EEE 304 **Test 2** **Name: __Solutions______**

Closed book-closed notes, transform tables allowed, 30'

Problem 1:

For the continuous-time causal system with transfer function $H(s) = \frac{-s}{(s+1)}$ compute the steady-state response to the input *x(t) = sin(t)u(t-10).*

$$1. |H(jw)| = \frac{\sqrt{w^2}}{\sqrt{w^2+1}} = \frac{1}{\sqrt{2}}$$

$$\angle H(jw) = -180 + \tan^{-1}\left(\frac{w}{0}\right) - \tan^{-1}\left(\frac{w}{1}\right) = -90 - \tan^{-1}(1) = -135$$

$$\Rightarrow \quad y_{ss}(t) = |H(j1)| \sin(t + \angle H(j1)) = \frac{1}{\sqrt{2}} \sin(t - 135°)$$

Problem 2:

For the continuous-time causal system with transfer function $H(s) = \frac{(0.1s+1)}{(s+1)}$ compute the discrete-time equivalent, say G(z), using the Backward Euler Approximation and a sampling interval of T = 0.1s.

$$Backward - Euler: s = \frac{1-z^{-1}}{T} \Rightarrow H_d(z) = \frac{0.1\frac{z-1}{Tz}+1}{\left(\frac{z-1}{Tz}+1\right)} = \frac{01(2z-1)}{(1.1z-1)}$$

Problem 3:

For the discrete-time causal system with transfer function $H(z) = \frac{(0.05z)}{(z-0.95)}$ compute the steady-state response to the sinusoid *x(n) = sin[0.1n]u(n-10)*

$$|H(e^{j\Omega})| = \frac{|0.05e^{j\Omega}|}{|e^{j\Omega} - 0.95|} = \frac{0.05}{\sqrt{(\cos 0.1 - 0.95)^2 + (\sin 0.1)^2}} = 0.46$$

$$\angle H(e^{j\Omega}) = l180 + \Omega - \tan^{-1}\left(\frac{\sin\Omega}{\cos\Omega - 0.95}\right) = 0.1 - \tan^{-1}\left(\frac{\sin 0.1}{\cos 0.1 - 0.95}\right) = -60$$

$\Rightarrow$

$$y_{ss}(n) = \left|H\left(e^{j0.1}\right)\right| \sin\left(0.1n + \angle H\left(e^{j0.1}\right)\right) = 0.46 \sin\left(0.1n - 60°\right)$$