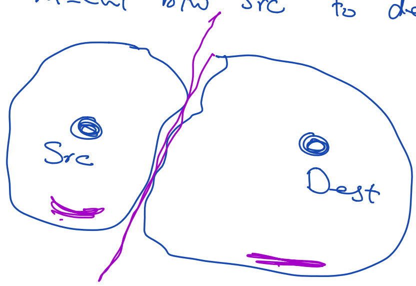
* We measure the quality of a monte-Carlo Alg. with Prob. of generating the Correct output

```
while check
{   output = MCAlg(...)
    check = verify(output)
}
```

---

A Monte Carlo Randomized Alg. for Min-Cut Problem

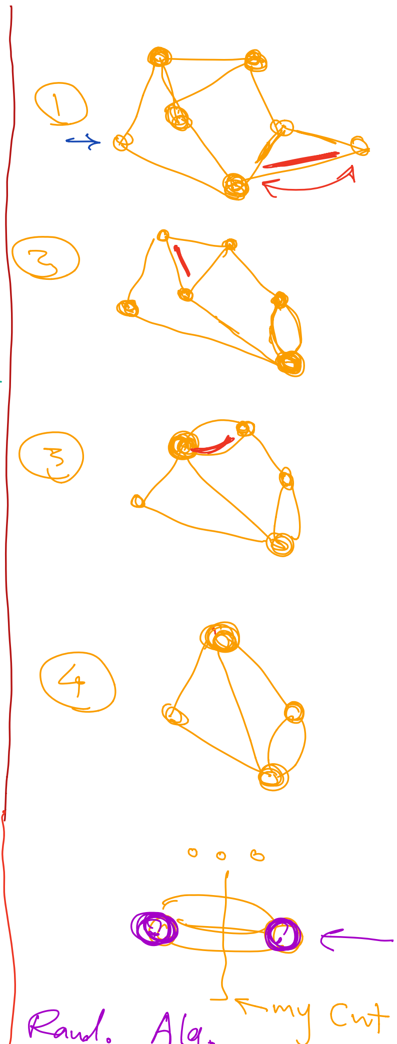
Min-Cut b/w Src to des




Min-Cut: we want to find the min. no. of edges that their removal disconnects the graph

Deterministic Alg.

```
minCut = ∞
for i = 1 to n-1
    for j = i+1 to n
        Cut = minCut(i, j)
        if (Cut < minCut)
        {
            minCut = Cut
        }
```




Rand. Alg.

while |v| > 2

  (u, v) = Select an edge uniformly at random

  update the graph by replacing (u, v) w/t a Supernode

the edges b/w the Survived pair of nodes is the Cut Set

* what is the Success prob.
what is the Prob. that this Algo. finds a Cut of Size at most $\boxed{k}$

* We assume min Cut Size $= k$

Consider an optimal Cut

* The min # edges

$nk/2$ : The degree of each node is $k$

what is the Prob. of selecting one edge from the Cut Set

$P(\mathcal{E}_i)$ : the prob. that at iteration $i$, none of the min-cut edges is Selected

$$P(\mathcal{E}_1) = 1 - \frac{k}{nk/2} = 1 - 2/n$$

* after every iteration the # nodes gets reduced by 1

| # nodes | iteration |
|---|---|
| $n$ | 1 |
| $n-1$ | 2 |
| | ... |
| $n-i+1$ | $i$ |

* The Size of opt. Cut does not reduce during the iterations

what is the min # edges at Step $i$

$$(n-i+1)k/2$$

$$P(\mathcal{E}_i \mid \mathcal{E}_1 \wedge \dots \wedge \mathcal{E}_{i-1}) = 1 - \frac{k}{(n-i+1)k/2}$$

$$= 1 - \frac{2}{n-i+1}$$

$$P(\text{Success}) = P(\mathcal{E}_1)P(\mathcal{E}_2 \mid \mathcal{E}_1) \cdots P(\mathcal{E}_{n-2} \mid \mathcal{E}_1 \cdots \mathcal{E}_{n-3})$$

$$= \prod_{i=1}^{n-2} P\Big(\mathcal{E}_i \,\Big|\, \bigwedge_{j=1}^{i-1} \mathcal{E}_j\Big)$$

$$= \prod_{i=1}^{n-2} \left(1 - \frac{2}{n-i+1}\right)$$

$$= \frac{2}{n(n-1)} > \boxed{2/n^2}$$

Run the algorithm $n^2/2$

$$P(\text{failure}) = \left(1 - 2/n^2\right)^{n^2/2}$$

$$\rightarrow \boxed{< 1/e} \checkmark$$