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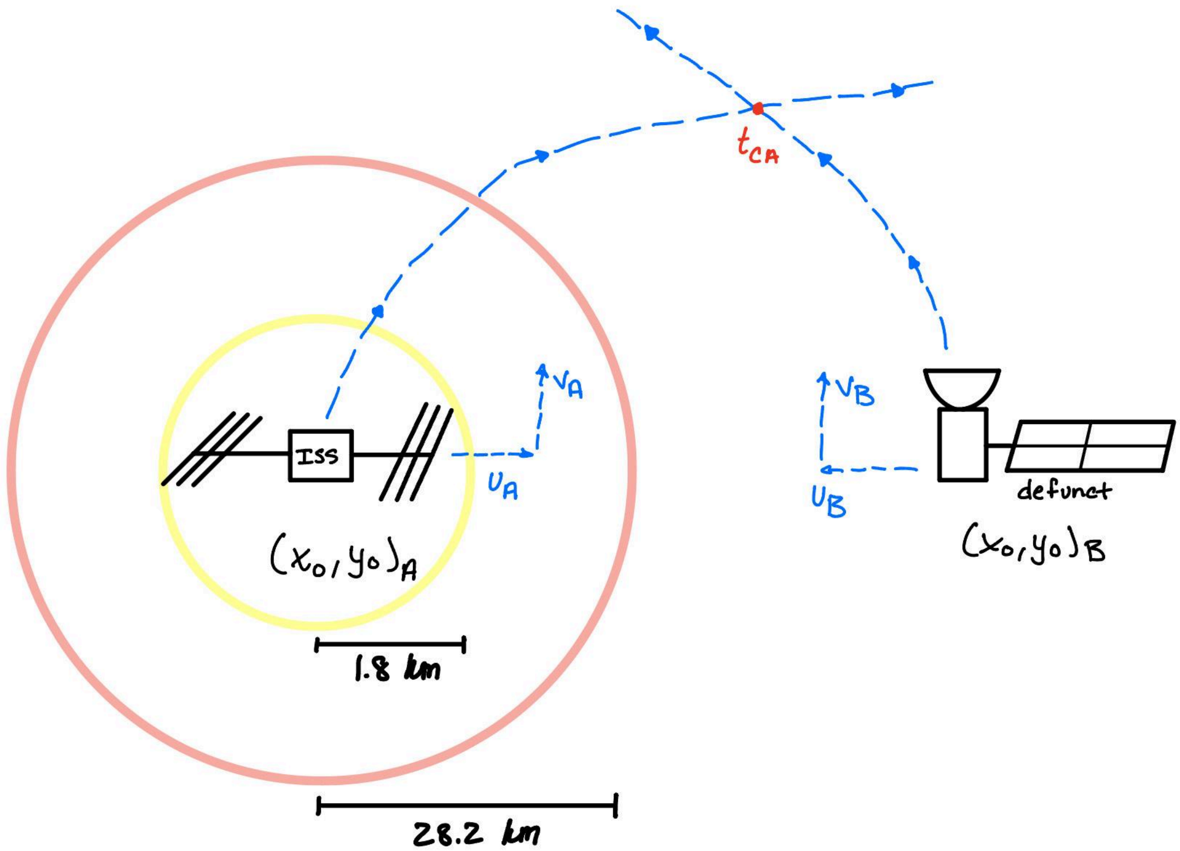
Project 1, Part 1 Deliverables

First 7 steps of Problem Solving Method

1. The purpose of this project is to analyze the positions of two spacecraft: ISS and a defunct satellite, in order to determine their trajectories and find the point at which they come closest to each other or collide. Using the data provided for both spacecraft and using specific methodologies such as error propagation and least squares linear fitting to extrapolate their positions.
2. What I know:
    a. The motion of both of the spacecraft is modeled as uniform rectilinear motion in circular orbits at an altitude of 410 km.
    b. Position Functions:
        i. $x_A(t) = x_{A,0} + u_A t$
        ii. $y_A(t) = y_{A,0} + v_A t$
        iii. $x_B(t) = x_{B,0} + u_B t$
        iv. $y_B(t) = y_{B,0} + v_B t$
    c. Distance between spacecraft:
        i. $D(t) = \sqrt{(x_B(t) - x_A(t))^2 + (y_B(t) - y_A(t))^2}$
    d. Derive the error propagation for this model, particularly for equation (3), which determines the time of closest approach t_{CA}. Account for uncertainties in initial positions and velocities.
    e. A linear fit will be applied separately to the x and y coordinates of both the spacecraft to extrapolate their trajectories and estimate the time of closest approach.
    f. Collision Warnings:
        i. Code Warning "Red": Minimum distance $D_{min} < 1.8\ km$
        ii. Code Warning "Yellow": Minimum distance $D_{min} < 28.2\ km$
3. Unknowns include:
    a. Time of closest approach (t_{CA}) - find exact time where the two spacecraft are closest together
    b. Minimum distance at time t_{CA}
    c. Velocities of both spacecraft: u_A & v_A (ISS) and u_B& v_B (defunct satellite)
    d. Errors in position, velocities, closest approach time, and minimum distance

4. Assumptions:
    a. Both spacecraft follow linear, uniform motion in 2D (x,y) space, which means that movement in the z-direction is negligible
    b. There are no external forces acting on the spacecrafts
    c. Since we are treating motion as uniform rectilinear, we assume constant velocity
    d. The positions of both spacecraft can be linearly extrapolated using the least squares fit
    e. The errors are normally distributed meaning we can treat them independently
5. Sketch



6. Equations
    a. $x_A(t) = x_{A,0} + u_A t$
    b. $y_A(t) = y_{A,0} + v_A t$
    c. $x_B(t) = x_{B,0} + u_B t$
    d. $y_B(t) = y_{B,0} + v_B t$
    e. $D(t) = \sqrt{(x_B(t) - x_A(t))^2 + (y_B(t) - y_A(t))^2}$ (2)
    f.

i. Steps to get to eq. (3)

ii. $D^2(t) = [(x_{B,0} - x_{A,0}) + (u_B - u_A)t]^2 + [(y_{B,0} - y_{A,0}) + (v_B - v_A)t]^2$

iii. $\frac{d}{dt}D^2(t) = 2[(x_{B,0} - x_{A,0}) + (u_B - u_A)t](u_B - u_A) + 2[(y_{B,0} - y_{A,0}) + (v_B - v_A)t](v_B - v_A)$

iv. $0 = [(x_{B,0} - x_{A,0}) + (u_B - u_A)t_{CA}](u_B - u_A) + [(y_{B,0} - y_{A,0}) + (v_B - v_A)t_{CA}](v_B - v_A)$

v. Solving for t_{CA}:

vi. $t_{CA} = \frac{-(x_{B,0}-x_{A,0})(u_B-u_A)-(y_{B,0}-y_{A,0})(v_B-v_A)}{(u_B-u_A)^2+(v_B-v_A)^2} = 0$

7. Alternative approaches:
   a. A couple ways that we could do this differently would be a monte carlo simulation for error estimation and using an orbital mechanics model. For the monte carlo, we could estimate the uncertainties in the time of closest approach and the minimum distance by randomly sampling from the error distributions of the initial conditions and velocities. We could also use Kepler's laws to model the spacecraft's orbit more accurately.
8. Flowchart
9. Steps for analytical derivation of error propagated for Eq. (3)
   a. $t_{CA} = \frac{-(x_{B,0}-x_{A,0})(u_B-u_A)-(y_{B,0}-y_{A,0})(v_B-v_A)}{(u_B-u_A)^2+(v_B-v_A)^2}$
   b. $N(x) =- (x_{B,0} - x_{A,0})(u_B - u_A) - (y_{B,0} - y_{A,0})(v_B - v_A)$
   c. $D(x) = (u_B - u_A)^2 + (v_B - v_A)^2$
   d. Partial Derivatives will be of the form: $\frac{\partial t_{CA}}{\partial -} = \frac{N^{'}(x)*D(x)-N(x)*D^{'}(x)}{D(x)^2}$
   e. $\frac{\partial t_{CA}}{\partial x_{A,0}} = \frac{(u_B-u_A)}{(u_B-u_A)^2+(v_B-v_A)^2}$ ⇒ Partial derivative with respect to $x_{A,0}$
   f. $\frac{\partial t_{CA}}{\partial x_{B,0}} = \frac{-(u_B-u_A)}{(u_B-u_A)^2+(v_B-v_A)^2}$ ⇒ Partial derivative with respect to $x_{B,0}$
   g. $\frac{\partial t_{CA}}{\partial y_{A,0}} = \frac{(v_B-v_A)}{(u_B-u_A)^2+(v_B-v_A)^2}$ ⇒ Partial derivative with respect to $y_{A,0}$
   h. $\frac{\partial t_{CA}}{\partial y_{B,0}} = \frac{-(v_B-v_A)}{(u_B-u_A)^2+(v_B-v_A)^2}$ ⇒ Partial derivative with respect to $y_{B,0}$
   i. $\frac{\partial t_{CA}}{\partial u_B} = \frac{-(x_{B,0}-x_{A,0})((u_B-u_A)^2+(v_B-v_A)^2)+2(u_B-u_A)(-(x_{B,0}-x_{A,0})(u_B-u_A)-(y_{B,0}-y_{A,0})(v_B-v_A))}{((u_B-u_A)^2+(v_B-v_A)^2)^2}$

      Partial derivative with respect to u_B
   j. $\frac{\partial t_{CA}}{\partial v_B} = \frac{-(y_{B,0}-y_{A,0})((u_B-u_A)^2+(v_B-v_A)^2)+2(v_B-v_A)(-(x_{B,0}-x_{A,0})(u_B-u_A)-(y_{B,0}-y_{A,0})(v_B-v_A))}{((u_B-u_A)^2+(v_B-v_A)^2)^2}$

      Partial derivative with respect to v_B

k. $\frac{\partial t_{CA}}{\partial u_A} = \frac{(x_{B,0}-x_{A,0})((u_B-u_A)^2+(v_B-v_A)^2)-2(u_B-u_A)(-(x_{B,0}-x_{A,0})(u_B-u_A)-(y_{B,0}-y_{A,0})(v_B-v_A))}{((u_B-u_A)^2+(v_B-v_A)^2)^2}$

Partial derivative with respect to u_A

l. $\frac{\partial t_{CA}}{\partial v_A} = \frac{(y_{B,0}-y_{A,0})((u_B-u_A)^2+(v_B-v_A)^2)-2(v_B-v_A)(-(x_{B,0}-x_{A,0})(u_B-u_A)-(y_{B,0}-y_{A,0})(v_B-v_A))}{((u_B-u_A)^2+(v_B-v_A)^2)^2}$

Partial derivative with respect to v_A

m. Final Derivation using general method:

$$\delta t_{CA} = \sqrt{(\frac{\partial t_{CA}}{\partial x_{A,0}}\delta x_{A,0})^2 + (\frac{\partial t_{CA}}{\partial x_{B,0}}\delta x_{B,0})^2 + (\frac{\partial t_{CA}}{\partial y_{A,0}}\delta y_{A,0})^2 + (\frac{\partial t_{CA}}{\partial y_{B,0}}\delta y_{B,0})^2 + (\frac{\partial t_{CA}}{\partial u_A}\delta u_A)^2 + (\frac{\partial t_{CA}}{\partial u_B}\delta u_B)^2 + (\frac{\partial t_{CA}}{\partial u_A}\delta v_A)^2 + (\frac{\partial t_{CA}}{\partial u_A}\delta v_B)^2}$$

9. I can determine the error for each position variable for each satellite by using:

$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}$ - using the data to first find $\bar{x}$, where n = amount of data points for each satellite and each direction (x,y)

$\delta = \frac{\sigma}{\sqrt{n}}$ - converting standard deviation to standard error

$\delta_{x_{A,0}} = x_{A,0} - \bar{x}, \delta_{y_{A,0}} = y_{A,0} - \bar{y},$ doing the same for the defunct satellite

We can then calculate the uncertainty of these velocities using:

$\delta_{u_A} = \frac{\delta_{x_A}}{t}$ and $\sigma_{u_B} = \frac{\delta_{x_B}}{t}$ - for x-direction, $t = t_f - t_i$

$\sigma_{v_A} = \frac{\delta_{y_A}}{t}$ and $\sigma_{v_B} = \frac{\delta_{y_B}}{t}$ - for y-direction, $t = t_f - t_i$