

George Boole (1815-1864)



George Boole was an English

- mathematician,
- philosopher and
- logician.

■ He worked in the fields of differential equations and algebraic logic

Boolean Algebra

- Invented by famous mathematician **George Boole**
- All operations are done by 2 states:
 - **True (1)**
 - **False (0)**
- Later **True** was represented by **1**
- **False** was represented by **0**
- So its become possible to implement Boolean algebra into computer circuit design

Boolean Operation

- Boolean Addition(Logical OR)
- Boolean Multiplication(Logical AND)
- Boolean Complement(Logical NOT)

Boolean Operation

- **Boolean Addition (Logical OR)**

- $0+0= 0$

- $0+1= 1$

- $1+0= 1$

- $1+1= 1$

Boolean Operation

- **Boolean Multiplication (Logical AND)**

- $0.0 = 0$

- $0.1 = 0$

- $1.0 = 0$

- $1.1 = 1$

Boolean Complement

- Complement of 1 is 0
 - i.e. $\bar{1} = 0$
- Complement of 0 is 1
 - i.e. $\bar{0} = 1$
- If $A=1$ Then $\bar{A}=0$
- If $A=0$ Then $\bar{A}=1$

Boolean Theorem

Logical OR

$$A + 0 = A$$

$$A + 1 = 1$$

$$A + A = A$$

$$A + \overline{A} = 1$$

Boolean Theorem

Logical AND

$$A \cdot 0 = 0$$

$$A \cdot 1 = A$$

$$A \cdot A = A$$

$$A \cdot \overline{A} = 0$$

Boolean Theorem

Logical NOT

$$\overline{\overline{A}} = A$$

$$\overline{\overline{AB}} = AB$$

$$\overline{\overline{A + B}} = A + B$$

XOR and XNOR

$$A \oplus B = \overline{A}B + A\overline{B}$$

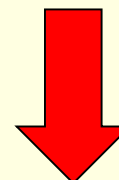
$$\overline{A \oplus B} = \overline{A}\overline{B} + AB$$

Truth Table (সত্যক সারণি)

- A **truth table** is a **table** showing the outputs for all possible combinations of inputs to a **logic gate** or **circuit**.

Input		Output
A	B	$A+B$
0	0	0
0	1	1
1	0	1
1	1	1

\overline{AB}



Input			Output
A	B	AB	$X = \overline{AB}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

$$F = AB + \bar{B}$$

A	B	\bar{B}	AB	$F = AB + \bar{B}$
0	0	1	0	1
0	1	0	0	0
1	0	1	0	1
1	1	0	1	1

$$X(\overline{Y+Z})$$

X	Y	Z	(Y+Z)	$\overline{(Y+Z)}$	$X \overline{(Y+Z)}$
0	0	0	0	1	0
0	0	1	1	0	0
0	1	0	1	0	0
0	1	1	1	0	0
1	0	0	0	1	1
1	0	1	1	0	0
1	1	0	1	0	0
1	1	1	1	0	0

De-Morgan's Theorems

$$\overline{A + B} = \overline{A} \cdot \overline{B}$$

$$\overline{AB} = \overline{A} + \overline{B}$$

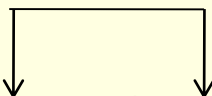
De-Morgan's Theorems(2 variables)

$$\overline{A + B} = \overline{A} \cdot \overline{B}$$

A	B	\overline{A}	\overline{B}	A+B	$\overline{A+B}$	$\overline{A} \cdot \overline{B}$
0	0	1	1	0	1	1
0	1	1	0	1	0	0
1	0	0	1	1	0	0
1	1	0	0	1	0	0

De-Morgan's Theorems(2 variables)

$$\overline{AB} = \overline{A} + \overline{B}$$



A	B	\overline{A}	\overline{B}	AB	\overline{AB}	$\overline{A+B}$
0	0	1	1	0	1	1
0	1	1	0	0	1	1
1	0	0	1	0	1	1
1	1	0	0	1	0	0

De-Morgan's Theorems with 3 variables

$$\overline{A+B+C} = \bar{A} \bar{B} \bar{C}$$

$$\overline{A B C} = \bar{A} + \bar{B} + \bar{C}$$

A	B	C	\bar{A}	\bar{B}	\bar{C}	A+B+C	$\overline{A+B+C}$	$\bar{A} \bar{B} \bar{C}$	ABC	\overline{ABC}	$\bar{A} + \bar{B} + \bar{C}$
0	0	0	1	1	1	0	1	1	0	1	1
0	0	1	1	1	0	1	0	0	0	1	1
0	1	0	1	0	1	1	0	0	0	1	1
0	1	1	1	0	0	1	0	0	0	1	1
1	0	0	0	1	1	1	0	0	0	1	1
1	0	1	0	1	0	1	0	0	0	1	1
1	1	0	0	0	1	1	0	0	0	1	1
1	1	1	0	0	0	1	0	0	1	0	0

De-Morgan's Theorems with N variables

$$\square \overline{A_1 + A_2 + A_3 + \dots + A_N} = \bar{A}_1 \bar{A}_2 \bar{A}_3 \dots \bar{A}_N$$

$$\square \overline{A_1 A_2 A_3 \dots A_N} = \bar{A}_1 + \bar{A}_2 + \bar{A}_3 + \dots + \bar{A}_N$$

Boolean Algebra(Simplification)

$$1) A + A\overline{B}$$

$$= A(1 + \overline{B})$$

$$= A(1)$$

$$= A$$

Boolean Algebra(Simplification)

$$\begin{aligned} 2) & A + \overline{A}B + \overline{A} \overline{B} \\ &= A + \overline{A} (B + \overline{B}) \\ &= A + \overline{A} (1) \\ &= 1 \end{aligned}$$

3) Proof that

$$(A + B)(A + \overline{B}) = A$$

$$L.H.S = (A + B)(A + \overline{B})$$

$$= A.A + A\overline{B} + BA + B.\overline{B}$$

$$= A + A\overline{B} + AB + 0$$

$$= A(1 + \overline{B}) + AB$$

$$= A(1) + AB$$

$$= A(1 + B)$$

$$= A$$

$$= R.H.S \quad (\text{Proved})$$

Simplification of Boolean Expressions

4. Prove that $(A + B)(A + C) = A + BC$

$$L.H.S = (A + B)(A + C)$$

$$= A.A + AC + BA + BC$$

$$= A + AC + AB + BC$$

$$= A(1 + C) + AB + BC$$

$$= A.1 + AB + BC$$

$$= A + AB + BC$$

$$= A(1 + B) + BC$$

$$= A.1 + BC$$

$$= A + BC = R.H.S \quad (\text{Proved})$$

Proof that , $\overline{X} + X = 1$

■ Let, $X=1$

$$\begin{aligned} &L.H.S, \\ &= \overline{X} + X \\ &= \overline{1} + 1 \\ &= 0 + 1 \\ &= 1 \end{aligned}$$

Let, $X=0$

$$\begin{aligned} &L.H.S, \\ &= \overline{X} + X \\ &= \overline{0} + 0 \\ &= 1 + 0 \\ &= 1 \end{aligned}$$

$$\begin{aligned}
& 7) B\bar{C} + \bar{B}\bar{C} + BC \\
&= \bar{C}(B + \bar{B}) + BC \\
&= \bar{C}.1 + BC \\
&= \bar{C} + BC \\
&= (\bar{C} + B)(\bar{C} + C) \\
&= (\bar{C} + B)(1) \\
&= \bar{C} + B
\end{aligned}$$

$$\begin{aligned}
& 8) ABC + A\bar{B}C + \bar{A} \\
&= AC(B + \bar{B}) + \bar{A} \\
&= AC + \bar{A} \\
&= \bar{A} + AC \\
&= (\bar{A} + A)(\bar{A} + C) \\
&= 1 (\bar{A} + C) \\
&= \bar{A} + C
\end{aligned}$$

$$(\overline{A+B})(A+B)$$

$$=\overline{A}A+\overline{A}B+\overline{B}A+\overline{B}B$$

$$=0+\overline{A}B+A\overline{B}+0$$

$$=\overline{A}B+A\overline{B}$$

$$= A \oplus B :$$

$$10)(A + \overline{B})(\overline{A} + B)$$

$$= A.\overline{A} + AB + \overline{A}.\overline{B} + B.\overline{B}$$

$$= 0 + AB + \overline{A}\overline{B} + 0$$

$$= AB + \overline{A}\overline{B}$$

$$= \overline{A \oplus B}$$

9. Proof that, $(A+B+C) BC = BC$

L.H.S,

$$= (A+B+C) BC$$

$$= A.BC + B.BC + C.BC$$

$$= ABC + BC + BC$$

$$= ABC + BC$$

$$= BC(A+1)$$

$$= BC(1)$$

$$= BC$$

$$= \text{R.H.S (Proved)}$$

$$11) AB\bar{C} + ABC + A\bar{B}C + ABC$$

$$= AB(\bar{C} + C) + AC(\bar{B} + B)$$

$$= AB(1) + AC(1)$$

$$= AB + AC$$

$$= A(B + C)$$

$$12) \overline{A}B\overline{C} + AB\overline{C} + B\overline{C}D$$

$$= B\overline{C}(\overline{A} + A) + B\overline{C}D$$

$$= B\overline{C}.1 + B\overline{C}D$$

$$= B\overline{C}(1 + D)$$

$$= B\overline{C}(1)$$

$$= B\overline{C}$$

$$XYZ + X\bar{Y}Z + \bar{X}YZ + \bar{X}\bar{Y}Z$$

$$= XZ(Y + \bar{Y}) + \bar{X}Z(Y + \bar{Y})$$

$$= XZ(1) + \bar{X}Z(1)$$

$$= XZ + \bar{X}Z$$

$$= Z(X + \bar{X})$$

$$= Z(1)$$

$$= Z$$

$$13) ABC + A\bar{B}C + ABC + \bar{A}C$$

$$= AC(B + \bar{B}) + ABC + \bar{A}C$$

$$= AC + ABC + \bar{A}C$$

$$= AC(1 + B) + \bar{A}C$$

$$= AC + \bar{A}C$$

$$= C(A + \bar{A})$$

$$= C.1$$

$$= C$$

$$15) z(y+z)(x+y+z)$$

$$= (zy + zz)(x + y + z)$$

$$= (zy + z)(x + y + z)$$

$$= z(y+1)(x+y+z)$$

$$= z(x+y+z)$$

$$= zx + zy + z.z$$

$$= zx + zy + z$$

$$= zx + z(y+1)$$

$$= zx + z$$

$$= z(x+1)$$

$$= z$$

$$14) \overline{\overline{A.B.C.D}}$$

$$= \overline{\overline{A}} + \overline{\overline{B}} + \overline{\overline{C}} + \overline{\overline{D}}$$

$$= A + B + C + D$$

$$\overline{\overline{(A+B)} \overline{(B+C)}}$$

$$\overline{\overline{(A+B)}} + \overline{\overline{(B+C)}}$$

$$A+B + B+C$$

$$A+B+C$$

Simplify

$$17) (\overline{\overline{A} + C}) + \overline{B + \overline{D}}$$

$$= (\overline{\overline{A} . \overline{C}}) + (\overline{\overline{B} . \overline{\overline{D}}})$$

$$= A\overline{C} + \overline{B}D$$

$$18) \overline{(M + \overline{N})(\overline{M} + N)}$$

$$= \overline{(M + \overline{N}) + (\overline{M} + N)}$$

$$= \overline{M} . \overline{\overline{N}} + \overline{\overline{M}} . \overline{N}$$

$$= \overline{M}N + M\overline{N}$$

$$= M \oplus N$$

Simplify

$$19) \overline{(A + B + \overline{C})} \overline{B} C$$

$$= \overline{A} . \overline{B} . \overline{\overline{C}} . \overline{B} . C$$

$$= \overline{A} . \overline{B} . \overline{B} . C . C$$

$$= \overline{A} . \overline{B} . C$$

Simplify

$$20) \overline{(A + B + \overline{C})} + \overline{B} C$$

$$= \overline{A} \overline{B} \overline{\overline{C}} + \overline{B} C$$

$$= \overline{A} . \overline{B} . C + \overline{B} C$$

$$= \overline{B} C (\overline{A} + 1)$$

$$= \overline{B} C (1)$$

$$= \overline{B} C$$

$$21) \overline{(A + \overline{B})} + \overline{(A + B)}$$

$$= \overline{A}.\overline{\overline{B}} + \overline{A}.\overline{B}$$

$$= \overline{A}.B + \overline{A}.\overline{B}$$

$$= \overline{A}.(B + \overline{B})$$

$$= \overline{A}.(1)$$

$$= \overline{A}$$

23) Proof that

$$\overline{A + \overline{B} + C\overline{D}} = \overline{A}B(\overline{C} + D)$$

$$L.H.S = \overline{A + \overline{B} + C\overline{D}}$$

$$= \overline{A}.\overline{\overline{B}}.\overline{C\overline{D}}$$

$$= \overline{A}B.(\overline{C} + \overline{\overline{D}})$$

$$= \overline{A}.B.(\overline{C} + D) = R.H.S$$

$$24) \overline{(\overline{A}.\overline{B}) + (AB)}$$

$$= \overline{(\overline{AB})(AB)}$$

$$= (\overline{\overline{A} + \overline{B}})(\overline{\overline{A} + \overline{B}})$$

$$= (A + B)(\overline{A} + \overline{B})$$

$$= A\overline{A} + A\overline{B} + B\overline{A} + B\overline{B}$$

$$= 0 + A\overline{B} + \overline{A}B + 0$$

$$= \overline{A}B + A\overline{B}$$

$$= A \oplus B$$

$$25) \overline{(A + B)} \cdot \overline{(\overline{A} + \overline{B})}$$

$$= (\overline{A}.\overline{B})(\overline{\overline{A}.\overline{B}})$$

$$= \overline{A}.\overline{B}.A.B$$

$$= \overline{A}.A.\overline{B}.B$$

$$= 0.0$$

$$= 0$$

$$\begin{aligned}
26) & \overline{(A + \bar{B})} \cdot \overline{(\bar{A} + \bar{B})} \\
&= \overline{(A + \bar{B}) + (\bar{A} + \bar{B})} \\
&= \bar{A}.\bar{\bar{B}} + \bar{\bar{A}}.\bar{\bar{B}} \\
&= \bar{A}.B + A.B \\
&= B(\bar{A} + A) \\
&= B.1 \\
&= B
\end{aligned}$$

$$\begin{aligned}
27) & \overline{(A + \bar{B}) + (A + B)} \\
&= \overline{(A + \bar{B})} \cdot \overline{(A + B)} \\
&= (\bar{A}.\bar{\bar{B}})(\bar{A}.\bar{B}) \\
&= \bar{A}.B.\bar{A}.\bar{B} \\
&= \bar{A}\bar{A}.B\bar{B} \\
&= \bar{A}.B.\bar{B} \\
&= \bar{A}.0 \\
&= 0
\end{aligned}$$

If $F = \overline{\overline{\overline{A+B}} + B + \overline{C}}$ then when $F=1$

$$F = \overline{\overline{\overline{A+B}} + B + \overline{C}}$$

$$\overline{\overline{\overline{A+B}}} \cdot \overline{B} + \overline{C}$$

$$(\overline{A+B})\overline{B}C$$

$$\overline{A}\overline{B}C + B\overline{B}C$$

$$\overline{A}\overline{B}C + 0$$

$$\overline{A}\overline{B}C$$

$$\overline{0}\overline{0}.1$$

$$\therefore A=0, B=0, C=1$$

$$\begin{aligned}
28) & \overline{(A + B).A\bar{B}} \\
&= \overline{(A + B)} + \overline{A\bar{B}} \\
&= \bar{A}.\bar{B} + (\bar{A} + \bar{\bar{B}}) \\
&= \bar{A}.\bar{B} + \bar{A} + B \\
&= \bar{A}(\bar{B} + 1) + B \\
&= \bar{A}.1 + B \\
&= \bar{A} + B
\end{aligned}$$

$$\begin{aligned}
29) & \overline{(\bar{A} + B) + (A\bar{B})} \\
&= \overline{(\bar{A} + B)}.\overline{(A\bar{B})} \\
&= (A.\bar{B})(A\bar{B}) \\
&= AA\bar{B}\bar{B} \\
&= A.\bar{B}
\end{aligned}$$

Proof that, 30) $\overline{x + \bar{y}(z + \bar{x})} = \bar{x}y$

$$L.H.S = \overline{x + \bar{y}(z + \bar{x})}$$

$$= \bar{x}.\overline{\bar{y}(z + \bar{x})}$$

$$= \bar{x}(\overline{\bar{y}} + z + \bar{x})$$

$$= \bar{x}(y + \overline{(z + \bar{x})})$$

$$= \bar{x}(y + \bar{z}.\bar{\bar{x}})$$

$$= \bar{x}(y + \bar{z}.x)$$

$$= \bar{x}y + x.\bar{x}.\bar{z}$$

$$= \bar{x}y + 0.\bar{z}$$

$$= \bar{x}y + 0$$

$$= \bar{x}y = R.H.S$$

$$\begin{aligned}
& 31) \overline{RST}(\overline{R+S+T}) \\
&= (\overline{R} + \overline{S} + \overline{T}) (\overline{R} . \overline{S} . \overline{T}) \\
&= \overline{R}(\overline{RST}) + \overline{S}(\overline{RST}) + \overline{T}(\overline{RST}) \\
&= \overline{R}\overline{R}\overline{S}\overline{T} + \overline{R}\overline{S}\overline{S}\overline{T} + \overline{R}\overline{S}\overline{T}\overline{T} \\
&= \overline{R}\overline{S}\overline{T} + \overline{R}\overline{S}\overline{T} + \overline{R}\overline{S}\overline{T} \\
&= \overline{R} \overline{S} \overline{T}
\end{aligned}$$

$$\begin{aligned}
32) & \overline{A\bar{B} + (C + A) + \bar{B}C} \\
&= \overline{(A\bar{B})(C + A) + \bar{B} + \bar{C}} \\
&= \overline{(\bar{A} + \bar{\bar{B}})(\bar{C}.\bar{A}) + B + \bar{C}} \\
&= \overline{(\bar{A} + B)(\bar{A}.\bar{C}) + B + \bar{C}} \\
&= \bar{A}.\bar{A}.\bar{C} + \bar{A}.B.\bar{C} + B + \bar{C} \\
&= \bar{A}.\bar{C} + \bar{A}.B.\bar{C} + B + \bar{C} \\
&= \bar{A}.\bar{C}(1 + B) + B + \bar{C} \\
&= \bar{A}.\bar{C} + B + \bar{C} \\
&= \bar{A}.\bar{C} + \bar{C} + B \\
&= \bar{C}(\bar{A} + 1) + B \\
&= \bar{C} + B
\end{aligned}$$

33) if $x \oplus y = \bar{x}y + x\bar{y}$ Proof $\overline{x \oplus y} = xy + \bar{x}\bar{y}$

$$x \oplus y = \bar{x}y + x\bar{y}$$

$$\overline{x \oplus y} = \overline{\bar{x}y + x\bar{y}}$$

$$= (\overline{\bar{x}y})(\overline{x\bar{y}})$$

$$= (\bar{\bar{x}} + \bar{y})(\bar{x} + \bar{\bar{y}})$$

$$= (x + \bar{y})(\bar{x} + y)$$

$$= x.\bar{x} + xy + \bar{x}\bar{y} + \bar{y}y$$

$$= 0 + xy + \bar{x}\bar{y} + 0$$

$$= xy + \bar{x}\bar{y}$$

Proof that,

$$34) \overline{A}.\overline{B}C + \overline{A}B\overline{C} + A\overline{B}.\overline{C} + ABC = A \oplus B \oplus C$$

$$\text{L.H.S, } \overline{\overline{A}}\overline{\overline{B}}C + \overline{\overline{A}}B\overline{\overline{C}} + A\overline{\overline{B}}\overline{\overline{C}} + ABC$$

$$= \overline{A}(\overline{B}C + B\overline{C}) + A(\overline{B}\overline{C} + BC)$$

$$= \overline{A}(B \oplus C) + A(\overline{B \oplus C})$$

$$= \overline{A}P + A\overline{P} \quad [Let, B \oplus C = P]$$

$$= A \oplus P$$

$$= A \oplus B \oplus C \quad [\text{Substitute the value of P}]$$

35) Proof that,

$$\overline{A}BC + A\overline{B}C + AB\overline{C} + ABC = AB + BC + CA$$

$$\begin{aligned}\text{L.H.S} &= \overline{A}BC + A\overline{B}C + AB\overline{C} + ABC \\&= \overline{A}BC + A\overline{B}C + AB\overline{C} + ABC + ABC + ABC \\&= \overline{A}BC + ABC + A\overline{B}C + ABC + AB\overline{C} + ABC \\&= BC(\overline{A} + A) + AC(\overline{B} + B) + AB(\overline{C} + C) \\&= BC.1 + AC.1 + AB.1 \\&= AB + BC + CA\end{aligned}$$

36) Proof that, $ABC + A\overline{B}C + AB\overline{C} = A(B + C)$

$$\begin{aligned}LH.S &= \underline{ABC} + A\overline{B}C + AB\overline{C} \\&= \underline{ABC} + A\overline{B}C + AB\overline{C} + \underline{ABC} \\&= AC(B + \overline{B}) + AB(\overline{C} + C) \\&= AC + AB \\&= A(C + B) \\&= A(B + C)\end{aligned}$$

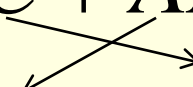
Prove that, $AB + \overline{A}C + BC = AB + \overline{A}C$

$$L.H.S = AB + \overline{A}C + BC$$

$$= AB + \overline{A}C + BC(1)$$

$$= AB + \overline{A}C + BC(A + \overline{A})$$

$$= AB + \overline{A}C + ABC + \overline{A}BC$$


$$= AB + A\overline{B}C + \overline{A}C + \overline{A}BC$$

$$= AB(1 + C) + \overline{A}C(1 + B)$$

$$= AB + \overline{A}C$$

$$= R.H.S$$