

Name:

Score:

UIN:

1. Problem X is NP-complete if any problem  $Y \in \text{NP}$  can be reduced to it. (5p no p/c)
  - (a) True
  - (b) False
  - (c) None of the above
2.  $P = \text{NP}$ . (5p no p/c)
  - (a) True
  - (b) False
  - (c) None of the above
3. Choose three of the following problems and prove those are in NP-complete. (90 pts)
  - a. [**Undirected Feedback Set Problem**] Given an undirected graph  $G = (V, E)$ , the goal is to find the smallest subset of vertices such that the graph  $G - X$  has no cycles.
  - b. [**Longest-simple-cycle Problem**] is the problem of determining a simple cycle (no repeated vertices) of max. length in an unweighted graph.
  - c. [**Hitting-set Problem**] Consider a set of elements  $A = \{a_1, \dots, a_n\}$ , and a collection of subsets  $B = \{B_1, \dots, B_m\}$  where each  $B_i$  is a subset of  $A$ . The goal is to find the minimum number of elements that hit all the sets in  $B$ . (an element  $a$  hits a set  $B_i$ , if  $a \in B_i$ )
  - d. [**Fair Max. Cover Problem**] Consider a set of elements  $U = \{u_1, \dots, u_n\}$ , and a collection of subsets  $S = \{S_1, \dots, S_m\}$  where each  $S_i$  is a subset of  $U$ . Every element  $u_i$  is associated with a binary color  $c_i$  (0:red, 1:blue). The objective is to select  $k$  sets such that (i) the number of red elements that are covered by the selected sets is equal to the number of blue elements that are covered and (ii) the total number of covered elements is maximized.









Scratch paper 1 (do not detach)

Scratch paper 2 (do not detach)