An Introduction to Mathematical Reasoning with Applications to Induction and Contradiction Proofs

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Outline

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1. What is a proof?
2. Proof by contradiction
3. Proof by induction
4. Conclusions

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Then there exists a point c in the open interval (a, b) satisfying $f(c) = 0$.

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  - $\exists c \in (a, b) : f(c) = 0$

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Mistakes in proofs

Conjecture: $1 = 2$.

Assume a, b *are two equal positive integers:*

1. $a = b$
2. $a^2 = ab$
3. $a^2 - b^2 = ab - b^2$
4. $(a - b)(a + b) = b(a - b)$
5. $a + b = b$
6. $2b = b$
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Where is the error?

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Thank you!

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