EEE 202 HW#3 SOLUTIONS

6.13 $i = C\dfrac{dv}{dt}$ or $v(t) = \dfrac{1}{C}\displaystyle\int_{-\infty}^{t} i(\tau)d\tau = v(t_0) + \dfrac{1}{C}\int_{t_0}^{t} i(\tau)d\tau$

$\therefore$ for $t \in [0, 2m]$, $v(t) = \dfrac{1}{5\mu}\displaystyle\int_0^t \dfrac{15}{2}\tau\, d\tau = \underline{7.5E5\, t^2}$ (in V)

at $t = 2ms$, $v(2m) = 3V$

for $t \in [2m, 4m]$ $v(t) = v(2m) + \dfrac{15m}{5\mu}(t - 2m) = \underline{3 + 3E3(t-2m)}$

at $t = 4ms$, $v(4m) = 9V$

for $t \in [4m, 6m]$ $v(t) = v(4m) - \dfrac{5m}{5\mu}(t - 4m) = 9 - 1E3(t - 4m)$

at $t = 6m$, $v(6m) = 7V$

for $t \in [6m, 8m]$ $v(t) = v(6m) + \dfrac{1}{5\mu}\cdot\left[\dfrac{5m}{2m}\dfrac{(t-6m)^2}{2} - 5m(t-6m)\right]$

$= 7 + \frac{1}{4}E6(t-6m)^2 - 1E3(t-6m)$

at $t = 8$, $v(8) = 6V$

The energy stored in the capacitor is $\frac{1}{2}Cv^2$, so

at $t = 1.4ms$, $v(1.4m) = 1.47 \rightarrow E = \frac{1}{2}5\mu \cdot 1.47^2 = \underline{5.4\mu J}$

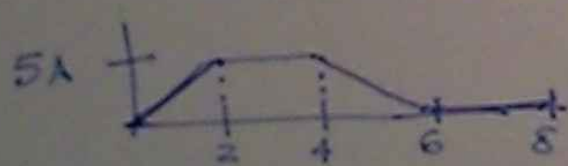
$t = 3.3ms$, $v(3.3m) = 6.9 \rightarrow \underline{E = 119\ \mu J}$

$t = 4.3ms$, $v(4.3m) = 8.7 \Rightarrow \underline{E = 189\ \mu J}$

$t = 6.7ms$, $v(6.7m) = 6.42 \Rightarrow \underline{E = 103\ \mu J}$

$t = 8.5ms$, $v(8.5m) = 6 \rightarrow \underline{E = 90\mu J}$

6.27 $v(t) = L\dfrac{di}{dt}$ or $i = \dfrac{1}{L}\int v \Rightarrow$ the graph for $i(t)$ is



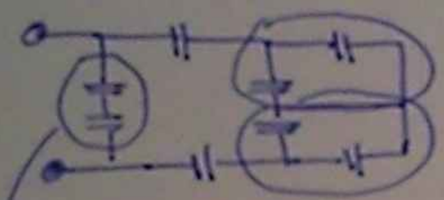
To find the peak, we note that the slope is $V/L = 5/2$ so $\frac{1}{2}\displaystyle\int_0^2 5\, d\tau = 5$ (A)

6.47

$$V_C = \frac{15}{15+10} 25 = 15(V) \Rightarrow E_C = \frac{1}{2} C v^2 = \frac{1}{2} \cdot 5E\text{-}3 \cdot 15^2 = 0.56\,J$$

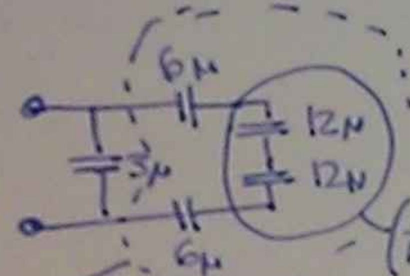
$$I_L = \frac{25\,(V)}{(10+15)(\Omega)} = 1\,(A) \Rightarrow E_L = \frac{1}{2} L I^2 = \frac{1}{2}\, 0.5 \cdot 1^2 = 0.25\,J$$

6.55



$\left(\frac{1}{6\mu}+\frac{1}{6\mu}\right)^{-1} = 3\mu$

$6\mu + 6\mu = 12\mu$ $\longrightarrow$

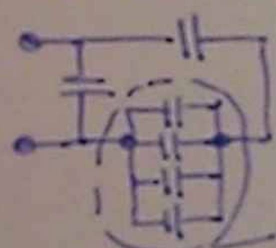
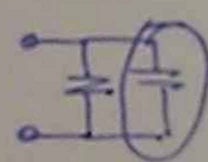


$\left(\frac{1}{12\mu}+\frac{1}{12\mu}\right)^{-1} = 6\mu$

$\left(\frac{1}{6\mu}+\frac{1}{6\mu}+\frac{1}{6\mu}\right)^{-1} = 2\mu$

$$\Rightarrow C_{eff} = 3\mu + 2\mu = \underline{\underline{5\,\mu F}}$$

6.56

 $\longrightarrow$  $\longrightarrow$ 

$4 \times 6\mu$

$\left(\frac{1}{24\mu}+\frac{1}{6\mu}\right)^{-1} = \frac{24}{5}\mu$

$$\Rightarrow C_{eff} = \frac{24}{5}\mu + 6\mu = \underline{\underline{\frac{54}{5}\mu F}}$$

7.46

The step-by-step method for discontinuous variables (i.e. other than capacitor voltages and inductor currents) can be described as follows

I. Solve for V_C (or I_L) & Replace Cap (or Ind.) by a Voltage (or Current) source $V_C(t)$ (or $I_L(t)$) and solve the resulting circuit.

II. 1) Find $V_C(0^-)$ (or $i_L(0^-)$) with the $t<0$-circuit.

2) Set $V_C(0^+) = V_C(0^-)$ (or $i_L(0^+) = i_L(0^-)$) and substitute the capacitor with a voltage source $V_C(0^+)$ (or, the inductor with a current source $i_L(0^+)$).

3) Compute the initial condition for the variable of interest, say x, such that $x(0^+)$ = value for the $t>0$-circuit (with the caps replaced by voltage sources).

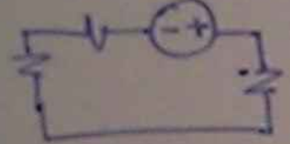
4). Compute $x(\infty)$ for the $t>0$ circuit

5) Compute R_{TH} as seen by the capacitor (or inductor) for the $t>0$ circuit.

6) $x(t) = x(\infty) + [x(0) - x(\infty)]\, e^{-t/\tau}$; $\tau = R_{TH} C$ (or R_{eq}/L).

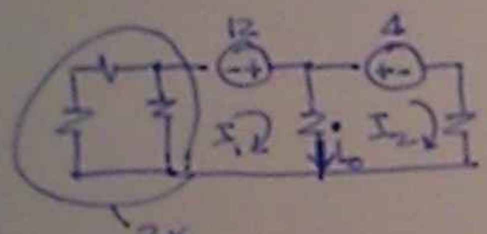
For our problem,

1) $t<0$



$V_{0^-} = \frac{2k}{6k} 12 = 4\,V$

3) $t>0$



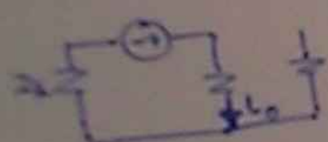
$I_1\, 2k - 12 + 2k\,(I_1 - I_2) = 0$

$4 + 2k I_2 + 2k\,(I_2 - I_1) = 0$

$\Rightarrow I_1 = \frac{10}{3}\,mA$, $I_2 = \frac{4}{6}\,mA$

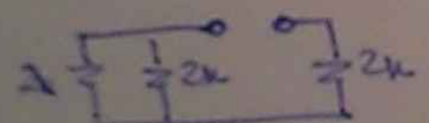
$\rightarrow \underline{i_0(0^+) = I_1 - I_2 = 8/3\,mA}$

4) $t>0$



$\Rightarrow i_0(\infty) = \frac{12}{4k} = 3\,mA$

5) $t>0$



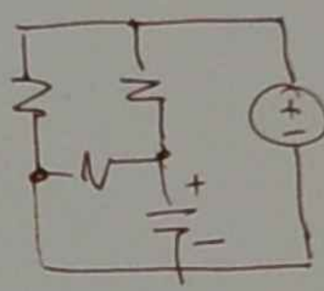
$\Rightarrow R_{TH} = \left(\frac{1}{2k} + \frac{1}{2k}\right)^{-1} + 2k = 3k\,\Omega$

6). $i_0(t) = 3m + \left(8/3\,m - 3m\right) e^{-t/0.6}$ (A)

where $0.6 = \tau = R_{eq} \cdot C = 3k\ 200\mu = 600\ ms$

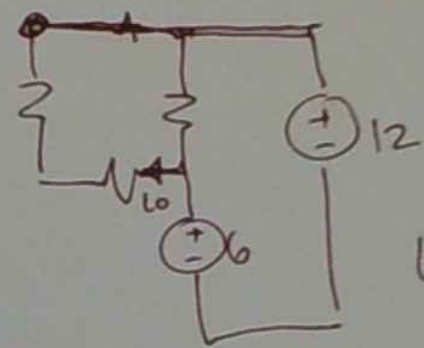
7.47

$t<0$



$$V_c(0) = \frac{(12)(6k)}{6k+6k} = 6\,(V)$$

$t>0$

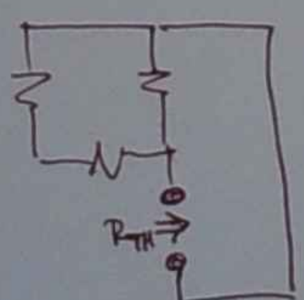


$$i_o(0) = -\frac{(12-6)}{6k+6k}$$

$$= \frac{1}{2}\,mA$$

$t>0 \quad i_o(\infty) = 0$

$$\Rightarrow \quad i_o(t) = \frac{1}{2}m\, e^{-t/\tau} \qquad \tau = R_{TH}C = 200m = 0.2\,s$$



$$R_{TH} = 6k \,\|\, (6k + 6k) = 4k$$

$$C = 50\mu \;\Rightarrow\; \tau = 200\,ms$$