# Adaptive/Self-Tuning PID Control by Frequency Loop-Shaping

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#### **Outline**

- Problem Description: PID Tuning from Input-Output data
- Frequency Loop Shaping
  - Off-line tuning
  - Target loop selection, 1st-2nd order targets
- Direct Adaptation of the PID parameters
  - Cost functional
  - Regressor generation via filter banks
  - Adaptation
  - Performance Monitoring Implications
- Simulation Results
- Conclusions

# **Problem Description**

- Industrial Applications
  - Large number of PID loops, often poorly tuned
  - Reliability and expediency requirements
- A Variety of PID Tuning Strategies
  - Complete or partial models. (System identification-based vs. crossover properties)
  - Control objectives (Time-Frequency domain)
  - Direct and indirect approaches to adaptation
- Frequency Loop Shaping
  - Accounting for uncertainty, several successful applications

## FLS PID Tuning (batch/off-line)

- System ID-modeling from I/O data
- Nominal model & uncertainty bounds
- Control Objective
  - Loop-shaping (sensitivity targets)
  - Disturbance attenuation subject to bandwidth constraints
  - Guide: "Robust Stability Condition"
- On-line version via indirect adaptation
  - Update plant model, re-tune controller
  - Complete solutions can be computationally demanding
  - Simple models => off-line construction of look-up table for the
     PID gains

# Target Loop Selection and FLS PID Tuning

- Typical Targets:  $\frac{\lambda}{s}$ ,  $\frac{\lambda(s+a)}{s^2}$ ,  $\frac{\lambda(s+a)}{s(s+\epsilon)}$ , ...
  - Target order depends open-loop/closed-loop bandwidth ratio (for input disturbance attenuation)
  - Uncertainty constraints and RHP pole-zero limitations
  - More difficult cases via LQ or full-order controller design methods e.g., K = lqr(A,B,Q,R), target: [A,B,K,0]
- FLS Tuning: convex optimization in the frequency domain

$$\begin{aligned} \min_{\theta_{pid}} \left\| S(GC(\theta_{pid}) - L) \right\|_{L_{\infty}} & \min_{\theta_{pid}} \left\| S(GC(\theta_{pid}) - L) \right\|_{L_{2}} \\ s.t. & \theta_{pid} \quad constr. & s.t. & \left\| S(GC(\theta_{pid}) - L) \right\|_{L_{2}} \end{aligned}$$

$$\min_{\theta_{pid}} \left\| S(GC(\theta_{pid}) - L) \right\|_{L_{2}}$$

$$s.t. \quad \left\| S(GC(\theta_{pid}) - L) \right\|_{L_{\infty}} \le b$$

$$\theta_{pid} \quad constr.$$

L=loop gain, S=sensitivity, T=complementary sensitivity

## Direct Adaptation with an FLS objective

• Construction of the estimation error (at the plant input)

$$e_{e} = S(CG - L)[u] = SC[y] - T[u]$$

$$\|S(CG - L)\|_{L_{\infty}} = \sup_{u \neq 0} \frac{\|e_{e}\|_{2}}{\|u\|_{2}}$$

Approximate sup by using a filter bank

$$\left\| S(CG - L) \right\|_{L_{\infty}} \cong \max_{i} \frac{\left\| SCF_{i}[y] - TF_{i}[u] \right\|_{2}}{\left\| F_{i}[u] \right\|_{2}} \leq \max_{i} \frac{\left\| SCF_{i}[y] - TF_{i}[u] \right\|_{2,\delta}}{\left\| F_{i}[u] \right\|_{2,\delta}}$$

-  $F_i$ : band-pass filters,  $\|.\|_{2,\delta}$ : exponentially weighted 2-norm

## Direct Adaptation with an FLS objective (cont.)

Optimization problem

$$\begin{aligned} & \min_{\theta \in M} \max_{i} \frac{J_{i,k}(\theta)}{m_{i,k}} \\ & J_{i,k}(\theta_{k}) = \sum_{n=0}^{k} \lambda^{k-n} |z_{i,n} - w_{i,n}^{T} \theta_{k}|^{2} \\ & m_{i,k} = \lambda m_{i,k-1} + |F_{i}[u]_{k}|^{2} \\ & z_{i,k} = TF_{i}[u]_{k}, \quad w_{i,k} = SC_{\theta} F_{i}[y]_{k} \end{aligned}$$

- Recursive computation of  $J_{i,k}$
- Optimization: min-max of quadratics

#### **Direct Adaptation details**

• Recursive computation of  $J_{i,k}$ 

$$J_{i,k+1}(\theta) = \hat{J}_{i,k+1} - S_{i,k+1}^{T}(\theta - \theta_k) + \frac{1}{2}(\theta - \theta_k)^{T} P_{i,k+1}(\theta - \theta_k)$$

$$\hat{J}_{i,k+1} = \lambda J_{i,k}(\theta_k) + |z_{i,k+1} - w_{i,k+1}^{T}\theta_k|^2$$

$$P_{i,k+1} = \lambda P_{i,k} + 2w_{i,k}w_{i,k}^T, \quad S_{i,k+1} = R_{i,k+1} - P_{i,k+1}\theta_k, \quad R_{i,k+1} = \lambda R_{i,k} + 2z_{i,k}w_{i,k}^T$$

• Each  $J_{i,k+1}$  is quadratic in the parameters: minimize the maximum by, e.g., computing a descent direction and performing a line search

## **Adaptive FLS Properties**

- Excitation requirements
- Effects of disturbances and unmodeled dynamics (SNR)
- A dead-zone condition: update when

$$S_{i,k}^T P_{i,k}^{-1} S_{i,k} - 2d_0^2 m_k > 0$$

- Update when the error operator gain drops by at least  $d_0$
- Input Saturation does not affect updates
- Linearization offsets (estimation or high-pass filtering)
- The cost functional provides a measure of tuning confidence
  - Feasibility of performance monitoring

#### **Example**

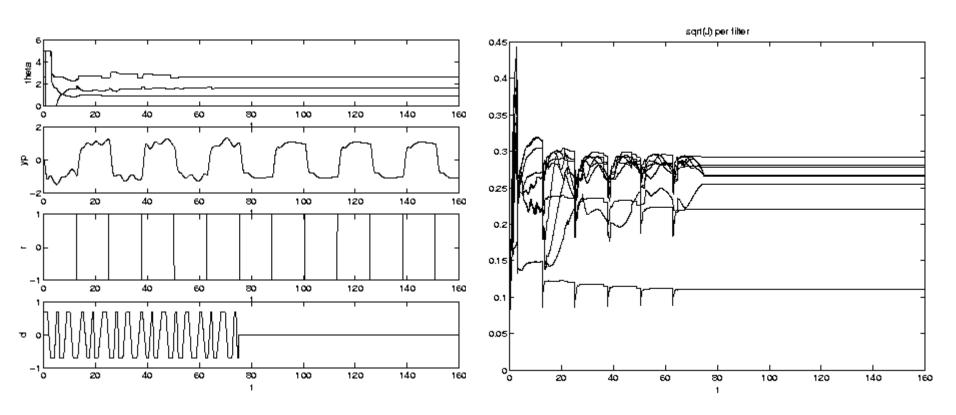
• Simulation Results for the following plant and target loop:

$$G(s) = \frac{1}{(s+1)^3}, \quad L(s) = \frac{1}{s}$$

- Square-wave reference input.
  - Excitation injected at the plant input for t<75.</li>
- PID gains converge approximately to the off-line tuning.
- Cost functional has a maximum of 0.3<sup>2</sup>, same as the off-line fitting error.

#### **Simulation Results**

- Left: Parameters, output, reference, excitation.
- Right: Square-root of cost functional.



#### **Conclusions**

- Direct adaptation of PID parameters with an FLS objective
  - FLS: Operator gain interpretation of fitting error
- Recursive implementation for on-line tuning
- Use of a filter bank to approximate the min-max objective
- Quantitative measures of tuning confidence
  - Gain of the error system

#### Future work:

- On-line monitoring of performance
- On-line adaptation of objective (target loop) based on the cost functional values