EEE 582, TEST 2 NAME:___SOLUTIONS_____

Nov. 16, 2011, 3:30-4:45, 4 Problems, Equal Credit, Closed-book, Closed-notes, calculator and 1 sheet of formulae allowed

Problem 1. Use the function $V = x^T P x$, $P = \begin{bmatrix} 1 & 1/4 \\ 1/4 & 1/2 \end{bmatrix}$, to find conditions on "a" such that the system $\dot{x} = \begin{bmatrix} 0 & 1 \\ a & -2 \end{bmatrix} x$ is asymptotically stable

1. Check P > 0: 1 > 0 and det(P) > 0.

2. Check $A^T P + PA = \begin{bmatrix} \frac{a}{2} & \frac{1}{2}+\frac{a}{2} \\ * & -\frac{3}{2} \end{bmatrix} < 0 \leftrightarrow \begin{bmatrix} -\frac{a}{2} & -\frac{1}{2}-\frac{a}{2} \\ * & \frac{3}{2} \end{bmatrix} > 0 \leftrightarrow \begin{Bmatrix} a < 0 \\ -\frac{3a}{4} - \left(\frac{1}{2}+\frac{a}{2}\right)^2 > 0 \end{Bmatrix} \leftrightarrow$
$\begin{Bmatrix} a < 0 \\ a^2 + 5a + 1 < 0 \end{Bmatrix} \leftrightarrow \begin{Bmatrix} a < 0 \\ -4.8 < a < -0.2 \end{Bmatrix} \leftrightarrow -4.8 < a < -0.2$

Problem 2. Show that *[A,B]* is completely controllable if and only if *[A-BK,B]* is completely controllable, where *K* is any compatible matrix.

[A,B] is c.c. iff for any set of n complex conjugate numbers G there exists Ko such that {eig(A-BKo)} = G. Define Ki = Ko-K; then {eig(A-BK-BKi)} = {eig(A-BKo)} = G, which is equivalent to [A-BK,B] is c.c.

Problem 3. It is easy to show that if the eigenvalues of *A* have negative real parts, then the system *[A,B,C,D]* (standard state-space description) is BIBO stable. Give an example where the converse is not true, i.e., find a BIBO stable system whose realization has system matrix *A* with some eigenvalues with positive real parts.

The poles of $G(s) = C(sI - A)^{-1}B + D$ are a subset of the eigenvalues of A; any uncontrollable or unobservable eigenvalues of A will not be poles of G(s). Let, $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C = [1 \ \ 0], D = 0$; Then, $G(s) = \frac{1}{s+1}$. G(s) is BIBO stable but eig(A)={-1, 1}.

Problem 4. Suppose $[A, B, C, D], [F, G, H, J]$ are two realizations of the same *n*-th order transfer function *P(s)*. Determine if the following statements are True or False. If False, provide a corrected or completed version:

- $[A, B, C, D], [F, G, H, J]$ are algebraically equivalent (related by a coordinate transformation), if dim(A) = dim(F) = n.
- $[A, B, C, D], [F, G, H, J]$ are exponentially stable if P(s) is BIBO stable and dim(A) = dim(F) = n.
- $[A, B, C, D], [F, G, H, J]$ are algebraically equivalent if they are both controllable and dim(A) = dim(F) = n.
- $[A, B, C, D], [F, G, H, J]$ are NOT algebraically equivalent (related by a coordinate transformation), if dim(A) > dim(F).

1. False; A true statement would have the additional condition [A,B,C,D] is minimal.
2. False; A true statement would have the additional condition [A,B,C,D] is minimal.
3. False; A true statement would have the additional condition [A,B,C,D] is observable.
4. True. (Algebraic equivalence preserves the system order.