Summer Project

DECIPHERING DECISIONS

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Abstract

The goals of this project is to learn about the basics of the financial markets, to get a good grasp about behavioural finance and game theory related to finance and to understand the implications of both these in the markets. These goals were met by group discussions, research paper discussions and implications. In this project we have focused on few notable research papers such as A Game-Theoretical Approach for Designing Market Trading Strategies by Garrison W. Greenwood and Richard Tymerski, Sudarshan Kumar, Avijit Bansal & Anindya S. Chakrabarti (2020): Ripples on financial networks, The European Journal of Finance. Multifractal properties of the Indian financial market by Sunil Kumar Nivedita Deo.

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Chapter 1

Introduction to financial markets

1.1 Stock market

The stock market is a marketplace, either physical or virtual, where investors can buy and sell stocks, which represent fractional ownership in companies.

While stocks are the most widely recognized instruments, other securities are also traded on stock exchanges. These include bonds, which are essentially IOUs issued by companies or governments, and derivatives, which are financial contracts whose value is derived from underlying assets like stocks or bonds.

The stock market plays a vital role in facilitating economic growth. It allows companies to raise capital for expansion, fostering innovation and job creation. For investors, it provides a means to grow their wealth and participate in the success of businesses.

1.2 Efficient market hypothesis

Efficient market hypothesis states that the stock prices reflect all the information and it is impossible to outperform the overall market by stock selection and market timing and only way an investor could have higher return is by taking higher risks. But EMH was proved to be wrong as there are biases in the market and market inefficiencies do exist. So by analysing and timing an investor could outperform the market and have higher returns. For this purpose we learn about Fundamental and Technical analysis theories.

1.3 Technical analysis

Technical Analysis is a research technique to identify trading opportunities in the market based on market participants' actions. The actions of market participants can be visualized in stock charts. Over time, patterns form in these charts, and each pattern conveys a certain message. The job of a technical analyst is to identify these patterns and develop a point of view.

Candle sticks are more significant way of looking into the charts. It is based on open, high, low, close price of the stock on that particular time interval. These candle sticks are individually or cumulatively observed to make inferences and predict the future movement of the market.



1.4 Technical indicators

In the previous section we saw about candle stick patterns but these candle stick pattern doesn't have a statistical significance. To understand the charts we consider few statistical indicators such as MACD, Relative strength index, Bollinger bands and more.

1.5 Futures & derivatives

The future, by its very nature, holds a certain degree of uncertainty. In the realm of finance, this translates to potential price fluctuations of various assets, be it commodities like oil or financial instruments like stocks. Futures contracts emerge as a powerful tool to navigate this uncertainty, offering a standardized agreement for buying or selling an asset at a predetermined price on a specific future date.

1.5.1 The Essence of Futures

At its core, a futures contract is a legally binding agreement between two parties. One party agrees to buy a specific quantity of an underlying asset (e.g., corn, gold, or a stock) at a predetermined price on a future date (settlement date). The other party agrees to sell the same quantity at the same price. This price is established at the time the contract is initiated and remains fixed throughout its duration.

The Underlying Asset:

The underlying asset in a futures contract can be a diverse range of financial instruments or physical commodities. Common examples include:

- Commodities: Futures contracts play a vital role in the agricultural sector, allowing farmers (producers) and consumers (e.g., food processing companies) to lock in prices for agricultural products like wheat or corn. This protects producers from price drops and ensures a steady supply for consumers. Similarly, energy futures contracts for oil or gas allow for price hedging in volatile markets.
- Financial Instruments: Stock index futures, for instance, track the performance of entire market indices like the S&P 500. Investors can utilize these contracts to speculate on the future movement of the index, potentially profiting from anticipated price increases or decreases.

1.5.2 The Mechanism of Futures Trading

Futures contracts are typically traded on designated exchanges, ensuring a transparent and regulated environment. Standardized features like contract size and settlement dates streamline trading and enhance market liquidity. Unlike traditional buying and selling of assets, which involve immediate exchange, futures contracts require an initial margin deposit – a small percentage of the total contract value – from both parties. This serves as a form of security and mitigates the risk of default.

Mark-to-Market:

A unique feature of futures contracts is the daily settlement process known as mark-to-market. This ensures that any fluctuations in the underlying asset's price are reflected in the daily settlement price of the contract. Unrealized gains or losses are credited or debited to the respective margins of each party on a daily basis. This system minimizes the risk of default by constantly adjusting the margin requirement based on market movements.

Hedging vs. Speculation:

Futures contracts cater to two primary functions in the financial landscape: hedging and speculation.

Hedging: Producers and consumers of underlying assets can utilize futures contracts to hedge against price fluctuations. For instance, a farmer concerned about a potential drop in corn prices can enter a futures contract to sell corn at a predetermined price in the future. This ensures a guaranteed income, regardless of the actual price at the time of harvest.

Speculation: Futures contracts can also be used for speculative purposes. Investors can take a directional view on the future price movement of the underlying asset. If they anticipate a price increase, they can buy a futures contract, aiming to profit by selling it later at a higher price. Conversely, if they expect a price decrease, they can enter a sell contract, hoping to profit by repurchasing it later at a lower price.

1.5.3 Advantages and Considerations

Futures contracts offer several advantages, including:

- Reduced Risk: Hedging allows producers and consumers to manage the risk of price fluctuations in the underlying asset.
- Price Discovery: Futures markets contribute to the efficient discovery of future asset prices by reflecting the collective expectations of market participants.
- Leverage: Margin requirements allow for greater exposure to the underlying asset compared to buying the asset outright, potentially amplifying profits (or losses).

However, it is crucial to acknowledge the inherent complexities and risks associated with futures trading:

- Price Volatility: Futures contracts are exposed to the same price fluctuations as the underlying assets, potentially leading to significant losses.
- Margin Calls: If the market price moves against a position, traders may face margin calls requiring them to deposit additional funds to maintain their position.

• Delivery Risk: While most futures contracts are settled through cash settlements, there is a possibility of physical delivery of the underlying asset on the expiry date, which requires storage and logistical considerations.

In conclusion, futures contracts represent a sophisticated instrument within financial markets. Their ability to manage risk and facilitate speculation makes them valuable tools for a variety of market participants.

1.6 Options

Financial markets thrive on the constant dance of risk and reward. Stock options, a versatile type of derivative contract, empower investors to navigate this dynamic landscape by offering the right, but not the obligation, to buy or sell a stock at a predetermined price by a specific date. This section delves into the core functionalities of stock options, exploring their applications and considerations within the realm of investment strategies.

1.6.1 The essence of stock Options

A stock option grants the holder the right, but not the obligation, to buy (call option) or sell (put option) a specific number of shares of a particular stock at a predetermined price (strike price) by a specific expiry date. Unlike buying the actual stock, options only require an upfront premium payment, a fraction of the stock price. This upfront cost reflects the potential value of the option contract.

Call Options and Put Options:

Call Options: These options grant the holder the right to buy a specific number of shares of a stock at the strike price by the expiry date. Investors typically purchase call options when they are bullish on a stock's future price movement, anticipating it to rise above the strike price before expiry. By exercising the call option, the investor can buy the stock at a predetermined lower price (the strike price) and then sell it at the higher market price, profiting from the difference.

Put Options: Put options provide the holder with the right to sell a specific number of shares of a stock at the strike price by the expiry date. Investors typically purchase put options when they are bearish on a stock's future price movement, anticipating it to fall below the strike price before expiry. By exercising the put option, the investor can sell the stock at the predetermined higher price (the strike price) even if the market price has fallen, mitigating potential losses.

1.6.2 Key Features of Stock Options

Strike Price: This is the predetermined price at which the holder has the right to buy (call) or sell (put) the underlying stock.

Expiry Date: This is the specific date by which the option contract must be exercised or expires, rendering it worthless. Options contracts come with various expiry dates, offering investors flexibility in their investment strategies.

Premium: This is the upfront cost paid by the option buyer to the option seller for the right, but not the obligation, to exercise the option. The premium price reflects factors like the underlying stock price, volatility, time to expiry, and interest rates.

1.6.3 Applications of Stock Options

Stock options cater to a diverse range of investment strategies, including:

Hedging Existing Holdings: Investors can utilize put options to hedge their existing stock holdings. By purchasing put options, they gain downside protection, ensuring a minimum selling price for their shares if the market price falls.

Income Generation: Option sellers (writers) can collect premium income by selling options contracts. This strategy generates income even if the option is not exercised. However, option writers bear the obligation to buy or sell the underlying stock at the strike price if the option is exercised.

Speculation on Stock Price Movements: Investors can utilize options to speculate on the future price movements of stocks. Call options allow for profiting from anticipated price increases, while put options enable profiting from anticipated price decreases.

1.6.4 Considerations and Risks

Stock options, while offering a multitude of benefits, are not without their inherent complexities and risks:

- Time Decay (Theta): The value of an option contract inherently decays over time as it approaches its expiry date. This is known as time decay or theta. Investors need to factor in this time value erosion when making investment decisions.
- Volatility Risk: Options are particularly sensitive to the underlying stock's price volatility. Increased volatility can lead to significant gains (or losses) on option positions.
- Assignment Risk (for Option Sellers): Option sellers (writers) bear the obligation to buy or sell the underlying stock at the strike price if the option is exercised. This can lead to unintended stock purchases or sales if not carefully managed.

In conclusion Stock options offer a sophisticated tool for investors seeking to navigate the complexities of the financial markets. By understanding the core functionalities, applications, and inherent risks associated with options, investors can leverage them to potentially enhance returns, hedge existing holdings, or generate income. However, a thorough understanding of options strategies and risk management techniques is crucial before actively incorporating them into an investment portfolio

Chapter 2

Ripple effect in Financial markets

This chapter briefly discuses about learning and implications from the discussions on the research paper Sudarshan Kumar, Avijit Bansal & Anindya S. Chakrabarti (2020): Ripples on financial networks, The European Journal of Finance.

2.1 Abstract

The research paper focuses on understanding the dynamics of shock propagation in financial networks, particularly examining how volatility shocks in one asset transmit across interconnected assets. The study addresses the challenge of identifying the paths through which shocks spread in a network of multiple traded assets. By constructing a network based on conditional volatility series estimated from asset returns, the paper employs a many-dimensional VAR model with unique identification criteria derived from network topology. This approach allows for tracing the exact path of ripple effects through the entire asset network. Drawing on a comprehensive time series dataset spanning 16 years and covering 100 major stocks of NYSE, the analysis delves into the intricate relationships and spillover effects within the financial market. The methodology developed in the paper offers insights into how shocks originating from individual assets propagate through the network, shedding light on the interconnected nature of financial markets and the ripple effects of volatility shocks.

After understanding this research paper we were interested to do a similar analysis using Indian stocks from NSE .We created a similar model and cross verified with the data of NYSE data's implication to check the efficiency of the model. We also tried categorising the Nifty 50 stocks based on its influence in market.

2.2 Introduction

Aftermath of financial crisis of 2008 there has been significant development in the theoretical and empirical analysis as it brought forward the importance of connections across economic and financial entities. A. Mullaly in the congressional hearing (Mullaly 2008) during the crisis: 'If any one of the domestic companies should fail, we believe there is a strong chance that the entire industry would face severe disruption'. In the research paper, they have focused on the nature of inter connectivity and model the phenomenon of shock propagation from an econometric point of view where one has to estimate the network from underlying process and data, in line with the reduced form approach.

2.3 Empirical data and methodology used in the research paper

The research paper considers data from 100 stocks with largest market capitalization in the New York Stock Exchange over a period of 16 years (2002-2017). the 16 years have been split into 4 periods of 4 years each. This data is processed by the model described below, which gives us the eigenvector centrality which gives us how influential is that stock and is ordered in descending values. later part of the research paper is utilizing the processed data of correlation matrices which is observed for how the whole market reacts by a short impulse (Ripple) on each asset.

Here, we provide a step-by-step recipe to construct the asset network and characterize the shock propagation mechanism.

- 1. We choose N number of stocks to create return series over T time periods such that $T\gg N$.
- 2. Next, we estimate latent volatility series from each of the return series by using generalized autoregressive conditional heteroscedasticity model.
- 3. We estimate a vector autoregression model on the log of latent volatility series across all stocks in the sample. The VAR model is identified through Cholesky decomposition of the error covariance matrix, by imposing an ordering on the stocks obtained from the eigenvector centrality ceigr of the return correlation matrix.
- 4. Now one can characterize the shock propagation over the stocks by using estimated impulse response functions obtained from the identified VAR model.
- 5. For visualization of shock propagation, we construct a network (G based on the correlation structure of the stocks and extract the minimum spanning tree .We plot the impulse responses across the network emanating from chosen epicenters.

In the following, we utilize the algorithm to characterize the ripples across the financial networks, emanating from chosen epicenters. We also characterize how the magnitudes of the ripples diminish over time.

2.4 Creation of a similar model

We have created a similar model by the use of python libraries such as

- yfinance: To collect the past market data
- numpy: To calculate the required terms and data
- pandas: to process the data as data frames
- arch: To create a GARCH model from the data

Here is the Google colab link for the model created: https://colab.research.google.com/drive/1KqnkKqjV1vsharingscroll $To = 7b3xJRs_ToMv$

• we import all the modules required to the model

```
[] pip install arch

[] import yfinance as yf import pandas as pd import numpy as np import plotly.express as px from arch import arch_model

[] import seaborn as sns from numpy.linalg import eig, inv
```

• We consider the companies whose data has to be analysed

```
comp = ["ABT", "AEP", "AET", "AFL", "AIG", "ALL", "AMT", "APD",

"AXP", "BA", "BAX", "BDX", "BLK", "BMY", "BSX", "CAT", "CB",

"CCI", "CCL", "CI", "COP", "CVS", "CVX", "D", "DE", "DHR",

"DIS", "DUK", "ECL", "EL", "EMR", "EOG", "ETN", "EXC", "F", "FDX",

"GD", "GE", "GIS", "GS", "HAL", "HD", "HON", "HUM", "IBM", "ITW", "JNJ",

"KMB", "KO", "LLY", "LOW", "LUV", "MCK", "MDT", "MET", "MMC", "MMM",

"MO", "MRK", "MS", "NEE", "NKE", "NOC", "NSC", "ORCL", "OXY", "PCG",

"PFE", "PG", "PRU", "PSA", "PXD", "SCHW", "SHW", "SLB",

"SO", "SPG", "SPGI", "STZ", "SYK", "T", "TGT", "TJX", "TMO", "TRV",

"UNH", "UNP", "UPS", "VLO", "VZ", "WM", "WMT", "XOM"]
```

• In the following step we consider an array of data frames which we calculate daily returns. After going through that GARCH model gives us conditional volatility

```
companies_data = []
for stock in comp:
    data = yf.download(stock, start="2002-01-01", end="2005-12-31")

    data["retser"] = np.log(data['Close'] / data['Close'].shift(1))
    data['retser'] = data['retser'].fillna(0)
    model = arch_model(data["retser"].dropna(), p=1, q=1)
    model_fit = model.fit(disp='off')
    data = data.assign(conditional_volatility=np.sqrt(model_fit.conditional_volatility))
    companies_data.append(data)
```

• In the following step we find the correlation matrix for each stock and proceed to find the eigenvalue centrality of each stock.

• We do this same process for other 3 time periods

```
companies_data = 🚺
for stock in comp:
   data = yf.download(stock, start="2006-01-01", end="2009-12-31")
   data["retser"] = np.log(data['Close'] / data['Close'].shift(1))
   data['retser'] = data['retser'].fillna(0)
   model = arch_model(data["retser"].dropna(), p=1, q=1)
model_fit = model.fit(disp='off')
   data = data.assign(conditional volatility=np.sqrt(model fit.conditional volatility))
   companies_data.append(data)
retser_data = pd.concat([df["retser"] for df in companies_data], axis=1)
retser_data.columns = comp
correlation_matrix = retser_data.corr()
eigenvalues, eigenvectors = eig(correlation matrix)
max eigenvalue index = np.argmax(eigenvalues)
eigenvector_centrality = np.abs(eigenvectors[:, max_eigenvalue_index])
centrality_df = pd.DataFrame({'Stock': correlation_matrix.columns, 'Centrality': eigenvector_centrality})
centrality_df.sort_values(by='Centrality', ascending=False, inplace=True)
ordered_stocks2 = pd.DataFrame(centrality_df['Stock'].values)
ordered stocks2
x.append( (centrality_df['Stock'].values))
```

```
companies data = []
for stock in comp:
    data = yf.download(stock, start="2010-01-01", end="2013-12-31")
    data["retser"] = np.log(data['Close'] / data['Close'].shift(1))
    data['retser'] = data['retser'].fillna(0)
   model = arch_model(data["retser"].dropna(), p=1, q=1)
model_fit = model.fit(disp='off')
    data = data.assign(conditional volatility=np.sqrt(model fit.conditional volatility))
    companies_data.append(data)
retser_data = pd.concat([df["retser"] for df in companies_data], axis=1)
retser data.columns = comp
correlation_matrix = retser_data.corr()
eigenvalues, eigenvectors = eig(correlation_matrix)
max_eigenvalue_index = np.argmax(eigenvalues)
eigenvector_centrality = np.abs(eigenvectors[:, max_eigenvalue_index])
centrality df = pd.DataFrame({'Stock': correlation matrix.columns, 'Centrality': eigenvector centrality})
centrality_df.sort_values(by='Centrality', ascending=False, inplace=True)
ordered stocks3 = pd.DataFrame(centrality df['Stock'].values)
ordered_stocks3
x.append( (centrality_df['Stock'].values))
companies data = []
for stock in comp:
    data = yf.download(stock, start="2014-01-01", end="2017-12-31")
    data["retser"] = np.log(data['Close'] / data['Close'].shift(1))
    data['retser'] = data['retser'].fillna(0)
    model = arch_model(data["retser"].dropna(), p=1, q=1)
    model fit = model.fit(disp='off'
    data = data.assign(conditional_volatility=np.sqrt(model_fit.conditional_volatility))
    companies_data.append(data)
retser data = pd.concat([df["retser"] for df in companies data], axis=1)
retser data.columns = comp
correlation_matrix = retser_data.corr()
eigenvalues, eigenvectors = eig(correlation_matrix)
max_eigenvalue_index = np.argmax(eigenvalues)
eigenvector_centrality = np.abs(eigenvectors[:, max_eigenvalue_index])
centrality df = pd.DataFrame({'Stock': correlation matrix.columns, 'Centrality': eigenvector centrality})
centrality_df.sort_values(by='Centrality', ascending=False, inplace=True)
x.append( (centrality_df['Stock'].values))
```

• Now we arrange the stocks by its eigenvalue centrality based on returns in each period and compare it with all other 3 periods to understand how this order has changed.

```
total_data = pd.DataFrame(x)
total_data

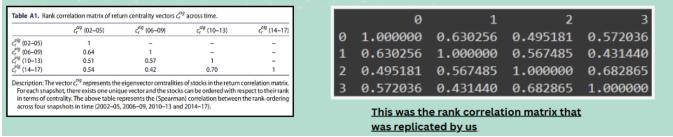
df = pd.DataFrame(total_data)
df_transposed = df.T
df_transposed.reset_index(drop=True, inplace=True)
df_transposed
```

```
for m in range(4):
 x=[]
 for i in temp:
   1=0
    for j in df transposed[m]:
      if i==j:
        x.append(1)
        l=1+1
       break
      l=1+1
 print(x)
 f.append(x)
df = pd.DataFrame(f)
df_transposed = df.T
df_transposed.reset_index(drop=True, inplace=True)
df transposed
correlation2 matrix = df transposed.corr()
print(correlation2 matrix)
```

2.5 Implications

2.5.1 Testing efficiency of the model

To check the efficiency of the created model that is described in the research paper we tried to replicate the data arrived using their model and here is the comparison



The vector ceigr represents the eigenvector centralities of stocks in the return correlation matrix. For each snapshot, there exists one unique vector and the stocks can be ordered with respect to their rank in terms of centrality. The above table represents the (Spearman) correlation between the rank-ordering across four snapshots in time (2002–05, 2006–09, 2010–13 and 2014–17).

Inference: By this we can understand that the model is efficient enough data obtained from our model has less deviation from the original model.

2.5.2 Influential stocks in Nifty 50

For this part of analysis we only consider the time period of 4 years (2020-2023) for 50 stocks that are in Nifty50 and arrange them by their eigenvalue centrality based on returns. This order gives us the stocks which has high influence across the market (any change/ripple/shock would affect other stocks a lot.)

HCLTECH.NS ADANIENT.NS INFY.NS ADANIPORTS.NS WIPRO.NS TITAN.NS TECHM.NS SHRIRAMFIN.NS BPCL.NS HINDUNILVR.NS BAJAJFINSV.NS COALINDIA.NS HINDALCO.NS RELIANCE.NS TATASTEEL.NS NESTLEIND.NS SUNPHARMA.NS BAJFINANCE.NS DIVISLAB.NS KOTAKBANK.NS BAJAJ-AUTO.NS DRREDDY.NS ULTRACEMCO.NS APOLLOHOSP.NS MARUTI.NS TATAMOTORS.NS LT.NS CIPLA.NS NTPC.NS TATACONSUM.NS INDUSINDBK.NS EICHERMOT.NS TCS.NS M&M.NS ASIANPAINT.NS GRASIM.NS SBIN.NS BRITANNIA.NS POWERGRID.NS HEROMOTOCO.NS JSWSTEEL.NS ITC.NS AXISBANK.NS HDFCBANK.NS ONGC.NS ICICIBANK.NS BHARTIARTL.NS

Inference: By this we understand that the ADANIENT has been highly influential in the market and any changes in the ADANIENT reflected throughout the market. Similarly we could understand the inter connectivity that exist in the market.

Chapter 3

Multifractal nature of Indian market

This chapter is based on the discussions and understandings from the research paper Multifractal properties of the Indian financial market by Sunil Kumar Nivedita Deo

3.1 Abstract

The research paper explore the multifractal characteristics of the logarithmic returns of the Indian financial indices (BSE & NSE) through the application of multifractal detrended fluctuation analysis. Their findings are compared with those of the US S&P 500 index. It is observed numerically that the qth-order generalized Hurst exponents h(q) and $\tau(q)$ exhibit variations with the moments q. The interrelation between these scaling exponents and the singularity spectrum $f(\alpha)$ indicates the presence of multifractality within the returns. A comparison between the MF-DFA outcomes of the original series and the shuffled series reveals that the multifractality is a result of both long-range correlations and a wide probability density function. The financial markets under examination are contrasted with the Binomial Multifractal Model (BMFM) and display a lesser degree of multifractal strength in comparison to the BMFM.

3.2 Introduction

Financial markets are intricate systems where physicists apply statistical mechanics to understand economic dynamics. These markets are open systems with nonlinear interactions and feedback, leading to non-stationary time series analysis. Standard multifractal analysis faces challenges with non-stationary time series affected by trends, leading to the development of improved methods like WTMM and MF-DFA. WTMM involves wavelet analysis, while MF-DFA identifies scaling of moments without requiring modulus maxima procedure. Distinguishing trends from intrinsic fluctuations in data is crucial for reliable correlation detection. Hurst's rescaled-range analysis and DFA are used to analyze noisy data in the presence of trends. Multifractality in time series can be due to broad probability density functions or different long-range correlations for small and large fluctuations. Hurst exponent provides insights into long-term memory and fractality of time series data, aiding in understanding stochastic phenomena behind fluctuations .

3.3 Method

They have used 5 step Multifractal-DFA method to analyse the nature of multifractality.

. Calculate Logarithmic Returns and Normalize them.

$$g\left(t\right)=\frac{\left(logp\left(t+1\right)-logp\left(t\right)\right)}{\sigma}$$
 2. Calculate the Profile. (from k=1 to i)

$$Y\left(i
ight) =\sum_{k=1}^{i}[\left(g\left(k
ight) -\left\langle g
ight
angle
ight]$$

- 3. Segment the dataset into intervals of length s , since the entire length may not be completely divisible , take sections from the beginning and from the End and then analyze.
- 4. Calculate the local trend for each of the segment by a least square fit and then calculate the variance for every segment and average it over all the segments to get the mean fluctuation.

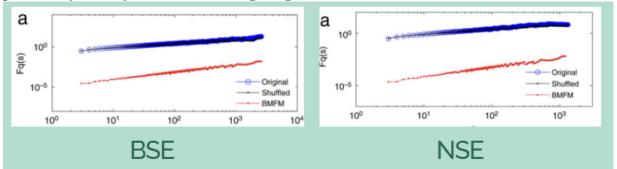
$$F^{2}\left(s,v
ight) =\sum_{i=1}^{s}\left(Y\left(\left(v-1
ight) s+i
ight) -yv\left(i
ight)
ight) ^{2}$$

5. Average over all the segments to get the q-th order fluctuation function.

$$Fq\left(s
ight)=\{rac{1}{2Ns}\sum_{v=1}^{2Ns}[F^{2}\left(s,v
ight)]^{rac{q}{2}}\}^{rac{1}{q}}$$

3.4 Multifractal nature observed in market

Multifractal analysis was conducted on the Indian financial markets (BSE and NSE) and the US market (S&P 500 index) using the multifractal detrended fluctuation analysis method. The Hurst exponent was utilized to predict future price changes and differentiate between emerging and mature markets, identifying the Indian markets as emerging and the US market as mature. The BSE index exhibited a change in the Hurst exponent near the 9/11 crash, indicating a significant shift compared to the S&P 500 index. The multifractal behavior of the BSE index was confirmed through the analysis of scaling exponents h(q) and (q), showing a nonlinear dependence and multifractal structure resembling an inverted parabola. The strength of multifractality () for the BSE index was determined to be 0.4415, indicating the richness of multifractality in the index. Random shuffling of log-return series for the indices revealed weaker multifractality in the shuffled series compared to the original series, highlighting the presence of multifractality in the BSE index due to broad probability density function and long-range correlations.



Time window (years)	Hurst exponent BSE	
1997-1999	0.5908	
1999-2001	0.3310	
2001-2003	0.5744	
2003-2005	0.6005	
2005-2007	0.4173	
Average	0.5028	

3.5 Conclusion

The MF-DFA method effectively characterizes multifractality in Indian and US financial markets. Nonlinear scaling exponents and singularity spectrum confirm multifractality in the markets. Multifractality in financial data is attributed to long-range correlations and broad probability density functions. Financial markets exhibit multifractality, but with lesser strength compared to the Binomial Multifractal Model.

Chapter 4

Understanding Game theory

4.1 Introduction

Game theory is a branch of mathematics and economics that studies strategic decision-making in competitive situations where the outcome of one participant's decision depends on the decisions of others. It provides a framework to analyse how rational individuals or entities make choices to maximize their own outcomes, considering the actions and responses of others. Central concepts in game theory include players (decision-makers), strategies (possible actions or choices), payoffs (outcomes associated with different strategy combinations), and equilibrium (stable states where no player can improve their payoff by changing their strategy unilaterally). Game theory has applications in diverse fields such as economics (pricing strategies, auctions), political science (voting behaviour, international relations), biology (evolutionary dynamics), and computer science (algorithm design). Its models range from simple, two-player games to complex scenarios involving multiple players and repeated interactions, offering insights into strategic behaviour and optimal decision-making in competitive environments.

4.2 Strategic interactions(Game)

Strategic interaction involves at least two decision makers whose choices impact each other's well-being and utility, distinguishing it from individual decision-making scenarios. A Strategy is a complete contingent plan for that planer in the game. In perfectly competitive markets, individual decisions do not influence market prices or outcomes, as each buyer and seller is a small unit with no strategic impact on others. In contrast, industries with few players like cell phone producers exhibit strategic interactions, where one player's decisions directly affect others' profitability and market behaviour.

Strategic interactions or game consists of five parts:1.Players 2.Actions 3.Information 4.Outcomes 5.Preferences

It is important to be specific about these parts.

- 1. Players: People involved in the strategic interaction are players.
- 2. Actions: Moves the players have are actions
- 3. Information: What the players know when they take an action or move
- 4. Outcomes: Effects of moves taken by the players.
- 5. Preferences: Outcomes favoured by each player are their preferences.

4.3 Types of games

• Cooperative vs non-cooperative:

Cooperative games allow for binding agreements between players, while non-cooperative games do not.

• Zero-sum vs Non-zero-sum:

In zero-sum games, one player's gain is exactly another's loss. In non-zero-sum games, the total gain or loss can vary.

• Simultaneous vs Sequential:

Simultaneous games involve players making decisions at the same time, while sequential games involve players making decisions one after another.

• Static vs Dynamic games:

In static games, players make their decisions (or choose their strategies) simultaneously, or if they move sequentially, they do so without knowledge of the other players' choices. In dynamic games, players make decisions at different points in time, allowing them to observe the choices of others before making their own.

Feature	Static Games	Dynamic Games
Timing of Decisions	Simultaneous or without observation	Sequential, with observation
Structure	Single stage	Multiple stages
Representation	Payoff matrix	Game tree
Examples	Prisoner's Dilemma, Battle of the Sexes	Chess, Stackelberg Competition
Solution Concepts	Nash Equilibrium	Subgame Perfect Equilibrium, Backward Induction
Strategic Complexity	Simpler, focusing on immediate decisions	More complex, involving planning and adaptation over time

• Repeated games:

When games played over multiple rounds, where players can condition their strategies on past actions. Cooperation can emerge in repeated games through strategies like 'tit- for-tat'.

4.4 Types of Information

Complete vs Incomplete info: A game of complete information is one in which all players know the structure of the game, including the payoffs, strategies, and types of all other players. A game of incomplete information is one in which some aspects of the game or the characteristics of the players are unknown to some or all players.

Feature	Games of Complete Information	Games of Incomplete Information
Knowledge	Full knowledge of game structure and players	Partial or hidden knowledge about some aspects
Strategic Complexity	Simpler due to transparency	More complex due to uncertainty
Beliefs	Not required	Required for decision-making
Examples	Chess, perfect market competition	Poker, auctions, diplomatic negotiations
Solution Concepts	Nash Equilibrium	Bayesian Nash Equilibrium

4.5 Types of strategies

Pure vs Mixed strategies:

A pure strategy is a specific, deterministic course of action that a player follows in a game. It involves choosing one particular action from the set of all possible actions available to the player. A mixed strategy is a probabilistic approach to decision-making where a player chooses among available actions according to a specific probability distribution.

Feature	Pure Strategies	Mixed Strategies
Nature of Choice	Deterministic (specific action)	Probabilistic (distribution over actions)
Predictability	Predictable	Unpredictable
Complexity	Simpler to understand and implement	More complex and flexible
Adaptability	Less adaptable	Highly adaptable
Common Use Cases	Simple games, dominant strategy scenarios	Complex games, multiple equilibria, need for unpredictability

4.6 Beliefs

In game theory, beliefs refer to a player's subjective probability assessments regarding the unknown factors in the game, including the strategies and types of other players.

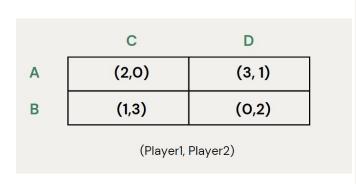
Prior Beliefs: The initial probability distribution over possible types or actions.

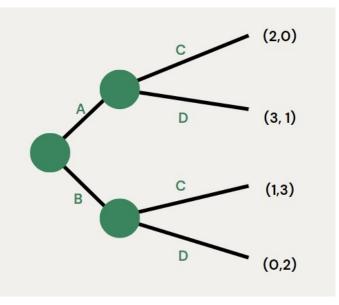
Likelihood: The probability of observing certain actions or signals given the possible types.

Posterior Beliefs: Updated beliefs after observing actions, calculated using Bayes' rule Posterior is proportional to Prior $\times Likelihood$

4.7 Game tree

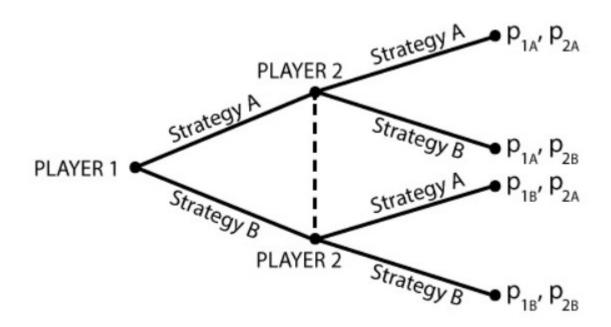
A game tree is a graphical representation used to map out the sequence of possible moves and outcomes in a strategic game. It starts with a decision node representing the initial decision point where players choose among various strategies. From each decision node, branches (or edges) extend outward, representing the possible actions each player can take.





As the game progresses, decision nodes lead to further branches, forming a tree-like structure that captures all possible sequences of moves and their resulting outcomes. Terminal nodes (or leaves) of the tree represent final outcomes or payoffs for each player based on the sequence of actions taken by all players.

Game trees are essential tools for analysing strategic interactions, helping to determine optimal strategies, predict outcomes, and understand the dynamics of competitive decision-making. They are used extensively in fields such as economics, biology, computer science, and political science to model and study a wide range of strategic situations from simple games to complex scenarios with multiple players and stages of decision-making.



Let us consider an example where Player 1 -> (A, B) and Player 2 -> (C, D) both have 2 possible moves. For this situation the game tree would be:

Here first the decision of player 1 is considered. Then based on those, 2 branches arise from each decision of player 1 which are decisions of player 2.

4.8 Dominant strategy

A dominant strategy refers to a strategy that is the best choice for a player regardless of the strategies chosen by other players. This means a dominant strategy will always yield the highest payoff or utility for a player, irrespective of what actions the other players take. Players choose dominant strategies based solely on maximizing their own outcomes, without needing to consider what others might do. Dominant strategies are crucial in analysing strategic interactions and predicting outcomes in various scenarios

In case of prisoner's dilemma:

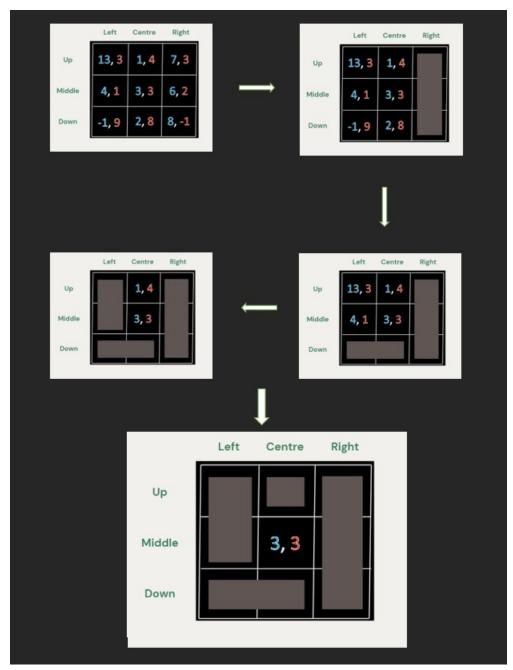
	1	Player2	
		Confess	Silent
Dlaver1	Confess	(5,5)	(0, 10)
Player1	Silent	(10,0)	(2,2)
	·		

Here,

- If both confess, they get 5 years jail time each.
- If one player confesses and the other stays silent, the player who confessed get 10 years jail time while the player who stayed silent is free to go.
- If both stays silent then they get 2 years jail time each.

Here the dominant strategy in one player's perspective will be that player stays silent and the other confesses. But in overall view, the dominant strategy is that both stays silent. And regardless of other player's choice, staying silent is the best option of a player.

How to find dominant strategy? A simple elimination method to find dominant strategy for simple cases: Let us take an example:



Here, step by step elimination takes place to find the dominant strategy. We compare each column followed by each row.

- First red numbers and columns centre and right are compared. Column right is eliminated.
- Blue numbers of rows middle and down are compared. Row down is eliminated
- Red numbers of left and centre columns are compared, and column left is eliminated.
- Finally blue numbers of up and middle rows are compared and row up is eliminated.

Therefore, we are left with the dominant strategy

4.9 Bayesian game

A Bayesian game is a type of game in game theory where players have incomplete information about the other players. Specifically, each player does not know the exact type (characteristics or

payoffs) of the other players. Instead, they have beliefs about the probability distribution over the possible types of the other players. These beliefs are represented by a common prior probability distribution.

In a Bayesian game, players use their beliefs to update their strategies based on the observed actions and types, aiming to maximize their expected utility given their information.

Chapter 5

Behavioural finance

5.1 Introduction

Behavioral finance is a field of study that examines how psychological influences and biases affect the financial behaviors of investors and financial markets. It challenges the traditional assumption of rationality in economics and finance, recognizing that humans often make irrational decisions due to cognitive biases, emotions, and social factors.

What is the need for behavioral finance:

The sole reason why one can make money in the stock market is because someone else loses it. Every stock that one buy is sold by someone and vice versa. What this means is that even with well defined technical analysis; opposing views exist in the market and the reason behind this is that people react differently to different changes in the market.

The efficient market theory, which is the basis of technical analysis, states that all equities are priced fairly based on all available public information, meaning the price of a stock incorporates into it all the available information about it. But the domain of behavioural finance argues that this theory fails to consider irrational emotional behaviour

5.2 Why do biases occur

Biases can occur for a variety of reasons. Some key factors causing biases are:

- Mental Accounting: Mental accounting refers to the propensity for people to allocate money for specific purposes. In other words people treat different money differently even though there is no logical reason behind it.
- Herd behavior: Herd behavior states that people tend to mimic the financial behaviors of the majority of the herd. Herding is notorious in the stock market as the cause behind dramatic rallies and sell-offs
- Emotional gap: The emotional gap refers to decision-making based on extreme emotions or emotional strains such as anxiety, anger, fear, or excitement. Oftentimes, emotions are a key reason why people do not make rational choices.
- Anchoring: Anchoring refers to attaching a spending level to a certain reference. Examples may include spending consistently based on a budget level or rationalizing spending based

on different satisfaction utilities. Anchoring means attaching special importance to a specific figure when in fact conditions change continuously

• Self-attribution: Self-attribution refers to a tendency to make choices based on overconfidence in one's own knowledge or skill. Self-attribution usually stems from an intrinsic knack in a particular area. Within this category, individuals tend to rank their knowledge higher than others, even when it objectively falls short.

5.3 Specific biases

- Confirmation bias: This bias states that one is often looking for and willing to accept information that confirms the already existing belief one has even if the information provided is false.
- Experiency bias or recency bias: Many of your decisions are influenced by the outcome of similar decisions you took in the recent past. A classic example is the time following the 2008-09 recession. Many were dismal about the market conditions and unwilling to take any risk in the fear of a similar event occurring again when in fact the market did rally and bounce back after the recession.
- Loss aversion bias: Traders place a greater weighting on the concern for losses than the pleasure from market gains. We see how different reactions to the same market change can affect how we fare in this game of the market.
- Familiarity bias: Many investors prefer to invest in companies they are familiar with. For example, among two companies, even if one has decently better numbers than the other, an investor might choose to invest in the other if it is a well known company or it has been around for a long time. This can lead to investors not diversifying to reduce risk when in fact they should.

5.4 Some important indicators

In this section we would discuss about few indicators

5.4.1 On Balance Volume

- If today's closing price is higher than yesterday's closing price, then: Current OBV = Previous OBV + today's volume
- \bullet If today's closing price is lower than yesterday's closing price, then: Current OBV = Previous OBV today's volume
- If today's closing price equals yesterday's closing price, then: Current OBV = Previous OBV

OBV helps us differentiate between smart money or institutional investors and retail investors. Institutional investors are assumed to make very efficient trading decisions and influence the market hugely. Say a stock is trading within a range for some time but its OBV has been steadily increasing. This could mean that smart money is consolidating its position while being careful not to drive the prices up. Retail investors are going to notice this in some time and then buy themselves thus driving up the price with a sharp rise in OBV. After this rally, the institutional investors will book

their profits thus leading to a dip in OBV. OBV is thus a leading indicator. Its absolute value is not very important but attention is to be paid on how it changes.

5.4.2 Accumulation/Distribution Indicator

The accumulation/distribution indicator (A/D) is a cumulative indicator that uses volume and price to assess whether a stock is being accumulated or distributed.



Therefore, when a stock closes near the high of the period's range and has high volume, it will result in a large A/D jump. Alternatively, if the price finishes near the high of the range but volume is low, or if the volume is high but the price finishes more toward the middle of the range, then the A/D will not move up as much. If a security's price is in a downtrend while the A/D line is in an uptrend, then the indicator shows there may be buying pressure and the security's price may reverse to the upside. Conversely, if a security's price is in an uptrend while the A/D line is in a downtrend, then the indicator shows there may be selling pressure, or higher distribution. This warns that the price may be due for a decline.

5.4.3 Open interest

Open interest only rises or falls when a new contract is created or destroyed—one long and one short seller must enter the market to increase the open interest, and one long and one short seller must close their positions for open interest to fall. A falling open interest shows that losers are exiting positions while winners are taking profits. It also shows there are no additional losers to take the place of those who have given up. Falling open interest is a clear signal that winners are taking their profits and running for the border while losers are giving up hope. A loss of a contract (and a declining open interest) points to the likely end of a trend. The open interest that remains relatively constant during a market uptrend indicates that the supply of losers has stopped growing as the only potential candidates to enter into a contract are previous buyers who are looking to profit from their position. In this case, the uptrend is likely nearing its end.

5.5 Fear and greed and how to asses them using indicators

Fear and greed can lead to overreactions and experienced traders can make use of this to profit. For example one can buy shares when others are panicking too much. If one recognises that the panic is short term and the market will most probably bounce back, one can make use of the low price of

shares due the current panic and buy cheap and sell high later. Similarly if one recognises that the market is over-euphotic and that the current surge in prices will eventually come back down, one can short stocks and make money.

5.6 Support and resistance

Technical analysts use support and resistance levels to identify price points on a chart where the probabilities favor a pause, or reversal, of a prevailing trend. Support occurs where downtrend is expected to pause, due to a concentration of demand. Resistance occurs where an uptrend is expected to pause temporarily, due to a concentration of supply. These levels, while they may appear arbitrary at first sight, are based on market sentiment and anchoring. Here, we examine how support and resistance zones are largely shaped by human emotion and psychology.

It is important to note that support and resistance levels are not set in stone. Once the price goes through these levels, these levels change their roles, that is support becomes resistance and vice versa.

At any point of time there are 3 types of traders in the market - those who have gone long, those who have gone short and those who are yet to enter a trade. Each of these either looks to increase their profits or cut their losses when price reaches a support or resistance. This leads to buying pressure at support and selling pressure at resistance.

Fear and greed, for example, are seen in the market participants' behavior outlined above. As price falls back to a support level, the traders who are already long will add to positions to make more money. Meanwhile, the traders who are short will buy to cover, because they are afraid of losing money. This behaviour is also a classic example of anchoring.

Once it so happens that price bounces back from a specific level, traders associate special importance to this level without any definitive logic and thus if a resistance or support level has been established in the past, it can create a shared anchor where those same levels will be met with resistance or support in the future.

$$ext{MFM} = rac{(ext{Close} - ext{Low}) - (ext{High} - ext{Close})}{ ext{High} - ext{Low}}$$

where:

MFM = Money Flow Multiplier

Close = Closing price

Low = Low price for the period

High = High price for the period

Money Flow Volume = $MFM \times Period Volume$

A/D = Previous A/D + CMFV

where:

CMFV = Current period money flow volume

Other support and resistance levels that are influenced by human emotion include round numbers, 52 week highs and lows, and historic events such as new market highs. One reason for this is that these prices have been significant in the past and traders know they are likely to be again.

Market participants often gauge future expectations based on what has happened in the past; if a support level worked in the past, the trader may assume that it will provide solid support again

5.7 Volatility and associated strategies

Volatility is a measure of how much the price of a security, such as a stock or an index, fluctuates within a certain period. High volatility means that the price can change dramatically in a short time span, while low volatility indicates that the price remains relatively stable.

Volatility can be historical or implied, expressed on an annualized basis in percentage terms. Historical volatility (HV) is the actual volatility demonstrated by the underlying asset over some time, such as the past month or year. Implied Volatility (IV) is the level of volatility of the underlying implied by the current option price.

5.7.1 VIX index

VIX measures the fear in the market. It measures fear by calculating the implied volatility in the near term expiration of the S&P 500 options contracts. Thus, the implied calculation is a forecast of the market's aggregate expectations of the volatility in the coming few weeks. The more fearful investors and traders are, the more they are willing to pay for the cost of insurance (options). Therefore, it is called the fear index. Uncertainty is the enemy for most investors and traders, both in the market and in everyday life, and therefore we are willing to pay for reducing uncertainty. The VIX, also known as the fear index, is a tool that measures how much investors and traders expect the S&P 500 (and the stock market in general) to fluctuate in the upcoming 30 days. When the VIX number is low, the market is expected to have low volatility in the coming days. And when the VIX is high, the market is expected to have high volatility in the coming days. When the VIX is low, then the market is in risk-on mode, meaning stock markets are rising, and the economy is usually booming. On the other hand, when the VIX is rising, stock indices are falling, and investors may consider a risk-off mode. A high VIX figure indicates that the S&P 500 and the general U.S. stock market will likely become more volatile within a month. It also indicates that the markets are likely to drop since investors' fear is rising. A low VIX figure signals a potential low volatility in the S&P 500 within the next 30 days. Generally, it is said that the market is at increasing risk when the VIX rises above 30. On the other hand, when VIX is trading below 20, investors interpret it as a low risk market condition.



20 day ema on vix and inverse proportionality



But, like many other indicators, it can be prone to many false signals. So, to find the most accurate trading combination, we try other moving average periods, other

So, to find the most accurate trading combination, we try other moving average periods, other indicators, or other trading strategies altogether. Several indicators can be used to get a view on future trends of VIX and strategies can be developed accordingly.

Chapter 6

Reaction to earnings report

This chapter briefly covers about the learnings from the discussion on the working paper on topic Stock price reaction to earnings announcements.

6.1 Introduction

The study of stock price reactions to earnings announcements has revealed several anomalies that challenge the efficient market hypothesis. This paper reviews recent evidence indicating that stock prices often underreact or overreact to earnings announcements, leading to persistent anomalies.

6.2 Post-Earnings-Announcement Drift

One of the most studied anomalies is the post-earnings-announcement drift, where stock prices continue to move in the direction of the earnings surprise for several months following the announcement. This drift suggests that the initial reaction to earnings news is incomplete. Studies by Bernard and Thomas (1989, 1990) and others have documented that the cumulative abnormal returns (CARs) for firms with positive earnings surprises continue to rise, while those for firms with negative surprises continue to fall. This behavior is inconsistent with the efficient market hypothesis, which would predict an immediate and complete adjustment of stock prices to new information.

6.3 Evidence of Underreaction

Several studies have documented underreactions to earnings announcements. For example, Bernard and Thomas (1989) showed that the initial response to earnings announcements is often too small and that stock prices continue to adjust over a period of six months or more. This underreaction is evident in the post-earnings-announcement drift, where CARs continue to drift upwards for firms with good news and downwards for firms with bad news. One potential explanation for this underreaction is that investors fail to fully incorporate the implications of current earnings for future earnings. Studies have shown that stock prices often do not reflect the autocorrelation structure of earnings, leading to predictable stock price movements following earnings announcements.

6.4 Overreaction to Earnings

In contrast to the evidence of underreaction, some studies have found evidence of overreaction to earnings announcements. DeBondt and Thaler(1987) documented that stocks with extreme prior returns tend to experience reversals, suggesting that prior price movements may have been

overreactions. Similarly, Ou and Penman (1989) found that firms with high earnings-toprice (EP) ratios tend to have higher future returns, which could be indicative of an overreaction to past earnings trends. However, the evidence for overreaction is mixed, and some studies have found that these patterns can be explained by risk factors or other anomalies.

6.5 Reconciling Underreaction and Overreaction

The seemingly contradictory evidence of underreaction and overreaction can potentially be reconciled by considering the different contexts and time horizons of these phenomena. For example, while post-earnings-announcement drift suggests underreaction in the short term, long-term reversals may indicate overreaction to longer-term earnings trends. Furthermore, the evidence suggests that different market participants may react differently to earnings news. Institutional investors and analysts may underreact due to conservative forecasting, while individual investors may overreact to salient news, leading to the observed anomalies.

6.6 Functional Fixation Hypothesis

The functional fixation hypothesis posits that investors fixate on reported earnings figures without fully adjusting for the economic implications of accounting methods. Hand (1990) provided evidence for this hypothesis by showing that stock prices react to earnings components that should have been anticipated by the market, such as accounting gains from debt-equity swaps.

6.7 Conclusion

The evidence reviewed in this paper suggests that stock prices do not always react efficiently to earnings announcements. Both under reaction and overreaction have been documented, challenging the efficient market hypothesis. Understanding these anomalies and their underlying causes is crucial for investors and researchers in the field of behavioral finance.

Chapter 7

A Game Theoretical approach for Designing Market Trading Strategies

This chapter is based upon the understanding and discussions of the research paper A Game-Theoretical Approach for Designing Market Trading Strategies by Garrison W. Greenwood and Richard Tymerski.

7.1 Abstract

Investors are constantly seeking effective stock market trading strategies to maximize profits. In technical analysis, trading rules are developed by examining historical market data to identify trends that can be leveraged. However, these trends often manifest only partially, complicating analysis. This paper introduces a method to co-evolve fuzzy trading rules based on market trend features. Fuzzy membership functions are used to effectively manage partial features in historical data. The co-evolutionary process is modeled as a zero-sum competitive game, reflecting how brokerage firms evaluate trading strategies. Experimental results demonstrate that the co-evolutionary approach generates trading rule-bases that yield positive returns when tested with actual stock market data.

7.2 Introduction

The recent emergence of online trading has made the stock market accessible to small investors. Some discount brokerage firms provide little or no investment advice, which means it is up to the small investor to come up with their own investment strategies. Other brokerage firms do advise investors when to buy or sell stock, but the underlying strategy is proprietary and for that reason is not disclosed to outsiders.

Fundamental Approach: This method involves making investment decisions based on a detailed analysis of a company's financial health, including its earnings, expenses, assets, and liabilities. The idea is to determine the intrinsic value of a company's stock.

Technical Approach: Contrarily, this approach relies on analyzing past trading activity and price movements to predict future stock prices. It involves identifying patterns or trends in historical market data that can be exploited for profitable trading.

Fuzzy logic:

Fuzzy logic is a form of many-valued logic that deals with reasoning that is approximate rather than fixed and exact. In classical set theory, an element either belongs to a set or it does not (binary membership). In fuzzy set theory, an element can partially belong to a set with a degree of mem-

bership ranging from 0 to 1.Membership function defines the degree to which an element belongs to a fuzzy set. For instance, the membership function A(x) gives the degree of membership of element x in fuzzy set A.

7.3 Why use Game theory

Investment counselors at brokerage firms develop various trading strategies to handle different market conditions, such as bull and bear markets. These strategies are collectively offered to investors, with their performance continuously monitored and adapted. The effectiveness of an investment strategy is traditionally measured by its returns. However, assessing fitness solely based on returns can be misleading without comparative benchmarks. Relative fitness evaluates strategies within a single firm, while true fitness compares them across different firms. True fitness, which measures the ability to attract investor dollars, is a more accurate indicator of a strategy's effectiveness. Stock market investment is modeled as a zero-sum game where brokerage firms compete for investor dollars. Strategies evolve through competitive coevolution, reflecting real-world dynamics where successful strategies attract more investments, and poor strategies lose investor dollars.

7.4 Strategic Formulation

NRk days, DOJI patterns, and Hook days are technical indicators used in market analysis to identify potential changes in market trends and volatility. NRk days focus on volatility contraction, DOJI patterns indicate market indecision, and Hook days signal potential reversals.

By using these membership functions, we can effectively convert crisp historical stock market data into fuzzy values that reflect the degree to which specific features are present. This allows for a more nuanced and flexible analysis, which is crucial for making informed investment decisions in the inherently uncertain environment of the stock market.

• We import all the modules required to the model and downloaded historical stock data for Apple (AAPL) from Yahoo Finance then calculates the daily trading range

```
import yfinance as yf
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt

nifty_data = yf.download("AAPL", start="2022-01-01", end="2024-01-01")
nifty_data['Range'] = nifty_data['High'] - nifty_data['Low']
nifty_data.head()
```

• DOJI: A DOJI is a candlestick pattern where the opening and closing prices of the trading day are very close to each other. The DOJI condition can be represented as a predicate function that returns 1 (TRUE) if the absolute difference between the open and close prices is within a small percentage x of the trading range of the day. Otherwise, it returns 0 (False).

```
DOJI(x) = 1 in case of |O - C| \le x(H - L)else0
```

Significance: A DOJI signifies market indecision, as reflected by the small difference between the open and close prices. It can often signal a potential major reversal in the market trend.

```
def calculate_candlestick_patterns(data):
    data['Doji'] = np.abs(data['Close'] - data['Open'] / data['High']- data['Low']) < 0.1
    return data
nifty_data = calculate_candlestick_patterns(nifty_data)</pre>
```

 NRk (Narrow Range Days): NRk represents days where the range (difference between high and low prices) is narrower compared to the ranges of the previous k-1 days. These periods of low volatility often precede periods of higher volatility (volatility expansion), where prices may experience wider swings.

Application: Identifying NRk days can help traders anticipate potential breakout or breakdown scenarios. After a series of NRk days, traders might expect a subsequent wide-ranging day (either up or down), leading to potential trading opportunities

```
nifty_data['Range'] = nifty_data['High'] - nifty_data['Low']
df = pd.DataFrame(nifty_data)
df.reset_index(inplace=True)
def narrow_range(data, k):

    result_column_name = f'NR{k}'
    data[result_column_name] = False

    for i in range(k - 1, len(data)):
        current_range = data.loc[i,'Range']
        previous_ranges = data.loc[i - k + 1:i - 1, 'Range']

        if current_range < previous_ranges.min():
            data.loc[i, result_column_name] = True

narrow_range(df, 4)
narrow_range(df, 6)
nifty_data = df
nifty_data</pre>
```

• Fuzzification is the process that maps days (D) onto the unit interval via a membership function $\mu(D)$. More precisely, D represents the number of previous days that a particular feature is satisfied. For instance, for the feature NR7 D \in 0, 1, . . . , 6. Then $\mu(0) = 0$ means the NR7 definition was not satisfied during any of the six previous days, $\mu(6) = 1$ means the definition was satisfied during all six previous days (i.e., NR7 is definitely present) and $0 < \mu(D) < 1$ means NR7 was satisfied for some D < 6 days. Trapezoidal membership functions are most appropriate for the most of the features (see Figure 1).

• NRk

For this feature the equation is slightly different for each value of k. Let $\mu k(x)$ denote the membership function for NRk. Then

$$\iota_k(x) = \begin{cases} c(x - v_{min}) & x < v_{min} \\ 1 & x \ge v_{max} \end{cases}$$

with parameter values as shown in Table 1.

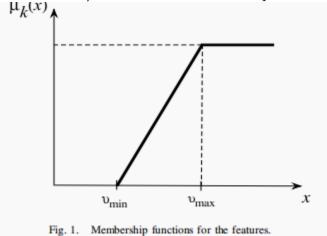
k	С	v_{min}	U_{ma}
4	1/2	2	4
6	1/3	3	6
7	1/3	4	7
TAI	BĽE I		

PARAMETER VALUES FOR NARROW RANGE FEATURES

In the above equation $x = D + \eta$ where $D \le k$ the number of days where (3) holds and, with $\overline{R} \max(1 < j < k) R[j]$.

$$\eta = \frac{\tilde{R} - R[0]}{\tilde{R}}$$

Notice that η increases the membership value for smaller previous day ranges.



```
def fuzzify_nrk(data, k, c, vmin, vmax):
    fuzzy values = np.zeros(len(data))
    for i in range(k, len(data)):
        D = sum(data['Range'].iloc[i-k:i] < data['Range'].iloc[i-k:i].max())</pre>
        R_max = data['Range'].iloc[i-k:i].max()
        R 0 = data['Range'].iloc[i]
        eta = (R max - R 0) / R max if R max != 0 else 0
        x = D + eta
        if x < vmin:
            fuzzy values[i] = 0
        elif x < vmax:
            fuzzy values[i] = c * (x - vmin)
        else:
            fuzzy values[i] = 1
    return fuzzy values
params = {
    'NR4': {'k': 4, 'c': 1/2, 'vmin': 2, 'vmax': 4},
    'NR6': {'k': 6, 'c': 1/3, 'vmin': 3, 'vmax': 6}, 'NR7': {'k': 7, 'c': 1/3, 'vmin': 4, 'vmax': 7},
for k in params:
    k_value = params[k]['k']
    c value = params[k]['c'
    vmin value = params[k]['vmin']
    vmax value = params[k]['vmax']
    nifty data[f'Fuzzy {k}'] = fuzzify nrk(nifty data, k value, c value, vmin value, vmax value)
print(nifty data.head())
```

• DOJI For this feature the membership function equation is $\mu(x) = \text{is equal to } 1 \text{ x}/\rho \text{ if } 0 \leq x \leq \rho$ else equal to 0 where typically $\rho \in [0.05, 0.30)$. x represents the percent difference between O and C and ρ represents the threshold percentage

• Hook Day A hook day is characterized by the market opening outside the previous day's range and then reversing direction. There are two types of hook days:

Up Hook Day: The market opens below the previous day's low and then moves upward. O[-1] <

 $L[0] - \delta$

Down Hook Day: The market opens above the previous day's high and then moves downward. O[-1] $> H[0] + \delta$

```
def fuzzify uphook(data,delta):
  fuzzy values = np.zeros(len(data))
  for i in range(1,len(data)):
    if(data['Open'][i]>data['High'][i-1]):
      x=data['Open'][i]-data['High'][i-1]-delta
      if(x < -0.5):
        fuzzy values[i]=0
      elif(x \ge -0.5 and x < 0):
        fuzzy values[i]=x
      elif(x>=0):
        fuzzy values[i]=1
    elif(data['Open'][i]<data['Low'][i-1]):
      x=data['Low'][i-1]-data['Open'][i]-delta
      if(x < -0.5):
        fuzzy values[i]=0
      elif(x \ge -0.5 and x < 0):
        fuzzy values[i]=x
      elif(x>=0):
        fuzzy values[i]=1
  return fuzzy values
nifty data['Fuzzy Hook-Day']=fuzzify_uphook(nifty_data,delta=0.5)
nifty_data
```

• Unfortunately, just detecting the presence or absence of a single feature is not a very good trend day predictor. The problem is to find combinations of features that make a good trend day predictor. If there are N total features, then there are N total rules in the rule-base. Consider the rule if x is NR4 then output is up-trend day

The semantics of this rule is as follows. The term "x is NR4" means ranges for the current and the previous three days are computed. x represents how many of those days meet the NR4 definition. This crisp data value is the argument for the NR4 membership functions which returns a number between 0 and 1 to give the degree of membership for NR4. The output value is the degree of membership for an up-trend day.

A singleton fuzzifier is used for the inputs. The set of possible outputs is

A =(0.25 0.5 0.75 1.0) These output values represent the likelihood an investor would buy stock because a given fuzzy rule had fired. The fuzzy rule-base is encoded as a matrix M with one column for every λ i \in A and one row for every fuzzy rule [10]. Each rule is of the form "if x is feature then y is an up-trend day". In this work we investigated 5 features so M has 5 rows corresponding to, from top to bottom, the features NR4, NR6, NR7, DOJI, and Up Hook Day (with $\delta = 0.5$). There are four columns, one for each value in A. A typical matrix might look like

$$M = \begin{pmatrix} 0 & 0.6 & 0.5 & 0 \\ 0 & 0.33 & 0.33 & 0.33 \\ 0 & 0.1 & 1.0 & 0.9 \\ 0 & 0.44 & 0.5 & 0.1 \\ 0 & 0.1 & 0.2 & 0.7 \end{pmatrix}$$

where each mij \in M is a weight. The first row in the above matrix is thus interpreted as the investor saying If the current day is the feature NR4, then I think the likelihood of an up-trend day tomorrow is 0.5 or 0.75 (out of 1) with strengths 0.6 and 0.5, respectively. Notice the strengths do not have to sum to 1.0.

Rule processing is straightforward. Given the rule-base matrix M, the vector of fuzzy numbers $A = (\mu NR4 , \mu NR6 , \mu NR7 , \mu DOJI , \mu hook)$ is created by running the training data through the

individual membership functions. A new A vector is created for each trading day. For example, on the 50th training day the ranges R(47), R(48) and R(49) are computed and μ 4(x) is computed using (5). Similarly the other membership function values are determined using (6) and (7). Then $A \circ M = B$ where

$$b_j \ = \ \max_{1 \le i \le 5} \min \left\{ a_i \, , \, m_{ij} \right\}$$

The fuzzy output vector B is then defuzzified to get a crisp output value indicating the desirability of buying shares of stock. A center of average defuzzification was used. That is,

$$U = \frac{\sum_{i=1}^{N} b_i \cdot \lambda_i}{\sum_{i=1}^{N} \lambda_i}$$

nifty_data[nifty_data['crisp']>0.8]

where λ i is the i-th singleton from A and N is the number of active rules

```
def crisp output(data):
  singleton=[0.25,0.5,0.75,1]
  crisp=np.zeros(len(data))
  m=[
      [0,0.6,0.5,0],
      [0,0.33,0.33,0.33],
      [0,0.1,1,0.9],
      [0,0.44,0.5,1],
      [0,0.1,0.2,0.7]
  a=np.zeros(5)
  b=np.zeros(4)
  for i in range(len(data)):
    u=0
    a[0]=data['Fuzzy NR4'][i]
    a[1]=data['Fuzzy NR6'][i]
    a[2]=data['Fuzzy NR7'][i]
    a[3]=data['Fuzzy Doji'][i]
    a[4]=data['Fuzzy Hook-Day'][i]
    for p in range(4):
      b[p] = \max(\min(a[0], m[0][p]), \min(a[1], m[1][p]), \min(a[2], m[2][p]), \min(a[3], m[3][p]), \min(a[4], m[4][p]))
    for d in range(4):
      u=u+(b[d]*singleton[d])/1.1
    crisp[i]=u
    data['crisp']=crisp
crisp_output(nifty_data)
```

The crisp output $U \in [0, 1]$ indicates the likelihood of an up-trend day or, equivalently, the desirability of purchasing more stock shares. However, stock is only purchased if the desirability exceeds a user-selected threshold. We chose 0.8 for this threshold. If the output value is greater than 0.8, then a 'buy' signal is generated

```
def signals(data):
    signal=np.zeros(len(data))
    for i in range(len(data)-1):
        if(data['crisp'][i]>0.8):
            signal[i+1]=data['crisp'][i]-0.8
        else:
            continue
        data['signal']=signal

signals(nifty_data)

nifty_data
```

the amount of stock purchased increased linearly the higher the output was above 0.8, subject to sufficient funds in the bank account, up to a maximum of 20 shares. Stock was bought at the opening price and sold at the closing price. All proceeds were deposited into the bank account at the end of each trading day

```
def transact(data,capital):
    initial=capital
    for i in range(len(data)):
        if(data['signal'][i]>0):
            noofshare=(capital*data['signal'][i]//data['Open'][i])
            if(noofshare>20):
                capital=capital-20*(data['Open'][i]-data['Close'][i])
            else:
                capital=capital-noofshare*(data['Open'][i]-data['Close'][i])
        returns=((capital-initial)*100)/initial
        return returns
```

7.5 Inferences

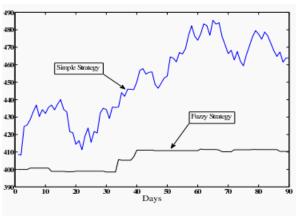


Fig. 5. Bank account balances for the top α firm strategy and the simple investment strategy over a 90 day period of test market data starting at the 1200th trading day. Both accounts started with a \$400 balance.

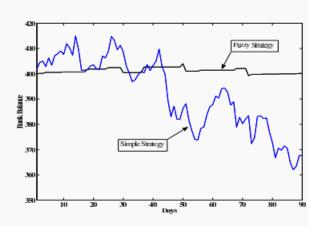


Fig. 6. Bank account balances for the top α firm strategy and the simple investment strategy over a 90 day period of test market data starting at the 1400th trading day. Both accounts started with a \$400 balance.

- The fuzzy indicator showed a modest +2.5% return during the first trading period and avoided losses in the second trading period, demonstrating its effectiveness in protecting capital during market downturns
- It performed well when the market was down, indicating its ability to handle volatile market conditions and protect investments during challenging times
- The fuzzy indicator's strength lies in its ability to provide fuzzy outputs, allowing for nuanced decision-making based on the likelihood of certain events happening, unlike crisp rules that only offer binary choices
- However, the fuzzy indicator may not perform as well when the market is up, as the returns were not as significant during upward market trends
- One of the key advantages of the fuzzy indicator is its capability to handle uncertainties and partial form features in historical data, which are challenging for traditional crisp rules to address effectively
- The fuzzy indicator's use of fuzzy if-then rules based on membership functions enables it to provide more flexible and nuanced trading strategies compared to rigid, deterministic approaches

Bibliography

- [1] To build python and programming skills (required for algo trading)
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 - Link to numpy tutorial https://youtu.be/ZB7BZMhfPgk?si=DW3pYrqdhmNDmatp
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- [5] For understanding the basics of Game Theory
 - 'Game theory for applied economists' by Robert Gibbons, is a superb book to get hold of the fundamentals and know the math behind them.