Modeling predator-prey interaction between Polar Bears and Seals in Arctic ecosystems

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Model parameters and calculations

Polar bears (predator)

$P0$ = 300 polar bears

Death rate ⇒ d = 1/15 years = 0.067 per year

Seals (prey)

$N0$ = 50,000 seals

Birth rate of seals ⇒ $r = 0.5$

Consumption ⇒ C = 50 seals per year (consumed by one polar bear)

Attack rate ⇒ $b = C/N_0 = 0.001$

On average, if one polar bear couple produces 2 cubs per year: then Reproductive Efficiency is:
E = 2 cubs / 100 seals = 0.02 cubs per seal consumed

Rate of change in prey population:

$dN/dt = (r \times N) - (b \times N \times P)$
= 10,000 seals per year

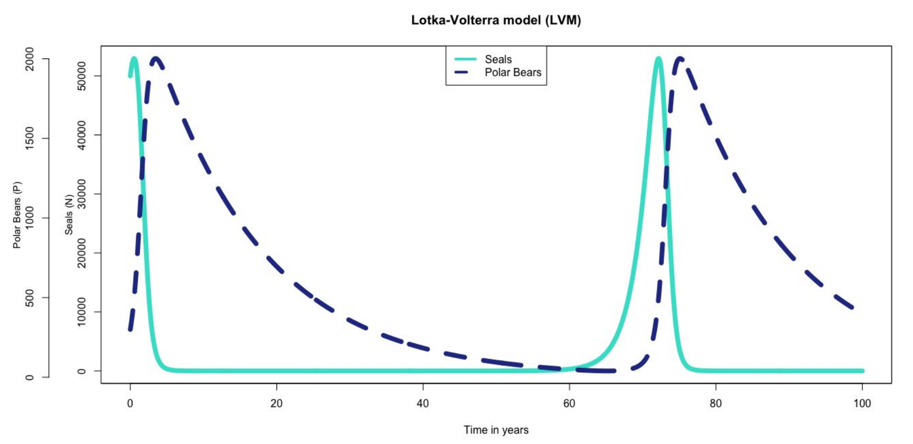
Rate of change in predator population:

$dP/dt = (E \times b \times N \times P) - (d \times P)$
= 280 polar bears per year

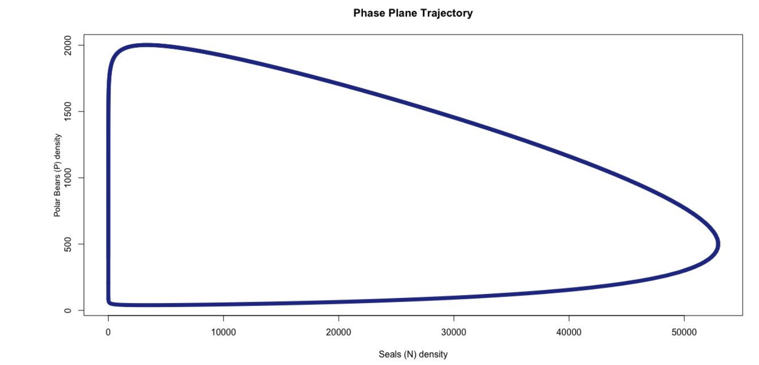
Cyclic fluctuations in predator-prey populations

Prey increase leads to predator increase, followed by prey decrease and then predator decrease, creating a repeating cycle.

Cyclic trends but out of phase with one another



Prey versus Predator densities



- Initial equilibrium with stable predator-prey populations
- Prey population increases, driving predator population growth
- Predators consume more prey, causing prey numbers to decline
- Predator population lags and decreases with reduced prey population due to lack of enough preys to feed on, leading to reduced predation pressure on prey – thereby initiating a new cycle of increase in prey numbers.
- Prey-Predator densities fluctuate in a manner so as to balance out each other.

Extended model with logistic growth and carrying capacity

Extended model

K for seals = 100,000

Rate of change in prey population:

dN/dt = (r x N x (**1 – N/K**)) – (b x N x P)

= – 2,500 seals per year

K for polar bears = 1000

Rate of change in predator population:

dP/dt = (E x b x N x P x (**1 – P/K**)) – (d x P)

= 190 polar bears per year

More realistic in nature

Considering that both prey and predator populations will show logistic growth in nature, the model can be tweaked to get realistic estimates.

Previous model

Rate of change in prey population:

dN/dt = (r x N) – (b x N x P)

= 10,000 seals per year

Rate of change in predator population:

dP/dt = (E x b x N x P) – (d x P)

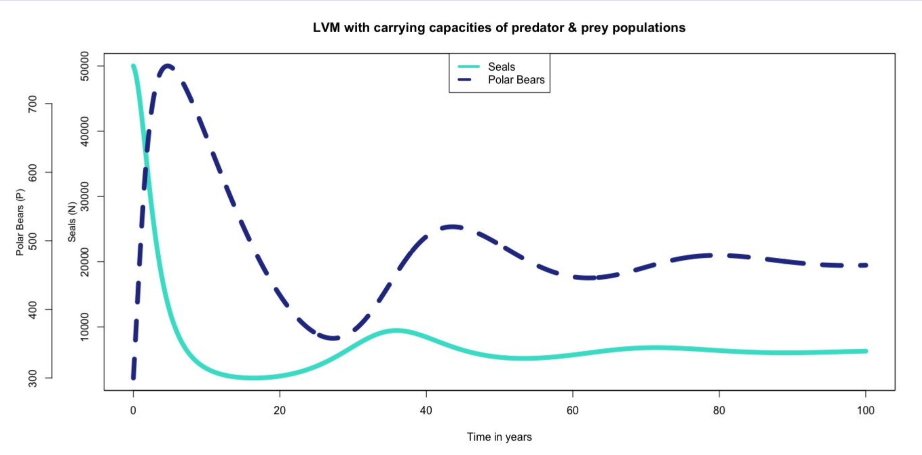
= 280 polar bears per year

The Seal population should in fact decrease per year by a rate of negative 2500, and the Polar Bear population should increase with a relatively lower rate of 190 polar bears per year — owing to their respective logistic growths and their carrying capacity in their respective niches.

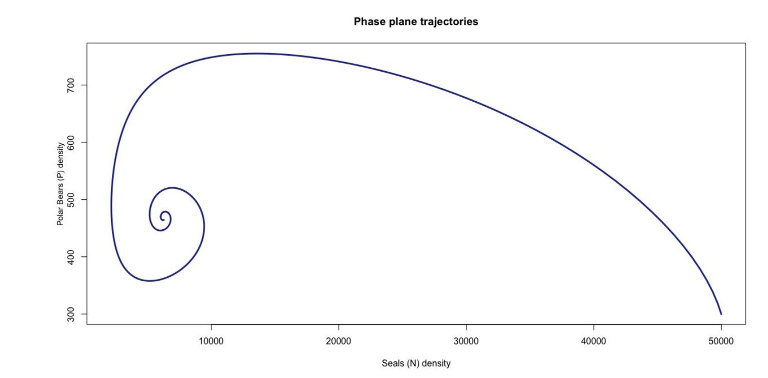
Extended model with logistic growth and carrying capacity

Logistic growth regulates both the populations over time, leading to equilibrium

Populations attain equilibrium over time



Prey versus Predator densities



- Prey population grows until they approach their carrying capacity, which reduces the rate of increase.
- Predator population grows until they approach their carrying capacity, reducing their predation pressure.
- This regulates both the populations over time, making them reach equilibrium with regard to their respective carrying capacities.
- Prey-Predator densities also gradually attain a single point of equilibrium.

Mathematical Biology

Weekly Exercise 3

Swati Tak

2023-09-17

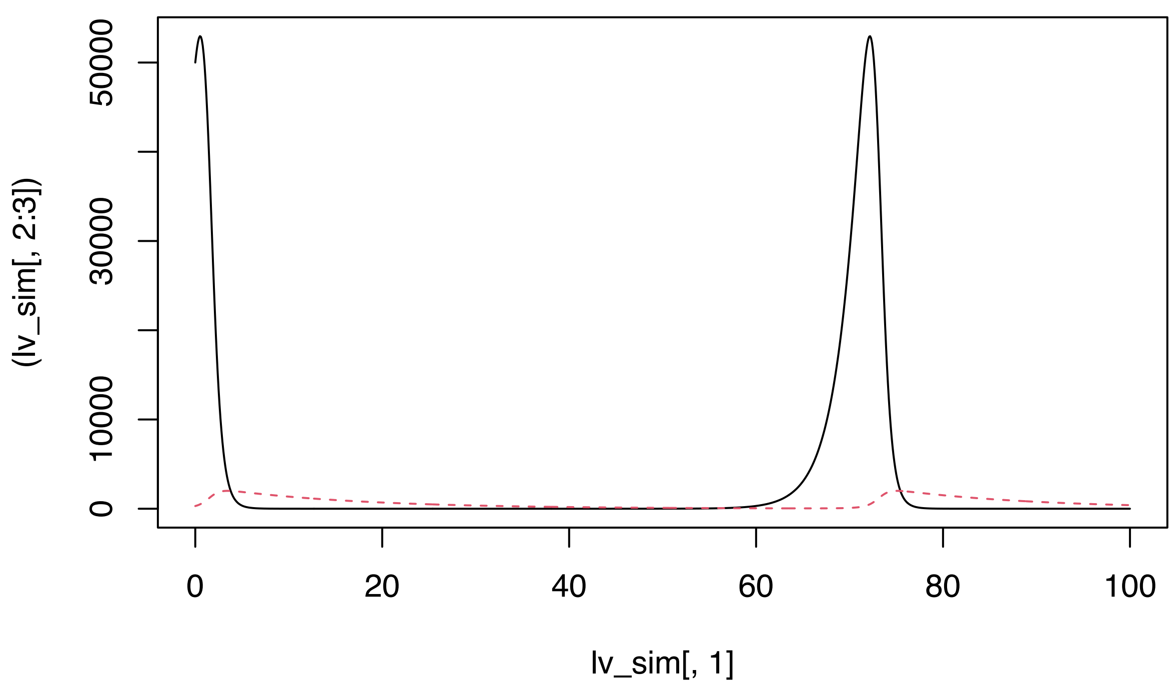
```
## Predator: Polar Bears
## Prey: Seals

setwd("/Users/swati/Desktop/Mathematical Biology/")
library(deSolve)
```

Setting the parameters and initial state

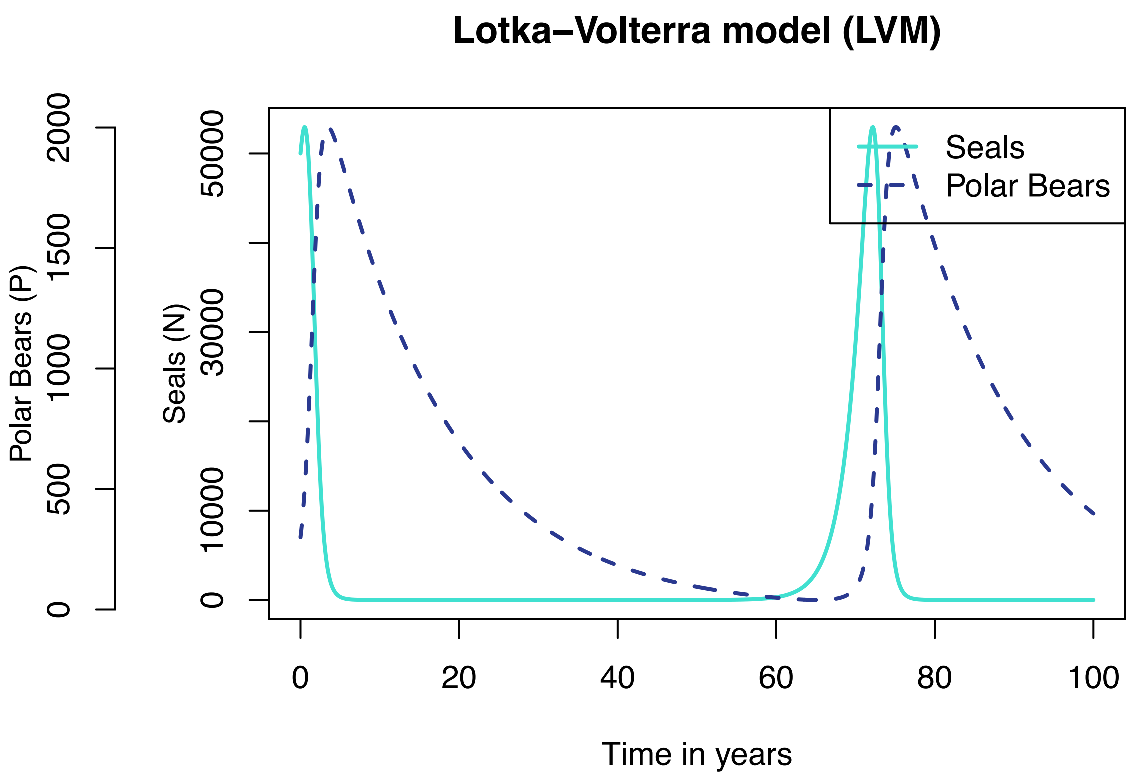
```
# # Number of predators
P0 <- 300

# # Number of preys
N0 <- 50000

# Consumption of preys by one predator per year
# Supp. one polar bear needs to consume 50 seals per year to survive
C <- 50

# Average production of surviving predator offsprings in one year
# Considering one on average produces 1 to 3 cubs per year and if 2 survive on average
y <- 2

# Reproductive efficiency of predator per one seal consumed
E <- y/(C*2)

# Attack rate per year
b <- C/N0

# 2 prey individuals (male and female) gives birth to 1 offspring per year
r <- 1/2

# Death rate for predator using the average life span
# With one predator surely dying in every 15 years due to natural causes
d <- 1/15
```

Defining the ODEs for Lotka-Volterra model

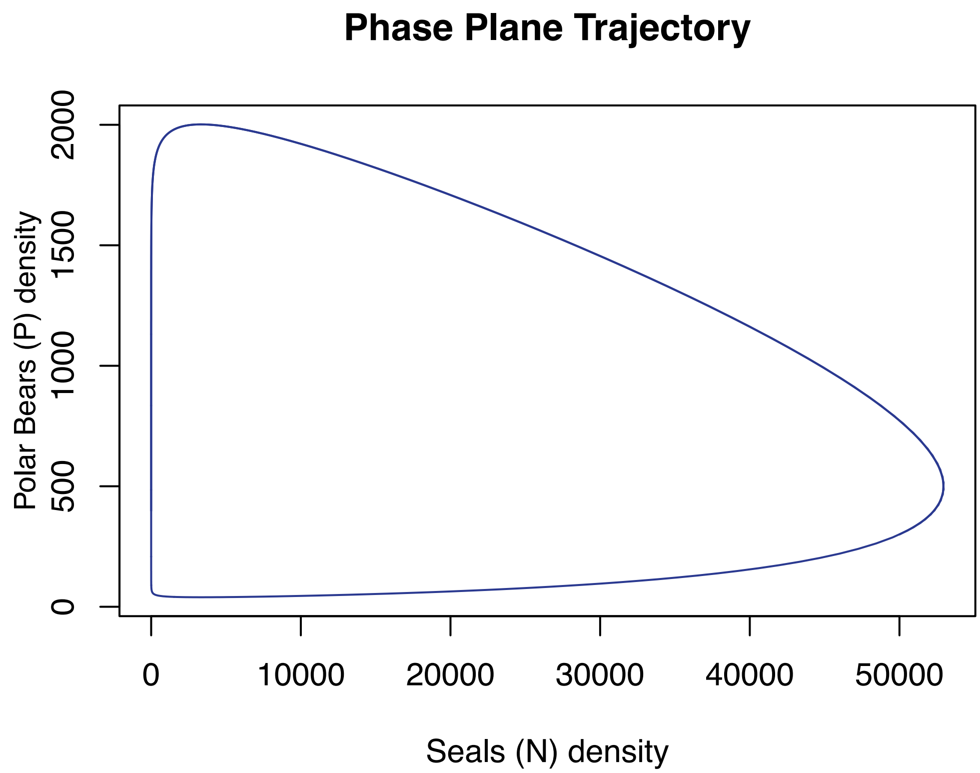
```r
# Defining a function for the Lotka Volterra model using the equations
lv_model <- function(time, state, params) {
  with(as.list(c(state, params)), {
    dN <- (r*N)-(b*N*P)
    dP <- (E*b*N*P)-(d*P)
    return(list(c(dN, dP)))
  })
}

# Defining initial conditions and parameters
initial_state <- c(N=N0, P=P0)
parameters <- c(E=E, b=b, r=r, d=d)

# Defining the times
times <- seq(0, 100, by = 0.1)

# Applying the ode function to solve the equations
lv_sim <- ode(y = initial_state, times = times, func = lv_model, parms = parameters)

matplot(lv_sim[,1], (lv_sim[,2:3]), type="l")
```



```r
# plot(lv_sim[,2], lv_sim[,3], type="l")

# Plot the results for population size with time
matplot.0D(lv_sim, which = list(c(1), c(2)),
           lty = c(1,2), col=c("turquoise", "#2a3990"),
           lwd=c(2,2),
           xlab="Time in years",
           ylab = c("Seals (N)", "Polar Bears (P)"),
           main="Lotka-Volterra model (LVM)",
           legend = "none")

# Define legend labels
legend_labels <- c("Seals", "Polar Bears")

# Change legend position and labels
legend("topright", legend = legend_labels, col=c("turquoise", "#2a3990"), lty=c(1,2), lwd=c(2, 2))
```




```r
# Plot the results for phase plane trajectory
matplot.0D(lv_sim[, c("N", "P")], type = "l",
           col="#2a3990",
           lwd=1,
           xlab = "Seals (N) density", ylab = "Polar Bears (P) density",
           main = "Phase Plane Trajectory",
           legend=FALSE)
```



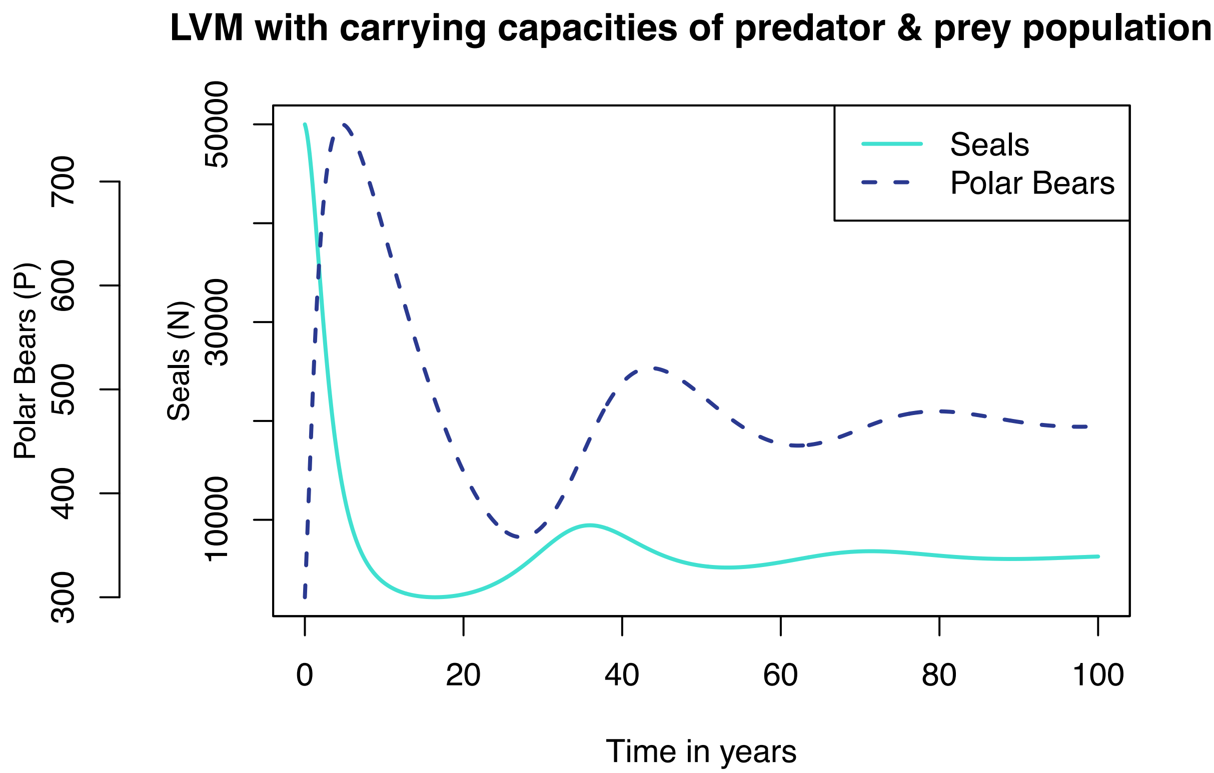
Adding carrying capacity to predator and prey populations

```
# Carrying capacities for two populations
K_pb <-1000
K_seals <- 100000 # a 100 thousand

# Defining a function for the Lotka Volterra model using the equations
lv_model_2 <- function(time, state, params) {
    with(as.list(c(state, params)), {
      dN <- (r*N*((1-N/K_seals)))-(b*N*P)
      dP <- (E*b*N*P*(1-(P/K_pb)))-(d*P)
      return(list(c(dN, dP)))
    })}

# Defining initial conditions and parameters
initial_state <- c(N=N0, P=P0)
parameters <- c(E=E, b=b, r=r, d=d, K_seals=K_seals, K_pb=K_pb)

# Defining the times
# times <- seq(0, 100, by = 0.1)

# Applying the ode function to solve the equations
lv_sim_2 <- ode(y = initial_state, times = times, func = lv_model_2, parms = parameters)
```

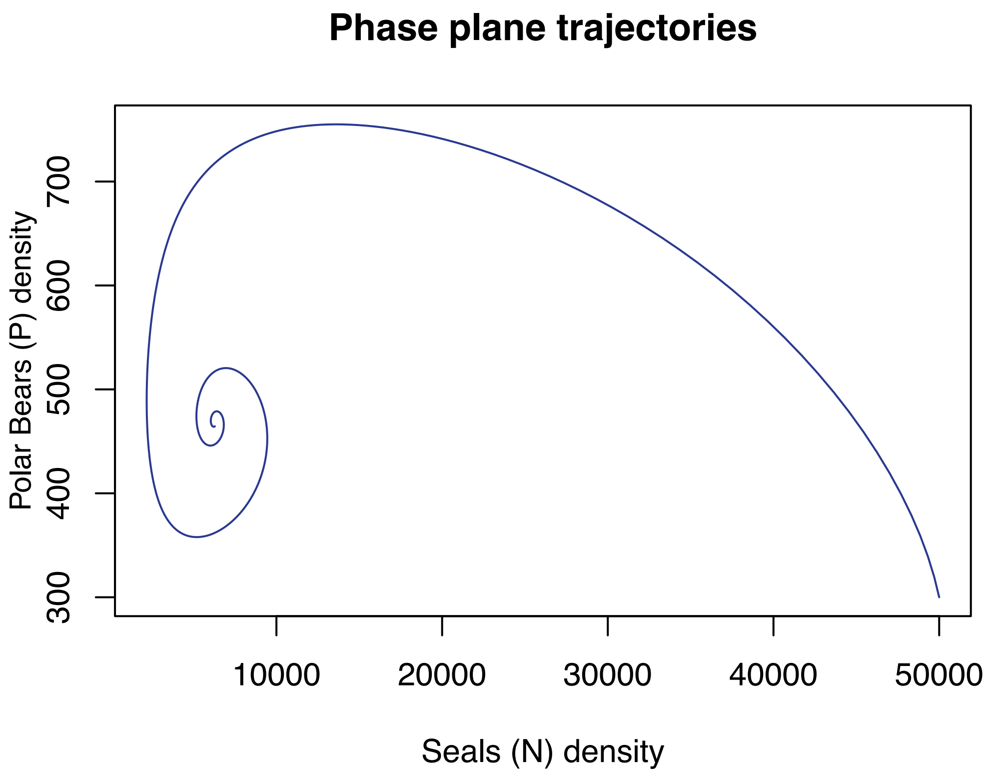
```r
head(lv_sim_2)
```

```
##      time        N        P
## [1,]  0.0 50000.00 300.0000
## [2,]  0.1 49706.44 319.2470
## [3,]  0.2 49326.80 338.9172
## [4,]  0.3 48863.38 358.8959
## [5,]  0.4 48319.37 379.0626
## [6,]  0.5 47698.83 399.2927
```

```r
# matplot(lv_sim_2[,1], (lv_sim_2[,2:3]), type="l")
# plot(lv_sim_2[,2], lv_sim_2[,3], type="l")

# Plot the results for population size with time
matplot.0D(lv_sim_2, which = list(c(1), c(2)),
           lty = c(1, 2), col=c("turquoise", "#2a3990"),
           lwd=c(2,2),
           xlab="Time in years",
           ylab = c("Seals (N)", "Polar Bears (P)"),
           main="LVM with carrying capacities of predator & prey populations",
           legend = FALSE)

# Define legend labels
legend_labels <- c("Seals", "Polar Bears")

# Change legend position and labels
legend("topright", legend = legend_labels, col=c("turquoise", "#2a3990"), lty=c(1, 2), lwd=c(2, 2))
```




```r
# Plot the results for phase plane trajectory
matplot.0D(lv_sim_2[, c("N", "P")], type = "l",
           col="#2a3990",
           lwd=1,
           xlab = "Seals (N) density", ylab = "Polar Bears (P) density",
           main = "Phase plane trajectories",
           legend=FALSE)
```


Phase plane trajectories
Polar Bears (P) density
300
400
500
600
700
10000
20000
30000
40000
50000
Seals (N) density