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EEE 304 **HW 1**

Problem 1:

Consider the filter with impulse response $h(t) = e^{-t}u(t-1)$.

1. Find the transfer function

2. Find the Laplace transform of the output when $x(t) = \sin(5t)u(t)$

3. Find the output by taking the inverse Laplace transform of your answer to part 2.

4. Can you obtain the same result using Fourier Transforms?

1. $h(t) = e^{-t}u(t-1) = \frac{1}{e}e^{-(t-1)}u(t-1) \Rightarrow H(s) = \frac{1}{e}\frac{e^{-s}}{s+1},\ ROC = \{\operatorname{Re} s > -1\}$

2. $L\{x\} = \frac{5}{s^2+25},\ ROC = \{\operatorname{Re} s > 0\};$

$$Y(s) = H(s)X(s) = \frac{1}{e}\frac{e^{-s}}{s+1}\frac{5}{s^2+25},\ ROC = \{\operatorname{Re} s > 0\}$$

3. $y(t) = L^{-1}\{Y(s)\} = L^{-1}\left\{\frac{1}{e}\frac{e^{-s}}{s+1}\frac{5}{s^2+25}\right\} = \frac{1}{e}L^{-1}\left\{\frac{A}{s+1} + \frac{Bs+C}{s^2+25}\right\}\Bigg|_{t-1}$

$Note: A = 5/[(-1)^2+25] = 5/26;\ B,C: As^2+25A+Bs^2+Bs+Cs+C = 5 \Rightarrow B = -5/26,\ C = 5/26$

$$= \frac{1}{e}L^{-1}\left\{\frac{5/26}{s+1}\right\}_{t-1} + \frac{1}{e}L^{-1}\left\{\frac{-5/26s}{s^2+25} + \frac{5/26}{s^2+25}\right\}_{t-1} = \left\{\frac{5}{26e}e^{-(t-1)}u(t-1) - \frac{5}{26e}\cos(5(t-1))u((t-1)) + \frac{4}{26e}\sin(5(t-1))u(t-1)\right\}$$

$Alt:$

$$y(t) = L^{-1}\{Y(s)\} = L^{-1}\left\{\frac{1}{e}\frac{e^{-s}}{s+1}\frac{5}{s^2+25}\right\} = \frac{1}{e}L^{-1}\left\{\frac{A}{s+1} + \frac{B}{s-5j} + \frac{C}{s+5j}\right\}\Bigg|_{t-1} = \frac{1}{e}L^{-1}\left\{\frac{5/26}{s+1} + \frac{5/((5j+1)(10j))}{s-5j} + \frac{B*}{s+5j}\right\}\Bigg|_{t-1}$$

$$= \frac{1}{e}L^{-1}\left\{\frac{5/26}{s+1} + \frac{-j/(10j+2)}{s-5j} + \frac{B*}{s+5j}\right\}\Bigg|_{t-1} = \left\{\frac{5}{26e}e^{-(t-1)}u(t-1) + 2\operatorname{Re}\left[\frac{-j}{(10j+2)e}e^{5j(t-1)}u(t-1)\right]\right\}$$

$$= \left\{\frac{5}{26e}e^{-(t-1)}u(t-1) - \frac{2}{e}\operatorname{Re}\left[\frac{1}{\sqrt{104}}\angle 90^\circ - \tan^{-1}(5)e^{5j(t-1)}u(t-1)\right]\right\}$$

$$= \left\{\frac{5}{26e}e^{-(t-1)}u(t-1) - \frac{2}{e}\operatorname{Re}\left[\frac{1}{\sqrt{104}}e^{j[5(t-1)+90^\circ-\tan^{-1}(5)]}u(t-1)\right]\right\}$$

$$= \left\{\frac{5}{26e}e^{-(t-1)}u(t-1) - \frac{2}{e}\left[\frac{1}{\sqrt{104}}\cos[5(t-1)+90^\circ-\tan^{-1}(5)]u(t-1)\right]\right\}$$

4. Yes. But finding the F{sin5t u(t)} is very involved (take the convolution F{sin5t} F{u(t)}) and group terms appropriately. For a complete solution, consult the sample old homework sets. The Fourier approach would fail if the jw-axis is not inside the Laplace ROC nor on its boundary.

Problem 2:

Consider the continuous time causal filter with transfer function

$$H(s) = \frac{3(s+1)}{(s+2)(s-1)}$$

1. Compute the response of the filter to $x[t] = u[t]$

2. Compute the response of the filter to $x[t] = u[-t]$

3. Repeat parts 1 and 2 for a stable system with the same transfer function.

$$1.\, Causality \Rightarrow ROC_H = \{\mathrm{Re}\, s > 1\}$$

$$Y(s) = H(s)X(s) = \frac{3(s+1)}{(s+2)(s-1)s};\ ROC \supseteq \{\mathrm{Re}\, s > 1\} \cap \{\mathrm{Re}\, s > 0\} = \{\mathrm{Re}\, s > 1\}$$

$$= \left\{\frac{-3/2}{s}\right\}_{ROC=\mathrm{Re}\, s>0} + \left\{\frac{-3/2}{(s+2)}\right\}_{ROC=\mathrm{Re}\, s>-2} + \left\{\frac{2}{(s-1)}\right\}_{ROC=\mathrm{Re}\, s>1}$$

$$y(t) = -\frac{3}{2}u(t) - \frac{3}{2}e^{-2t}u(t) + 2e^{t}u(t)$$

$$2.\, Y(s) = H(s)X(s) = \frac{3(s+1)}{(s+2)(s-1)(-s)};\ ROC \supseteq \{\mathrm{Re}\, s > 1\} \cap \{\mathrm{Re}\, s < 0\} = \{\varnothing\}$$

$y(t)$ not well - defined.

$$3.\, Stability \Rightarrow ROC_H = \{-2 < \mathrm{Re}\, s < 1\}$$

$$3.1: Y(s) = H(s)X(s) = \frac{3(s+1)}{(s+2)(s-1)s};\ ROC \supseteq \{-2 < \mathrm{Re}\, s < 1\} \cap \{\mathrm{Re}\, s > 0\} = \{0 < \mathrm{Re}\, s < 1\}$$

$$= \left\{\frac{-3/2}{s}\right\}_{ROC=\mathrm{Re}\, s>0} + \left\{\frac{-3/2}{(s+2)}\right\}_{ROC=\mathrm{Re}\, s>-2} + \left\{\frac{2}{(s-1)}\right\}_{ROC=\mathrm{Re}\, s<1}$$

$$y(t) = -\frac{3}{2}u(t) - \frac{3}{2}e^{-2t}u(t) + 2[-e^{t}u(-t)] = -\frac{3}{2}u(t) - \frac{3}{2}e^{-2t}u(t) - 2e^{t}u(-t)$$

$$3.2: Y(s) = H(s)X(s) = \frac{3(s+1)}{(s+2)(s-1)(-s)};\ ROC \supseteq \{-2 < \mathrm{Re}\, s < 1\} \cap \{\mathrm{Re}\, s < 0\} = \{-2 < \mathrm{Re}\, s < 0\}$$

$$= -\left\{\frac{-3/2}{s}\right\}_{ROC=\mathrm{Re}\, s<0} - \left\{\frac{-3/2}{(s+2)}\right\}_{ROC=\mathrm{Re}\, s>-2} - \left\{\frac{2}{(s-1)}\right\}_{ROC=\mathrm{Re}\, s<1}$$

$$y(t) = \frac{3}{2}[-u(-t)] + \frac{3}{2}e^{-2t}u(t) - 2[-e^{t}u(-t)] = -\frac{3}{2}u(-t) + \frac{3}{2}e^{-2t}u(t) + 2e^{t}u(-t)$$

Problem 3:

Consider the discrete time stable filter with transfer function

$$H(z) = \frac{z-1}{(z-0.5)(z-1.5)}$$

1. Compute the response of the filter to $x[n] = u[n]$.

2. Repeat part 1 for a causal filter with the same transfer function.

$$1.\ Stability \Rightarrow ROC_H = \{0.5 < |z| < 1.5\}$$

$$Y(z) = H(z)X(z) = \frac{(z-1)z}{(z-0.5)(z-1.5)(z-1)};\ ROC \supseteq \{0.5 < |z| < 1.5\} \cap \{1 < |z|\}$$

$$= \left\{\frac{-1/2}{z-0.5}\right\}_{ROC=0.5<|z|} + \left\{\frac{3/2}{z-1.5}\right\}_{ROC=|z|<1.5} + \left\{\frac{0}{z-1}\right\}_{ROC=1<|z|}$$

$$y(n) = -\frac{1}{2}0.5^{n-1}u(n-1) - \frac{3}{2}1.5^{n-1}u(-n)$$

$$2.\ Causality \Rightarrow ROC_H = \{1.5 < |z|\}$$

$$Y(z) = H(z)X(z) = \frac{(z-1)z}{(z-0.5)(z-1.5)(z-1)};\ ROC \supseteq \{1.5 < |z|\} \cap \{1 < |z|\}$$

$$= \left\{\frac{-1/2}{z-0.5}\right\}_{ROC=0.5<|z|} + \left\{\frac{3/2}{z-1.5}\right\}_{ROC=2<|z|} + \left\{\frac{0}{z-1}\right\}_{ROC=1<|z|}$$

$$y(n) = -\frac{1}{2}0.5^{n-1}u(n-1) + \frac{3}{2}1.5^{n-1}u(n-1)$$