EEE 582 HW#4 SOLUTIONS

PR. 19

A. $Q_c = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 3 \end{pmatrix} \Rightarrow \det Q_c = -1 \Rightarrow$ c.c.

$Q_o = \begin{pmatrix} 1 & 2 & 1 \\ -1 & -2 & -1 \\ 1 & 2 & 1 \end{pmatrix} \Rightarrow \det Q_o = 0 \Rightarrow$ not c.o.

B. $Q_c = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 2 & 0 & -2 & 0 \end{pmatrix} \Rightarrow \det Q_c Q_c^T = 28 \Rightarrow$ c.c.

$Q_o = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 3 & -1 \\ 0 & -2 & 4 \end{pmatrix} \Rightarrow \det Q_o = 10 \Rightarrow$ c.o.

PR. 20 It is not a well-posed problem.. The given condition is necessary but not sufficient. Eg. for $(A|B) =$

$\left(\begin{array}{ccc|c} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{array}\right)$ is cc. but $\left(\begin{array}{ccc|c} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{array}\right)$ is not cc.

Notice that both have (A_{11}, B_1) not cc..

There can be two possible fixes:

1. $A_{12} = 0$ then $\{(A_{11}\ B_1)$ cc. + (A_{22}, A_{21}) cc$\} \Leftrightarrow (A,B)$ cc
2. (A, B) c.c. $\Rightarrow (A_{22}, A_{21})$ cc.

For both cases, we note that controllability is equivalent to $\text{rk}\,(A - sI, B) = n$ or $\text{rk}\begin{pmatrix} A_{11}-sI & A_{12} & B_1 \\ A_{21} & A_{22}-sI & 0 \end{pmatrix} = n$ for all s. Then we must have $\text{rk}\,([A_{21}, A_{22} - sI]) = n$ forall s. which is equivalent to (A_{22}, A_{21}) c.c.

If $A_{12} = 0$ then (A_{11}, B_1) must be cc. and the converse will also be true ((A_{11}, B_1) cc and (A_{22}, A_{21}) cc $\Rightarrow (A,B)$ c.c.)

Pr.21: We must have $\det \begin{pmatrix} b_1 & a_1 b_1 + d_1 b_2 \\ b_2 & -d_1 b_1 + a_1 b_2 \end{pmatrix} \neq 0$

or, $-d_1 b_1^2 + \underbrace{a_1 b_1 b_2} - \underbrace{a_1 b_1 b_2} - d_1 b_2^2 \neq 0$

or $-d_1 (b_1^2 + b_2^2) \neq 0$

Equivalently (A, B) is c.c. $\iff$ $d_1 \neq 0$ and $\|B\| \neq 0$

Pr.22 Constructed the TV controllability matrix,

$Q_C = (B, -AB + \mathring{B}) = \begin{bmatrix} 0 & -1 \\ 1 & -t \end{bmatrix}$ whose det. is $+1 \neq 0$
$\Rightarrow$ c.c.

The TV observability matrix is $\begin{bmatrix} 0 & 1 \\ 0 & t \end{bmatrix}$ whose det $= 0$
$\Rightarrow$ not c.o.

Pr.23

c.c. realization: $[A, B, C, D] = \left[\begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, [1 \;\; 1], [0] \right]$

c.o. realization $= \left[\begin{pmatrix} -3 & 1 \\ -2 & 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}, (1 \;\; 0), (0) \right]$

Pr.24 There are several possible proofs, e.g. via eig$(A+BK)$, contr. matrix, etc.

Here, we use the definition: If (A, B) is c.c. then for any x_0, x_f there exists $\tilde{u}_0$ s.t. $x_0 \to x_f$. Let $\tilde{x}_0$ be the corresponding state trajectory. Then, the system $\mathring{x} = Ax + BKx + B\tilde{u}$ with $\tilde{u} = \tilde{u}_0 - K\tilde{x}_0$ has the obvious solution $x = \tilde{x}_0$ which must also be unique by existence & uniqueness of ODE solutions. Hence $(A+BK, B)$ is c.c. Similarly for the converse –
