b) Define perceptron learning rule

Ans In perceptron network, the learning rule is difference b/w the desired & actual response of a neuron

Learning rule explained as follows:

- Consider a finite 'n' number of i/p training vectors, $x(n)$ with their associated target values $t(n)$, where n ranges from 1 to n
- The target is either +1 or -1
- The o/p y is obtained on the basis of net i/p calculated & activation function being applied over net i/p.

$$y = f(y_{in}) = \begin{cases} 1 & \text{if } y_{in} \geq \theta \\ 0 & \text{if } -\theta \leq y_{in} \leq \theta \\ -1 & \text{if } y_{in} < -\theta \end{cases}$$

weight updation in case of perceptron learning is as shown

if $y \neq t$ then:

$$w_i(new) = w_i(old) + \alpha t x_i$$

$$b(new) = b(old) + \alpha t$$

where, t = target value (+1 or -1)

α = learning rate

else:

$$w(new) = w(old)$$

→ for the second i/p pattern, $x_1=1$, $x_2=-1$ & $t=-1$ with weight & bias $w_1=1$, $w_2=1$ & $b=-1$

net i/p : $y_{in} = b + x_1 w_1 + x_2 w_2 = -1 + 1\times 1 + 1\times -1 = -1$

then o/p y over activation function : $y = f(y_{in}) = \begin{cases} 1 & \text{if } y_{in} > 0 \\ 0 & \text{if } y_{in} = 0 \\ -1 & \text{if } y_{in} < 0 \end{cases}$

here $y_{in} = -1$, $\therefore \underline{y = -1}$

then $t=-1$ & $y=-1$, $\therefore$ no weight updation takes place

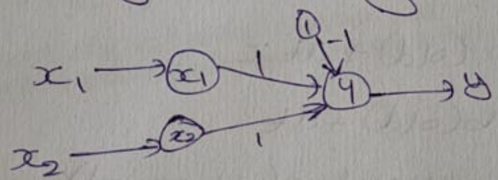
→ for third i/p : $x_1=-1$, $x_2=1$ & $t=-1$ with $w_1=1$, $w_2=1$ & $b=-1$

net i/p : $y_{in} = b + x_1 w_1 + x_2 w_2 = -1 + -1\times 1 + 1\times 1 = -1$

then o/p y over activation function : $y = f(y_{in}) = \begin{cases} 1 & \text{if } y_{in} > 0 \\ 0 & \text{if } y_{in} = 0 \\ -1 & \text{if } y_{in} < 0 \end{cases}$

here $y_{in} = -1$, $\therefore y = -1$.

so $t=-1$ & $y=-1$, ie $t=y$ $\therefore$ no weight updation takes place



Input			target (t)	Net i/p (y_{in})	Calculated o/p (y)	weight changes			weight		
x_1	x_2	1				Δw_1	Δw_2	Δb	w_1	w_2	b
Epoch-1											
1	1	1	1	0	0	1	1	1	1	1	1
1	-1	1	-1	1	1	-1	1	-1	0	2	0
-1	1	1	-1	2	1	+1	-1	-1	1	1	-1
-1	-1	1	-1	-3	-1	0	0	0	1	1	-1
Epoch-2											
1	1	1	1	1	1	0	0	0	1	1	-1
1	-1	1	-1	-1	-1	0	0	0	1	1	-1
-1	1	1	-1	-1	-1	0	0	0	1	1	-1
-1	-1	1	-1	-3	-1	0	0	0	1	1	-1.

So final weights: $w_1=0$, $w_2=2$, $b=0$

→ for third i/p pattern: $x_1=-1$, $x_2=1$, $t=-1$ with $w_1=0$, $w_2=2$, $b=0$

then net i/p: $y_{in}=b+x_1w_1+x_2w_2=0+-1\times0+1\times2=2$

∴ o/p y by applying activation function over net i/p:

$$y=f(y_{in})=\begin{cases}1 & \text{if } y_{in}>0\\ 0 & \text{if } y_{in}=0\\ -1 & \text{if } y_{in}<0\end{cases}$$ here $y_{in}=2$ ∴ $y=1$

here $t=-1$, $y=1$ ∴ $y\neq t$ so weight updation takes place:

$w_1(new)=0+1\times-1\times-1=1$

$w_2(new)=2+1\times-1\times1=1$

$b(new)=0+1\times-1=-1$

∴ Final weights & bias are $w_1=1$, $w_2=1$, $b=-1$

→ for fourth i/p: $x_1=1$, $x_2=1$, and $t=-1$ with $w_1=1$, $w_2=1$ & $b=-1$

here i/p: $y_{in}=b+x_1w_1+x_2w_2=-1+-1\times1+-1\times1=-3$

then o/p y ~~is~~ over applying activation function.

$$y=f(y_{in})=\begin{cases}1 & \text{if } y_{in}>0\\ 0 & \text{if } y_{in}=0\\ -1 & \text{if } y_{in}<0\end{cases}$$ here $y_{in}=-3$ ∴ $y=-1$

here $t=y$, hence no weight updation takes place.

Epoch 2

for the first i/p $x_1=1$, $x_2=1$ & $t=1$ with $w_1=1$, $w_2=1$ & $b=1$

net i/p: $y_{in}=b+x_1w_1+x_2w_2=-1+1\times1+1\times1=1$

∴ o/p y after applying activation function.

$$y=f(y_{in})=\begin{cases}1 & \text{if } y_{in}>0\\ 0 & \text{if } y_{in}=0\\ -1 & \text{if } y_{in}<0\end{cases}$$ ∴ $y=1$

here $t=1$, $y=1$ ∴ $t=y$ so no weight updation takes place.

Q) Implement the AND logic function using the perception network algorithm for bipolar inputs and targets.

Ans: Assume $w_1 = w_2 = b = 0$ & $\theta = 0$
then $\alpha = 1$

x_1	x_2	t
1	1	1
1	-1	-1
-1	1	-1
-1	-1	-1

Epoch 1

→ For $x_1 = 1, x_2 = 1$ & $t = 1$ with $w_1 = 0, w_2 = 0$ & $b = 0$

i/p: $y_{in} = b + x_1 w_1 + x_2 w_2$
$= 0 + 1 \times 0 + 1 \times 0 = 0$

o/p: $y = f(y_{in}) = \begin{cases} 1 & \text{if } y_{in} > 0 \\ 0 & \text{if } y_{in} = 0 \quad [\text{where } \theta = 0] \\ -1 & \text{if } y_{in} < 0 \end{cases}$

Here $y_{in} = 0$, $\therefore f(y_{in}) = y = 0$.

here, $t \neq y$, hence updation takes place

$w_i(new) = w_i(old) + \alpha t x_i$
$b_i(new) = b_i(old) + \alpha t$

then $w_1(new) = 0 + 1 \times 1 \times 1 = 1$
$w_2(new) = 0 + 1 \times 1 \times 1 = 1$
$b(new) = 0 + 1 \times 1 = 1$

so final weight after first i/p is presented are
$w_1 = 1, w_2 = 1, b = 1$

→ For the second i/p pattern, $x_1 = 1, x_2 = -1$, & $t = -1$ with $w_1 = 1, w_2 = 1$, & $b = 1$

then, net i/p: $y_{in} = b + x_1 w_1 + x_2 w_2$
$= 1 + 1 \times 1 + -1 \times 1 = 1$

then o/p by applying activation: $y = f(y_{in}) = \begin{cases} 1 & \text{if } y_{in} > 0 \\ 0 & \text{if } y_{in} = 0 \\ -1 & \text{if } y_{in} < 0 \end{cases}$ here $y_{in} = 1$ $\therefore y = 1$

here $t = -1$ & $y = 1$, $\therefore t \neq y$ so weight updation takes place.

$\therefore w_1(new) = 1 + 1 \times -1 \times 1 = 0$
$w_2(new) = 1 + 1 \times -1 \times -1 = 2$
$b(new) = 1 + 1 \times -1 = 0$