Algo Focus

sum of numbers less than given limit n

$$\text{sum} = \frac{n(n+1)}{2}$$

let $n = 5$

sum $= 1+2+3+4+5$

$= 15$

sum of squares

$$f(n) = \frac{n}{6}(2n+1)(n+1)$$ formula

limit

let $n = 5$

$f(n) = 1^2+2^2+3^2+4^2+5^2$

$= \ldots$

$\cancel{d=0}$

$a+b+c+\cancel{d}=1$ (1)

$8a+4b+2c+\cancel{d}=5$ (2)

$27a+9b+3c+\cancel{d}=14$ (3)

REF

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 5 \\ 27 & 9 & 3 & 14 \end{array}\right] \qquad x = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$-8R_1+R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -4 & -6 & -3 \\ 27 & 9 & 3 & 14 \end{array}\right]$$

$-27R_1+R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1\ \cancel{-4}/{-4} & \frac{3}{2}\ \cancel{-6}/{-4} & -3/{-4} \\ 0 & -18 & -24 & -13 \end{array}\right]$$

$R_2/{-4}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3/2 & 3/4 \\ 0 & -18 & -24 & -13 \end{array}\right]$$

$+18R_2+R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3/2 & 3/4 \\ 0 & 0 & 3 & 0.5 \end{array}\right]$$

$R_3/3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3/2 & 3/4 \\ 0 & 0 & 1 & 1/6 \end{array}\right] \begin{array}{l} -a \\ -b \\ -c \end{array}$$

$$f(n+1) = f(n) + (n+1)^2$$

$$\frac{(n+1)(2(n+1)+1)(n+1+1)}{6} = \frac{n(2n+1)(n+1)}{6} \quad (n+1)^2$$

$$\frac{(n+1)(2n+3)(n+2)}{6} = \frac{1}{3}n^3 + \cancel{3}\frac{1n^2}{2} + \frac{1}{6}n + (n+1)^2$$

$$= \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n + n^2 + 2n + 1^2$$

$$\frac{2n(n^2+3n+2) + 3(n^2+3n+2)}{6}$$

$$= \frac{1}{3}n^3 + \frac{3}{2}n^2 + \frac{13}{2}n + 1$$

$$= \frac{2n^3 + 6n^2 + 4n + 3n^2 + 9n + 6}{6}$$

$$= \frac{2n^3 + 9n^2 + 13n + 6}{6}$$

$$\boxed{= \frac{n^3}{3} + \frac{3n^2}{2} + \frac{13n}{6} + 1}$$

$\therefore f(n)$ is correct

equals

Remember
n is limit
sum_square_dif
for $n = 100$
etc.

$\therefore$ $\boxed{\frac{n}{6}(2n+1)(n+1)}$ is used to get sum of square
ie $1^2 + 2^2 \ldots 100^2$

$\therefore f(n) = an^3 + bn^2 + cn + d$ $(2n+1)(n+1)$

$= \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$

$$\begin{array}{r|l} 2 & 6, 2, 3 \\ 3 & 3, 1, 3 \\ & 1, 1, 1 \end{array}$$

$= \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$

LCM $= 6$

$= \frac{2n^3 + 3n^2 + n}{6}$

$= \frac{n(2n^2+3n+1)}{6} = \frac{n(2n^2+3n+1)}{6}$

$= \frac{n}{6}(2n+1)(n+1)$

Proff by induction

show $f(n+1)$ works

ex let $n = 5$

$f(5) = 1^2 + 2^2 + 3^2 + 4^2 + 5^2$

if $n = 6 = \underset{5}{n} + 1$

$f(6) = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + \underset{(n+1)^2}{6^2}$

ie $n+1$

$\therefore f(n+1) = f(n) + (n+1)^2$