# Economic Dispatch Problem

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The economic dispatch problem is one of the most used models in energy systems. Particularly, the aim of this problem is to allocate the total demand among generating units so that the production cost is minimized. Specifically, for a given set of $N$ generating units, $i = 1, \ldots, N$, situated in a network with $K$ nodes, $k = 1, \ldots, K$ and $L$ lines, $l = 1, \ldots, L$ and defined over $T$ time periods, $t = 1, \ldots, T$, the optimization problem we want to solve is:

$$
\left\{
\begin{array}{lll}
\min\limits_{\substack{g_{it}, \forall i, \forall t, \\ f_{lt}, \forall i, \forall t \\ \delta}} & \sum\limits_{t=1}^{T} \sum\limits_{i=1}^{N} c_i g_{it} & \\
\text{s.t.} & g_i^{min} \leq g_{it} \leq g_i^{max}, & \forall i, \forall t \\
 & \sum\limits_{i \in \phi_k} g_{it} = d_{kt} + \sum\limits_{l \in \mathcal{O}_k} f_{lt} - \sum\limits_{l \in \mathcal{E}_k} f_{lt}, & \forall k, \forall t \\
 & -f_{lt}^{max} \leq f_{lt} \leq f_{lt}^{max}, & \forall l, \forall t \\
 & f_{lt} = X_{lt}(\delta_{\mathcal{O}_k} - \delta_{\mathcal{E}_k}), & \forall l, \forall t \\
 & R_i^{down} \leq g_{it} - g_{i\,t-1} \leq R_i^{up}, & \forall i, t = 2, \ldots, T \\
 & g_{it} \geq 0, & \forall i, \forall t
\end{array}
\right. \tag{1}
$$

where $g_{it}$ is the power produced by the generating unit $i$ at the time period $t$, $c_i$ is the cost of producing power in the generating unit $i$, $d_{kt}$ is the power demand of node $k$ at the time instant $t$. The energy flow through the line $l$ at time $t$ is denoted by $f_{lt}$. It is bounded by $f_{lt}^{max}$ and the sign indicates the movement sense of the energy. The values $R_i^{down}$ and $R_i^{up}$ are respectively the minimum and maximum values of the ramp limits. Moreover,

$$\begin{aligned}
\phi_k &= \{i : \text{the generating unit} i \text{gives power to the node } k\} \\
\mathcal{O}_k &= \{l : \text{the origin of the line is at node } k\} \\
\mathcal{E}_k &= \{l : \text{the end of the line is at node } k\}.
\end{aligned}$$

# 1 Goals

The aim of this document is to formally state the optimization problem we want to solve.

Particularly, we focus on the Unit Commitment problem. It is known that such a problem is very hard to solve for large instances, like the ones that appear in real-life applications. Hence, it is desirable to develop new strategies to simplify the model in order to obtain good solutions in a reasonable time. Our proposal is to identify the congested lines in the network to then remove the associated inactive constraints as well as fix the corresponding variables. This way, a simpler model, with a lower number of constraints and decision variables will be solved.

The remaining of this manuscript is structured as follow: Section 2 formulate the standard Unit Commitment problem and set notation whereas Section 3 details the optimization problem we want to solve, i.e., the problem where we identify if a line is congested or not. Finally, Section 4 briefly outlines the solving strategy.

# 2 Unit Commitment Problem

The Unit Commitment problem (UC) consists in allocating the total demand among generating units so that the production cost is minimized. Specifically, for a given set of $N$ generating units, $i = 1, \ldots, N$, situated in a network with $K$ nodes, $k = 1, \ldots, K$ and $L$ lines, $l = 1, \ldots, L$ and defined over $T$ time periods, $t = 1, \ldots, T$, the optimization problem we want to solve is:

$$
\left\{
\begin{array}{lll}
\min\limits_{\substack{g_{it},\forall i,\forall t,\\ f_{lt},\forall i,\forall t\\ \delta,\\ u_i,\forall i}} & \sum\limits_{t=1}^{T}\sum\limits_{i=1}^{N} c_i g_{it} & \\
\text{s.t.} & u_i g_i^{min} \leq g_{it} \leq u_i g_i^{max}, & \forall i, \forall t \\
 & \sum\limits_{i\in\phi_k} g_{it} = d_{kt} + \sum\limits_{l\in\mathcal{O}_k} f_{lt} - \sum\limits_{l\in\mathcal{E}_k} f_{lt}, & \forall k, \forall t \\
 & -f_{lt}^{max} \leq f_{lt} \leq f_{lt}^{max}, & \forall l, \forall t \\
 & f_{lt} = X_{lt}(\delta_{\mathcal{O}_k} - \delta_{\mathcal{E}_k}), & \forall l, \forall t \\
 & R_i^{down} \leq g_{it} - g_{i\,t-1} \leq R_i^{up}, & \forall i, t = 2, \ldots, T \\
 & g_{it} \geq 0, & \forall i \forall t \\
 & u_i \in \{0, 1\} & \forall i
\end{array}
\right. \tag{2}
$$

where $g_{it}$ is the power produced by the generating unit $i$ at the time period $t$, $c_i$ is the cost of producing power in the generating unit $i$, $u_i$ denotes if the generating unit $i$ is switched on, $u_i = 1$ or not, $u_i = 0$, $d_{kt}$ is the power demand of node $k$ at the time instant $t$. The energy flow through the line $l$ at time $t$ is denoted by $f_{lt}$. It is bounded by $f_{lt}^{max}$ and the sign indicates the movement sense of the energy. The values $R_i^{down}$ and $R_i^{up}$ are respectively the minimum and maximum values of the ramp limits. Moreover,

$$
\begin{array}{rcl}
\phi_k & = & \{i : \text{the generating unit} i \text{gives power to the node } k\} \\
\mathcal{O}_k & = & \{l : \text{the origin of the line is at node } k\} \\
\mathcal{E}_k & = & \{l : \text{the end of the line is at node } k\}.
\end{array}
$$

Once the notation used along the document and the Unit Commitment problem are detailed, the objective of the next section is to formally state the formulation problem to be solved.

# 3 Identifying Congested Lines

As mentioned above, our aim is to identify the congested lines of the network, in order to remove decision variables and inactive constraints, yielding to a reduced problem of type (2) but easier to solve. To formulate the corresponding optimization problem, the inequalities $-f_{lt}^{max} \leq f_{lt} \leq f_{lt}^{max}, \forall l, \forall t$ are rewritten to be equalities, since the line $l$ at time $t$ will be congested if and only if $f_{lt} = \pm f_{lt}^{max}$. To do this, a new set of variables are introduced. Particularly, the slack variables $r_{lt}, s_{lt}, \forall l, \forall t$ are defined, as well as the discrete variables $a_{lt} \in \{-1, 0, 1\}, \forall l, \forall t$, indicating if a line $l$ at time period $t$ is congested, $a_{lt} = \pm 1$ or not, $a_{lt} = 0$. With the help of such new variables, now we can formulate our proposed optimization problem:

$$
\left\{
\begin{array}{lll}
\min\limits_{\substack{g_{it},\forall i,\forall t,\\ f_{lt},\forall i,\forall t,\\ \delta,\\ u_i,\forall i,\\ a_{lt},\forall l,\forall t,\\ r_{lt},\forall l,\forall t,\\ s_{lt},\forall l,\forall t}} & \sum_{t=1}^{T}\sum_{i=1}^{N} c_i g_{it} & \\
\text{s.t.} & u_i g_i^{min} \leq g_{it} \leq u_i g_i^{max}, & \forall i, \forall t \\
& \sum_{i\in\phi_k} g_{it} = d_{kt} + \sum_{l\in\mathcal{O}_k} f_{lt} - \sum_{l\in\mathcal{E}_k} f_{lt}, & \forall k, \forall t \\
& f_{lt} = -f_{lt}^{max} + r_{lt}(1 + a_{lt}), & \forall l, \forall t \\
& f_{lt} = f_{lt}^{max} + s_{lt}(1 - a_{lt}), & \forall l, \forall t \\
& f_{lt} = X_{lt}(\delta_{\mathcal{O}_k} - \delta_{\mathcal{E}_k}), & \forall l, \forall t \\
& R_i^{down} \leq g_{it} - g_{i\,t-1} \leq R_i^{up}, & \forall i, t = 2, \dots, T \\
& g_{it} \geq 0, & \forall i \forall t \\
& u_i \in \{0, 1\} & \forall i \\
& a_{lt} \in \{-1, 0, 1\}, & \forall l, \forall t \\
& r_{lt}, s_{lt} \geq 0, & \forall l, \forall t
\end{array}
\right. \tag{3}
$$

# 4 Sketch of the Resolution Strategy

Problem (3) is very hard to solve in general due, among other reasons, to the large number of discrete variables. Hence, we propose to solve it with a two-step algorithm.

The first step is based on a machine learning procedure. More specifically, using a classification algorithm, e.g., $k-$nn, we will know which are the congested lines, i.e., the values of $a_{lt}$, based on the demand power values $d_{kt}$.

In the second step, the values of $a_{lt}$ are fixed, and therefore, we can delete the associated constraints and set to the maximum capacity some of the decision variables of type $f_{lt}$, yielding a reduced unit commitment problem which can now be solved using the solvers as Cplex.

Note that, since our main objective is to know if a line is congested or not, the values of the slack variables, $r_{lt}$ and $s_{lt}$ are not important for us.