EEE 481 **Test 1** **NAME __SOLUTIONS____**

Open Book, Closed Notes, 30'

Problem 1

In a data acquisition application we would like to use the Diamond MM board to sample 16 channels with range 0-5V, and transmit the results over the RS-232 serial port.

1. What is the minimum Baud rate required so that the transmission takes less than 0.1sec?
2. What is the maximum error in the A/D conversion?

1. The Diamond MM has 12bit A/D so the total number of bits to be transmitted is 192. At best, we need the serial rate set higher than 1920 Baud to achieve transmission in 0.1sec.

In a more realistic situation, each number will occupy two bytes, each using 10bits (1start+1stop) for a total of 320 bits. So the minimum rate should be 3200 Baud (or the next highest standard rate which is 4800).

2. The maximum error is 5/2^bits = 5/4096 = 0.0012V.

Problem 2

The first-principles model of a temperature control system is $\dot{Y} = -0.1(Y - 300) + Q$, where Y is the Temperature (Kelvin) and Q is the supplied heat (Watts).

1. Use the Forward Euler approximation of derivative $\dot{Y}(t_k) \cong \dfrac{Y(t_{k+1}) - Y(t_k)}{T_s}$ to write a corresponding discrete time state-space model for a sampling time of 2sec.
2. What is the discrete-time transfer function of the system?
3. What are the limitations (if any) of this discretization method.

1. Using the FE, we have

$$Y(k+1) - Y(k) = -0.1T_s Y(k) + 30T_s + T_s Q(k)$$

There are two approaches now to derive a state space model. One is to define a new effective input (30+Q) and use the temperature Y as an output and state. The other is to consider the deviations from the steady state temperature 300(K). Notice, when Y=300, Q=0, then Y is constant. The latter method has better interpretations for linearization and is preferred. So, we define

$$y = Y - 300, \quad u = Q$$

and the state $x = y$ (first order system, one state). The above equation yields

$$x(k+1) = (1 - 0.1T_s)x(k) + T_s u(k), \quad y(k) = x(k) + 0u(k)$$

$$\Rightarrow A = (1 - 0.1T_s), B = T_s, C = 1, D = 0$$

2. The transfer function (from u to y, that is, from Q to Y-300) is

$$C(zI - A)^{-1}B + D = \frac{T_s}{z - (1 - 0.1T_s)} = \frac{2}{z - 0.8}$$

3. Forward Euler is limited to sampling rates faster than the pole bandwidth to ensure that stable CT transform to stable DT system. More precisely, it should hold

$$|1 - 0.1T_s| < 1 \Leftrightarrow 0 < 0.1T_s < 2 \Leftrightarrow \frac{1}{T_s} > \frac{0.1}{2}$$

Notice that this is not exactly the Nyquist theorem. The pole is in rad/s so if interpreted as a lowpass filter, the sampling rate should be faster than 1/6.28 of the corresponding Nyquist rate. It does not guarantee the ability to reconstruct signals accurately, just stability of the DT system.