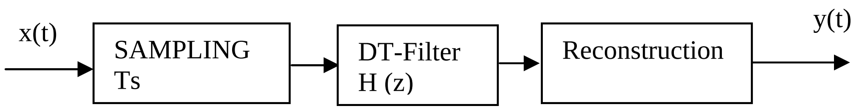
EEE 304 **Test 3** **NAME:__________________**

30min, Closed-book, closed-notes, calculators and transform tables allowed

Problem 1:
Suppose that a continuous-time signal x(t) is bandlimited to 10Hz and it is processed by DT system with ideal sampling and reconstruction. Suppose $H(z) = \frac{0.2}{z-0.9}$ and Ts = 0.01s. Find the output y(t) when x(t) = sin(t).



The equivalent c-t transfer function for a bandlimited input below 50Hz is $H_{ct}(jw) = H\left(e^{jwT_s}\right)$. Hence, for the given values $H_{ct}(j1) = H\left(e^{j0.01}\right) = \frac{0.2}{z-0.9} = \frac{0.2}{e^{j0.01}-0.9} = \frac{0.2}{cos0.01-0.9+jsin0.01} = \frac{0.2}{0.0999+j0.01} = 1.99e^{j0.1}$. So, $y(t) = |H_{ct}(jw)|\sin\left(t + \arg\left(H_{ct}(jw)\right)\right) = 1.99\sin\left(t - 5.7^{o}\right)$

Problem 2:
Find the largest sampling interval T_s to allow perfect reconstruction of the signals

Here we use the shortcuts to find upper bound estimates for w_{max}

1. $\frac{\sin^2(t)}{t} = \frac{\sin(t)}{t}\sin(t) \Rightarrow w_{\max} \le 1+1 \Rightarrow \max T_s = \frac{\pi}{2}$
2. $\frac{\sin(t)}{t^2} = \frac{\sin(t)}{t}\frac{1}{t} \Rightarrow w_{\max} \le 1+\infty \Rightarrow \max T_s = 0$ (1/t is not bandlimited)
3. $\sin(t)\cos(2t) \Rightarrow w_{\max} \le 1+2 \Rightarrow \max T_s = \frac{\pi}{3}$