ECE 201 HW # 1 SOLUTIONS

Problem 1.12

Current is $I = (72)(24) = 3A$.
Element dissipates power $p = (-6)(4) = -24W$, or it supplies 24W.

Problem 1.17

Element 1 dissipates power $p = (6)(2) = 12W$.
Element 2 dissipates power $p = (4)(-2) = -8W$, or it supplies 8W.

Problem 2.7

Applying KVL clockwise around the loop we have $-1.5 - 1.5 + V_R = 0$ (in V), hence, $V_R = 3$V. Then, $p = V_R I_R = V_R^2/R = 9/1 = 9W$.

Problem 2.10

Applying KCL at the top two nodes we get:
left node: $I_s = I_1 + I_2 = 12mA$
right node: $I_2 = 4mA + 2mA = 6mA$
Hence, $I_1 = 12 - 6 = 6mA$.

Problem 2.27

Applying KVL clockwise in the loop with a clockwise current I we get:
$-12 + (30k)I + 2V_x + V_x = 0$ (in V)
$V_x = (10k)I$ (Ohm's law)
Substituting V_x in the first equation we get: $-12 + (60k)I = 0$. Hence, $I = 0.2mA$

ECE 201 HW # 2 SOLUTIONS

Problem 2.29

Let I be the current flowing clockwise in the loop. Then

$$P_3 = 12mW = V_x(V_x/3k) \Rightarrow V_x = 6V$$

. Then, $I = 6/3k = 2mA$. Next, apply KVL:

$$-V_S + V_x + (6k)I + 2V_x \quad = \quad 0$$

Hence, $V_S = 30V$

Problem 2.38

The three resistors are connected in parallel with an effective resistance

$$R_{eff} = \frac{1}{\frac{1}{2[10^3]} + \frac{1}{3[10^3]} + \frac{1}{6[10^3]}} = 1[10^3]\Omega$$

So the voltage drop across the three resistors is $V_L = R_{eff}I = (1[10^3])((5-2)[10^{-3}]) = 3V$. From this, the current flowing through the $6k\Omega$ resistor is

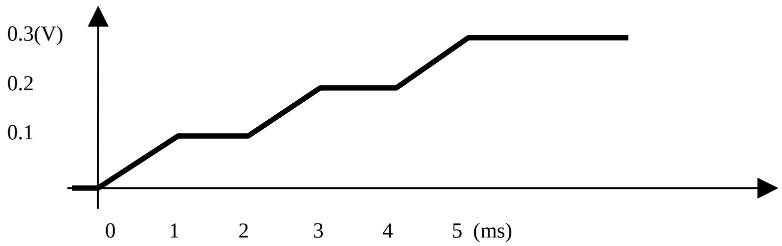
$$I_L = V_L/6[10^3] = 3/(6[10^3]) = 0.5[10^{-3}]A$$

Alternatively, one could use current division (notice the formula) to find

$$I_L = \frac{\frac{1}{6k}}{\frac{1}{2k} + \frac{1}{3k} + \frac{1}{6k}} I_s = 0.5mA$$

ECE 201 HW#3 Solutions

6.18 $i(t) = C\dfrac{dv}{dt}(t) \Rightarrow v(t) = v(0) + \dfrac{1}{C}\int_0^t i(\tau)d\tau$, where *v(0)* represents the initial voltage of the capacitor. Assuming that *v(0) = 0*, we graphically integrate the plot of *i(t) vs. t* to get v(t).
v(t) increases 10mA/100uF in 1ms, i.e., 0.1V in 10ms.



v(t) stays constant between 1-2ms,3-4 ms, and aftert 5ms.

6.21 Assuming that the current change is linear (constant slope and constant voltage), we have that

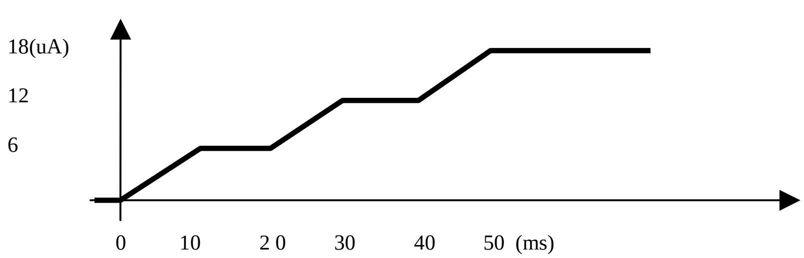
$$L\frac{di}{dt} = v \Rightarrow L\frac{200(10^{-3})}{4(10^{-3})} = 100(10^{-3}) \Rightarrow L = \frac{(4)(100)}{200}(10^{-3}) = 2mH$$, (all units in SI).

6.27 $v(t) = L\dfrac{di}{dt}(t)$. Calculating slopes from the figure, all slopes are the same in absolute value and equal to $100(10^{-3})\big/2(10^{-3}) = 50$. Then the voltage is piecewise constant, according to the following rule

$$v(t) = (50.10^{-3})x\begin{cases} 0 & t < 2(10^{-3}) \\ -50 & 2(10^{-3}) < t < 4(10^{-3}) \\ 50 & 4(10^{-3}) < t < 8(10^{-3}) \\ -50 & 8(10^{-3}) < t < 10(10^{-3}) \\ 0 & 10(10^{-3}) < t \end{cases} = \begin{cases} 0 & t < 2(10^{-3}) \\ -2.5 & 2(10^{-3}) < t < 4(10^{-3}) \\ 2.5 & 4(10^{-3}) < t < 8(10^{-3}) \\ -2.5 & 8(10^{-3}) < t < 10(10^{-3}) \\ 0 & 10(10^{-3}) < t \end{cases}$$

in V and with *t* in s.

6.34 $v(t) = L\dfrac{di}{dt}(t) \Rightarrow i(t) = i(0) + \dfrac{1}{L}\int_0^t v(\tau)d\tau$, so the current will have a slope of 2.4mV/4H = 0.6mA/s or 0, depending on the interval. Hence, when v(t) = 2.4mV, the current increases by 6mA in 10ms.
We assume that *i*(0) = 0 (0 voltage for t<0 and circuit starting at rest at t = - infinity).



ECE 201 HW # 4 SOLUTIONS

Problem 7.9

First, we determine the initial condition for v_c by solving the steady-state problem of the $t < 0$ network.

At steady-state with a constant source, the capacitor will act as an infinite resistance (open) and the voltage across its ends will be the same as the voltage across the 4k resistor. After simplifying the two resistors in parallel

$$12k//4k = \left(\frac{1}{12k} + \frac{1}{4k}\right)^{-1} = 3k$$

a simple voltage division yields

$$v_c(0) = v_{c,ss-} = (6)\left(\frac{4k}{2k + 4k + 3k}\right) = \frac{8}{3}(V)$$

Next, for $t > 0$ the left part of the circuit, including the source, is disconnected and the capacitor discharges through the 4k resistor. Applying KVL, $v_c = v_R = Ri_R$, while $i_R = i_c = -C\frac{dv_c}{dt}$. Hence,

$$RC\frac{dv_c}{dt} + v_c = 0$$

whose solution is

$$v_c(t) = v_c(0)e^{-t/RC}$$

and using the given numerical values

$$v_c(t) = \frac{8}{3}e^{-t/0.8}(V)$$

with t in s.

Problem 7.18

First, we determine the initial condition for the inductor current i_L by solving the steady-state problem of the $t < 0$ network.

At steady-state (with a constant source) the inductor will act as an zero resistance (short) and the current will be the same as the current through the 3Ω resistor. A "brute-force" approach to compute this, is:

- The current through the source is $v_s/(2 + [3//(4 + 2)]) = 12/4 = 3(A)$.
- The voltage at the middle top node is $12 - 2 * 3 = 6(V)$.
- The current through the 3Ω resistor and the inductor is $6/3 = 2(A)$.
- It now follows that the current through the 4-2 resistors is $i(0^-) = 1(A)$ in the direction shown.

Next, for $t > 0$ the left part of the circuit, including the source, is shorted-out and the inductor dissipates energy through the series combination of the 3-4-2 resistors. The current through the resistors is the same as the current through the inductor. So $i(0^+) = -2(A)$. (Notice the discontinuity at 0 as this is not an inductor current.)

The final current in the loop is 0, as it contains no sources. So,

$$i(t) = i(0^+)e^{-t/(L/R)} = -2e^{-t/(2/9))} = -2e^{-4.5t)}(A)$$

Problem 7.44

The general solution for the inductor current will be of the form

$$i_L(t) = K_1e^{-t/\tau} + K_2$$

where $K_1 + K_2 = i_L(0^+)$, $K_2 = i_L(\infty)$ and $\tau = L/R_{Th}$, with R_{Th} being the effective (Thevenin) resistance seen by the inductor.

For the $t < 0$-network, the steady state inductor current is found by replacing the inductor with a wire ("short-circuit"). This implies that the inductor current is the same as the top-4k resistor, while there is no current flowing through the 6k resistor.

Applying "brute-force" simplification of the $t < 0$-network we find:

- Total effective resistance as seen by the source: $12k + [4k//4k//4k] = \frac{40}{3}k$.
- Voltage across the current source: $12m\frac{40}{3}k = 160(V)$ (bottom-to-top).
- Voltage drop across 12k: $(12k)(12m) = 144(V)$.
- Voltage at middle top node: $-16(V)$ relative to bottom node (notice the current direction).
- Current through 4k and coil: $i_0(0^-) = -\frac{16}{4k} = -4mA$ (here i_0 is the inductor current).

Next, for the $t > 0$-network, the steady-state is found similarly, by removing the side-6k resistor and replacing the inductor by a "short." The same computation yields $i_0(\infty) = -4mA$

Hence, $K_1 + K_2 = -4m$ and $K_2 = -4m$, from which, $K_1 = 0$. Since $i_0(0^+) = i_0(0^-)$, this yields immediately that the inductor current is constant $i_0(t) = -4m\ (A)$ for $t > 0$.

In other words, when this circuit is at steady-state the addition or removal of the side-6k resistor does not affect the inductor current. This resistance will play a role during transients or changes in the source current (e.g., AC sources).

While it is not necessary for the solution of this problem, we can compute R_{Th} for completeness. The resistance seen by the inductor in the $t > 0$ network is $4k + (4k//4k//12k) = \frac{40}{7}k\Omega$. This yields a time constant $T = L/R_{Th} = 7m/4k = 1.75\mu s$.

Problem 7.74

The characteristic equation is $s^2 + 8s + 16 = 0$. Hence, the undamped resonant frequency is $w_0 = 4$ ($w_0^2 = 16$). The damping ratio is $\zeta = 8/(2w_0) = 1$. The system is critically damped (double root of the characteristic equation at -4), and the natural frequencies are $-4,\ -4$, implying that the solution has the form

$$i_o(t) = K_1 e^{-4t} + K_2 t e^{-4t}$$

Problem 7.77

Following the derivation of the textbook, the common voltage v satisfies the differential equation

$$C\frac{d^2v}{dt^2} + \frac{1}{R}\frac{dv}{dt} + \frac{1}{L}v = \frac{di_S}{dt}$$

Normalizing the leading coefficient to one,

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v = \frac{1}{C}\frac{di_S}{dt}$$

From this we identify the undamped resonant frequency $w_0 = 1/\sqrt{LC}$ and the damping ratio

$$\zeta = \frac{1}{2}\sqrt{\frac{L}{R^2C}} = \frac{1}{2}\frac{\sqrt{L/R}}{\sqrt{RC}}$$

Substituting the given numerical values we find $w_0 = \frac{1}{\sqrt{1/4}} = 2(rad/s)$, $\sqrt{L/R} = \frac{1}{\sqrt{2}}$, $\sqrt{RC} = \frac{1}{\sqrt{2}}$, and $\zeta = \frac{1}{2}$. Since $\zeta < 1$, the circuit exhibits underdamped behavior.

ECE 201 HW # 5 SOLUTIONS

Problem 8.1

The period of $5\cos(400t - 120^o)$ is $2\pi/400 = 0.0157s$. The frequency is $400/(2\pi) = 63.66Hz$.

Problem 8.2

General procedure:

1. Convert to the same function form (sin or cos; use eqn 8.7-8.8).
2. Subtract phases ($\phi_1 - \phi_2$ = phase by which $i_1(t)$ leads $i_2(t)$).
3. Extract modulo 2π ($mod_{2\pi}(\phi) \in (-\pi, \pi]$ (or $[0, 2\pi)$) and $\phi = 2\pi n + mod_{2\pi}(\phi)$ where n is an integer).

In our problem both signals are cos of the same frequency. $\phi_1 - \phi_2 = -30 - 90 = -120$. Extracting the mod, $mod_{2\pi}(-120) = -120$. So $i_1(t)$ lags 120deg behind $i_2(t)$.

ECE 201 HW # 6 SOLUTIONS

Problem 8.5

a. In phasor notation, $V_1 = 10\angle 180^o$. Then $i_1 = v_1/R = \frac{10}{2}\cos(377t + 180^o) = 5\angle 180^o(A)$.

b. In phasor notation, $V_2 = 12\angle(45-90)^o$. $i_2 = v_2/R = \frac{12}{2}\sin(377t+45^o) = 6\cos(377t-45^o) = 6\angle -45^o(A)$.

Problem 8.7

a. In phasor notation $V_1 = 10\angle -30^o\ (V)$. Capacitor impedance: $Z_c = \frac{1}{jwC} = \frac{1}{j(377)(1\mu)} = 2652.5\angle -90^o\ (\Omega)$ Then, $I_1 = \frac{10\angle -30^o}{2652.5\angle -90^o} = 0.00377\angle 60^o\ (A)$. In the time domain, $i_1(t) = 3.77\cos(377t + 60^o)\ (mA)$.

b. In phasor notation $V_2 = 5\angle(60-90)^o\ (V)$. Capacitor impedance: $Z_c = \frac{1}{jwC} = \frac{1}{j(377)(1\mu)} = 2652.5\angle -90^o\ (\Omega)$ Then, $I_2 = \frac{5\angle -30^o}{2652.5\angle -90^o} = 0.00189\angle 60^o\ (A)$. In the time domain, $i_2(t) = 1.89\cos(377t + 60^o)\ (mA)$.

Problem 8.18 Apply KCL at the top node

$$\begin{aligned}
I_s &= I_R + I_L + I_C = \frac{V}{Z_R} + \frac{V}{Z_L} + \frac{V}{Z_C} \\
20\angle 120^o &= V\left(\frac{1}{1} + \frac{1}{j100[10^{-3}]377} + \frac{1}{\frac{1}{j10[10^{-3}]377}}\right) \\
&= V(1 + 3.74j) \\
&= V(3.87\angle 75.5^o) \\
V &= \frac{20\angle 120^o}{3.87\angle 75.5^o} = 5.16\angle 44.5^o \\
v(t) &= 5.16\cos(377t + 44.5^o)\ \ (V)
\end{aligned}$$

Problem 8.31 The circuit is purely resistive when the overall impedance Z is a positive real number.

$$\frac{1}{Z} = \frac{1}{1} + \frac{1}{j\omega 5m} + \frac{1}{\frac{1}{j\omega 1m}} = \frac{j\omega 5m + 1 + j\omega 1m j\omega 5m}{j\omega 1m} = \frac{\omega 1m - j + j\omega^2 5\mu}{\omega 1m}$$

So Z is real if $1 = \omega^2 5\mu$ or $\omega = \sqrt{0.2}k(rad/s) = 447.2(rad/s) = 71.2(Hz)$

Problem 8.34

Current division yields

$$I = \frac{1/1}{1/1 + 1/j1}100\angle 0^o = \frac{1}{1-j}100\angle 0^o = \frac{1}{\sqrt{2}}\angle 45^o 100\angle 0^o = 70.7\angle 45^o(A)$$

Problem 8.31

Voltage division yields

$$V = \frac{-j1}{1+(-j1)}100\angle 0^o = \frac{-j}{1-j}100\angle 0^o = \frac{1}{\sqrt{2}}\angle -45^o 100\angle 0^o = 70.7\angle -45^o(V)$$

ECE 201 HW # 7 SOLUTIONS

Problem 2.49

Reducing the circuit from the far right:
$2k$ series $2k = 4k$
$4k//4k = 2k$
$2k$ series $2k = 4k$
$4k//12k = 3k$
$3k$ series $9k = 12k$
Thus, $R_{AB} = 12k\Omega$.

Problem 2.51

As in the previous problem:
$12k//4k = 3k$
$3k$ series $9k = 12k$
$12k//6k = 4k$
$4k$ series $4k = 8k$
$8k//8k//12k = 3k$
$3k$ series $2k = 5k$
Thus, $R_{AB} = 5k\Omega$.

Problem 2.102

Let us find the total current through the source first: The two 6k resistors with the 12k form a Y which we change to Δ. That yields two 30k resistors in parallel with the 2k and the 18k and one 15k across the ends of the voltage source.

The equivalent total resistance is $[(30k//2k) + (18k//30k)]//15k = 7k$.

Now, total absorbed power is $V^2/R = 21^2/7k = 63m(W)$

Problem 2.110

Applying KVL,

$$-12 + 3kI_s - 2000I_s + 1kI_s = 0 \Rightarrow I_s = 6m(A)$$

Then $V_o = 1kI_s = 6(V)$.

ECE 201 HW # 8 SOLUTIONS

Problem 2.116

First, we find V_{in} by voltage division (since the two parts of the circuit are decoupled):

$$V_{in} = \frac{R_{in}}{R_s + R_{in}} V_s$$

Similarly,

$$V_o = \frac{R_L}{R_o + R_L} \mu V_{in} = \frac{R_L}{R_o + R_L} \frac{R_{in}}{R_s + R_{in}} \mu V_s$$

The maximum V_o occurs when as $R_{in} \to \infty$ and $R_o = 0$. In such a case, the ratio V_o/V_s approaches μ, regardless of the value of R_L.

Problem 2.117

First, we find I_b: The total resistance seen by the source is $(5k//500) + 100 = 554.54\Omega$. Hence the total current through the source is 0.4508m(A), and the voltage drop across the 100Ω resistor is 0.0451(V). So the voltage drop across the 500Ω is 0.2049(V) and $I_b = 0.4098m(A)$.

Next, we apply voltage division to find V_o:

$$V_o = \frac{300}{300 + 4000}(4e5)(-0.4098m) = -11.436(V)$$

Hence, the amplifier gain is $G = -11.436/0.25 = -45.74$.

Problem M1 *A voltmeter with internal resistance 10-MΩ and a 10-bit, ±10V A/D converter is used to measure a voltage of 0.95V and a voltage of 1.05V. Find the worst-case percent-error in the voltage reading due to quantization.*

The A/D converter quantizes its entire range into 2^{10} different states, so its resolution is (10-(-10))/1024=0.02V. For the first measurement, we assume that an internal amplifier is used to amplify voltages less than 1V to the range of the A/D. After the measurement is obtained the voltmeter processor divides the measurement by 10 to produce the measured value. This means that the quantization error is also divided by 10, i.e., it is 0.002V. (The same result we get if we quantize the interval ±1 into 2^{10} states.) Now, the maximum quantization error is 1 bit, corresponding to 0.002V for a measured value 0.95V. That is approximately 0.2% error.

On the other hand, to measure the second voltage we must use the ±10 scale in the voltmeter. For this, the internal amplifier gain is 1, the quantization error is 0.02V and for a measured value 1.05V, this translates into a 2% error.

Problem M2 *A voltmeter with internal resistance 10-MΩ is used to measure the voltage across a 1-MΩ resistance, driven by an 1-μA current source. Find the percent-error in the voltage reading introduced by the use of the meter.*

Before using the meter, the voltage across the resistor is $v_{act} = (1M)(1\mu) = 1V$. After introducing the meter (connecting the resistances in parallel) the effective resistance is $\{1M//10M\} = 10/11M = 0.91M$ and the measured voltage is $v_{meas} = (0.91M)(1\mu) = 0.91V$. This represents a $(1 - 0.91)/0.91 \simeq 10\%$ error on the measurement.

Problem M3 *For the same setting as in Problem M2, suppose that the meter uses a 12-bit A/D converter, with a range of ± 10V. Find the value of the internal resistance such that the error in the voltage is comparable with the resolution of the meter. Assume first that the meter is used at the 1V setting and repeat for the 10V setting.*

The resolution of the meter is $20/(2^{12}) = 20/4096 \simeq 0.005V$ at 10V. If the meter is used at the 1V setting, the entire range corresponds to $\pm 1V$ measurement, i.e., the resolution is 0.0005V. Following the M2 computations, to have a comparable measurement error the measured voltage should be at least 0.9995V. Hence, the combined resistance should be at least 0.9995MΩ. Solving the parallel connection formula for the internal resistance (i.e., $\{1M//R_{met}\} = 0.9995M$) we find $R_{met} = 1999M \simeq 2G\Omega$, at least.

For the 10V setting, the resolution is 0.005V. Now the measured voltage should be at least 0.995V, the combined resistance 0.995MΩ and the internal resistance $199M\Omega$.

Problem M4 *A meter with internal resistance 10-MΩ, and a 10-bit, $\pm$10V, A/D converter is used to measure the current through a 100Ω resistance, driven by an 10-V voltage source. Find the value of the shunt resistance so that the error introduced by the meter is comparable to the resolution of the meter.*

This is the current-analog of M3 but its solution is more involved. First, let us denote by v_0 and R_0 the voltage of the source and the 100Ω resistance. Then, denote by i the current that flows through the shunt resistance, when connected, and v the voltage across it. And let $\hat{i}, \hat{v}$ be the measured values of the same quantities and δ be the quantization error. Notice that the optimal value of $\delta = v/2^{10}$ is not always feasible since such an adjustment of the range may require unreasonably large amplification gains. In the following we assume that the amplifier gain is set at 10, so $\delta = 0.02$.

When we connect the meter, the shunt resistance is in series and the current flowing through it is $i = v_0/(R_0 + R)$, or the voltage is $v = v_0 R/(R_0 + R)$. What we measure is $\hat{v} = v + \delta$ (more precisely $|\hat{v} - v| \leq \delta$). After the current computation, this yields $\hat{i} = \hat{v}/R = i + \delta/R$. Notice that the last term is the effective error in the current due to quantization and becomes smaller for bigger R. (Why?)

Next, substitute the expression for the original current i_0

$$\begin{aligned} \hat{i} &= \frac{i_0}{1 + R/R_0} + \frac{\delta/R_0}{R/R_0} \\ &= i_0 - \frac{i_0(R/R_0)}{1 + R/R_0} + \frac{\delta/R_0}{R/R_0} \end{aligned}$$

Observe that the use of normalized quantities brings out nicely the fact that the important quantity is the ratio of the shunt vs. the circuit resistance. Let us denote this ratio by x and define $\Delta = \delta/(i_0 R_0)$. The latter is the voltage quantization error normalized by the original voltage across the resistor R_0.

So,

$$\hat{i} = i_0 - i_0 \left[\frac{x}{1 + x} - \frac{\Delta}{x} \right]$$

The two bracketed terms are the measurement error. The first one is the error due to the introduction of the shunt resistance and it increases with x (the ratio shunt-to-original). The second term is due to quantization and decays with x. Our problem is now to find the value of x where these two terms have equal magnitude. (Notice that Δ is random, so this does not mean that the two errors will offset each other.) Now Δ is usually small and that implies that x is also small so $1 + x \simeq 1$. From this, it follows that $x \simeq \sqrt{\Delta}$.

Recalling our definitions of variables,

$$R = R_0 \sqrt{\frac{\delta}{i_0 R_0}} = R_0 \sqrt{\frac{\delta}{v_0}}$$

Finally, bringing in the given numerical values

$$R = 100 \sqrt{\frac{0.02}{10}} = 4.5\Omega$$

For this value, the worst-case error in the current measurement is approximately 9% (each source of error contributes 4.5%).

ECE 201 HW # 9 SOLUTIONS

Problem 3.7

Define the voltages at the top nodes as v_1, v_2, v_3 from left to right. Then, $v_0 = v_2 - v_3$ and nodal analysis yields the equations:

$$\begin{aligned} v_1 &= 12 \\ \frac{v_2 - v_1}{6k} + \frac{v_2}{6k} + \frac{v_2 - v_3}{12k} &= 0 \\ v_3 &= 6 \end{aligned}$$

From these, we easily get that $v_2 = 6$ (V) and $v_0 = 0$ (V).

Problem 3.64

Define the clockwise currents i_1, i_2 in the left and right loops, respectively. Then, loop analysis yields the equations:

$$\begin{aligned} 2ki_1 + 4ki_1 + 6k(i_1 - i_2) + 12 &= 0 \\ 6k(i_2 - i_1) + 4ki_2 + 2ki_2 - 12 &= 0 \end{aligned}$$

From which we find $i_2 = \frac{2}{3}[10^{-3}](A)$ and, therefore, $v_0 = 2ki_2 = \frac{4}{3}(V)$

Problem 8.48

Define the voltages at the top nodes as v_1, v_2 from left to right. Then, $i_0 = v_1/2$ and nodal analysis yields the equations:

$$\begin{aligned} \frac{v_1 - 12\angle 0^o}{2} + 2\angle 0^o \frac{v_1}{2} + \frac{v_1 - v_2}{1} &= 0 \\ \frac{v_2 - v_1}{1} + \frac{v_2}{-j} - 4\angle 0^o &= 0 \end{aligned}$$

From these, using MATLAB, we get that $v_1 = 1.6 - 1.2j = 4\angle - 36.87^o$ (V) and $i_0 = 2\angle - 36.87^o$ (V).

Problem 8.63

Define the clockwise currents i_1, i_2 in the left and right loops, respectively. Then, loop analysis yields the equations:

$$\begin{aligned} -6\angle 0^o - jI_1 + 2j(I_1 - I_2) &= 0 \\ 2j(I_2 - I_1) - 12\angle 45^o + 2I_2 &= 0 \end{aligned}$$

From which, using MATLAB, we find $i_2 = -3 + 1.243j = 3.247\angle 157.5^o(A)$ (remember to add 180^o correction for the negative real part) and, therefore, $v_0 = 2i_2 = 6.494\angle 157.5^o(V)$

Problem 8.67

Define the clockwise currents i_1, i_2, i_3, clockwise from the top loop. Then, loop analysis yields the equations:

$$\begin{aligned} I_3 - I_1 &= 2\angle 0^o \\ I_2 &= 4\angle 180^o \\ 2I_1 + j1(I_1 - I_2) + 2(I_2 - I_3) - j2I_3 &= 12\angle 0^o \end{aligned}$$

We find the solution using Matlab:

$$I_1 = 0 + j0(A), \quad I_2 = -4 + j0(A), \quad I_3 = 2 + j0(A)$$

Then, $I_0 = I_3 - I_2 = 6 + j0 = 6\angle 0^o(A)$.

ECE 201 HW # 10 SOLUTIONS

Problem 4.22

First, apply KCL at the inverting input node. This yields:

$$\frac{V_- - V_i}{2} + \frac{V_- - V_o}{10} = 0$$

Now, from the ideal op-amp assumptions, $V_- = V_+$ and $i_+ = 0$. The latter implies that $V_+ = 0$ (KCL at the non-inverting input node) and, hence,

$$V_o = -\frac{10}{2}V_i = -10(V)$$

This is the well-known ideal op-amp equation and is independent of R_1. The reason is that regardless of the value of the resistance V_+ is kept at zero. (This is true as long as R_1 is small relative to the internal op-amp impedance; as R_1 grows there will be a discrepancy both in the op-amp gain and its dynamic response.)

Problem 4.25

First, apply KCL at the inverting input node. This yields:

$$\frac{V_- - V_5}{1} + \frac{V_- - V_o}{4} = 0$$

where $V_5 = 5$ (V). Now, from the ideal op-amp assumptions, $V_- = V_+ = V_4 = 4$ (V). Solving for V_o we get

$$V_o = -\frac{4}{1}V_5 + \left(1 + \frac{4}{1}\right)V_4 = -20 + 20 = 0(V)$$

The last equation shows the general dependence of the op-amp output to voltages at both inverting and non-inverting inputs.

Problem 4.50

The first op-amp performs the addition so that

$$V_x = -\sum_{i=1}^{3} \frac{R_f}{R_i} V_i$$

The second op-amp acts as an inverting gain to recover the positivity of the voltage.

$$V_o = -\frac{R_B}{R_A} V_x$$

To average the three measurements, one option is to choose $R_f/R_i = 1/3$ for all i. Further, to achieve 1V/10deg slope (or 0.1V/deg) while the original thermocouples produce 0.025V/deg, we need an overall gain of 4. For that we could choose $R_B/R_A = 4$. Trying to keep a relatively high input impedance we can choose $R_i = 30k$, $R_f = R_A = 10k$, $R_B = 40k$ (all in Ω).

(Even better, take the desired ratios and input impedance constraints and visit `http://www.eecircle.com/applets/008/ResPicker.html`).

Problem 6.73

The input and the output of the op-amp are related by $V_o = -\frac{Z_o}{Z_i}V_i$. where $Z_o = \frac{1}{j\omega C} = \frac{1}{j(377)(20\mu)}$ and $Z_i = 4k$. Expressing the input voltage in phasor notation,

$$V_i = 115\sin(377t) = 115\cos(377t - 90^o) = 115\angle - 90^o$$

So,

$$V_o = -\frac{1}{j(377)(20\mu)(4k)} 115\angle - 90^o = (0.033\angle 90^o)(115\angle - 90^o) = 3.81\angle 0^o$$

Or, $V_o(t) = 3.81\cos(377t)$.

ECE 201 HW # 11 SOLUTIONS

Problem 5.7

Keeping the voltage source only and replacing the current source by an open circuit, we can reduce the circuit and find the voltage across the $2k\Omega$ resistor with voltage division: The total resistance seen by the source is $R_{TOT} = 6k + \{2k//[3k + 3k]\} = 7.5k\ \Omega$. Using voltage division the voltage across the resistor is

$$V_2 = \frac{3/2k}{6k + 3/2k} V_{in} = 6/5(V) \Rightarrow I_{0,1} = \frac{6/5}{2k} = 0.6m\ (A)$$

Keeping the current source only and replacing the voltage source by a short circuit, we can use nodal analysis to find the voltage across the $2k\Omega$ resistor:

$$\begin{aligned} \frac{V_1}{3k} + \frac{V_1 - V_2}{3k} - 2m &= 0 \\ \frac{V_2}{2k} + \frac{V_2}{6k} + \frac{V_2 - V_1}{3k} &= 0 \end{aligned}$$

From this, $V_2 = 1.2\ (V)$ and $I_{0,2} = 1.2/2k = 0.6m\ (A)$.

Then, using superposition, $I_0 = I_{0,1} + I_{0,2} = 1.2m\ (A)$.

Problem 5.10

First, keep the left V-source and replace the top V-source by a short and the I-source by an open. Then the voltage across the 4k resistor of interest is $\frac{4k}{4k+4k}12 = 6(V)$ and $I_o = \frac{6}{4k} = 1.5m(A)$.

Next, keep the top V-source and replace the left V-source by a short and the I-source by an open. Then the two resistors-in series including the resistor of interest are shorted out, the voltage drop across their ends is zero and therefore, $I_o = 0$.

Finally, keep the I-source and replace the V-sources by shorts. Then $6mA$ current flows through the parallel combination of the two 4k resistors including the resistor of interest. Hence, $I_o = \frac{4k}{4k+4k}6m = 3m(A)$.

Hence, the total current is $I_o = 1.5m + 0 + 3m = 4.5m(A)$.

Problem 5.67

There can be several different "applications" of Thevenin, but the one that makes the most sense is to find the equivalent of the circuit that includes the left portion up to the Ix-resistor. (Notice that in any case, the dependent source and the Ix resistor cannot be separated.)

Finding V_{OC}: Let V_1 the voltage of the left top node. Then

$$\begin{aligned} 2I_x + \frac{V_1}{1k} + \frac{V_{OC}}{1k} &= 0 \\ V_{OC} - V_1 &= 12 \\ I_x &= \frac{V_{OC}}{1k} \end{aligned}$$

From which, $V_{OC} = 3(V)$.

Finding I_{SC}: $I_x = 0$ because the Ix-resistor is shorted-out. So, the current dependent source gives zero current (open circuit) and $I_{SC} = \frac{12}{1k} = 12m(A)$.

Thus, $R_{Th} = \frac{V_{OC}}{I_{SC}} = \frac{3}{12m} = 250(\Omega)$. Finally, by voltage division,

$$V_o = \frac{1k}{R_{Th} + 1k + 1k} V_{OC} = 1.33(V)$$

Problem 8.74

Working with the voltage source first, we set the current source to zero, to obtain a circuit with the capacitor in series with the inductor, both in parallel with the 1Ω resistor, then in series with the 1Ω resistor. So, using the voltage division formula,

$$V_{o1} = -\frac{1}{1 + [1\|(j1 + -j1)]} 6\angle 0 = -6\angle 0^o = 6\angle 180^o(V)$$

Next, setting the voltage source to zero, the two resistors become connected in parallel and $V_o = 1/2 I_R$, where I_R is found through current division:

$$V_{o2} = \frac{1}{2} \frac{\frac{1}{1/2+j}}{\frac{1}{1/2+j} + \frac{1}{-j}} 6\angle 0 = -j6\angle 0^o = 6\angle -90^o (V)$$

Adding the two,

$$V_o = (-1-j)6\angle 0^o = 6\sqrt{2}\angle(-180+45)^o = 6\sqrt{2}\angle -135^o = 8.49\angle -135^o (V)$$

ECE 201 HW # 12 SOLUTIONS

Problem 5.69

To apply Thevenin, consider the circuit without the 1k resistor corresponding to V_o. This circuit has dependent and independent sources, so we need to compute V_{oc} and I_{sc} to find the Thevenin equivalent.

Let us first denote the clockwise loop currents by $I_1 - I_4$, clockwise from the top left.

Finding V_{oc} is rather easy, since clearly $I_2 = 0$, implying that $V_x = 0$, and then the dependent source produces zero current. So we only have two independent loops and their currents are specified by the sources: $I_1 = 1m\ (A)$ and $I_3 = -4m\ (A)$. Now we compute the voltage drops across the resistances through KVL, yielding $V_{oc} = -8\ (V)$.

Finding I_{sc} is more complicated, and we need to perform a complete loop analysis. The equations are

$$\begin{aligned}
&I_1 - I_2 = 1m\\
&I_3 = -4m\\
&I_4 = \frac{2V_x}{1000} = \frac{2(-I_2 1k)}{1000} = -2I_2\\
&(I_1 - I_4)1k + I_1 2k + I_1 1k + I_2 1k + (I_2 - I_3)1k = 0
\end{aligned}$$

the last being the supermesh equation. Using Matlab to compute the solution, we find the current of interest $I_{sc} = I_2 = -1m\ (A)$.

Hence, the Thevenin equivalent is a -8V source in series with an $R_{TH} = \frac{V_{oc}}{I_{sc}} = 8k\ \Omega$ resistor. This is now in series with the 1k resistor of the complete network and V_o is the voltage across the 1k resistor. This can be found by voltage division so $V_o = \frac{1k}{1k+8k}V_{oc} = -8/9\ (V)$.

Unfortunately, Thevenin's Theorem did not save us any work in this problem. A straightforward loop analysis yields the same answer and the amount of work is equivalent to finding I_{sc}.

Problem 5.87

Transform the 6V voltage source into a 1.5mA current source in parallel with a $4k\Omega$ resistor and the combined series of the two $6k\Omega$'s. All these resistors combine into a $3k\Omega$ resistor.

Transform the 1.5mA current source to a 4.5V voltage source, in series with the $3k\Omega$ resistor. Now combine the series of the two $3k\Omega$'s. Convert back to a 3/4mA current source in parallel with a $6k\Omega$ resistor.

Add the two current sources connected to the same two nodes. Convert back to a 33/2V voltage source and a $6k\Omega$ resistor in series.

Now the voltage sources add to 9/2V (opposite polarity) and transform to a 3/4mA current source with a $6k\Omega$ resistor in parallel.

The rest of the circuit is the $3k\Omega$ and a $6k\Omega$ effective resistance from the series of the $3k\Omega$'s. The final answer is now obtained by current division, $I_0 = -0.375mA$.

Problem 8.87

Break the circuit at the capacitor. Then $V_{OC} = V_x$. Nodal analysis yields

$$2V_{OC} + \frac{V_{OC} + V_1}{1} + \frac{V_{OC}}{-j1} = 0 \Rightarrow V_{OC} = -3.6 + j1.2(V)$$

To find I_{SC} short the capacitor and observe that $V_x = 0$. Then it is simply $I_{SC} = \frac{-12\angle 0^o}{1} = -12\angle 0^o(A)$.

The Thevenin impedance is

$$Z_{Th} = \frac{V_{OC}}{I_{SC}} = 0.3 - j0.1\Omega$$

and it is connected in series with the voltage source V_{OC}, the inductor and the output resistance. After voltage division, we get $V_o = 2.4\angle 127^o(V)$

Problem 8.90

Break the circuit after the voltage source and find I_{SC} using nodal analysis:

$$I_{SC} + 2\angle 0^o + \frac{11.3\angle 45^o}{4\angle 90^o} = 0 \Rightarrow I_{SC} = -0.0024 - j1.9976(A) \simeq -j2(A)$$

Because there are only independent sources $Z_{Th} = j4(\Omega)$ (replace v-sources by shorts and i-sources by opens).

Thus, using Norton's theorem, the circuit transforms to an I_{SC} current source in parallel with Z_{Th} and in parallel with the resistance and the capacitor. Then

$$V_x = I_{SC}\,[Z_{Th}||10|| - j3] = -9.84 - j11.78 = 15.35\angle - 129.88^o(V)$$

Problem 9.26

For complex impedances, the maximum average power transfer occurs at $Z_{Load} = Z^*_{Th}$, the complex conjugate of the Thevenin equivalent impedance.

For this circuit (no dependent sources), $Z_{Th} = j1 + 1 \Rightarrow Z_{Load} = 1 - j(\Omega)$.

After transforming to a $6\angle 0^o(V)$ voltage source in series with 1Ω resistor, the total impedance is 2Ω and the total current flowing through the load is

$$I_{Load} = \frac{6\angle 0^o(V)}{2\Omega} = 3\angle 0^o(A)$$

Thus, $P_{max} = \frac{1}{2}I_M^2 R_L = 4.5(W)$, where I_M is the current amplitude and R_L is the resistive part of the load (see Chapter 9 for details).

Problem 9.27

First, find the Thevenin equivalent of the circuit without the load. Applying superposition, $V_{OC} = V_S + (R_1 + Z_C)I_S = 12\angle 30^o + (1 - j1)4\angle 0^o = 14.53\angle 7.91^o(V)$.

Then, by inspection, $Z_{Th} = 1 + (-j1) + 4 + (j1) = 5(\Omega)$.

Z_{Th} is purely resistive so the optimum $Z_L = 5(\Omega)$ is also purely resistive. Then $I = V_{OC}/(Z_{Th} + Z_L) = 1.45\angle 7.91^o(V)$, and $P_{max} = \frac{1}{2}I_M^2 R_L = 5.26(W)$.

ECE 201 HW # 13 SOLUTIONS

Problem 4.60

To apply Thevenin, consider the circuit without the 1k resistor corresponding to V_o. This circuit has dependent and independent sources, so we need to compute V_{oc} and I_{sc} to find the Thevenin equivalent.

Let us first denote the clockwise loop currents by $I_1 - I_4$, clockwise from the top left.

Finding V_{oc} is rather easy, since clearly $I_2 = 0$, implying that $V_x = 0$, and then the dependent source produces zero current. So we only have two independent loops and their currents are specified by the sources: $I_1 = 1m\ (A)$ and $I_3 = -4m\ (A)$. Now we compute the voltage drops across the resistances through KVL, yielding $V_{oc} = -8\ (V)$.

Finding I_{sc} is more complicated, and we need to perform a complete loop analysis. The equations are

$$
\begin{aligned}
&I_1 - I_2 = 1m \\
&I_3 = -4m \\
&I_4 = \frac{2V_x}{1000} = \frac{2(-I_2 1k)}{1000} = -2I_2 \\
&(I_1 - I_4)1k + I_1 2k + I_1 1k + I_2 1k + (I_2 - I_3)1k = 0
\end{aligned}
$$

the last being the supermesh equation. Using Matlab to compute the solution, we find the current of interest $I_{sc} = I_2 = -1m\ (A)$.

Hence, the Thevenin equivalent is a -8V source in series with an $R_{TH} = \frac{V_{oc}}{I_{sc}} = 8k\ \Omega$ resistor. This is now in series with the 1k resistor of the complete network and V_o is the voltage across the 1k resistor. This can be found by voltage division so $V_o = \frac{1k}{1k+8k} V_{oc} = -8/9\ (V)$.

Unfortunately, Thevenin's Theorem did not save us any work in this problem. A straightforward loop analysis yields the same answer and the amount of work is equivalent to finding I_{sc}.

Problem 7.70

The Norton equivalent of this circuit is a current source I_{sc} with an impedance in parallel Z_{TH}. Then $V_x = V_{oc} = I_{sc} Z_{TH}$. To find I_{sc} we short the terminals of V_x and find the current that passes through. This eliminates the resistor and the capacitor from the computation. Now, using a source transformation on the current source and the inductor, we get a voltage source $(4\angle 0)(j4)$ in series with the same $j4$ inductor. These are also in series with the voltage source $11.3\angle 45^o$, The current that flows through this loop is

$$I_{sc} = \frac{(4\angle 0)(j4) - 11.3\angle 45^o}{j4} = 4\angle 0 - \frac{11.3}{4}\angle -45^o$$

For the Thevenin impedance Z_{TH}, we short the voltage sources and open the current sources and find the effective impedance from the V_x terminals. This is valid since there are no dependent sources in the circuit. It is also very easy since this impedance is the parallel connection of the coil, resistor, and capacitor impedances:

$$Z_{TH} = \frac{1}{\frac{1}{Z_L} + \frac{1}{Z_R} + \frac{1}{Z_C}} = \frac{1}{\frac{1}{j4} + \frac{1}{10} + \frac{1}{-j3}} = \frac{12}{1.2 + j}$$

Substituting the numerical values we get

$$
\begin{aligned}
V_x &= I_{sc} Z_{TH} \\
&= \left(4\angle 0 - \frac{11.3}{4}\angle -45^o\right) \frac{12}{1.2 + j} \\
&= (4 - 2.825(0.707 - j0.707)) \frac{12}{1.2 + j} 1\angle 0 \\
&= (21.642 + 1.941j)1\angle 0 \\
&= 21.73\angle 5.1^o
\end{aligned}
$$

ECE 201 HW Measurements SOLUTIONS

Problem M1 *A voltmeter with internal resistance 10-MΩ and a 10-bit, $\pm$10V A/D converter is used to measure a voltage of 0.95V and a voltage of 1.05V. Find the worst-case percent-error in the voltage reading due to quantization.*

The A/D converter quantizes its entire range into 2^{10} different states, so its resolution is (10-(-10))/1024=0.02V. For the first measurement, we assume that an internal amplifier is used to amplify voltages less than 1V to the range of the A/D. After the measurement is obtained the voltmeter processor divides the measurement by 10 to produce the measured value. This means that the quantization error is also divided by 10, i.e., it is 0.002V. (The same result we get if we quantize the interval ± 1 into 2^{10} states.) Now, the maximum quantization error is 1 bit, corresponding to 0.002V for a measured value 0.95V. That is approximately 0.2% error.

On the other hand, to measure the second voltage we must use the ± 10 scale in the voltmeter. For this, the internal amplifier gain is 1, the quantization error is 0.02V and for a measured value 1.05V, this translates into a 2% error.

Problem M2 *A voltmeter with internal resistance 10-MΩ is used to measure the voltage across a 1-MΩ resistance, driven by an 1-μA current source. Find the percent-error in the voltage reading introduced by the use of the meter.*

Before using the meter, the voltage across the resistor is $v_{act} = (1M)(1\mu) = 1V$. After introducing the meter (connecting the resistances in parallel) the effective resistance is $\{1M//10M\} = 10/11M = 0.91M$ and the measured voltage is $v_{meas} = (0.91M)(1\mu) = 0.91V$. This represents a $(1 - 0.91)/0.91 \simeq 10\%$ error on the measurement.

Problem M3 *For the same setting as in Problem M2, suppose that the meter uses a 12-bit A/D converter, with a range of $\pm$ 10V. Find the value of the internal resistance such that the error in the voltage is comparable with the resolution of the meter. Assume first that the meter is used at the 1V setting and repeat for the 10V setting.*

The resolution of the meter is $20/(2^{12}) = 20/4096 \simeq 0.005V$ at 10V. If the meter is used at the 1V setting, the entire range corresponds to $\pm 1V$ measurement, i.e., the resolution is 0.0005V. Following the M2 computations, to have a comparable measurement error the measured voltage should be at least 0.9995V. Hence, the combined resistance should be at least 0.9995MΩ. Solving the parallel connection formula for the internal resistance (i.e., $\{1M//R_{met}\} = 0.9995M$) we find $R_{met} = 1999M \simeq 2G\Omega$, at least.

For the 10V setting, the resolution is 0.005V. Now the measured voltage should be at least 0.995V, the combined resistance 0.995MΩ and the internal resistance $199M\Omega$.

Problem M4 *A meter with internal resistance 10-MΩ, and a 10-bit, $\pm$10V, A/D converter is used to measure the current through a 100Ω resistance, driven by an 10-V voltage source. Find the value of the shunt resistance so that the error introduced by the meter is comparable to the resolution of the meter.*

This is the current-analog of M3 but its solution is more involved. First, let us denote by v_0 and R_0 the voltage of the source and the 100Ω resistance. Then, denote by i the current that flows through the shunt resistance, when connected, and v the voltage across it. And let $\hat{i}, \hat{v}$ be the measured values of the same quantities and δ be the quantization error. Notice that the optimal value of $\delta = v/2^{10}$ is not always feasible since such an adjustment of the range may require unreasonably large amplification gains. In the following we assume that the amplifier gain is set at 10, so $\delta = 0.02$.

When we connect the meter, the shunt resistance is in series and the current flowing through it is $i = v_0/(R_0 + R)$, or the voltage is $v = v_0 R/(R_0 + R)$. What we measure is $\hat{v} = v + \delta$ (more precisely $|\hat{v} - v| \leq \delta$). After the current computation, this yields $\hat{i} = \hat{v}/R = i + \delta/R$. Notice that the last term is the effective error in the current due to quantization and becomes smaller for bigger R. (Why?)

Next, substitute the expression for the original current i_0

$$\begin{aligned}
\hat{i} &= \frac{i_0}{1 + R/R_0} + \frac{\delta/R_0}{R/R_0} \\
&= i_0 - \frac{i_0(R/R_0)}{1 + R/R_0} + \frac{\delta/R_0}{R/R_0}
\end{aligned}$$

Observe that the use of normalized quantities brings out nicely the fact that the important quantity is the ratio of the shunt vs. the circuit resistance. Let us denote this ratio by x and define $\Delta = \delta/(i_0 R_0)$. The latter is the voltage quantization error normalized by the original voltage across the resistor R_0.

So,

$$\hat{i} \;=\; i_0 - i_0 \left[\frac{x}{1+x} - \frac{\Delta}{x}\right]$$

The two bracketed terms are the measurement error. The first one is the error due to the introduction of the shunt resistance and it increases with x (the ratio shunt-to-original). The second term is due to quantization and decays with x. Our problem is now to find the value of x where these two terms have equal magnitude. (Notice that Δ is random, so this does not mean that the two errors will offset each other.) Now Δ is usually small and that implies that x is also small so $1 + x \simeq 1$. From this, it follows that $x \simeq \sqrt{\Delta}$.

Recalling our definitions of variables,

$$R = R_0 \sqrt{\frac{\delta}{i_0 R_0}} = R_0 \sqrt{\frac{\delta}{v_0}}$$

Finally, bringing in the given numerical values

$$R \;=\; 100\sqrt{\frac{0.02}{10}} = 4.5\Omega$$

For this value, the worst-case error in the current measurement is approximately 9% (each source of error contributes 4.5%).

ECE 201 HW Ch.1 SOLUTIONS

Problem 1.12

Current is $I = (72)(24) = 3A$.
Element dissipates power $p = (-6)(4) = -24W$, or it supplies 24W.

Problem 1.17

Element 1 dissipates power $p = (6)(2) = 12W$.
Element 2 dissipates power $p = (4)(-2) = -8W$, or it supplies 8W.

ECE 201 HW Ch.2 SOLUTIONS

Problem 2.7

Applying KVL clockwise around the loop we have $-1.5 - 1.5 + V_R = 0$ (in V), hence, $V_R = 3$V. Then, $p = V_R I_R = V_R^2/R = 9/1 = 9W$.

Problem 2.10

Applying KCL at the top two nodes we get:
left node: $I_s = I_1 + I_2 = 12mA$
right node: $I_2 = 4mA + 2mA = 6mA$
Hence, $I_1 = 12 - 6 = 6mA$.

Problem 2.27

Applying KVL clockwise in the loop with a clockwise current I we get:
$-12 + (30k)I + 2V_x + V_x = 0$ (in V)
$V_x = (10k)I$ (Ohm's law)
Substituting V_x in the first equation we get: $-12 + (60k)I = 0$. Hence, $I = 0.2mA$

Problem 2.29

Let I be the current flowing clockwise in the loop. Then

$$P_3 = 12mW = V_x(V_x/3k) \Rightarrow V_x = 6V$$

. Then, $I = 6/3k = 2mA$. Next, apply KVL:

$$-V_S + V_x + (6k)I + 2V_x \quad = \quad 0$$

Hence, $V_S = 30V$

Problem 2.38

The three resistors are connected in parallel with an effective resistance

$$R_{eff} = \frac{1}{\frac{1}{2[10^3]} + \frac{1}{3[10^3]} + \frac{1}{6[10^3]}} = 1[10^3]\Omega$$

So the voltage drop across the three resistors is $V_L = R_{eff}I = (1[10^3])((5-2)[10^{-3}]) = 3V$. From this, the current flowing through the $6k\Omega$ resistor is

$$I_L = V_L/6[10^3] = 3/(6[10^3]) = 0.5[10^{-3}]A$$

Alternatively, one could use current division (notice the formula) to find

$$I_L = \frac{\frac{1}{6k}}{\frac{1}{2k} + \frac{1}{3k} + \frac{1}{6k}} I_s = 0.5mA$$

Problem 2.43

Reducing the circuit from the far right:
$2k$ series $2k = 4k$
$4k//2k = \frac{4}{3}k$
$\frac{4}{3}k$ series $2k = \frac{10}{3}k$
$\frac{10}{3}k//2k = \frac{10}{8}k$
$\frac{10}{8}k$ series $5k = \frac{50}{8}k$
Thus, $R_{AB} = \frac{25}{4}k\Omega$.

Problem 2.46

As in the previous problem:
$12k//4k = 3k$
$3k$ series $9k = 12k$
$12k//6k = 4k$
$4k$ series $4k = 8k$
$8k//8k//12k = 3k$
$3k$ series $2k = 5k$
Thus, $R_{AB} = 5k\Omega$.

Problem 2.50

Let us find the total current through the source first: $R_{tot} = [\{8k + 4k\}//6k] + 2k = 6k$. So, $I_{tot} = 12/6k = 2[10^{-3}](A)$. Then the voltage drop across the 2k resistor is $I_{tot}(2k) = 4(V)$, hence the voltage across the 6k resistor is $12 - 4 = 8(V)$. From this:

1. The current through the 6k resistor is $I_1 = 8/6k = 4/3(mA)$.
2. The voltage across the 4k resistor is $V_0 = 8\frac{4k}{8k+4k} = 8/3(V)$

Problem 2.65

First, we find I_b (since the two parts of the circuit are decoupled): $I_b = V_s/(R_s + R_b) = 0.2[10^{-3}](A)$.
Then, the voltage across the load is $V_L = \{100I_b\}\{4k//400\} = 0.02\frac{4000}{11} = 7.27(V)$.
From this, $I_0 = -\frac{7.27}{400} = -0.018(A)$.

Problem 2.49

Reducing the circuit from the far right:
$2k$ series $2k = 4k$
$4k//4k = 2k$
$2k$ series $2k = 4k$
$4k//12k = 3k$
$3k$ series $9k = 12k$
Thus, $R_{AB} = 12k\Omega$.

Problem 2.51

As in the previous problem:
$12k//4k = 3k$
$3k$ series $9k = 12k$
$12k//6k = 4k$
$4k$ series $4k = 8k$
$8k//8k//12k = 3k$
$3k$ series $2k = 5k$
Thus, $R_{AB} = 5k\Omega$.

Problem 2.102

Let us find the total current through the source first: The two 6k resistors with the 12k form a Y which we change to Δ. That yields two 30k resistors in parallel with the 2k and the 18k and one 15k across the ends of the voltage source.

The equivalent total resistance is $[(30k//2k) + (18k//30k)]//15k = 7k$.

Now, total absorbed power is $V^2/R = 21^2/7k = 63m(W)$

Problem 2.110

Applying KVL,

$$-12 + 3kI_s - 2000I_s + 1kI_s = 0 \Rightarrow I_s = 6m(A)$$

Then $V_o = 1kI_s = 6(V)$.

Problem 2.116

First, we find V_{in} by voltage division (since the two parts of the circuit are decoupled):

$$V_{in} = \frac{R_{in}}{R_s + R_{in}} V_s$$

Similarly,

$$V_o = \frac{R_L}{R_o + R_L} \mu V_{in} = \frac{R_L}{R_o + R_L} \frac{R_{in}}{R_s + R_{in}} \mu V_s$$

The maximum V_o occurs when as $R_{in} \to \infty$ and $R_o = 0$. In such a case, the ratio V_o/V_s approaches μ, regardless of the value of R_L.

Problem 2.117

First, we find I_b: The total resistance seen by the source is $(5k//500) + 100 = 554.54\Omega$. Hence the total current through the source is 0.4508m(A), and the voltage drop across the 100Ω resistor is 0.0451(V). So the voltage drop across the 500Ω is 0.2049(V) and $I_b = 0.4098m(A)$.

Next, we apply voltage division to find V_o:

$$V_o = \frac{300}{300 + 4000}(4e5)(-0.4098m) = -11.436(V)$$

Hence, the amplifier gain is $G = -11.436/0.25 = -45.74$.

ECE 201 HW Ch.3 SOLUTIONS

Problem 3.7

Define the voltages at the top nodes as v_1, v_2, v_3 from left to right. Then, $v_0 = v_2 - v_3$ and nodal analysis yields the equations:

$$\begin{aligned} v_1 &= 12 \\ \frac{v_2 - v_1}{6k} + \frac{v_2}{6k} + \frac{v_2 - v_3}{12k} &= 0 \\ v_3 &= 6 \end{aligned}$$

From these, we easily get that $v_2 = 6$ (V) and $v_0 = 0$ (V).

Problem 3.64

Define the clockwise currents i_1, i_2 in the left and right loops, respectively. Then, loop analysis yields the equations:

$$\begin{aligned} 2ki_1 + 4ki_1 + 6k(i_1 - i_2) + 12 &= 0 \\ 6k(i_2 - i_1) + 4ki_2 + 2ki_2 - 12 &= 0 \end{aligned}$$

From which we find $i_2 = \frac{2}{3}[10^{-3}](A)$ and, therefore, $v_0 = 2ki_2 = \frac{4}{3}(V)$

ECE 201 HW Ch.4 SOLUTIONS

Problem 4.22

First, apply KCL at the inverting input node. This yields:

$$\frac{V_- - V_i}{2} + \frac{V_- - V_o}{10} = 0$$

Now, from the ideal op-amp assumptions, $V_- = V_+$ and $i_+ = 0$. The latter implies that $V_+ = 0$ (KCL at the non-inverting input node) and, hence,

$$V_o = -\frac{10}{2}V_i = -10(V)$$

This is the well-known ideal op-amp equation and is independent of R_1. The reason is that regardless of the value of the resistance V_+ is kept at zero. (This is true as long as R_1 is small relative to the internal op-amp impedance; as R_1 grows there will be a discrepancy both in the op-amp gain and its dynamic response.)

Problem 4.25

First, apply KCL at the inverting input node. This yields:

$$\frac{V_- - V_5}{1} + \frac{V_- - V_o}{4} = 0$$

where $V_5 = 5$ (V). Now, from the ideal op-amp assumptions, $V_- = V_+ = V_4 = 4$ (V). Solving for V_o we get

$$V_o = -\frac{4}{1}V_5 + \left(1 + \frac{4}{1}\right)V_4 = -20 + 20 = 0(V)$$

The last equation shows the general dependence of the op-amp output to voltages at both inverting and non-inverting inputs.

Problem 4.50

The first op-amp performs the addition so that

$$V_x = -\sum_{i=1}^{3} \frac{R_f}{R_i} V_i$$

The second op-amp acts as an inverting gain to recover the positivity of the voltage.

$$V_o = -\frac{R_B}{R_A}V_x$$

To average the three measurements, one option is to choose $R_f/R_i = 1/3$ for all i. Further, to achieve 1V/10deg slope (or 0.1V/deg) while the original thermocouples produce 0.025V/deg, we need an overall gain of 4. For that we could choose $R_B/R_A = 4$. Trying to keep a relatively high input impedance we can choose $R_i = 30k$, $R_f = R_A = 10k$, $R_B = 40k$ (all in Ω).

(Even better, take the desired ratios and input impedance constraints and visit `http://www.eecircle.com/applets/008/ResPicker.html`).

Problem 4.60

To apply Thevenin, consider the circuit without the 1k resistor corresponding to V_o. This circuit has dependent and independent sources, so we need to compute V_{oc} and I_{sc} to find the Thevenin equivalent.

Let us first denote the clockwise loop currents by $I_1 - I_4$, clockwise from the top left.

Finding V_{oc} is rather easy, since clearly $I_2 = 0$, implying that $V_x = 0$, and then the dependent source produces zero current. So we only have two independent loops and their currents are specified by the sources: $I_1 = 1m$ (A) and $I_3 = -4m$ (A). Now we compute the voltage drops across the resistances through KVL, yielding $V_{oc} = -8$ (V).

Finding I_{sc} is more complicated, and we need to perform a complete loop analysis. The equations are

$$
\begin{aligned}
&I_1 - I_2 = 1m \\
&I_3 = -4m \\
&I_4 = \frac{2V_x}{1000} = \frac{2(-I_2 1k)}{1000} = -2I_2 \\
&(I_1 - I_4)1k + I_1 2k + I_1 1k + I_2 1k + (I_2 - I_3)1k = 0
\end{aligned}
$$

the last being the supermesh equation. Using Matlab to compute the solution, we find the current of interest $I_{sc} = I_2 = -1m\ (A)$.

Hence, the Thevenin equivalent is a -8V source in series with an $R_{TH} = \frac{V_{oc}}{I_{sc}} = 8k\ \Omega$ resistor. This is now in series with the 1k resistor of the complete network and V_o is the voltage across the 1k resistor. This can be found by voltage division so $V_o = \frac{1k}{1k+8k} V_{oc} = -8/9\ (V)$.

Unfortunately, Thevenin's Theorem did not save us any work in this problem. A straightforward loop analysis yields the same answer and the amount of work is equivalent to finding I_{sc}.

ECE 201 HW Ch.5 SOLUTIONS

Problem 5.7

Keeping the voltage source only and replacing the current source by an open circuit, we can reduce the circuit and find the voltage across the $2k\Omega$ resistor with voltage division: The total resistance seen by the source is $R_{TOT} = 6k + \{2k//[3k+3k]\} = 7.5k\ \Omega$. Using voltage division the voltage across the resistor is

$$V_2 = \frac{3/2k}{6k+3/2k} V_{in} = 6/5(V) \Rightarrow I_{0,1} = \frac{6/5}{2k} = 0.6m\ (A)$$

Keeping the current source only and replacing the voltage source by a short circuit, we can use nodal analysis to find the voltage across the $2k\Omega$ resistor:

$$\begin{aligned} \frac{V_1}{3k} + \frac{V_1 - V_2}{3k} - 2m &= 0 \\ \frac{V_2}{2k} + \frac{V_2}{6k} + \frac{V_2 - V_1}{3k} &= 0 \end{aligned}$$

From this, $V_2 = 1.2\ (V)$ and $I_{0,2} = 1.2/2k = 0.6m\ (A)$.

Then, using superposition, $I_0 = I_{0,1} + I_{0,2} = 1.2m\ (A)$.

Problem 5.10

First, keep the left V-source and replace the top V-source by a short and the I-source by an open. Then the voltage across the 4k resistor of interest is $\frac{4k}{4k+4k}12 = 6(V)$ and $I_o = \frac{6}{4k} = 1.5m(A)$.

Next, keep the top V-source and replace the left V-source by a short and the I-source by an open. Then the two resistors-in series including the resistor of interest are shorted out, the voltage drop across their ends is zero and therefore, $I_o = 0$.

Finally, keep the I-source and replace the V-sources by shorts. Then $6mA$ current flows through the parallel combination of the two 4k resistors including the resistor of interest. Hence, $I_o = \frac{4k}{4k+4k}6m = 3m(A)$.

Hence, the total current is $I_o = 1.5m + 0 + 3m = 4.5m(A)$.

Problem 5.67

There can be several different "applications" of Thevenin, but the one that makes the most sense is to find the equivalent of the circuit that includes the left portion up to the Ix-resistor. (Notice that in any case, the dependent source and the Ix resistor cannot be separated.)

Finding V_{OC}: Let V_1 the voltage of the left top node. Then

$$\begin{aligned} 2I_x + \frac{V_1}{1k} + \frac{V_{OC}}{1k} &= 0 \\ V_{OC} - V_1 &= 12 \\ I_x &= \frac{V_{OC}}{1k} \end{aligned}$$

From which, $V_{OC} = 3(V)$.

Finding I_{SC}: $I_x = 0$ because the Ix-resistor is shorted-out. So, the current dependent source gives zero current (open circuit) and $I_{SC} = \frac{12}{1k} = 12m(A)$.

Thus, $R_{Th} = \frac{V_{OC}}{I_{SC}} = \frac{3}{12m} = 250(\Omega)$. Finally, by voltage division,

$$V_o = \frac{1k}{R_{Th} + 1k + 1k} V_{OC} = 1.33(V)$$

Problem 5.69

To apply Thevenin, consider the circuit without the 1k resistor corresponding to V_o. This circuit has dependent and independent sources, so we need to compute V_{oc} and I_{sc} to find the Thevenin equivalent.

Let us first denote the clockwise loop currents by $I_1 - I_4$, clockwise from the top left.

Finding V_{oc} is rather easy, since clearly $I_2 = 0$, implying that $V_x = 0$, and then the dependent source produces zero current. So we only have two independent loops and their currents are specified by the sources: $I_1 = 1m\ (A)$ and $I_3 = -4m\ (A)$. Now we compute the voltage drops across the resistances through KVL, yielding $V_{oc} = -8\ (V)$.

Finding I_{sc} is more complicated, and we need to perform a complete loop analysis. The equations are

$$\begin{aligned}
&I_1 - I_2 = 1m \\
&I_3 = -4m \\
&I_4 = \frac{2V_x}{1000} = \frac{2(-I_2 1k)}{1000} = -2I_2 \\
&(I_1 - I_4)1k + I_1 2k + I_1 1k + I_2 1k + (I_2 - I_3)1k = 0
\end{aligned}$$

the last being the supermesh equation. Using Matlab to compute the solution, we find the current of interest $I_{sc} = I_2 = -1m\ (A)$.

Hence, the Thevenin equivalent is a -8V source in series with an $R_{TH} = \frac{V_{oc}}{I_{sc}} = 8k\ \Omega$ resistor. This is now in series with the 1k resistor of the complete network and V_o is the voltage across the 1k resistor. This can be found by voltage division so $V_o = \frac{1k}{1k+8k} V_{oc} = -8/9\ (V)$.

Unfortunately, Thevenin's Theorem did not save us any work in this problem. A straightforward loop analysis yields the same answer and the amount of work is equivalent to finding I_{sc}.

Problem 5.87

Transform the 6V voltage source into a 1.5mA current source in parallel with a $4k\Omega$ resistor and the combined series of the two $6k\Omega$'s. All these resistors combine into a $3k\Omega$ resistor.

Transform the 1.5mA current source to a 4.5V voltage source, in series with the $3k\Omega$ resistor. Now combine the series of the two $3k\Omega$'s. Convert back to a 3/4mA current source in parallel with a $6k\Omega$ resistor.

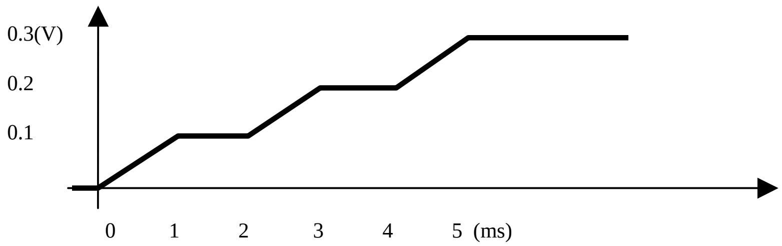
Add the two current sources connected to the same two nodes. Convert back to a 33/2V voltage source and a $6k\Omega$ resistor in series.

Now the voltage sources add to 9/2V (opposite polarity) and transform to a 3/4mA current source with a $6k\Omega$ resistor in parallel.

The rest of the circuit is the $3k\Omega$ and a $6k\Omega$ effective resistance from the series of the $3k\Omega$'s. The final answer is now obtained by current division, $I_0 = -0.375mA$.

ECE 201 HW Solutions, Ch.6

6.18 $i(t) = C\frac{dv}{dt}(t) \Rightarrow v(t) = v(0) + \frac{1}{C}\int_0^t i(\tau)d\tau$, where *v(0)* represents the initial voltage of the capacitor. Assuming that *v(0) = 0*, we graphically integrate the plot of *i(t) vs. t* to get v(t).
v(t) increases 10mA/100uF in 1ms, i.e., 0.1V in 10ms.



v(t) stays constant between 1-2ms,3-4 ms, and aftert 5ms.

6.21 Assuming that the current change is linear (constant slope and constant voltage), we have that

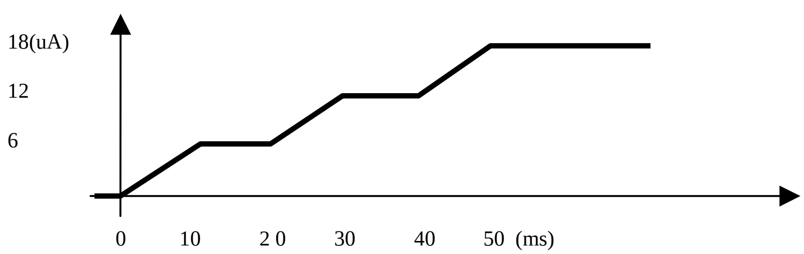
$$L\frac{di}{dt} = v \Rightarrow L\frac{200(10^{-3})}{4(10^{-3})} = 100(10^{-3}) \Rightarrow L = \frac{(4)(100)}{200}(10^{-3}) = 2mH$$, (all units in SI).

6.27 $v(t) = L\frac{di}{dt}(t)$. Calculating slopes from the figure, all slopes are the same in absolute value and equal to $100(10^{-3}) \big/ 2(10^{-3}) = 50$. Then the voltage is piecewise constant, according to the following rule

$$v(t) = (50.10^{-3})x\begin{cases} 0 & t < 2(10^{-3}) \\ -50 & 2(10^{-3}) < t < 4(10^{-3}) \\ 50 & 4(10^{-3}) < t < 8(10^{-3}) \\ -50 & 8(10^{-3}) < t < 10(10^{-3}) \\ 0 & 10(10^{-3}) < t \end{cases} = \begin{cases} 0 & t < 2(10^{-3}) \\ -2.5 & 2(10^{-3}) < t < 4(10^{-3}) \\ 2.5 & 4(10^{-3}) < t < 8(10^{-3}) \\ -2.5 & 8(10^{-3}) < t < 10(10^{-3}) \\ 0 & 10(10^{-3}) < t \end{cases}$$

in V and with *t* in s.

6.34 $v(t) = L\frac{di}{dt}(t) \Rightarrow i(t) = i(0) + \frac{1}{L}\int_0^t v(\tau)d\tau$, so the current will have a slope of 2.4mV/4H = 0.6mA/s or 0, depending on the interval. Hence, when v(t) = 2.4mV, the current increases by 6mA in 10ms.
We assume that *i*(0) = 0 (0 voltage for t<0 and circuit starting at rest at t = - infinity).



ECE 201 HW Ch.6 SOLUTIONS

Problem 6.73

The input and the output of the op-amp are related by $V_o = -\frac{Z_o}{Z_i} V_i$. where $Z_o = \frac{1}{j\omega C} = \frac{1}{j(377)(20\mu)}$ and $Z_i = 4k$. Expressing the input voltage in phasor notation,

$$V_i = 115 \sin(377t) = 115 \cos(377t - 90^o) = 115\angle - 90^o$$

So,

$$V_o = -\frac{1}{j(377)(20\mu)(4k)} 115\angle - 90^o = (0.033\angle 90^o)(115\angle - 90^o) = 3.81\angle 0^o$$

Or, $V_o(t) = 3.81 \cos(377t)$.

ECE 201 HW Ch.7 SOLUTIONS

Problem 7.9

First, we determine the initial condition for v_c by solving the steady-state problem of the $t < 0$ network.

At steady-state with a constant source, the capacitor will act as an infinite resistance (open) and the voltage across its ends will be the same as the voltage across the 4k resistor. After simplifying the two resistors in parallel

$$12k//4k = \left(\frac{1}{12k} + \frac{1}{4k}\right)^{-1} = 3k$$

a simple voltage division yields

$$v_c(0) = v_{c,ss-} = (6)\left(\frac{4k}{2k + 4k + 3k}\right) = \frac{8}{3}(V)$$

Next, for $t > 0$ the left part of the circuit, including the source, is disconnected and the capacitor discharges through the 4k resistor. Applying KVL, $v_c = v_R = Ri_R$, while $i_R = i_c = -C\frac{dv_c}{dt}$. Hence,

$$RC\frac{dv_c}{dt} + v_c = 0$$

whose solution is

$$v_c(t) = v_c(0)e^{-t/RC}$$

and using the given numerical values

$$v_c(t) = \frac{8}{3}e^{-t/0.8}(V)$$

with t in s.

Problem 7.18

First, we determine the initial condition for the inductor current i_L by solving the steady-state problem of the $t < 0$ network.

At steady-state (with a constant source) the inductor will act as an zero resistance (short) and the current will be the same as the current through the 3Ω resistor. A "brute-force" approach to compute this, is:

- The current through the source is $v_s/(2 + [3//(4 + 2)]) = 12/4 = 3(A)$.
- The voltage at the middle top node is $12 - 2 * 3 = 6(V)$.
- The current through the 3Ω resistor and the inductor is $6/3 = 2(A)$.
- It now follows that the current through the 4-2 resistors is $i(0^-) = 1(A)$ in the direction shown.

Next, for $t > 0$ the left part of the circuit, including the source, is shorted-out and the inductor dissipates energy through the series combination of the 3-4-2 resistors. The current through the resistors is the same as the current through the inductor. So $i(0^+) = -2(A)$. (Notice the discontinuity at 0 as this is not an inductor current.)

The final current in the loop is 0, as it contains no sources. So,

$$i(t) = i(0^+)e^{-t/(L/R)} = -2e^{-t/(2/9))} = -2e^{-4.5t)}(A)$$

Problem 7.44

The general solution for the inductor current will be of the form

$$i_L(t) = K_1e^{-t/\tau} + K_2$$

where $K_1 + K_2 = i_L(0^+)$, $K_2 = i_L(\infty)$ and $\tau = L/R_{Th}$, with R_{Th} being the effective (Thevenin) resistance seen by the inductor.

For the $t < 0$-network, the steady state inductor current is found by replacing the inductor with a wire ("short-circuit"). This implies that the inductor current is the same as the top-4k resistor, while there is no current flowing through the 6k resistor.

Applying "brute-force" simplification of the $t < 0$-network we find:

- Total effective resistance as seen by the source: $12k + [4k//4k//4k] = \frac{40}{3}k$.
- Voltage across the current source: $12m\frac{40}{3}k = 160(V)$ (bottom-to-top).
- Voltage drop across 12k: $(12k)(12m) = 144(V)$.
- Voltage at middle top node: $-16(V)$ relative to bottom node (notice the current direction).
- Current through 4k and coil: $i_0(0^-) = -\frac{16}{4k} = -4mA$ (here i_0 is the inductor current).

Next, for the $t > 0$-network, the steady-state is found similarly, by removing the side-6k resistor and replacing the inductor by a "short." The same computation yields $i_0(\infty) = -4mA$

Hence, $K_1 + K_2 = -4m$ and $K_2 = -4m$, from which, $K_1 = 0$. Since $i_0(0^+) = i_0(0^-)$, this yields immediately that the inductor current is constant $i_0(t) = -4m\ (A)$ for $t > 0$.

In other words, when this circuit is at steady-state the addition or removal of the side-6k resistor does not affect the inductor current. This resistance will play a role during transients or changes in the source current (e.g., AC sources).

While it is not necessary for the solution of this problem, we can compute R_{Th} for completeness. The resistance seen by the inductor in the $t > 0$ network is $4k + (4k//4k//12k) = \frac{40}{7}k\Omega$. This yields a time constant $T = L/R_{Th} = 7m/4k = 1.75\mu s$.

Problem 7.70

The Norton equivalent of this circuit is a current source I_{sc} with an impedance in parallel Z_{TH}. Then $V_x = V_{oc} = I_{sc}Z_{TH}$. To find I_{sc} we short the terminals of V_x and find the current that passes through. This eliminates the resistor and the capacitor from the computation. Now, using a source transformation on the current source and the inductor, we get a voltage source $(4\angle 0)(j4)$ in series with the same $j4$ inductor. These are also in series with the voltage source $11.3\angle 45^o$, The current that flows through this loop is

$$I_{sc} = \frac{(4\angle 0)(j4) - 11.3\angle 45^o}{j4} = 4\angle 0 - \frac{11.3}{4}\angle - 45^o$$

For the Thevenin impedance Z_{TH}, we short the voltage sources and open the current sources and find the effective impedance from the V_x terminals. This is valid since there are no dependent sources in the circuit. It is also very easy since this impedance is the parallel connection of the coil, resistor, and capacitor impedances:

$$Z_{TH} = \frac{1}{\frac{1}{Z_L} + \frac{1}{Z_R} + \frac{1}{Z_C}} = \frac{1}{\frac{1}{j4} + \frac{1}{10} + \frac{1}{-j3}} = \frac{12}{1.2 + j}$$

Substituting the numerical values we get

$$\begin{aligned}
V_x &= I_{sc}Z_{TH} \\
&= \left(4\angle 0 - \frac{11.3}{4}\angle - 45^o\right)\frac{12}{1.2 + j} \\
&= (4 - 2.825(0.707 - j0.707))\frac{12}{1.2 + j}1\angle 0 \\
&= (21.642 + 1.941j)1\angle 0 \\
&= 21.73\angle 5.1^o
\end{aligned}$$

Problem 7.74

The characteristic equation is $s^2 + 8s + 16 = 0$. Hence, the undamped resonant frequency is $w_0 = 4$ ($w_0^2 = 16$). The damping ratio is $\zeta = 8/(2w_0) = 1$. The system is critically damped (double root of the characteristic equation at -4), and the natural frequencies are $-4,\ -4$, implying that the solution has the form

$$i_o(t) = K_1 e^{-4t} + K_2 t e^{-4t}$$

Problem 7.77

Following the derivation of the textbook, the common voltage v satisfies the differential equation

$$C\frac{d^2v}{dt^2} + \frac{1}{R}\frac{dv}{dt} + \frac{1}{L}v = \frac{di_S}{dt}$$

Normalizing the leading coefficient to one,

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v = \frac{1}{C}\frac{di_S}{dt}$$

From this we identify the undamped resonant frequency $w_0 = 1/\sqrt{LC}$ and the damping ratio

$$\zeta = \frac{1}{2}\sqrt{\frac{L}{R^2C}} = \frac{1}{2}\frac{\sqrt{L/R}}{\sqrt{RC}}$$

Substituting the given numerical values we find $w_0 = \frac{1}{\sqrt{1/4}} = 2(rad/s)$, $\sqrt{L/R} = \frac{1}{\sqrt{2}}$, $\sqrt{RC} = \frac{1}{\sqrt{2}}$, and $\zeta = \frac{1}{2}$. Since $\zeta < 1$, the circuit exhibits underdamped behavior.

ECE 201 HW Ch.8 SOLUTIONS

Problem 8.1

The period of $5\cos(400t - 120^o)$ is $2\pi/400 = 0.0157s$. The frequency is $400/(2\pi) = 63.66Hz$.

Problem 8.2

General procedure:

1. Convert to the same function form (sin or cos; use eqn 8.7-8.8).
2. Subtract phases ($\phi_1 - \phi_2 =$ phase by which $i_1(t)$ leads $i_2(t)$).
3. Extract modulo 2π ($mod_{2\pi}(\phi) \in (-\pi, \pi]$ (or $[0, 2\pi)$) and $\phi = 2\pi n + mod_{2\pi}(\phi)$ where n is an integer).

In our problem both signals are cos of the same frequency. $\phi_1 - \phi_2 = -30 - 90 = -120$. Extracting the mod, $mod_{2\pi}(-120) = -120$. So $i_1(t)$ lags 120deg behind $i_2(t)$.

Problem 8.5

a. In phasor notation, $V_1 = 10\angle 180^o$. Then $i_1 = v_1/R = \frac{10}{2}\cos(377t + 180^o) = 5\angle 180^o(A)$.
b. In phasor notation, $V_2 = 12\angle(45-90)^o$. $i_2 = v_2/R = \frac{12}{2}\sin(377t+45^o) = 6\cos(377t-45^o) = 6\angle -45^o(A)$.

Problem 8.7

a. In phasor notation $V_1 = 10\angle -30^o$ (V). Capacitor impedance: $Z_c = \frac{1}{jwC} = \frac{1}{j(377)(1\mu)} = 2652.5\angle -90^o$ (Ω) Then, $I_1 = \frac{10\angle -30^o}{2652.5\angle -90^o} = 0.00377\angle 60^o$ (A). In the time domain, $i_1(t) = 3.77\cos(377t + 60^o)$ (mA).
b. In phasor notation $V_2 = 5\angle(60-90)^o$ (V). Capacitor impedance: $Z_c = \frac{1}{jwC} = \frac{1}{j(377)(1\mu)} = 2652.5\angle -90^o$ (Ω) Then, $I_2 = \frac{5\angle -30^o}{2652.5\angle -90^o} = 0.00189\angle 60^o$ (A). In the time domain, $i_2(t) = 1.89\cos(377t + 60^o)$ (mA).

Problem 8.18 Apply KCL at the top node

$$\begin{aligned}
I_s &= I_R + I_L + I_C = \frac{V}{Z_R} + \frac{V}{Z_L} + \frac{V}{Z_C} \\
20\angle 120^o &= V\left(\frac{1}{1} + \frac{1}{j100[10^{-3}]377} + \frac{1}{\frac{1}{j10[10^{-3}]377}}\right) \\
&= V(1 + 3.74j) \\
&= V(3.87\angle 75.5^o) \\
V &= \frac{20\angle 120^o}{3.87\angle 75.5^o} = 5.16\angle 44.5^o \\
v(t) &= 5.16\cos(377t + 44.5^o) \quad (V)
\end{aligned}$$

Problem 8.31 The circuit is purely resistive when the overall impedance Z is a positive real number.

$$\frac{1}{Z} = \frac{1}{1} + \frac{1}{j\omega 5m} + \frac{1}{\frac{1}{j\omega 1m}} = \frac{j\omega 5m + 1 + j\omega 1m j\omega 5m}{j\omega 1m} = \frac{\omega 1m - j + j\omega^2 5\mu}{\omega 1m}$$

So Z is real if $1 = \omega^2 5\mu$ or $\omega = \sqrt{0.2}k(rad/s) = 447.2(rad/s) = 71.2(Hz)$

Problem 8.34

Current division yields

$$I = \frac{1/1}{1/1 + 1/j1}100\angle 0^o = \frac{1}{1-j}100\angle 0^o = \frac{1}{\sqrt{2}}\angle 45^o 100\angle 0^o = 70.7\angle 45^o(A)$$

Problem 8.35

Voltage division yields

$$V = \frac{-j1}{1+(-j1)}100\angle 0^o = \frac{-j}{1-j}100\angle 0^o = \frac{1}{\sqrt{2}}\angle -45^o 100\angle 0^o = 70.7\angle -45^o (V)$$

Problem 8.48

Define the voltages at the top nodes as v_1, v_2 from left to right. Then, $i_0 = v_1/2$ and nodal analysis yields the equations:

$$\begin{aligned} \frac{v_1 - 12\angle 0^o}{2} + 2\angle 0^o \frac{v_1}{2} + \frac{v_1 - v_2}{1} &= 0 \\ \frac{v_2 - v_1}{1} + \frac{v_2}{-j} - 4\angle 0^o &= 0 \end{aligned}$$

From these, using MATLAB, we get that $v_1 = 1.6 - 1.2j = 4\angle -36.87^o$ (V) and $i_0 = 2\angle -36.87^o$ (V).

Problem 8.63

Define the clockwise currents i_1, i_2 in the left and right loops, respectively. Then, loop analysis yields the equations:

$$\begin{aligned} -6\angle 0^o - jI_1 + 2j(I_1 - I_2) &= 0 \\ 2j(I_2 - I_1) - 12\angle 45^o + 2I_2 &= 0 \end{aligned}$$

From which, using MATLAB, we find $i_2 = -3 + 1.243j = 3.247\angle 157.5^o(A)$ (remember to add 180^o correction for the negative real part) and, therefore, $v_0 = 2i_2 = 6.494\angle 157.5^o(V)$

Problem 8.67

Define the clockwise currents i_1, i_2, i_3, clockwise from the top loop. Then, loop analysis yields the equations:

$$\begin{aligned} I_3 - I_1 &= 2\angle 0^o \\ I_2 &= 4\angle 180^o \\ 2I_1 + j1(I_1 - I_2) + 2(I_2 - I_3) - j2I_3 &= 12\angle 0^o \end{aligned}$$

We find the solution using Matlab:

$$I_1 = 0 + j0(A), \quad I_2 = -4 + j0(A), \quad I_3 = 2 + j0(A)$$

Then, $I_0 = I_3 - I_2 = 6 + j0 = 6\angle 0^o(A)$.

Problem 8.74

Working with the voltage source first, we set the current source to zero, to obtain a circuit with the capacitor in series with the inductor, both in parallel with the 1Ω resistor, then in series with the 1Ω resistor. So, using the voltage division formula,

$$V_{o1} = -\frac{1}{1 + [1\|(j1 + -j1)]}6\angle 0 = -6\angle 0^o = 6\angle 180^o(V)$$

Next, setting the voltage source to zero, the two resistors become connected in parallel and $V_o = 1/2I_R$, where I_R is found through current division:

$$V_{o2} = \frac{1}{2}\frac{\frac{1}{1/2+j}}{\frac{1}{1/2+j} + \frac{1}{-j}}6\angle 0 = -j6\angle 0^o = 6\angle -90^o(V)$$

Adding the two,

$$V_o = (-1-j)6\angle 0^o = 6\sqrt{2}\angle(-180+45)^o = 6\sqrt{2}\angle -135^o = 8.49\angle -135^o(V)$$

Problem 8.87

Break the circuit at the capacitor. Then $V_{OC} = V_x$. Nodal analysis yields

$$2V_{OC} + \frac{V_{OC} + V_1}{1} + \frac{V_{OC}}{-j1} = 0 \Rightarrow V_{OC} = -3.6 + j1.2(V)$$

To find I_{SC} short the capacitor and observe that $V_x = 0$. Then it is simply $I_{SC} = \frac{-12\angle 0^o}{1} = -12\angle 0^o(A)$.
The Thevenin impedance is

$$Z_{Th} = \frac{V_{OC}}{I_{SC}} = 0.3 - j0.1\Omega$$

and it is connected in series with the voltage source V_{OC}, the inductor and the output resistance. After voltage division, we get $V_o = 2.4\angle 127^o(V)$

Problem 8.90

Break the circuit after the voltage source and find I_{SC} using nodal analysis:

$$I_{SC} + 2\angle 0^o + \frac{11.3\angle 45^o}{4\angle 90^o} = 0 \Rightarrow I_{SC} = -0.0024 - j1.9976(A) \simeq -j2(A)$$

Because there are only independent sources $Z_{Th} = j4(\Omega)$ (replace v-sources by shorts and i-sources by opens).

Thus, using Norton's theorem, the circuit transforms to an I_{SC} current source in parallel with Z_{Th} and in parallel with the resistance and the capacitor. Then

$$V_x = I_{SC}\left[Z_{Th}||10|| - j3\right] = -9.84 - j11.78 = 15.35\angle - 129.88^o(V)$$

Problem 9.26

For complex impedances, the maximum average power transfer occurs at $Z_{Load} = Z_{Th}^*$, the complex conjugate of the Thevenin equivalent impedance.

For this circuit (no dependent sources), $Z_{Th} = j1 + 1 \Rightarrow Z_{Load} = 1 - j(\Omega)$.

After transforming to a $6\angle 0^o(V)$ voltage source in series with 1Ω resistor, the total impedance is 2Ω and the total current flowing through the load is

$$I_{Load} = \frac{6\angle 0^o(V)}{2\Omega} = 3\angle 0^o(A)$$

Thus, $P_{max} = \frac{1}{2}I_M^2 R_L = 4.5(W)$, where I_M is the current amplitude and R_L is the resistive part of the load (see Chapter 9 for details).

Problem 9.27

First, find the Thevenin equivalent of the circuit without the load. Applying superposition, $V_{OC} = V_S + (R_1 + Z_C)I_S = 12\angle 30^o + (1 - j1)4\angle 0^o = 14.53\angle 7.91^o(V)$.

Then, by inspection, $Z_{Th} = 1 + (-j1) + 4 + (j1) = 5(\Omega)$.

Z_{Th} is purely resistive so the optimum $Z_L = 5(\Omega)$ is also purely resistive. Then $I = V_{OC}/(Z_{Th} + Z_L) = 1.45\angle 7.91^o(V)$, and $P_{max} = \frac{1}{2}I_M^2 R_L = 5.26(W)$.