EEE 304 **Test 1** **NAME: ________________**

Closed book-closed notes, transform tables allowed, 30'

Problem 1:

Consider the continuous time system with transfer function $H(s)=\dfrac{1}{s^2+s+1}$.

1. Sketch the ROC if the system is causal.

Poles are at $-\frac{1}{2}\pm j\frac{\sqrt{3}}{2}$. The causal system has ROC = {s: Re s > -1/2}

2. Sketch the ROC if the system is stable.

The stable system has ROC = {s: Re s > -1/2}

Problem 2:

For the causal discrete time system with transfer function $H(z)=\dfrac{2(z-1)}{z^2(z-0.8)}$

1. Sketch the region of convergence of H(z). Is the system stable?

   Poles are at 0, 0, 0.8. The causal ROC is ROC = {z: |z| > 0.8}
   It contains the unit circle, hence the system is stable.

2. Compute the unit step response (x(n)=u(n)).

$$Y(z)=H(z)X(z)=\frac{2(z-1)z}{z^2(z-0.8)(z-1)}=\frac{2}{z(z-0.8)};\ ROC=\{z:1<|z|\}$$

$$=\left\{\frac{2/(-0.8)}{z}\right\}_{RS}+\left\{\frac{2/0.8}{z-0.8}\right\}_{RS}$$

$$=\left\{\frac{-2.5}{z}\right\}_{RS}+\left\{\frac{2.5}{z-0.8}\right\}_{RS}$$

$$y(n)=-2.5\delta(n-1)+2.5(0.8)^{n-1}u(n-1)$$

$Alt.:$

$$Z^{-1}\left\{\frac{2}{z(z-0.8)}\right\}=Z^{-1}\left\{\frac{2}{(z-0.8)}\right\}_{n=n-1}=2(0.8)^{n-1}u(n-1)\Big|_{n=n-1}=2(0.8)^{n-2}u(n-2)=2.5(0.8)^{n-1}u(n-2)$$

The two expressions are equivalent since the first one yields y(1) = -2.5+2.5 = 0.