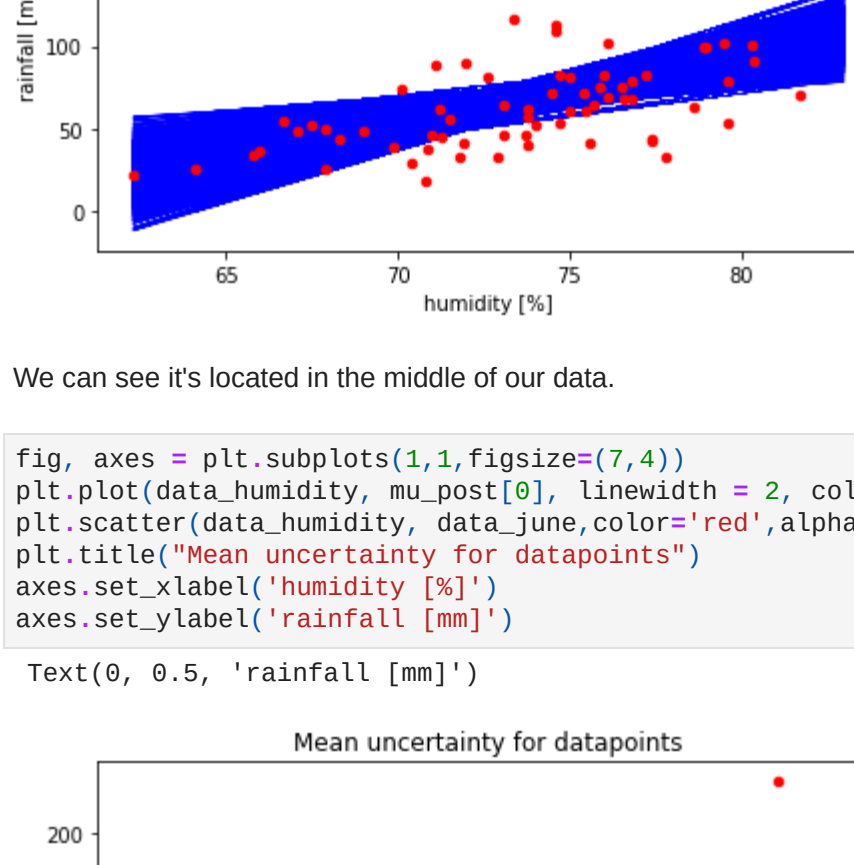



```
Text(0, 0.5, 'rainfall [mm]')
```

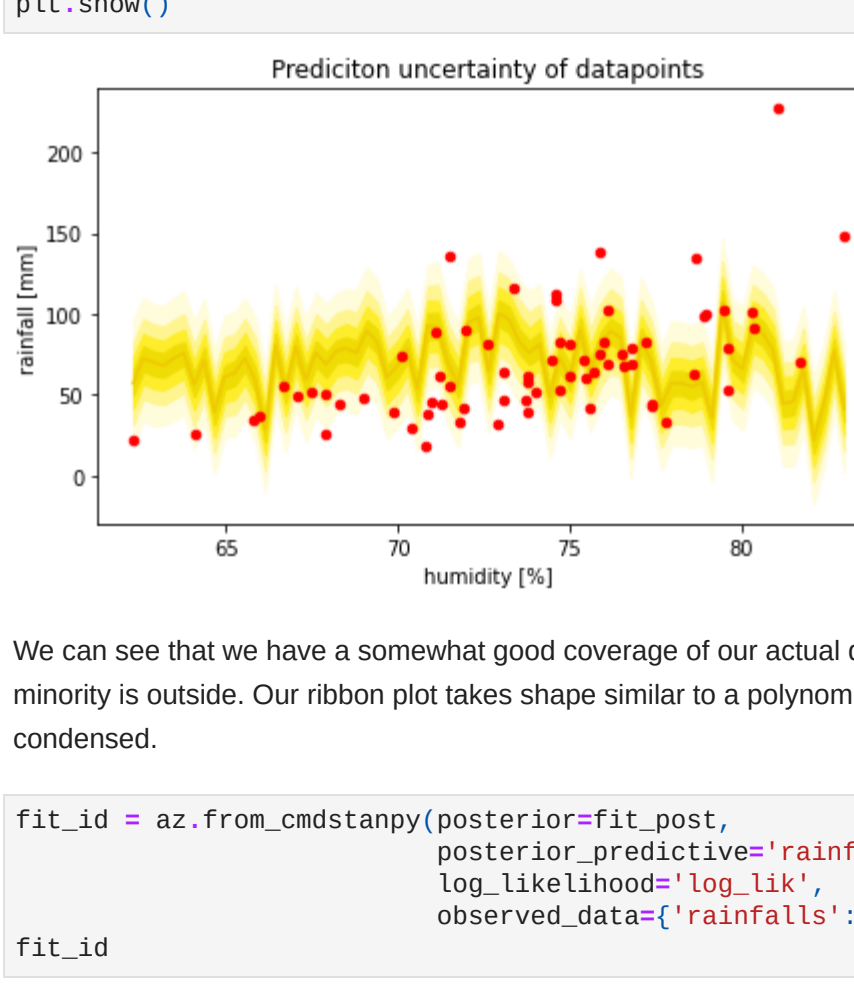
Mean samples for datapoints



We can see it's located in the middle of our data.

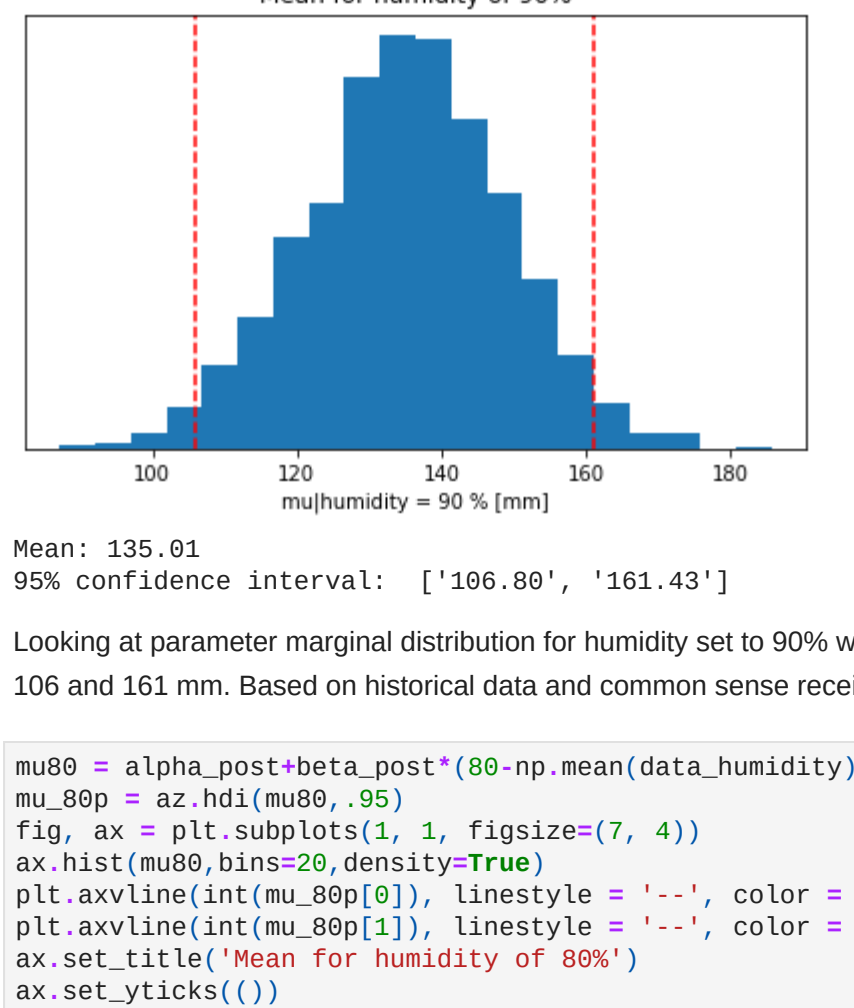
```
In [794]: fig, axes = plt.subplots(1,1,figsize=(7,4))
plt.plot(data_humidity, mu_post[0], linewidth=2, color='b',zorder=1)
plt.scatter(data_humidity, data_june,color='red',alpha=1,s=20,zorder=2)
plt.title("Mean uncertainty for datapoints")
axes.set_xlabel('humidity [%]')
axes.set_ylabel('rainfall [mm]')
```

```
Out[794]: Text(0, 0.5, 'rainfall [mm]')
```



The blue line is the distribution of the pressure on the mean of rainfall distribution. This is not an actual prediction but parameter of the distribution.

```
In [795]: fig, axes = plt.subplots(1,1,figsize=(7,4))
axes = ribbon_plot(data_sim_hum('humidity')*np.mean(data_humidity),rainfall_pred_post,axes)
axes.scatter(data_humidity, data_june,color='red',alpha=1,s=20)
plt.title("Prediction uncertainty of datapoints")
axes.set_xlabel('humidity [%]')
axes.set_ylabel('rainfall [mm]')
plt.show()
```



We can see that we have a somewhat good coverage of our actual datapoints. Not all of them are covered in this interval but only minority is outside. Our ribbon plot takes shape similar to a polynomial function which makes sense as our data isn't really condensed.

```
In [796]: fit_id = az.from_cmdstanpy(posterior=fit_post,
                                posterior_predictive='rainfall',
                                log_likelihood='log_lik',
                                observed_data=('rainfall',data_june) )
fit_id
```

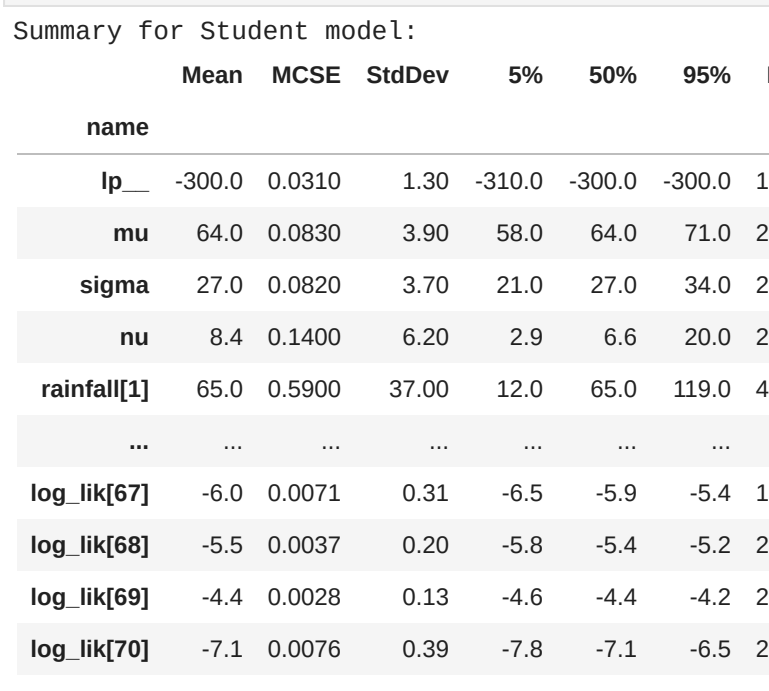
```
Out[796]: arviz.InferenceData
```

- posterior
- posterior_predictive
- log_likelihood
- sample_stats
- observed_data

Marginal for selected humidity

```
In [797]: mu90 = alpha_post+beta_post*(90-np.mean(data_humidity))
mu_90p = az.hdi(mu90,.95)
fig, ax = plt.subplots(1,1,figsize=(7,4))
ax.hist(mu90,bins=20,density=True)
ax.scatter(data_humidity, data_june,color='red',alpha=1,s=20)
plt.axvline(int(mu_90p[0]), linestyle = '--', color = 'r')
plt.axvline(int(mu_90p[1]), linestyle = '--', color = 'r')
ax.set_title("Mean for humidity of 90%")
ax.set_xlabel('mu[humidity = 90 % [mm]')
plt.show()
```

```
print('Mean: {:4.2f}'.format(np.mean(mu90)))
print('95% confidence interval: ',f'{:4.2f}'.format(k) for k in az.hdi(mu90,.95))
```

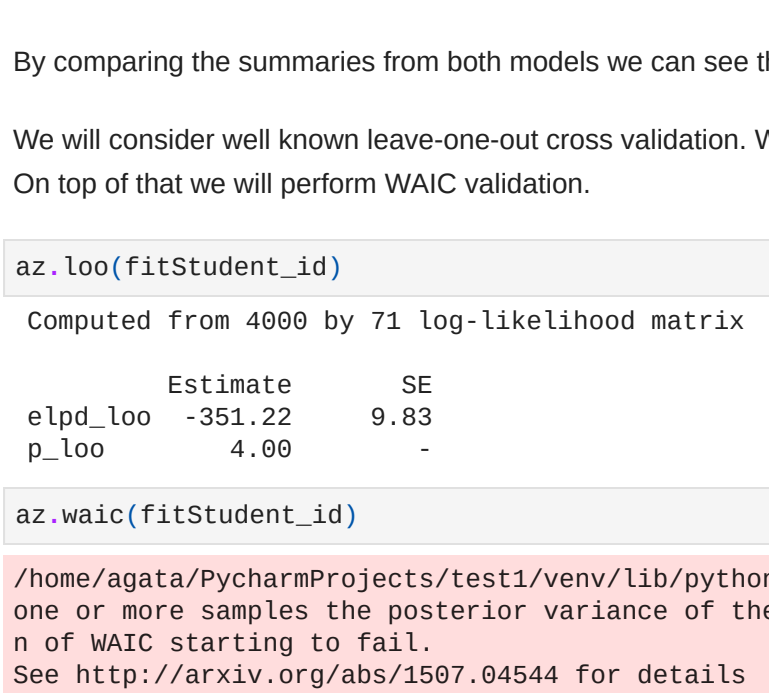


Mean: 135.01
95% confidence interval: ['106.80', '161.43']

Looking at parameter marginal distribution for humidity set to 90% we can see that the is 95 % probability that rainfall will be between 106 and 161 mm. Based on historical data and common sense received values are reasonable.

```
In [798]: mu80 = alpha_post+beta_post*(80-np.mean(data_humidity))
mu_80p = az.hdi(mu80,.95)
fig, ax = plt.subplots(1,1,figsize=(7,4))
ax.hist(mu80,bins=20,density=True)
plt.axvline(int(mu_80p[0]), linestyle = '--', color = 'r')
plt.axvline(int(mu_80p[1]), linestyle = '--', color = 'r')
ax.set_title("Mean for humidity of 80%")
ax.set_xlabel('mu[humidity = 80 % [mm]')
plt.show()
```

```
print('Mean: {:4.2f}'.format(np.mean(mu80)))
print('95% confidence interval: ',f'{:4.2f}'.format(k) for k in az.hdi(mu80,.95))
```

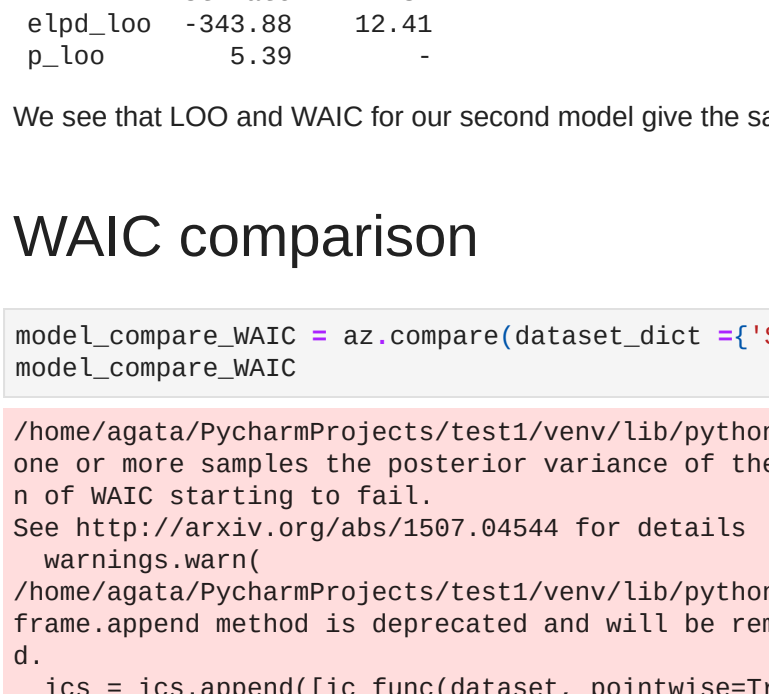


Mean: 93.95
95% confidence interval: ['61.74', '106.18']

Looking at parameter marginal distribution for humidity set to 80% we can see that the is 95 % probability that rainfall will be between 82 and 106 mm. Based on historical data and common sense received values are reasonable.

```
In [799]: mu70 = alpha_post+beta_post*(70-np.mean(data_humidity))
mu_70p = az.hdi(mu70,.95)
fig, ax = plt.subplots(1,1,figsize=(7,4))
ax.hist(mu70,bins=20,density=True)
plt.axvline(int(mu_70p[0]), linestyle = '--', color = 'r')
plt.axvline(int(mu_70p[1]), linestyle = '--', color = 'r')
ax.set_title("Mean for humidity of 70%")
ax.set_xlabel('mu[humidity = 70 % [mm]')
plt.show()
```

```
print('Mean: {:4.2f}'.format(np.mean(mu70)))
print('95% confidence interval: ',f'{:4.2f}'.format(k) for k in az.hdi(mu70,.95))
```



Mean: 52.99
95% confidence interval: ['43.67', '61.63']

Looking at parameter marginal distribution for humidity set to 70% we can see that the is 95 % probability that rainfall will be between 44 and 62 mm. Based on historical data and common sense received values are reasonable.

Moreover, comparing results from all three histograms we see that the higher the predicted rain values are, the higher the humidity. Obtained results are true to nature's laws.

Model comparison

Posterior summary statistics

```
In [800]: print("Summary for Student model:")
fit_post.summary()
```

```
Out[800]: Summary for Student model:
```

	name	Mean	MCSE	StdDev	5%	50%	95%	N_Eff	N_Effs	R_hat
Student model	lp_	-300.0	0.0310	1.30	-310.0	-300.0	-300.0	1700.0	1800.0	1.0
	mu	64.0	0.0630	3.90	58.0	64.0	71.0	2200.0	3100.0	1.0
	sigma	27.0	0.0620	3.70	21.0	27.0	34.0	2100.0	2200.0	1.0
	nu	8.4	0.1400	6.20	2.9	6.6	20.0	2000.0	2100.0	1.0
	rainfall[1]	65.0	0.5900	37.00	12.0	65.0	119.0	4008.0	4273.0	1.0
Student model
	log_lik[67]	-6.0	0.0071	0.31	-6.5	-5.9	-5.4	1971.0	2101.0	1.0
	log_lik[68]	-5.5	0.0037	0.20	-5.8	-5.4	-5.2	2853.0	3042.0	1.0
	log_lik[69]	-4.4	0.0028	0.13	-4.6	-4.4	-4.2	2047.0	2183.0	1.0
	log_lik[70]	-7.1	0.0076	0.39	-7.8	-7.1	-6.5	2662.0	2838.0	1.0
Student model
	log_lik[71]	-4.4	0.0028	0.13	-4.6	-4.4	-4.2	2069.0	2205.0	1.0

146 rows x 9 columns

```
In [801]: print("Summary for Gaussian model:")
fit_post.summary()
```

```
Out[801]: Summary for Gaussian model:
```

	name	Mean	MCSE	StdDev	5%	50%	95%	N_Eff	N_Effs	R_hat
Gaussian model	lp_	-280.0	0.029	1.30	-280.0	-280.0	-280.0	1900.0	1500.0	1.0
	alpha	69.0	0.055	3.50	63.0	69.0	74.0	3900.0	3100.0	1.0
	beta	4.1	0.014	0.83	2.7	4.1	5.4	3500.0	2700.0	1.0
	sigma	29.0	0.045	2.50	26.0	29.0	34.0	3200.0	2500.0	1.0
	mu[1]	57.0	0.067	4.10	51.0	57.0	64.0	3680.0	2900.0	1.0
Gaussian model
	rainfall[67]	66.0	0.480	30.00	18.0	66.0	116.0	4015.0	3164.0	1.0
	rainfall[68]	21.0	0.510	31.00	-29.0	22.0	72.0	3568.0	2812.0	1.0
	rainfall[69]	44.0	0.500	30.00	-6.1	44.0	94.0	3752.0	2957.0	1.0
	rainfall[70]	77.0	0.480	29.00	29.0	77.0	126.0	3975.0	3132.0	1.0
Gaussian model
	rainfall[71]	41.0	0.480	31.00	-9.0	41.0	92.0	4086.0	3220.0	1.0

217 rows x 9 columns

By comparing the summaries from both models we can see that the mean of sigma parameter is lower for our second model.

We will consider well known leave-one-out cross validation. We will use an estimator based on Pareto Smoothed Importance Sampling. On top of that we will perform WAIC validation.

```
In [802]: az.loo(fitStudent_id)
```

```
Out[802]: Computed from 4000 by 71 log-likelihood matrix
```

	Estimate	SE
elpd_loo	-351.22	9.63
p_loo	4.68	-

```
In [803]: az.waic(fitStudent_id)
```

```
/home/agata/PycharmProjects/test1/venv/lib/python3.9/site-packages/arviz/stats/stats.py:1458: UserWarning: For one or more samples the posterior variance of the log predictive densities exceeds 0.4. This could be indication of WAIC starting to fail.
See http://arxiv.org/abs/1507.04544 for details
warnings.warn(
Computed from 4000 by 71 log-likelihood matrix
```

	Estimate	SE
elpd_waic	-351.11	9.75
p_waic	3.89	-

There has been a warning during the calculation. Please check the results.

We see that LOO and WAIC for our first model gives a very similar result (as they should). It's important to note that LOO has a better performance across a wider variety of models.

```
In [804]: az.loo(fit_id)
```

```
/home/agata/PycharmProjects/test1/venv/lib/python3.9/site-packages/arviz/stats/stats.py:1458: UserWarning: For one or more samples the posterior variance of the log predictive densities exceeds 0.4. This could be indication of WAIC starting to fail.
See http://arxiv.org/abs/1507.04544 for details
warnings.warn(
Computed from 4000 by 71 log-likelihood matrix
```

	Estimate	SE
elpd_loo	-343.88	12.42
p_loo	5.39	-

There has been a warning during the calculation. Please check the results.

```
In [805]: az.loo(fit_id)
```

```
Out[805]: Computed from 4000 by 71 log-likelihood matrix
```

	Estimate	SE
elpd_loo	-343.88	12.41
p_loo	5.39	-

We see that LOO and WAIC for our second model give the same exact result.

WAIC comparison

```
In [806]: model_compare_WAIC = az.compare(dataset_dict = {'Student model':fitStudent_id, 'Gaussian model':fit_id, ic='waic'}, model_compare_WAIC)
```

```
/home/agata/PycharmProjects/test1/venv/lib/python3.9/site-packages/arviz/stats/stats.py:1458: UserWarning: For one or more samples the posterior variance of the log predictive densities exceeds 0.4. This could be indication of WAIC starting to fail.
See http://arxiv.org/abs/1507.04544 for details
warnings.warn(
/home/agata/PycharmProjects/test1/venv/lib/python3.9/site-packages/arviz/stats/stats.py:248: FutureWarning: The frame.append method is deprecated and will be removed from pandas in a future version. Use pandas.concat instead.
ics = ics.append([ic_func(dataset, pointwise=True, scale=scale, var_name=var_name)])
/home/agata/PycharmProjects/test1/venv/lib/python3.9/site-packages/arviz/stats/stats.py:1458: UserWarning: For one or more samples the posterior variance of the log predictive densities exceeds 0.4. This could be indication of WAIC starting to fail.
See http://arxiv.org/abs/1507.04544 for details
warnings.warn(
/home/agata/PycharmProjects/test1/venv/lib/python3.9/site-packages/arviz/stats/stats.py:248: FutureWarning: The frame.append method is deprecated and will be removed from pandas in a future version. Use pandas.concat instead.
ics = ics.append([ic_func(dataset, pointwise=True, scale=scale, var_name=var_name)])
```

	rank	waic	p_waic	d_waic	weight	se	dse	warning	waic_scale
Gaussian model	0	-343.880162	5.390936	0.000000	0.824591	12.421013	0.000000	True	log
Student model	1	-351.109292	3.898955	7.227212	0.173409	9.746650	5.742774	True	log

According to results obtained from using LOO information criterion we can see that our second model has a rank 0 which means it's the best model. Higher IC for the second model indicates higher out-of-sample predictive fit ("better" model).

Unfortunately for both models we observe a warning which indicated that the computation of IC is starting to fail.

```
In [807]: az.plot_compare(model_compare_WAIC)
```

```
Out[807]: <AxesSubplot:xlabel='Log'>
```



LOO Comparison

```
In [808]: model_compare_LOO = az.compare(dataset_dict = {'Student model':fitStudent_id, 'Gaussian model':fit_id, ic='loo'}, model_compare_LOO)
```

```
/home/agata/PycharmProjects/test1/venv/lib/python3.9/site-packages/arviz/stats/stats.py:248: FutureWarning: The frame.append method is deprecated and will be removed from pandas in a future version. Use pandas.concat instead.
ics = ics.append([ic_func(dataset, pointwise=True, scale=scale, var_name=var_name)])
/home/agata/PycharmProjects/test1/venv/lib/python3.9/site-packages/arviz/stats/stats.py:248: FutureWarning: The frame.append method is deprecated and will be removed from pandas in a future version. Use pandas.concat instead.
ics = ics.append([ic_func(dataset, pointwise=True, scale=scale, var_name=var_name)])
```

	rank	waic	p_loo	d_loo	weight	se	dse	warning	loo_scale
Gaussian model	0	-343.880162	5.390936	0.000000	0.825623	12.411448	0.000000	False	log
Student model	1	-351.223216	4.003779	7.343055	0.173377	9.834404	5.663364	False	log

According to results obtained from using LOO information criterion we can see that our second model has a rank 0 which means it's the best model. Higher IC for the second model indicates higher out-of-sample predictive fit ("better" model).

There are no warnings.

```
In [809]: az.plot_compare(model_compare_LOO)
```

```
Out[809]: <AxesSubplot:xlabel='Log'>
```



Both WAIC and PSIS-LOO information criterion indicate supremacy of our Gaussian model over the Student-T model.

Using Gaussian distribution and adding new parameter to our model (humidity) along with adjusting the prior (Beta from normal to lognormal) resulted in obtaining better results overall. Student-T distribution seemed like a good model to try based on nature of our dataset but ended up not performing as well as the second model.

Since our intention while adding new parameter was to improve our model, we expected the result of the comparison to be to the advantage of the second model. Obtained comparison results confirm our expectations.