HW # Minimal Realization, Solutions

1. The minimal realization has dimension 4.

2. The Gramians of the balanced realization are diagonals with entries

9.5135e-001 1.5584e-001 8.9122e-002 1.4599e-002

We can, therefore, eliminate the last balanced state with additive error at most

2*1.4599e-002 = 0.029

and the last two states with additive error at most

2*(8.9122e-002 + 1.4599e-002) = 0.207

Let H be the original system, Hm the minimal realization, Hb the balanced realization, Hb2 the balanced truncation with two states and Hb3 the balanced truncation with three states. The maximum singular value of the difference transfer function is the induced L-2 norm of the difference (error) system. We can plot the singular values in MATLAB using sigma.

Hb2=ss(a(1:2,1:2),b(1:2,:),c(:,1:2), d), Hb3=ss(a(1:3,1:3),b(1:3,:),c(:,1:3),d)

sigma(H-Hm,Hm-Hb,Hb-Hb2,Hb-Hb3)

The first two are numerically the same so the singular values of the difference are numerically zero. Hb and Hb3 are different only in one state so only one singular value is essentially different from zero. Zooming in, we observe that the peak sigma (Hb-Hb2) is -15dB ~ 0.178 (<=0.207) and the peak sigma (Hb-Hb3) is -30.5dB ~ 0.029 (<=0.029), as expected.