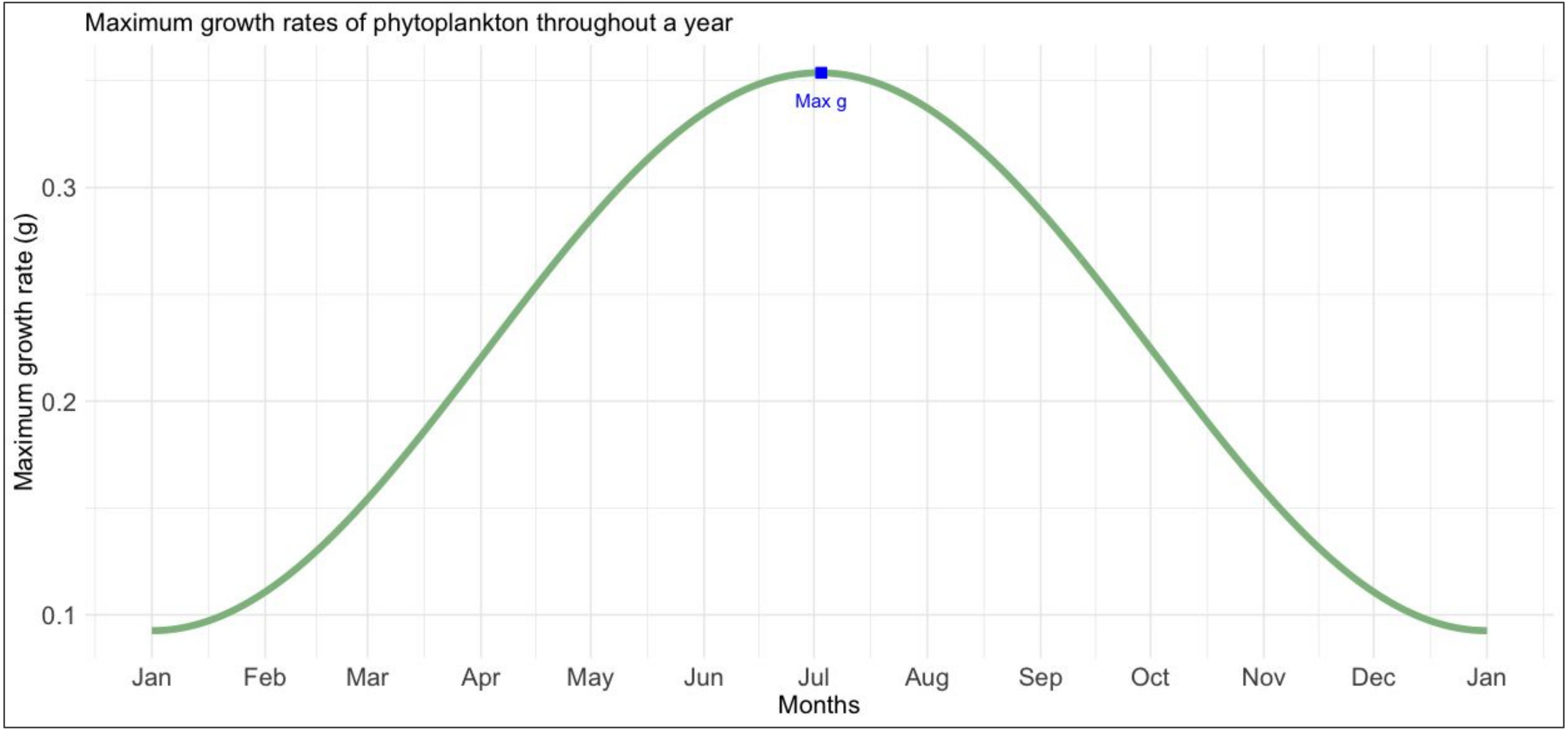
Exploring the seasonal succession of nutrients, phytoplanktons, and zooplanktons in a temperate ecosystem

Phytoplankton: Cyanobacteria (e.g., Nodularia spumigena)

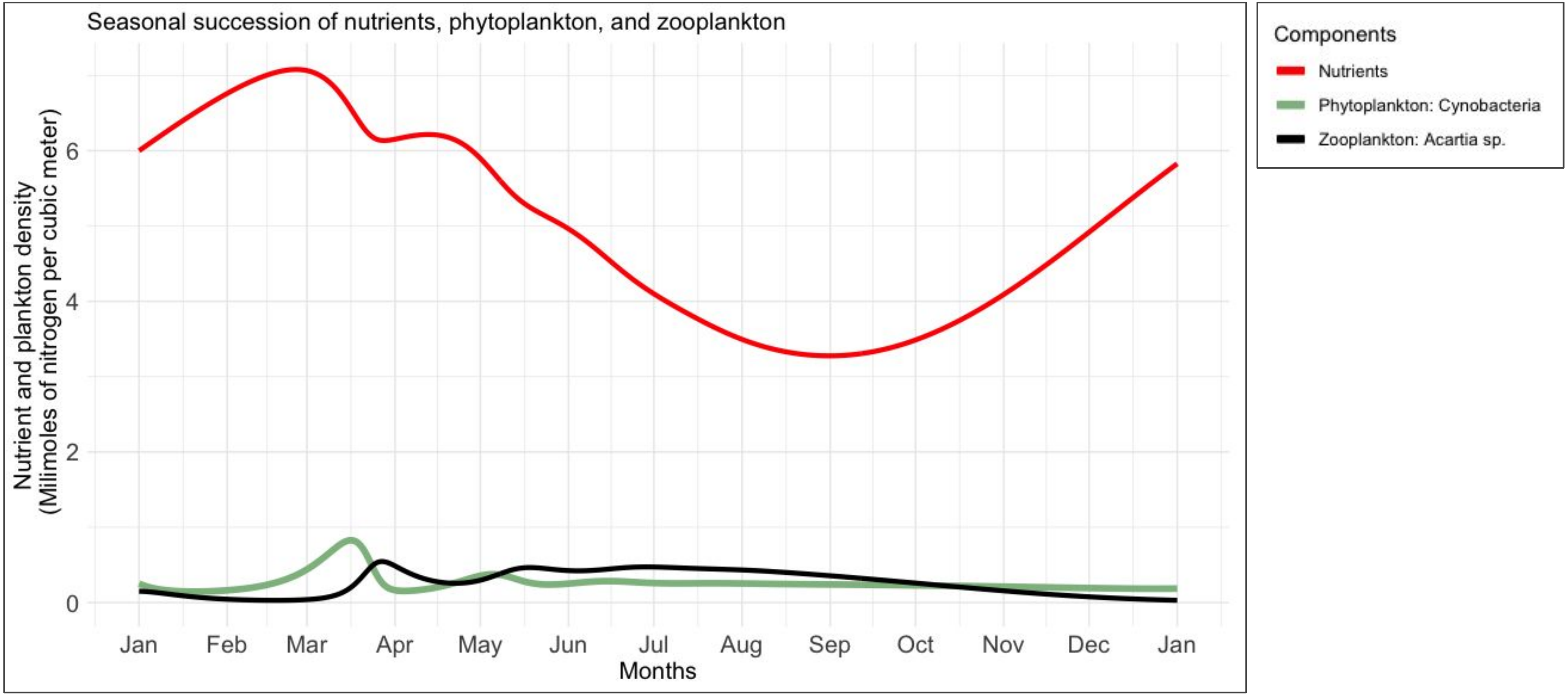
Zooplankton: Acartia sp.

Swati Tak (s220868)

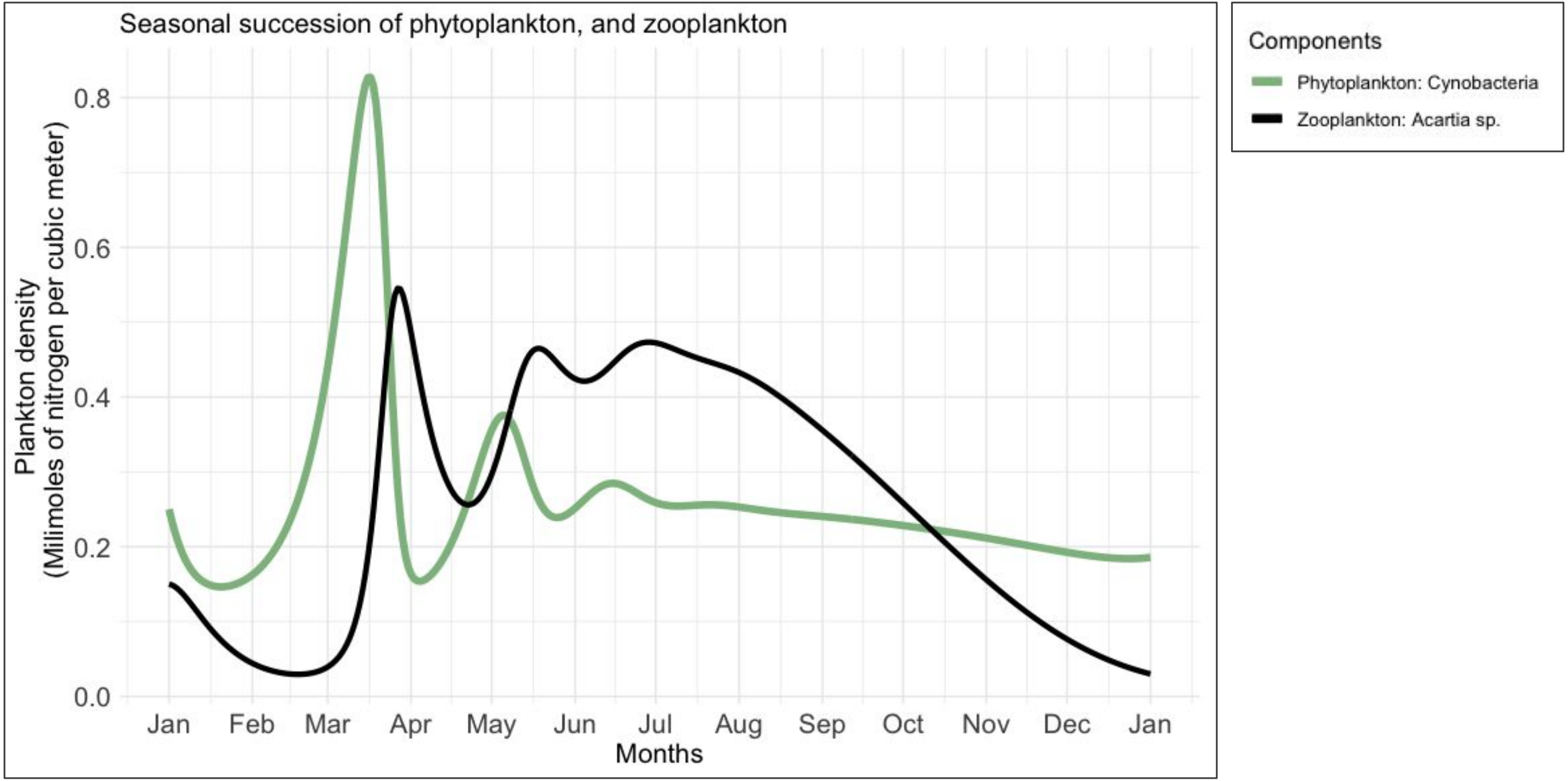
Maximum growth rate peaks at a time when daylight is usually abundant



Nutrients reach a peak in Spring, planktons follow suit in Summer



There are significant seasonal variations and patterns. And cyclic lags.



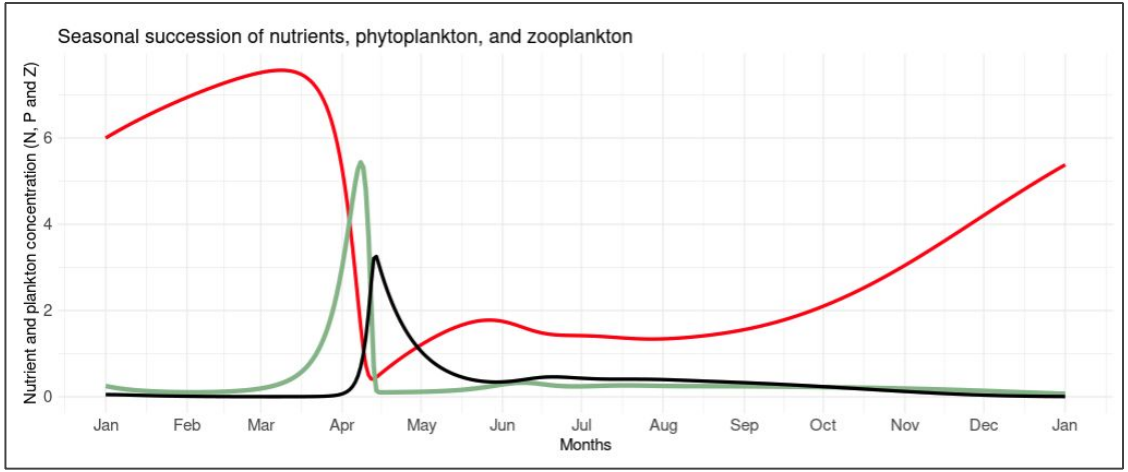
Latitude influences the prominence of seasonal succession

Web App: https://aswatib.shinyapps.io/SeasonalSuccession/

Latitude variations influence nutrient cycles and growth patterns in marine ecosystems. Clearer patterns are observed at higher latitudes.

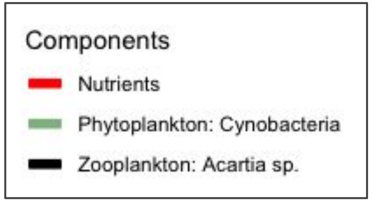
When latitude is changed from 47 to 80 degrees:

- Nutrient availability peaks in Spring (mid-March onward) – melting sea ice enriches the sea with nutrients.
- Phytoplankton numbers grow around Summer – abundant nutrients support increased growth leading to a reduction in nutrient availability thereafter.
- Zooplankton gets enough to graze on too with phytoplankton growth.



Beyond 66.5 degrees latitude:

- Seasonal succession favors phytoplankton over zooplankton
- Longer daylight hours in summers enhance phytoplankton photosynthesis
- Abundant phytoplankton growth



Mathematical Biology

Weekly Exercise 10: Seasonal Sucession in a Plankton System

Swati Tak

2023-11-13

Work environment

```
# Seasonal Succession in a plankton system -- forced dynamical system controlled by sunlight
# Location: Temperate regions (latitude 47 degrees)
# Organisms:
  # Phytoplankton: Cyanobacteria (e.g., Nodularia spumigena)
  # Zooplankton: Acartia spp.

# Set the working directory
setwd("/Users/swati/Desktop/Mathematical Biology/SeasonalSuccession")

# Load the required packages
library(deSolve)
library(ggplot2)
library(lubridate)
```

```
##
## Attaching package: 'lubridate'

## The following objects are masked from 'package:base':
##
##     date, intersect, setdiff, union
```

Initial states and parameters

```
# Initial states
N0 <-  6 # Nutrients in mixed layers -- milimoles of nitrogen per cubic meter
P0 <- 0.25 # phytoplankton -- milimoles of nitrogen per cubic meter
Z0 <- 0.15 # zooplankton -- milimoles of nitrogen per cubic meter

# Model parameters
N_deep <- 12 # Deep nutrients -- 12 milimoles of nitrogen per cubic meter
a <- 0.025
m <- 0.3 # Mixing rate or diffusion rate, per day
M <- 60 # Depth in meters
```

```r
phi <- 47 # Latitude in degrees

# Death rates
d_p <- 0.07 # Death rate of phytoplankton
d_z <- 0.07 # Death rate of zooplankton

# Grazing by Z on P
Cmax <- 1 # Maximum consumption of phytoplankton by zooplankton -- maximum grazing rate
P_low <- 0.1 # Grazing threshold, P concentration below which   grazing by Z stops
Hp <- 0.5 # Half-saturation coefficient for phytoplankton -- uptake half saturation
Hz <- 1 # Half-saturation coefficient for zooplankton -- grazing half-saturation
e <- 0.5 # Zooplankton's grazing efficiency

# Vector for parameters
parameters <- c(N_deep = N_deep, a = a, m = m, M = M, phi = phi, d_p = d_p,
                d_z = d_z, Cmax = Cmax, P_low = P_low, Hp = Hp, Hz = Hz, e = e,
                P_low = P_low)

# Vector for initial states
initial_state <- c(N=N0, P=P0, Z=Z0)

# Time span in days
times <- seq(0, 365, by=1)
```

Model

```r
# Define the model
ss_model <- function(time, state, params){
  with(as.list(c(state, params)), {

    # Time in months
    time_in_months <- time

    # Maximum growth rate of phytoplankton
    g <- exp(- a * M) * (1 - 0.8 * sin((pi * phi) / 180) * cos(2 * pi * (time / 365)))

    # Grazing by zooplankton as function of phytoplankton density
    grazing_z <- Cmax * (P - P_low) / (Hz + P - P_low)

    # Type II functional response
    fcmax <- max(0, grazing_z)

    # Calculate time in months
    time_in_months <- as.Date("2023-01-01") + months(round(time * 12))

    # ODEs
    dNdt <- - (g * N / (Hp + N) - d_p) * P + (m / M) * (N_deep - N)
    dPdt <- (g * N / (Hp + N) - d_p) * P - Z * fcmax
    dZdt <- e * fcmax * Z - d_z * Z

    return(list(c(dNdt, dPdt, dZdt), g=g))
```

```r
  })
}

# Apply the model
out <- ode(y=initial_state, times=times, func=ss_model, parms=parameters)
```

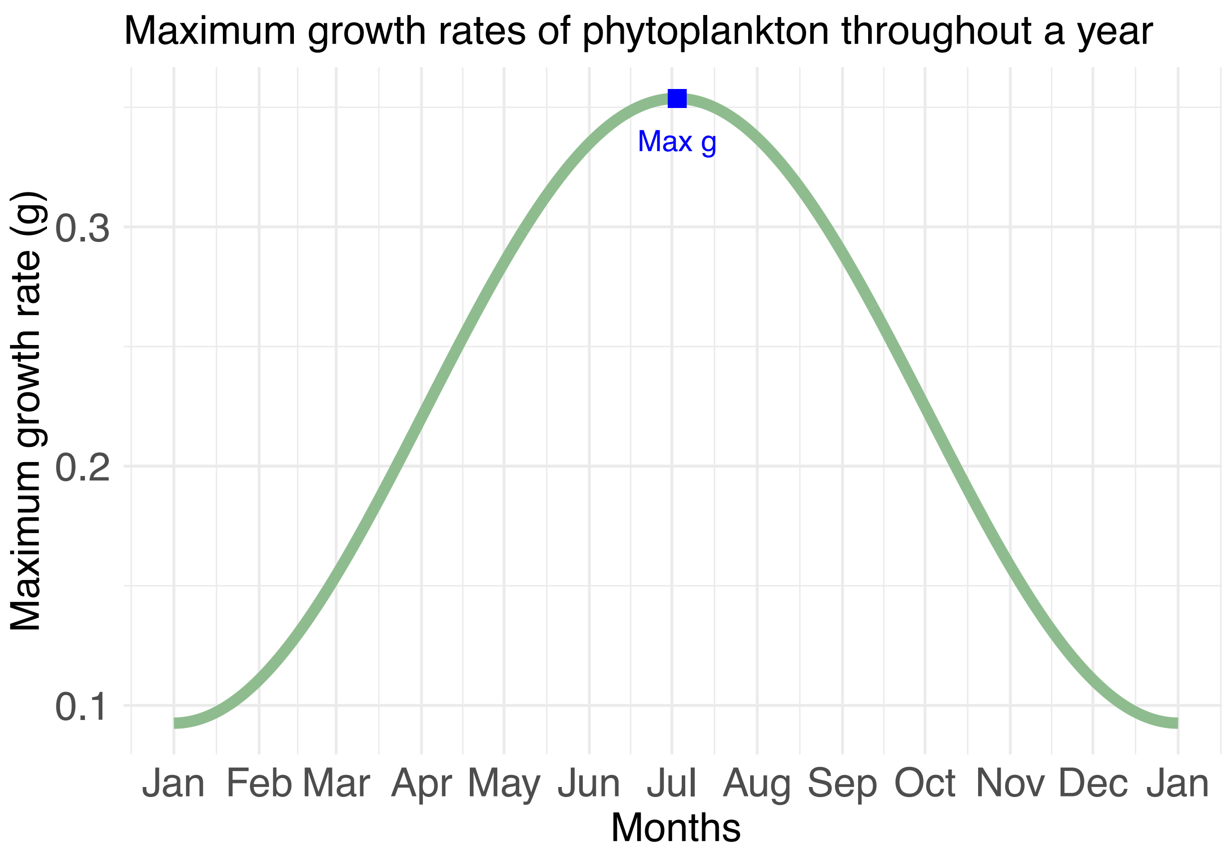
Results

```r
# Wrangle the results
out <- as.data.frame(out)
out$date <- as.Date(out$time, origin = as.Date("2023-01-01"))

# Customizing the display format
#out$date <- format(out$date, format="%b %d")
class(out$date)
```

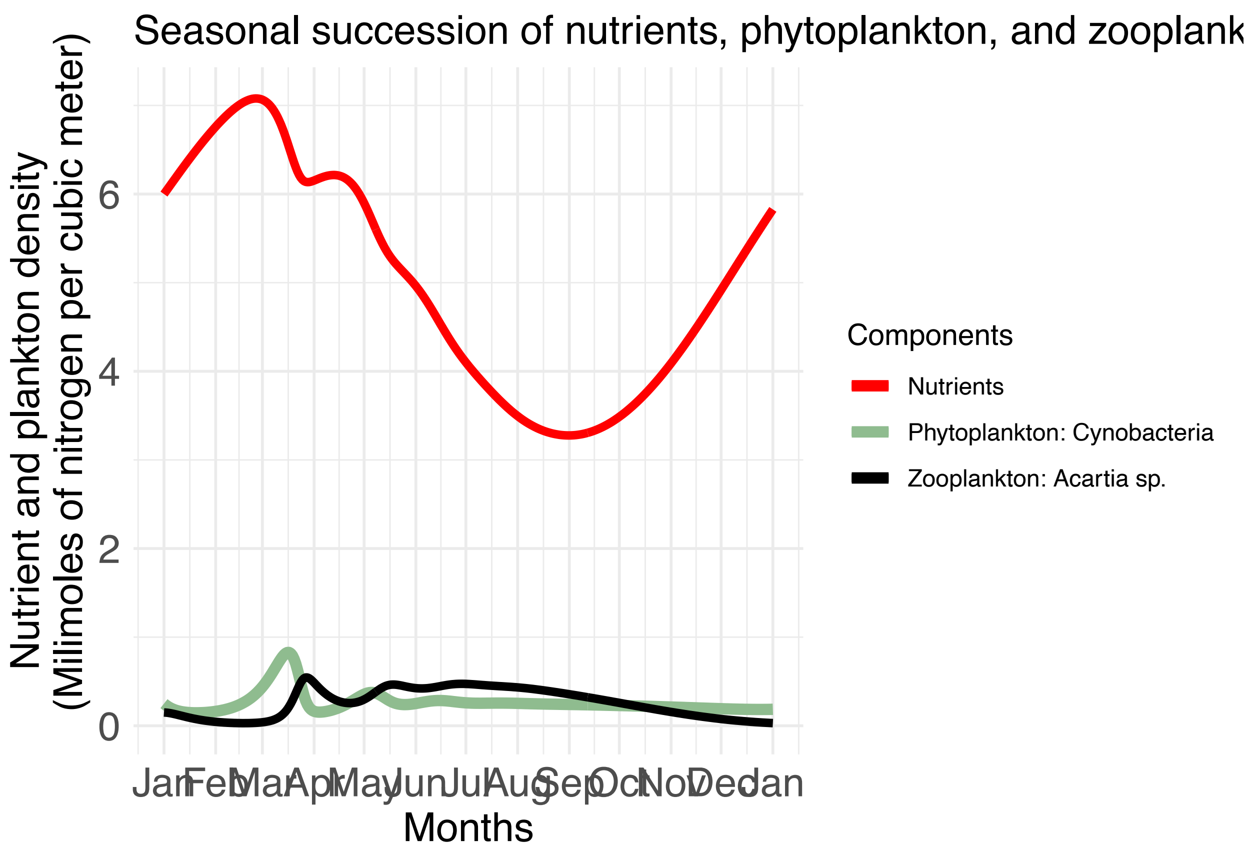
```
## [1] "Date"
```

```r
# Creating the custom theme function
mytheme <- function() {
  theme_minimal() +
    theme(
      axis.text.x = element_text(size = 15),
      axis.text.y = element_text(size = 15),
      axis.title.x = element_text(size = 15),
      axis.title.y = element_text(size = 15),
      plot.title = element_text(size = 15)
    )
}

# Finding the peak sunlight availability and nutrient availability
peak_row_g <- which.max(out$g)
peak_row_N <- which.max(out$N)
```

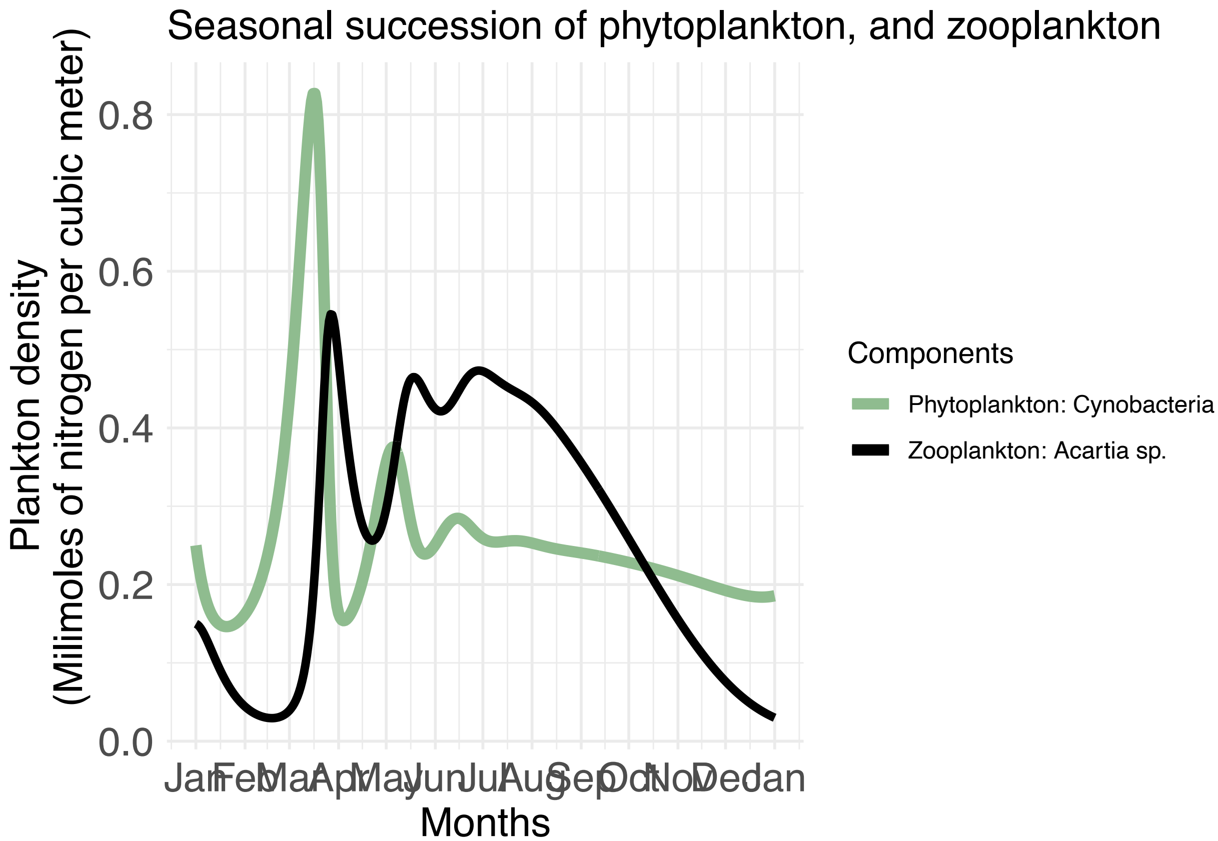
```r
# Plot  the maximum growth rate of  phytoplankton   as  a   function    of  time
ggplot(out, aes(x = date, y = g)) +
  geom_line(color = "darkseagreen", linewidth = 2) +
  geom_point(data = subset(out, time == peak_row_g), aes(x = date, y = g),
             col = "blue", pch = 15, size=3) +
  geom_text(data = subset(out, time == peak_row_g), aes(x = date, y = g,
                                                        label = "Max g"),
            vjust = 2.5, hjust = 0.5, col = "blue", size = 4) +
  scale_x_date(date_breaks = "1 month", date_labels = "%b") +
  labs(x = "Months", y = "Maximum growth rate (g)",
       title = "Maximum growth rates of phytoplankton throughout a year") +
  theme_minimal() +
  mytheme()
```




```
# Plotting monthly levels according to sunlight availability
ggplot(out, aes(x = date)) +
  geom_line(aes(y = N, color = "Nutrients"), linewidth = 1.5) +
  geom_line(aes(y = P, color = "Phytoplankton: Cynobacteria"), linewidth = 2) +
  geom_line(aes(y = Z, color = "Zooplankton: Acartia sp."), linewidth = 1.5) +
  scale_x_date(date_breaks = "1 month", date_labels = "%b") +
  scale_color_manual(name = "Components",
                     values = c("Nutrients" = "red",
                                "Phytoplankton: Cynobacteria" = "darkseagreen",
                                "Zooplankton: Acartia sp." = "black")) +
  theme(legend.text = element_text(size = 10)) +
  labs(x = "Months", y = "Nutrient and plankton density
     (Milimoles of nitrogen per cubic meter)",
     title = "Seasonal succession of nutrients, phytoplankton, and zooplankton") +
  theme_minimal() +
  mytheme()
```




```r
# Plotting monthly levels according to sunlight availability only for planktons
ggplot(out, aes(x = date)) +
  geom_line(aes(y = P, color = "Phytoplankton: Cynobacteria"), linewidth = 2) +
  geom_line(aes(y = Z, color = "Zooplankton: Acartia sp."), linewidth = 1.5) +
  scale_x_date(date_breaks = "1 month", date_labels = "%b") +
  scale_color_manual(name = "Components",
                     values = c("Phytoplankton: Cynobacteria" = "darkseagreen",
                                "Zooplankton: Acartia sp." = "black")) +
  theme(legend.text = element_text(size = 10)) +
  labs(x = "Months", y = "Plankton density
     (Milimoles of nitrogen per cubic meter)",
     title = "Seasonal succession of phytoplankton, and zooplankton") +
  theme_minimal() +
  mytheme()
```


Seasonal succession of phytoplankton, and zooplankton
Plankton density
(Milimoles of nitrogen per cubic meter)
0.8
0.6
0.4
0.2
0.0
Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec Jan
Months
Components
Phytoplankton: Cynobacteria
Zooplankton: Acartia sp.

Model equations:

$$\frac{dN}{dt} = -\left(g\frac{N}{H_p+N} - d_p\right)P - \frac{m}{M}\left(N_{deep} - N\right)$$

$$\frac{dP}{dt} = \left(g\frac{N}{H_p+N} - d_p\right)P - Z f_{cmax}$$

$$\frac{dZ}{dt} = e f_{cmax} Z - d_z Z$$

where,

$$f_{cmax} = max\left\{0,\ Cmax\frac{P-P_{low}}{H_z+P-P_{low}}\right\}$$

Equilibria:

Starting with the third equation (dZ/dt), solving it for equilibrium:

$$e f_{cmax} Z - d_z Z = 0$$

$$\Rightarrow e f_{cmax} Z = d_z Z$$

$$\Rightarrow e f_{cmax} = d_z$$

$$\Rightarrow f_{cmax} = \frac{d_z}{e}$$

Inputting the value above for f_{cmax} into the equation for dP/dt and solving it for P:

$$\left(g\frac{N}{H_p+N} - d_p\right)P - Z\frac{d_z}{e} = 0$$

$$\Rightarrow \left(g\frac{N}{H_p+N} - d_p\right)P = Z\frac{d_z}{e}$$

$$\Rightarrow \left(\frac{g\,N - d_p\,(H_p + N)}{H_p + N}\right) P \;=\; Z\,\frac{d_z}{e}$$

$$\Rightarrow P \;=\; \frac{(H_p + N)\,Z\,d_z}{e\,(g\,N - d_p\,(H_p + N))}$$

Inputting the value of P at equilibrium in dN/dt and solving it for N:

$$-\;\left(g\,\frac{N}{H_p + N} - d_p\right) P \;=\; \frac{m}{M}\,(N_{deep} \;-\; N)$$

$$\Rightarrow -\;\left(g\,\frac{N}{H_p + N} - d_p\right) \frac{(H_p + N)\,Z\,d_z}{e\,(g\,N - d_p\,(H_p + N))} \;=\; \frac{m}{M}\,(N_{deep} \;-\; N)$$

$$\Rightarrow -\,\frac{g\,N - d_p\,(H_p + N)}{(H_p + N)}\;.\;\frac{(H_p + N)\,Z\,d_z}{e\,(g\,N - d_p\,(H_p + N))} \;=\; \frac{m}{M}\,(N_{deep} \;-\; N)$$

$$\Rightarrow -\;\frac{Z\,d_z}{e} \;=\; \frac{m}{M}\,(N_{deep} \;-\; N)$$

$$\Rightarrow -\;\frac{Z\,d_z\,M}{e\,m} = N_{deep} \;-\; N$$

$$\Rightarrow N \;=\; N_{deep} \;+\; \frac{Z\,d_z\,M}{e\,m}$$

Solving the equation further for Z:

$$Z \;=\; -\;\frac{e\,m\,(N_{deep} - N)}{M\,d_z}$$

Solving further for N_{deep}:

$$N_{deep} \;=\; N \;-\; \frac{Z\,d_z\,M}{e\,m}$$