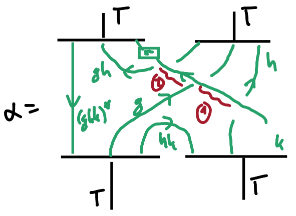
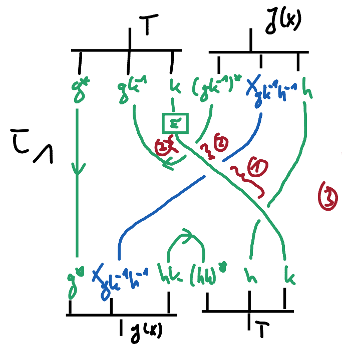
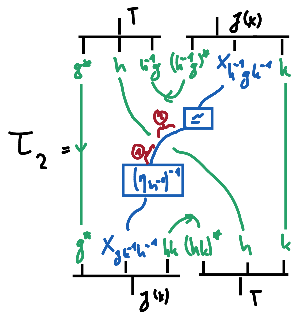


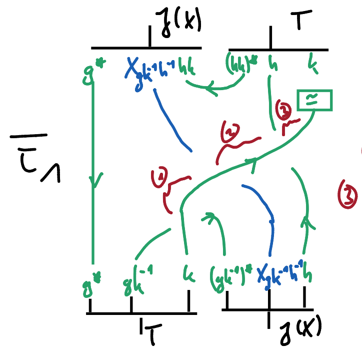
① $\varphi(h^{-1})(k)$
② $\varphi(g^{-1})\,\varphi(h^{-1})(k)$
③ $\underbrace{\varphi(gh)\,\varphi(g^{-1})\,\varphi(h^{-1})(k)}_{}$ $= \varphi(h^{-1}g^{-1}gh) = id$



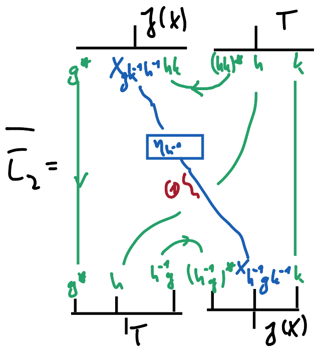
① $\varphi(h^{-1})(k)$
② $\varphi(hkg^{-1})\,\varphi(h^{-1})(k)$
③ $\underbrace{\varphi(gk^{-1})\,\varphi(hkg^{-1})\,\varphi(h^{-1})(k)}_{}$ $= \varphi(h^{-1}hkg^{-1}gk^{-1})$ $= id$ ✓



① $\varphi(h^{-1})(X_{h^{-1}gk^{-1}})$
② $\underbrace{\varphi(h)\,\varphi(h^{-1})}_{}(X_{h^{-1}gk^{-1}})$ $\cong \varphi(h^{-1}h)$ $= id$

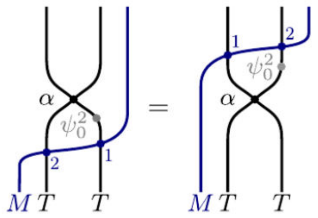


① $\varphi(kg^{-1})(k)$
② $\varphi(g^{-1}h)\,\varphi(kg^{-1})(k)$
③ $\varphi(h)\,\varphi(gk^{-1}h^{-1})\,\varphi(kg^{-1})(k)$ $= \varphi(kg^{-1}gk^{-1}h^{-1}h)$ $= id$ ✓



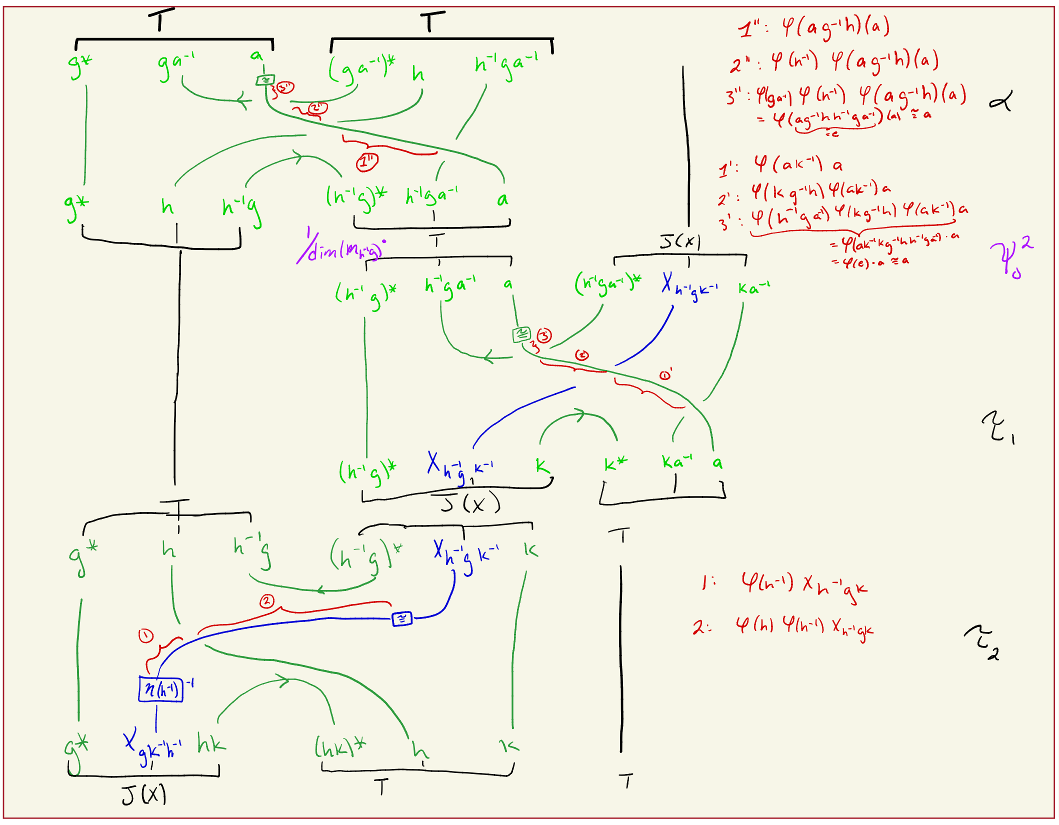
① $\varphi(h^{-1})(X_{h^{-1}gk^{-1}})$

TODO
E0
ribbon
check with isomorphism



(T2)

(lhs)



1'': $\varphi(ag^{-1}h)(a)$
2'': $\varphi(h^{-1})\,\varphi(ag^{-1}h)(a)$
3'': $\varphi(ga^{-1})\,\varphi(h^{-1})\,\varphi(ag^{-1}h)(a)$ $= \varphi(ag^{-1}hh^{-1}ga^{-1})(a) \cong a$

α

1': $\varphi(ak^{-1})\,a$
2': $\varphi(kg^{-1}h)\,\varphi(ak^{-1})\,a$
3': $\varphi(h^{-1}ga)\,\varphi(kg^{-1}h)\,\varphi(ak^{-1})\,a$ $= \varphi(ak^{-1}kg^{-1}hh^{-1}ga)\cdot a$ $= \varphi(e)\cdot a \cong a$

ψ_0^2

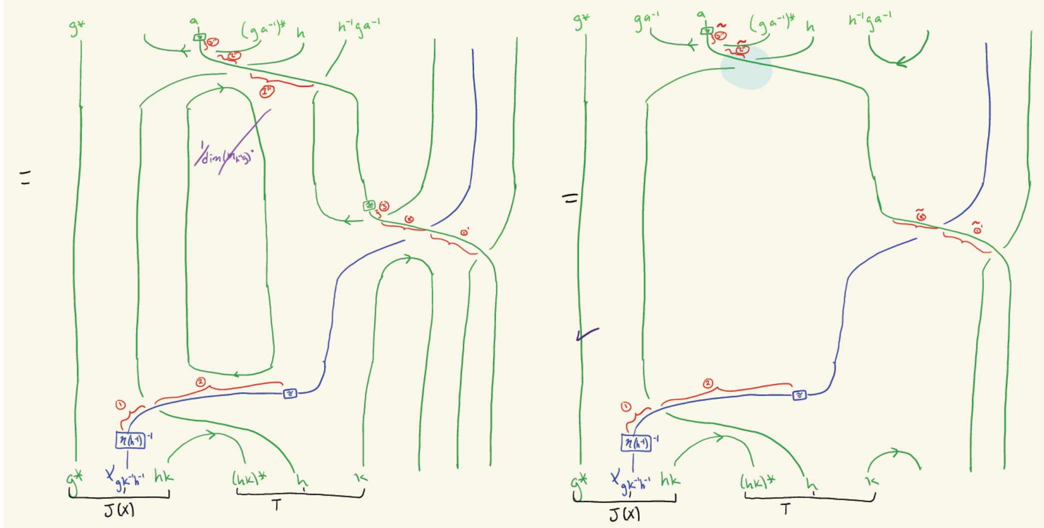
$1/\dim(m_{h^{-1}g})$

τ_1

1: $\varphi(h^{-1})\,X_{h^{-1}gk}$
2: $\varphi(h)\,\varphi(h^{-1})\,X_{h^{-1}gk}$

τ_2

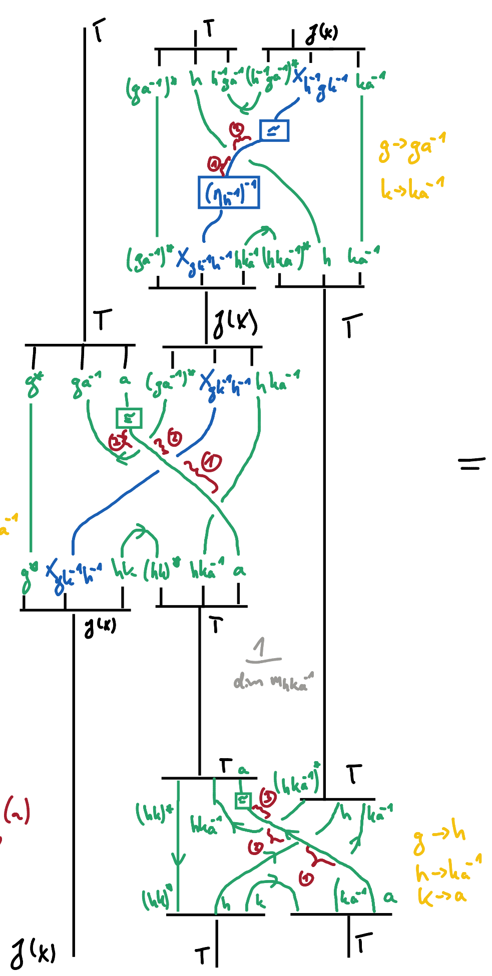
=



$\tilde{1}'$: $\varphi(ak^{-1})\cdot a$
$\tilde{2}'$: $\varphi(kg^{-1}h)\,\varphi(ak^{-1})\,a$
$\tilde{2}''$: $\varphi(h^{-1})\,\varphi(kg^{-1}h)\,\varphi(ak^{-1})\,a$
$\tilde{3}''$: $\varphi(ga^{-1})\,\varphi(h^{-1})\,\varphi(kg^{-1}h)\,\varphi(ak^{-1})\,a$ $= \varphi(ak^{-1}kg^{-1}hh^{-1}ga^{-1})\,a \cong a$ ✓

My lhs = Sebastian's rhs

(T2) (rhs)



① $\varphi(h^{-1})(X_{h^{-1}gk^{-1}})$
② $\varphi(h)\,\varphi(h^{-1})(X_{h^{-1}gk^{-1}})$ $\cong \varphi(h^{-1}h)$ $= id$

$g \to ga^{-1}$
$k \to ka^{-1}$

① $\varphi(ak^{-1})(a)$
② $\varphi(hkg^{-1})\,\varphi(ak^{-1}h^{-1})(a)$
③ $\underbrace{\varphi(ga^{-1})\,\varphi(hkg^{-1})\,\varphi(ak^{-1}h^{-1})(a)}_{}$ $= \varphi(ak^{-1}h^{-1}hkg^{-1}ga^{-1})(a)$ $= id$ ✓

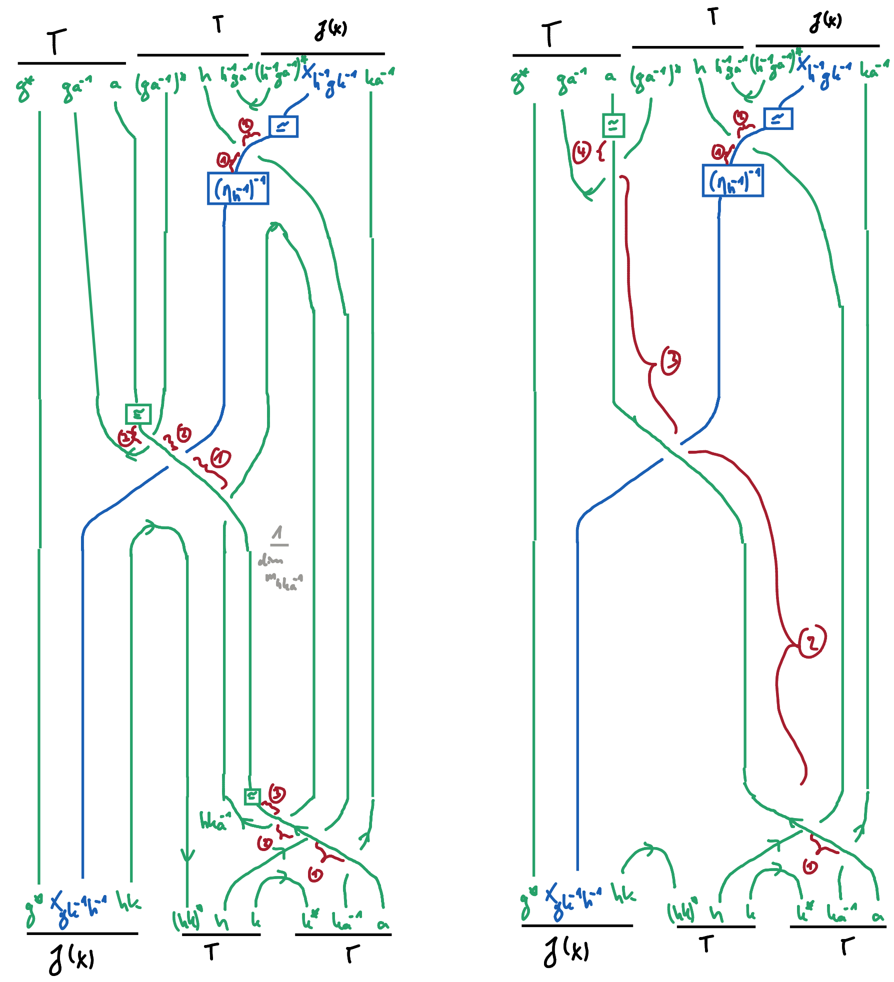
$h \to hka^{-1}$
$k \to a$

$\frac{1}{\dim m_{hka^{-1}}}$

① $\varphi(ak^{-1})(a)$
② $\varphi(h^{-1})\,\varphi(ak^{-1})(a)$
③ $\underbrace{\varphi(hka^{-1})\,\varphi(h^{-1})\,\varphi(ak^{-1})(a)}_{}$ $\cong \varphi(ak^{-1}h^{-1}hka^{-1})$ $= id$ ✓

$g \to h$
$h \to ka^{-1}$
$k \to a$

=

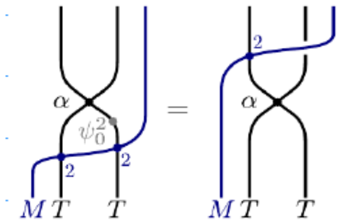


④ $\varphi(ga^{-1})\,\varphi(hkg^{-1})\,\varphi(h^{-1})\,\varphi(ak^{-1})(a)$ $\cong \varphi(ak^{-1}h^{-1}hkg^{-1}ga^{-1})$ $= id$ ✓

③ $\varphi(hkg^{-1})\,\varphi(h^{-1})\,\varphi(ak^{-1})(a)$

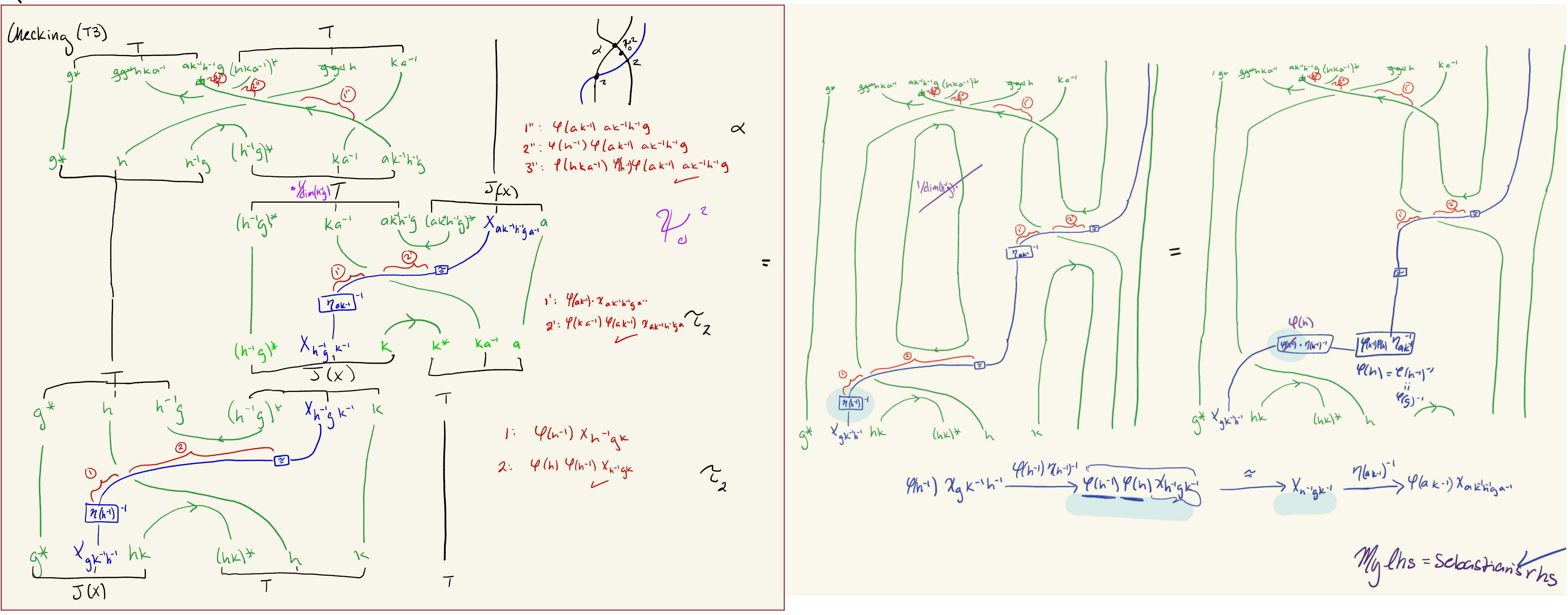
② $\varphi(h^{-1})\,\varphi(ak^{-1})(a)$

① $\varphi(ak^{-1})(a)$

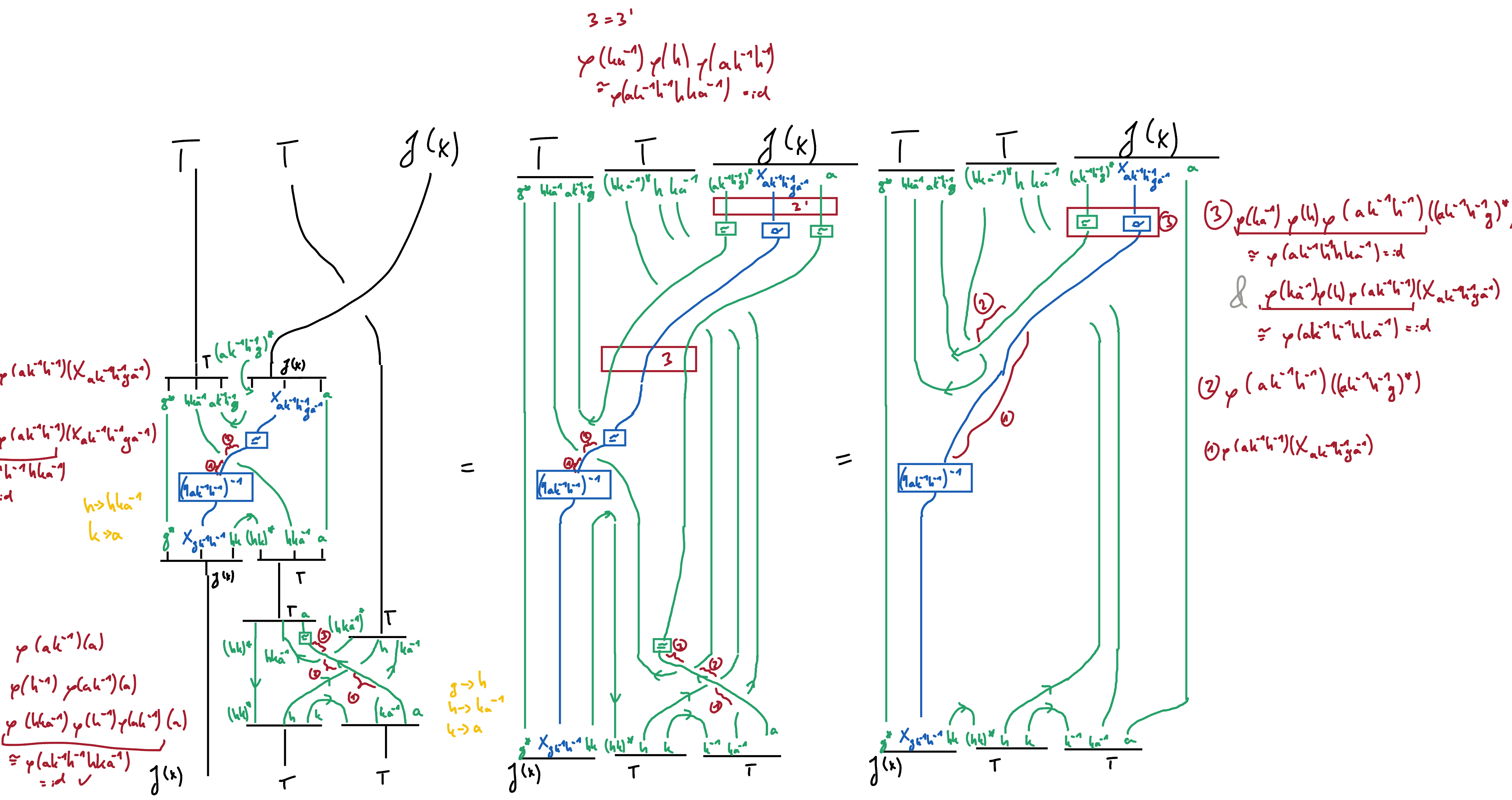

α
ψ_0^2
2
$M\ T\quad T$
$=$

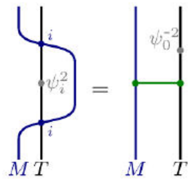
(T3)

(lhs)


Checking (T3)
α
τ_2
My lhs = Sebastian's rhs

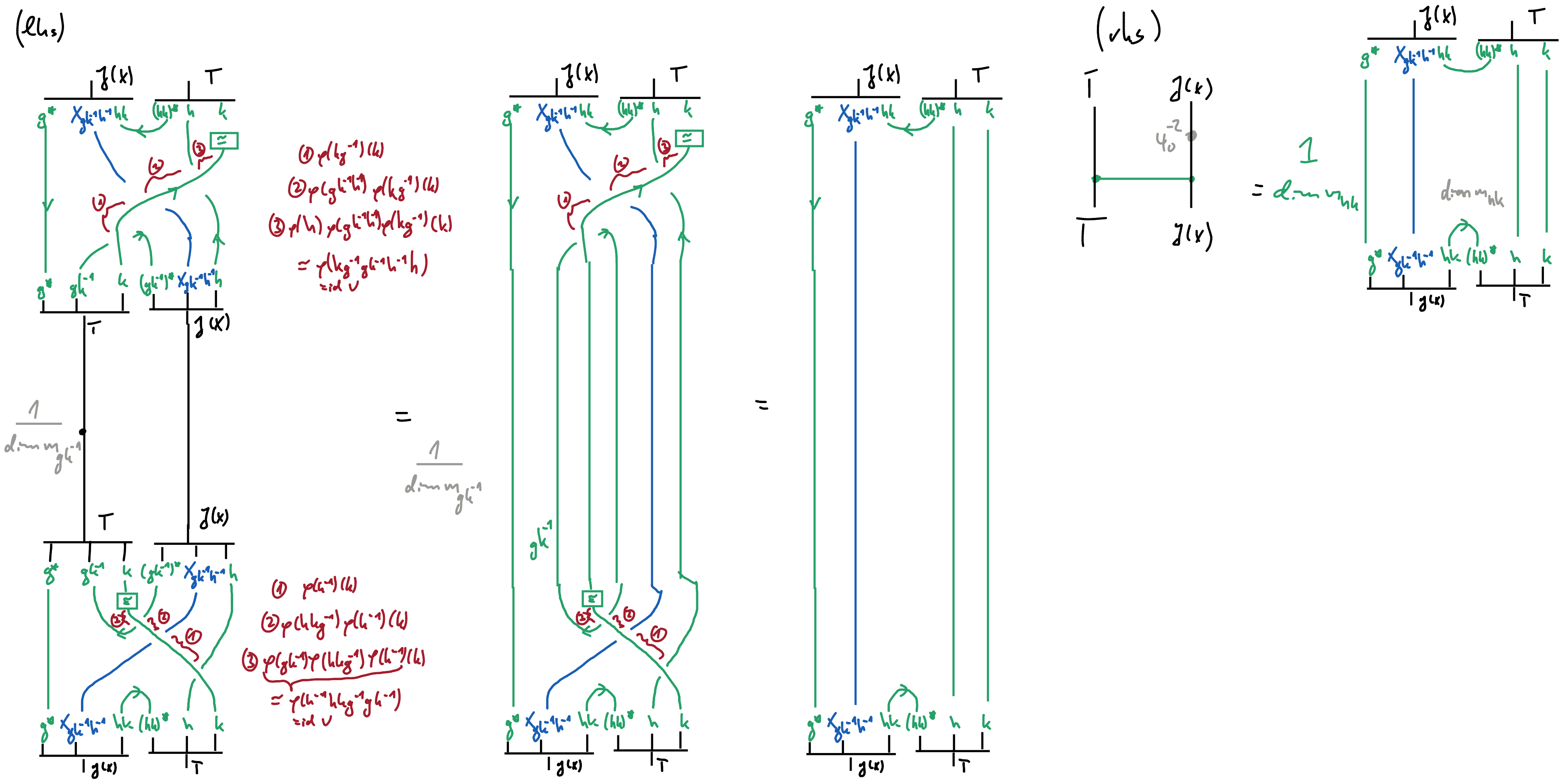
(rhs)

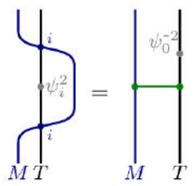

3 = 3'
$=$



(T4) :=1

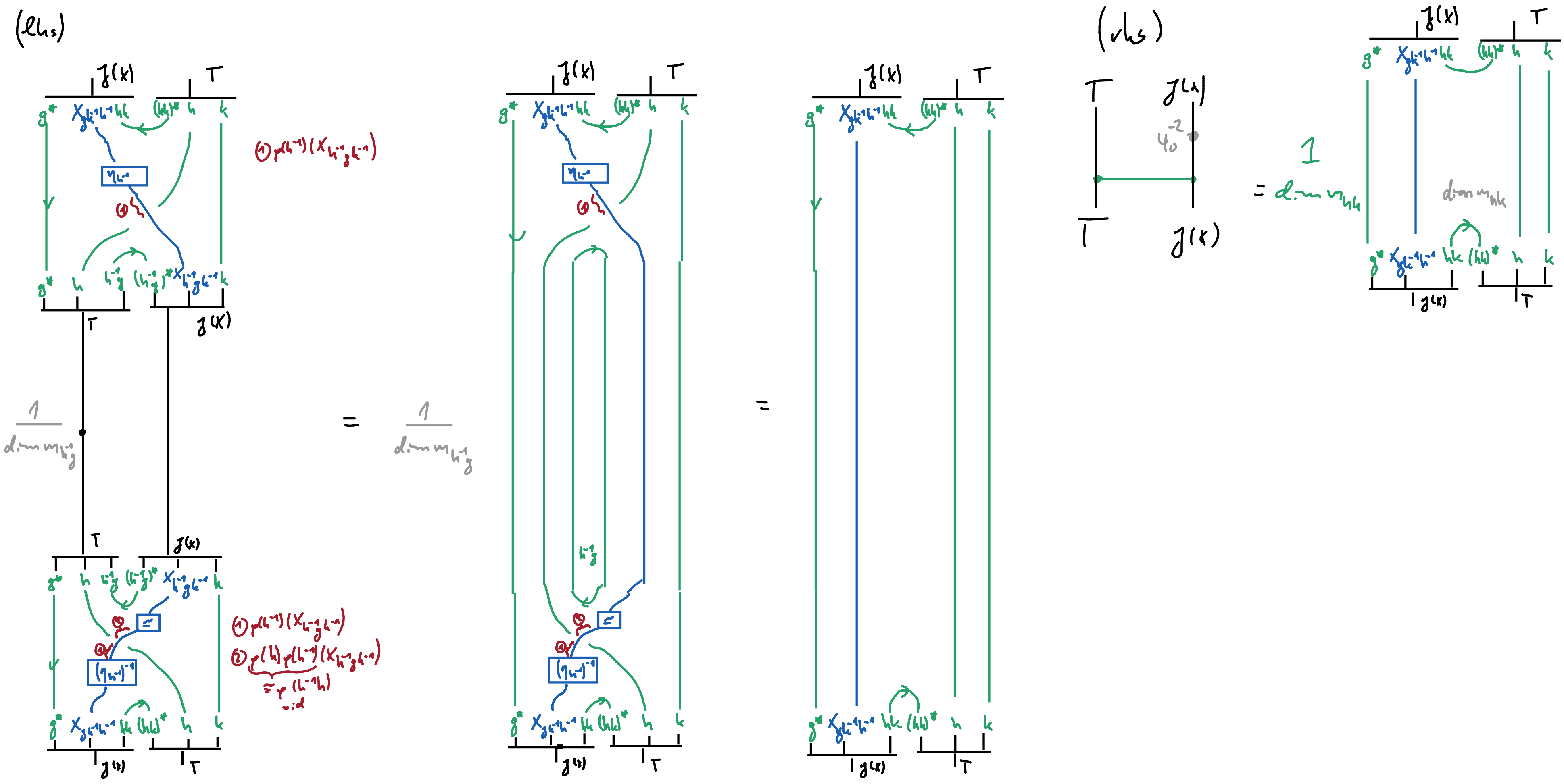
(lhs) (rhs)

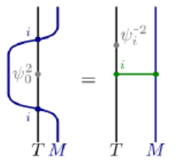




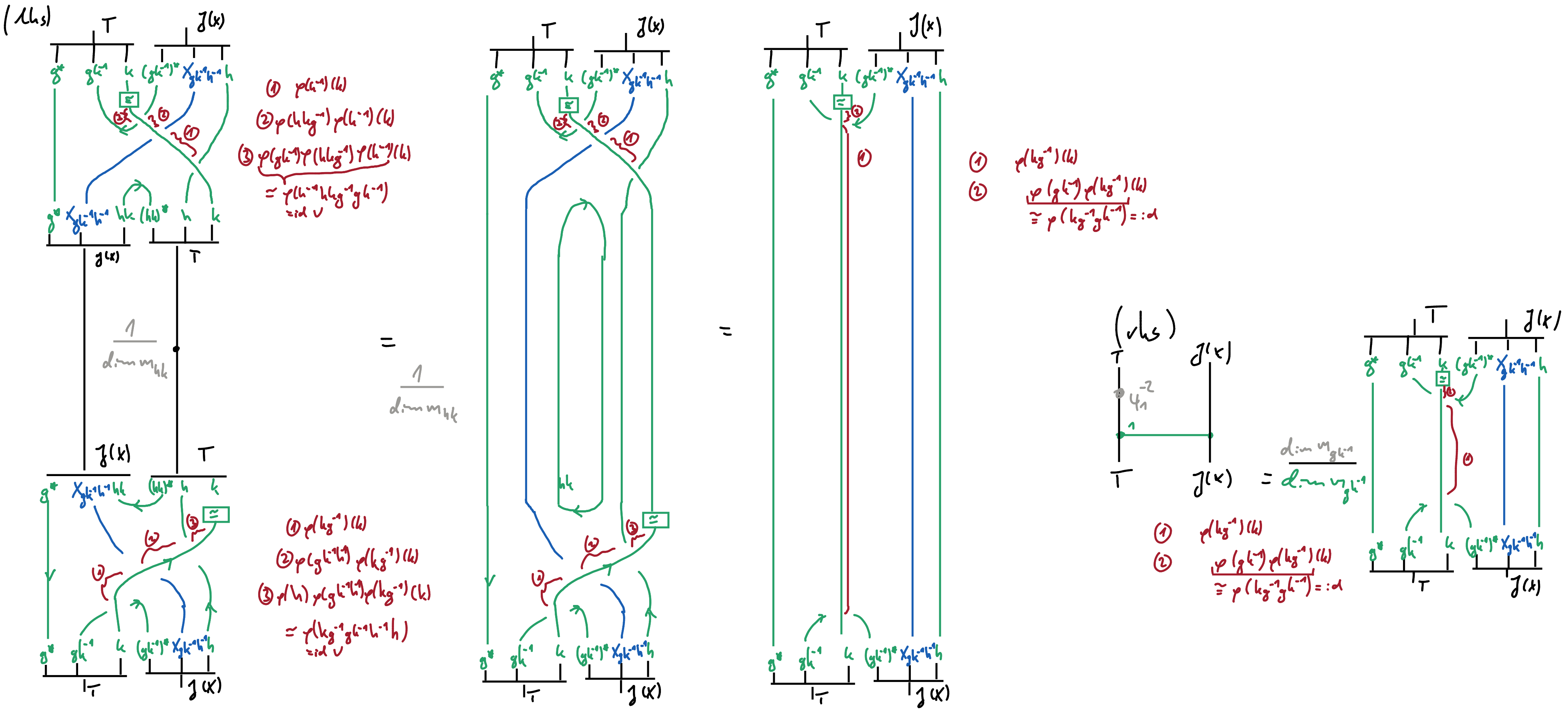
(T4) :=2

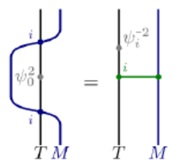
(lhs) (rhs)



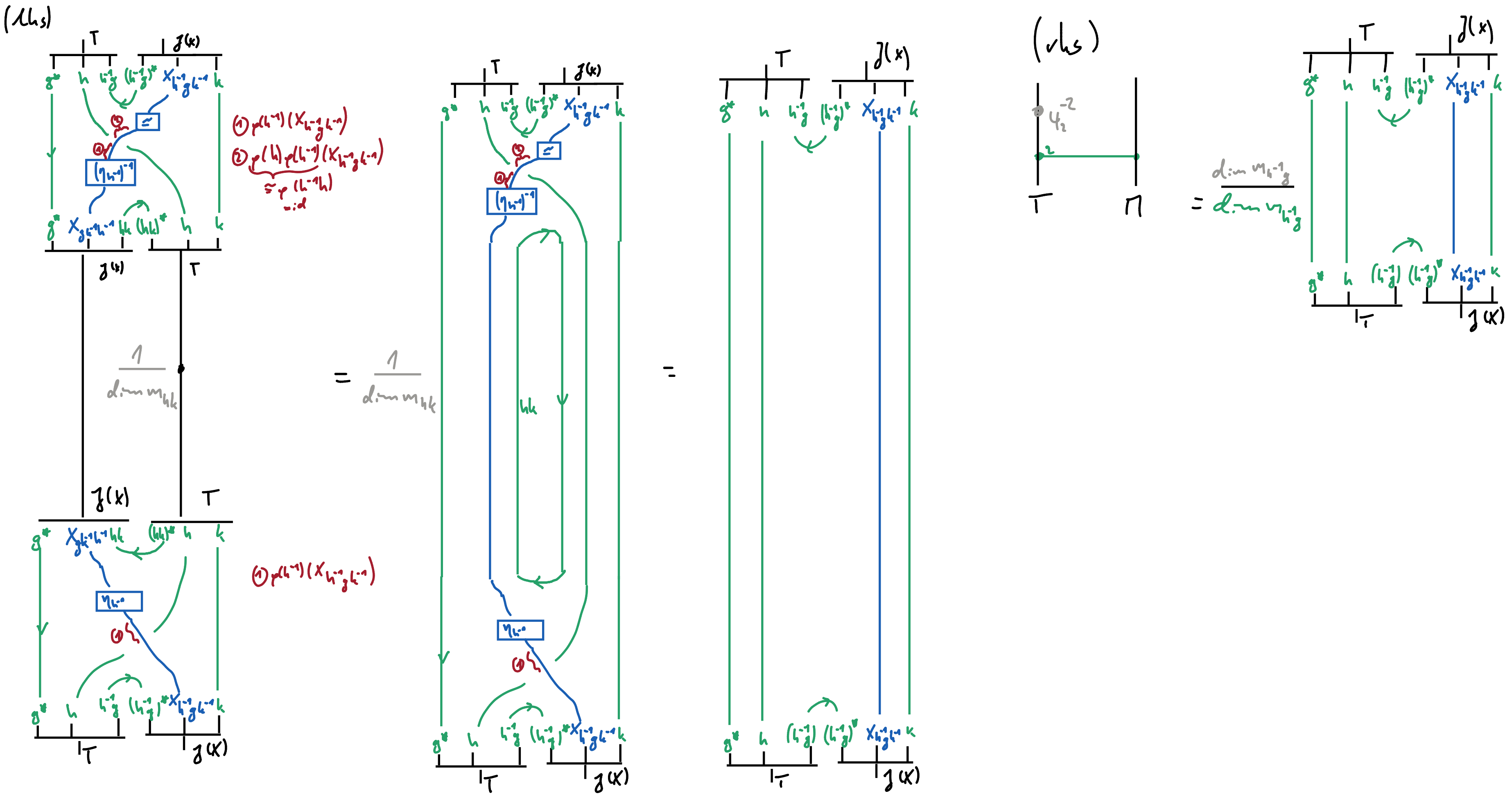


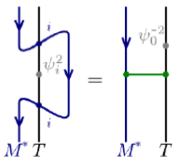
(T5) i=1





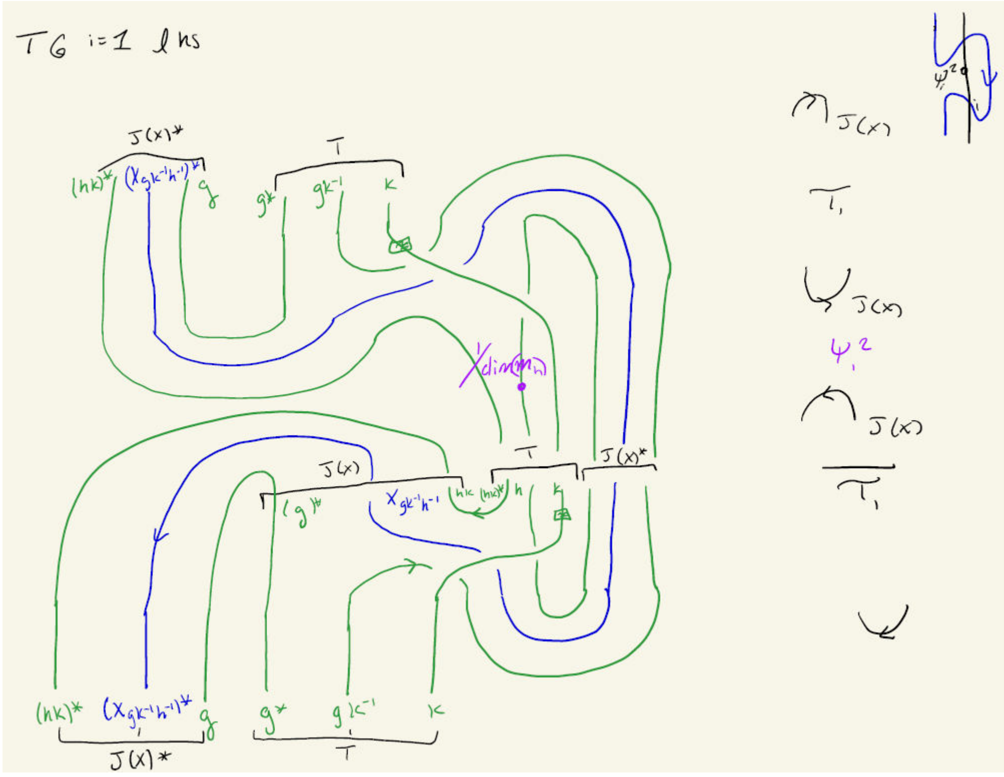
(T5) i=2

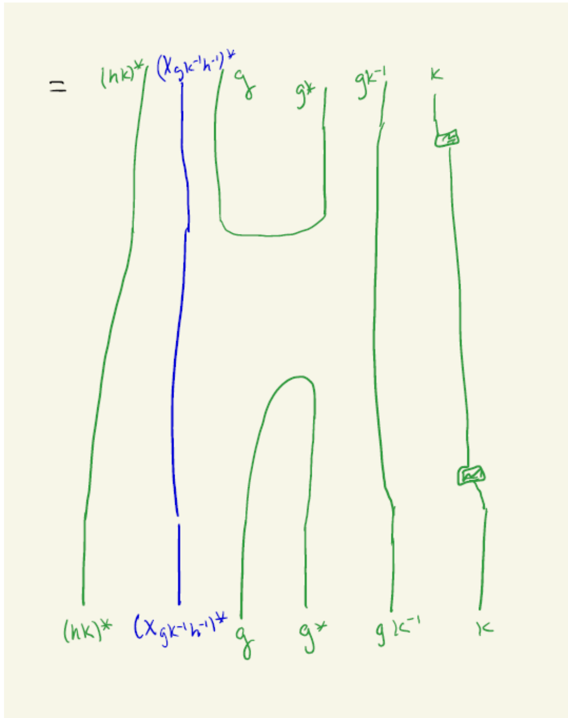




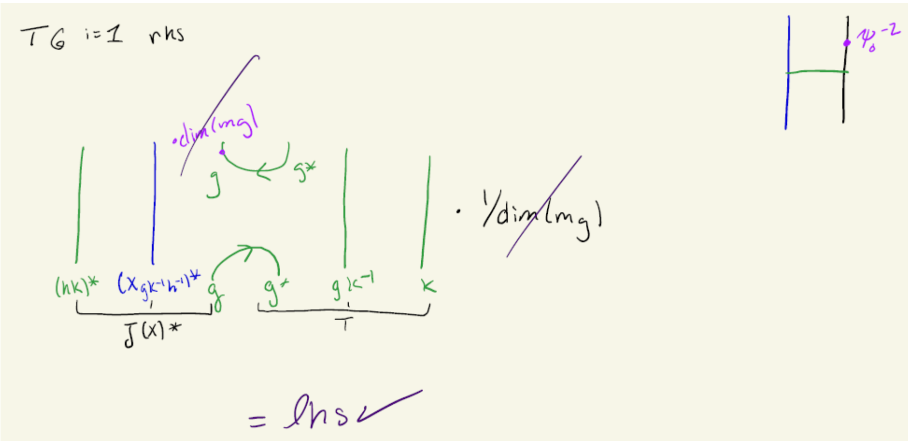
(T6)

T6 i=1 lhs

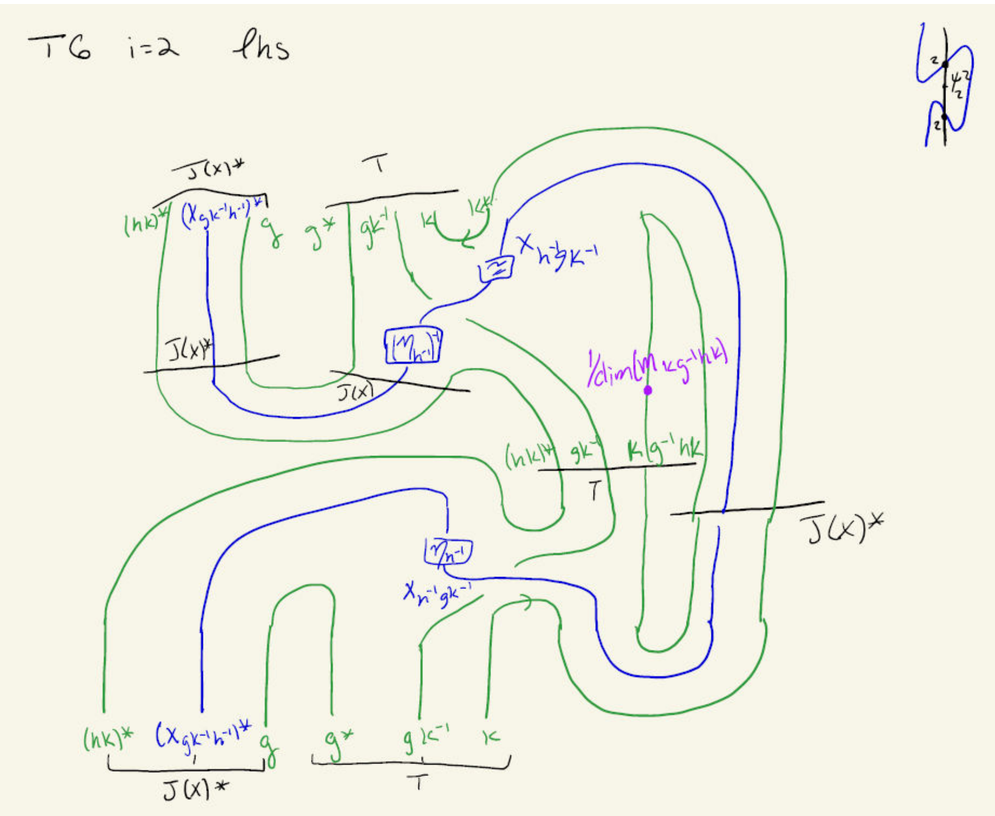


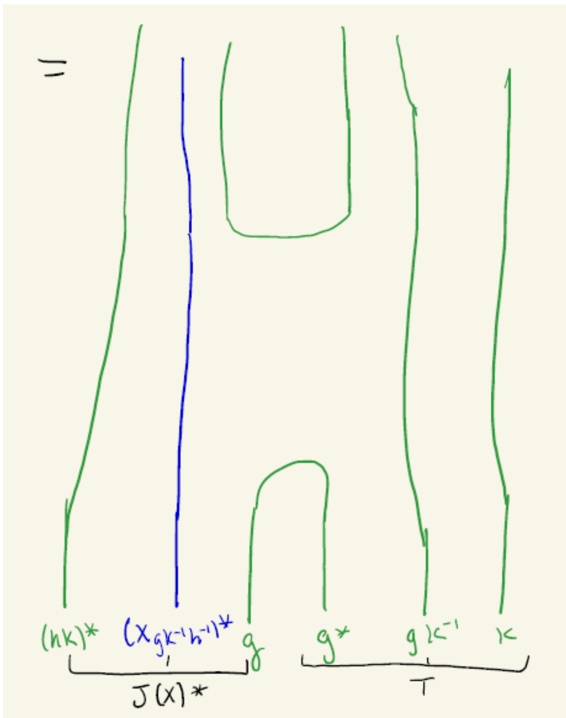


T6 i=1 rhs

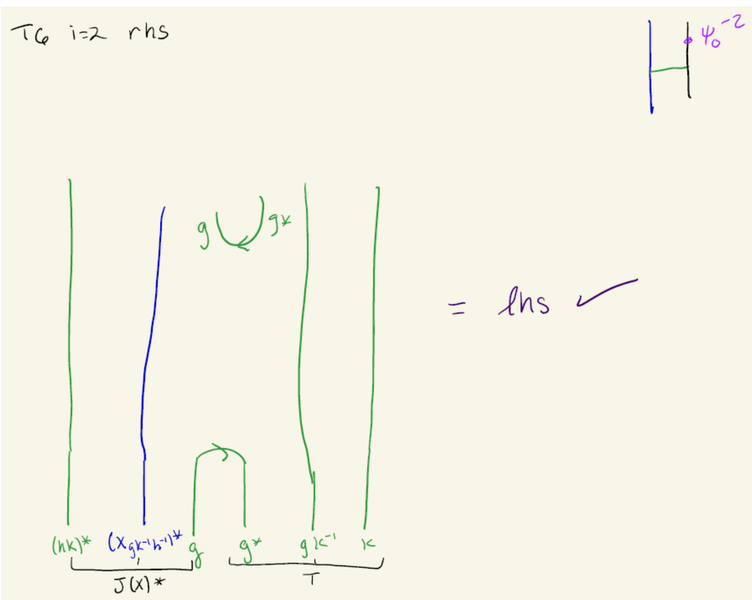


T6 i=2 lhs

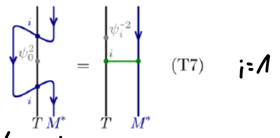




T6 i=2 rhs

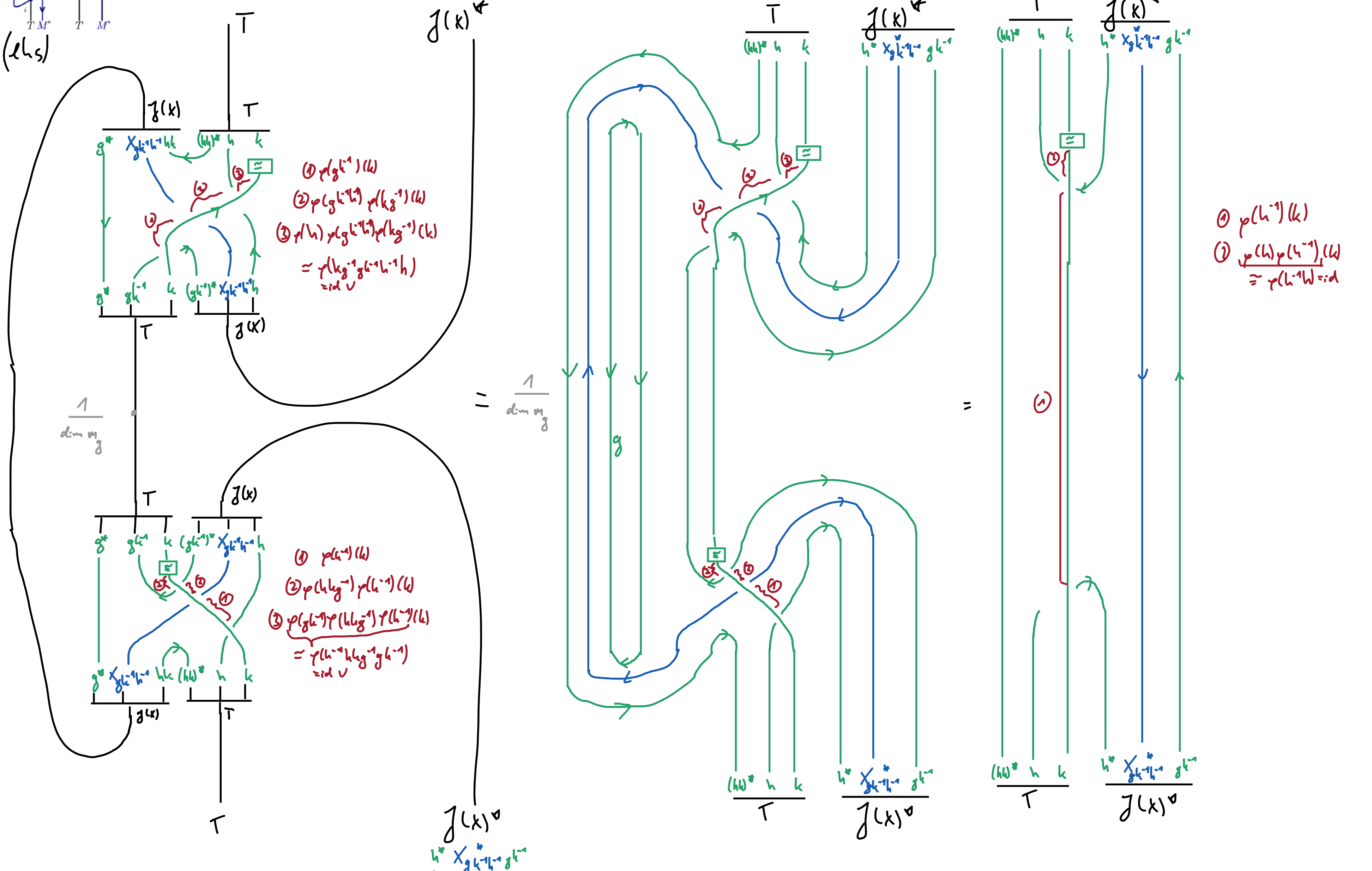


(T7)



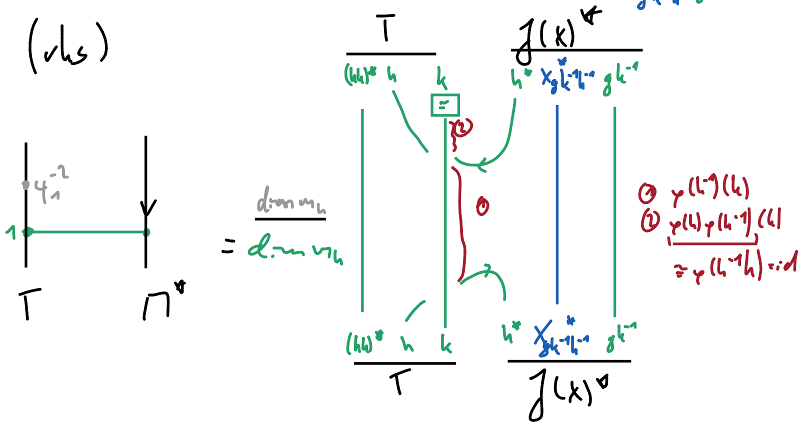
(T7)
i=1

(lhs)

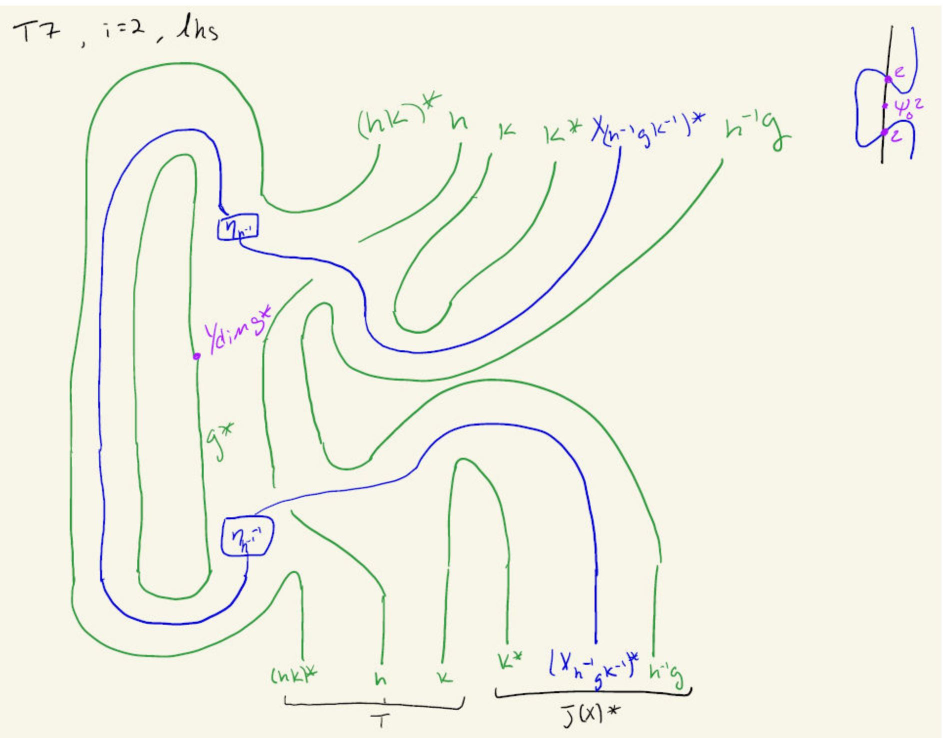


T
J(K)*
J(K)
$\frac{1}{\dim m_g}$
① φ(g^{-1})(k)
② φ(gk^{-1}h) φ(kg^{-1})(k)
③ φ(h) φ(gk^{-1}h) φ(kg^{-1})(k) = φ(kg^{-1}gk^{-1}h) =id ✓
① φ(k^{-1})(k)
② φ(hkg^{-1}) φ(k^{-1})(k)
③ φ(gk^{-1}) φ(hkg^{-1}) φ(k^{-1})(k) = φ(k^{-1}hkg^{-1}gk^{-1}) =id ✓
g
① φ(h^{-1})(k)
① φ(h) φ(h^{-1})(k) = φ(h^{-1}h) =id

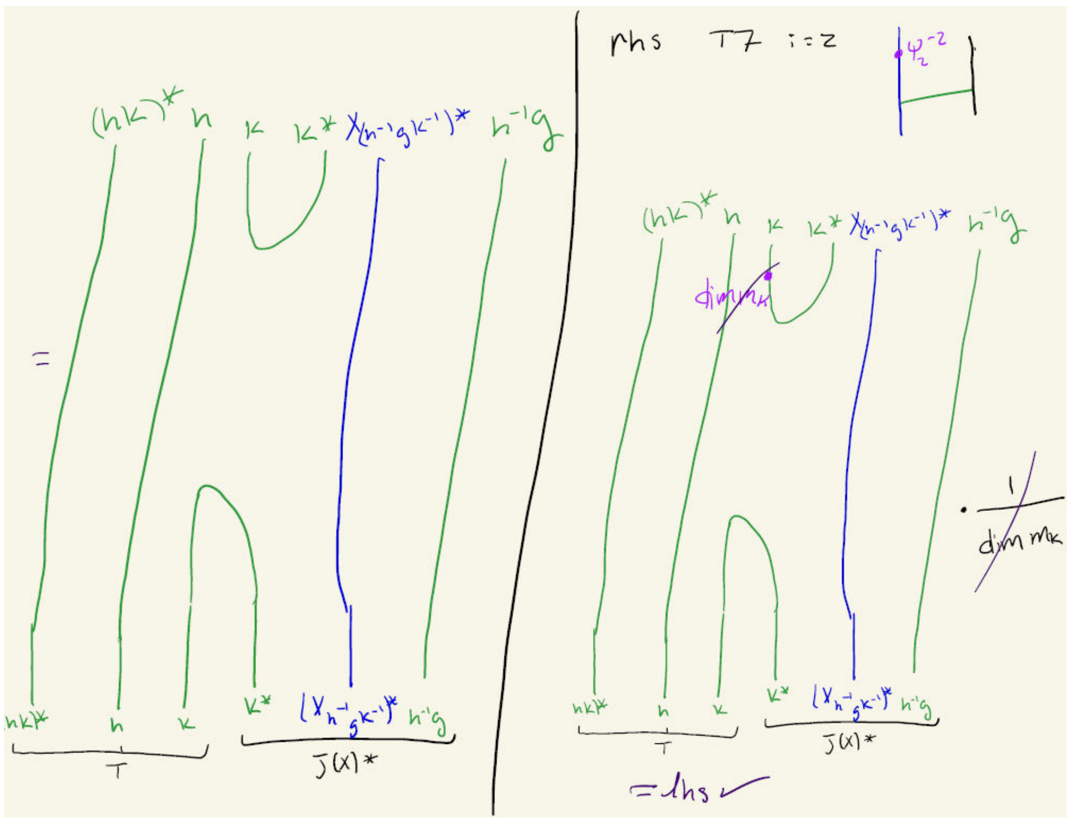
(rhs)



T
M*
$\frac{\dim m_h}{\dim m_h}$
J(K)*
① φ(h^{-1})(k)
① φ(h) φ(h^{-1})(k) = φ(h^{-1}h) =id



T7, i=2, lhs
(hk)* h k k* X_{(h^{-1}gk^{-1})*} h^{-1}g
1/dim g*
g*
T
J(K)*



rhs T7 i=2
(hk)* h k k* X_{(h^{-1}gk^{-1})*} h^{-1}g
dim m_K
1/dim m_K
T
J(K)*
= lhs ✓

Monoidal

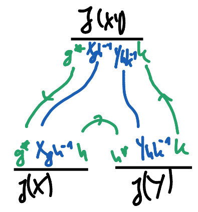
TODO: Think about E^(0).

$E^{(2)}: E(X,\eta)\underset{A}{\otimes}E(Y,\psi) \longrightarrow E(\underbrace{(X,\eta)\otimes^{G}(Y,\psi)}_{:=(X\otimes Y,\ \varphi(\ )(X\otimes Y)\xrightarrow{\tau_2^{-1}}\varphi(\ )(X)\otimes\varphi(\ )(Y)\xrightarrow{\eta\otimes\psi} X\otimes Y)})$

$(J(X),\tau_1^X,\tau_2^X)\underset{A}{\otimes}(J(Y),\tau_1^Y,\tau_2^Y)$

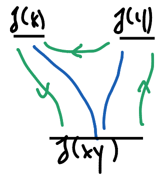
$(J(X)\underset{A}{\otimes}J(Y),\ \tau_1^{J(X)\otimes J(Y)},\ \tau_2^{J(X)\otimes J(Y)}) \longrightarrow (J(X\otimes Y),\ \tau_1^{J(X\otimes Y)},\ \tau_2^{J(X\otimes Y)})$

$\widehat{E^{(2)}} = (\dim h)^{-\frac{1}{2}}$



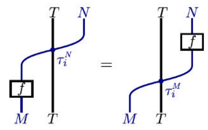
Are these inverse to each other?

$\widehat{E^{(2)}}^{-1} = (\dim h)^{-\frac{1}{2}}$



[CR2 3]

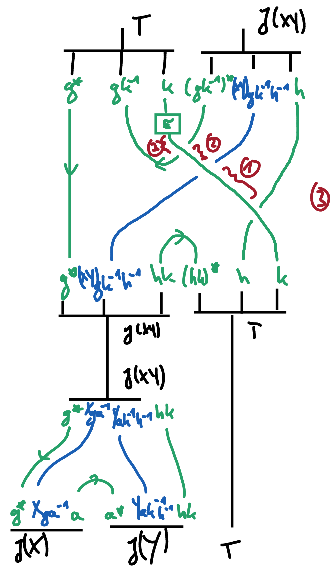
J is a morphism in $\mathcal{B}_A$, i.e.



$i = 1,2$. (M)

$\boxed{i=1}$

(lhs)



$(\dim a)^{-\frac{1}{2}}$

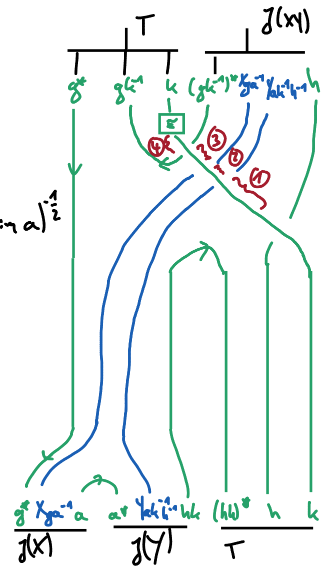
h→a, k→hk

① $\varphi(k^{-1})(k)$

② $\varphi(hkg^{-1})\varphi(k^{-1})(k)$

③ $\underbrace{\varphi(gk^{-1})\varphi(hkg^{-1})\varphi(k^{-1})(k)}_{=\varphi(k^{-1}hkg^{-1}gk^{-1}) = \mathrm{id}\ V}$

$= (\dim a)^{-\frac{1}{2}}$



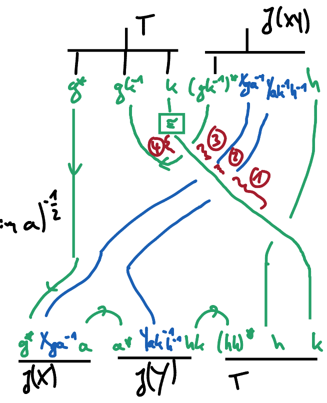
① $\varphi(k^{-1})(k)$

② $\varphi(hka^{-1})\varphi(k^{-1})(k)$

③ $\varphi(ag^{-1})\varphi(hka^{-1})\varphi(k^{-1})(k)$

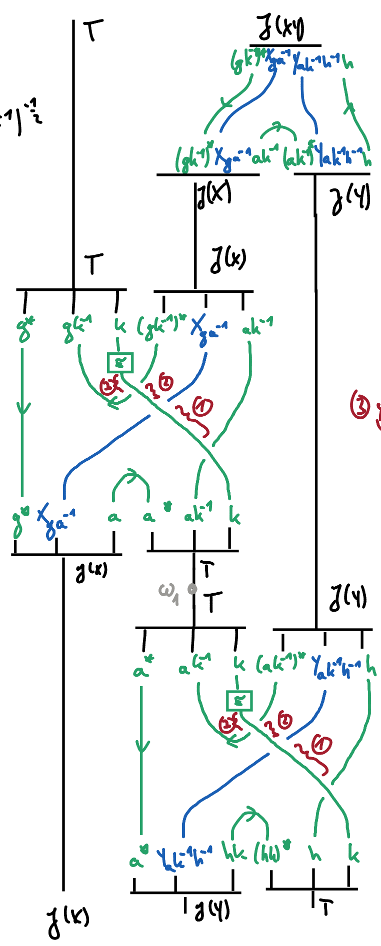
④ $\underbrace{\varphi(gk^{-1})\varphi(ag^{-1})\varphi(hka^{-1})\varphi(k^{-1})(k)}_{=\varphi(k^{-1}hka^{-1}ag^{-1}gk^{-1}) = \mathrm{id}\ V}$

$= (\dim a)^{-\frac{1}{2}}$



(rhs)

$\dim(hk^{-1})^{-\frac{1}{2}}$



k→h, h→ak^{-1}, g→gk^{-1}

h→ak^{-1}

① $\varphi(ka^{-1})(k)$

② $\varphi(ag^{-1})\varphi(ka^{-1})(k)$

③ $\underbrace{\varphi(gk^{-1})\varphi(ag^{-1})\varphi(ka^{-1})(k)}_{=\varphi(k^{-1}ag^{-1}gk^{-1}) = \mathrm{id}\ V}$

h→ak^{-1}

① $\varphi(k^{-1})(k)$

② $\varphi(hka^{-1})\varphi(k^{-1})(k)$

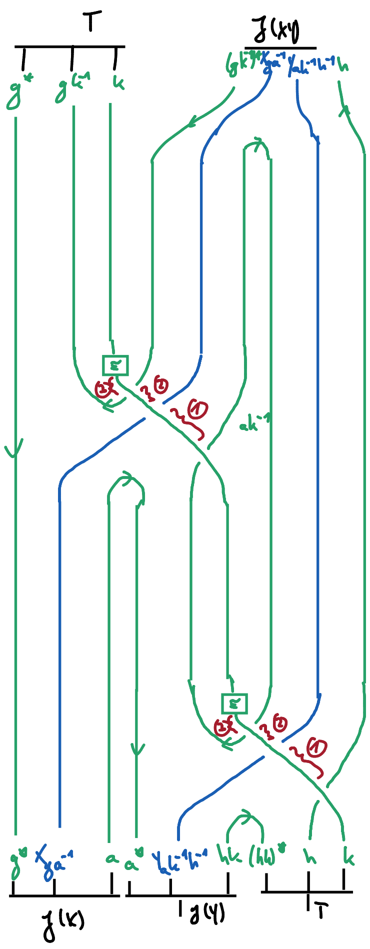
③ $\underbrace{\varphi(ak^{-1})\varphi(hka^{-1})\varphi(k^{-1})(k)}_{=\varphi(k^{-1}hka^{-1}k^{-1}) = \mathrm{id}\ V}$

g→a

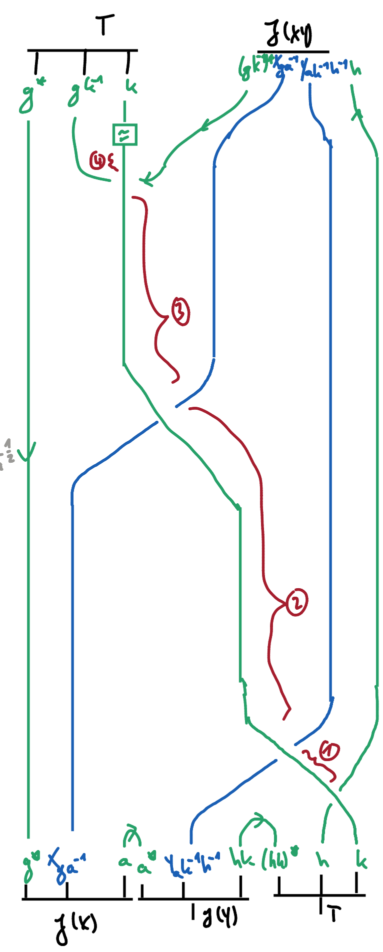
$= \dim(hk^{-1})^{-\frac{1}{2}}$

$\dim a^{-\frac{1}{2}}$

$\dim(ak^{-1})^{-\frac{1}{2}}$



$= \dim a^{-\frac{1}{2}}$



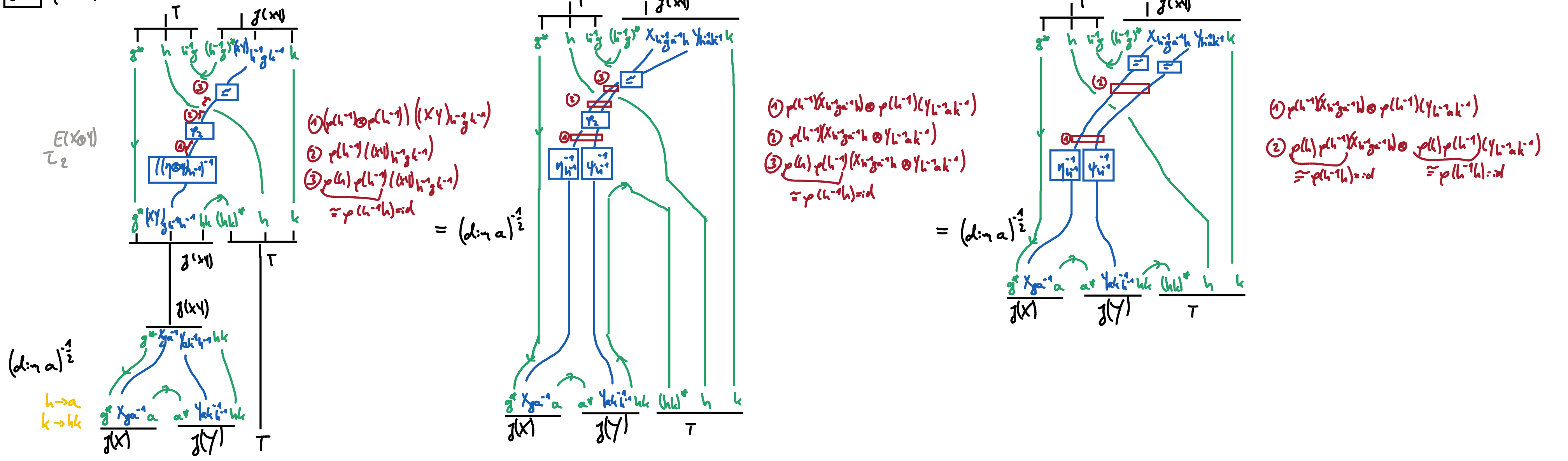
① $\varphi(k^{-1})(k)$

② $\varphi(hka^{-1})\varphi(k^{-1})(k)$

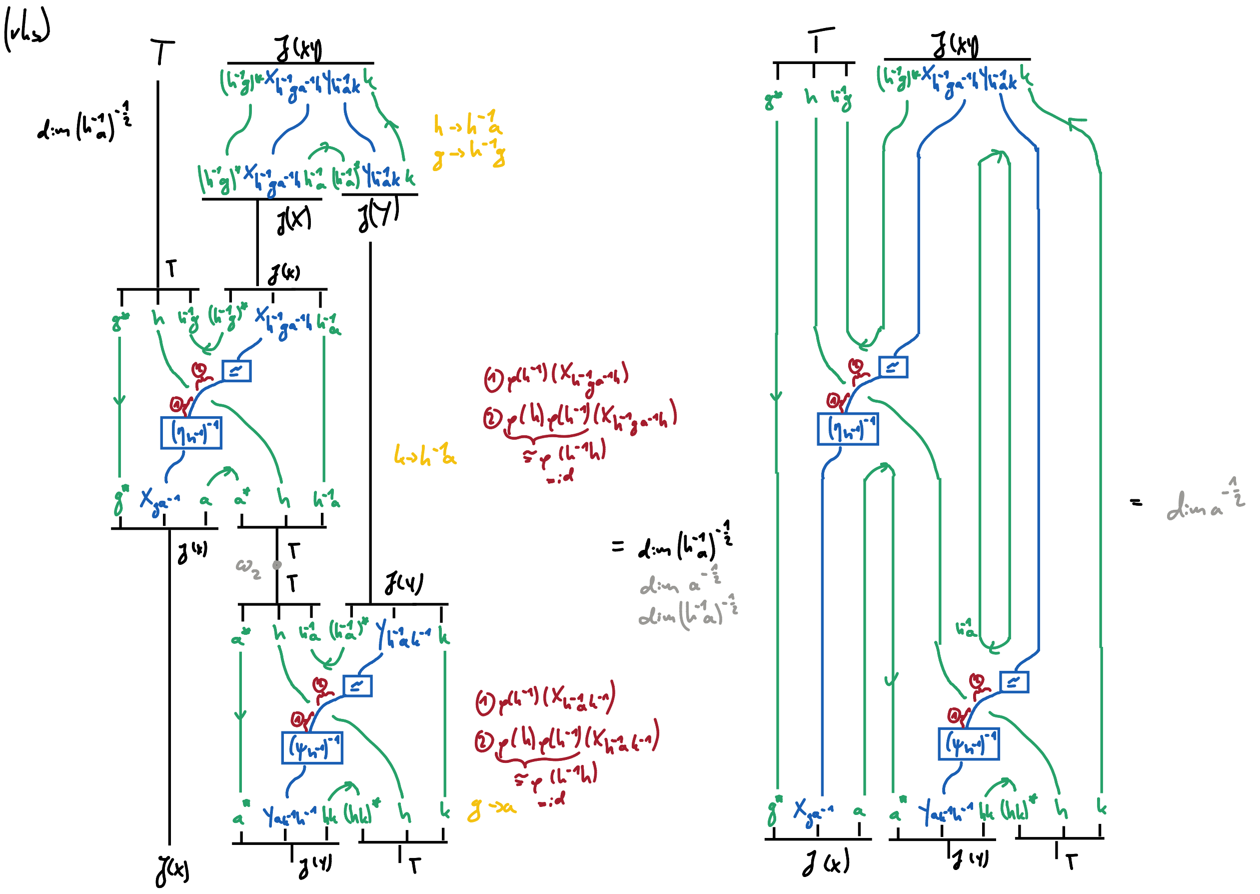
③ $\varphi(ag^{-1})\varphi(hka^{-1})\varphi(k^{-1})(k)$

④ $\underbrace{\varphi(gk^{-1})\varphi(ag^{-1})\varphi(hka^{-1})\varphi(k^{-1})(k)}_{=\varphi(k^{-1}hka^{-1}ag^{-1}gk^{-1}) = \mathrm{id}\ V}$

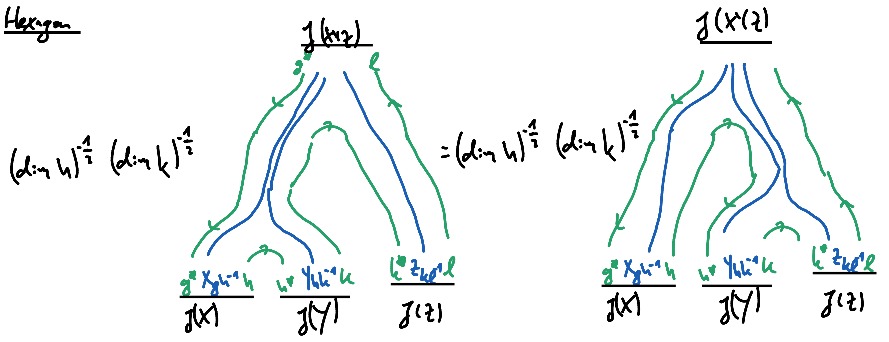
$\boxed{i=2}$ (lhs)



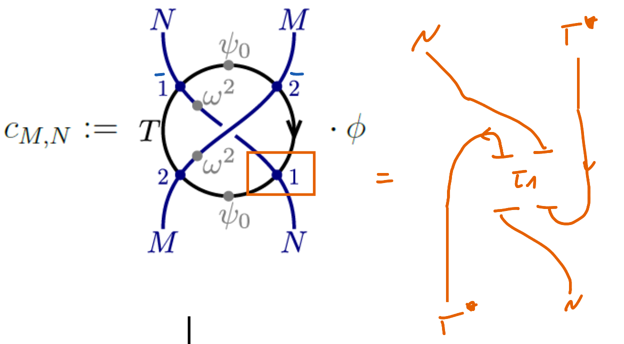
(rhs)



Hexagon

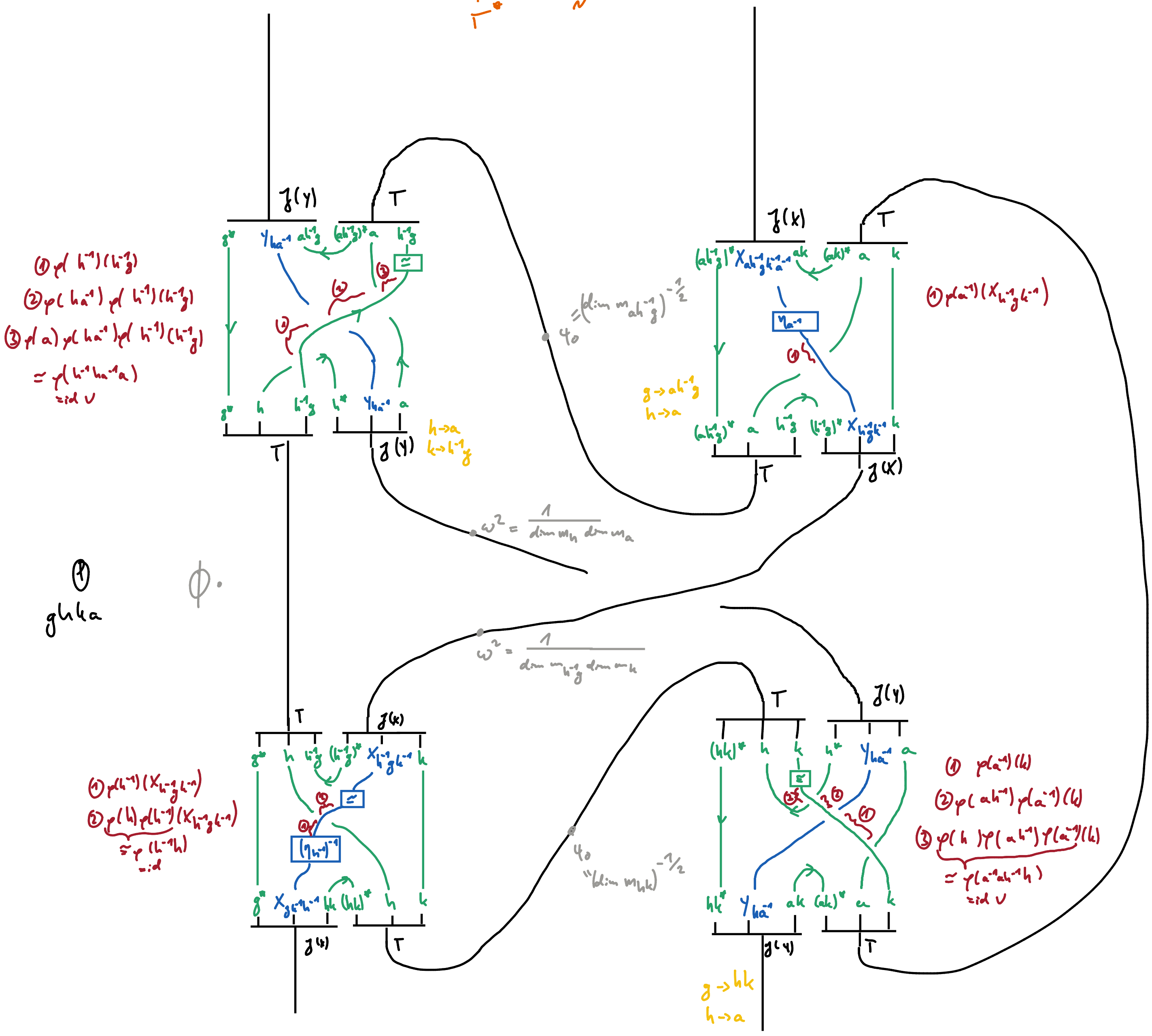


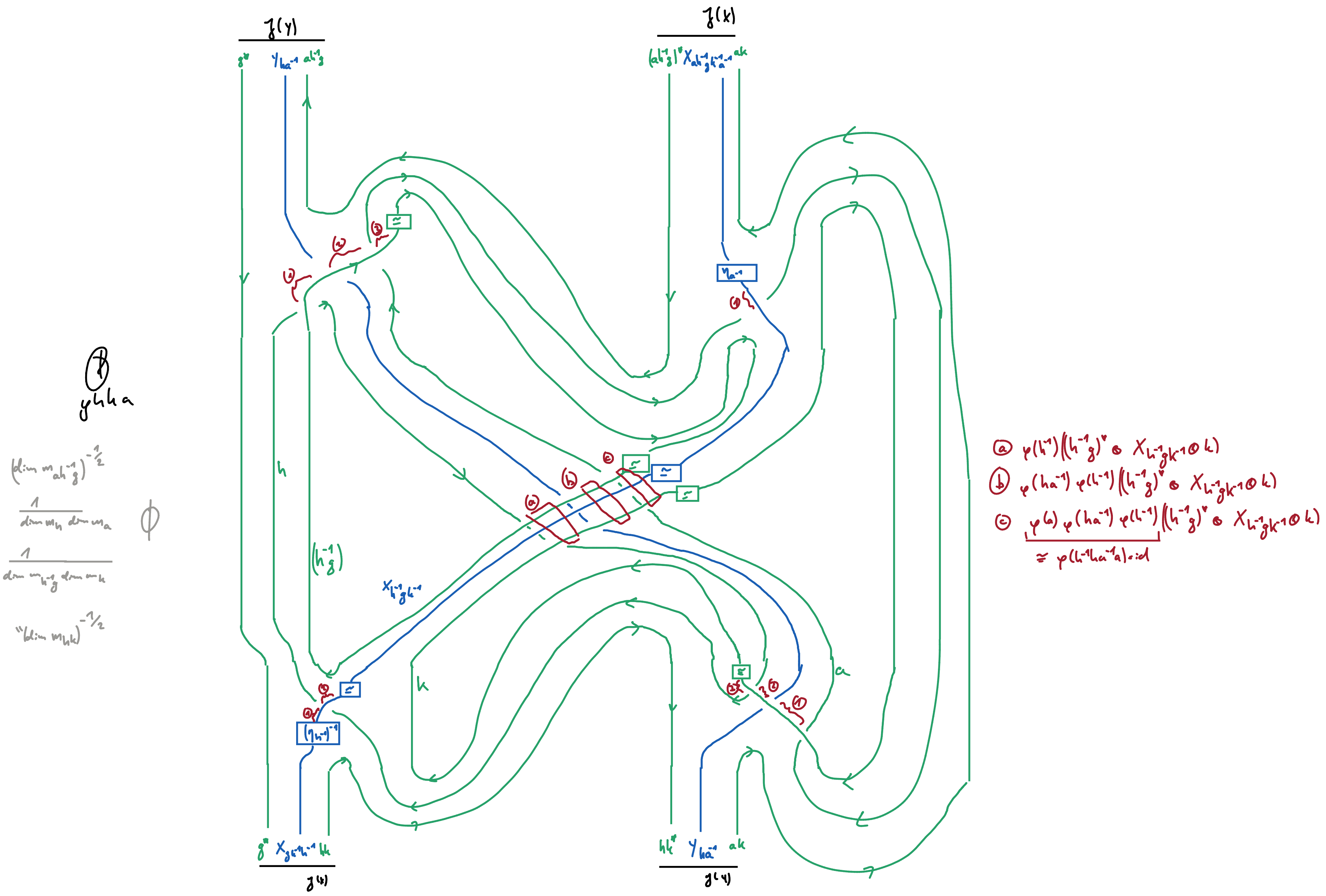
Thus, E is monoidal.



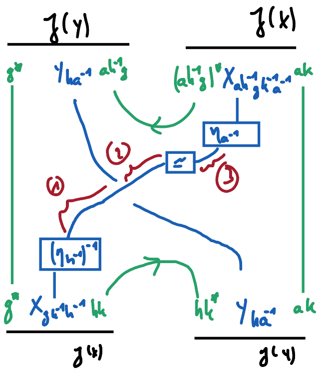
[MR, 1.1.3]

$\phi = \frac{1}{|G|}$





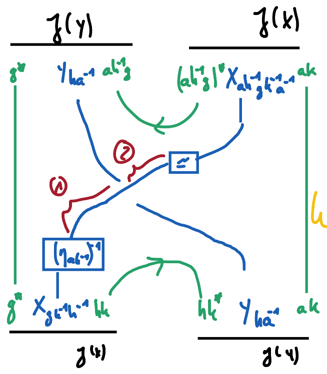
$\bigoplus_{g,h,k,a}$ $(\dim m_{ah^{-1}g})^{-\frac{1}{2}} (\dim m_{hk})^{-1/2} \frac{1}{|G|}$



① $\varphi(k^{-1})(X_{h^{-1}gk^{-1}})$
② $\varphi(ha^{-1})\varphi(k^{-1})(X_{h^{-1}gk^{-1}})$
③ $\cong \varphi(k^{-1}ha^{-1})(X_{h^{-1}gk^{-1}})$
$= \varphi(a^{-1})(X_{h^{-1}gk^{-1}})$

= see picture below

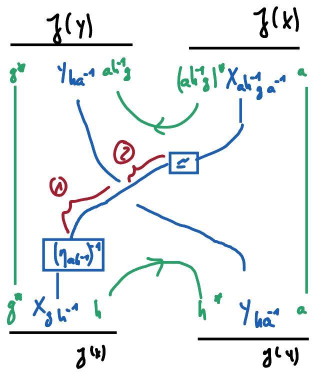
$\bigoplus_{g,h,k,a}$ $(\dim m_{ah^{-1}g})^{-\frac{1}{2}} (\dim m_{hk})^{-1/2} \frac{1}{|G|}$



① $\varphi(ak^{-1})(X_{h^{-1}gk^{-1}})$
② $\varphi(ha^{-1})\varphi(ak^{-1})(X_{h^{-1}gk^{-1}})$
$\cong \varphi(ak^{-1}ha^{-1})(X_{h^{-1}gk^{-1}})$

= $k \rightsquigarrow e$

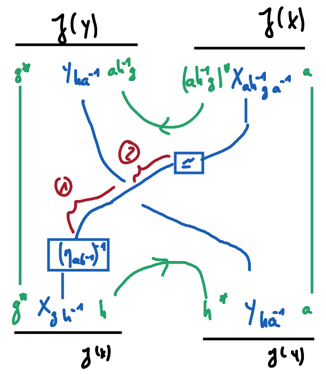
$\bigoplus_{g,h,a}$ $(e \rightsquigarrow |G|)$ $(\dim m_{ah^{-1}g})^{-\frac{1}{2}} (\dim m_{h})^{-1/2} \frac{1}{|G|}$



① $\varphi(ak^{-1})(X_{h^{-1}g})$
② $\varphi(ha^{-1})\varphi(ak^{-1})(X_{h^{-1}g})$
$\cong \varphi(ak^{-1}ha^{-1})(X_{h^{-1}g})$

||

$\bigoplus_{g,h,a}$ $(\dim m_{ah^{-1}g})^{-\frac{1}{2}} (\dim m_{h})^{-1/2}$



① $\varphi(ak^{-1})(X_{h^{-1}g})$
② $\varphi(ha^{-1})\varphi(ak^{-1})(X_{h^{-1}g})$
$\cong \varphi(ak^{-1}ha^{-1})(X_{h^{-1}g})$

---

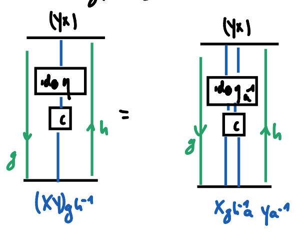
braiding on $\mathcal{B}^G$: $c^{\mathcal{B}^G}: (X,\eta)\otimes(Y,\psi) = (\bigoplus_g X\otimes Y,\ \rho(g)(X\otimes Y) \xrightarrow{\rho_2^{-1}} \rho(g)(X)\otimes\rho(g)(Y) \xrightarrow{\eta\otimes\psi} X\otimes Y)$

$\downarrow$

$(Y,\psi)\otimes(X,\eta) = (\bigoplus_g Y\otimes X,\ \rho(g)(Y\otimes X) \xrightarrow{\rho_2^{-1}} \rho(g)(Y)\otimes\rho(g)(X) \xrightarrow{\psi\otimes\eta} Y\otimes X)$

$c^{\mathcal{B}^G} := \bigoplus_{g,h} c^{\mathcal{B}^G}_{g,h}$, $\quad c^{\mathcal{B}^G}_{g,h}: X_g\otimes Y_h \xrightarrow{c_{X,Y}} Y_h\otimes\varphi(h)(X_g) \xrightarrow{\mathrm{id}\otimes\eta_h} Y_h\otimes X_g$

$E(c^{\mathcal{B}^G})$:



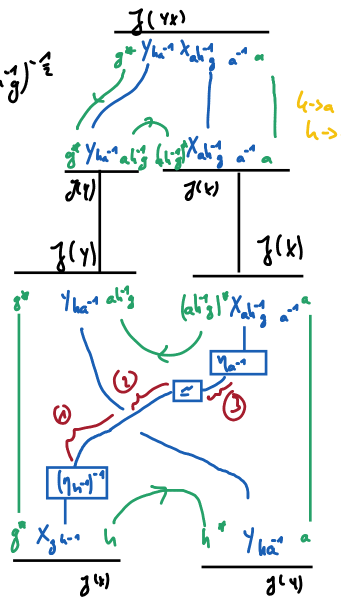
braided:

$$\begin{tikzcd} E(X)\otimes E(Y) \arrow[r, "c^{\mathcal{B}_M}"] \arrow[d, "E^2"'] & E(Y)\otimes E(X) \arrow[d, "E^2"] \\ E(X\otimes Y) \arrow[r, "E(c^{\mathcal{B}^G})"'] & E(Y\otimes X) \end{tikzcd}$$

$(\dim m_{ah^{-1}g})^{-\frac{1}{2}}$

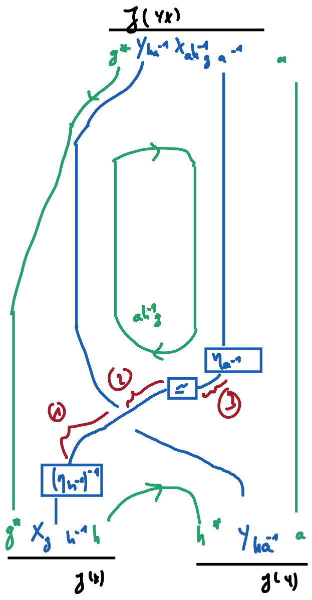
$k \to a$
$h \to ah^{-1}g$

$(\dim m_{ah^{-1}g})^{-\frac{1}{2}} (\dim m_{h})^{-1/2}$

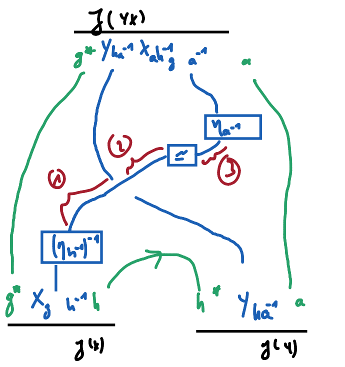


① $\varphi(k^{-1})(X_{h^{-1}g})$
② $\varphi(ha^{-1})\varphi(k^{-1})(X_{h^{-1}g})$
③ $\cong \varphi(k^{-1}ha^{-1})(X_{h^{-1}g})$
$= \varphi(a^{-1})(X_{h^{-1}g})$

= $(\dim m_{ah^{-1}g})^{-\frac{1}{2}}$ $(\dim m_{ah^{-1}g})^{-\frac{1}{2}} (\dim m_{h})^{-1/2}$

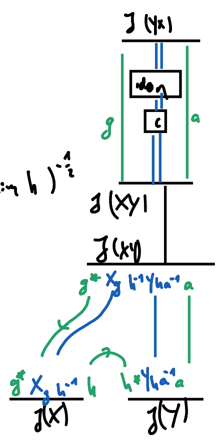


= $(\dim m_{h})^{-1/2}$



||

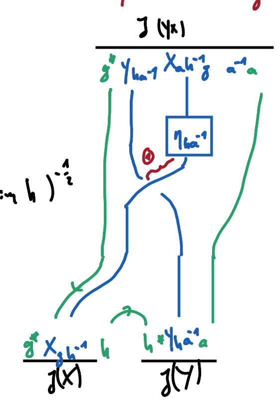
$(\dim h)^{-\frac{1}{2}}$



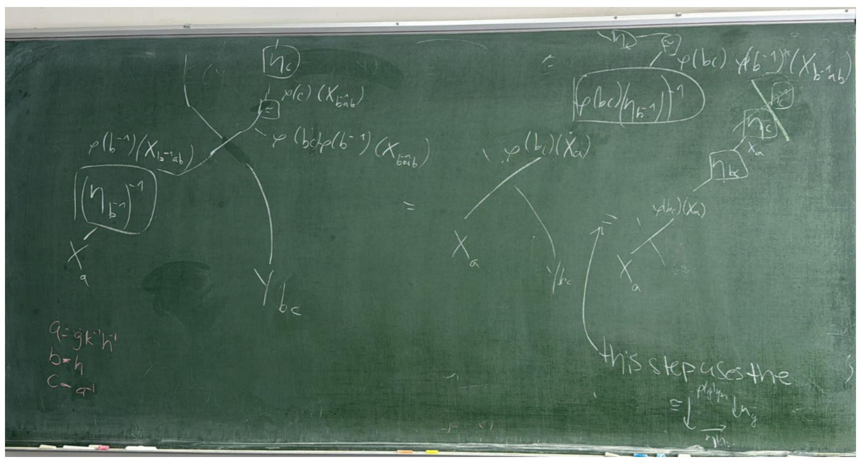
$k \to a$

= $(\dim h)^{-\frac{1}{2}}$

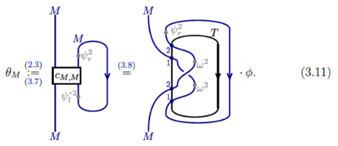
① $\varphi(ha^{-1})(X_{ah^{-1}g}\ a^{-1}a)$



=

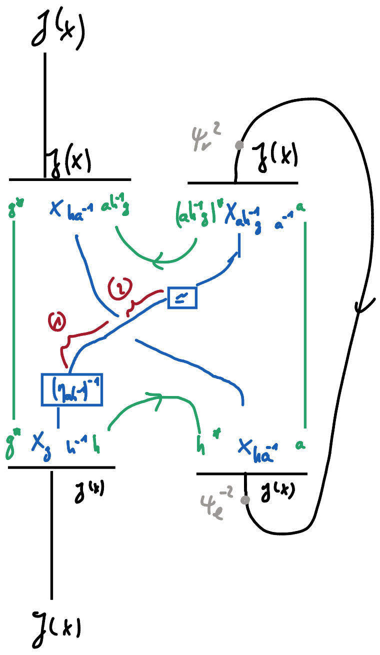


ribbon



$\Theta^{B/A}_{E(x)} =$

$(\dim m_{ah^{-1}g})^{-\frac{1}{2}} \ (\dim m_{hk})^{-1/2}$



① $\varphi(h^{-1})(X_{h^{-1}gk^{-1}})$

② $\varphi(ha^{-1})\varphi(h^{-1})(X_{h^{-1}gk^{-1}})$

③ $\cong \varphi(h^{-1}ha^{-1})(X_{h^{-1}gk^{-1}})$

$= \varphi(a^{-1})(X_{h^{-1}gk^{-1}})$

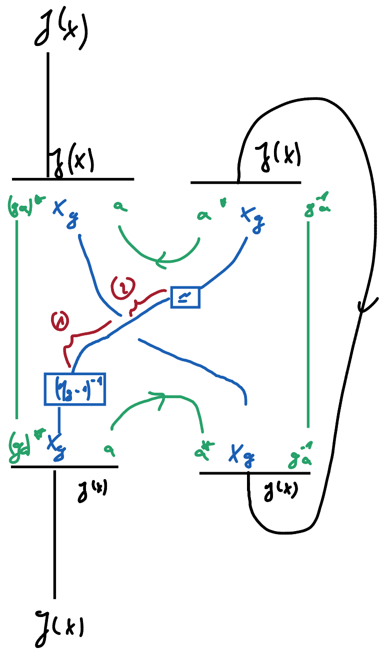
$= (\dim m_{ga^{-1}})^{-1}$

$(\dim m_a)^{-\frac{1}{2}}$

$(\dim m_a)^{-1/2}$

only works for

$ga \ X_g \ a \ a \ Y_g \ g a^{-1}$ $\dim m_a$

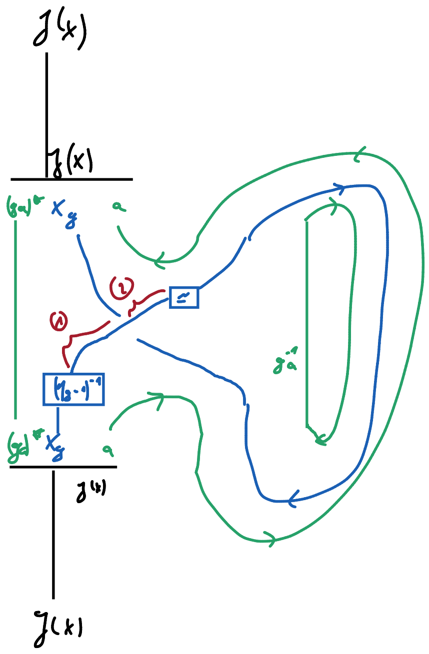


① $\varphi(g^{-1})(X_g)$

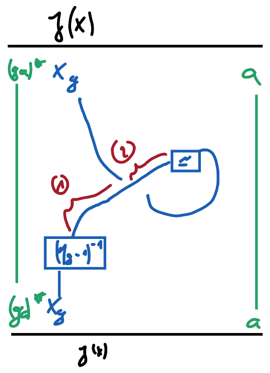
② $\varphi(g)\varphi(g^{-1})(X_g)$

$\cong id$

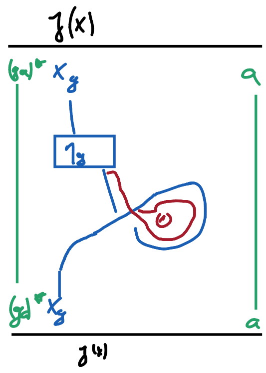
$=$ $(\dim m_{ga^{-1}})^{-1}$



$=$



$E(\Theta_{(X,M)}) =$



⓪ $\varphi(g)(X_g)$

Essential surjectivity

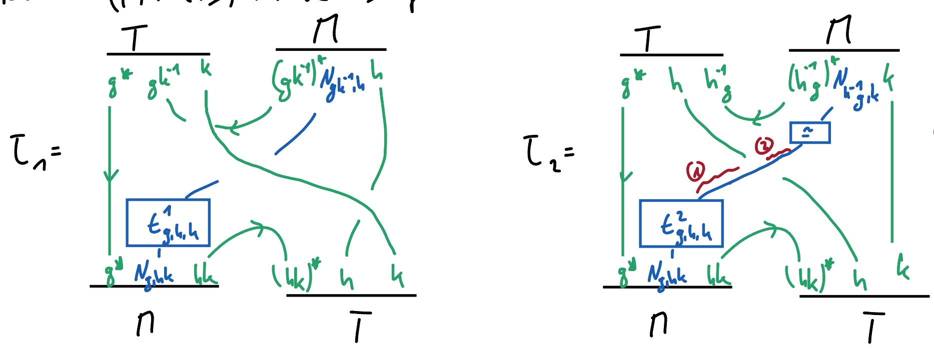
① $X := \bigoplus_{b\in G} M_{b,e}$

$\eta_a : \varphi(a)(X) \xrightarrow{\simeq} X$

defined via

$$\eta_a|_{a^{-1}ba} = \left[\varphi(a)(M_{b,e}) \xrightarrow{(t^2_{a^{-1}b,a^{-1},e})^{-1}} M_{a^{-1}b,a^{-1}} \xrightarrow{(t^1_{a^{-1}ba,a^{-1},a})^{-1}} M_{a^{-1}ba,e}\right]$$

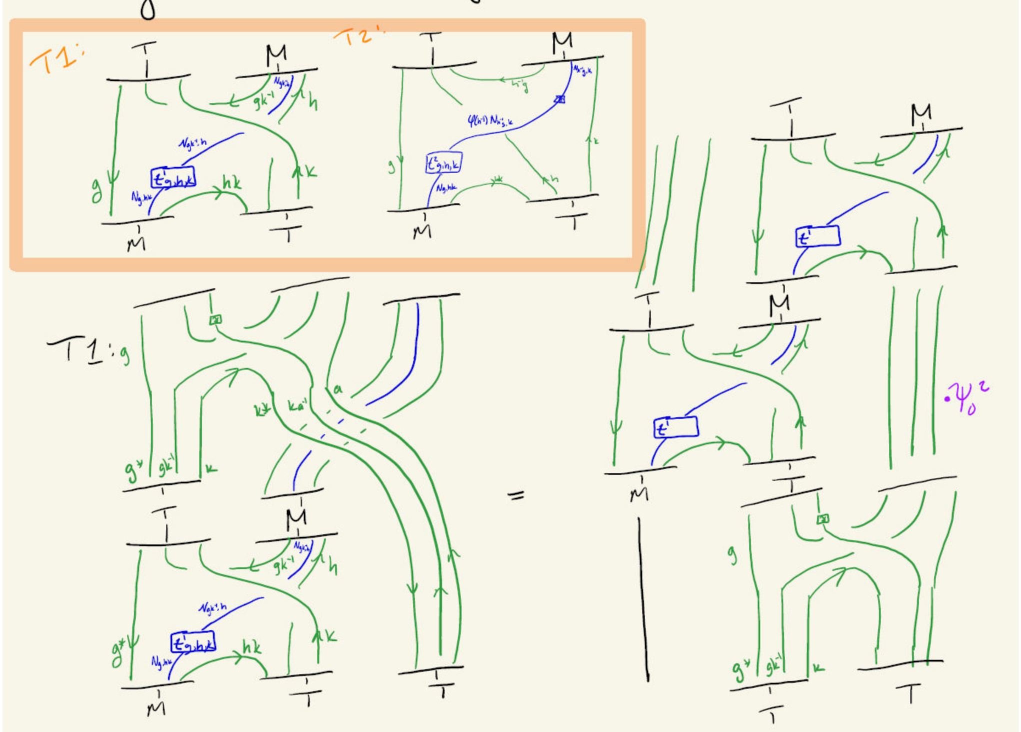
Rewrite (T1)-(T3) in terms of

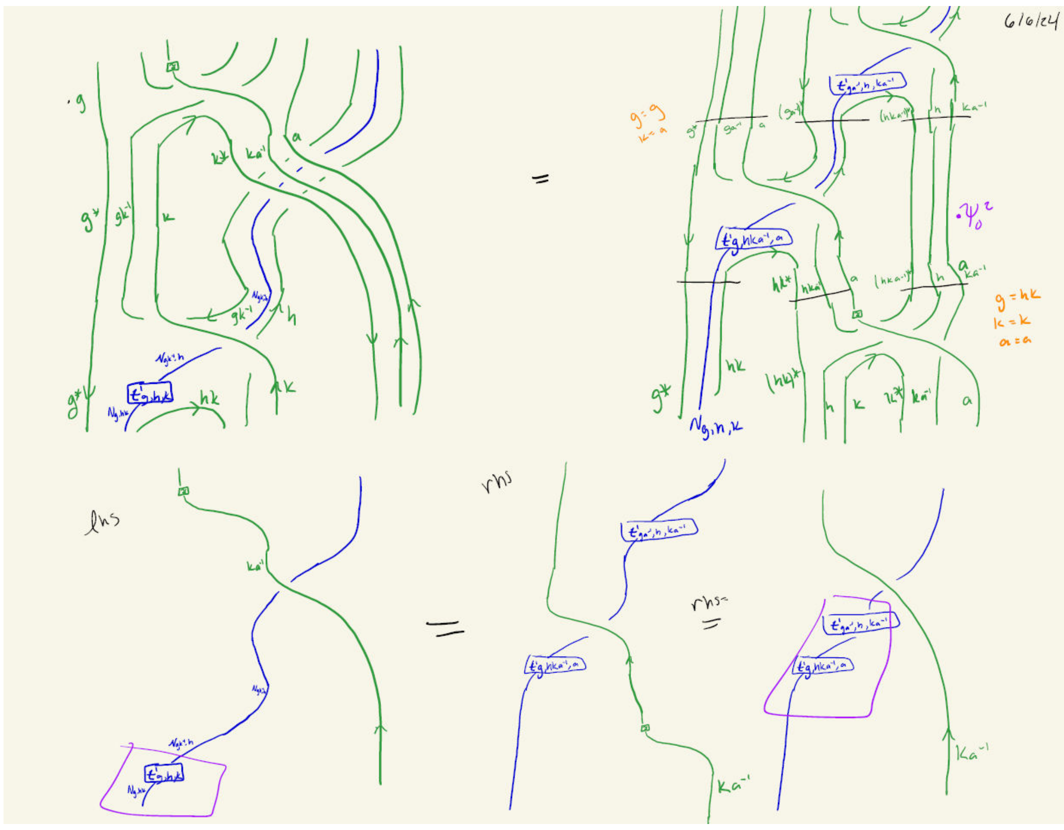


① $\varphi(h^{-1})(N_{h^{-1}g,k})$

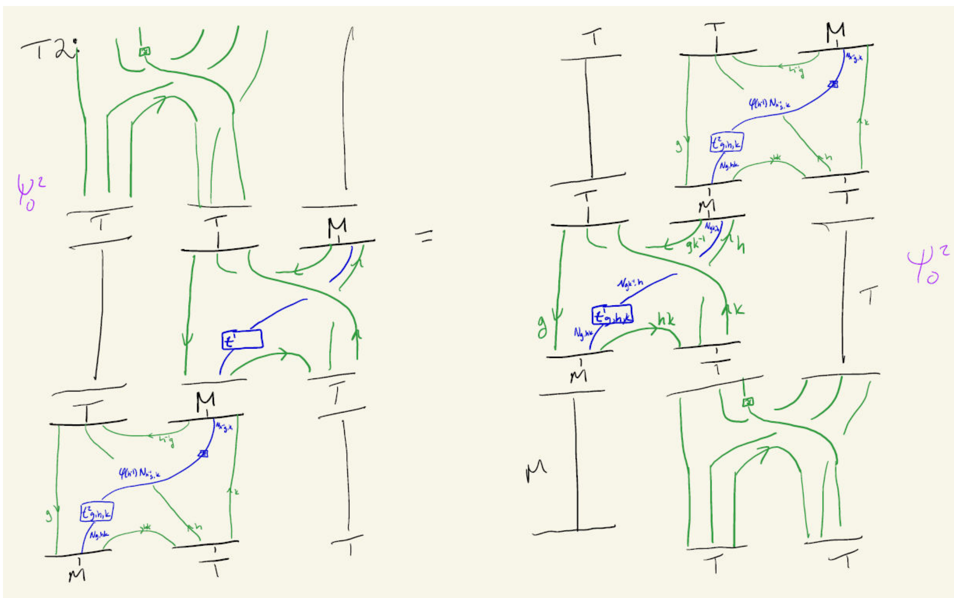
② $\underbrace{\varphi(h)\varphi(h^{-1})}_{\simeq\ \varphi(h^{-1}h)=id}(N_{h^{-1}g,k})$

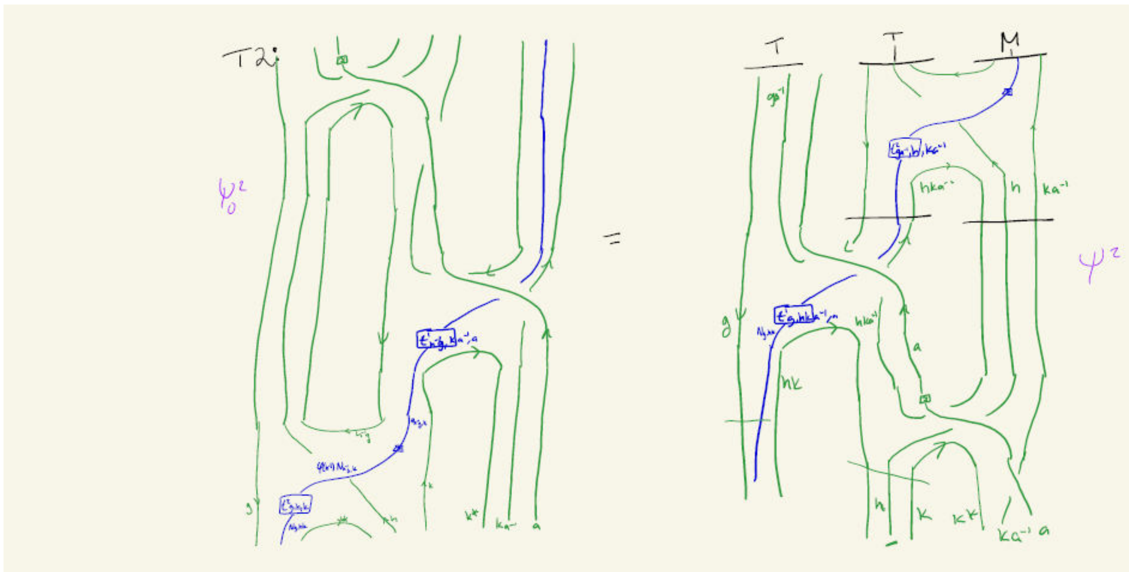
Rewriting T1-T3 with t' diagrams 6/6/24

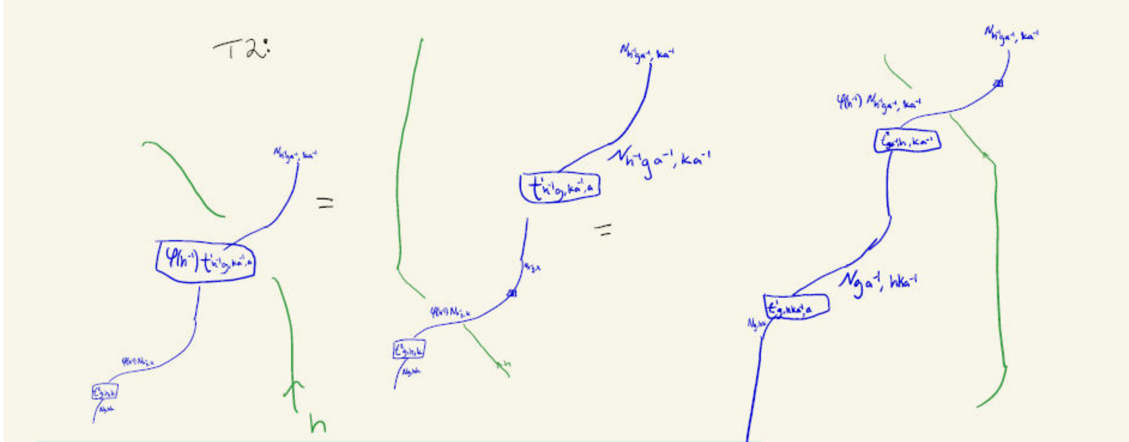




So, (T1) yields $\boxed{t'_{g,h,k} = t'_{ga^{-1},h,ka^{-1}}(t'_{g,hka^{-1},a})}$

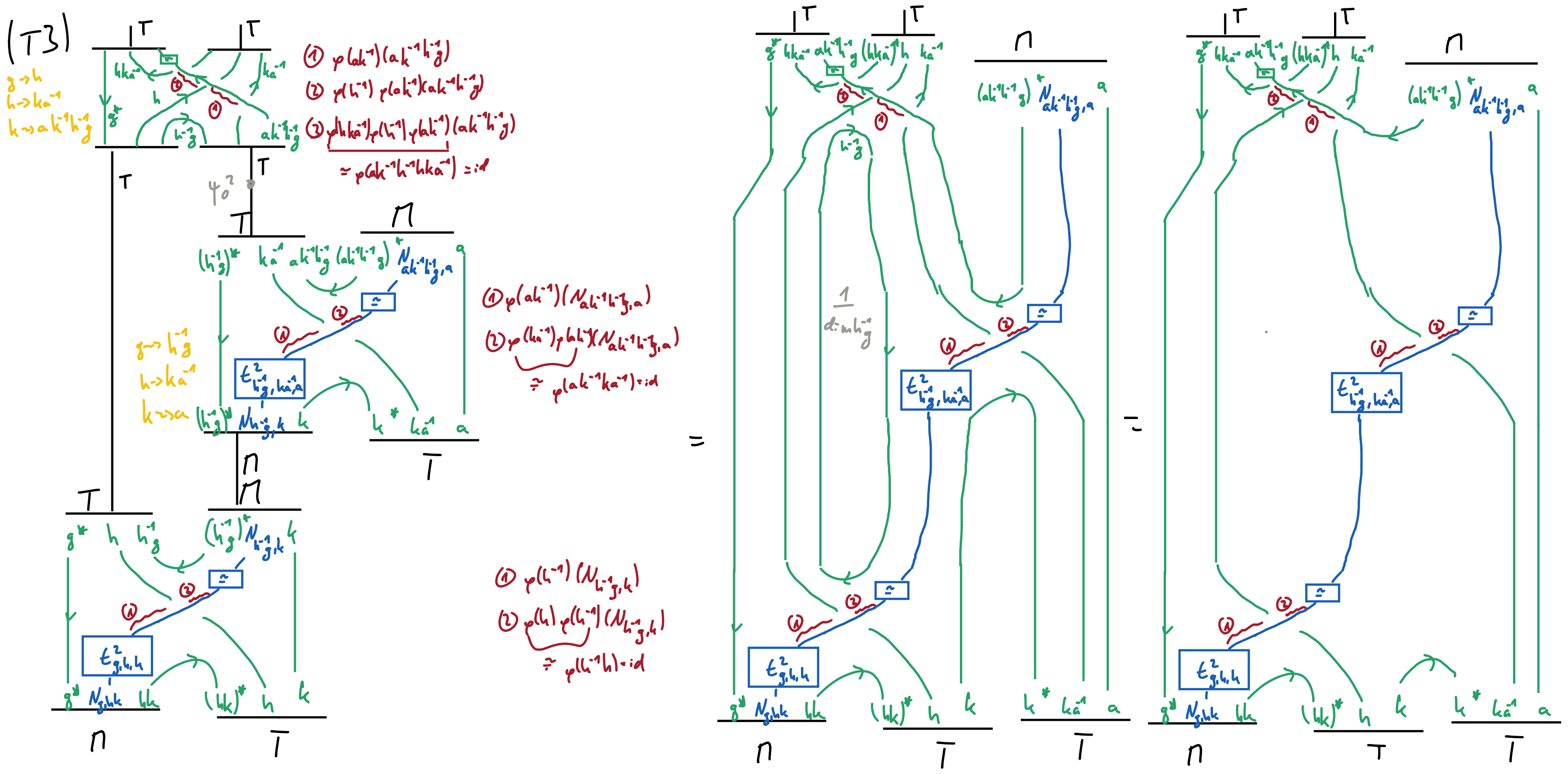




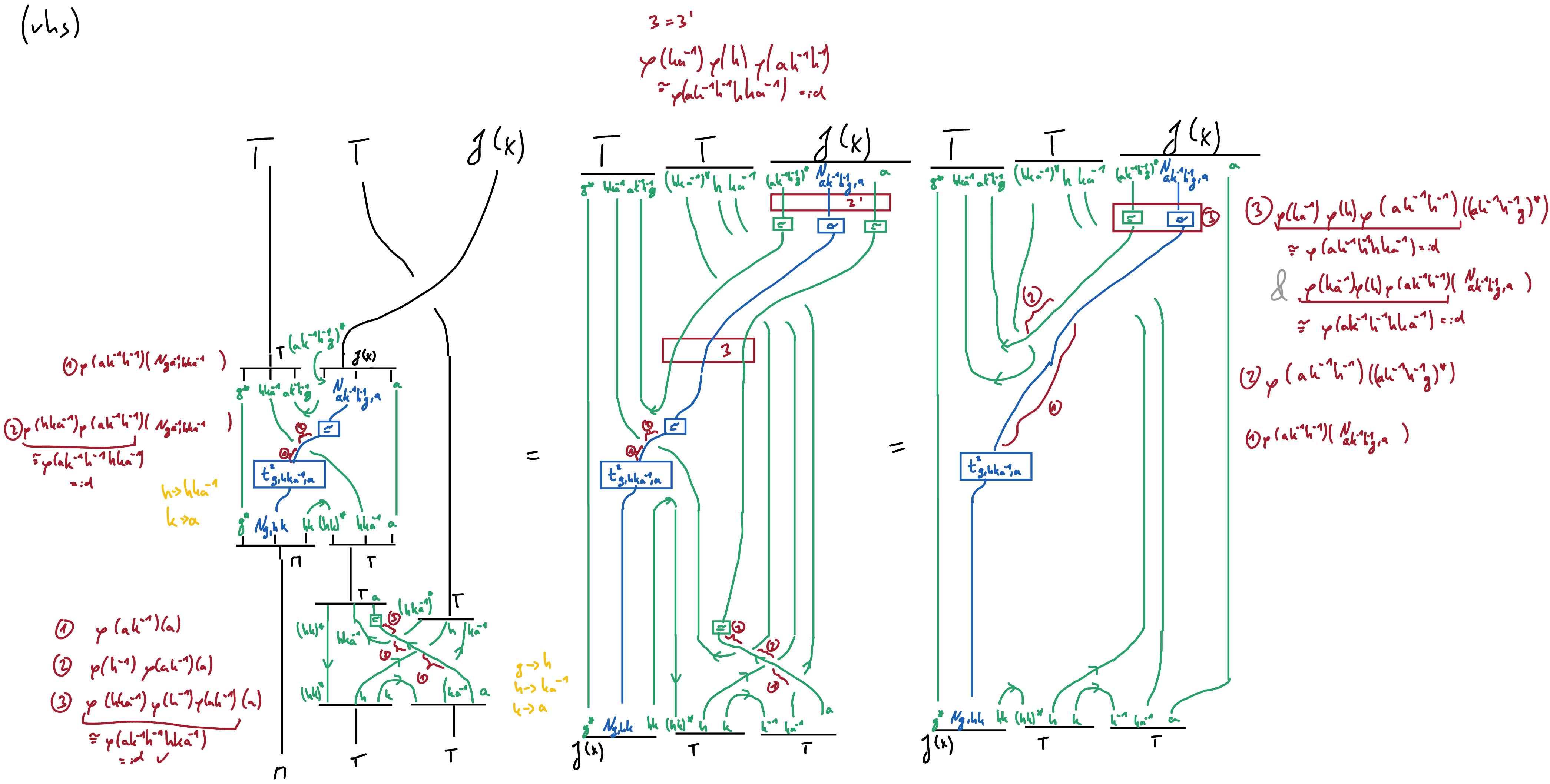


$$\boxed{\varphi(h^{-1})\, t'_{h^{-1}g,ka^{-1},a} \circ t^2_{g,h,k} = t^2_{ga^{-1},h,ka^{-1}} \circ t'_{g,hka^{-1},a}}$$

(T3)



(rhs)



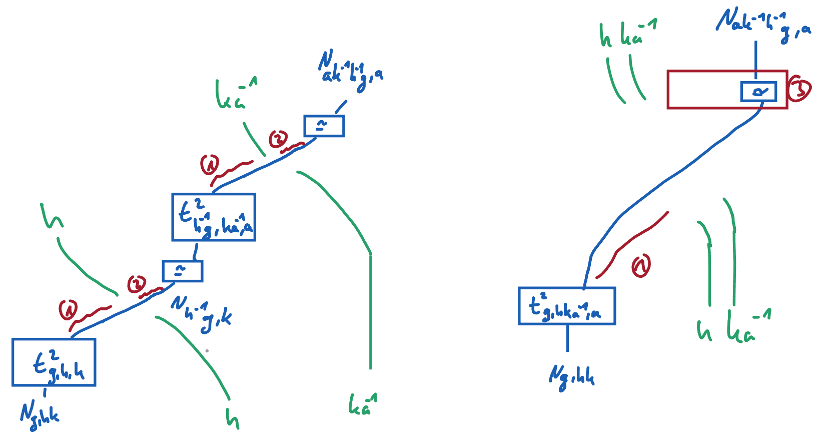
So (T3) implies

① $\rho(ak^{-1})(N_{ak^{-1}h^{-1}g,a})$

② $\underbrace{\rho(hk^{-1})\rho(kh^{-1})}_{\cong\, \rho(ak^{-1}ka^{-1}) = id}(N_{ak^{-1}h^{-1}g,a})$

① $\rho(h^{-1})(N_{h^{-1}g,k})$

② $\underbrace{\rho(h)\rho(h^{-1})}_{\cong\, \rho(h^{-1}h) = id}(N_{h^{-1}g,k})$



$$\boxed{\left(\rho(h^{-1})\, t^2_{h^{-1}g,\, hka^{-1},\, a}\right) \circ \left(t^2_{g,h,k}\right) = t^2_{g,\, hka^{-1},\, a}}$$

TODO: Check with isomorphism

Check:

$$\begin{array}{ccc} \varphi(g)\varphi(h)(X) & \xrightarrow{\varphi(g)(\eta_h)} & \varphi(g)(X) \\ \downarrow & & \downarrow \eta_g \\ \varphi(hg)(X) & \xrightarrow{\eta_{hg}} & X \end{array}$$

" $\lrcorner$ "

$$\varphi(g)\varphi(h)(N_{b,e}) \xrightarrow{\varphi(g)\left(t^2_{h^{-1}b,h^{-1},e}\right)^{-1}} \varphi(g)N_{h^{-1}b,h} \xrightarrow{\varphi(g)\left(t'_{h^{-1}bh,h^{-1},h}\right)^{-1}} \varphi(g)N_{h^{-1}bh,e} \xrightarrow{\left(t^2_{g^{-1}h^{-1}bh,g^{-1},e}\right)^{-1}} N_{g^{-1}h^{-1}bh,g^{-1}} \xrightarrow{\left(t'_{g^{-1}h^{-1}bhg,g^{-1},g}\right)^{-1}} N_{g^{-1}h^{-1}bhg,e}$$

Eqn. from (T2)

$h \to g^{-1}$

$k \to e$

$g \to g^{-1}h^{-1}bh$

$a \to h$

$$\varphi(g)\,\underbrace{t'_{g\,g^{-1}h^{-1}bh,h^{-1},h}}_{t'_{h^{-1}bh,h^{-1},h}} \circ t^2_{g^{-1}h^{-1}bh,g^{-1},e}$$

$$= t^2_{g^{-1}h^{-1}b,g^{-1}h^{-1}} \circ t'_{g^{-1}h^{-1}bh,g^{-1}h^{-1},h}$$

|| (via T2)

$$\varphi(g)\varphi(h)(N_{b,e}) \xrightarrow{\varphi(g)\left(t^2_{h^{-1}b,h^{-1},e}\right)^{-1}} \varphi(g)N_{h^{-1}b,h} \xrightarrow{\left(t^2_{g^{-1}h^{-1}b,g^{-1},h^{-1}}\right)^{-1}} N_{g^{-1}h^{-1}b,g^{-1}h^{-1}} \xrightarrow{\left(t'_{g^{-1}h^{-1}bh,g^{-1}h^{-1},h}\right)^{-1}} N_{g^{-1}h^{-1}bh,g^{-1}} \xrightarrow{\left(t'_{g^{-1}h^{-1}bhg,g^{-1},g}\right)^{-1}} N_{g^{-1}h^{-1}bhg,e}$$

Via (T1) $t'_{g,h,k} = t'_{g^{a^{-1}},h,ka^{-1}}\left(t'_{g,hka^{-1},a}\right)$ $g \rightsquigarrow g^{-1}h^{-1}bhg$, $h \rightsquigarrow g^{-1}h^{-1}$, $k \rightsquigarrow hg$, $a \rightsquigarrow g$

$$\varphi(g)\varphi(h)(N_{b,e}) \xrightarrow{\varphi(g)\left(t^2_{h^{-1}b,h^{-1},e}\right)^{-1}} \varphi(g)N_{h^{-1}b,h} \xrightarrow{\left(t^2_{g^{-1}h^{-1}b,g^{-1},h^{-1}}\right)^{-1}} N_{g^{-1}h^{-1}b,g^{-1}h^{-1}} \xrightarrow{\left(t'_{g^{-1}h^{-1}bhg,g^{-1}h^{-1},hg}\right)^{-1}} N_{g^{-1}h^{-1}bhg,e}$$

Via (T3)

$$\varphi(g)\varphi(h)(N_{b,e}) \xrightarrow{\cong} \varphi(gh)N_{b,e} \xrightarrow{\left(t^2_{g^{-1}h^{-1}b,g^{-1}h^{-1},e}\right)^{-1}} N_{g^{-1}h^{-1}b,g^{-1}h^{-1}} \xrightarrow{\left(t'_{g^{-1}h^{-1}bhg,g^{-1}h^{-1},hg}\right)^{-1}} N_{g^{-1}h^{-1}bhg,e}$$

$\left(\varphi(h)\,t^2_{h^{-1}g,ka^{-1},a}\right) t^2_{g,h,k} = t^2_{g,hka^{-1},a}$ ($\cong$

$h \rightsquigarrow g^{-1}$

$g \rightsquigarrow g^{-1}h^{-1}b$

$k \rightsquigarrow h^{-1}$

$a \rightsquigarrow e$

= Sebastian's leg of the commutative diagram!

So (X,η) is an object in $\mathcal{C}^{G}$! (1) ✓

" $\llcorner$ "

$$\varphi(g)\varphi(h)(N_{b,e}) \xrightarrow{\cong} \varphi(hg)(N_{b,e}) \xrightarrow{\left(t^2_{g^{-1}h^{-1}b,g^{-1}h^{-1},e}\right)^{-1}} N_{g^{-1}h^{-1}b,g^{-1}h^{-1}} \xrightarrow{\left(t'_{g^{-1}h^{-1}bhg,g^{-1}h^{-1},hg}\right)^{-1}} N_{g^{-1}h^{-1}bhg,e}$$

Essential Surjectivity (Q2)

$t'_{g,e,k} : N_{g,k} \xrightarrow{\sim} N_{gk^{-1},e}$

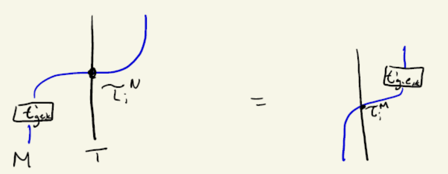
Given τ_1, τ_2,

$M = \bigoplus_{gk \in G} m_g^* \otimes N_{g,k} \otimes m_k$

Apply $E\left(\bigoplus N_{g,e},\ \eta|_{B_{a^{-1}ba}} = t'^{-1} \circ t^{2^{-1}}\right)$

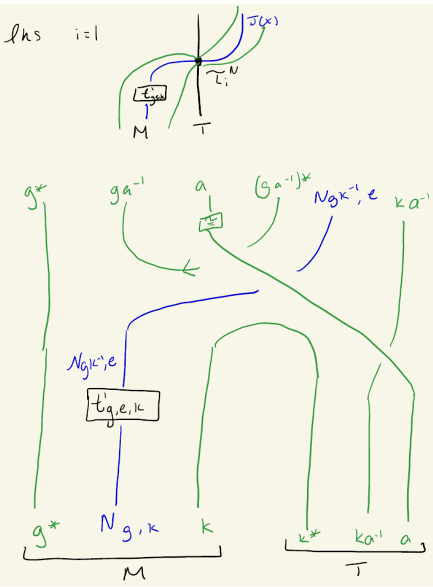
Check that the iso t' lifts to an iso in the cat. $B_{\mathcal{A}}$

I.e. Check



for $i=1,2$

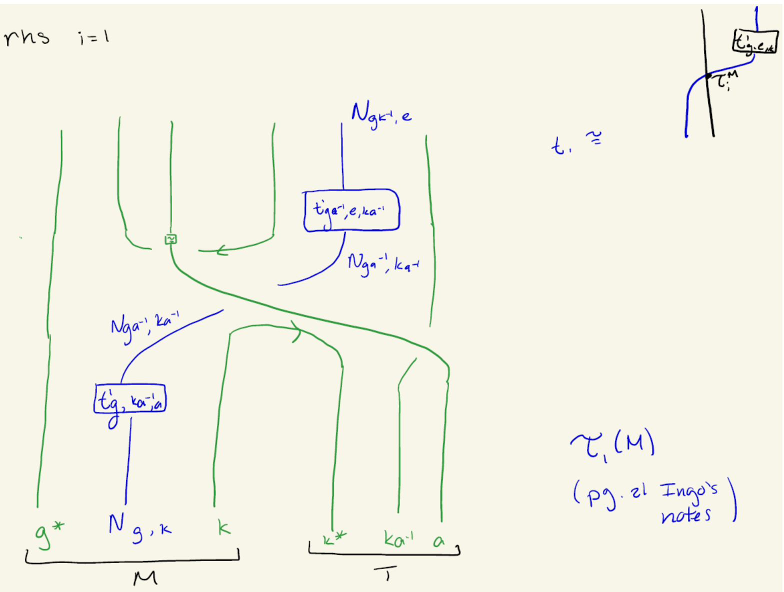
lhs $i=1$



$\tau_1(J(x))$

$t, \cong$

rhs $i=1$



$t, \cong$

$\tau_1(M)$

(pg. 21 Ingo's notes)

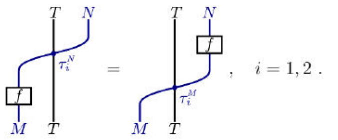
For $i=1$, clearly rhs=lhs if $t'_{g,e,k} = t'_{ga^{-1},e,ka^{-1}} \circ t'_{g,ka^{-1},a}$

this should be a consequence of T1, and, indeed, T1 states

$t'_{g,h,k} = t'_{g^{a^{-1}},h,ka^{-1}}\left(t'_{g,hka^{-1},a}\right)$ So taking $g \rightsquigarrow g$, $h \rightsquigarrow e$, $k \rightsquigarrow k$, $a \rightsquigarrow a$ in this eqn.

gives: $t'_{g,e,k} = t'_{ga^{-1},e,ka^{-1}}\left(t'_{g,ka^{-1},a}\right)$ ✓

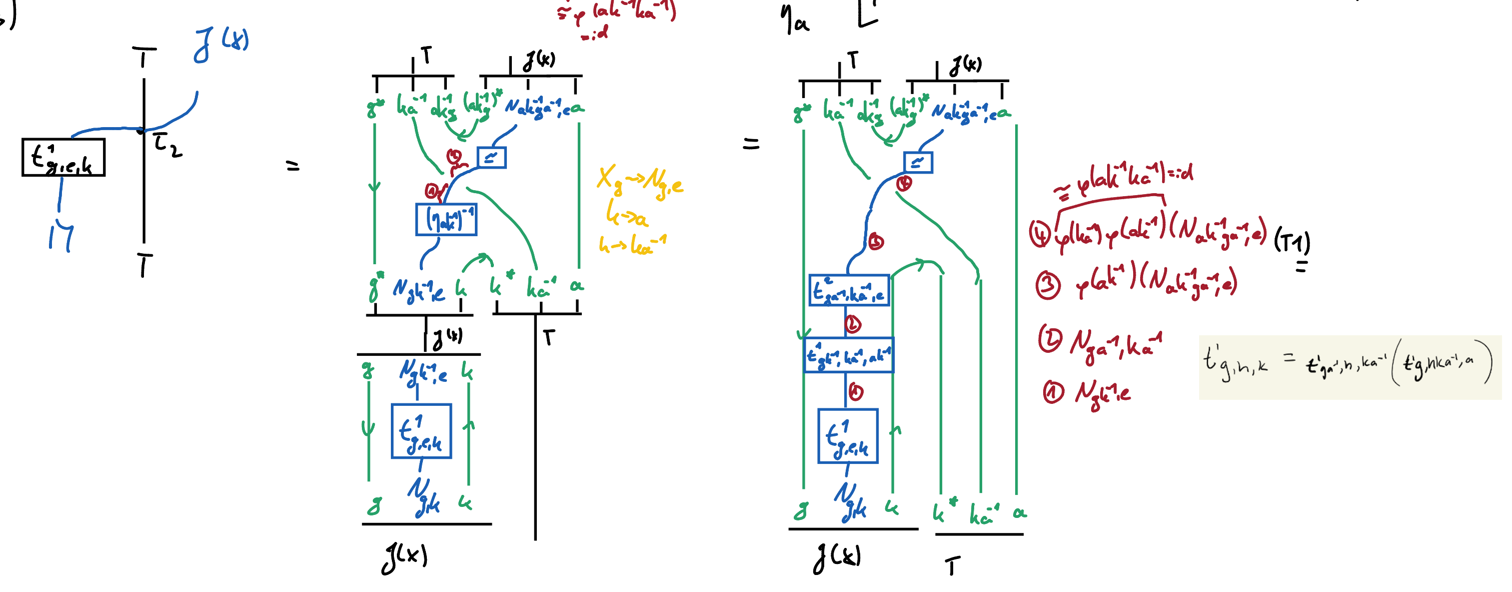
$\boxed{i=2}$



$, \quad i = 1, 2 .$ (M)

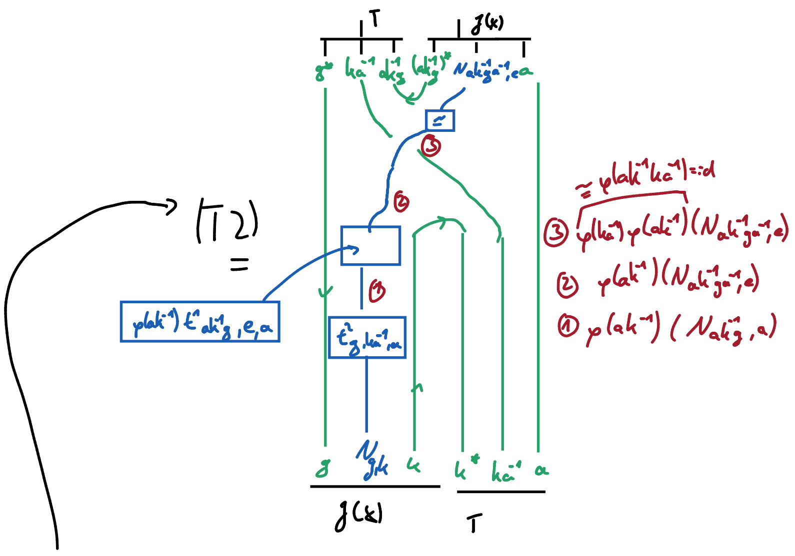
(lhs)

$\eta_a \left[\varphi(a)(N_{b,c}) \xrightarrow{(t^2_{a^{-1},ha^{-1},e})^{-1}} N_{a^{-1}b,a^{-1}} \xrightarrow{(t^1_{a^{-1}ba,a^{-1},a})^{-1}} N_{a^{-1}ba,e} \right]$



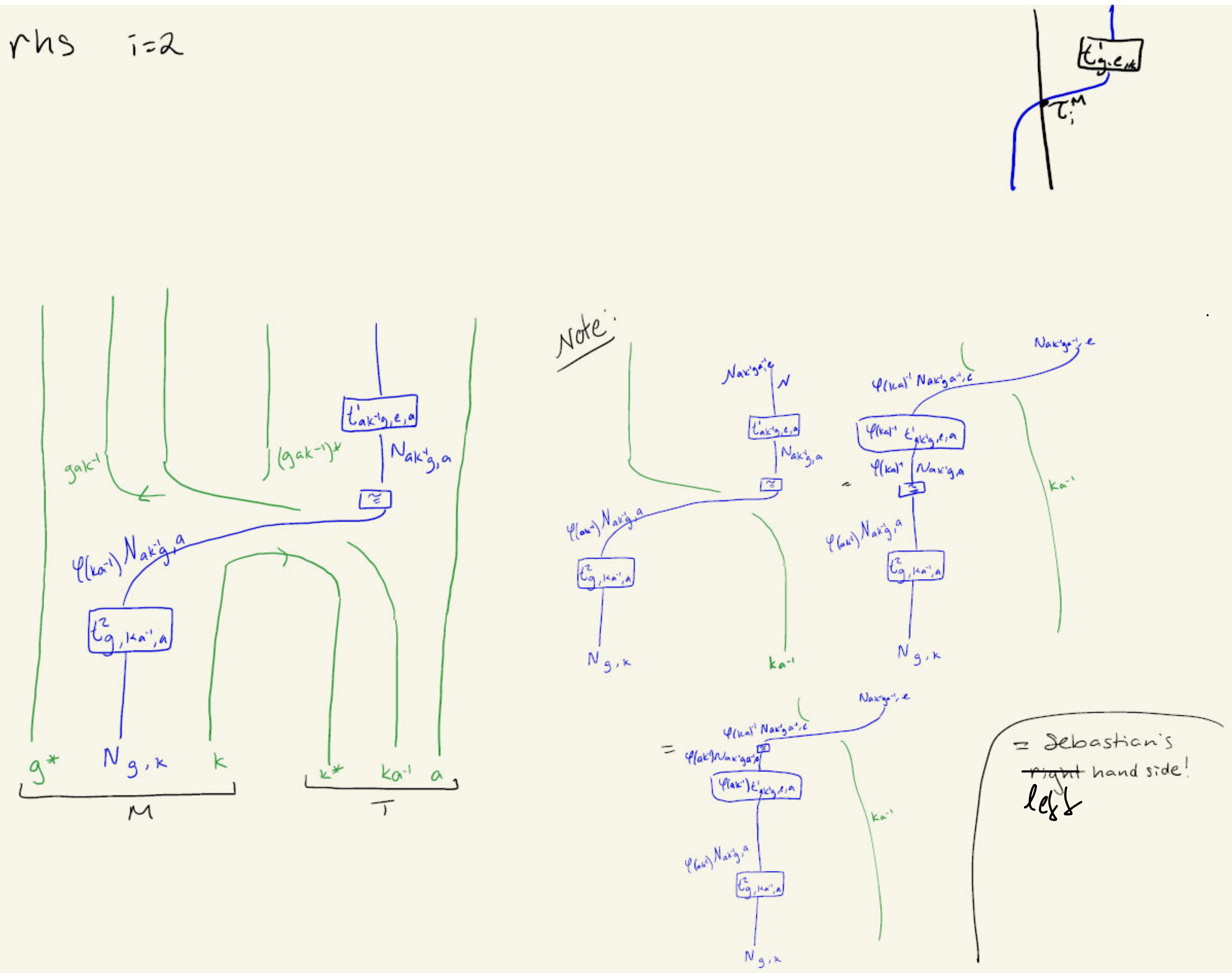
$t'_{g,h,k} = t'_{ga^{-1},h,ka^{-1}}\left(t_{g,hka^{-1},a}\right)$

(T2)



$\varphi(h^{-1})\, t'_{h^{-1}g,ka^{-1},a} \circ t^2_{g,h,k} = t^2_{ga^{-1},h,ka^{-1}} \circ t'_{g,hka^{-1},a}$

rhs $\quad i=2$



= Sebastian's ~~right~~ left hand side!