Nucleation of Market Bubbles

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Abstract

This project explores the extent to which homogenious nucleation equations can be used to model financial bubbles. In doing so, it translates various aspects of classical and modern nucleation theory of bubbles into the language of financial markets - in order to better study the indicators of a financial collapse, and thereby better predict potential market collapses that may occur in the future. The project builds a model using training data from the housing bubble of 2007. Upon sufficient training/extrapolation, the model will be applied to current market trends to understand if we are currently in a bubble economy. We seek to see whether there exists a link between the model used for nucleation and growth of physical bubbles, and economic bubbles

1 Introduction

The field of econo-physics is one that is rapidly growing and is the center for a lot of active research. This project in particular explores the applications of nucleation theory to financial market bubbles. In order to keep the project self contained, we introduce a few simple definitions.

Definition 1. ***Financial Bubble*** *A period of time in which market speculation causes the price of an asset to inflate far past its intrinsic value.*

Definition 2. ***Market crash*** *The financial market is said to crash when the market value of an asset sharply returns to the price before the bubble.*

Before understanding what nucleations of bubbles are in the physical sense, we introduce a priliminary definition that will aid our understanding. Consider a physical system, typically just matter in some state

Definition 3. ***First order phase transitions*** *A transition of phase during which the system either absorbs or releases a fixed amount of energy per unit volume. The temperature of the system stays constant as heat is added; the system is in a "mixed" phase regime in which some parts of the system have completed the transition and others have not.*

First order Phase transitions play an important role in science, and are useful in many technical applications. Simple examples are condensations, evaporation, boiling, etc.

Definition 4. ***Nucleation*** *The first step of the phase transition is essentially overcoming a free-energy barrier, which is the work done towards the formation of a small "embryo" and evolution of the embryo to become the nucleus of the new phase. This nucleus can only emerge from random thermal fluctuations within the old metastable phase. This initiating process of a first order phase transition is called nucleation.*

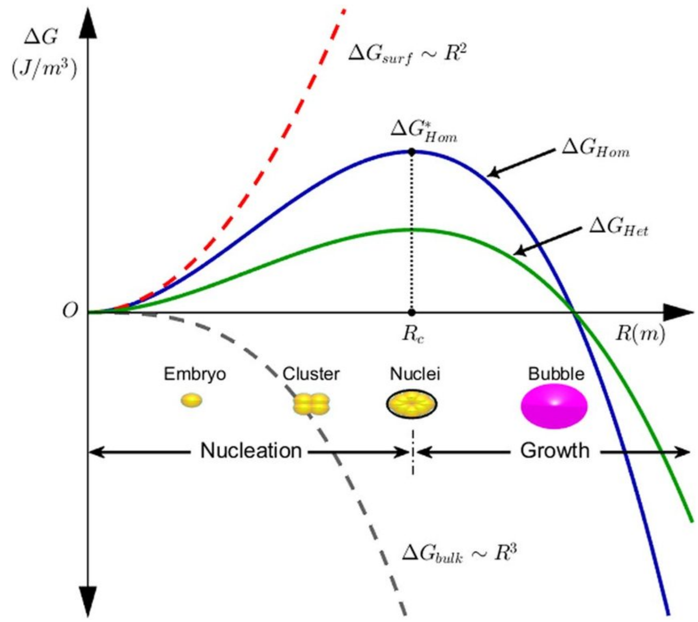

Figure 1: Physical formation of bubbles

The following infographic provides a lot of insight into the nucleation process and the growth of bubbles.
A nucleation is said to be *Homogeneous* if the new phase is solely formed from the fluctuations within the old phase.

2 Building the model

2.1 Physical set up

In our project, we wish to use homogeneous/heterogeneous nucleation models, namely the ones pertaining to the change in Gibbs' free energy in a material going through a phase change, to gain a better understanding of how bubbles form in financial markets over time.
We will consider using the JMAK equation to model the phenomenon more effectively with respect to time. Presented below is the JMAK equation in its generality.

$$\frac{V_\beta}{V} = Y = 1 - e^{-K(t)^4} \tag{1}$$

and

$$K = \frac{\pi}{3}\dot{N}\dot{G}^3 \tag{2}$$

where,
V_β is the proportion of the volume that has already undergone nucleation,
V is the total volume of the substance
$\dot{N}$ is the rate of nucleation
$\dot{G}$ is the rate of growth of radius.

The equations for $\dot{N}$ and $\dot{G}$ stem from equations of their own. But before listing them, we introduce a few more preliminary equations.

$$\Delta G_V = G_V^{P_1} - G_V^{P_2} \tag{3}$$

is the change in Gibbs free energy per unit volume as the substance transitions from one phase P_1 to another P_2.

We denote by γ the interfacial free energy. With the help of this and ΔG_V, we can now identify a critical radius r^* that would differentiate an embryo from a nucleus. Thus, this is also the radius at which "bubbles form"

$$r^* = \frac{2\gamma}{\Delta G_V} \tag{4}$$

and the associated critical change in gibbs energy (barrier for bubble formation) is given by

$$\Delta G^* = \frac{16\pi\gamma^3}{3(\Delta G_V)^2} \tag{5}$$

Finally, assuming that we have a spherical bubble of volume V, we have

$$r = \sqrt[3]{\frac{3V}{4\pi}} \tag{6}$$

as the radius of the bubble.

With all of these equations, we are finally ready to define $\dot{N}$ and $\dot{G}$. They are defined as follows.

$$\dot{N} = ae^{\frac{\Delta G^*}{f_0 T}} \tag{7}$$

$$\dot{G} = \frac{dr}{dt} \tag{8}$$

where, a, f_0 are some positive scaling factors involving fractional multiples of π, which upon inclusion would only further complicate the equation.

Presented below is a plot of the Avrami equation.

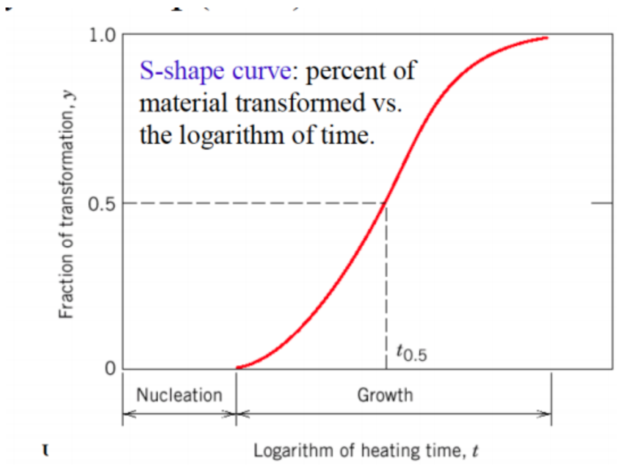

Figure 2: Plot of Y vs $\log(t)$

2.2 Translating to economics terms

We devote this section to translate the equations seen in the above subsection, into economic terms - by identifying economic variables and their correlation to physical variables. [1].

Notation 1. *Denote by* V_j $j \in \overline{1,n}$ *the market cap of a company in an industry, and denote by* V_i $i \in \overline{1,m}$ *the market cap of an industry.*

A simple observation is that

$$\sum_{j=1}^{n} V_j = V_i \tag{9}$$

In this sense, we can redefine our physical variables as follows:
V_β: The summed market cap of companies (in an industry) in the bubble phase
V: The summed market cap of the industry.

It is clear that $0 \leq \frac{V_\beta}{V} \leq 1$.

This covers the left hand side of the JMAK equation. For the right hand side, we study k. We can then redefine $\dot{N}$ and $\dot{G}$ as follows

[1] It appears that there are a few changes yet to be made in this section, in how we define certain variables. Upon future revisions, these changes will be reflected and the model would be reworked into a better one.

$\dot{N}$: Rate at which stocks transition into the bubble phase.
$\dot{G}$ Rate at which the market cap of a company grows.

This allows us to think of bubble nucleation as companies entering the bubble phase. We can also define a quantity called "prepice" that is analogous to the radius of the bubble in the physical sense

$$r = \sqrt[3]{\frac{3V_j}{4\pi}} \tag{10}$$

that is proportional to the cube-root of the market cap.
We can also redefine γ the surface interface energy as market forces that would prevent bubbles from forming, and in the same spirit, redefine ΔG_V, the change in gibbs free energy per unit volume, as factors that assist in the growth of a bubble.
In a more formal economic sense, we can define

$$\gamma_j = \frac{B_j}{V_j} \sqrt[3]{C_i} \tag{11}$$

as the cube root of the total book value of the company added with the average Cost (barriers to market entry) of the industry to which it belongs. The bigger the value of γ (or, in other words, the more stable a company/industry is) the harder it should be for the company to become a bubble. ΔG_V can also be modified as

$$\Delta G_V = |\nu_{actual} + \nu_{implied}| \left| \frac{F_j}{V_j} \right| \tag{12}$$

where $\mathcal{L}$ is the leverage of the company and ν is its volatility. The higher the value of ΔG_V (that is to say, the more the levarage or volatility of a company), the easier it should be for the company to become a bubble.

With these definitions in hand, we can now define the "critical prepice" as the price value (or the critical radius in the physical sense) at which a company enters a bubble.

$$r^* = \frac{\zeta \gamma_j}{\Delta G_V j} \tag{13}$$

Along with this definition, we also make an implicit assumption that a company can either be inside a bubble ($r > r^*$) or outside a bubble ($r < r^*$). We can also associate a ΔG^* to this set up, which is more or less defined through r^* (or can be thought of as the critical combination of $\mathcal{L}$ and ν at which the company enters the bubble phase).

$$\Delta G^* = b \frac{\gamma^3}{\Delta G_V^2} \tag{14}$$

where $\mathcal{K}$ is a positive scaling factor.

With this we are finally ready to define $\dot{N}$ and $\dot{G}$ in a way that is analogous to the physical set-up. Their equations are as follows.

$$\dot{N} = f c_0 e^{\frac{\Delta G^*}{T}} \tag{15}$$

$$\dot{G} = \frac{dr}{dt} \tag{16}$$

Putting it all together, we have our JMAK equation for financial market bubble nucleation as

$$\frac{V_\beta}{V} = Y = 1 - e^{-k^2(t)^4} \tag{17}$$

where

$$k = a\dot{N}\dot{G}^3 \tag{18}$$

3 Data

In order to apply our model to historical financial bubbles, we obtained data from Fred Economic data, a resource powered by the federal reserve bank of St. Louis. An example plot of the change in the volatility index as a time series data.

Figure 3: Time series data of the volatility index

The grey bars observed pertain to recessions that the US economy went through, and is typically correlated with sharp spikes in the volatility index. This confirms our intuition that a large volatility index might lead to the easier formation of a financial bubble. Similar data for other variables have been procured, they are yet to be cleaned and studied thoroughly.

3.1 Implementation

We aim to implement our model using python with the help of libraries such as numpy, scipy, matplotlib and/or pandas. After cleaning the data, we aim to perform non-linear least squares to fit our model to the data in order to obtain the value of all the scaling coefficients that were introduced in the financial reformulation of the JMAK equation. Upon deriving the value of these constants, we also want to perturb the model to see if the values of these constants give us insight into the effectiveness of the fit. We also want to apply our model to the current market trend to address the speculation that we are currently living in a bubble economy.

References

[1] A. Kiselev, L. Ryzhik, *A Simple Model For Asset Price Bubble Formation And Collapse*, Stanford University

Appendix

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Table 1: Variables

Symbol	**Physics Meaning**	**Economics Parallel**
$\mathbf{Y} = \frac{V_\beta}{V}$	Ratio of the volume of bubbles in a system to the volume of the entire system.	Ratio of the market cap of bubbling assets in an industry to the market cap of the whole industry.
$\dot{N}$	Rate of nucleation (i.e. rate of new bubbles forming in the system).	Rate of companies changing from normal phase to bubble phase.
$\dot{G}$	Rate of radius growth of bubbles in the system.	Rate of "prepice" growth of stocks in the bubble phase.
$V = \frac{4\pi r^3}{3}$	Volume as a function of the radius.	Market cap as a function of the "prepice."
γ	Interfacial free energy; multiplied by area to give surface tension.	Market forces that prevent bubbles from forming.
ΔG_V	Free energy per unit volume	Market forces that cause bubbles to form.
r	Radius of bubble	The "prepice" is a variable proportional to the cubic root of market cap.
r^*	Critical radius; the radius at which a bubble is formed.	Critical prepice; the prepice at which an asset transitions from to the bubble phase.
ΔG^*	Critical free energy; the free energy at which a bubble is formed.	The ratio of balancing markets forces that indicates a transition to the bubble phase.
L_V	Latent heat of fusion per unit volume.	N/A
ΔT	Undercooling	N/A
T_m	Melting temperature.	N/A
V_B	N/A	Book Value
ν	N/A	Volatility (implied or actual)
F_j	N/A	Free Cash Flow
C_j	N/A	Cost of revenue

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Table 2: Variables

Variable	**Positive or Negative**	**Correlation Justification**	**Qualitative Justification**
V_i	Positive	$V_i \uparrow \Rightarrow T \uparrow \Rightarrow \dot{N} \uparrow$	An increase in the market cap of an industry should increase the rate of new bubbles forming because there is more total capital to supply each bubble.
V_j	Positive	$V_j \uparrow \Rightarrow \gamma_j \downarrow \Rightarrow \Delta G^* \downarrow \Rightarrow \dot{N} \uparrow$	An increase in the market cap of a company should increase the rate of nucleation because a bubble forms when market cap reaches a critical point.
F_j	Positive	$F_j \uparrow \Rightarrow \Delta G_V \uparrow \Rightarrow \Delta G^* \downarrow \Rightarrow \dot{N} \uparrow$	An increase in free cash flow should increase the rate of nucleation because this is a commonly used metric that is known to inflate the value of a stock.
ν	Positive	$\nu \uparrow \Rightarrow \Delta G_V \uparrow \Rightarrow \Delta G^* \downarrow \Rightarrow \dot{N} \uparrow$	An increase in volatility (both implied and actual) should increase the rate of nucleation because historical data shows that volatility is increased during asset bubbles.
C_j	Negative	$C_j \uparrow \Rightarrow \gamma_j \uparrow \Rightarrow \Delta G^* \uparrow \Rightarrow \dot{N} \downarrow$	An increase in the cost of revenue decreases the rate of nucleation because this metric is hypothesized to deflate the value of a stock.
B_j	Negative	$B_j \uparrow \Rightarrow \gamma_j \uparrow \Rightarrow \Delta G^* \uparrow \Rightarrow \dot{N} \downarrow$	An increase in book value decreases the rate of nucleation because companies with a higher value of tangible assets are less likely to develop into bubbles.