

George Boole (1815-1864)



George Boole was an English

- mathematician,
- philosopher and
- logician.

- He worked in the fields
of differential
equations and algebraic logic

Boolean Algebra

- Invented by famous mathematician **George Boole**
- All operations are done by 2 states:
 - **True (1)**
 - **False (0)**
- Later **True** was represented by **1**
- **False** was represented by **0**
- So its become possible to implement Boolean algebra into computer circuit design

Boolean Operation

- Boolean Addition(Logical OR)
- Boolean Multiplication(Logical AND)
- Boolean Complement(Logical NOT)

Boolean Operation

■ Boolean Addition (Logical OR)

$$\blacksquare \quad 0+0= \quad 0$$

$$\blacksquare \quad 0+1= \quad 1$$

$$\blacksquare \quad 1+0= \quad 1$$

$$\blacksquare \quad 1+1= \quad 1$$

Boolean Operation

- Boolean Multiplication (Logical AND)
- $0.0 = 0$
- $0.1 = 0$
- $1.0 = 0$
- $1.1 = 1$

Boolean Complement

- Complement of 1 is 0

 - i.e. $\bar{1} = 0$

- Complement of 0 is 1

 - i.e. $\bar{0} = 1$

- If $A=1$ Then $\bar{A}=0$

- If $A=0$ Then $\bar{A}=1$

Boolean Theorem

Logical OR

$$A + 0 = A$$

$$A + 1 = 1$$

$$A + A = A$$

$$A + \overline{A} = 1$$

Boolean Theorem

Logical AND

$$A \cdot 0 = 0$$

$$A \cdot 1 = A$$

$$A \cdot A = A$$

$$A \cdot \overline{A} = 0$$

Boolean Theorem

Logical NOT

$$\overline{\overline{A}} = A$$

$$\overline{\overline{AB}} = AB$$

$$\overline{\overline{A + B}} = A + B$$

XOR and XNOR

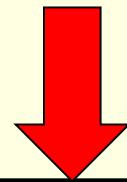
$$A \oplus B = \overline{\overline{A}B + A\overline{B}}$$

$$\overline{A \oplus B} = \overline{\overline{A} \cdot \overline{B}} + A\overline{B}$$

Truth Table (সত্যক সারণি)

- A **truth table** is a **table** showing the outputs for all possible combinations of inputs to a **logic gate** or **circuit**.

Input		Output
A	B	$A+B$
0	0	0
0	1	1
1	0	1
1	1	1

\overline{AB} 

Input		AB	Output
A	B		$X = \overline{AB}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

$$F = AB + \bar{B}$$

A	B	\bar{B}	AB	$F=AB+\bar{B}$
0	0	1	0	1
0	1	0	0	0
1	0	1	0	1
1	1	0	1	1

$$X(\overline{Y+Z})$$

X	Y	Z	(Y+Z)	$\overline{(Y+Z)}$	X $\overline{(Y+Z)}$
0	0	0	0	1	0
0	0	1	1	0	0
0	1	0	1	0	0
0	1	1	1	0	0
1	0	0	0	1	1
1	0	1	1	0	0
1	1	0	1	0	0
1	1	1	1	0	0

De-Morgan's Theorems

$$\overline{A + B} = \overline{A} \cdot \overline{B}$$

$$\overline{AB} = \overline{A} + \overline{B}$$

De-Morgan's Theorems(2 variables)

$$\overline{A + B} = \overline{A} \cdot \overline{B}$$

A	B	\overline{A}	\overline{B}	$A+B$	$\overline{A+B}$	$\overline{A} \cdot \overline{B}$
0	0	1	1	0	1	1
0	1	1	0	1	0	0
1	0	0	1	1	0	0
1	1	0	0	1	0	0

De-Morgan's Theorems(2 variables)

$$\overline{AB} = \overline{A} + \overline{B}$$



A	B	\overline{A}	\overline{B}	AB	\overline{AB}	$\overline{A+B}$
0	0	1	1	0	1	1
0	1	1	0	0	1	1
1	0	0	1	0	1	1
1	1	0	0	1	0	0

De-Morgan's Theorems with 3 variables

$$\overline{A+B+C} = \overline{A} \cdot \overline{B} \cdot \overline{C}$$

$$\overline{ABC} = \overline{A} + \overline{B} + \overline{C}$$

A	B	C	\bar{A}	\bar{B}	\bar{C}	$A+B+C$	$\overline{A+B+C}$	\overline{ABC}	ABC	\overline{ABC}	$\overline{A}+\overline{B}+\overline{C}$
0	0	0	1	1	1	0	1	1	0	1	1
0	0	1	1	1	0	1	0	0	0	1	1
0	1	0	1	0	1	1	0	0	0	1	1
0	1	1	1	0	0	1	0	0	0	1	1
1	0	0	0	1	1	1	0	0	0	1	1
1	0	1	0	1	0	1	0	0	0	1	1
1	1	0	0	0	1	1	0	0	0	1	1
1	1	1	0	0	0	1	0	0	1	0	0

De-Morgan's Theorems with N variables

$$\overline{A_1 + A_2 + A_3 + \dots + A_N} = \overline{\overline{A}_1} \ \overline{\overline{A}_2} \ \overline{\overline{A}_3} \dots \ \overline{\overline{A}_N}$$

$$\overline{A_1 \ A_2 \ A_3 \dots \ A_N} = \overline{\overline{A}_1} + \overline{\overline{A}_2} + \overline{\overline{A}_3} + \dots + \overline{\overline{A}_N}$$

Boolean Algebra(Simplification)

$$1) A + A\bar{B}$$

$$= A(1 + \bar{B})$$

$$= A(1)$$

$$= A$$

Boolean Algebra(Simplification)

$$\begin{aligned}2) A + \overline{A}B + \overline{A} \overline{B} \\&= A + \overline{A} (B + \overline{B}) \\&= A + \overline{A}(1) \\&= 1\end{aligned}$$

3) Proof that

$$(A + B)(A + \bar{B}) = A$$

$$L.H.S = (A + B)(A + \bar{B})$$

$$= A.A + A\bar{B} + BA + B.\bar{B}$$

$$= A + A\bar{B} + AB + 0$$

$$= A(1 + \bar{B}) + AB$$

$$= A(1) + AB$$

$$= A(1 + B)$$

$$= A$$

$$= R.H.S \quad (\text{Proved})$$

Simplification of Boolean Expressions

4. Prove that $(A + B)(A + C) = A + BC$

$$\begin{aligned}L.H.S &= (A + B)(A + C) \\&= A.A + AC + BA + BC \\&= A + AC + AB + BC \\&= A(1 + C) + AB + BC \\&= A.1 + AB + BC \\&= A + AB + BC \\&= A(1 + B) + BC \\&= A.1 + BC \\&= A + BC \quad = R.H.S \quad (\text{Proved})\end{aligned}$$

Proof that , $\overline{X} + X = 1$

■ Let, $X=1$

$$L.H.S,$$

$$= \overline{X} + X$$

$$= \bar{1} + 1$$

$$= 0 + 1$$

$$= 1$$

Let, $X=0$

$$L.H.S,$$

$$= \overline{X} + X$$

$$= \bar{0} + 0$$

$$= 1 + 0$$

$$= 1$$

$$\begin{aligned}
& 7) \overline{B}\overline{C} + \overline{B}\overline{\cdot}\overline{C} + BC \\
&= \overline{C}(B + \overline{B}) + BC \\
&= \overline{C} \cdot 1 + BC \\
&= \overline{C} + BC \\
&= (\overline{C} + B)(\overline{C} + C) \\
&= (\overline{C} + B)(1) \\
&= \overline{C} + B
\end{aligned}$$

$$\begin{aligned}
& 8) ABC + A\overline{B}\overline{C} + \overline{A} \\
&= AC(B + \overline{B}) + \overline{A} \\
&= AC + \overline{A} \\
&= \overline{A} + AC \\
&= (\overline{A} + A)(\overline{A} + C) \\
&= 1 (\overline{A} + C) \\
&= \overline{A} + C
\end{aligned}$$

$$(\overline{A} + \overline{B})(A + B)$$

$$= \overline{A}\overline{A} + \overline{A}B + \overline{B}A + \overline{B}\overline{B}$$

$$= 0 + \overline{A}B + A\overline{B} + 0$$

$$= \overline{A}B + A\overline{B}$$

$$= \underline{\underline{A}} \oplus B :$$

$$10)(A + \overline{B})(\overline{A} + B)$$

$$= A.\overline{A} + AB + \overline{A}.\overline{B} + B.\overline{B}$$

$$= 0 + AB + \overline{A}\overline{B} + 0$$

$$= AB + \overline{A}\overline{B}$$

$$=\overline{A\oplus B}$$

9. Proof that, $(A+B+C) BC = BC$

L.H.S,

$$=(A+B+C) BC$$

$$=A.BC+B.BC+C.BC$$

$$=ABC+BC+BC$$

$$=ABC+BC$$

$$=BC(A+1)$$

$$=BC(1)$$

$$=BC$$

=R.H.S (Proved)

$$11) AB\bar{C} + ABC + A\bar{B}\bar{C} + ABC$$

$$= AB(\bar{C} + C) + AC(\bar{B} + B)$$

$$= AB(1) + AC(1)$$

$$= AB + AC$$

$$= A(B + C)$$

$$12)\overline{A}\overline{B}\overline{C}+A\overline{B}\overline{C}+B\overline{C}D$$

$$=B\overline{C}(\overline{A}+A)+B\overline{C}D$$

$$=B\overline{C}.1+B\overline{C}D$$

$$=B\overline{C}(1+D)$$

$$=B\overline{C}(1)$$

$$=B\overline{C}$$

$$XYZ + X\bar{Y}Z + \bar{X}YZ + \bar{X}\bar{Y}Z$$

$$= XZ(Y + \bar{Y}) + \bar{X}Z(Y + \bar{Y})$$

$$= XZ(1) + \bar{X}Z(1)$$

$$= XZ + \bar{X}Z$$

$$= Z(X + \bar{X})$$

$$= Z(1)$$

$$= Z$$

$$13) ABC + A\bar{B}C + ABC + \bar{A}C$$

$$= AC(B + \bar{B}) + ABC + \bar{A}C$$

$$= AC + ABC + \bar{A}C$$

$$= AC(1 + B) + \bar{A}C$$

$$= AC + \bar{A}C$$

$$= C(A + \bar{A})$$

$$= C.1$$

$$= C$$

$$15)z(y+z)(x+y+z)$$

$$= (zy + zz)(x + y + z)$$

$$= (zy + z)(x + y + z)$$

$$= z(y+1)(x + y + z)$$

$$= z(x + y + z)$$

$$= zx + zy + z.z$$

$$= zx + zy + z$$

$$= zx + z(y+1)$$

$$= zx + z$$

$$= z(x+1)$$

$$= z$$

$$14)\overline{\overline{A}}.\overline{\overline{B}}.\overline{\overline{C}}.\overline{\overline{D}}$$

$$=\overline{\overline{A}}+\overline{\overline{B}}+\overline{\overline{C}}+\overline{\overline{D}}$$

$$=A+B+C+D$$

$$\overline{\overline{(A+B)}} \; \overline{\overline{(B+C)}}$$

$$\overline{\overline{(A+B)}} + \overline{\overline{\overline{(B+C)}}}$$

$$A+B + B+C$$

$$A+B+C$$

Simplify

$$17) \overline{(\bar{A} + C)} + \overline{B + \bar{D}}$$

$$= (\bar{\bar{A}}.\bar{C}) + (\bar{B}.\bar{\bar{D}})$$

$$= A\bar{C} + \bar{B}D$$

$$18) \overline{(M + \bar{N})(\bar{M} + N)}$$

$$= \overline{(M + \bar{N})} + \overline{(\bar{M} + N)}$$

$$= \overline{M}\overline{N} + \overline{M}\overline{N}$$

$$= \overline{M}N + M\overline{N}$$

$$= M \oplus N$$

Simplify

$$19) \overline{(A + B + \overline{C})} \overline{B} C$$

$$= \overline{A} \cdot \overline{B} \cdot \overline{\overline{C}} \cdot \overline{B} \cdot C$$

$$= \overline{A} \cdot \overline{B} \cdot \overline{B} \cdot C \cdot C$$

$$= \overline{A} \cdot \overline{B} \cdot C$$

Simplify

$$20) \overline{(A + B + \overline{C})} + \overline{B} C$$

$$= \overline{A} \overline{B} \overline{\overline{C}} + \overline{B} C$$

$$= \overline{A} \cdot \overline{B} \cdot C + \overline{B} C$$

$$= \overline{B} C(\overline{A} + 1)$$

$$= \overline{B} C(1)$$

$$= \overline{B} C$$

$$21) \overline{(A + \overline{B})} + \overline{(A + B)}$$

$$= \overline{\overline{A}} \overline{B} + \overline{A} \overline{\overline{B}}$$

$$= \overline{\overline{A}} \overline{B} + \overline{\overline{A}} \overline{\overline{B}}$$

$$= \overline{\overline{A}} \cdot (B + \overline{B})$$

$$= \overline{\overline{A}} \cdot (1)$$

$$= \overline{A}$$

23) Proof that

$$\overline{A + \overline{B} + C\overline{D}} = \overline{A}B(\overline{C} + D)$$

$$L.H.S = \overline{A + \overline{B} + C\overline{D}}$$

$$= \overline{\overline{A}} \overline{\overline{B}} \overline{\overline{C\overline{D}}}$$

$$= \overline{A}B(\overline{C} + \overline{\overline{D}})$$

$$= \overline{A}B(\overline{C} + D) \quad = R.H.S$$

$$24) \overline{\overline{A} \overline{B}} + (AB)$$

$$= \overline{\overline{AB}} (\overline{AB})$$

$$= (\overline{\overline{A}} + \overline{\overline{B}})(\overline{\overline{A}} + \overline{\overline{B}})$$

$$= (A + B)(\overline{\overline{A}} + \overline{\overline{B}})$$

$$= A\overline{\overline{A}} + A\overline{\overline{B}} + B\overline{\overline{A}} + B\overline{\overline{B}}$$

$$= 0 + A\overline{\overline{B}} + \overline{\overline{A}}B + 0$$

$$= \overline{\overline{A}}B + A\overline{\overline{B}}$$

$$= A \oplus B$$

$$25) \overline{(A + B)} \left(\overline{\overline{A}} + \overline{\overline{B}} \right)$$

$$= (\overline{\overline{A}} \cdot \overline{\overline{B}}) \left(\overline{\overline{A}} \cdot \overline{\overline{B}} \right)$$

$$= \overline{\overline{A}} \cdot \overline{\overline{B}} \cdot A \cdot B$$

$$= \overline{\overline{A}} \cdot A \cdot \overline{\overline{B}} \cdot B$$

$$= 0.0$$

$$= 0$$

$$26) \overline{(A + \bar{B})} \cdot \overline{(\bar{A} + \bar{B})}$$

$$= \overline{(A + \bar{B})} + \overline{(\bar{A} + \bar{B})}$$

$$= \bar{A} \cdot \bar{\bar{B}} + \bar{\bar{A}} \cdot \bar{\bar{B}}$$

$$= \bar{A} \cdot B + A \cdot B$$

$$= B(\bar{A} + A)$$

$$= B \cdot 1$$

$$= B$$

$$27) \overline{(A + \bar{B})} + \overline{(A + B)}$$

$$= \overline{(A + \bar{B})} \cdot \overline{(A + B)}$$

$$= (\bar{A} \cdot \bar{\bar{B}}) \cdot (\bar{A} \cdot \bar{B})$$

$$= \bar{A} \cdot B \cdot \bar{A} \cdot \bar{B}$$

$$= \bar{A} \bar{A} \cdot B \bar{B}$$

$$= \bar{A} \cdot B \cdot \bar{B}$$

$$= \bar{A} \cdot 0$$

$$= 0$$

If $F = \overline{\overline{(\bar{A} + B)} + B + \bar{C}}$ then when F=1

$$F = \overline{\overline{(\bar{A} + B)} + B + \bar{C}}$$

$$\overline{\overline{A + B}} \cdot \overline{B + \bar{C}}$$

$$(\bar{A} + B)\bar{B}\bar{C}$$

$$\bar{A}\bar{B}\bar{C} + B\bar{B}.C$$

$$\bar{A}\bar{B}\bar{C} + 0$$

$$\bar{A}\bar{B}\bar{C}$$

$$\bar{0}\bar{0}1$$

$$\therefore A = 0, B = 0, C = 1$$

$$28) \overline{(A + B).A\bar{B}}$$

$$= \overline{(A + B)} + \overline{A\bar{B}}$$

$$= \overline{A}.\overline{B} + (\overline{A} + \overline{\overline{B}})$$

$$= \overline{A}.\overline{B} + \overline{A} + B$$

$$= \overline{A}(\overline{B} + 1) + B$$

$$= \overline{A}.1 + B$$

$$= \overline{A} + B$$

$$29) \overline{(\overline{A} + B)} + \overline{(A\bar{B})}$$

$$= \overline{(\overline{A} + B)} \overline{(A\bar{B})}$$

$$= (A.\overline{B})(A\bar{B})$$

$$= AAB\bar{B}\bar{B}$$

$$= A.\overline{B}$$

Proof that, 30) $\overline{x + \bar{y}(z + \bar{x})} = \bar{x}y$

$$\begin{aligned} L.H.S &= \overline{x + \bar{y}(z + \bar{x})} \\ &= \bar{x} \cdot \overline{\bar{y}(z + \bar{x})} \\ &= \bar{x} \left(\overline{\bar{y}} + z + \bar{x} \right) \\ &= \bar{x} \left(y + (z + \bar{x}) \right) \\ &= \bar{x} \left(y + \bar{z} \cdot \bar{\bar{x}} \right) \\ &= \bar{x} \left(y + \bar{z} \cdot x \right) \\ &= \bar{x}y + x \cdot \bar{x} \cdot \bar{z} \\ &= \bar{x}y + 0 \cdot \bar{z} \\ &= \bar{x}y + 0 \\ &= \bar{x}y \quad = R.H.S \end{aligned}$$

$$\begin{aligned}
& 31) \overline{\overline{R}} \overline{\overline{S}} \overline{\overline{T}}(R + S + T) \\
&= (\overline{\overline{R}} + \overline{\overline{S}} + \overline{\overline{T}}) (\overline{\overline{R}} \cdot \overline{\overline{S}} \cdot \overline{\overline{T}}) \\
&= \overline{\overline{R}}(\overline{\overline{\overline{R}} \overline{\overline{S}} \overline{\overline{T}}}) + \overline{\overline{S}}(\overline{\overline{\overline{R}} \overline{\overline{S}} \overline{\overline{T}}}) + \overline{\overline{T}}(\overline{\overline{\overline{R}} \overline{\overline{S}} \overline{\overline{T}}}) \\
&= \overline{\overline{\overline{R}} \overline{\overline{R}} \overline{\overline{S}} \overline{\overline{T}}} + \overline{\overline{\overline{R}} \overline{\overline{S}} \overline{\overline{S}} \overline{\overline{T}}} + \overline{\overline{\overline{R}} \overline{\overline{S}} \overline{\overline{T}} \overline{\overline{T}}} \\
&= \overline{\overline{R}} \overline{\overline{S}} \overline{\overline{T}} + \overline{\overline{R}} \overline{\overline{S}} \overline{\overline{T}} + \overline{\overline{R}} \overline{\overline{S}} \overline{\overline{T}} \\
&= \overline{\overline{R}} \overline{\overline{S}} \overline{\overline{T}}
\end{aligned}$$

$$\begin{aligned}
32) & \overline{A}\overline{B} + \overline{(C+A)} + \overline{\overline{B}C} \\
&= \overline{(AB)}\overline{(C+A)} + \overline{\overline{B}} + \overline{C} \\
&= (\overline{A} + \overline{\overline{B}})(\overline{C}\cdot\overline{A}) + B + \overline{C} \\
&= (\overline{A} + B)(\overline{A}\cdot\overline{C}) + B + \overline{C} \\
&= \overline{A}\cdot\overline{A}\cdot\overline{C} + \overline{A}\cdot B\cdot\overline{C} + B + \overline{C} \\
&= \overline{A}\cdot\overline{C} + \overline{A}\cdot B\cdot\overline{C} + B + \overline{C} \\
&= \overline{A}\cdot\overline{C}(1+B) + B + \overline{C} \\
&= \overline{A}\cdot\overline{C} + B + \overline{C} \\
&= \overline{A}\cdot\overline{C} + \overline{C} + B \\
&= \overline{C}(\overline{A} + 1) + B \\
&= \overline{C} + B
\end{aligned}$$

33) if $x \oplus y = \bar{x}y + x\bar{y}$ Proof $\overline{x \oplus y} = xy + \bar{x}\bar{y}$

$$\begin{aligned}x \oplus y &= \bar{x}y + x\bar{y} \\ \overline{x \oplus y} &= \overline{\bar{x}y + x\bar{y}} \\ &= (\overline{\bar{x}y})(\overline{x\bar{y}}) \\ &= (\bar{\bar{x}} + \bar{y})(\bar{x} + \bar{\bar{y}}) \\ &= (x + \bar{y})(\bar{x} + y) \\ &= x.\bar{x} + xy + \bar{x}\bar{y} + \bar{y}y \\ &= 0 + xy + \bar{x}\bar{y} + 0 \\ &= xy + \bar{x}\bar{y}\end{aligned}$$

Proof that,

$$34) \overline{A}.\overline{B}C + \overline{A}\overline{B}\overline{C} + A\overline{B}.\overline{C} + ABC = A \oplus B \oplus C$$

L.H.S, $\overline{\overline{ABC}} + \overline{\overline{ABC}} + \overline{\overline{ABC}} + ABC$

$$\begin{aligned} &= \overline{A}(\overline{BC} + \overline{BC}) + A(\overline{BC} + BC) \\ &= \overline{A}(B \oplus C) + A(\overline{B \oplus C}) \\ &= \overline{AP} + A\overline{P} \quad [Let, B \oplus C = P] \\ &= A \oplus P \\ &= A \oplus B \oplus C \quad [\text{Substitute the value of P}] \end{aligned}$$

35) Proof that,

$$\overline{A}\overline{B}C + A\overline{\overline{B}}C + A\overline{B}\overline{C} + ABC = AB + BC + CA$$

$$\begin{aligned}\text{L.H.S} &= \overline{A}\overline{B}C + A\overline{\overline{B}}C + A\overline{B}\overline{C} + ABC \\&= \overline{A}\overline{B}C + A\overline{\overline{B}}C + A\overline{B}\overline{C} + ABC + ABC + ABC \\&= \overline{A}\overline{B}C + ABC + A\overline{\overline{B}}C + ABC + A\overline{B}\overline{C} + ABC \\&= BC(\overline{A} + A) + AC(\overline{B} + B) + AB(\overline{C} + C) \\&= BC.1 + AC.1 + AB.1 \\&= AB + BC + CA\end{aligned}$$

36) Proof that, $ABC + A\bar{B}C + AB\bar{C} = A(B + C)$

$$\begin{aligned}LHS &= \underline{\underline{ABC}} + A\bar{B}C + AB\bar{C} \\&= \underline{\underline{ABC}} + A\bar{B}C + AB\bar{C} + \underline{\underline{ABC}} \\&= AC(B + \bar{B}) + AB(\bar{C} + C) \\&= AC + AB \\&= A(C + B) \\&= A(B + C)\end{aligned}$$

Prooфе that, $AB + \overline{AC} + BC = AB + \overline{AC}$

$$L.H.S = AB + \overline{AC} + BC$$

$$= AB + \overline{AC} + BC(1)$$

$$= AB + \overline{AC} + BC(A + \overline{A})$$

$$= AB + \overline{AC} + \cancel{ABC} + \overline{ABC}$$

$$\xrightarrow{\quad\quad\quad}\quad\quad\quad = AB + A\cancel{BC} + \overline{AC} + \overline{ABC}$$

$$= AB(1+C) + \overline{AC}(1+B)$$

$$= AB + \overline{AC}$$

$$= R.H.S$$