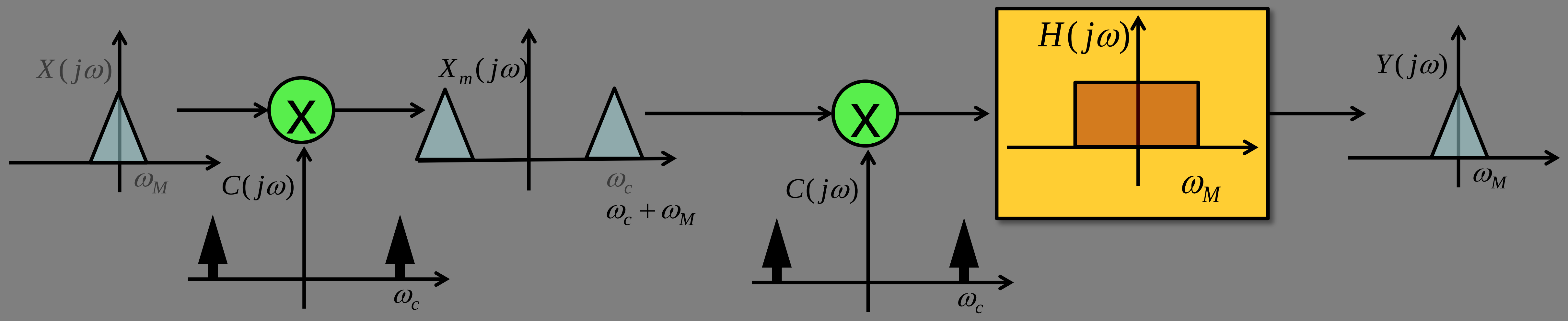
# EEE304

## Week 7: Communication Systems: Applications of Modulation

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## Week 7: Communication Systems: Applications of Modulation

**Learning Objectives:**

- Develop a general understanding of communications applications of modulation
- AM basics and derivations
- Asynchronous demodulation and envelope detection
- Multiplexing (Frequency Division, Time Division)

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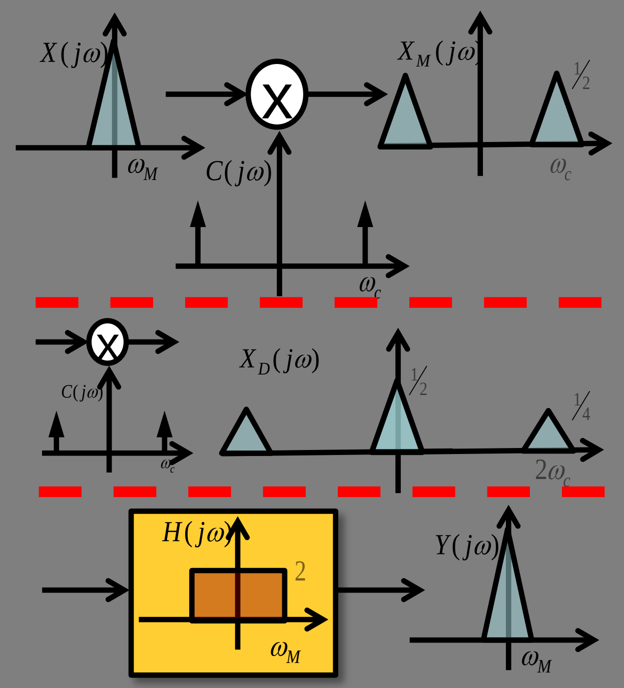
## Lecture 7.1: Modulation and Demodulation Generalities



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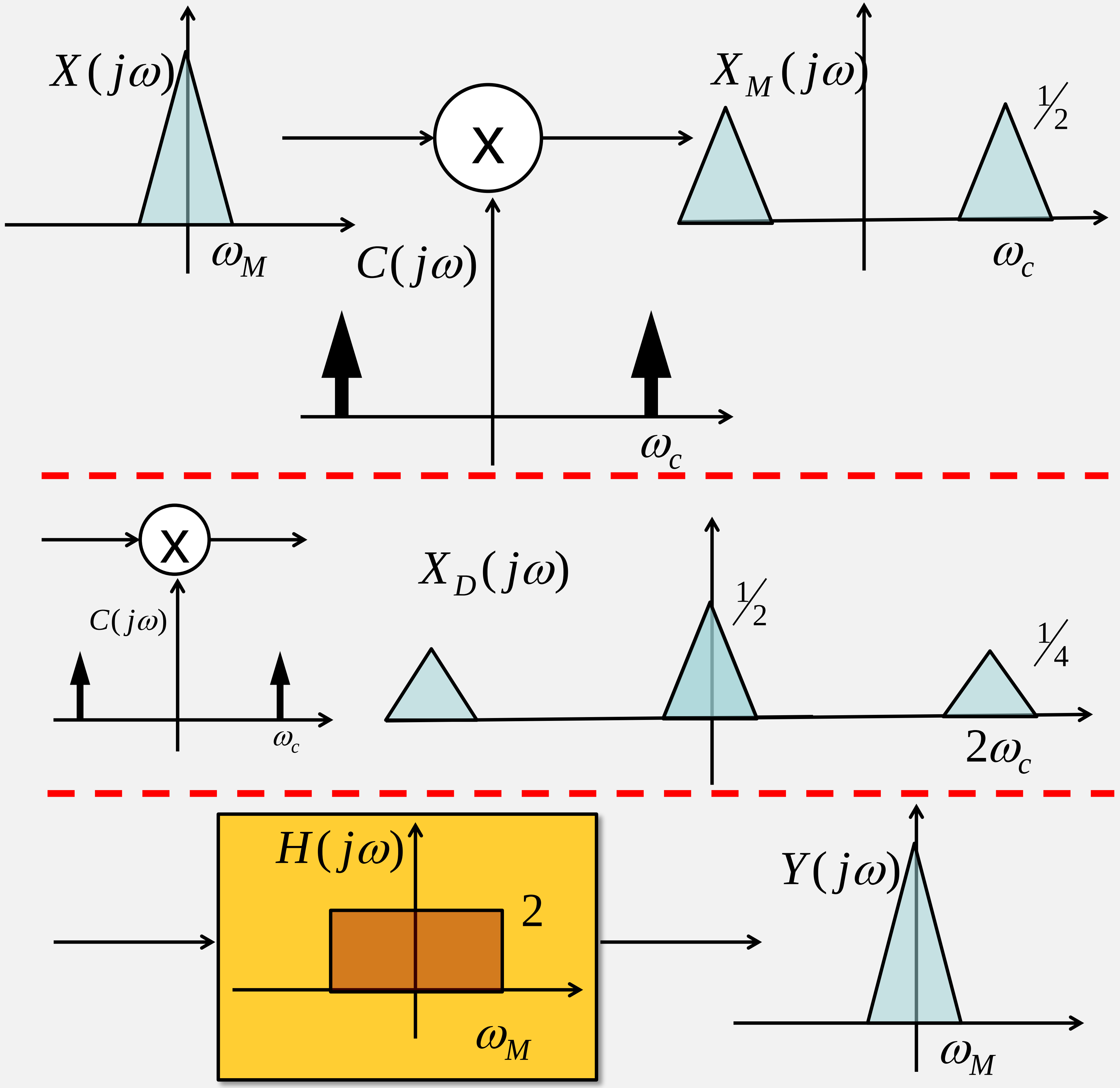
# Benefits of Modulation

- Shift transmitted signal spectrum to a frequency inside the operating band of a communication channel.
  - Speech signal 200Hz-4kHz. AM modulation ~1MHz, Cellular radio 800-900MHz, Microwave Link 300MHz-300GHz
  - High-speed digital data (Mbps) using optical carrier frequencies (200THz) : Huge Bandwidth, low transmission losses, immune to EM interference, small size, rugged and flexible.
- Reduced susceptibility to noise and interference
  - Use of certain modulation forms can trade off bandwidth for noise immunity, e.g. FM, PCM
- Multiplexing
  - Simultaneous transmission of signals (frequency division, time division, code division)
- Physical size of equipment
  - Antenna size comparable to wavelength

# Modulation and Demodulation

- A carrier wave (e.g., sinusoid) is modulated with a bandlimited signal
- The result contains frequency shifted signal replicas centered at the modulating frequency.
- The modulated signal is suitable for transmission over a communication channel. It is recovered by a "demodulation process" (modulation + lowpass, envelope detector).



# Standard Amplitude Modulation: Basic Equations

- Fourier transform of signals and their operations to account for the lowpass filter at the end.

$$c(t)=\cos(\omega_c t) \Rightarrow C(j\omega)=\pi\{\delta(\omega-\omega_c)+\delta(\omega+\omega_c)\}$$

- Modulation property

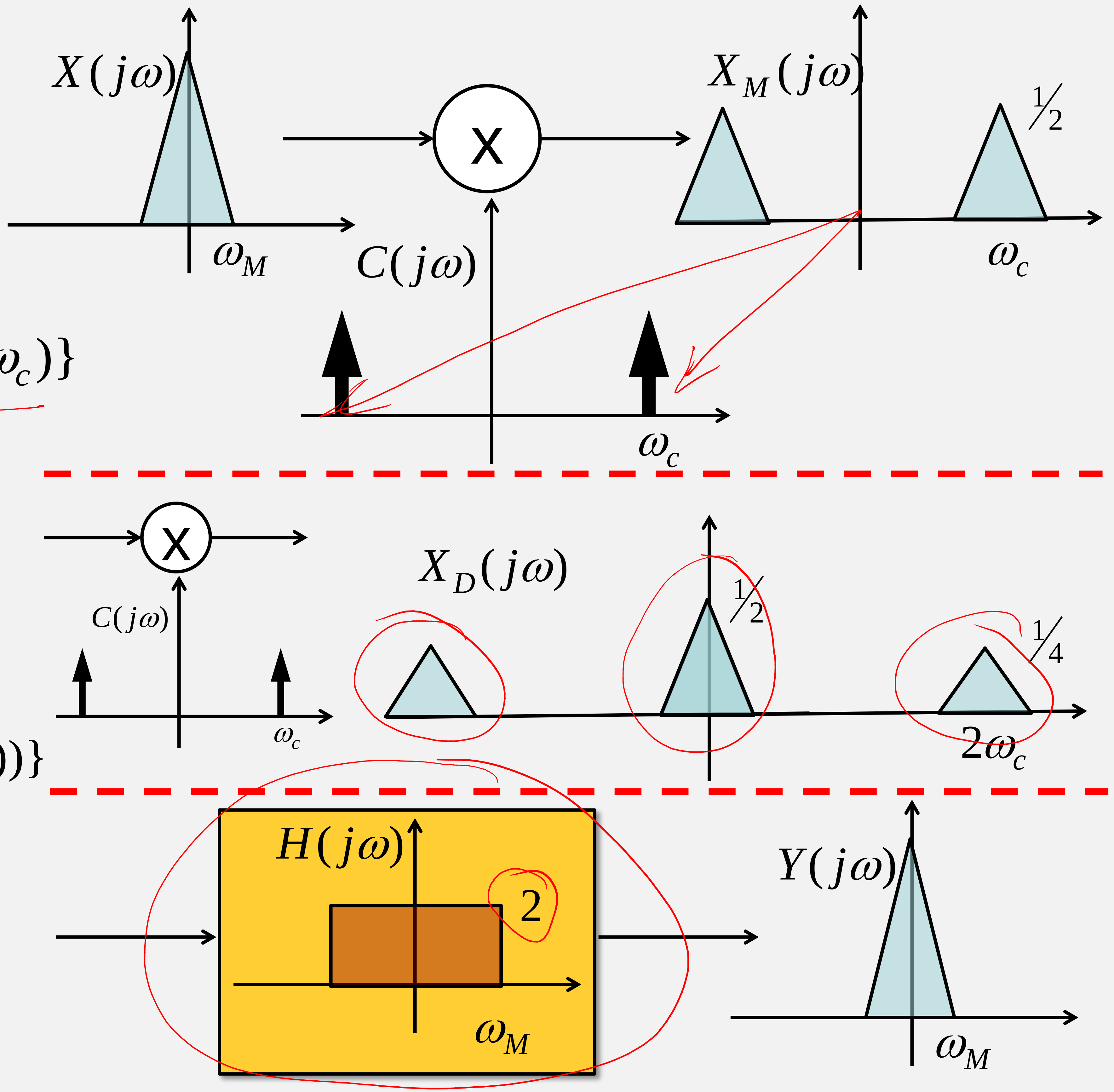
$$X_M(j\omega)=\frac{1}{2\pi}X(j\omega)*C(j\omega)$$
$$=\frac{1}{2}\{X(j(\omega-\omega_c))+X(j(\omega+\omega_c))\}$$

- Demodulation: Second Modulation

$$X_D(j\omega)=\frac{1}{4}\{X(j(\omega-2\omega_c))+2X(j\omega)+X(j(\omega+2\omega_c))\}$$

- Lowpass Filtering

$$Y(j\omega)=H(j\omega)X_D(j\omega)=X(j\omega)$$



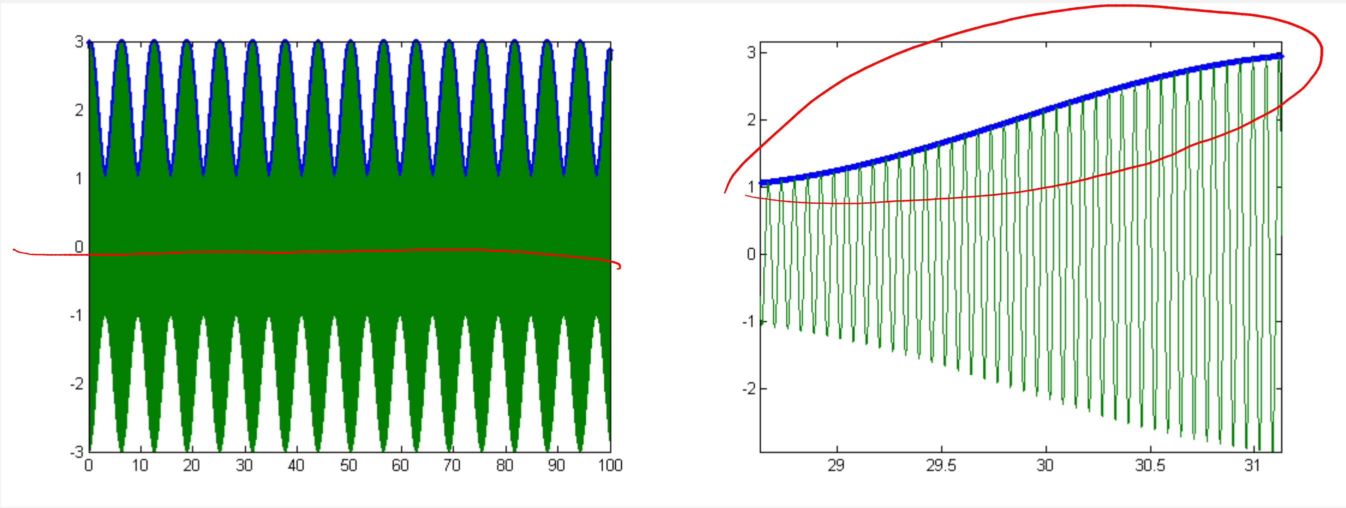
# Standard Amplitude Modulation: Asynchronous Demodulation

- Translate signal into a positive envelope

  $$x(t) \leftarrow x(t) + A > 0$$

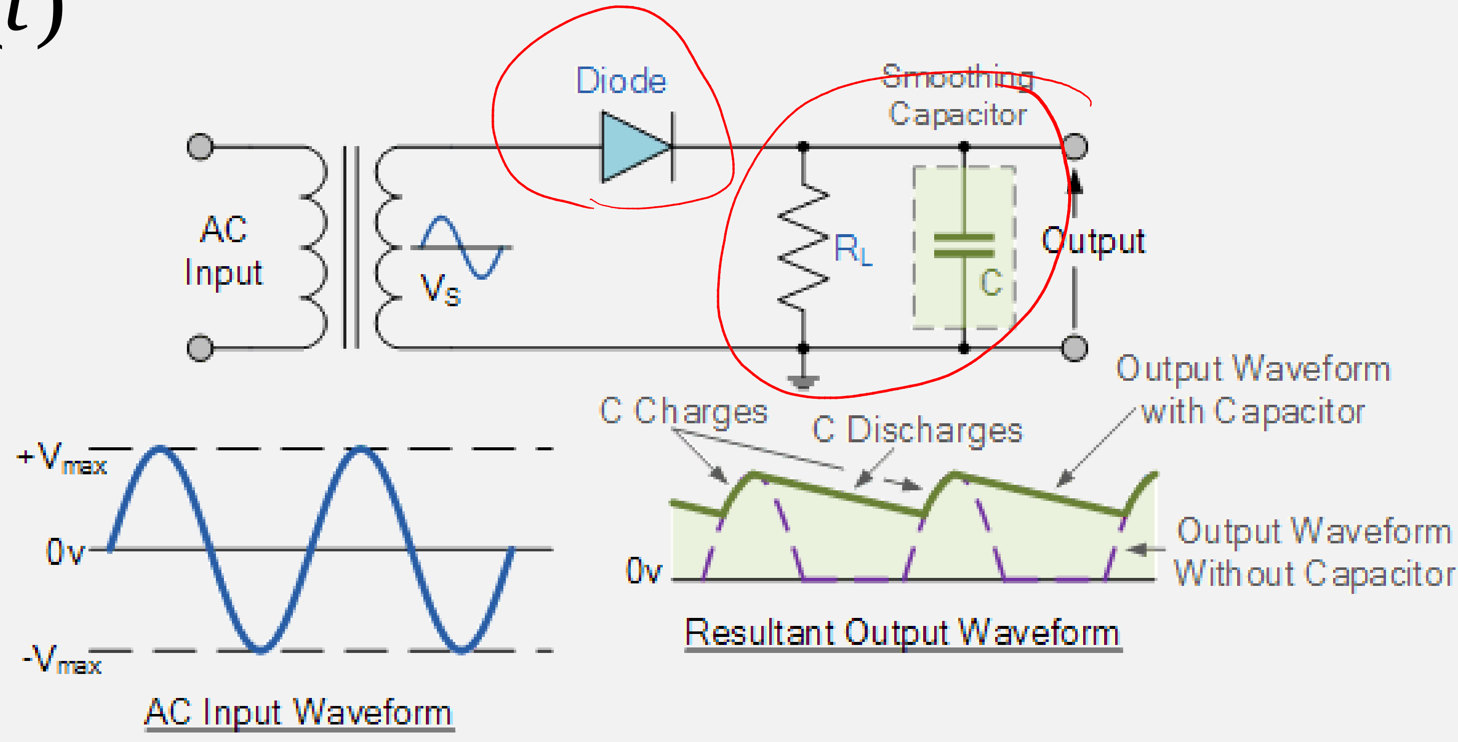
  $$\frac{\max(|x|)}{A} = m = \text{Modulation Index}$$

- Modulate $y_M(t) = \cos(\omega_c t)x(t)$
- Use a half-wave rectifier $y_R(t) = \max(\cos(\omega_c t),0)x(t)$
- and a smoothing filter $Y(j\omega) = H(j\omega)Y_R(j\omega)$
  - When $y(t) > y_R(t)$, $y(t) = e^{-(t-t_i)/RC}\, y(t_i)$
  - 1/RC << carrier frequency
  - 1/RC >> signal frequency



Common app:
Wm ~ 1e3 Hz
Wc ~ 1e5 Hz

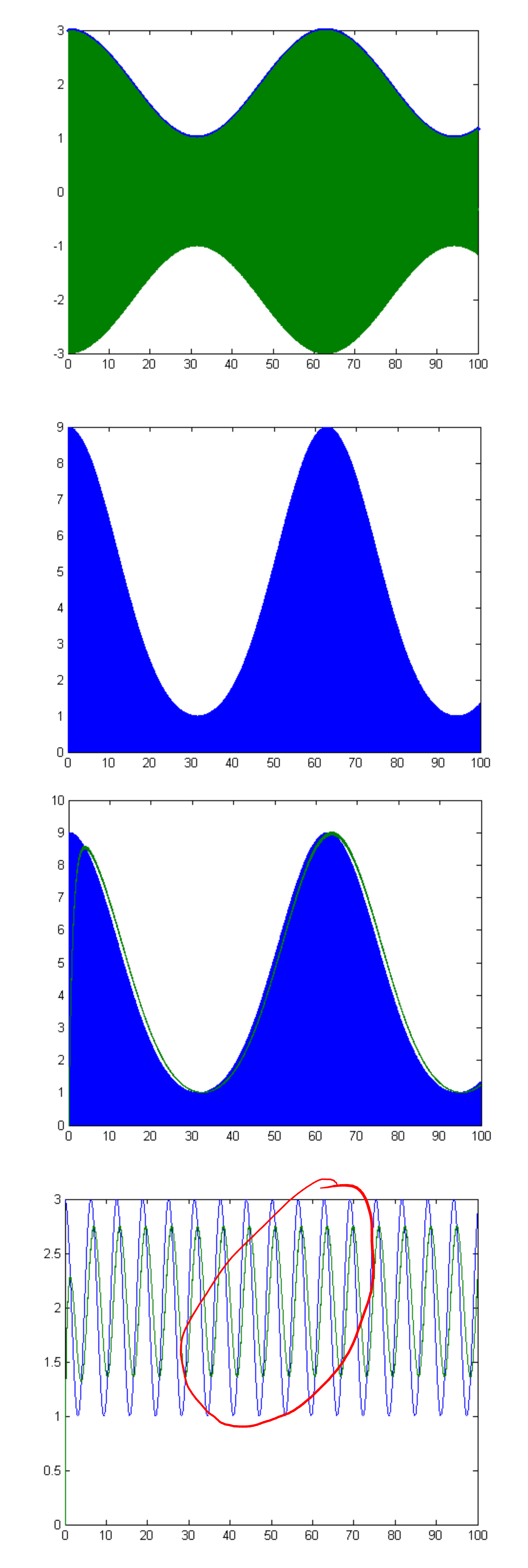
```
t=[0:.001:100];
x=cos(1*t);
c=sin(100*t);
plot(t,x+2,'.',t,(x+2).*c)
```

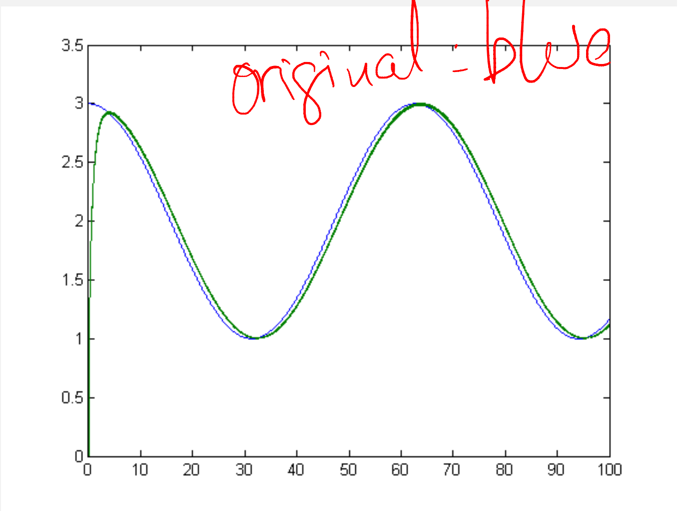


# Standard Amplitude Modulation: Asynchronous Demodulation

Alternative scheme for a rectifier:

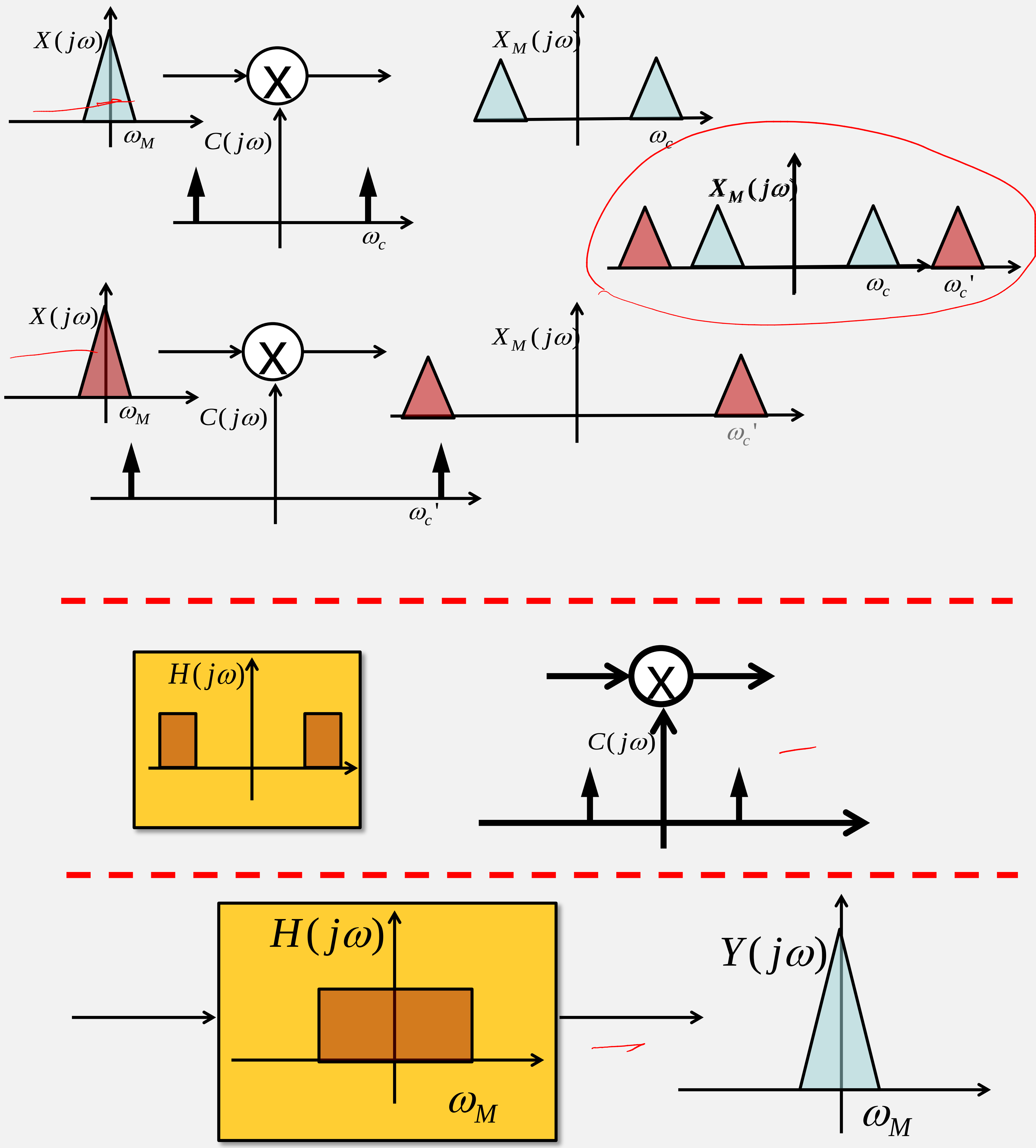
- Modulate $y_M(t) = \cos(\omega_c t)x(t)$
- Use a square nonlinearity $y_D(t) = \cos^2(\omega_c t)x^2(t) = \frac{1}{2}x^2(t) + \frac{1}{2}\cos(2\omega_c t)x^2(t)$
- Then a lowpass filter $Y(j\omega) = H(j\omega)Y_R(j\omega)$ ⇐ Band limited to $2\omega_m$ $\sim 2\omega_c$
- And a square-root nonlinearity to recover x(t) ⇐ Lowpass cutoff $> 2\omega_m$
    - Note: the observed difference is the same as filtering x with H. Left: x(t) = cos(0.1t), Right: x(t) = cos(1t). H(s)= 1/(s+1)





# Multiplexing

- Signals modulated with different carrier frequencies and simultaneously transmitted (Frequency Division Multiplexing –carefully regulated BW)
- Bandpass filter and Demodulation for recovery of a specific signal.
- Practical inexpensive systems include an Intermediate Frequency stage where the selected signal is shifted to a fixed frequency where it is bandpass-filtered and demodulated by a fixed-frequency demodulator.



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## Lecture 3.2: Other Modulation Schemes

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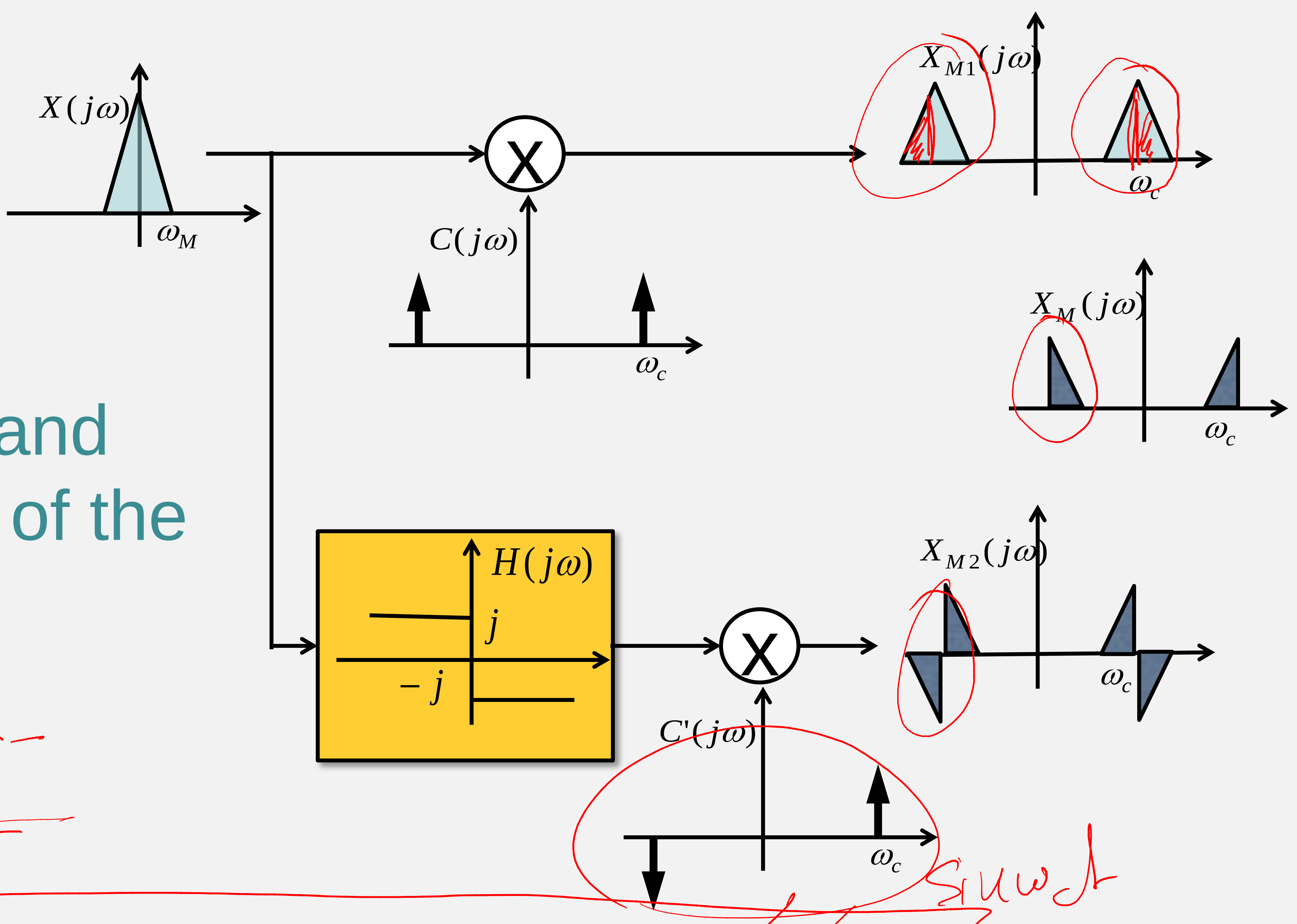
# Single-Side Band AM

- Use of a 90 deg. phase shifting filter and sin/cos modulation to retain only one of the sidebands and make a more efficient bandwidth utilization.





$$X(j\omega)=X_+(j\omega)+X_-(j\omega)$$

$$X_{M1}(j\omega)=\frac{1}{2\pi}X(j\omega)*C(j\omega)=\frac{1}{2}\{X_+(j(\omega-\omega_c))+X_-(j(\omega-\omega_c))+X_+(j(\omega+\omega_c))+X_-(j(\omega+\omega_c))\}$$

$$X_H(j\omega)=H(j\omega)[X_+(j\omega)+X_-(j\omega)]=-jX_+(j\omega)+jX_-(j\omega)$$

$$X_{M2}(j\omega)=\frac{1}{2\pi}X(j\omega)*C'(j\omega)=\frac{1}{2j}\{X_H(j(\omega-\omega_c))-X_H(j(\omega+\omega_c))\}$$

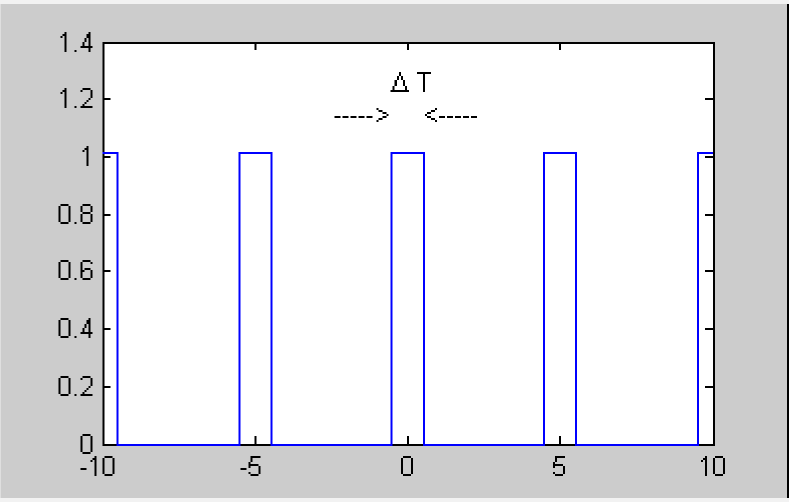
$$=\frac{1}{2j}\{-jX_+(j(\omega-\omega_c))+jX_-(j(\omega-\omega_c))+jX_+(j(\omega+\omega_c))-jX_-(j(\omega+\omega_c))\}$$

$$=\frac{1}{2}\{X_-(j(\omega-\omega_c))-X_+(j(\omega-\omega_c))+X_+(j(\omega+\omega_c))-X_-(j(\omega+\omega_c))\}$$

$$X_M(j\omega)=X_{M1}(j\omega)+X_{M2}(j\omega)=X_-(j(\omega-\omega_c))+X_+(j(\omega+\omega_c))$$

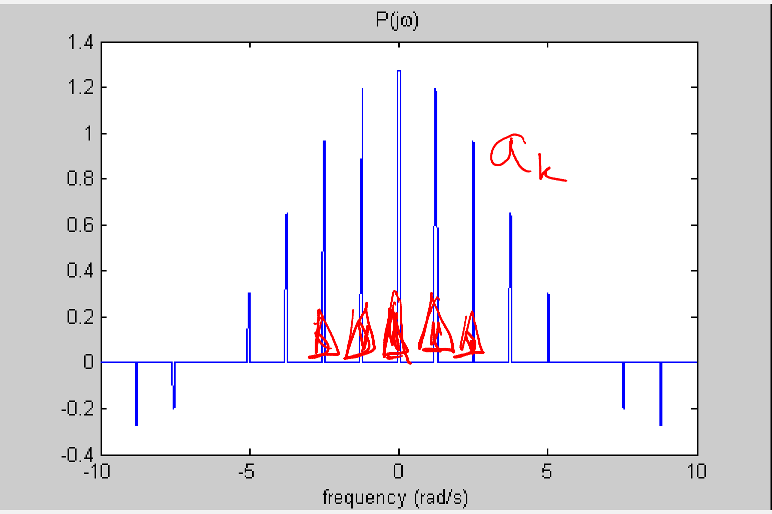
# Pulse-Train Carrier Modulation

- Use of a pulse train to transmit snapshots of the signal. As long as the Nyquist theorem is satisfied, reconstruction of the signal is possible.



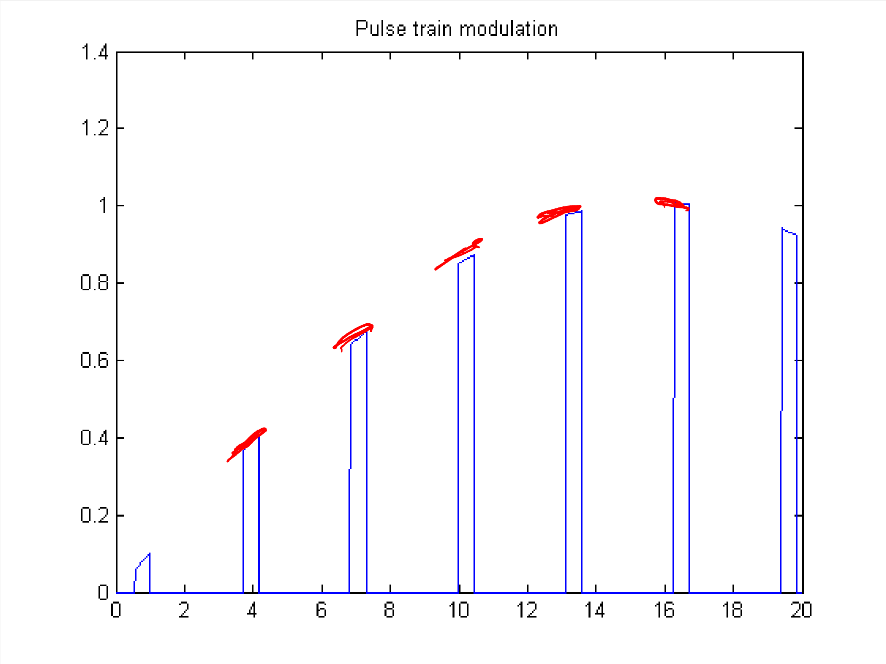

$$p(t) = \sum_n s_\Delta(t - nT); \quad \text{where } s_\Delta \text{ is a pulse of duty cycle } \Delta$$

$$P(j\omega) = 2\pi \sum_k a_k \delta(\omega - k\omega_S); \quad \omega_S = \frac{2\pi}{T}, \; a_k = \mathcal{FS}\{s_\Delta\}$$




$$X_M(j\omega) = \frac{1}{2\pi} P(j\omega) * X(j\omega) = \sum_k a_k X(j(\omega - k\omega_S))$$

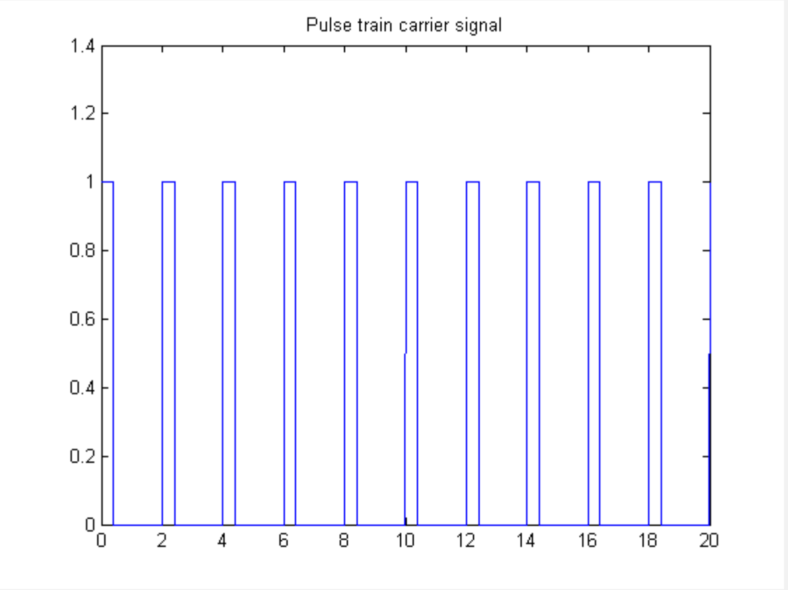
$$a_k = \frac{\sin\left(k \frac{2\pi}{T} \cdot \frac{\Delta T}{2}\right)}{k\pi} = \frac{\sin(k\pi\Delta)}{k\pi}$$

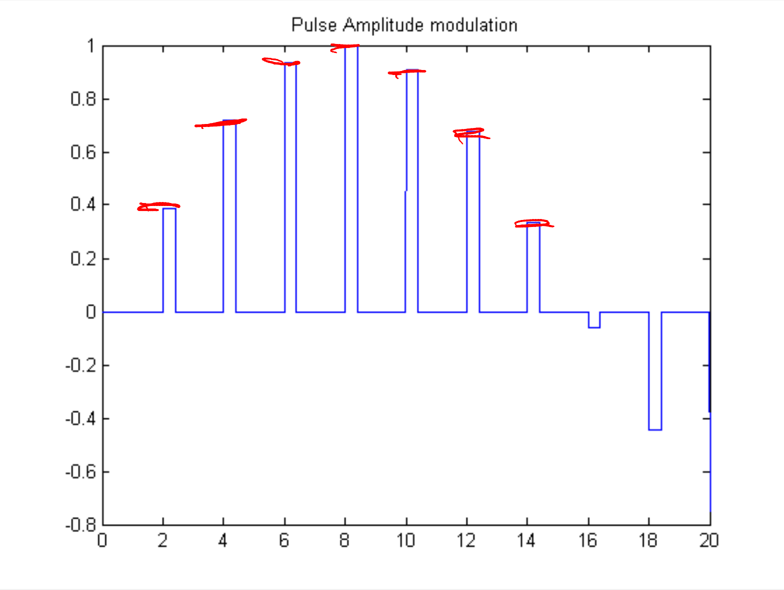



- Reconstruction is possible from the center replica as long as the DC coefficient is nonzero, or any harmonic using a bandpass filter and sinusoidal demodulation, regardless of the duty cycle value.

# Pulse Amplitude Modulation

- Similar to Pulse Train Carrier but sending only the sampled values of the signal, x(nT).
  - Here, instead of an impulse with weight x(nT), we transmit a ZOH-pulse with the value x(nT).
  - The reconstruction is again similar, through lowpass filtering or bandpass filtering and sinusoidal demodulation.
  - Requires a timing sequence to identify the correct sampling instants
  - Quantization offers some noise immunity trading off accuracy

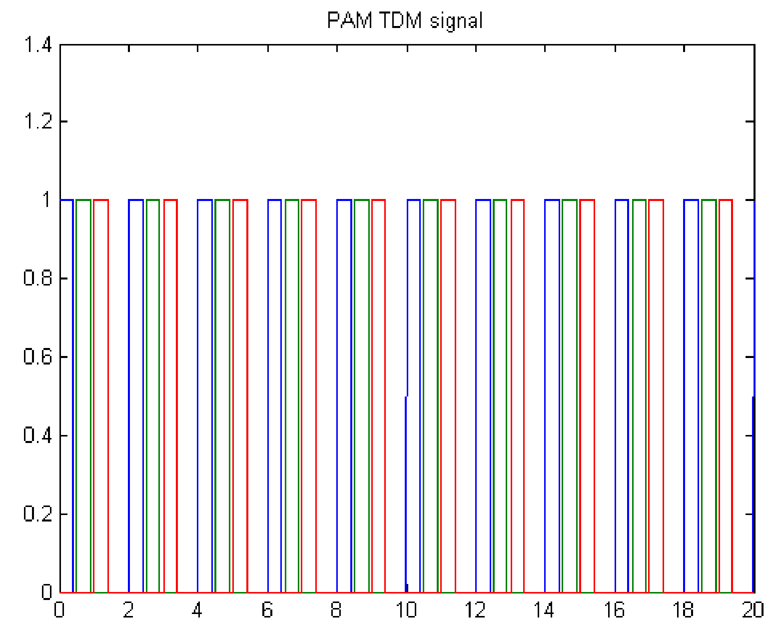


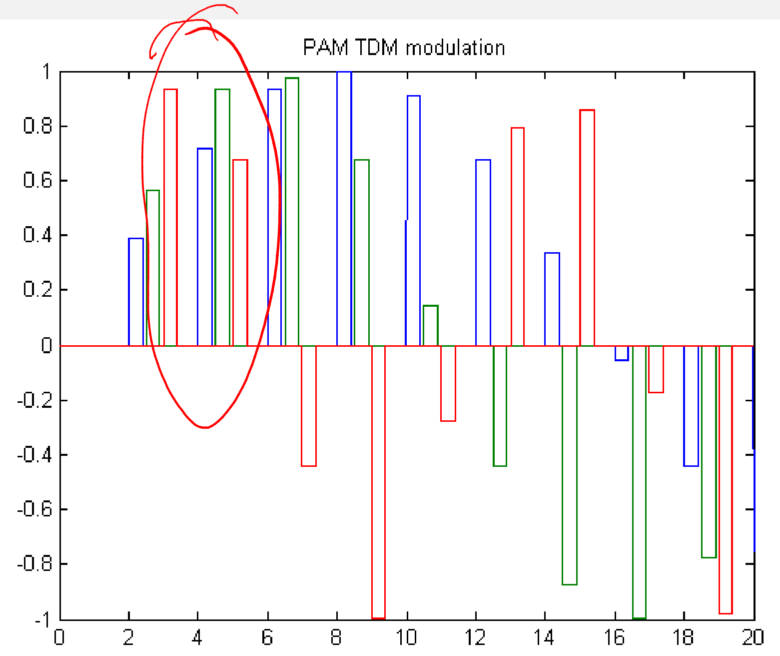



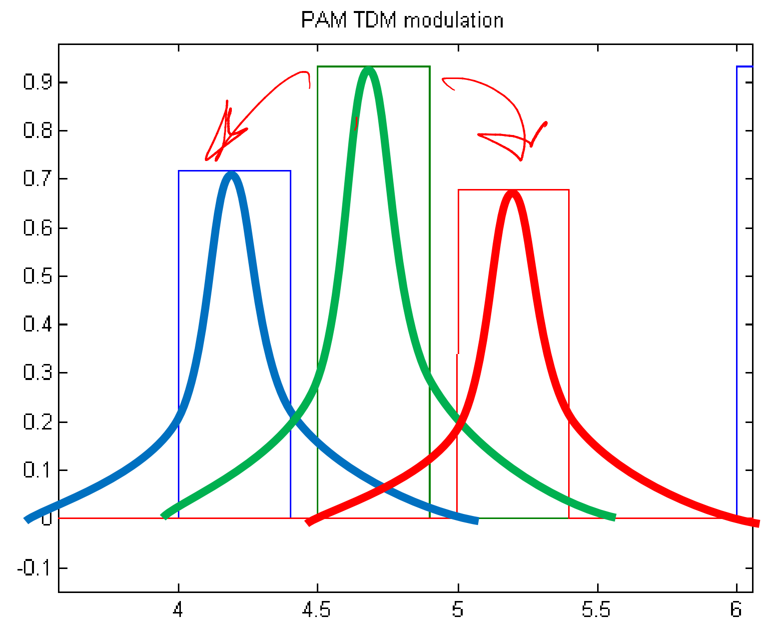
# Pulse Amplitude Modulation

Time Division Multiplexing in PAM systems

- Each time slot allocated to one signal, deciphered by a timing sequence (using additional bandwidth)
- Each pulse is affected by channel distortion (smearing effect) causing “Intersymbol Interference”
- Digital Pulse Amplitude Modulation
  - Sending only quantized values, e.g. 0-1 reduces intersymbol interference but increases the bandwidth of the transmission
- Pulse Code Modulation
  - Encoding the transmitted binary signal with error correction and encryption codes. Transmitted code can be a vector selected to reduce susceptibility to noise.



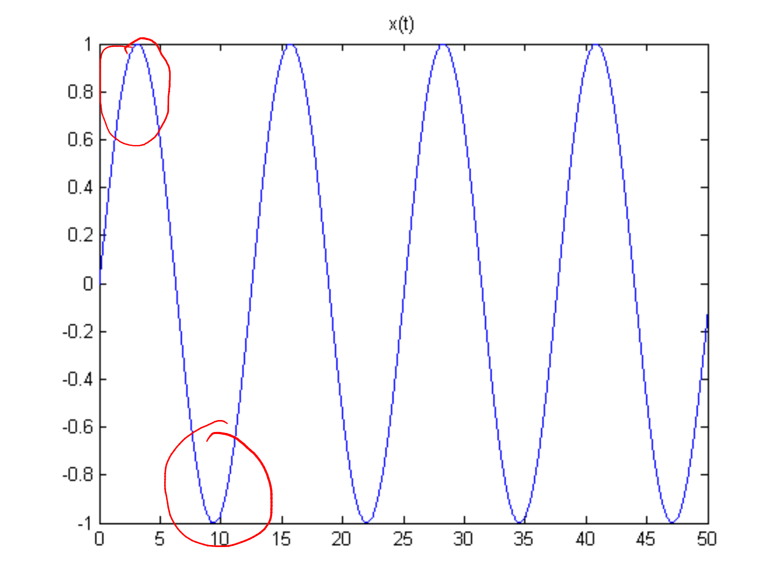



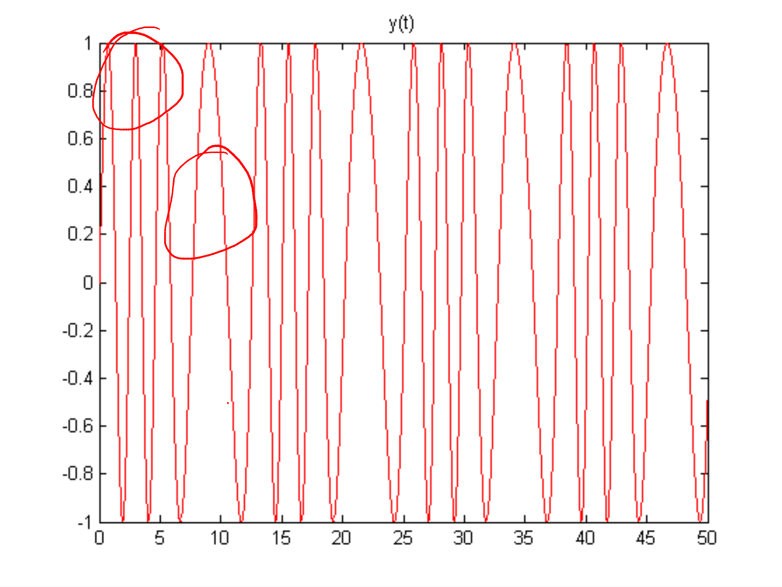
# TDM-FDM Example

- 4 voice signals with bandwidth 4kHz are transmitted over a TDM and an FDM communication system. How much bandwidth is required by each system?
  - The TDM system multiplexes 4 signals using PAM plus synchronizing pulse train sampled at 4kHz x 2 = 8kHz minimum rate. The sampling time is 1/8kHz = 125us. The samples of the five signals must be included in this interval, implying that each is allocated 125/5 = 25us. Therefore, the channel bandwidth should be 1/25us = 40kHz.
  - The FDM system uses 4kHz for each signal in adjacent frequency intervals. The total bandwidth used is 4 x 4 = 16kHz.
  - Notes: Practical PAM would typically use more code for error correction. FDM would also require a fraction of the bandwidth for filter roll-off.

# Sinusoidal Frequency Modulation

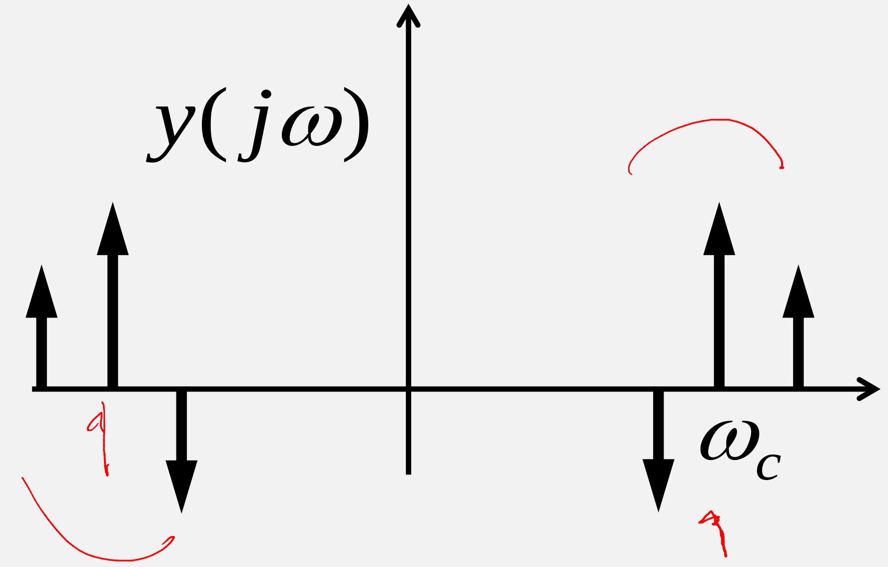
- Angle Modulation schemes: $c(t) = A\cos(\omega_c t + \theta_c) = A\cos(\theta(t))$
    - Modulate the angle or frequency of the sinusoid with the signal.
    - Have the advantage of constant amplitude transmission, which allows the transmitter to operate at peak efficiency.
    - Use more bandwidth than AM but with better quality
- Phase Modulation $\theta_c(t) = \theta_0 + K_p x(t)$
- Frequency Modulation $\dfrac{d\theta}{dt} = \omega_c + K_f x(t)$
    - Phase and Frequency modulation are related: PM of $x(t)$ is equivalent to FM of $\dfrac{dx(t)}{dt}$, and FM of $x(t)$ is equivalent to PM of $\int x(t)$



# Frequency Modulation details

- Instantaneous frequency: $y(t) = A\cos(\theta(t)) \Rightarrow \omega_i = \dfrac{d\theta(t)}{dt}$
- Narrowband FM




$$\omega_i = \omega_c + K_f \underbrace{A\cos\omega_m t}_{x(t)}$$

$$\Delta\omega = K_f A, \quad m = \frac{\Delta\omega}{\omega_m} = \text{modulation index}$$

$$y(t) = \cos\left[\omega_c t + \int x\right] = \cos\left[\omega_c t + \frac{\Delta\omega}{\omega_m}\sin\omega_m t\right]$$

$$= \cos[\omega_c t]\cos[m\sin\omega_m t] - \sin[\omega_c t]\sin[m\sin\omega_m t]$$

$$\approx \cos[\omega_c t]\times 1 - \sin[\omega_c t]m\sin\omega_m t$$

- Looks like AM but modulated signal has constant amplitude

# Frequency Modulation details

- Wideband FM

$$\omega_i = \omega_c + K_f \underbrace{A\cos\omega_m t}_{x(t)}$$

$$\Delta\omega = K_f A, \quad m = \frac{\Delta\omega}{\omega_m} = \text{modulation index}$$

$$y(t) = \cos\left[\omega_c t + \int x\right] = \cos\left[\omega_c t + \frac{\Delta\omega}{\omega_m}\sin\omega_m t\right]$$

$$= \cos[\omega_c t]\cos[m\sin\omega_m t] - \sin[\omega_c t]\sin[m\sin\omega_m t]$$

$$=> \text{Bessel functions}$$

- The communication system bandwidth is much larger than the carrier frequency and depends on the amplitude of the signal.

# Modulation Applications

- Implementation of Modulation $x_m(t) = \cos(\omega_c t)x(t)$

$$x_1(t) = \boxed{\cos(\omega_c t) + x(t)}$$

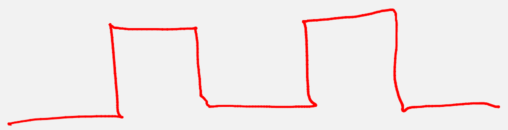
$$x_2(t) = \boxed{x_1^2(t)} = (\cos(\omega_c t) + x(t))^2 = \underbrace{\frac{1}{2}\cos(2\omega_c t)}_{\text{Hi-freq}} + \underbrace{\frac{1}{2} + x^2(t)}_{\text{Low-freq}} + 2\cos(\omega_c t)x(t)$$

$$H_{bandpass}[x_2] = \cos(\omega_c t)x(t)$$

$2\omega_M$

$\omega_c \pm \omega_M$

- Chopper Amplifier: AC amplification is easier. With s(.) square wave, e.g., from a switching transistor (common in modulation/demodulation)



$s(t)$

$$x_m(t) = s(\omega_c t)x(t)$$

$$H_{amplify\text{-}bandpass}[x_m] = Ax_m(t)$$

$$x_d(t) = s(\omega_c t)Ax_m(t)$$

$$x_a(t) = H_{lowpass}[x_d]$$