EEE 582 HW 5 SOLUTIONS

6.2 $[B, AB] = \left[\begin{array}{cc:cc} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right]$ has full row rank

$\Rightarrow$ completely controllable

(A^2B not needed here)

$\begin{bmatrix} C \\ AC \\ A^2C \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 3 & -1 \\ 0 & -2 & 4 \end{bmatrix}$ has rank 3

$\Rightarrow$ completely observable

6.4 (A, B) is c.c. $\Leftrightarrow$ $\text{rank}\begin{bmatrix} A_{11}-sI & A_{12} & B_1 \\ A_{21} & A_{22}-sI & 0 \end{bmatrix} = n$

for all s. Hence $[A_{21}, A_{22}-sI]$ should have full row rank

for all s $\Leftrightarrow$ (A_{22}, A_{21}) is c.c.

6.8 $\dot{x} = \begin{bmatrix} -1 & 4 \\ 4 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u, \quad y = [1 \;\; 1] x$

$Q_c = \begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix}$. Select $P^{-1} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \Rightarrow P = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$

$\Rightarrow P^{-1}AP = \begin{bmatrix} 3 & 4 \\ 0 & -5 \end{bmatrix} \quad PB = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad CP^{-1} = [2, 1]$

$\Rightarrow \dot{\bar{x}} = \begin{bmatrix} 3 & 4 \\ 0 & -5 \end{bmatrix} \bar{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u, \quad y = [2, 1] \bar{x}$

Obviously, the equation can be reduced (zero state equivalent system) to: $\dot{x}_1 = 3x_1 + 1u, \quad y = 2x_1$ which is c.o.

6.15 For controllability (JCF test) the rows $\begin{bmatrix} b_{21} & b_{22} \\ b_{41} & b_{42} \\ b_{61} & b_{62} \end{bmatrix}$ should be independent; this is not possible.

For observability, the columns of $\begin{bmatrix} c_{11} & c_{13} & c_{15} \\ c_{21} & c_{23} & c_{25} \\ c_{31} & c_{33} & c_{35} \end{bmatrix}$ should be independent; this is possible e.g. I (identity)

(7.56)

7.10 $a_1 = 2,\ a_2 = 1,\quad h(1) = 0,\quad h(2) = 1 \Rightarrow$

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

This is a companion form realization.

7.11 $T = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} \quad \underset{(TT)}{\tilde{T}} = \begin{bmatrix} 1 & -2 \\ -2 & 3 \end{bmatrix}$

Using MATLAB,

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[k,S,L] = svd(T)
  S1 = sqrtm(S)
QO = k * S1 ; QC = S1 * L ;
a = inv(QO) * TT * inv(QC) ;
b = QC(1:2,1) ; c = QO(1, 1:2)
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$$\Rightarrow \dot{x} = \begin{bmatrix} -1.707 & 0.707 \\ -0.707 & -0.293 \end{bmatrix} x + \begin{bmatrix} 0.595 \\ -0.595 \end{bmatrix} u$$

$$y = \begin{bmatrix} -0.595, & -0.595 \end{bmatrix} x$$

This is a balanced realization in the sense of the controllability/observability matrices.
