EEE 582, Test 1 **NAME: ___SOLUTIONS__**

30min, 1 Problem, Closed-book, Closed-notes, calculator and 1 sheet of formulae allowed

Problem 1.

The model of a robotic manipulator is given as

$$\frac{d^2\theta}{dt^2} + 0.1\frac{d\theta}{dt} + \sin\theta = 3T$$

Where T is the Torque applied at the pivot point and θ is the angle.

Linearize the system around the constant steady-state where $\theta = 90^o$. Write a state-space realization for the linearized model and find its transfer function.

1. We solve for a steady-state solution $\frac{d\theta}{dt} = 0, \theta = 90^o = \frac{\pi}{2} rad, (=> \sin\theta = 1)$ which for the given parameters results in $T = \frac{1}{3}$. Next, we define the output and input variations $y = \theta - \frac{\pi}{2}$, $u = T - \frac{1}{3}$, and the state $x = [y; \dot{y}]$, for which we have the linearized model

$$\frac{dx}{dt} = \left[x_2;\ -0.1x_2 - \cos\frac{\pi}{2}x_1\right] + 3u$$

$$\frac{dx}{dt} = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 3 \end{bmatrix}, \qquad y = [1 \quad 0]x$$

2. We now have $[A, B, C, D]$, as given above, and applying $H(s) = C(sI - A)^{-1}B + D$, we find the transfer function

$$H(s) = \frac{3}{s(s + 0.1)}$$