EEE 481 Test 2

.1

1. Design and compare a PI and a PID controller to achieve a crossover of 0.02Hz, 50deg phase margin, and discretized with a sampling rate of 0.2Hz for the system with transfer function

$$P_1(s) = \frac{2.2}{(6s+1)(s^2+2.2s+1)}$$

2. Comment on closed-loop bandwidth, disturbance attenuation at 0.002Hz, overshoot and overall set-point tracking capabilities of the two controllers.

Note: For the PID, use a pseudo-derivative time constant T/2 (half-sample time).

Solution:

The crossover 0.02Hz corresponds to $\omega_c = 0.126 rad/s$. At that frequency the plant has phase [m,p]=bode(P,.126); p = -52.8 deg.

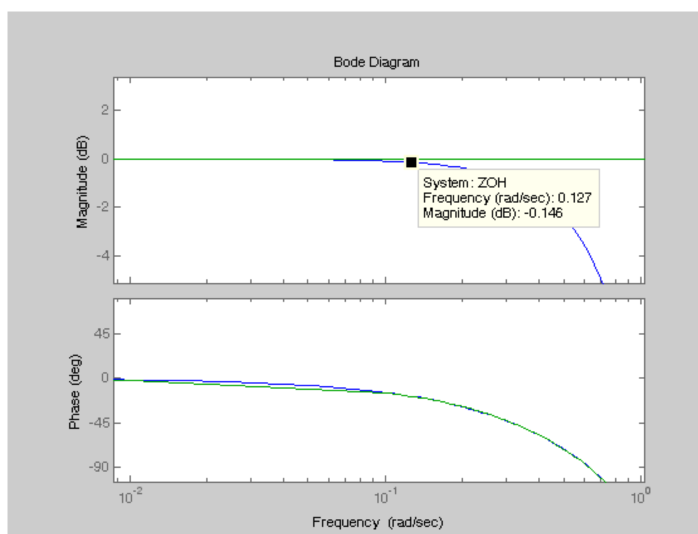
Furthermore, the ZOH will contribute $-\frac{\omega_c T_s}{2} = \frac{2\pi(0.02)}{(0.2)(2)}\frac{180}{\pi} = -18deg.$ of phase lag.

The controllers should therefore contribute $-180 + 50 - (-52.8 - 18) = -59.2deg$ of phase.

For the PI, $angle(PI) = -90 + \tan^{-1}\frac{\omega_c}{a} = -59.2 \Rightarrow a = \frac{\omega_c}{\tan 30.8} = 0.21.$

Next, we compute the gain of the controller so that the magnitude of PC is 1 at the crossover.

Notice that here only the magnitude of the plant transfer function is used which is the same as the magnitude of the plant augmented by a delay approximating the ZOH. The approximation is quite accurate one decade below the Nyquist frequency, as shown in the attached plot. The magnitudes differ by 0.15dB and the phases are nearly identical. This observation simplifies the design computations.




```
C=tf([1 a],[1 0])
[m,p]=bode(P*C,.126); m = 3.6
C=tf([1 a],[1 0])/m
Transfer function:
0.2989 s + 0.06301
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        s
```

And finally, discretize using the Tustin bilinear transformation

```
Cd=c2d(C,1/.2,'tustin')

Transfer function:
0.4564 z - 0.1414
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     z - 1
```
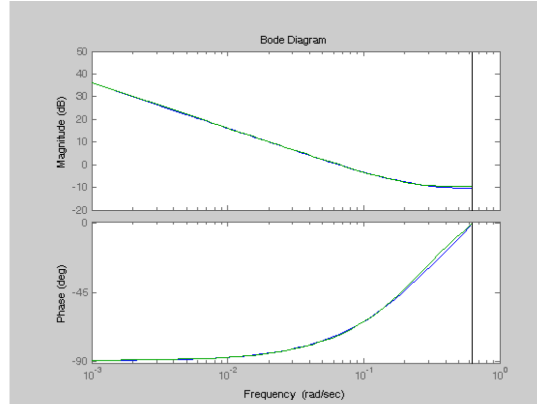

We repeat the same steps for the PID, using a pseudo-differentiator time constant Ts/2 = 2.5 and a double zero: $angle(PID) = -90 - \tan^{-1} T\omega_c + 2\tan^{-1}\frac{\omega_c}{a} = -59.2 \Rightarrow a = \frac{\omega_c}{\tan\frac{48.3}{2}} = 0.28.$

Next, we compute the gain of the controller so that the magnitude of PC is 1 at the crossover.

```
>> C=tf(conv([1 a],[1 a]),[2.5 1 0])
```



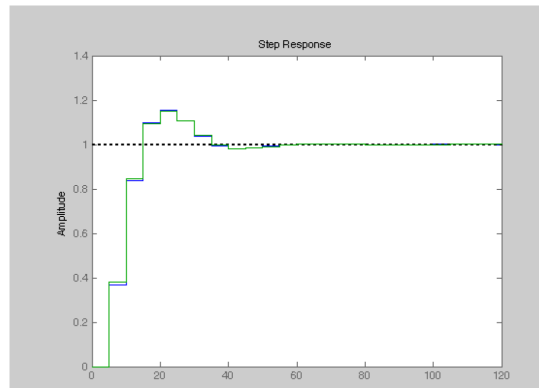
Transfer function:
s^2 + 0.5605 s + 0.07855

2.5 s^2 + s

>> [m,p]=bode(P*C,.126); m = 1.23e+000
>> C=tf(conv([1 a],[1 a]),[2.5 1 0])/m

Transfer function:
0.8151 s^2 + 0.4569 s + 0.06402

2.5 s^2 + s



And finally, discretize using the Tustin bilinear transformation
Cd=c2d(C,1/.2,'tustin')

Transfer function:
0.4715 z^2 - 0.166 z + 0.01461
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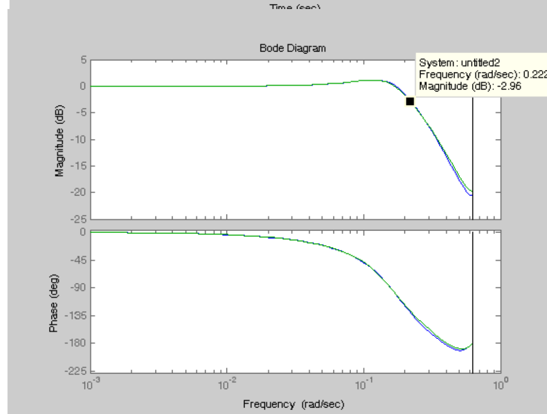
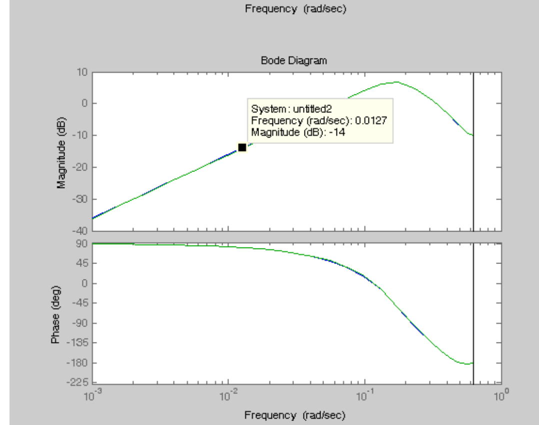
z^2 – z
Sampling time: 5



Comparing the two controllers we observe that, after making the denominators equal their numerator coefficients differ in the second significant digit and their bode plots (top graph) are quite similar. While differences in the controllers do not immediately translate into closed-loop differences (they must be weighted by appropriate sensitivity functions P/(1+PC)), their closeness in the entire frequency range suggests that the closed loop transfer functions will be very similar.

Next, we simulate the responses with the two different controllers. The quantities of interest are in the closed loop step response from the reference (or command or set point) and the frequency response from the input disturbance to the output.

The step responses are nearly identical, with rise time 2 samples and settling time 7 samples. The rise time of about 10s is consistent with the crossover frequency (2/BW ~ 1.3/wc = 10.3s). It is also too small for the differences between continuous and discrete designs to be ignored. (The “six-samples-per-rise-time” rule would require about 3-times faster sampling.) The overshoots are 15%, also consistent with the design parameter of 50 deg. phase margin. (The PID zeros are just outside the bandwidth and their effect on the overshoot is not as pronounced.) The actual bandwidth is 0.22rad/s, slightly larger than the estimate 1.5wc.



Finally, for the input disturbance attenuation we evaluate the bode plot of the sensitivity transfer function from the input disturbance to the output.
bode(fbk(c2d(P,5)*Cd,1),fbk(c2d(P,5)*CCd,1))
At 0.002Hz = 0.0126rad/s the magnitude of this transfer function is -14dB, signifying a factor of 5 attenuation of input disturbances at that frequency. The attenuation factor increases for lower frequencies and becomes 0 at DC (as expected, because of the integral action in the controller).