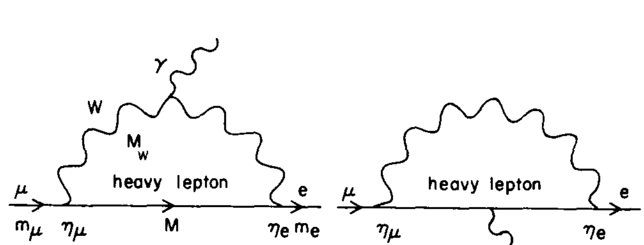

Fig. 3. One-loop $\mu \to e\gamma$ diagrams.

Our conclusions so far depend only on the assumption that $\Delta I = 0$ fermions are much heavier than normal fermions. To make quantitative predictions we must specify the light mass m which enters in the mixing angles. This is important given, for instance, the range in lepton masses from m_e to m_τ. Determining this light mass requires further assumptions about the fermion mass matrix. We will assume that the low-energy ($\Delta I = \frac{1}{2}$) spectrum (in particular the light mass hierarchy) does not depend on a special choice of the large mass ($\Delta I = 0$) parameters, and will henceforth refer to this assumption as the "hierarchy principle". The hierarchy principle implies that the mixing angle η_a for a normal fermion of mass m_a and a $\Delta I = 0$ one of mass M is $\sim m_a/M$ (see appendix A). Grand unified models exist where some low-mass hierarchies, for instance $m_e \ll m_\mu$, are a consequence of particular relations between large-M and small-m mass parameters. However one needs to restrict the particle content and the allowed fermion mass-generating mechanisms to enforce these relations. In the cases we know these relations are lost when one adds extra higgses or further $\Delta I = 0$ fermions (see sect. 4). Many models simply do not allow any linear relation among M and m entries. We stress, however, that the hierarchy principle is not a necessary assumption. The restrictions on $\Delta I = 0$ masses are still not severe without it: the masses can be low. For this reason we present our analysis for both the general case and the specific case resulting from use of the hierarchy principle.

To discuss the Higgs mixing angles we must first specify the Higgs content whether or not we consider the higgses elementary. We assume that there is only one Higgs doublet giving $\Delta I = \frac{1}{2}$ mass to each set of fermions with the same C-quantum numbers. Owing to model dependence we cannot make any general statement if there are more Higgs particles. Conflict with the experimental limits on rare processes is likely, however, [13] and specific models should be carefully analysed. Often the only way to avoid this conflict is to banish the extra Higgs particles to high mass.

From now on we regard the physical Higgs as neutral but the conclusions for charged higgses are the same, up to Cabibbo mixing in the normal fermion sector. For a given Higgs field the mixing angles among fermions are obtained by expressing the corresponding Yukawa matrix in the mass eigenstates. This matrix can only be simultaneously diagonalized with the mass matrix if the two are proportional. This requires there to be only one source of fermion masses, with all the Yukawa couplings constrained to reproduce the fermion spectrum. When there is more than one source the Yukawa couplings cannot all be constrained by the fermion spectrum and there can be large FCNC effects. Because we also allow for $\Delta I = 0$ mass terms, there will be induced mixing. However we prove in appendix B a theorem which shows that the mixing angles in this case are adequately small.

Higgs bosons change the fermion helicity. Thus the FFH vertices involve LH and RH fermions.

2.2. FFH MIXING ANGLE THEOREM

Theorem: Light normal fermions mix among themselves with mixing angles η^2 (except for charged Higgs vertices in which case the mixing is Cabibbo-like). Light normal fermions and heavy $\Delta I=0$ fermions or heavy $\Delta I=0$ fermions among themselves mix with angles of order unity if their LH and RH parts differ in isospin by $\frac{1}{2}$, with angles η if they have the same isospin and with angles η or smaller if they differ in isospin by 1 or more.

A similar comment to that of theorem 2.1 follows when two heavy $\Delta I=0$ fermions with the same C- but different NC-quantum numbers have nearly degenerate masses.

The hierarchy principle implies that η, or the Yukawa coupling y, is proportional to the mass of the light fermion that is mixing. In FFG vertices the mixing angles are multiplied by the gauge coupling constant g. In FFH vertices they are multiplied by a Yukawa coupling $y=m/v$ where m is a small normal mass m and $v\sim 250$ GeV is the Higgs vacuum expectation value. Consequently y is typically much smaller than the gauge coupling constant. Since the mixing-angle suppression is less in Higgs diagrams, however, their contributions are not negligible. In the cases we study it is possible to set the constraints on $\Delta I=0$ masses by considering only the gauge diagrams. We discuss essentially three classes of diagrams. The first class is one-loop diagrams where fig. 1 applies. Here the Higgs diagram corresponds to replacing the gauge bosons by scalars and the ratio of Higgs to gauge contributions in the amplitude is obtained by replacing the factor $m_\mu\eta_\mu\eta_e g^2$ by $m_\mu y_\mu y_e$ according to the FFH theorem. Since

$$\frac{m_\mu\eta_\mu\eta_e g^2}{m_\mu y_\mu y_e}\sim\frac{g^2v^2}{M^2}\sim\left(\frac{2M_W}{M}\right)^2\sim\left(\frac{155\ \text{GeV}}{M}\right)^2, \tag{2.1}$$

the Higgs contribution is less important than the gauge one for $M<155$ GeV and the Higgs mass order M_W. For small Higgs mass both contributions are at most comparable. Henceforth we use the gauge contributions to estimate the constraints on M. For higher-M values both contributions are comparable but well below the experimental limits. (Note that in (2.1) we have not written the expression for the one-loop diagrams [4].) The second class is tree-level diagrams where fig. 2 applies. For these processes the Higgs diagram is suppressed by the same mixing angle and by an extra factor y/g. Finally in discussing the characteristics of $\Delta I=0$ fermions we consider tree diagrams where a single $\Delta I=0$ fermion is produced or decays. In this case the vertex emitting a Higgs is in the ratio

$$\frac{y}{\eta g}\sim\frac{M}{gv}\sim\frac{M}{155\ \text{GeV}}, \tag{2.2}$$

to the vertex emitting a gauge boson. For $M>155$ GeV, then, Higgs contributions will dominate. We note though that if the Higgs and gauge bosons are virtual and decay to a pair of light fermions, the Higgs contribution will be suppressed by an extra factor y/g. In conclusion we consider the Higgs contributions only when we discuss the production and decay of heavy $\Delta I = 0$ fermions (see below). The Higgs contributions to FCNC are considered in detail in ref. [4].

2.3. CHARACTERISTICS OF $\Delta I = 0$ FERMIONS

The weak interactions of $\Delta I = 0$ fermions are determined by their multiplet assignments and mixing angles. In tables 1 and 2 we gather the salient characteristics of $\Delta I = 0$ pseudoexotic leptons and quarks respectively. For each possible multiplet assignment we give the relevant non-diagonal couplings, an estimate of the lifetime (assuming the hierarchy principle), the first SU(5) representation in which the multiplet occurs and the signatures which distinguish them from normal fermions. These are the mass degeneracy within each multiplet, the typical splitting being mm/M, and a larger value of the neutral to charged decay ratio. For estimating the production of these new $\Delta I = 0$ fermions and their detection signatures the usual diagonal couplings following from their multiplet structure and the dominant non-diagonal couplings given in tables 1 and 2 should be used. The diagonal couplings are of strength unity, the non-diagonal ones are suppressed by mixing angles $\eta \sim m/M$, where m is a typical light mass for the $\Delta I = \frac{1}{2}$ fermion entering or leaving the vertex and M the typical large mass of the new heavy $\Delta I = 0$ fermion. If the hierarchy principle applies, the mixing and then the signals are expected to be larger for processes involving the heaviest normal generations. Higgs and gauge vertices are in the ratio $y/g\eta \sim M/155$ GeV*.

3. Experimental constraints

What are the experimental constraints on the mass of $\Delta I = 0$ fermions? We study first the constraints from the GWS phenomenology of FCNC, second the constraints from the usual GUT predictions and finally those derived from standard cosmology.

3.1. STANDARD PHENOMENOLOGY

Since fermions with $\Delta I = 0$ masses have not been observed we expect them to be heavier than ~20 GeV [14]. We now study their effect on rare processes which are forbidden (or nearly so) in the minimal $SU(2)_L \times U(1)_Y$ model. The strong upper bounds on lepton-number violating processes, such as $\mu \to e\gamma$, $\mu \to ee\bar{e}$ or $\mu N \to eN$,

* The Jade collaboration at PETRA [13] has searched for heavy neutral leptons: $e^+e^- \to \nu E^0 \to e^+\tau^-\nu^0$. Neutral $\Delta I = 0$ fermions will have similar signatures. However, the correct incorporation of mixing angles is essential. In the $\Delta I = 0$ case this means that we expect heavy generations to be preferred in the final state.

and the high suppression of FCNC in the quark sector, as in $K \to \mu\bar{\mu}$ and the small mass differences between K_L, K_S and D_L, D_S, restrict the possible quantum numbers of fermions embedded in the GWS model. Fermions which mix with the first two families (e, ν_e, u, d; μ, ν_μ, c, s) with Cabibbo-like mixing angles and moderate mass must have the same C- and NC-quantum numbers as these families to be consistent with the magnitude of the above processes. Fermions with $\Delta I = 0$ mass evade this restriction. Their naturally small mixing angles alone suppress rare processes.

The neutral currents in the GWS model are diagonal at tree level (this is the GIM mechanism [12]). In our case we have order η and η^2 non-diagonal currents. There are tree-level FCNC effects. One-loop contributions are down by at least a factor α/π with respect to the tree-level ones. We neglect *CP* violation.

3.1.1. Lepton number non-conservation. No process violating lepton number has been observed. At present the best bounds on these processes are [15] (and see also ref. [11]):

$$B(\mu \to e\gamma) < 2\times 10^{-10}, \qquad B(\mu \to ee\bar{e}) < 2\times 10^{-9}, \qquad B(\mu N \to eN) < 4\times 10^{-10}. \tag{3.1}$$

Let us examine the most important contributions of $\Delta I = 0$ fermions to these processes.

Process $\mu \to e\gamma$. This is forbidden at the tree level because electromagnetic interactions conserve flavour. The typical one-loop gauge diagrams are shown in fig. 3, where M is the $\Delta I = 0$ mass, η_μ and η_e are the mixing angles of the muon and electron with the mediating $\Delta I = 0$ mass fermion and $M_w, M \gg m_\mu \gg m_e$. The decay rate is

$$\Gamma(\mu \to e\gamma) \sim \Gamma(\mu \to e\nu\bar{\nu})\frac{\alpha}{\pi}\eta_\mu^2\eta_e^2, \tag{3.2}$$

where α/π comes from the two vertices and subsequent loop itegration. We have calculated this rate for arbitrary M including the Higgs contribution [4] and verified (3.2) up to a factor order 1.

The branching ratio is then

$$B(\mu \to e\gamma) \sim \frac{\alpha}{\pi}\eta_\mu^2\eta_e^2 \equiv \frac{\alpha}{\pi}\frac{\delta^4}{M^4}; \tag{3.3}$$

(3.3) defines the mixing parameter $\delta = \sqrt{\eta_\mu\eta_e}M$. For M to be ~20 GeV and the branching ratio less than the experimental limit δ must be ≤ 0.4 GeV. The hierarchy principle gives

$$\eta_\mu \sim \frac{m_\mu}{M}, \qquad \eta_e \sim \frac{m_e}{M}, \tag{3.4}$$

and therefore

$$\delta \sim \sqrt{m_e m_\mu}. \tag{3.5}$$

TABLE 1

$\Delta I = 0$ lepton characteristics

	LH and RH parts	Masses (M large)	Gauge L	Gauge R	Higgs RL (ℓ is a doublet)	Higgs LR (ℓ is a singlet)
singlets	N	no constraint	$L \overset{\eta}{—} \ell$ Z,W	(N will be in general a self-conjugate Majorana spinor)	$L — \ell$ H	(N will be in general a self-conjugate Majorana spinor)
	E		$L \overset{\eta}{—} \ell$ Z,W	η^2 mixing (no mixing if no other $\Delta I = 0$ multiplets)	$L — \ell$ H	$L \overset{\eta}{—} \ell$ H
doublets	N E	particles in the same multiplet are nearly degenerate in mass	η^2 mixing (no mixing in NC if no other $\Delta I = 0$ multiplets)	$L \overset{\eta}{—} \ell$ Z,W	$L \overset{\eta}{—} \ell$ H	$L — \ell$ H
	E ·		η^2 mixing			
triplets	E^c N N , E E ·	$\Delta M \sim m\frac{m}{M}$	$L \overset{\eta}{—} \ell$ Z,W	negligible	$L — \ell$ H	$L \overset{\eta}{—} \ell$ H
	E · ·		negligible		negligible	
other multiplets			negligible		negligible	

Non-diagonal couplings with normal generations (We indicate only the mixing angles ($\eta \sim m/M$) suppressing the gauge and Yukawa couplings)

The lifetimes should be interpreted as order of magnitude estimates. m_τ is the mass of the tau lepton.

This gives

$$B(\mu \to e\gamma) \sim 4 \times 10^{-17}\left(\frac{20\ \text{GeV}}{M}\right)^4, \tag{3.6}$$

which is much less than the experimental limit for $M \gtrsim 20$ GeV.

TABLE 1—(*continued*)

Lifetimes $M<M_{W,Z}$	Lifetimes $M_{W,Z}<M$	Usual SU(5) representations containing these $\Delta I=0$ leptons	Other characteristics	
$\tau \lesssim \tau(L \to \ell_\tau e \nu_e) \sim \left[\frac{G^2}{192\pi^3} m_\tau^2 M^3\right]^{-1} \sim 10^{-15}\left(\frac{20}{M \text{ in GeV}}\right)^3$ sec	$\tau \lesssim \tau(L \to \ell_\tau Z) \sim \left[\frac{G}{8\sqrt{2}\pi} m_\tau^2 M\right]^{-1} \sim 10^{-20}\left(\frac{100}{M \text{ in GeV}}\right)$ sec	1_F and 24_F		no sequential character or associated nearly massless neutrino or conserved quantum number, ratio of neutral to charge decays large
		$10_F+\overline{10}_F$		
		$5_F+\bar{5}_F$ (used in supersymmetry) and $45_F+\overline{45}_F$		
		other representations	in general imply exotics	
		24_F, other representations		
10^{-16} sec $<\tau<1$ sec		other representations		
		difficult to embed		

Process $\mu \to ee\bar{e}$. This process is allowed at the tree level by Z exchange (fig. 4). The decay rate is

$$\Gamma(\mu \to ee\bar{e}) \sim \Gamma(\mu \to e\nu\bar{\nu})\tfrac{1}{2}\eta_e^2\eta_\mu^2 \equiv \Gamma(\mu \to e\nu\bar{\nu})\frac{1}{2}\frac{\delta^4}{M^4}. \tag{3.7}$$

For M to be ~20 GeV and the branching ratio less than the experimental limit, δ

TABLE 2
$\Delta I = 0$ quark characteristics.

	LH and RH parts	Masses (M large)	Gauge L	Gauge R	Higgs RL (q is a doublet)	Higgs LR (q is a singlet)
singlets	U	no constraint	Q —η— q Z,W	η^2 mixing (no mixing if no other $I=0$ multiplets)	Q —— q H	Q —η— q H
	D					
doublets	U D	particles in the same multiplet are nearly degenerate in mass	η^2 mixing (no mixing in NC if no other $I=0$ multiplets)	Q —η— q Z,W	Q —η— q H	Q —— q H
	· D , U ·		η^2 mixing			
triplets	U · D U · D	$\Delta M \sim m\frac{m}{M}$	Q —η— q Z,W	negligible	Q —— q H	Q —η— q H
	· D · · U ·		negligible		negligible	
other multiplets			negligible		negligible	

The lifetimes should be interpreted as order of magnitude estimates. Non-diagonal couplings with normal generations (We indicate only the mixing angles ($\eta \sim m/M$) suppressing the gauge and Yukawa couplings) For instance, replacing the bottom mass m_b by a probable top mass m_t to estimate the mixing with the third family will decrease these estimates more than one order of magnitude.

must be $\leqslant 0.16$ GeV. The hierarchy principle gives again (3.5) and thus

$$B(\mu \to ee\bar{e}) \sim 10^{-14}\left(\frac{20\ \text{GeV}}{M}\right)^4 , \tag{3.8}$$

i.e. approximately 5 orders of magnitude below the experimental limit for $M = 20$ GeV.

TABLE 2—(*continued*)

Lifetimes $M < M_{W,Z}$	Lifetimes $M_{W,Z} < M$	Usual SU(5) representations containing these $\Delta I = 0$ quarks	Other characteristics
$\tau \leqslant \tau(Q \to q_b e \nu_e) \sim \left[\frac{G^2}{192\pi^3} m_b^2 M^3\right]^{-1} \sim 10^{-17}\left(\frac{50}{M \text{ in GeV}}\right)^3$ sec	$\tau \leqslant \tau(Q \to q_b Z) \sim \left[\frac{G}{8\sqrt{2}\pi} m_b^2 M\right]^{-1} \sim 10^{-21}\left(\frac{100}{M \text{ in GeV}}\right)$ sec	$10_F + \overline{10}_F$	ratio of neutral to charged decays large
		$5_F + \bar{5}_F$ (used in supersymmetry) and $45_F + \overline{45}_F$	
		$10_F + \overline{10}_F$	
		$45_F + \overline{45}_F$, 24_H	imply exotics
		$45_F + \overline{45}_F$, other representations	
10^{-18} sec $< \tau < 1$ sec		other representations	
		difficult to embed	

Process $\mu N \to eN$. This process gives a branching ratio similar to that of $\mu \to ee\bar{e}$ in (3.7) and (3.8). Since the experimental bound is lower (though we think more uncertain) it would give a better constraint on M.

In the last two processes one-loop contributions are down by $(\alpha/\pi)^2$.

With our assumption on the origin of the normal fermion mass hierarchy the experimental limits on all these lepton number non-conserving processes require

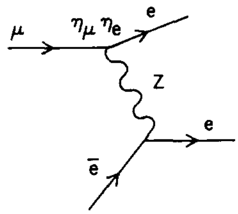

Fig. 4. Tree-level $\mu \to ee\bar{e}$ transition.

considerable improvement to indicate $\Delta I = 0$ leptons at 20 GeV. Because mixing angles are proportional to lepton masses experiments on τ and μ non-diagonal decays would be more restrictive*.

3.1.2. FCNC in the quark sector. In the quark sector the best limits on FCNC come from $K \to \mu\bar{\mu}$ decays and the mass differences $m_{K_L} - m_{K_S}$, $m_{D_L} - m_{D_S}$. Since the minimal GWS model is in agreement with experiment we must check that quarks with low $\Delta I = 0$ masses do not invalidate these predictions.

The experimental values are [11, 16]

$$B(K \to \mu\bar{\mu}) \sim 9 \times 10^{-9}, \qquad \delta m_K / m_K \sim 7 \times 10^{-15}, \qquad \delta m_D / m_D \leq 10^{-12}. \tag{3.9}$$

Process $K \to \mu\bar{\mu}$. This process goes at the tree level by interchanging a Z via a $\bar{d}sZ$ vertex proportional to $\mu_d \eta_s$ (see fig. 5). This gives a decay rate

$$\Gamma(K \to \mu\bar{\mu}) \sim \Gamma(K^+ \to \mu^+ \nu_\mu) \left(\frac{1}{\sin \theta_C} \eta_d \eta_s \right)^2 \equiv \Gamma(K^+ \to \mu^+ \nu_\mu) \frac{1}{\sin^2 \theta_C} \left(\frac{\delta^4}{M^4} \right). \tag{3.10}$$

For a $\Delta I = 0$ mass of 20 GeV, δ must be ≤ 0.09 GeV to prevent the branching ratio exceeding the experimental value (3.9). The hierarchy principle yields

$$\delta \sim \sqrt{m_d m_s} \approx 0.04 \text{ GeV}, \tag{3.11}$$

and thus

$$B(K \to \mu\bar{\mu}) \sim 3 \times 10^{-10} \left(\frac{20 \text{ GeV}}{M} \right)^4. \tag{3.12}$$

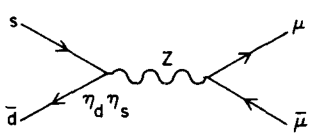

Fig. 5. Tree-level $K \to \mu\bar{\mu}$ transition.

* Note also that there will be contributions to the electron and muon anomalous magnetic moments $(g-2)$ from the exchange of $\Delta I = 0$ mass fermions. Since the amplitude is the relevant quantity here one might expect a better limit on M. However the $\mu \to e\gamma$ rate turns out to be more restrictive [4].

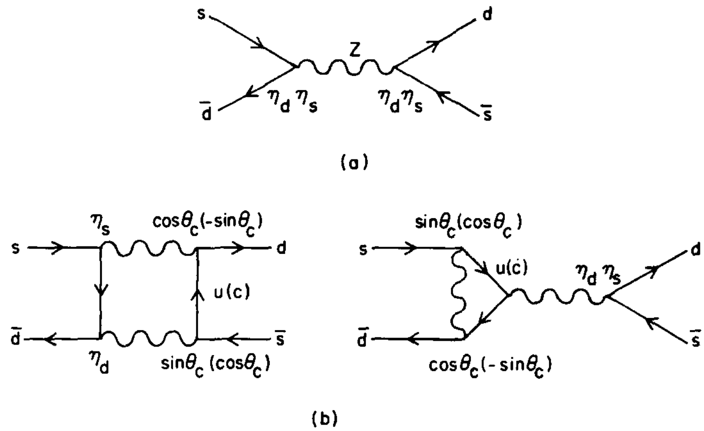

Fig. 6. $K^0 \leftrightarrow \bar{K}^0$ transition: (a) tree diagram; (b) typical one-loop diagram.

One-loop contributions are down by $(\alpha/\pi)^2$. As in all hadron processes one needs a model to estimate the effect of non-free quarks inside the hadron.

The mass difference: $K_L - K_S$. This mass difference is related to the transition amplitude $K^0 \leftrightarrow \bar{K}^0$. Fig. 6a shows the tree level contribution

$$\frac{\delta m_K}{m_K} \sim \tfrac{2}{3} f_K^2 \frac{G_F}{2\sqrt{2}} (\eta_d \eta_s)^2 \equiv \tfrac{2}{3} f_K^2 \frac{G_F \delta^4}{2\sqrt{2} M^4} . \tag{3.13}$$

With $M \sim 20$ GeV, $\delta \leqslant 1.1$ GeV gives a contribution to $\delta m_K / m_K$ smaller than the measured value. The hierarchy principle (3.11) gives

$$\frac{\delta m_K}{m_K} \sim 9 \times 10^{-19} \left(\frac{20\ \text{GeV}}{M} \right)^4 . \tag{3.14}$$

One-loop contributions can be important to the $K_L - K_S$ mass difference since one-loop diagrams such as those shown in fig. 6b have only one factor $\eta_d \eta_s$. However, the GIM mechanism [3, 12] still works in the line interchanging u or c. Thus the one-loop contribution is

$$\frac{\delta m_K}{m_K} \sim \tfrac{2}{3} f_K^2 \frac{C_F}{2\sqrt{2}} \frac{\alpha}{\pi} \frac{m_c^2}{\sin^2 \theta_w M_W^2} \sin \theta_C \cos \theta_C \frac{\delta^2}{M^2} , \tag{3.15}$$

where $\delta^2 = \eta_d \eta_s M^2$ as above. Thus

$$\frac{\delta m_K}{m_K} \sim 5 \times 10^{-14} \frac{\delta^2}{M^2} , \tag{3.16}$$

requiring δ to be $\leqslant 7.5$ GeV for an M of 20 GeV. With the hierarchy principle

$$\frac{\delta m_K}{m_K} \sim 2 \times 10^{-19} \left(\frac{20\ \text{GeV}}{M} \right)^2 , \tag{3.17}$$

which dominates (3.14) for large M but is still well below the experimental value for possible M's.

We have assumed no CP phase is present. The experimental value of CP violation in the K-system is very small i.e. $\mathrm{Im}\,(\delta m_{\mathrm{K}}/m_{\mathrm{K}}) \sim 10^{-17}$*. We note that there is no contribution from tree level diagrams to the $\mathrm{Im}\,\delta m_{\mathrm{K}}$. The one-loop amplitude (fig. 6b) on the other hand is proportional to $\eta_{\mathrm{d}}\eta_{\mathrm{s}}$ alone and gives a contribution to $\mathrm{Im}\,\delta m_{\mathrm{K}}$ of order (3.16, 17) or smaller.

The mass difference: $D_L - D_S$. This is analogous to the $\mathrm{K_L}-\mathrm{K_S}$ case if we replace d and s by u and c. Then

$$\frac{\delta m_{\mathrm{D}}}{m_{\mathrm{D}}} \sim \tfrac{2}{3} f_{\mathrm{D}}^2 \frac{G_{\mathrm{F}}}{2\sqrt{2}} \frac{\delta^2}{M^2}, \tag{3.18}$$

with $\delta^2 = \eta_{\mathrm{u}}\eta_{\mathrm{c}} M^2$. We then have

$$\frac{\delta m_{\mathrm{D}}}{m_{\mathrm{D}}} \sim 9\times 10^{-8}\left(\frac{\delta}{M}\right)^4, \qquad (\text{i.e. } \delta \leqslant 1.2\ \mathrm{GeV}, \qquad \text{for } M = 20\ \mathrm{GeV}), \tag{3.19}$$

for the tree level and

$$\frac{\delta m_{\mathrm{D}}}{m_{\mathrm{D}}} \sim 7\times 10^{-14}\left(\frac{\delta}{M}\right)^2, \qquad (\text{i.e. } \delta < 380\ \mathrm{GeV}, \qquad \text{for } M = 20\ \mathrm{GeV}), \tag{3.20}$$

for the one-loop level.

The hierarchy principle gives

$$\frac{\delta m_{\mathrm{D}}}{m_{\mathrm{D}}} \sim 2\times 10^{-17}\left(\frac{20\ \mathrm{GeV}}{M}\right)^4, \tag{3.21}$$

for the tree level and

$$\frac{\delta m_{\mathrm{D}}}{m_{\mathrm{D}}} \sim 10^{-18}\left(\frac{20\ \mathrm{GeV}}{M}\right)^2, \tag{3.22}$$

for the one-loop level.

Note that we have used a $\Delta I = 0$ mass M of 20 GeV merely for illustration; it is a reasonable lower limit since otherwise such fermions would probably already have been seen in experiments. In any given case one must ensure that $M \gg m$ for the mixing angles to follow our theorems.

We have discussed only those FCNC which give the severest constraints on the $\Delta I = 0$ masses. We can similarly estimate the contributions to other processes such as $\mathrm{K} \to \mathrm{e}\mu$ [11], but they are far below the experimental limits.

In conclusion the most restrictive process is $\mathrm{K} \to \mu\bar{\mu}$. Without some additional assumption about the form of the mass matrix we can only place a lower limit on the $\Delta I = 0$ mass for a given mixing parameter δ. However worst-case estimates

* For estimating an upper limit on $\delta m_{\mathrm{D}}/m_{\mathrm{D}}$ see ref. [16].

($\delta \approx 1$ GeV) still give a minimum M of the order of hundreds of GeV which is tantalizingly low. If the mass matrix has the form dictated by the hierarchy principle, then we find that $\Delta I = 0$ fermion masses are not constrained by the experimental magnitudes and limits of rare processes. In all cases the masses of the normal fermions involved are small enough to give very small mixing with the heavier $\Delta I = 0$ mass fermions.

3.2. GUT PREDICTIONS

Grand unified theories [17], apart from incorporating nicely the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ model, predict $\sin^2 \theta_w$, $M_{\text{unification}}$ and m_b/m_τ [18, 19]. The values of $\sin^2 \theta_w$ and m_b/m_τ are in good agreement with experiment. These predictions are unqualified only in the minimal SU(5) model. When new fermions or higgses are added to SU_5 or the gauge group is enlarged (SO(10), E_6 etc), these quantities can be adjusted and statements are much less rigorous. (For supersymmetric models see ref. [20].)

$\sin^2 \theta_w$ and M_x. We find that the usual SU(5) predictions of $\sin^2 \theta_w$ and M_x can each be separately increased or decreased for different values of the $\Delta I = 0$ fermion masses and/or representations. In fact we can obtain any reasonable value consistent with the measured value of $\sin^2 \theta_w$ and the lower bound on the proton lifetime. The only general observation is that the addition of $\Delta I = 0$ fermions in complete SU(5) representations with the different $SU(3)_C \times SU(2)_L \times U(1)_Y$ multiplets degenerate does not change $\sin^2 \theta_w$ and M_x to first order. Thus the deviations of $\sin^2 \theta_w$ and M_x from their standard value are due to mass differences between $SU(3)_C \times SU(2)_L \times U(1)_Y$ multiplets. Adding new fermions always tends to increase the coupling constant α_{Gum} at the unification mass, and therefore decreases the proton lifetime. The lifetime is much more sensitive to M_x, however, than α_{Gum} and so increases in M_x overwhelm increases in α_{Gum}, giving a longer proton lifetime.

m_b/m_τ. The prediction of the bottom quark mass [19] was a triumph of the minimal SU(5) model. In this model m_b/m_τ is one at the unification mass and there is no extra structure until the GWS scale. More complicated models need not share these features but, given the success of the minimal SU(5) predictions of m_b, one may worry about the effects of new $\Delta I = 0$ fermions. We find that m_b/m_τ can be increased or decreased depending on the $\Delta I = 0$ fermion masses and/or representations.

In table 3 we list the changes in the SU(5) predictions of $M_{\text{unification}}$, $\sin^2 \theta_w$, α_{Gum}, τ_p and m_b/m_τ for the extreme case where one $SU(3)_C \times SU(2)_L \times U(1)_Y$ multiplet of the SU(5) representations $5_F + \bar{5}_F$ and $10_F + \overline{10}_F$ is at M_W and for the case where the splitting within the $5_F + \bar{5}_F$ representations is an order of magnitude. The quoted values were obtained from one-loop formulae [17]. (The $SU(3)_C \times SU(2)_L \times U(1)_Y$ content of these SU(5) representations is given in table 4.) In the extreme cases

TABLE 3

Illustrative examples of the effects of new $\Delta I = 0$ fermions at M_w on $\sin^2\theta_w$, τ_p and m_b/m_τ

$SU(3)_C \times SU(2)_L \times U(1)_Y$ multiplets at M_W $(r+\bar{r})$	(M'_x/M_x)	$\sin^2\theta'_w - \sin^2\theta_w$	$\alpha'_{Gum}/\alpha_{Gum}$	τ'_p/τ_p	m'_b/m_b
$(3,1)_{-2}$	6.6	−0.017	1.03	1.5×10^3	1.06
$(1,2)_3$	0.2	0.015	1.05	1.6×10^{-3}	0.98
$(3,2)_1$	6.6	0.035	1.12	$1.2+10^3$	1.13
$(\bar{3},1)_{-4}$	1	−0.025	1.09	0.8	1.04
$(1,1)_6$	0.2	−0.009	1.05	1.1×10^{-3}	0.96
$5_F+\bar{5}_F$ with $m_{(3,1)}=10m_{(1,2)}=10M_W$	0.9	0.001	1.08	0.5	1.04
$5_F+\bar{5}_F$ with $m_{(1,2)}=10m_{(3,1)}=10M_W$	1.1	−0.001	1.08	1.4	1.03

Primed denotes new values and unprimed the values for the minimal model.
$\alpha^{-1}=128$ and $\alpha_3=0.12$ at M_W.

we see that almost any combination of changes is possible but the changes are usually small. In the nearly degenerate case the changes are even smaller.

3.3. COSMOLOGICAL CONSTRAINTS

The standard model of the early universe [21] does not constrain very short-lived particles i.e. those with a lifetime of $\lesssim 1$ sec. Thus pseudoexotic fermions, which mix with and decay into normal fermions with lifetimes much smaller than 1 sec (see tables 1 and 2), are not constrained by standard cosmology. Exotic fermions like those in the SU(5) representations 45_F and 24_F (see table 4) can be short or long-lived. Those which interact weakly with pseudoexotic fermions and then decay into normal ones have very short lifetimes, comparable to those of the pseudoexotics. For example, there is a quark doublet in the 45_F which contains a U antiquark of charge $-\frac{2}{3}$ and a G antiquark of charge $-\frac{5}{3}$; the decay rate of $G\to t\bar{e}\nu$ is then of the same order as $U\to b\bar{e}\nu$ (see table 2). Those exotic fermions which

TABLE 4

$SU(3)_C\times SU(2)_L\times U(1)_Y$ content of several SU(5) representations

$5=(3,1)_{-2}+(1,2)_3$
$10=(3,2)_1+(\bar{3},1)_{-4}+(1,1)_6$
$24=(8,1)_0+(3,2)_{-5}+(\bar{3},2)_5+(1,3)_0+(1,1)_0$
$45=(8,2)_3+(\bar{6},1)_{-2}+(3,3)_{-2}+(\bar{3},2)_{-7}+(3,1)_{-2}+(\bar{3},1)_8+(1,2)_3$

The electric charge is defined $Q=I_3+\frac{1}{6}Y$.

do not have weak interactions with the pseudoexotic fermions, such as the neutral octet in the 24_F, would be as stable as the proton if their masses were very low. For an asymmetric matter–antimatter universe, the masses of stable fermions must be $\leq 10 m_{\text{nucleon}}$ ($\simeq 10$ GeV) otherwise they will contribute too much mass to the universe. Accelerator searches thus seem to rule them out. For a more general discussion and examples see ref. [22].

4. An example

We illustrate with an example the plausibility of low-mass (<1 TeV) $\Delta I = 0$ fermions that decouple from normal fermions because they mix with very small angles. The example is taken from the EGZ model [7] (see (1.1)), in which we assume that the Yukawa and Higgs sectors respect two continuous symmetries to be identified with $B-L$ and Peccei–Quinn (PQ) [23]. These may be regarded as effective symmetries arising from $N=8$ supergravity and so perhaps not exact*. We deal with two normal ($\bar{5}_F + 10_F$) fermion families plus pseudoexotic $5_F + \bar{5}_F$ and $10_F + \overline{10}_F$ fermion families. There are three 5_H, one 10_H and a complex 24_H Higgs representation, all of which appear in the EGZ model. This Higgs sector is minimal for our purposes. The number of normal and $\Delta I = 0$ fermion families does not affect the analysis. We restrict their number for clarity and simplicity. One can imagine that the two normal families correspond to the e and μ families and the $\Delta I = 0$ families to new fermions. Since nothing depends crucially on having two families we may consider this example as realistic.

The imposition of continuous symmetries is a general technique [8, 9]. It can be applied to any fermion content provided one introduces an appropriate Higgs sector. In our case the Yukawa couplings are

$$a_{ij}\bar{5}_{Fi}10_{Fj}\bar{5}_H + 10_{Fi}10_{Fj}(b_{ij}5'_H + c_{ij}5''_H) + d_j 5_F 10_{Fj} 10_H + g_j \bar{5}_{Fj}\overline{10}_F\overline{10}_H + h\, 5_F\overline{10}_F 5_H\,, \tag{4.1}$$

where we sum over $i, j = 1, 2, 3$, and a_{ij}, b_{ij}, c_{ij}, d_j, g_j and h are arbitrary constants. The Higgs sector contains the quartic couplings

$$\overline{10}_H 24_H 5'_H 5''_H\,, \qquad 24_H 24_H 5_H \bar{5}''_H\,, \tag{4.2}$$

in addition to the usual potential terms. The coupling $24_H 24_H 5_H \bar{5}'_H$ is also allowed.

The corresponding lagrangian has two continuous symmetries:

	$\bar{5}_{Fi}$	10_{Fi}	5_F	$\overline{10}_F$	10_H	5_H	$5'_H, 5''_H$	24_H
X_1	1	−1	−4	4	5	0	2	1
X_2	−3	1	3	−1	−4	−2	−2	0

* Domain walls could be a problem for exact global symmetries. In many cases and in particular for axion models [24] the vacuum degeneracy implies the existence of domains in the recent past and hence an excess of energy in domain walls.

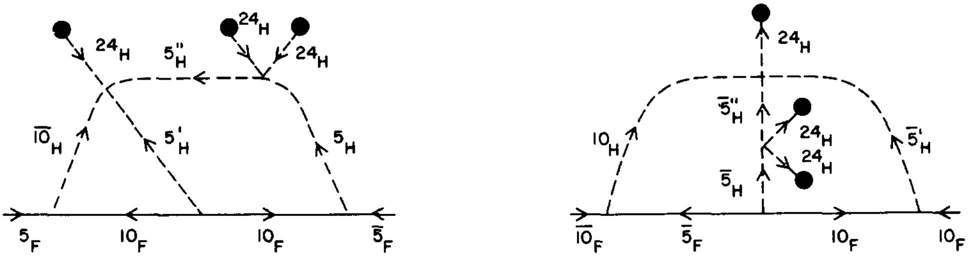

Fig. 7. Two-loop Higgs diagrams giving radiative masses.

X_1 is a chiral symmetry and is broken at 10^{15} GeV when 24_H acquires a v.e.v. This produces an invisible axion. X_2 corresponds to $B-L$ after the breaking of WS by the $\langle 5_H\text{'s}\rangle$.

Even when 24_H acquires a v.e.v. and breaks SU(5) to $SU(3)_C \times SU(2)_L \times U(1)_Y$ the fermions in 5_F and $\overline{10}_F$ are still massless, at the tree level, since the couplings $5_F\bar{5}_F24_H$ and $10_F\overline{10}_F24_H$ are forbidden by the symmetry X_1. Radiative corrections give them calculable masses, the most important contributions coming from all-Higgs two-loop diagrams (see fig. 7). In addition to the two-loop suppression we have an extra suppression from the smallness of the Yukawa couplings. Estimating these diagrams [9] we find that the masses of the $\Delta I = 0$ fermions can be 1 TeV or lower. We have an effective $SU(3)_C \times SU(2)_L \times U(1)_Y$ theory with e and μ families of normal mass m (after GWS breaking) and a group of $\Delta I = 0$ fermions with masses M larger than m(they have $SU(2)_L$ invariant mass terms). In this model the different M masses are related since the fermion multiplets couple to the same expectation value $\langle 24_H\rangle$. Also some large-M entries are correlated with low-m entries. In what follows we assume that M parameters are large but arbitrary*.

The fermion content of the effective theory is

$$2\begin{pmatrix} N_L \\ E_L \end{pmatrix}, 2E_R + \begin{pmatrix} N_L \\ E_L \end{pmatrix}, \begin{pmatrix} N_R \\ E_R \end{pmatrix}, E_L, E_R,$$

$$2\begin{pmatrix} U_L \\ D_L \end{pmatrix}, 2U_R, 2D_R + D_L, D_R, \begin{pmatrix} U_L \\ D_L \end{pmatrix}, \begin{pmatrix} U_R \\ D_R \end{pmatrix}, U_L, U_R, \tag{4.3}$$

where we have not distinguished the current eigenstates from the $\bar{5}_F$, 10_F, 5_F or $\overline{10}_F$. When necessary we will use subindices for LH doublets and RH singlets.

* If there was no $10_F + \overline{10}_F$ of fermions in this example and the most important contribution to the $5_F\bar{5}_F$ $\Delta I = 0$ mass came from the diagram of fig. 7, a linear relation would exist between the large-M and small-m entries in the mass matrix for leptons of charge -1 and quarks of charge $-\frac{1}{3}$. This relation would give m_e, $m_d = 0$. It is sufficient to add another 5_H of higgses coupling to 5_F10_F to destroy this relation.

The electroweak interaction lagrangian for leptons is

$$\begin{aligned}\mathscr{L}_{\mathrm{FFG}}+\mathscr{L}_{\mathrm{FFH}}=&\sqrt{\tfrac{1}{2}}gW^{+}_{\mu}(\bar{N}_{\mathrm{L}i}\gamma^{\mu}E_{\mathrm{L}i}+\bar{N}_{\mathrm{R}}\gamma^{\mu}E_{\mathrm{R}})+\mathrm{h.c.}\\&+\tfrac{1}{2}(g^{2}+g'^{2})^{1/2}Z_{\mu}(\bar{N}_{\mathrm{L}i}\gamma^{\mu}N_{\mathrm{L}i}-\bar{E}_{\mathrm{L}i}\gamma^{\mu}E_{\mathrm{L}i}\\&+\bar{N}_{\mathrm{R}}\gamma^{\mu}N_{\mathrm{R}}-\bar{E}_{\mathrm{R}}\gamma^{\mu}E_{\mathrm{R}}-2J^{\mu}_{\mathrm{EM}}\sin^{2}\theta_{\mathrm{w}})+eA_{\mu}J^{\mu}_{\mathrm{EM}}\\&-[\sqrt{\tfrac{1}{2}}(v+\chi)(c^{\mathrm{e}}_{ij}\bar{E}_{\mathrm{L}i}E_{\mathrm{R}j}+c^{\mathrm{e}}\bar{E}_{\mathrm{L}}E_{\mathrm{R}})+M^{\mathrm{e}}_{i}\bar{E}_{\mathrm{L}i}E_{\mathrm{R}}\\&+M^{\mathrm{e}'}_{j}\bar{E}_{\mathrm{L}}E_{\mathrm{R}j}+M^{\mathrm{n}}_{i}\bar{N}_{\mathrm{L}i}N_{\mathrm{R}}]+\mathrm{h.c.}\,,\end{aligned}\tag{4.4}$$

where we sum over $i, j = 1, 2, 3$ and $M^{\mathrm{n}}_i = M^{\mathrm{e}}_i$ (they correspond to the same bare mass term). χ is the physical Higgs of the single doublet; $m^{\mathrm{e}}_{ij} = \sqrt{\tfrac{1}{2}}c^{\mathrm{e}}_{ij}v$ and $m^{\mathrm{e}} = \sqrt{\tfrac{1}{2}}c^{\mathrm{e}}v$, where v is the vacuum expectation value, are elements of the mass matrix.

The lagrangian for quarks is obtained by replacing N with U and E with D and by writing the corresponding Yukawa and bare mass terms for U. The fields in (4.4) are current eigenstates. To write the lagrangian in terms of physical fields we must diagonalize the corresponding mass matrices. These are for leptons (ignoring any CP phase)

$$\begin{array}{c}\bar{N}_{\mathrm{L}i}(M^{\mathrm{n}}_{i})\\ N_{\mathrm{R}}\end{array},\qquad ij=1,2,3\,,\tag{4.5a}$$

$$\begin{array}{c}\bar{E}_{\mathrm{L}}\\ \bar{E}_{\mathrm{L}i}\\ \end{array}\!\!\begin{pmatrix}m^{\mathrm{e}} & M^{\mathrm{e}'}_{j}\\ M^{\mathrm{e}}_{i} & m^{\mathrm{e}}_{ij}\end{pmatrix},\quad \begin{array}{cc}\bar{E}_{\mathrm{R}} & E_{\mathrm{R}j}\end{array}\tag{4.5b}$$

where $\Delta I = 0$ entries are order M and $\Delta I = \tfrac{1}{2}$ entries order m, with $m \ll M$, and $\Delta I = 1$ entries are 0. For the U and D quarks the general mass matrices are of the form (4.5b) with E changed to U and D respectively.

Diagonalizing as indicated in the appendices and writing the mass eigenstates as $n_{\mathrm{L(R)}}$, $e_{\mathrm{L(R)}}$, $u_{\mathrm{L(R)}}$, $d_{\mathrm{L(R)}}$, we find that the interaction lagrangian for leptons to first order in $\eta \sim m/M$ is

$$\begin{aligned}&\sqrt{\tfrac{1}{2}}gW^{+}_{\mu}[-(\eta^{\mathrm{e}}_{\mathrm{L}1}\bar{n}_{\mathrm{L}1}+\eta^{\mathrm{e}}_{\mathrm{L}2}\bar{n}_{\mathrm{L}2}+\eta^{\mathrm{e}}_{\mathrm{L}3}\bar{n}_{\mathrm{L}3})\gamma^{\mu}e_{\mathrm{L}4}+\bar{n}_{\mathrm{R}}\gamma^{\mu}(\eta^{\mathrm{e}}_{\mathrm{R}1}e_{\mathrm{R}1}+\eta^{\mathrm{e}}_{\mathrm{R}2}e_{\mathrm{R}2}+\eta^{\mathrm{e}}_{\mathrm{R}4}e_{\mathrm{R}4})]\\&\quad+\tfrac{1}{2}(g^{2}+g'^{2})^{1/2}Z_{\mu}[(\eta^{\mathrm{e}}_{\mathrm{L}1}\bar{e}_{\mathrm{L}1}+\eta^{\mathrm{e}}_{\mathrm{L}2}\bar{e}_{\mathrm{L}2}+\eta^{\mathrm{e}}_{\mathrm{L}3}\bar{e}_{\mathrm{L}3})\gamma^{\mu}e_{\mathrm{L}4}\\&\quad-(\eta^{\mathrm{e}}_{\mathrm{R}1}\bar{e}_{\mathrm{R}1}+\eta^{\mathrm{e}}_{\mathrm{R}2}\bar{e}_{\mathrm{R}2}+\eta^{\mathrm{e}}_{\mathrm{R}4}\bar{e}_{\mathrm{R}4})\gamma^{\mu}e_{\mathrm{R}3}]\\&\quad+\chi[\bar{e}_{\mathrm{L}1}(-y^{\mathrm{e}}_{13}\eta e_{\mathrm{R}3}+y^{\mathrm{e}}_{14}e_{\mathrm{R}4})+\bar{e}_{\mathrm{L}2}(-y^{\mathrm{e}}_{23}\eta e_{\mathrm{R}3}+y^{\mathrm{e}}_{24}e_{\mathrm{R}4})\\&\quad+\bar{e}_{\mathrm{L}3}(y^{\mathrm{e}}_{31}e_{\mathrm{R}1}+y^{\mathrm{e}}_{32}e_{\mathrm{R}2}+y^{\mathrm{e}}_{34}e_{\mathrm{R}4})-\bar{e}_{\mathrm{L}4}(y^{\mathrm{e}}_{41}\eta e_{\mathrm{R}1}+y^{\mathrm{e}}_{42}\eta e_{\mathrm{R}2}+y^{\mathrm{e}}_{43}e_{\mathrm{R}3})]+\mathrm{h.c.}\end{aligned}\tag{4.6}$$

The subindex 1 stands for electron, 2 for muon, 3 for $\Delta I = 0$ mass leptons with $I = \tfrac{1}{2}$ and 4 for $\Delta I = 0$ mass leptons with $I = 0$. The η parameters are of order m/M and the y parameters are typical Yukawa couplings of order m/v. (4.6) exemplifies the mixing-angle theorems of sect. 2. There is no Cabibbo angle in the charged sector because the e, μ neutrino masses are zero (degenerate).

FCNC are thus significantly suppressed in this model. If we adopt the hierarchy principle then

$$\eta_1^{\mathrm{e}} \sim \frac{m_{\mathrm{e}}}{M}, \qquad \eta_2^{\mathrm{e}} \sim \frac{m_\mu}{M}, \tag{4.7}$$

and the y parameters with subindices 1 and 2 go as m_{e}/v and m_μ/v respectively.

For quarks the interaction lagrangian to order η is

$$\begin{aligned} &-\sqrt{\tfrac{1}{2}}\, g W_\mu^+[\bar{u}_{\mathrm{L}\alpha}\gamma^\mu A^{\alpha\beta} d_{\mathrm{L}\beta} + \bar{u}_{\mathrm{R}\alpha}\gamma^\mu B^{\alpha\beta} d_{\mathrm{R}\beta}] + \text{h.c.} \\ &\quad + \tfrac{1}{2}\sqrt{g^2+g'^2}\, Z_\mu[\bar{u}_{\mathrm{L}\alpha}\gamma^\mu C_{\mathrm{u}}^{\alpha\beta} u_{\mathrm{L}\beta} - \bar{d}_{\mathrm{L}\alpha}\gamma^\mu C_{\mathrm{d}}^{\alpha\beta} d_{\mathrm{L}\beta} \\ &\quad + \bar{u}_{\mathrm{R}\alpha}\gamma^\mu D_{\mathrm{u}}^{\alpha\beta} u_{\mathrm{R}\beta} - \bar{d}_{\mathrm{R}\alpha}\gamma^\mu D_{\mathrm{d}}^{\alpha\beta} d_{\mathrm{R}\beta} - 2J_{\mathrm{EM}}^\mu \sin^2\theta_{\mathrm{w}}] \\ &\quad + eA_\mu J_{\mathrm{EM}}^\mu + \chi[\bar{u}_{\mathrm{L}\alpha} F_{\mathrm{u}}^{\alpha\beta} u_{\mathrm{R}\beta} + \bar{d}_{\mathrm{L}\alpha} F_{\mathrm{d}}^{\alpha\beta} d_{\mathrm{R}\beta}] + \text{h.c.} \end{aligned} \tag{4.8}$$

where

$$A^{\alpha\beta} = \left[\begin{array}{cc:c:c} c\theta_{\mathrm{C}} & s\theta_{\mathrm{C}} & & \begin{array}{c} -\eta_{\mathrm{L}1}^{\mathrm{d}} c\theta_{\mathrm{C}} \\ -\eta_{\mathrm{L}2}^{\mathrm{d}} c\theta_{\mathrm{C}} \end{array} \\ -s\theta_{\mathrm{C}} & c\theta_{\mathrm{C}} & & \begin{array}{c} \eta_{\mathrm{L}1}^{\mathrm{d}} s\theta_{\mathrm{C}} \\ -\eta_{\mathrm{L}2}^{\mathrm{d}} s\theta_{\mathrm{C}} \end{array} \\ \hdashline & & 1 & -\eta_{\mathrm{L}3}^{\mathrm{d}} \\ \hdashline \begin{array}{c} -\eta_{\mathrm{L}1}^{\mathrm{u}} c\theta_{\mathrm{C}} \\ +\eta_{\mathrm{L}2}^{\mathrm{u}} s\theta_{\mathrm{C}} \end{array} & \begin{array}{c} -\eta_{\mathrm{L}1}^{\mathrm{u}} s\theta_{\mathrm{C}} \\ -\eta_{\mathrm{L}2}^{\mathrm{u}} c\theta_{\mathrm{C}} \end{array} & -\eta_{\mathrm{L}3}^{\mathrm{u}} & \end{array}\right]$$

$$B^{\alpha\beta} = \left[\begin{array}{cc:c:c} & & \eta_{\mathrm{R}1}^{\mathrm{u}} & \\ & & \eta_{\mathrm{R}2}^{\mathrm{u}} & \\ \hdashline \eta_{\mathrm{R}1}^{\mathrm{d}} & \eta_{\mathrm{R}2}^{\mathrm{d}} & 1 & \eta_{\mathrm{R}4}^{\mathrm{d}} \\ \hdashline & & \eta_{\mathrm{R}4}^{\mathrm{u}} & \end{array}\right], \qquad C_{\mathrm{u,d}}^{\alpha\beta} = \left[\begin{array}{cc:c:c} 1 & & & -\eta_{\mathrm{L}1}^{\mathrm{u,d}} \\ & 1 & & -\eta_{\mathrm{L}2}^{\mathrm{u,d}} \\ \hdashline & & 1 & -\eta_{\mathrm{L}3}^{\mathrm{u,d}} \\ \hdashline -\eta_{\mathrm{L}1}^{\mathrm{u,d}} & -\eta_{\mathrm{L}2}^{\mathrm{u,d}} & -\eta_{\mathrm{L}3}^{\mathrm{u,d}} & \end{array}\right]$$

$$D^{\alpha\beta}_{\mathrm{u,d}} = \left(\begin{array}{cc|c|c} & & \eta^{\mathrm{u,d}}_{\mathrm{R1}} & \\ & & \eta^{\mathrm{u,d}}_{\mathrm{R2}} & \\ \hline \eta^{\mathrm{u,d}}_{\mathrm{R1}} & \eta^{\mathrm{u,d}}_{\mathrm{R2}} & 1 & \eta^{\mathrm{u,d}}_{\mathrm{R4}} \\ \hline & & \eta^{\mathrm{u,d}}_{\mathrm{R4}} & \end{array}\right), \quad F^{\alpha\beta}_{\mathrm{u,d}} = \left(\begin{array}{cc|cc} \frac{-m_{\mathrm{u,d}}}{v} & & -y^{\mathrm{u,d}}_{13}\eta & y^{\mathrm{u,d}}_{14} \\ & \frac{-m_{\mathrm{c,s}}}{v} & -y^{\mathrm{u,d}}_{23}\eta & y^{\mathrm{u,d}}_{24} \\ \hline y^{\mathrm{u,d}}_{31} & y^{\mathrm{u,d}}_{32} & -y^{\mathrm{u,d}}_{33}\eta & y^{\mathrm{u,d}}_{34} \\ -y^{\mathrm{u,d}}_{41}\eta & -y^{\mathrm{u,d}}_{42}\eta & -y^{\mathrm{u,d}}_{43} & -y^{\mathrm{u,d}}_{44}\eta \end{array}\right), \tag{4.9}$$

In eqs. (4.9) subindices 1 and 2 are family indices, and subindices 3 and 4 indicate the heavy $\Delta I = 0$ quarks with $I = \frac{1}{2}$ and $I = 0$ respectively. $A^{\alpha\beta}$ contains the usual Cabibbo mixing in the charged current. The orders of the parameters parallel those in the lepton case.

5. Final remarks and conclusions

Fermions with SU(2)-invariant ($\Delta I_{\mathrm{weak}} = 0$) mass arise in many current theories that introduce new mass scales. Such theories include GUTS and supersymmetric models. These fermions must be heavier than ~20 GeV since they have not been detected in accelerator experiments [14].

Assuming $\Delta I = \frac{1}{2}$ breaking of $\mathrm{SU(2)}_L \times \mathrm{U(1)}_Y$ to $\mathrm{U(1)}_Q$ and that each class of normal fermions acquire mass from only one Higgs doublet we show that the mixing of $\Delta I = 0$ fermions of mass M with conventional fermions of mass order $m_{\Delta I = 1/2}$ is order $m_{\Delta I = 1/2}/M_{\Delta I = 0}$. $\Delta I = 0$ Majorana masses for neutral fermions are covered by our analysis. Mixing them suppresses the amplitude of all the weak processes that $\Delta I = 0$ fermions induce or mediate. Sect. 2 lists the non-diagonal couplings and the salient characteristics of $\Delta I = 0$ fermions. The most restrictive flavour-changing neutral-current process is $\mathrm{K} \to \mu\bar{\mu}$ which goes at the tree level. In terms of the mixing parameter δ which modulates the strength of the $\bar{\mathrm{d}}\mathrm{s}$ vertex ($\delta = M_{\Delta I = 0}\sqrt{\eta_{\mathrm{d}}\eta_{\mathrm{s}}}$) we find that $M_{\Delta I = 0}$ must be $\gtrsim 220\,\delta$. At worst δ is of order the mass of the heaviest $\Delta I = \frac{1}{2}$ fermion in the problem, allowing $\Delta I = 0$ fermions in the Tev range. Adopting the hierarchy principle ($\eta_{\mathrm{f}} \sim m_{\mathrm{f}}/M_{\Delta I = 0}$) we find that all FCNC effects are well below the measured values or limits as the case may be*.

* The analysis for the fermionic partners of the usual Higgs and gauge bosons in supersymmetric models is sensitive to the particular model. If the scalar partners of the normal fermions are light (low-energy supersymmetry breaking) one should worry about their contributions to rare processes, particularly since they can have gauge, in contrast to Yukawa, couplings. Off-diagonal FCNC processes constrain the masses of scalar fermions of different generations to be nearly degenerate [25]. Diagonal process ($g-2$) may constrain more severely the gaugino masses [26].

Since these $\Delta I = 0$ fermions may populate any part of the desert it is exciting to suggest that they may be found in the next generation of accelerators. If produced they will have a distinct signature: very small mass splitting among the members of a given multiplet and enhanced ratios of neutral-to-charged current decays. More restrictive bounds on their mass would follow from improvements on the limits of FCNC processes involving the heaviest families (τ, b, t?, ...). If $\Delta I = 0$ fermions are very massive it will be difficult to establish their existence. Grand unified models with these fermions can reproduce the predictions of the minimal models.

The strength and pattern of the mixing angles dictated by our theorems is responsible for the effective decoupling of heavy $\Delta I = 0$ fermions [4]. If present at low energies $\Delta I = 0$ fermions could induce *CP* violation in the K system. Thus they may connect *CP* violation at the unification scale to that at low energy, as originally suggested by Sanda [27]. We discuss a specific model elsewhere [28].

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Appendix A

In this appendix we prove the theorem stated in sect. 2 on the mixing angles between $\Delta I = 0$ mass and $\Delta I = \frac{1}{2}$ mass fermions for the fermion–fermion–gauge vertices. These mixing angles are determined by diagonalizing a general mass matrix $\mathcal{M}$ for each set of particles with the same conserved C-quantum numbers. For a set of n Dirac particles $\mathcal{M}$ is an $n \times n$ matrix where the rows and columns correspond to LH and RH parts respectively. An arbitrary matrix $\mathcal{M}$ can always be diagonalized by two unitary matrices O_{L} and O_{R},

$$D = O_{\mathrm{L}} \mathcal{M} O_{\mathrm{R}}^{+} , \tag{A.1}$$

where D is diagonal and positive.

We will take $\mathcal{M}$ to be real and O_{L} and O_{R} orthogonal, assuming that no *CP* violation is present. In the current eigenstates, the general $\mathcal{M}$ is given by (A.2), where b_1 is the number of LH doublets, b_2 the number of LH singlets, a_1 the number of RH singlets, a_2 the number of RH doublets, a_3 the number of LH and RH triplets and so on. (A.2) has three kinds of entries; those which correspond to $\Delta I = 0$ and are order M, those which correspond to $\Delta I = \frac{1}{2}$ and are order $m \ll M$, and zero entries. (We are assuming that the $\mathrm{SU(2)_L}$ breaking is mainly $\Delta I = \frac{1}{2}$, as is experimentally known.) We will diagonalize $\mathcal{M}$ to order $\eta = m/M$.

$$
\begin{array}{c|ccccc|}
 & \ddots & & & & \\
\vdots & M & m & & & \\
a_3 & m & M & m & \sim 0 & \\
b_2 & & \sim 0 & m & M & \\
b_1 & & m & M & m & \\
\hline
 & \cdots & a_3 & a_2 & a_1 &
\end{array}
,\qquad
\begin{aligned}
n &= a_1 + a_2 + a_3 \\
&= b_1 + b_2 + a_3 \\
&\quad (a_1 > b_2)
\end{aligned}
\tag{A.2}
$$

$\mathcal{M}$ can be trivially diagonalized to first order. The matrix obtained from $\mathcal{M}$ by setting the $\Delta I = \frac{1}{2}$ entries to zero is diagonalized by orthogonal matrices which commute with the isospin generators since $SU(2)_L$ is unbroken; hence the weak current lagrangian is preserved in this new basis. $\mathcal{M}$ has the form

$$
\mathcal{M} = \begin{array}{|cccccc|c}
\ddots & & & & & & \\
 & D & m & & & & \vdots \\
 & m & D & 0 & m & 0 & a_3 \\
 & & 0 & D & m & 0 & b_2 \\
 & & m & m & D & m & a_2 \\
 & & m & m & 0 & m & b_1 - a_2 \\
\hline
 & \cdots & a_3 & b_2 & a_2 & a_1 - b_2 &
\end{array}
\,. \tag{A.3}
$$

To order zero in $\eta \sim m_{\Delta I = 1/2}/M_{\Delta I = 0}$, $b_1 - a_2$ and $a_1 - b_2$ will be the light LH doublets and RH singlets respectively (these correspond to the $\Delta I = \frac{1}{2}$ mass fermions), and $a_2, b_2, a_3, \ldots$ will be the heavy $\Delta I = 0$ doublets, singlets, triplets, ... D stands for diagonal matrices with large M eigenvalues.

The matrices O_L and O_R diagonalizing (A.3) and giving the mixing angles are obtained by solving

$$
D^2 = O_L \mathcal{M}\mathcal{M}^T O_L^T = O_R \mathcal{M}^T \mathcal{M} O_R^T \,. \tag{A.4}
$$

We discuss O_L; the argument for O_R is analogous. We address later the possibility of degeneracy in the large eigenvalues and the presence of zero eigenvalues to first order (corresponding to the light fermions). Expanding in η,

$$
\mathcal{M}\mathcal{M}^{\mathrm{T}} = \begin{array}{c}
\left[\begin{array}{c|ccccc}
\ddots & & & 0 & & \\
\hline
 & D^2+m^2 & Dm & 0 & m^2 & m^2 \\
 & Dm & D^2+m^2 & m^2 & Dm & Dm \\
0 & 0 & m^2 & D^2+m^2 & Dm & Dm \\
 & m^2 & Dm & Dm & D^2+m^2 & m^2 \\
 & m^2 & Dm & Dm & m^2 & m^2
\end{array}\right]
\begin{array}{l}
\\ \\ \\ a_3 \\ b_2 \\ a_2 \\ b_1-a_2
\end{array} \\
\begin{array}{ccccc}
\cdots & a_3 & b_2 & a_2 & b_1-a_2
\end{array}
\end{array}
\cdot \equiv A = A_0 + \eta A_1 + \eta^2 A_2, \tag{A.5}
$$

where

$$
A_0 = \left[\begin{array}{c|ccccc}
 & & & & & \\
\hline
 & D^2 & & & & \\
 & & D^2 & & & \\
 & & & D^2 & & \\
 & & & & D^2 & \\
 & & & & & 0
\end{array}\right], \quad
A_1 = \left[\begin{array}{c|ccccc}
 & & & & & \\
\hline
 & & M^2 & & & \\
 & M^2 & & & M^2 & M^2 \\
 & & & & M^2 & M^2 \\
 & & M^2 & M^2 & & \\
 & & M^2 & M^2 & &
\end{array}\right],
$$

$$
A_2 = \left[\begin{array}{c|ccccc}
 & & & & & \\
\hline
 & M^2 & & & M^2 & M^2 \\
 & & M^2 & M^2 & & \\
 & & M^2 & M^2 & & \\
 & M^2 & & & M^2 & M^2 \\
 & M^2 & & & M^2 & M^2
\end{array}\right], \tag{A.6}
$$

where we indicate only the orders of the entries.

$$D^2 = B_0 + \eta B_1 + \eta^2 B_2 + \cdots, \tag{A.7}$$

$$O_L = C_0 + \eta C_1 + \eta^2 C_2 + \cdots. \tag{A.8}$$

In this basis $C_0 = \mathbb{1}$ by definition ($B_0 = A_0$) since we have done the zeroth order diagonalization. From $D^2 O_L = O_L \mathcal{M}\mathcal{M}^T$ and $O_L^T O_L = \mathbb{1}$:

$$A_0 C_1 + B_1 = A_1 + C_1 A_0 , \tag{A.9}$$

$$C_1 + C_1^T = 0 . \tag{A.10}$$

In components

$$(C_1)_{ij} = \frac{(A_1)_{ij}}{(A_0)_{ii} - (A_0)_{jj}}, \qquad i \neq j , \tag{A.11}$$

$$(B_1)_{ii} = (A_1)_{ii} = (C_1)_{ii} = 0 . \tag{A.12}$$

Now C_1 gives the order-η mixing. From (A.11) C_1 is proportional to A_1, whose only non-zero entries are $\Delta I = \frac{1}{2}$ (A.6). Thus $\Delta I = \frac{1}{2}$ mixing is of order η. $\Delta I \neq \frac{1}{2}$ mixing is at most order η^2. We see also that the large eigenvalues get corrections of order ηm and not m (A.12).

When two large eigenvalues are degenerate (A.11) is divergent. We have analyzed this case more carefully with the result given in sect. 2. The physical consequences of the theorem are unchanged.

We next prove that the mixing among light fermions induced by the new $\Delta I = 0$ mass fermions is order η^2. In (A.3) one can make a rotation in the light LH and RH fields to diagonalize the $(b_1 - a_2) \times (b_1 - a_2)$ box and then make the perturbation expansion as before. This rotation gives, for instance, the usual Cabibbo mixing in the charged sector. Alternatively one can first make the order η rotations $(\mathbb{1} + \eta C_1, \ldots)$ and at the end rotate the LH and RH fields of the light sector $(b_1 - a_2) \times (b_1 - a_2)$. It is easy to convince oneself that both rotations in the light sector are equal because of the block structure of the order η rotations. This means that an initial diagonalization of the light sector is not undone by the order η rotations which mix $\Delta I = 0$ and $\Delta I = \frac{1}{2}$ fermions. This is equivalent to corrections in the light sector being of order η^2. It implies also that the small eigenvalues get corrections only of order $\eta^2 m$.

Our analysis includes also $\Delta I = 0$ Majorana masses for neutral fermions.

The m entries in the light box $(b_1 - a_2) \times (b_1 - a_2)$ of (A.3), which give the physical light masses to order η^2, are combinations of different m entries (A.2). These combinations depend on the large-M entries. If the small mass eigenvalues (in particular their hierarchy $m_e \ll m_\mu \ll m_\tau, \ldots$) are not to be the result of a special choice of large-M entries, the hierarchy principle, then all the elements in a row (column) of the $b_1 - a_2$ lines (A.3) must be of the same order. Thus the rows (columns) are in the same hierarchy. This immediately implies that each light fermion mixes in proportion to its mass, $\eta_f \sim m_f / M$.

As a simple example consider the case of two LH SU(2) doublets (call them e^0_L and μ^0_L) and a LH singlet (ℓ^0_L) with all their RH partners (e^0_R, μ^0_R and ℓ^0_R) being singlets. The mass matrix has the form $\bar{\psi}_L M \psi_R$ where

$$\psi_{L,R} = \begin{pmatrix} e^0 \\ \mu^0 \\ \ell^0 \end{pmatrix}_{L,R} \tag{A.13}$$

$$M \simeq \begin{pmatrix} m & m & m \\ m' & m' & m' \\ M & M & M \end{pmatrix} = \begin{pmatrix} m_i \\ m'_i \\ M_i \end{pmatrix}. \tag{A.14}$$

The first two steps in the diagonalization process described above cast M in the form

$$\begin{pmatrix} m_e & 0 & \delta_e \\ 0 & m_\mu & \delta_\mu \\ 0 & 0 & M_\ell \end{pmatrix} = \begin{pmatrix} c\theta & s\theta & \\ -s\theta & c\theta & \\ & & 1 \end{pmatrix} \begin{pmatrix} m_i \\ m'_i \\ M_i \end{pmatrix} \begin{pmatrix} \boldsymbol{u} = & \boldsymbol{u}' = & \boldsymbol{e} = \\ c\phi \boldsymbol{e}_1 & -s\phi \boldsymbol{e}_1 & M_i/M \\ +s\phi \boldsymbol{e}_2 & +c\phi \boldsymbol{e}_2 & \end{pmatrix}, \tag{A.15}$$

where θ and ϕ are the rotations necessary to cast M in the form above and $\boldsymbol{e}_1, \boldsymbol{e}_2$ are two arbitrary orthonormal vectors orthogonal to $\boldsymbol{e}$.

The initial mass matrix in the new basis is

$$\begin{pmatrix} m_i \\ m'_i \\ M_i \end{pmatrix} \equiv \begin{pmatrix} m'_e \boldsymbol{u} + m''_e \boldsymbol{u}' + \delta'_e \boldsymbol{e} \\ m'_\mu \boldsymbol{u} + m''_\mu \boldsymbol{u}' + \delta'_\mu \boldsymbol{e} \\ M_\ell \boldsymbol{e} \end{pmatrix}, \tag{A.16}$$

(this defines m'_e, m''_e, etc.).

Now

$$m_e = \frac{m'_e m''_\mu - m'_\mu m''_e}{\sqrt{m''^2_e + m''^2_\mu}}, \qquad \delta_e = \frac{\delta'_e m''_\mu - \delta'_\mu m''_e}{\sqrt{m''^2_e + m''^2_\mu}},$$

$$m_\mu = \sqrt{m''^2_e + m''^2_\mu}, \qquad \delta_\mu = \frac{\delta'_e m''_e + \delta'_\mu m''_\mu}{\sqrt{m''^2_e + m''^2_\mu}}. \tag{A.17}$$

Suppose now we keep $\boldsymbol{m}$ and $\boldsymbol{m}'$ fixed but realign $M\boldsymbol{e}$ to $M\boldsymbol{u}$. Then

$$m_e \leftrightarrow \delta_e, \qquad \delta_\mu \to \frac{m'_e m''_e + m'_\mu m''_\mu}{\sqrt{m''^2_e + m''^2_\mu}}. \tag{A.18}$$

Hence we require $\delta_e \sim m_e$ if m_e is to be stable under the realignment of $M\boldsymbol{e}$ to $M\boldsymbol{u}$. In a similar way one can prove $\delta_\mu \sim m_\mu$. In fact the assumption that the normal masses do not depend crucially on the large-mass parameters implies that $|\boldsymbol{m}| \sim m_e$ and $|\boldsymbol{m}'| \sim m_\mu$ up to an arbitrary rotation.

Appendix B

Here we prove the theorem on the mixing angles of the fermion–fermion–Higgs vertices stated in sect. 2. We draw on the results of appendix A. For the set of n fermions of appendix A and the neutral physical Higgs χ the Yukawa matrix $\mathcal{Y}$ is, in the current eigenstates

$$\mathcal{Y}=\frac{1}{v}\begin{array}{c|cccc|c} \ddots & & & 0 & & \\ 0 & m & & & & \\ m & 0 & 0 & m & 0 & a_3 \\ & 0 & 0 & m & 0 & b_2 \\ & m & m & 0 & m & a_2 \\ 0 & m & m & 0 & m & b_1-a_2 \\ \hline \cdots & a_3 & b_2 & a_2 & b_1-a_2 & \end{array}\,, \tag{B.1}$$

($\mathcal{L}_{\mathrm{FFH}}=-\sum_{i,j}(m_{ij}/v)\bar{F}_{\mathrm{L}i}F_{\mathrm{R}j}\chi+\mathrm{h.c.}$, where we assume that only one Higgs doublet gives $\Delta I=\frac{1}{2}$ mass to these fermions.) The matrix $\mathcal{Y}$ is proportional to (A.3) if we set the $\Delta I=0$ entries M of (A.3) to zero. In the mass eigenstates $\mathcal{Y}$ will be

$$Y=O_{\mathrm{L}}\mathcal{Y}O_{\mathrm{R}}^{+}\,, \tag{B.2}$$

where the O_{L} and O_{R} are those of appendix A. Defining

$$\mathcal{M}_0=\mathcal{M}-v\mathcal{Y}\,, \tag{B.3}$$

the Yukawa matrix is

$$Y=\frac{1}{v}D-\frac{1}{v}O_{\mathrm{L}}\mathcal{M}_0O_{\mathrm{R}}^{+}\,. \tag{B.4}$$

Using O_{L} and O_{R} determined from the steps of appendix A one finds the generic structure of Y to be

$$\mathcal{Y}=\frac{1}{v}\begin{array}{c|cccc|c} \ddots & & & 0 & & \vdots \\ \eta m & m & & & & \\ m & \eta m & \eta m & m & \eta m & a_3 \\ & \eta m & \eta m & m & \eta m & b_2 \\ & m & m & \eta m & m & a_2 \\ 0 & m & m & \eta m & \begin{smallmatrix} \ddots & & 0 \\ & m_2 & \\ 0 & & m_1 \end{smallmatrix} & b_1-a_2 \\ \hline \cdots & a_3 & b_2 & a_2 & b_1-a_2 & \end{array}\,, \tag{B.5}$$

This proves theorem 2.2. Note that the off-diagonal entries in the light sector are zero to order η^2. As in the gauge case, the mixing angle η will be proportional to the mass of the light fermion involved in the FFH vertex if we make the mass matrix assumption (hierarchy principle) outlined in appendix A.

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