VC

$V = \{v_1 \dots v_m\}$

$e = \{e_1 \dots e_n\}$

Set_Cover

$S = \{v_1 \dots v_m\}$

$U = \{e_1 \dots e_n\}$

2-approx Alg.

$A = \{\}$

- while an edge left
- e = Select one edge
- add both endnodes to A
- remove all edges hitted by the two ~~edd~~ nodes

$A = \{\}$

while an element left uncovered

- e = Select an element
- add all Sets containing $\underline{e}$ to A
- mark all elements in selected Sets as covered

X-approximation

* where X is the max number of Sets that contain an element e_i
* Can be as bad as selecting ALL Sets

GREEDY approx. Alg. for Set Cover

$C = \{\}$

$A = \{\}$

while $C \neq U$

* $S_i = \underset{\forall S_j \in (S - A)}{\operatorname{argmax}} (S_j - C)$
* for $u_j \in S_i$
  - add u_j to C
  - $e_j = \dfrac{1}{|S_i - C|}$
* Add S_i to A

$\boxed{\text{Amortized Cost Analysis for the approx. ratio}}$

$A = |Approx| = \sum_{i=1}^{n} e_i$

Consider the order based on which the elements are covered by the Alg.

$e_1, e_2, \dots, e_n$

u_n is the last covered element

$e_1 \cdots e_{k-1} \mid \boxed{e_k} \cdots e_n$

$\underbrace{e_1 \cdots e_{k-1}}_{k-1} \quad \underbrace{e_k \cdots e_n}_{n-k+1}$

Suppose Opt_k is to cover the $n-k+1$ elements

$\frac{Opt}{n-k+1}$ is the optimal cost for covering each element.

S_i is the selection by Greedy

$|S_i|$ is maximized

$|S_i - C|$

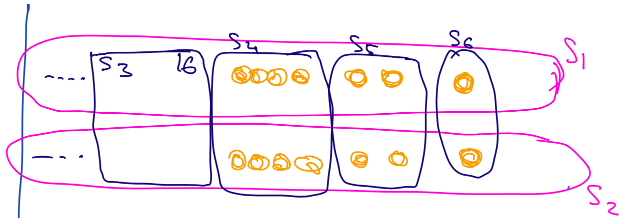
$$e_k \leq \frac{opt_k}{|U-C'|} \leq \frac{Opt_k}{n-k+1}$$

$$A = \sum_{k=1}^{n} e_k = \frac{opt}{n} + \frac{opt}{n-1} + \dots + \frac{opt}{1}$$

$$= Opt \sum_{i=1}^{n} 1/i = H_n \, Opt$$

$$* \sum_{i=1}^{n} 1/i \lessapprox \int_1^n \frac{1}{t}\, dt = \ln t \Big|_1^n = \ln(n)$$

$$A = O(\log n)\, Opt \Rightarrow \boxed{\frac{A}{Opt} \leq \log n}$$



Greedy: $S_3, S_4, S_5, \dots, S_6$

Opt: S_1, S_2

Greedy

$k = \log n$

$2^k \quad 2^{k-1} \quad \cdots \quad 2^1$

$k = (\log n)$