Modeling Sustainable Harvesting of Atlantic Cod

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Resource growth and yield

Initial state and parameters

 # Initial population size
N0 = 10,000 cod individuals per nm^2

 # Carrying capacity
K = N0 = 10,000 cod individuals per nm^2

 # Maximum intrinsic growth rate
r_max = 0.75 per year (up by 75% per year)

 # Effort or mortality rate
E = 0.25 fraction of N0 per year

 # Yield
Y = E × N0 = 2500 cod individuals per year

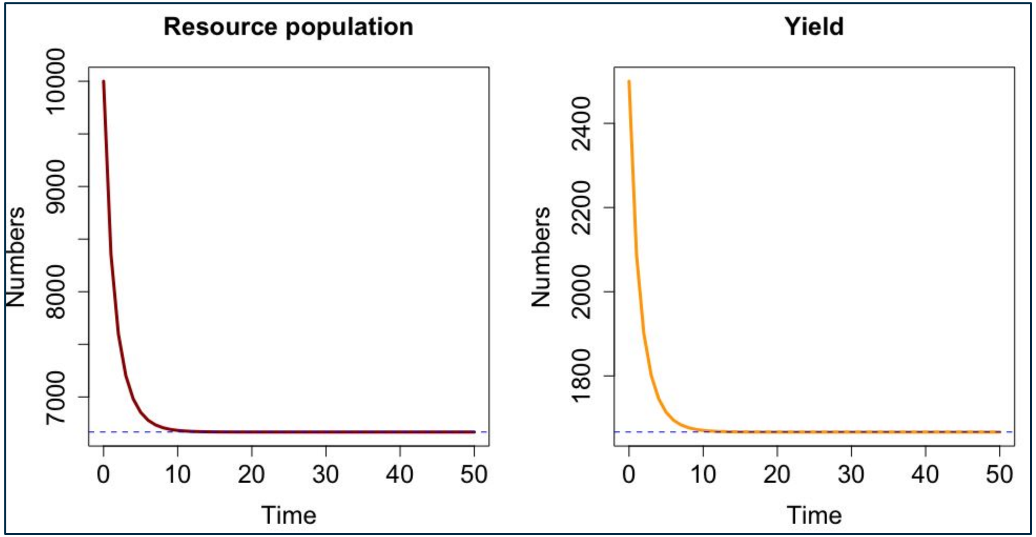
Model

$$\frac{dN}{dt} = r_{max} \times (1 - \frac{N}{K}) \times N - E \times N$$

$$= 0.75 \times (1 - \frac{10000}{10000}) \times 10000 - 0.25 \times 10000$$

$$= -2500 \textit{ resource units per year}$$

A decrease of 25% per year in resource population, despite a high per capita growth rate as the population was already at its carrying capacity



$$N_{equilibrium} = K \times [1 - (\frac{E}{r\_max})]$$

$$\Rightarrow N_{equilibrium} = 10000 \times [1 - (\frac{0.25}{0.75})] = 6666.667 \textit{ resource units}$$

$$Y_{equilibrium} = E \times N_{equilibrium}$$

$$\Rightarrow Y_{equilibrium} = 0.25 \times 6666.667 = 1666.667 \textit{ resource units per year}$$

Resource growth, yield and agents

Agent initial state and parameters

 # Agents per square kilometer of shore
A_0 = 10 agents per km^2

 # Clearance rate per agent per year
b = 0.05 fraction of resource cleared per year

 # Effort by all agents per year (mortality rate)
E_agents = b × A_0 = 0.5 per year

 # Total yield of all agents combined
Y_agents = E_agents × N_0 = 5000 cod individuals per year

 # Rate of increase of effort per year per agent
kappa = 0.25 per year (a 2.5% increase per year)

 # Price per unit of catch
Price = 50 DKK per unit of cod (one cod, arbitrary)

 # Cost of exploitation per agent per year
Cost = 3000 DKK (arbitrary)

 # Revenue per year per agent
Revenue = Price × b × N_0 = 25000 DKK per year

 # Profit per year per agent
Profit = Revenue — Cost = 22000 DKK per year

Model

$$\frac{dA}{dt} = kappa \times \left(\frac{profit}{cost}\right) = kappa \times \left(\frac{price \times b \times N}{cost} - 1\right)$$

$$\Rightarrow \frac{dA}{dt} = 0.25 \times \left(\frac{22000}{3000}\right)$$

$$\Rightarrow \frac{dA}{dt} = 1.8333\ agents\ per\ sq\ km\ per\ year$$

An increase of 18.33% per year in agent abundance

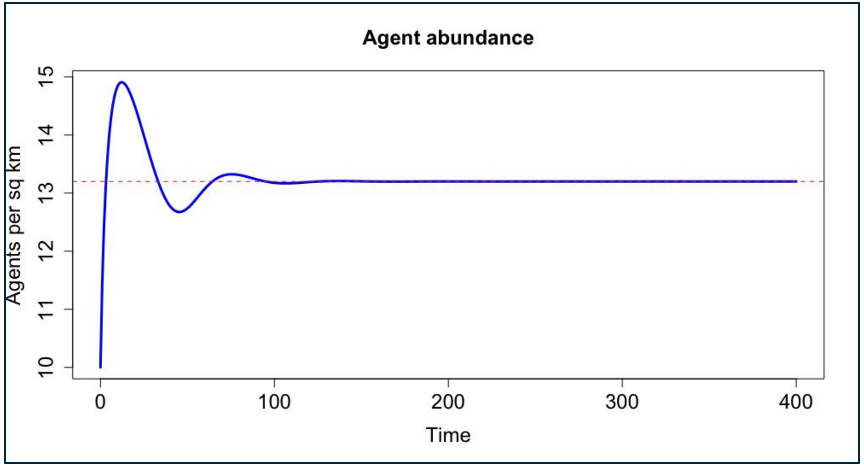
$$\frac{dN}{dt} = r_{max} \times \left(1 - \frac{N}{K}\right) \times N - b \times A \times N$$

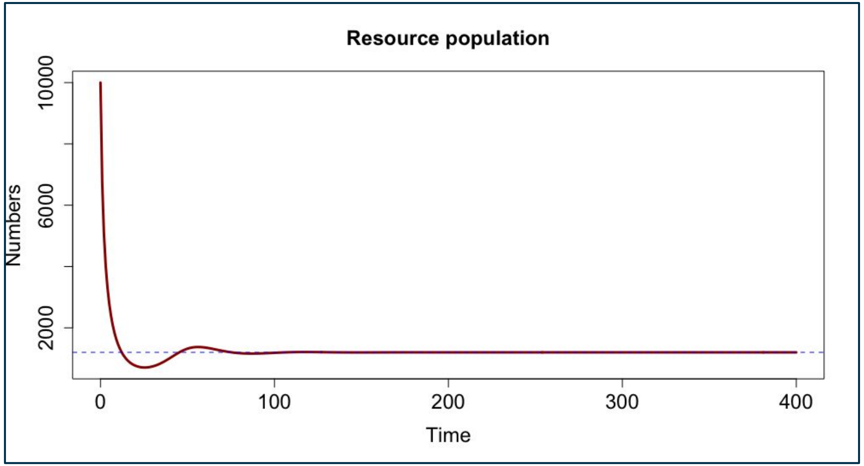
$$\Rightarrow \frac{dN}{dt} = 0.75 \times \left(1 - \frac{10000}{10000}\right) \times 10000 - 0.05 \times 10 \times 10000$$

$$\Rightarrow \frac{dN}{dt} = -5000\ resource\ units\ per\ year$$

A decrease of 50% per year in resource population

Resource growth, yield and agents: equilibrium





$$Agents_{equilibrium} = \frac{r_{max}}{b} \times \left(1 - \frac{cost}{price \times b \times K}\right)$$

$$\Rightarrow Agents_{equilibrium} = \frac{0.75}{0.05} \times \left(1 - \frac{3000}{50 \times 0.05 \times 10000}\right) = 13.2\ agents\ per\ sq\ km\ per\ year$$

$$N_{equilibrium} = K \times \left(1 - \frac{b \times A}{r_{max}}\right) = \frac{cost}{price \times b}$$

$$\Rightarrow N_{equilibrium} = 10000 \times \left(1 - \frac{0.05 \times 10}{0.75}\right) = \frac{3000}{50 \times 0.05} = 1200\ resource\ units$$

$$E_{equilibrium} = r_{max} \times \left(1 - \frac{N_{equilibrium}}{K}\right)$$

$$\Rightarrow E_{equilibrium} = 0.75 \times \left(1 - \frac{1200}{10000}\right) = 0.66\ per\ year$$

$$Y_{equilibrium} = E_{equilibrium} \times N_{equilibrium}$$

$$\Rightarrow Y_{equilibrium} = 0.66 \times 1200 = 792\ resource\ units\ per\ year$$

$$Revenue_{equilibrium} = price \times b \times N_{equilibrium}$$

$$\Rightarrow Revenue_{equilibrium} = 50 \times 0.05 \times 1200 = 3000\ DKK\ per\ year$$

Maximum sustainable yield vs equilibrium

$$Y_{msy} = \frac{r_{max} \times K}{4} = \frac{0.75 \times 10000}{4} = 1875\ resource\ units$$

$$E_{msy} = \frac{r_{max}}{2} = \frac{0.75}{2} = 0.375\ per\ year$$

$$N_{msy} = \frac{Y_{msy}}{E_{msy}} = \frac{K}{2} = 5000\ resource\ units$$

$$Y_{equilibrium} = E_{equilibrium} \times N_{equilibrium}$$

$$\Rightarrow Y_{equilibrium} = 0.66 \times 1200 = 792\ resource\ units\ per\ year$$

$$E_{equilibrium} = r_{max} \times (1 - \frac{N_{equilibrium}}{K})$$

$$\Rightarrow E_{equilibrium} = 0.75 \times (1 - \frac{1200}{10000}) = 0.66\ per\ year$$

$$N_{equilibrium} = K \times (1 - \frac{b \times A}{r_{max}}) = \frac{cost}{price \times b}$$

$$\Rightarrow N_{equilibrium} = 10000 \times (1 - \frac{0.05 \times 10}{0.75}) = \frac{3000}{50 \times 0.05} = 1200\ resource\ units$$

Here, the equilibrium values represent a simplified model where resource reproduction and harvesting are in balance, without considering other ecological factors. In contrast, MSY values focus on maximizing resource extraction but may lead to higher yields, potentially at the expense of ecological health.

Cost, price and rate of increase of effort

Changes in the cost, price and effort increase rate can be explored through this web app:

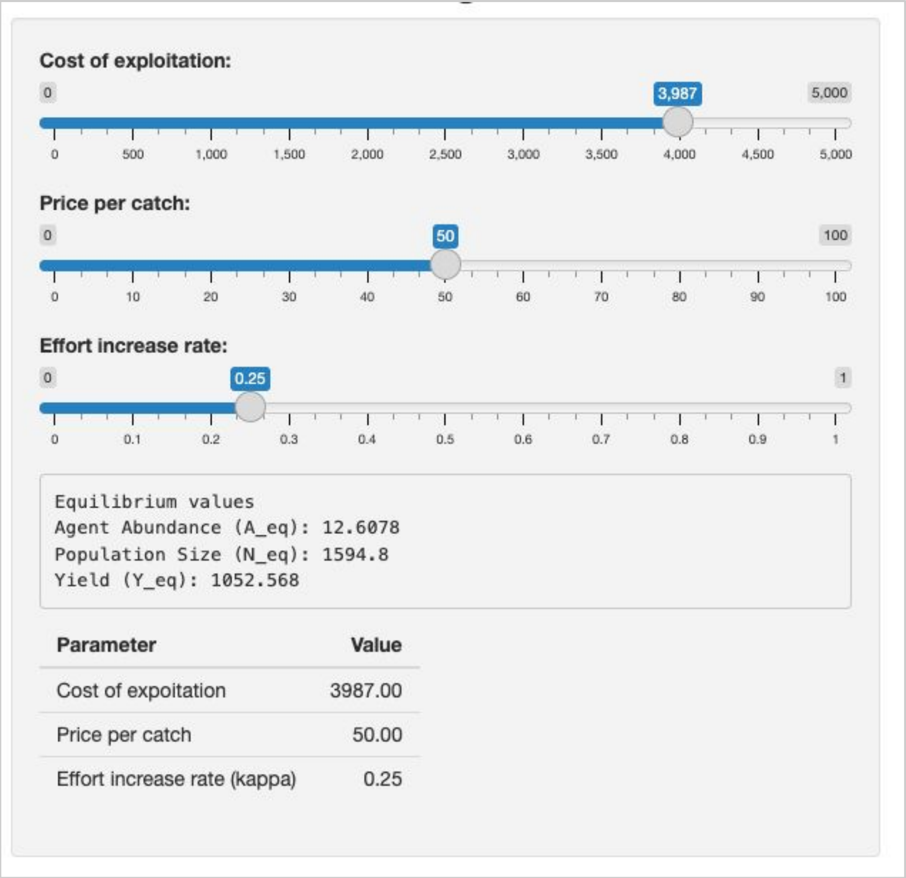
Atlantic Cod Harvesting Model

Link: https://aswatib.shinyapps.io/Harvest/

Increasing the Cost of exploitation leads to increase in equilibrium values for Population size and Yield, while causing a decrease in Agent abundance. With increasing exploitation costs, revenue levels go down and that directly impacts agent abundance with time.

Increasing the Price per catch leads to an increase in Agent abundance, while leading to a decrease in equilibrium levels for Population size and Yield.

Building up the rate of increase in effort causes equilibrium values to just arrive a bit late in the time, without causing any changes in the values as such.



Mathematical Biology

Weekly Exercise 5

Swati Tak

2023-10-03

```r
# Species of interest: Atlantic cod in inshore Greenland waters
library(knitr)
library(deSolve)

# Initial state in inshore waters, one of the fjords for example
N0 <- 10000 # 10,000 cod individuals per square nautical miles of area

# Carrying capacity inshore waters
K <- N0 # Keeping the natural carrying capacity same as the population size

# Initial state and parameters according to logistic growth
r_max <- 0.75 # arbitrarily chosen linear per capita growth, unit is per year
E <- 0.25  # keeping a constant effort or mortality rate, unit is per year (effort expended)
Y0 = E*N0 # harvesting yield, unit = fraction of population per year

# ------------------------------------------------------------------------------
# ------------------------------------------------------------------------------
# LOGISTIC GROWTH OF POPULATION AND YIELD

# Defining the model with logistic growth
harvest_model <- function(time, state, parameters) {
  with(as.list(c(state, parameters)), {
   growth_term = r_max*(1-(N/K))*N
   yield_term = E*N
   dN = growth_term - yield_term
   list(c(dN))
   })
}

initial_state <- c(N=N0)
parameters <- c(E=E, K=K, r_max=r_max)
times <- seq(0, 50, by=1)

# Applying the model with a constant effort rate or mortality rate
harvest_sim <- ode(y=initial_state, parms=parameters, func=harvest_model, times=times)

# Adding yield change with time as Y = E*N
harvest_sim <- as.data.frame(harvest_sim)
harvest_sim$Y <- harvest_sim$N * E
```

```r
# At equilibrium
N_eq = K * (1 - (E / r_max)) # Equilibrium population size
Y_eq = E * N_eq # Equilibrium yield

# Comparing analytical and simulated equilibrium values for resource population and yield
x <- data.frame(N_eq=N_eq, N_tail=tail(harvest_sim$N, 1), Y_eq=Y_eq, Y_tail=tail(harvest_sim$Y, 1))
kable(x)
```

N_eq	N_tail	Y_eq	Y_tail
6666.667	6666.667	1666.667	1666.667

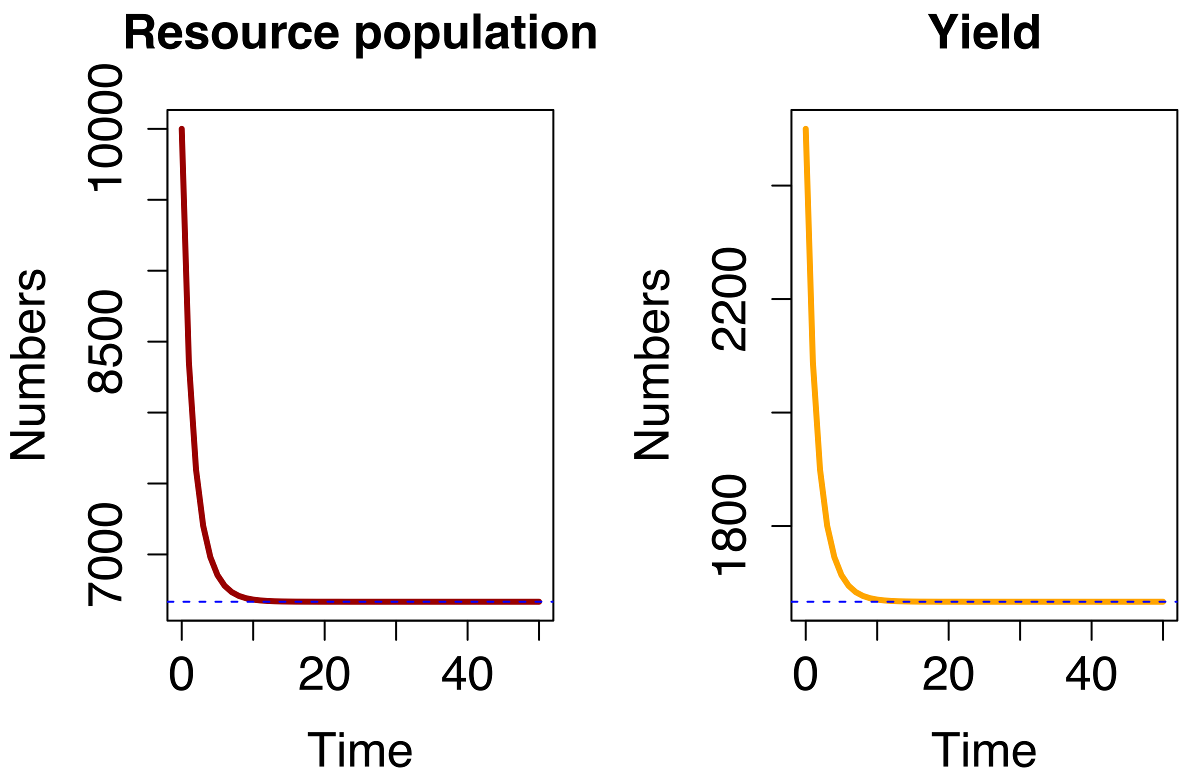
```r
# Plotting the results with equilibrium population size
par(mfrow=c(1,2))

# Plot resource population
plot(harvest_sim[,1], harvest_sim[,2], type="l", xlab="Time", ylab="Numbers",
     main="Resource population", lwd=3, cex.main=1.5,
     cex.lab=1.5, cex.axis=1.5, col="#990000")

abline(h=N_eq, col=c("blue"), lty=2)

# Plot yield
plot(harvest_sim[,1], harvest_sim[,3], type="l", xlab="Time", ylab="Numbers",
     main="Yield", lwd=3, cex.main=1.5, cex.lab=1.5, cex.axis=1.5, col="orange")

abline(h=Y_eq, col=c("blue"), lty=2)
```




```
graphics.off()
# ---------------------------------------------------------------------------------
# ---------------------------------------------------------------------------------

# LOGISTIC GROWTH, AGENT DYNAMICS, AND RETAIL PRICE AND REVENUE DYNAMICS

# Agents (fisheries) around inshore waters
A0 = 10 # 10 agents per square km of shore

# Clearance rate per agent
b <- 0.05 # Suppose 0.05 fraction of cod population is caught per year per agent

# Effort by all agents per year
E_agents <- b*A0 # mortality rate per year based on catch by all agents

# Total yield by all agents or catch per year
# Number of cod removed from the resource per square nautical miles
# of inshore waters per year
Y_agents <- E_agents*N0

# Cost of exploitation per agent per unit of time
cost <- 3000 # Danish Krone (DKK) per year

# Price per unit of yield per agents or per catch
price <- 50 # DKK
```

```r
# Revenue per unit of catch per year
revenue = price*b*N0

# Profit per agent per year
profit <- revenue - cost

# Rate of increase of effort per year per agent
kappa <- 0.25 # an increase of 2.5% per year

# QUESTION 6: Solving numerically for equilibrium

# Equilibrium solution for agent abundance
A_eq <- (r_max/b)*(1-(cost/(price*b*K)))

# Equilibrium calculations for logistic growth with agent dynamics
N_eq <- K*(1-((b*A_eq)/r_max))

# Equilibrium calculations for agent dynamics
Effort_eq <- b*A_eq

# Yield at equilibrium
Y_eq <- Effort_eq * N_eq

# Revenue at equilibrium
Revenue_eq = price * b * N_eq

# Defining the model with logistic growth and agent dynamics
harvest_model2 <- function(time, state, parameters) {
  with(as.list(c(state, parameters)), {
    growth_term = r_max * (1 - (N / K)) * N
    yield_term = b * A * N
    revenue = price * b * N
    profit = revenue - cost
    dN = growth_term - yield_term
    dA = kappa * (profit / cost)
    list(c(dN, dA))
  })
}

# Setting initial state and parameters for the model
initial_state <- c(N=N0, A=A0)
parameters <- c(K=K, r_max=r_max, b=b, price=price, cost=cost, kappa=kappa)
times <- seq(0, 400, by=1)

# Applying the model
harvest_sim2 <- ode(y=initial_state, parms=parameters, func=harvest_model2, times=times)
tail(harvest_sim2)
```

```
##        time    N    A
## [396,]  395 1200 13.2
## [397,]  396 1200 13.2
## [398,]  397 1200 13.2
## [399,]  398 1200 13.2
```

```
## [400,]  399 1200 13.2
## [401,]  400 1200 13.2
```

```r
head(harvest_sim2)
```

```
##      time         N        A
## [1,]    0 10000.000 10.00000
## [2,]    1  6712.025 11.44385
## [3,]    2  5056.330 12.40396
## [4,]    3  4031.611 13.09329
## [5,]    4  3325.632 13.60558
## [6,]    5  2807.270 13.99186
```

```r
# Wrangling the results to include other parameters and economic
harvest_sim2 <- as.data.frame(harvest_sim2)
harvest_sim2$Effort <- b * harvest_sim2$A
harvest_sim2$Yield <- b * harvest_sim2$A * harvest_sim2$N
harvest_sim2$Revenue <- price * b * harvest_sim2$N

# Plot the results
plot(harvest_sim2[,1], harvest_sim2[,2], type="l", xlab="Time", ylab="Numbers",
     main="Resource population", lwd=3,
     cex.main=1.5,
     cex.lab=1.5,
     cex.axis=1.5,
     col="#990000")

abline(h=N_eq, col=c("blue"), lty=2)

plot(harvest_sim2[,1], harvest_sim2[,5], type="l", ylab="Numbers",
     main="Yield", xlab="Time", lwd=3, cex.main=1.5, cex.lab=1.5,
     cex.axis=1.5, col="orange")

abline(h=Y_eq, col=c("blue"), lty=2)

plot(harvest_sim2[,1], harvest_sim2[,3], type="l", ylab="Agents per sq km",
     xlab="Time", main="Agent abundance", lwd=3, cex.main=1.5, cex.lab=1.5,
     cex.axis=1.5, col="blue")

abline(h=A_eq, col=c("red"), lty=2)

# Maximum s ustainable yield (MSY), effort and population
Y_msy = (r_max*K)/4
E_msy = (r_max)/2
N_msy = K/2

# Comparison of numerical solutions with equilibrium values
comparison_df <- data.frame(
  N_tail = tail(harvest_sim2$N, 1),
  N_eq = N_eq,
  N_msy = K/2,
  Effort_tail = tail(harvest_sim2$Effort, 1),
  Effort_eq = Effort_eq,
```

```r
  E_msy = (r_max)/2,
  A_tail = tail(harvest_sim2$A, 1),
  A_eq = A_eq,
  Yield_tail = tail(harvest_sim2$Yield, 1),
  Yield_eq = Y_eq,
  Y_msy = (r_max*K)/4,
  Revenue_tail = tail(harvest_sim2$Revenue, 1),
  Revenue_eq = Revenue_eq
)

kable(t(comparison_df))
```

N_tail	1200.000
N_eq	1200.000
N_msy	5000.000
Effort_tail	0.660
Effort_eq	0.660
E_msy	0.375
A_tail	13.200
A_eq	13.200
Yield_tail	792.000
Yield_eq	792.000
Y_msy	1875.000
Revenue_tail	3000.000
Revenue_eq	3000.000

```r
list(comparison_df)
```

```
## [[1]]
##   N_tail N_eq N_msy Effort_tail Effort_eq E_msy A_tail A_eq Yield_tail Yield_eq
## 1   1200 1200  5000         0.66      0.66 0.375   13.2 13.2        792      792
##   Y_msy Revenue_tail Revenue_eq
## 1  1875         3000       3000
```

```r
# install.packages("shiny")
library(shiny)
library(deSolve)


# Define the user interface
ui <- fluidPage(
  titlePanel("Atlantic Cod Harvesting Model"),

  # Sidebar layout with input and output definitions
  sidebarLayout(
    sidebarPanel(
      sliderInput("cost", "Cost of exploitation:", min = 0, max = 5000, value = 3000),
      sliderInput("price", "Price per catch:", min = 0, max = 100, value = 50),
      sliderInput("kappa", "Effort increase rate:", min = 0, max = 1, value = 0.25),
      verbatimTextOutput("solution_text"),
      tableOutput("solution_table")
```

```r
    ),
    mainPanel(
      plotOutput("population_plot"),
      plotOutput("agent_plot")
    )
  )
)

# Define the server logic
server <- function(input, output) {
  observe({
    # Update the parameters based on user input
    cost <- input$cost
    price <- input$price
    kappa <- input$kappa

    # Update the parameters in the model
    parameters <- c(K = K, r_max = r_max, b = b, price = price, cost = cost, kappa = kappa)

    # Apply the model
    harvest_sim2 <- ode(y = initial_state, parms = parameters, func = harvest_model2, times = times)

    # Wrangle the results
    harvest_sim2 <- as.data.frame(harvest_sim2)
    harvest_sim2$Effort <- b * harvest_sim2$A
    harvest_sim2$Yield <- b * harvest_sim2$A * harvest_sim2$N

    # Calculate equilibrium values
    A_eq <- (r_max/b)*(1-(cost/(price*b*K)))
    N_eq <- K*(1-((b*A_eq)/r_max))
    Y_eq <- Effort_eq*N_eq

    # Generate solutions text
    solutions_text <- paste("Equilibrium values\nAgent Abundance (A_eq):", A_eq)
    solutions_text <- paste(solutions_text, "\nPopulation Size (N_eq):", N_eq)
    solutions_text <- paste(solutions_text, "\nYield (Y_eq):", Y_eq)

    # Generate solutions data table
    solutions_table <- data.frame(
      Parameter = c("Cost of expoitation", "Price per catch", "Effort increase rate (kappa)"),
      Value = c(cost, price, kappa)
    )

    # Display solutions text
    output$solution_text <- renderText({
      solutions_text
    })

    # Display solutions data table
    output$solution_table <- renderTable({
      solutions_table
    })
```

```r
    # Plot population dynamics
    output$population_plot <- renderPlot({
      plot(harvest_sim2[,1], harvest_sim2[,2], type="l", xlab="Time", ylab="Numbers",
           main="Resource population", lwd=3, cex.main=1.5,
           cex.lab=1.5, cex.axis=1.5, col="#990000")
    })

    # Plot agent dynamics
    output$agent_plot <- renderPlot({
      plot(harvest_sim2[, 1], harvest_sim2[, 3], type = "l", xlab = "Time",
           lwd=3, ylab = "Numbers per sq km", main="Agent abundance",
           cex.main=1.5, cex.lab=1.5, cex.axis=1.5, col="blue")
    })
  })
}

# Run the Shiny app
shinyApp(ui = ui, server = server)
```

```r
# Loading app on shiny server
# install.packages('rsconnect')
# library(rsconnect)
# if(!require("devtools"))
#   install.packages("devtools")
# devtools::install_github("rstudio/rsconnect")

# install.packages("renv")
# renv::init()
# renv::snapshot()
# renv::restore()

# rsconnect::deployApp(appDir = "/Users/swati/Desktop/Mathematical Biology/Harvest-Model")
```