
Image source: https://www.artsculpturegallery.com/garden-decorative-bronze-bear-statue-with-eagle-fighting-fish-for-sale-bokk-292.html

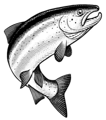
Brown Bears versus Bald Eagles

The Salmon Showdown during Spawning Migration in Southeast Alaska’s temperate rainforests

Swati Tak (s220868)

Competition model: initial states and parameters

Resource: Salmon



Initial size: R ⇒ 10,000 individuals

Daily input upstream: Input ⇒ 300 salmon per day

Rate of input: r_in ⇒ 1/300 salmon per day

Carrying capacity: K ⇒ 20,000 individuals

Consumer 1: Brown Bears

 # Initial population size:
C1 ⇒ 30 bears

 # Birth rate per day
birth_rate_1 <- (2 cubs / 2 adults) / 365 days
(if 2 cubs per year survive to adulthood)

 # Death rate per day
death_rate_1 <- 1/(20 × 365)
(bear lifespan: 20-25 years)

Consumer 2: Bald Eagles

 # Initial population size:
C2 ⇒ 30 eagles

 # Birth rate per day
birth_rate_2 <- (2 eaglets / 2 adults) / 365 days (if 2 eagle babies survive to adulthood)

 # Death rate per day
death_rate_2 <- 1/(15 × 365)
(eagle lifespan: 15-25 years)

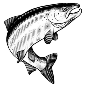
 # Maximum consumption possible for each consumer
Cmax1 <- 10 salmons per day for 1 bear
Cmax2 <- 3 salmons per day for 1 eagle

 # Consumption required for each consumer per day as an essential resource during the season
consumption_1 <- 4 (if 1 bear needs to eat 4 salmon per day)
consumption_2 <- 2 (if 1 eagle needs to eat 2 salmon per day)

 # Reproductive efficiency for each consumer species per unit of resource consumed
E1 <- birth_rate_1/(consumption_1) = 0.00068 surviving cub per unit consumption
E2 <- birth_rate_2/(consumption_2) = 0.00137 surviving eaglet per unit consumption

 # Attack rates per day (unit: resource fraction caught by a consumer)
attack_rate_1 <- (consumption_1)/input = 0.013 caught by a bear
attack_rate_2 <- (consumption_2)/input = 0.006 caught by an eagle

Competition model: calculations



Rate of change of resource: Salmon

$$\frac{dR}{dt} = [r_{in} \times (K - R)] - [Cmax1 \times \frac{attack\ rate\ 1 \times R}{Cmax1 + (attack\ rate\ 1 \times R)} \times C1] - [Cmax2 \times \frac{attack\ rate\ 2 \times R}{Cmax2 + (attack\ rate\ 2 \times R)} \times C2]$$

$$\Rightarrow \frac{dR}{dt} = [\frac{1}{400} \times (20000 - 10000)] - [10 \times \frac{0.013 \times 10^4}{10 + (0.013 \times 10^4)} \times 30] - [3 \times \frac{0.0067 \times 10^4}{3 + (0.0067 \times 10^4)} \times 30]$$

$$\Rightarrow \frac{dR}{dt} = -339.7\ salmon\ per\ day$$

A decrease of 3.397% per day in the resource

Rate of change of consumer 1: Brown Bears

$$\frac{dC1}{dt} = [E1 \times Cmax1 \times \frac{attack\ rate\ 1 \times R}{Cmax1 + (attack\ rate\ 1 \times R)} - death\ rate\ 1] \times C1$$

$$\Rightarrow \frac{dC1}{dt} = [0.00068 \times 10 \times \frac{0.013 \times 10^4}{10 + (0.013 \times 10^4)} - 0.000137] \times 30 = 0.185\ fraction\ of\ bear\ population\ per\ day$$

An increase of 0.61% per day

Rate of change of consumer 2: Bald Eagles

$$\frac{dC2}{dt} = [E2 \times Cmax2 \times \frac{attack\ rate\ 2 \times R}{Cmax2 + (attack\ rate\ 2 \times R)} - death\ rate\ 2] \times C2$$

$$\Rightarrow \frac{dC2}{dt} = [0.00137 \times 3 \times \frac{0.067 \times 10^4}{3 + (0.067 \times 10^4)} - 0.0001826] \times 30 = 0.112\ fraction\ of\ eagle\ population\ per\ day$$

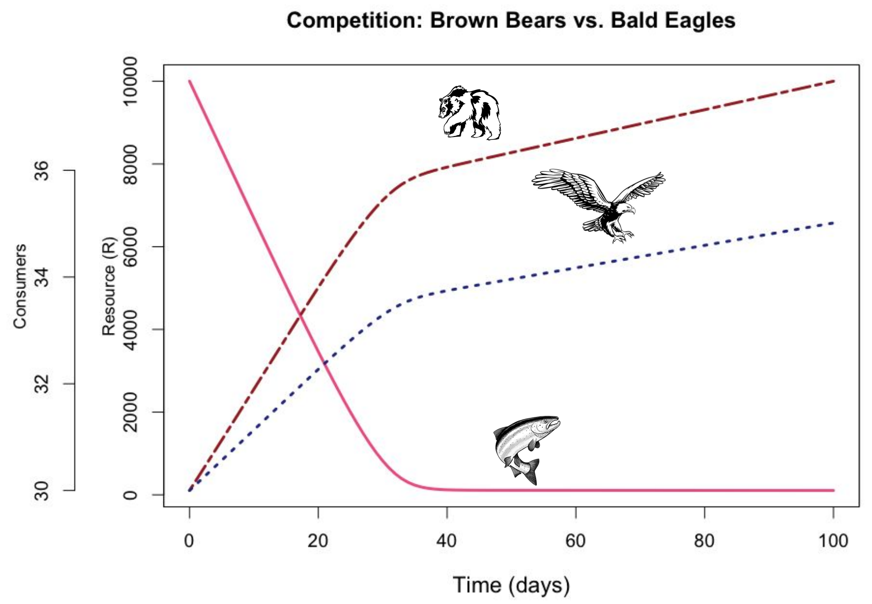
An increase of 0.37% per day

The initial growth rates for brown bears and bald eagles are quite different, and brown bears seem to be leading with an increase of 0.61% per day.

However, this does not mean that brown bears will necessarily prosper in steady state dynamics when there is very limited salmon available to consume (off season).

For that, the minimum concentration of resource for both the consumers need to be compared at steady state.

Competition model: transient and steady state



Minimum resource concentrations

R* for Brown Bears:

$$R^*_{bears} = \frac{1}{attack\ rate\ 1} \times \frac{Cmax1 \times death\ rate\ 1}{Cmax1 \times E1 - death\ rate\ 1} \simeq 15\ resource\ units$$

R* for Bald Eagles:

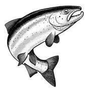
$$R^*_{eagles} = \frac{1}{attack\ rate\ 2} \times \frac{Cmax2 \times death\ rate\ 2}{Cmax2 \times E2 - death\ rate\ 2} \simeq 21\ resource\ units$$

With a lead on the plot for type II functional responses and also a lower R* value, brown bears seem to actually prosper the transient as well as steady state dynamics in this competition.

Legend
- Salmon (Resource)
- Brown Bears (Consumer 1)
- Eagles (Consumer 2)

R* → Minimum number of resource required to sustain a consumer population, that is the consumer population growth rate is zero (at equilibrium and does not change, especially negatively)

Competition model: foraging efficiency as trait for trade-off



Rate of change of resource: Salmon

$$\frac{dR}{dt} = [r_{in} \times (K - R)] - [Cmax1 \times \frac{attack\ rate\ 1 \times R}{Cmax1 + (attack\ rate\ 1 \times R)} \times C1] - [Cmax2 \times \frac{attack\ rate\ 2 \times R}{Cmax2 + (attack\ rate\ 2 \times R)} \times C2]$$

$$\Rightarrow \frac{dR}{dt} = [\frac{1}{400} \times (20000 - 10000)] - [10 \times \frac{0.013 \times 10^4}{10 + (0.013 \times 10^4)} \times 30] - [3 \times \frac{0.0067 \times 10^4}{3 + (0.0067 \times 10^4)} \times 30]$$

$$\Rightarrow \frac{dR}{dt} = -339.7\ salmon\ per\ day$$

A decrease of 3.397% per day in the resource

Rate of change of consumer 1: Brown Bears

$$\frac{dC1}{dt} = alpha \times [E1 \times Cmax1 \times \frac{attack\ rate\ 1 \times R}{Cmax1 + (attack\ rate\ 1 \times R)} - death\ rate\ 1] \times C1$$

$$\Rightarrow \frac{dC1}{dt} = 0.5 \times [0.00068 \times 10 \times \frac{0.013 \times 10^4}{10 + (0.013 \times 10^4)} - 0.000137] \times 30 = 0.092\ fraction\ of\ bears\ per\ day$$

An increase of approx. 0.31% per day

Rate of change of consumer 2: Bald Eagles

$$\frac{dC2}{dt} = beta \times [E2 \times Cmax2 \times \frac{attack\ rate\ 2 \times R}{Cmax2 + (attack\ rate\ 2 \times R)} - death\ rate\ 2] \times C2$$

$$\Rightarrow \frac{dC2}{dt} = 0.8 \times [0.00137 \times 3 \times \frac{0.067 \times 10^4}{3 + (0.067 \times 10^4)} - 0.0001826] \times 30 = 0.090\ eagle\ per\ day$$

An increase of 0.30% per day

Foraging efficiency is the ratio of energy gained from resource consumption to the energy expended in obtaining that resource. Brown bears, with a higher maximum consumption rate (Cmax) than bald eagles, face a trade-off in foraging efficiency, affecting factors like death rate and reproductive efficiencies.

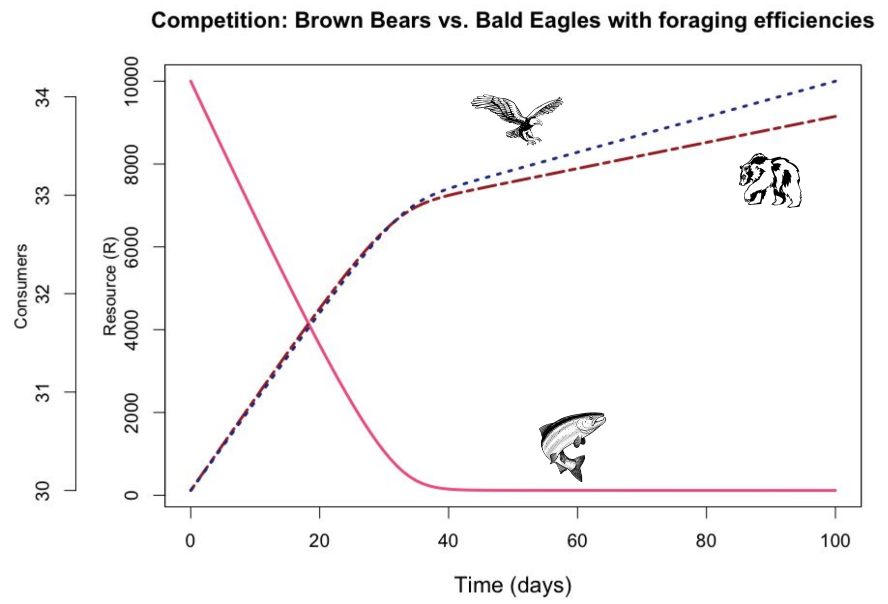
For Brown Bears (alpha = 0.5):

- Competition for prime catching spots among bears
- Need to maintain a strong presence at salmon-rich sites
- Exhaustion due to foraging in fast-running water currents

For Bald Eagles (beta = 0.8):

- Good at scavenging carrion
- Ability to make precise dives to catch salmon without much competition
- Diverse diet compared to brown bears

Competition model: foraging efficiency as trade-off trait



Minimum resource concentrations with trade-off

R* for Brown Bears:

$$R^*_{bears} = \frac{1}{alpha \times attack\ rate\ 1} \times \frac{Cmax1 \times death\ rate\ 1}{Cmax1 \times E1 - death\ rate\ 1} \simeq 30.61\ resource\ units$$

R* for Bald Eagles:

$$R^*_{eagles} = \frac{1}{beta \times attack\ rate\ 2} \times \frac{Cmax2 \times death\ rate\ 2}{Cmax2 \times E2 - death\ rate\ 2} \simeq 26.16\ resource\ units$$

When foraging efficiencies are included, Bald Eagles seem to be relatively doing better than Brown Bears during the transient and steady states.

Legend

- Salmon (Resource)
- Brown Bears (Consumer 1)
- Eagles (Consumer 2)

Inspiration: The Salmon Forest documentary https://www.youtube.com/watch?v=rm25cRi8TL8&t=1178s

Mathematical Biology

Weekly Exercise 4

Swati Tak

2023-09-24

```
setwd("/Users/swati/Desktop/Mathematical Biology/")
library(deSolve)

# Organisms portfolio
# Resource: Salmon
# Consumer 1: Brown Bears
# Consumer 2: Bald Eagles

# All values used for calculations are arbitrary

# Defining the function for the competition model
comp_model <- function(time, state, params) {
  with(as.list(c(state, params)), {
    dC1 <- ((E1 * Cmax1 * (b1 * R / ((b1 * R) + Cmax1))) - death_rate_1) * C1
    dC2 <- ((E2 * Cmax2 * (b2 * R / ((b2 * R) + Cmax2))) - death_rate_2) * C2
    dR <- (r_in * (K - R)) - (Cmax1 * (b1 * R / ((b1 * R) + Cmax1)) * C1) - (Cmax2 * (b2 * R / ((b2 * R)
    return(list(c(dR, dC1, dC2)))
  })
}

# Amount of resource available

## estimate of total spawning salmon per season
## (different salmon species have different spawning seasons in a year)
R <- 10000

# average number of salmon swimming upstream per day to go to spawning grounds
input <- 300

# average rate of resource input into the stream per day of the season
r_in <- 1/300

# Maximum number of live salmon individuals that can be accommodated by the stream
K <- 20000

# Initial number of consumers living around the stream
C1 <- 30 # bears
C2 <- 30 # eagles

# Maximum consumption possible for each species
```

```r
Cmax1 <- 10 # salmons per day for one bear
Cmax2 <- 3 # salmons per day for one eagle

# Number of births per year that leads to surviving consumer offsprings

## Birth rate per day if 2 bear cubs per year survive to adulthood (1 litter = 1 to 3 cubs)
birth_rate_1 <- (2/2)/365

## Birth rate per day if 2 eagle babies per year survive to adulthood (1 clutch = 1 to 3 eggs)
birth_rate_2 <- (2/2)/365


# Consumption required for each consumer per day as an essential resource during the season
consumption_1 <- 4 # if one bear needs to eat 4 salmon per day
consumption_2 <- 2 # if one eagle needs to eat one salmon per day


# Reproductive efficiency per day for each consumer species per unit of resource consumed
E1 <- birth_rate_1/consumption_1
E2 <- birth_rate_2/consumption_2

# # Attack rates per day
b1 <- consumption_1/input
b2 <- consumption_2/input

# Death rate of consumers per day
death_rate_1 <- 1/(20*365) # 1 bear dies in every 20-25 years
death_rate_2 <- 1/(15*365) # 1 eagle dies in every 15-25 years

# Steady-state dynamics for competition model
# Minimum resource concentration R* for brown bears
Rstar_C1 <- (1/b1)*((Cmax1*death_rate_1)/(Cmax1*E1-death_rate_1))
cat("Minimum required salmon concentration for Brown Bears =>", Rstar_C1)
```

```
## Minimum required salmon concentration for Brown Bears => 15.30612
```

```r
# Minimum resource concentration R* for bald eagles
Rstar_C2 <- (1/b2)*((Cmax2*death_rate_2)/(Cmax2*E2-death_rate_2))
cat("\nMinimum required salmon concentration for Bald Eagles =>", Rstar_C2)
```

```
##
## Minimum required salmon concentration for Bald Eagles => 20.93023
```

```r
# Set initial conditions, parameters and times
initial_state <- c(R = R, C1 = C1, C2 = C2)  # Initial populations
parameters <- c(E1 = E1, E2 = E2, Cmax1 = Cmax1, Cmax2 = Cmax2,
                b1 = b1, b2 = b2, death_rate_1 = death_rate_1,
                death_rate_2 = death_rate_2,
                r_in = r_in, K = K)

times <- seq(0, 100, by=1)
```

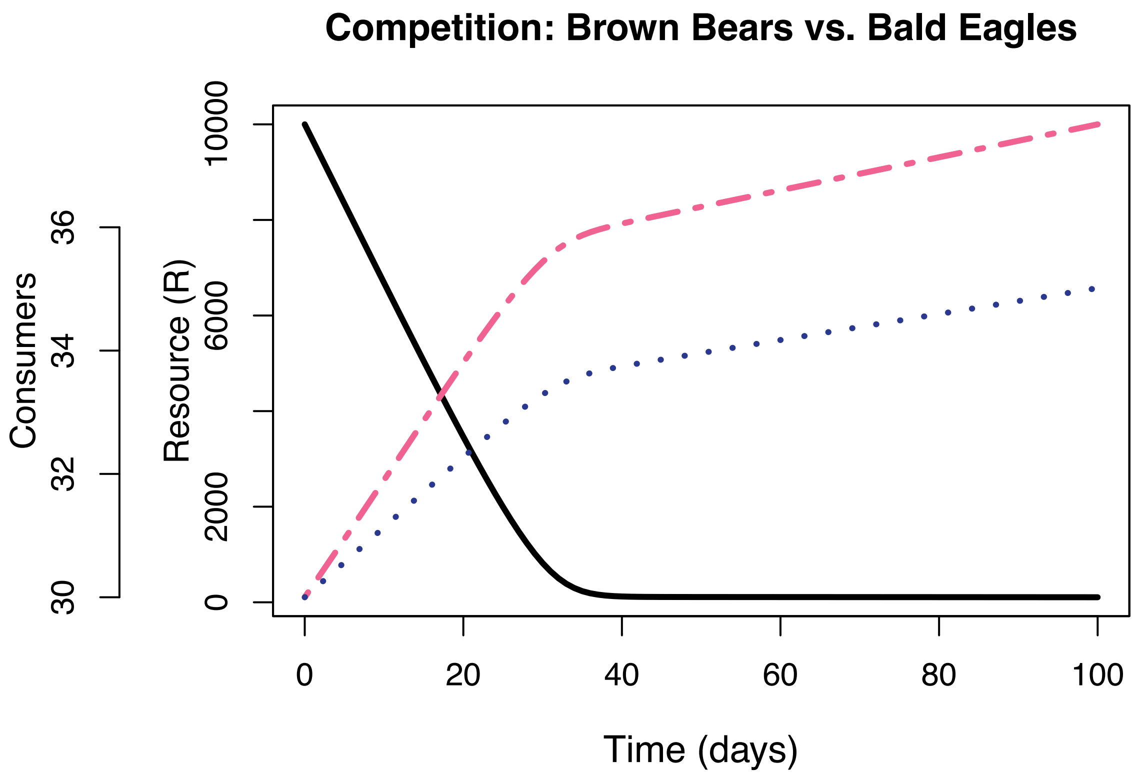
```r
# Applying the model
comp_sim <- ode(y = initial_state, times = times, func=comp_model, parms=parameters)
colnames(comp_sim) <- c("Time", "Resource", "Brown Bears", "Bald Eagles")
head(comp_sim)
```

```
##      Time  Resource Brown Bears Bald Eagles
## [1,]    0 10000.000    30.00000    30.00000
## [2,]    1  9668.058    30.18739    30.11262
## [3,]    2  9335.976    30.37547    30.22549
## [4,]    3  9003.833    30.56421    30.33858
## [5,]    4  8671.669    30.75358    30.45189
## [6,]    5  8339.572    30.94352    30.56539
```

```r
# Increasing the font size for axes annotations
# par(cex=1.2)

# Legend
Legend = c("Salmon (Resource)", "Brown Bears (Consumer 1)", "Eagles (Consumer 2)")

# Plotting the results
matplot.0D(comp_sim, which = list(c(1), c(2,3)),
           lty = c(1,6,3), col=c("black", "#f06292", "#2a3990"),
           lwd=c(3),
           xlab="Time (days)",
           ylab = c("Resource (R)", "Consumers"),
           main="Competition: Brown Bears vs. Bald Eagles",
           cex.lab=1.2,
           legend="none")
```




```
# legend("topleft", Legend, lty = c(1,6,3), col=c("black", "#f06292", "#2a3990"),
#        lwd=c(3), title= "Legend")


# Incorporating foraging efficiency based on hunting and scavenging by consumers
# as single trait as trade-off between parameters.

## Foraging efficiency can be defined as the ratio of energy gained to
## energy expended to acquire food

# Foraging efficiency of brown bears
# Low due to competition from other bears
# and due to exhaustion caused by fighting for prime catching spots
alpha <- 0.5

# Foraging efficiency of bald eagles
# Higher due to less energy expended in hunting
# as scavenging of carrion works well for eagles during peak spawning season
beta <- 0.8

# Re-defining the function for the competition model
# with foraging efficiency as the single trait for trade-off
comp_model_2 <- function(time, state, params) {
  with(as.list(c(state, params)), {
    dC1 = alpha * ((E1 * Cmax1 * (b1 * R / ((b1 * R) + Cmax1))) - death_rate_1) * C1
    dC2 = beta * ((E2 * Cmax2 * (b2 * R / ((b2 * R) + Cmax2))) - death_rate_2) * C2
```

```r
    dR <- (r_in * (K - R)) - (Cmax1 * (b1 * R / ((b1 * R) + Cmax1)) * C1) - (Cmax2 * (b2 * R / ((b2 * R)
    return(list(c(dR, dC1, dC2)))
  })
}

# Setting updated parameters
parameters2 <- c(E1 = E1, E2 = E2, Cmax1 = Cmax1, Cmax2 = Cmax2,
                 b1 = b1, b2 = b2, death_rate_1 = death_rate_1,
                 death_rate_2 = death_rate_2,
                 r_in = r_in, K = K, alpha=alpha, beta=beta)

# Applying the extended model with foraging efficiencies
comp_sim_2 <- ode(y = initial_state, times = times, func=comp_model_2, parms=parameters2)

# Updating column names of model results
colnames(comp_sim_2) <- c("Time", "Resource", "Brown Bears", "Bald Eagles")
head(comp_sim_2)
```

```
##      Time  Resource Brown Bears Bald Eagles
## [1,]    0 10000.000    30.00000    30.00000
## [2,]    1  9668.525    30.09355    30.09006
## [3,]    2  9337.839    30.18715    30.18026
## [4,]    3  9008.021    30.28079    30.27056
## [5,]    4  8679.100    30.37446    30.36097
## [6,]    5  8351.160    30.46812    30.45148
```

```r
# Steady-state dynamics for competition model with foraging efficiency included

# Calculating R star for both consumers
# R* for brown bears
Rstar_C1 <- (1 / (alpha * b1)) * ((Cmax1 * death_rate_1) / (Cmax1 * E1 - death_rate_1))
Rstar_C1
```

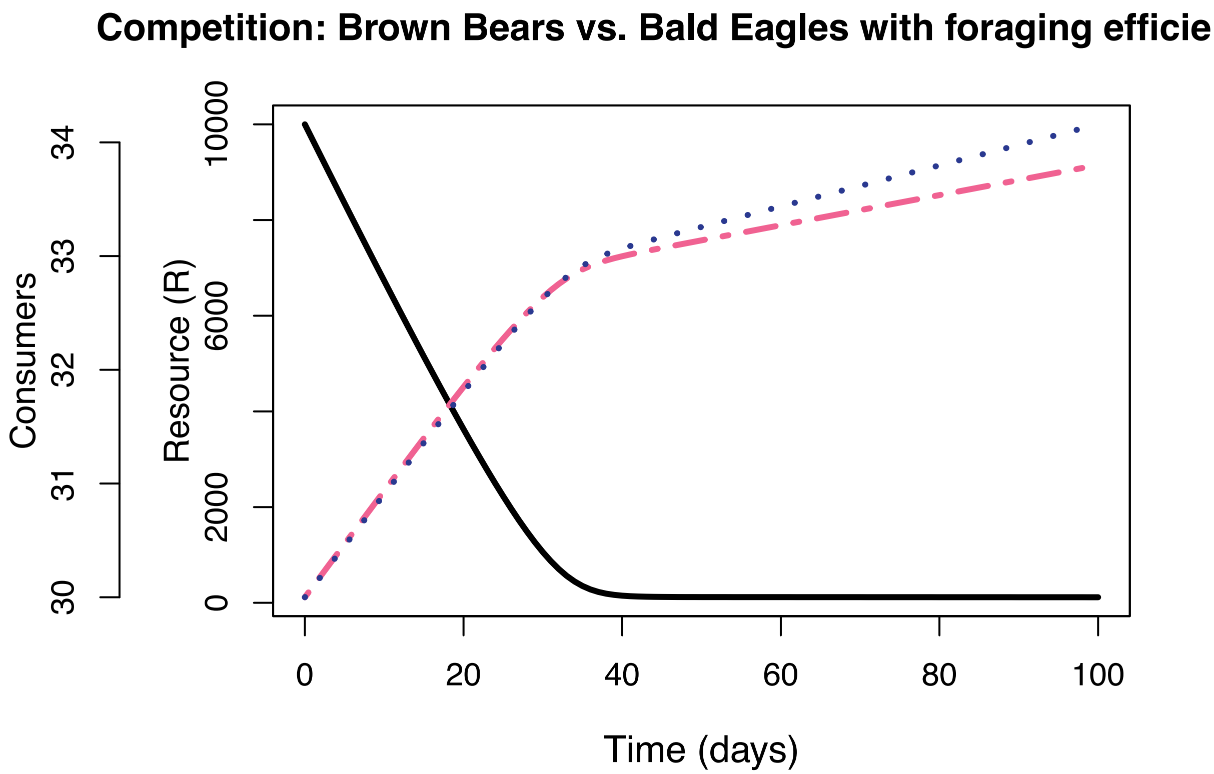
```
## [1] 30.61224
```

```r
# R* for bald eagles
Rstar_C2 <- (1 / (beta * b2)) * ((Cmax2 * death_rate_2) / (Cmax2 * E2 - death_rate_2))
Rstar_C2
```

```
## [1] 26.16279
```

```r
# Increasing the font size for axes annotations
# par(cex=1.2)

# Plotting the results
matplot.0D(comp_sim_2, which = list(c(1), c(2,3)),
           lty = c(1,6,3), col=c("black", "#f06292", "#2a3990"),
           lwd=c(3),
           xlab="Time (days)",
           ylab = c("Resource (R)", "Consumers"),
           main="Competition: Brown Bears vs. Bald Eagles with foraging efficiencies",
           cex.lab=1.2,
           legend="none")
```




```
# Printing results
if (Rstar_C1 < Rstar_C2) {
   cat("With foraging efficiency considered, Brown Bears prosper at lower salmon concentration of", Rsta
 } else if (Rstar_C1 > Rstar_C2) {
   cat("With foraging efficiency considered, Bald Eagles prosper at lower salmon concentration of", Rsta
 } else {
   cat("With foraging efficiency considered, Brown Bears and Bald Eagles have the same R* values and may
 }
```

```
## With foraging efficiency considered, Bald Eagles prosper at lower salmon concentration of 26.16279
```