EEE 582 HW # 4 SOLUTIONS

5.4

1) Impulse response $g(t) = \mathcal{L}^{-1}\{\hat{g}(s)\}$
$$= e^{-(t-2)} \quad (t \geq 2)$$

$$\int |g(t)|\,dt = 1 < \infty \Rightarrow \text{BIBO stable}$$

2) $\hat{g}(s)$ is a cascade interconnection of two stable systems e^{-2s} (shift/delay) and $\frac{1}{s+1}$

$\therefore$ stable

5.7

$$\hat{g}(s) = [-2, 3]\begin{bmatrix} s+1 & -10 \\ 0 & s-1 \end{bmatrix}^{-1}\begin{bmatrix} -2 \\ 0 \end{bmatrix} = \frac{4}{s+1}$$

$\Rightarrow$ BIBO stable.

(Notice that the unstable mode $\frac{1}{s-1}$ is canceled)

5.10

$\sigma(A) = \{-1, 0, 0\} \Rightarrow$ not Asymptotically Stable.

For marginal stability, we should have the max Jordan block for $\{0\}$-eigenvalue to have size 1.

$$\text{Null}(0I - A) = \text{Null}\begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \dim 2$$

$\Rightarrow$ there are 2 Jordan blocks for 0 so their size is 1 $\Rightarrow$ the homogeneous eqn is marginally stable

5.11

Just as in 5.10 $\dim \text{Null}(0I - A) = \dim \text{Null}\begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix} = 1$

$\Rightarrow$ there is 1 Jordan block for 0 $\Rightarrow$ the homogeneous eqn. is not stable.

5.12 This is the discrete time analog of 5.10.

There are three eigenvalues, 0.9, 1, 1 ; the first is A.S.

The two at 1 are marginally stable iff they correspond to different Jordan blocks. Since dim Null $(1I - A) = 2$, there are 2 Jordan blocks for $\{1\}$ $\Rightarrow$ equ. is marginally stable.

5.14
$$A^T M + MA = -I \Rightarrow M = \begin{bmatrix} 1.75 & 1 \\ 1 & 1.5 \end{bmatrix}$$

Hurwitz test: $1.75 > 0$

$$\det\begin{pmatrix} 1.75 & 1 \\ 1 & 1.5 \end{pmatrix} = 1.75 \times 1.5 - 1 > 0$$

$\Rightarrow M > 0 \Rightarrow \operatorname{Re} \operatorname{eig}(A) < 0.$

5.18
$$A^T M + MA + 2\mu M = -N \Rightarrow (A + \mu I)^T M + M(A + \mu I) = -N$$

since $M, N > 0$, we have $\operatorname{Re} \operatorname{eig}(A + \mu I) < 0$

$\Rightarrow \operatorname{Re} \operatorname{eig}(A) < -\mu$

5.19
$$\rho^2 M - A^T M A = \rho^2 N \Rightarrow M - \left(\tfrac{1}{\rho} A\right)^T M \left(\tfrac{1}{\rho} A\right) = N$$

$\Rightarrow |\operatorname{eig}(A \cdot 1/\rho)| < 1 \Rightarrow |\operatorname{eig}(A)| < \rho.$