Approximation Algorithms (Intro)

Given a Problem X, let the optimal value be O
an Algorithm Appx is called approximation for X if,
assuming that the value Appx finds is A,

$\max\left(\frac{A}{O}, \frac{O}{A}\right)$ is bounded

minimization

Maximization

Approximation Ratio

approx-ratio ≥ 1

optimal Alg $= 1$

1) There exist an Approx. Alg. w/t a fixed approx.-ratio

inP

e.g. Vertex-Cover

2) The problems for which one can prove there is no P-time approx. Alg., unless P = NP.

e.g. MIS

3) The tunable approx-ratio Algorithms

3-1) Polynomial-Time Approximation Schemes (PTAS)

Given a value $\varepsilon > 0$, the algorithms Satisfies
$(1+\varepsilon)$-approx. ratio and runs in p-time to the input

Can be exponential to $1/\varepsilon$

eg.: $O(n^{2/\varepsilon})$ ← ✓ PTAS ← ✗ FPTAS

3-2) Fully PTAS (FPTAS)

Given a value $\varepsilon > 0$, the alg. satisfies
$(1+\varepsilon)$ approx-ratio, and runs in P-time to the input
and $1/\varepsilon$.

$O(n^3 1/\varepsilon)$ ← FPTAS

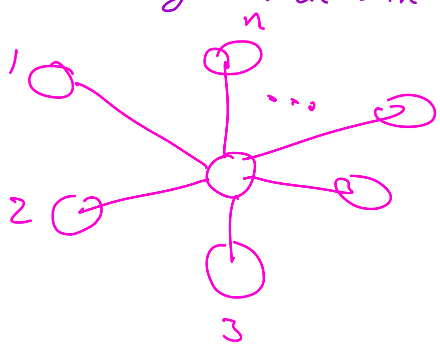
A 2-approximate alg. for V-C

First attempt:

observation:

for every edge, we need to select at least one vertex

X for all edges pick both end-nodes



opt = 1

A = n

X

The algorithm

$E' = E$; $VC = \{\}$

while E' is not ϕ:

Select an arbitrary edge $(u,v) \in E'$

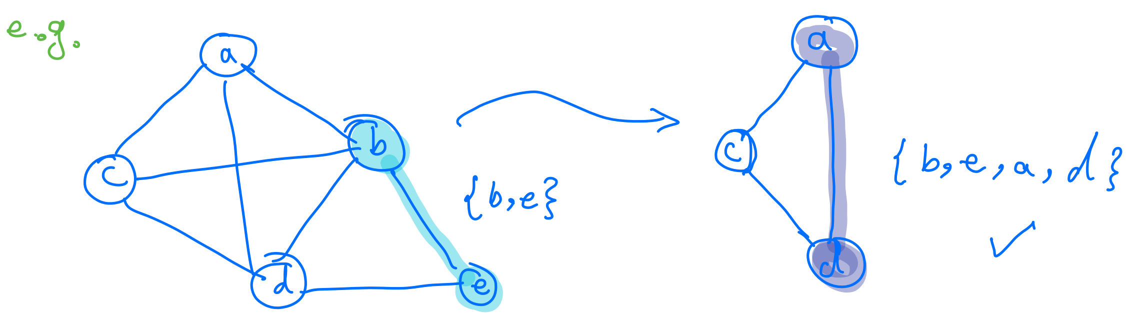
add $\{u, v\}$ to the VC

remove all edges incident to u or v from E'

return VC

$O(|E|)$

e.g.



Observation: Selected edges are disjoint

$\Rightarrow$ the optimal alg. must select at least one of the end-nodes of selected edges

$$\Rightarrow O \geq |\text{Selected Edges}|$$

,

$$A = 2\,|\text{Selected Edges}|$$

$$\Rightarrow 2O \geq 2\,|\text{Selected Edges}| = A$$

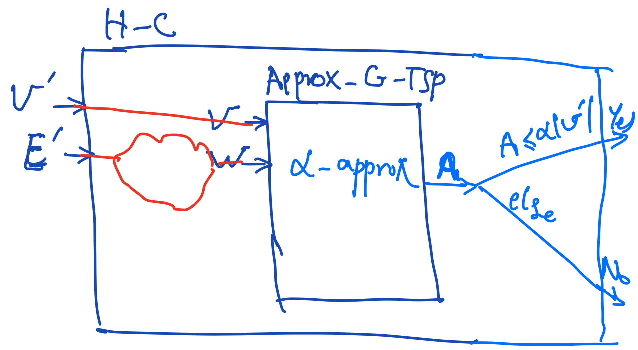
$$\Rightarrow A/O \leq 2$$

TSP:
- general TSP: edge weights can be any positive value
  - $\rightarrow$ No Approximation Alg., unless $P = NP$
- Metric TSP: edge weights follow Triangular inequality
  - $\rightarrow$ 2-approx
  - 1.5-approx
- Euclidian TSP: " " " Euclidian dist
  - $\rightarrow$ PTAS

General TSP (G-TSP)

* There is no Approximation Alg. for G-TSP, Unless $P = NP$

H-C $\le_p$ Approx-G-TSP



$\forall (u,v) \in E'$

add $w(u,v) = 1$ to W

$\forall (u,v) \notin E'$

add $w(u,v) = \alpha |V'|$

Is Approx-VC an approx. Alg. for MIS ? NO

	V-C	MIS
optimal	k	$n-k$
Approx	$k' = 2k$	$n-k'$

approx-ratio for MIS : $\dfrac{n-k}{n-k'}$

Unbounded

$k = n/2$

$k' = n$

$n - k' = 0$

$$\frac{n-k}{n-k'} = \infty$$