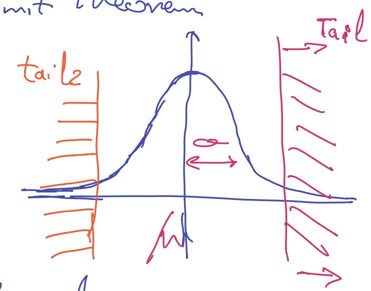
The Sum of indep. Random Variables

Central Limit Theorem



Chernoff Bound

$$P(X \ge (1+\delta)\mu) \le \left(\frac{e^{\delta}}{(1+\delta)^{\delta}}\right)^{\mu} \le e^{-\delta^2\mu/3} \quad (1)$$

$\delta > 0$

$$P(X \le (1-\delta)\mu) \le e^{-\delta^2\mu/2} \quad (2)$$

$\delta > 0$

e.g.: given n indep. flip of coins w/t $P(\text{head}) = 0.5$

what is the prob. that you will see at least 80% head?

Using Markov Ineq.

$$P(X \ge t) \le \mu/t$$

$t = 0.8n$, $\mu = 0.5n$

$$\Rightarrow \boxed{P(X \ge 0.8n) \le 5/8} \checkmark$$

Using Chernoff Bound

Using Eq.(1)

$P(X \ge (1+\delta)\mu)$

$\mu = 0.5n$

$(1+\delta).5n = .8n$

$\Rightarrow \delta = 8/5 - 1 = 0.6$

$$P(X \ge (1+\delta)\mu) \le e^{-\delta^2\mu/3}$$

$$= e^{-\frac{(0.6)^2 \times 0.5n}{3}}$$

$$= \boxed{\frac{1}{e^{0.06n}}} \checkmark$$

e.g. $n = 1000$

$$\frac{1}{e^{.06n}} \le \frac{1}{e^{60}}$$

Team A in NBA wins every game with Prob $\boxed{0.75}$

what is the prob that team A loses in more than 50% of the games

$P(X \le (1-\delta)\mu)$

$\mu = 0.75n$

$$(1-\delta)\mu = (1-\delta)\underset{3/4}{\cancel{.75}}\,n = .\underset{1/2}{\cancel{5}}\,n \Rightarrow \delta = 1 - 2/3 = \boxed{1/3}$$ ✓

Using Eq. (2)

$$P(X \le (1-\delta)\mu) \le e^{-\delta^2\mu/2}$$

$$= e^{-\frac{.5n}{9\times 2}} = \frac{1}{e^{\frac{n}{36}}}$$