

Max-flow($G(U, E), C, s, t$)

$O(m)$ ← for each $e \in E : f(e) = 0$

$O(nm)$ ← $G_f = \text{Residual}(G)$

while(\exists Path $P: s \rightsquigarrow t$ in G_f) $O(m)$

$f = \text{Augment}(G, P, f)$ $O(n)$

update(G_f, P) $O(n)$

return f

$O(Xm)$
iterations

Augment(G, P, f)

$b = \text{bottleneck}(P)$

for $e \in P$

if $e \in E$ // forward flow

$f(e) = f(e) + b$

else // backward flow

$f(e^R) = f(e^R) - b$

return f

update(G_f, P, G)

for each $e \in P$

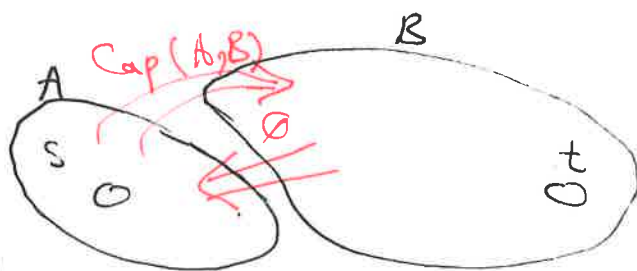
if $e \in E$

$w(e) = c(e) - f(e)$

else $w(e^R) = f(e)$

$w(e) = f(e)$

$w(e^R) = c(e) - f(e)$



$$v(f) = \sum_{\text{out of } A} f(e) - \sum_{\text{in } A} f(e)$$

$$= \sum_{\text{leaving } A} c(e)$$

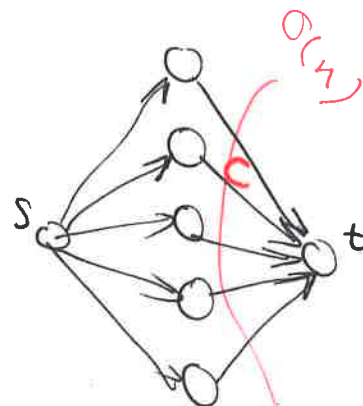
$$= \text{Cap}(A, B)$$

$O(Xm)$
↳ ?

$$X \leq v(f^*)$$

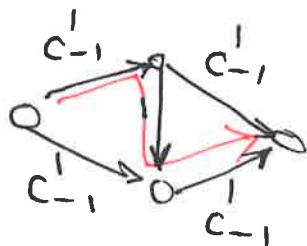
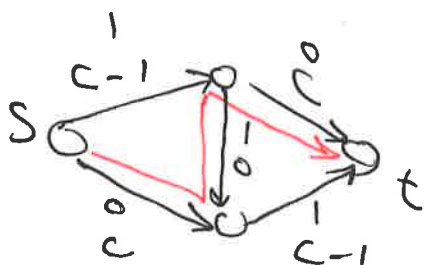
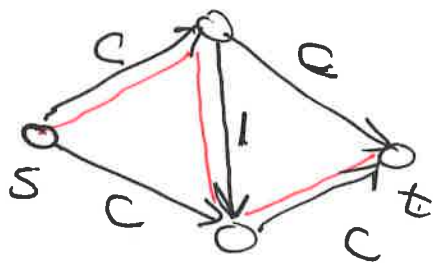
$$O(\underbrace{v(f^*)}_m)$$

Let C max Capacity



$$v(f^*) \leq nC$$

$$\Rightarrow O(nmC)$$



...

2C

↑

Example of Bad Path augmentation