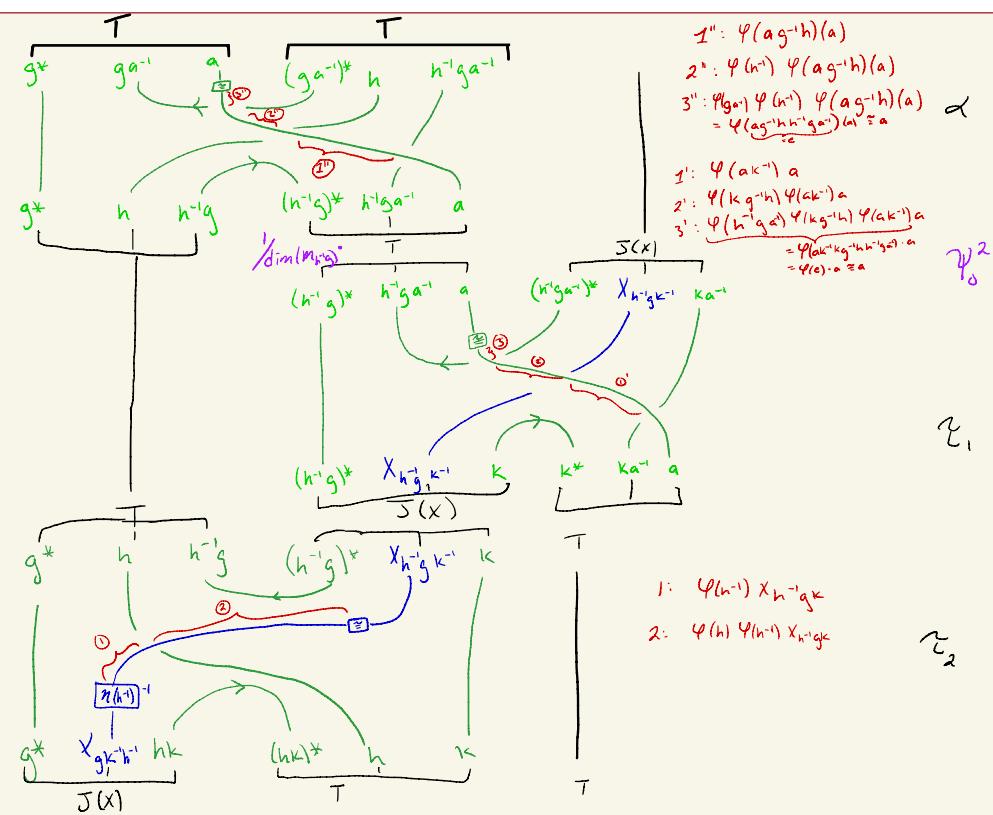


TO
EO
rib
ch

$$\begin{array}{c} \text{Diagram showing two configurations of a loop with labels } \alpha, \psi_0^2, 1, 2, M, T, T. \\ = \\ \text{Diagram showing the same configuration with labels } \alpha, \psi_0^2, 1, 2, M, T, T. \end{array} \quad (\text{T2})$$

(lhs)



My lhs = Sebastian's rhs

(T2) (rhs)

$\circledcirc \quad p(hg^{-1}) (X_{g^{-1}h^{-1}})$
 $\circledcirc \quad p(h) p(h^{-1}) (K_h g^{-1})$
 $\approx p(h^{-1}h)$
 $= id$

$(ga^{-1})^n X_{ga^{-1}h^{-1}ka^{-1}hka^{-1}}^n h^{-1}$
 $g \rightarrow ga^{-1}$
 $k \rightarrow ka^{-1}$

$p(ak^{-1})(a)$
 $(hg^{-1}) p(ak^{-1}h^{-1})(a)$
 $\cancel{p(hg^{-1}) p(ak^{-1}h^{-1})(a)}$
 $\cancel{(ak^{-1}h^{-1}hkga^{-1})(a) h \rightarrow hk a^{-1}}$
 $= id \checkmark$

$j(k)$

T

g^* ga^{-1} a $(ga^{-1})^n X_{ga^{-1}h^{-1}ka^{-1}hka^{-1}}^n a$

$g(x)$

T

1
 $\text{dim } V_{hk a^{-1}}$

$p(ak^{-1})(a)$
 $p(h^{-1}) p(ak^{-1})(a)$
 $p(hka^{-1}) p(h^{-1}) p(hk^{-1})(a)$
 $\approx p(ak^{-1}h^{-1}hk a^{-1})$
 $= id \checkmark$

$j(k)$

T

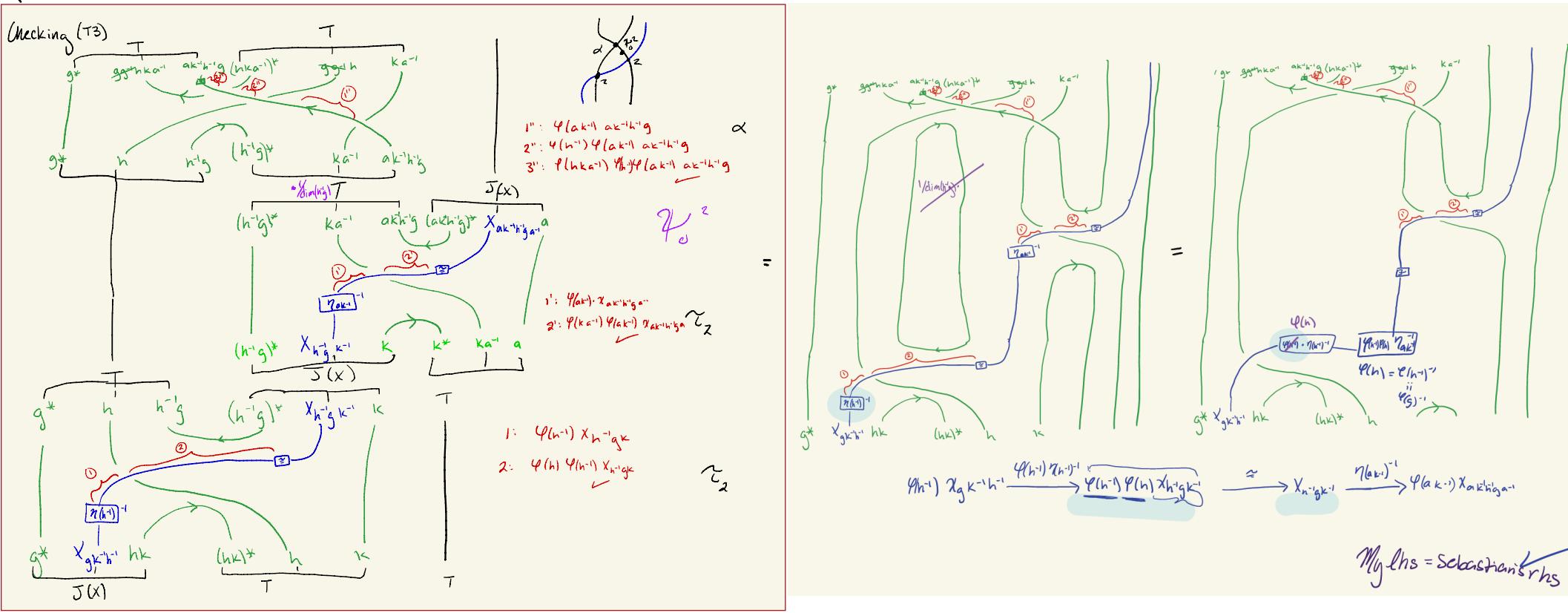
$(hk)^*$ $hk a^{-1}$ $(hk a^{-1})^n$ $h^{-1}ka^{-1}$
 $h \rightarrow h$
 $h \rightarrow hk a^{-1}$
 $k \rightarrow a$

$g \rightarrow h$
 $h \rightarrow hk a^{-1}$
 $k \rightarrow a$

$j(k)$

$$\text{Diagram showing two configurations of a loop with labels } \alpha, \psi_0^2, 2, M, T, T. \text{ The right configuration is labeled (T3).}$$

(lhs)

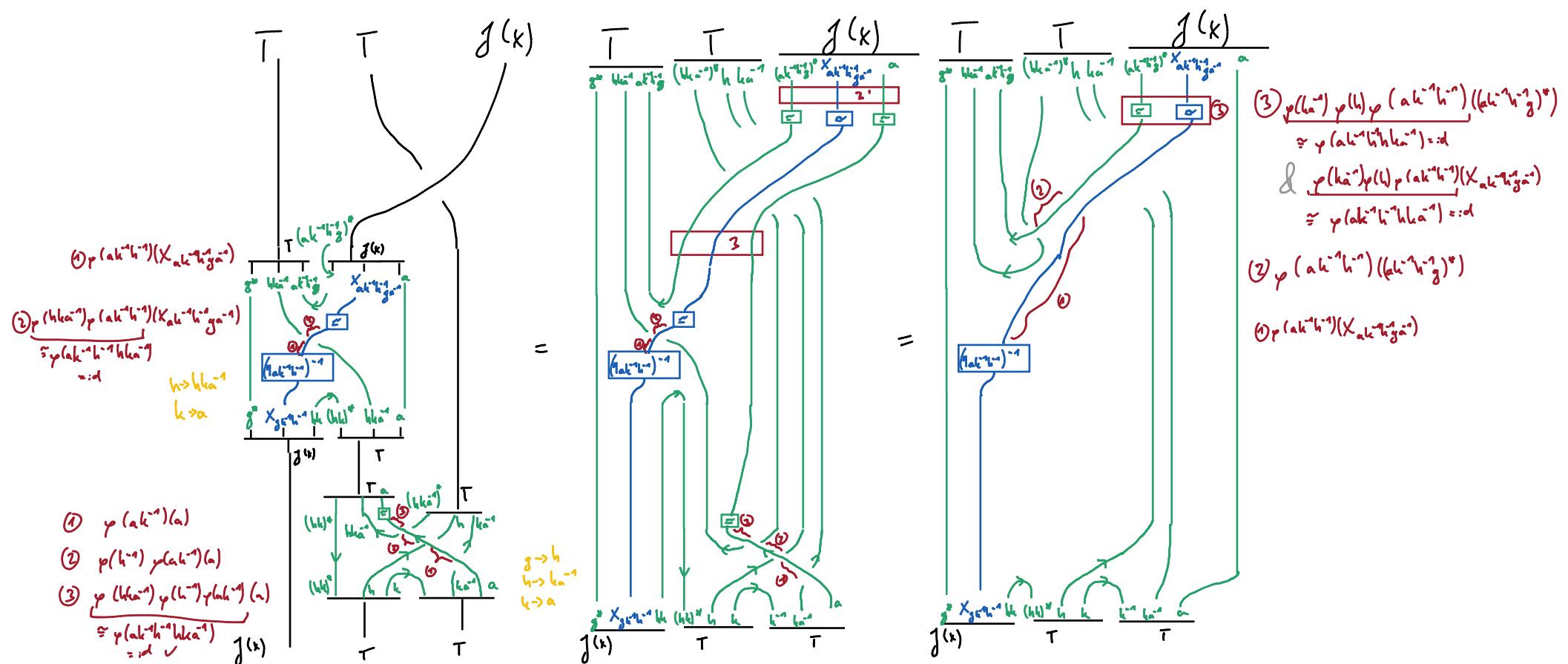


(vhs)

$$3 = 3^1$$

$$\varphi(h^{-1}) \varphi(h) \varphi(ah^{-1}h^{-1})$$

$$= \varphi(ah^{-1}h^{-1}hah^{-1}) = id$$



$$\begin{array}{c} \text{Diagram showing } M \xrightarrow{i} \psi_i^2 \text{ and } \psi_0^{-2} \xrightarrow{i} T \\ \text{with } M = T \end{array} \quad (\text{T4}) \quad := 1$$

$$M \xrightarrow[T]{\psi_i^2} i \xleftarrow[i]{\psi_0^{-2}} M = T \quad (\text{T4}) \quad := \mathfrak{I}$$

The figure illustrates the equivalence of two representations of a linear operator T . The top part shows a diagrammatic proof using commutative squares and the dimension formula. The bottom part shows an alternative representation where the operator T is decomposed into a sum of two operators, each with a different dimension formula.

Top Diagram:

- Left side: A commutative square with δ^{**} at the top-left, δ^* at the top-right, δ^{**} at the bottom-left, and δ at the bottom-right. The vertical arrow from δ^{**} to δ^* is labeled $\frac{1}{\dim m_{\delta^{**}}}$. The horizontal arrow from δ^* to δ is labeled T .
- Right side: A commutative square with δ^{**} at the top-left, δ^* at the top-right, δ^{**} at the bottom-left, and δ at the bottom-right. The vertical arrow from δ^{**} to δ^* is labeled $\frac{1}{\dim m_{\delta^{**}}}$. The horizontal arrow from δ^* to δ is labeled T .
- Bottom: A commutative square with δ^{**} at the top-left, δ^* at the top-right, δ^{**} at the bottom-left, and δ at the bottom-right. The vertical arrow from δ^{**} to δ^* is labeled $\frac{1}{\dim m_{\delta^{**}}}$. The horizontal arrow from δ^* to δ is labeled T .
- Annotations: Red text indicates the equivalence of the two top representations: $\textcircled{1} \rho^{(L^*)}(X_{L^*} g^{L^*})$. Red text also indicates the equivalence of the two bottom representations: $\textcircled{1} \rho^{(L^*)}(X_{L^*} g^{L^*})$, $\textcircled{2} \rho^{(L)} \rho^{(L^*)}(X_{L^*} g^{L^*})$, and $\tilde{\rho}^{(L^*)} \tilde{\rho}^{(L)} = 1$.

The diagram illustrates the decomposition of the right-hand side (rhs) of a linear system into two components: $\tilde{g}(x)$ and $g^*(x)$.

rhs

T

$\tilde{g}(x)$

g^*

x_{k+1}^{k+1-h}

$\tilde{g}(x)$

T

$\tilde{g}(x)$

x_{k+1}^{k+1-h}

$(\tilde{g}(x))^*$

T

k

h

1

$= \dim v_{m_k}$

$\dim w_{m_k}$

$\tilde{g}(x)$

x_{k+1}^{k+1-h}

$(\tilde{g}(x))^*$

T

k

h

$$\psi_0^2 = \begin{array}{c} i \\ \text{---} \\ TM \\ \text{---} \\ T \\ \text{---} \\ M \end{array} \quad (\text{T5}) \quad i=1$$

(lhs)

$$= \frac{1}{\dim m_{hk}}$$

rhs

$$= \frac{1}{\dim m_{hk}}$$

(lhs)

$$= \frac{1}{\dim m_{hk}}$$

rhs

$$\begin{aligned} & \text{① } p(l^{-1})(h) \\ & \text{② } p(g^{k^{-1}})p(l^{-1})(h) \\ & \approx p(kg^{-1}g^{k^{-1}}) = id \end{aligned}$$

$$\psi_0^2 = \begin{array}{c} i \\ \text{---} \\ TM \\ \text{---} \\ T \\ \text{---} \\ M \end{array} \quad (\text{T5}) \quad i=2$$

(lhs)

$$= \frac{1}{\dim m_{hk}}$$

rhs

$$= \frac{1}{\dim m_{hk}}$$

(lhs)

$$= \frac{1}{\dim m_{hk}}$$

rhs

$$\begin{aligned} & \text{① } p(l^{-1})(X_{l^{-1}g^{-1}h^{-1}}) \\ & \text{② } p(g^{k^{-1}})p(l^{-1})(X_{h^{-1}g^{-1}h^{-1}}) \\ & \approx p(kg^{-1}g^{k^{-1}}) = id \end{aligned}$$

(lhs)

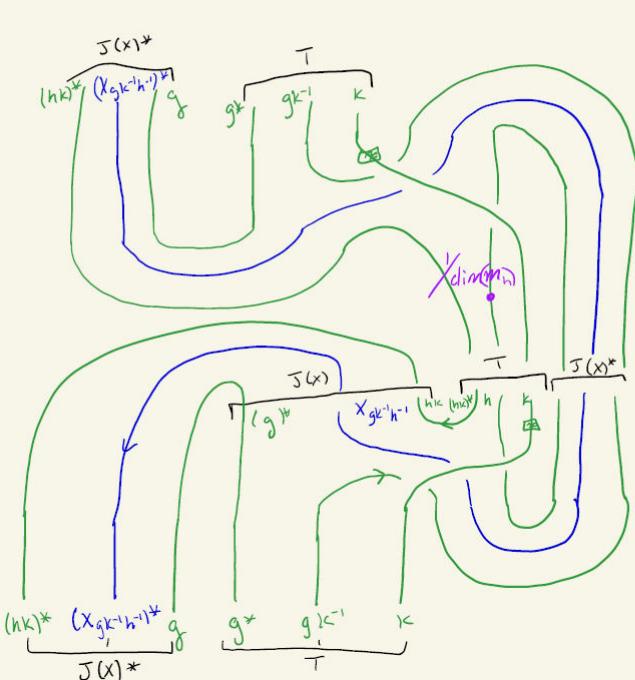
$$= \frac{1}{\dim m_{hk}}$$

rhs

$$\begin{aligned} & \text{① } p(l^{-1})(X_{l^{-1}g^{-1}h^{-1}}) \\ & \text{② } p(g^{k^{-1}})p(l^{-1})(X_{h^{-1}g^{-1}h^{-1}}) \\ & \approx p(kg^{-1}g^{k^{-1}}) = id \end{aligned}$$

$$\begin{array}{c} \psi_i^2 \\ \downarrow \\ M^* T \end{array} = \begin{array}{c} \psi_0^{-2} \\ \downarrow \\ M^* T \end{array} \quad (T6)$$

$TG_{i=1} \text{ lhs}$



$$\begin{array}{c} \psi_0^{-2} \\ \downarrow \\ T \end{array} \quad \begin{array}{c} \psi_0^{-2} \\ \downarrow \\ T \end{array}$$

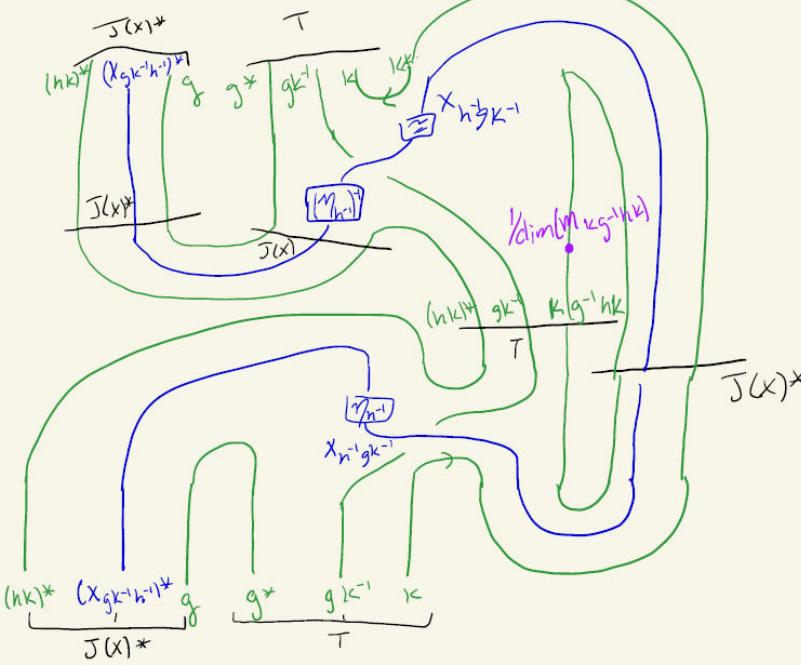
$$= \begin{array}{c} (hk)^* (x_{gk^{-1}h^{-1}})^* g \\ \downarrow \\ (hk)^* (x_{gk^{-1}h^{-1}})^* g \end{array} \quad \begin{array}{c} g \\ \downarrow \\ g^* \end{array} \quad \begin{array}{c} gk^{-1} \\ \downarrow \\ gk^{-1} \end{array} \quad \begin{array}{c} k \\ \downarrow \\ k \end{array}$$

$TG_{i=1} \text{ rhs}$

$$\begin{array}{c} \psi_0^{-2} \\ \downarrow \\ (hk)^* (x_{gk^{-1}h^{-1}})^* g \\ \downarrow \\ J(x)^* \end{array} \quad \begin{array}{c} g \\ \downarrow \\ g^* \end{array} \quad \begin{array}{c} gk^{-1} \\ \downarrow \\ T \end{array} \quad \begin{array}{c} k \\ \downarrow \\ k \end{array}$$

$= \text{lhs} \checkmark$

$TG_{i=2} \text{ lhs}$



$$\begin{array}{c} \psi_0^{-2} \\ \downarrow \\ T \end{array}$$

$$= \begin{array}{c} (hk)^* (x_{gk^{-1}h^{-1}})^* g \\ \downarrow \\ (hk)^* (x_{gk^{-1}h^{-1}})^* g \end{array} \quad \begin{array}{c} g \\ \downarrow \\ g^* \end{array} \quad \begin{array}{c} gk^{-1} \\ \downarrow \\ gk^{-1} \end{array} \quad \begin{array}{c} k \\ \downarrow \\ k \end{array}$$

$TG_{i=2} \text{ rhs}$

$$\begin{array}{c} \psi_0^{-2} \\ \downarrow \\ T \end{array}$$

$= \text{lhs} \checkmark$

$$\begin{array}{c} g \\ \downarrow \\ (hk)^* (x_{gk^{-1}h^{-1}})^* g \\ \downarrow \\ J(x)^* \end{array} \quad \begin{array}{c} g \\ \downarrow \\ g^* \end{array} \quad \begin{array}{c} gk^{-1} \\ \downarrow \\ T \end{array} \quad \begin{array}{c} k \\ \downarrow \\ k \end{array}$$

Non-abelian

TODO: Think about E^0 .

$$E^n : E(X, \eta) \otimes E(Y, \eta) \rightarrow E((X, \eta) \otimes (Y, \eta))$$

$$(J(X), T_1, T_2) \otimes (J(Y), T_1, T_2) = (K(X, Y), T_1 \otimes T_2, \rho(X) \otimes \rho(Y))$$

$$(J(X) \otimes J(Y), T_1 \otimes T_2, \rho(X) \otimes \rho(Y)) \rightarrow (J(XY), T_1 \otimes T_2, \rho(XY))$$

$$\widehat{E}^{(n)} = (\dim h)^{\frac{n}{2}}$$

Are these inverse to each other?

J is a morphism in \mathcal{B}_A , i.e.

$$\begin{array}{c} T \\ \square \\ M \end{array} = \begin{array}{c} T \\ \square \\ M \end{array}, \quad i = 1, 2. \quad (M)$$

[Cor 3]

i=1

(lhs)

$$\begin{array}{c} T \\ \square \\ M \end{array} = \begin{array}{c} T \\ \square \\ M \end{array}$$

$$\begin{array}{c} T \\ \square \\ M \end{array} = \begin{array}{c} T \\ \square \\ M \end{array}$$

$(\dim a)^{\frac{n}{2}}$

$$\begin{array}{c} T \\ \square \\ M \end{array} = \begin{array}{c} T \\ \square \\ M \end{array}$$

$$\begin{array}{c} T \\ \square \\ M \end{array} = \begin{array}{c} T \\ \square \\ M \end{array}$$

$(\dim a)^{\frac{n}{2}}$

$$\begin{array}{c} T \\ \square \\ M \end{array} = \begin{array}{c} T \\ \square \\ M \end{array}$$

$$\begin{array}{c} T \\ \square \\ M \end{array} = \begin{array}{c} T \\ \square \\ M \end{array}$$

(rhs)

$$\begin{array}{c} T \\ \square \\ M \end{array} = \begin{array}{c} T \\ \square \\ M \end{array}$$

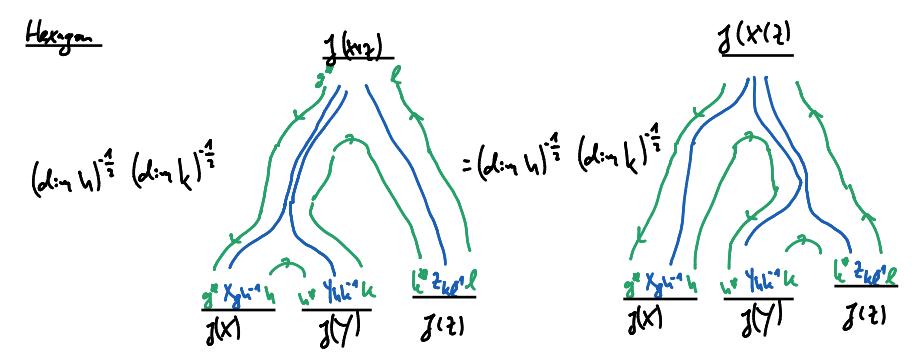
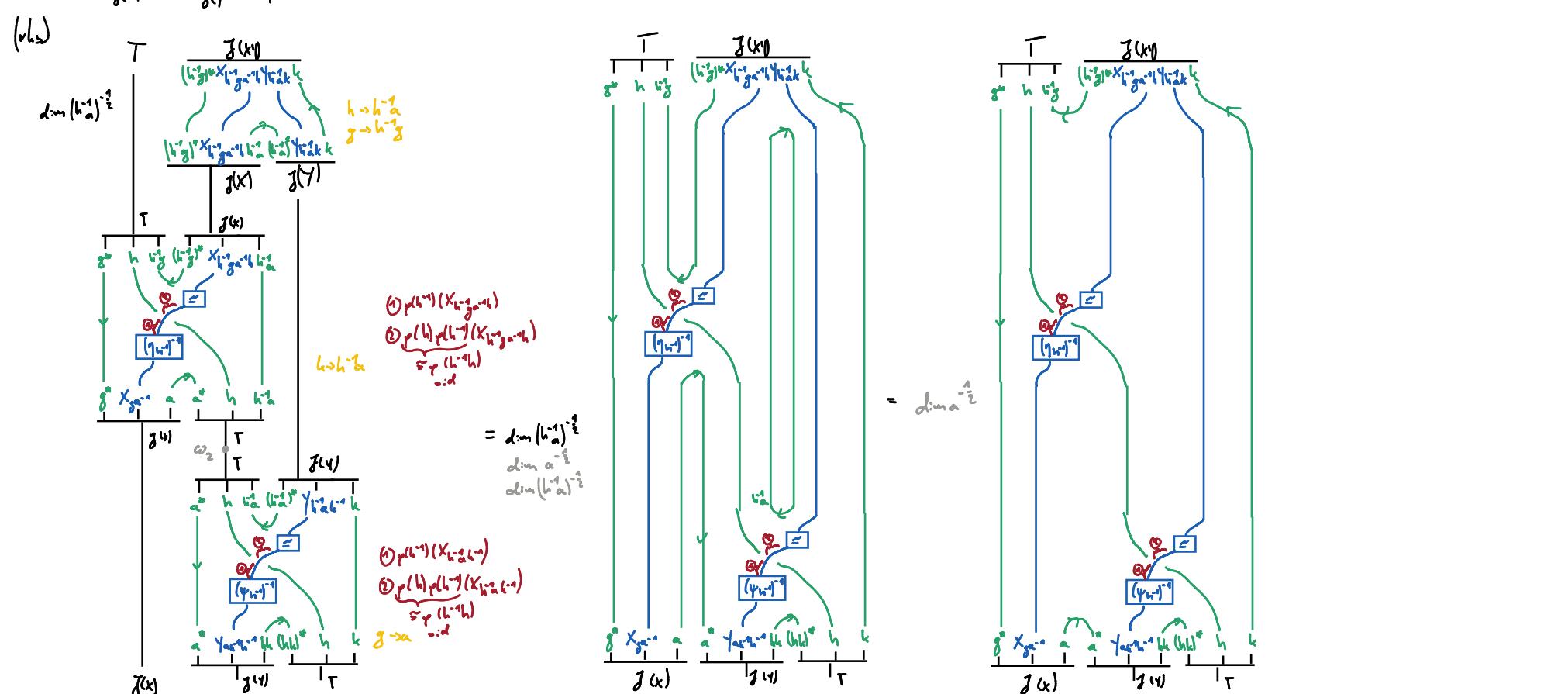
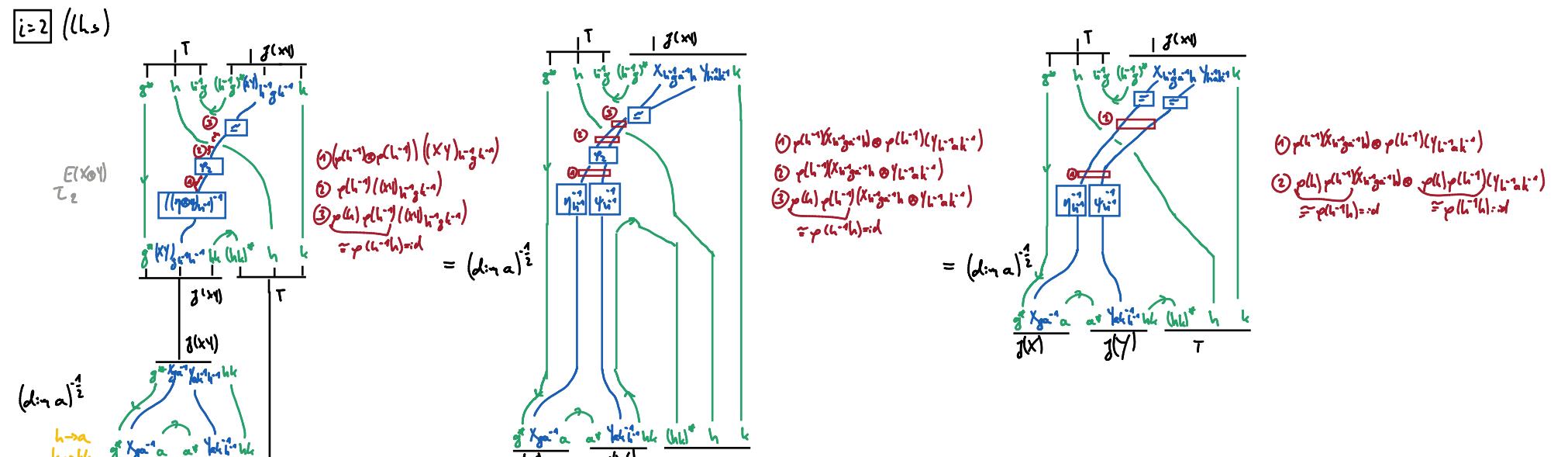
$$\begin{array}{c} T \\ \square \\ M \end{array} = \begin{array}{c} T \\ \square \\ M \end{array}$$

$$\begin{array}{c} T \\ \square \\ M \end{array} = \begin{array}{c} T \\ \square \\ M \end{array}$$

$$\begin{array}{c} T \\ \square \\ M \end{array} = \begin{array}{c} T \\ \square \\ M \end{array}$$

$(\dim a)^{\frac{n}{2}}$

$$\begin{array}{c} T \\ \square \\ M \end{array} = \begin{array}{c} T \\ \square \\ M \end{array}$$



Thus, E is monoidal.

$$c_{M,N} := T \cdot \phi = \begin{array}{c} \text{Diagram showing two circles } M \text{ and } N \text{ intersecting at points } \psi_0 \text{ and } \omega^2. \\ \text{The intersection point } \psi_0 \text{ is marked with a red box.} \\ \text{The angle between the radii to the intersection points is labeled } \omega^2. \\ \text{The diagram is multiplied by } \phi. \end{array}$$

$\text{① } \gamma(h^{-1})(h^{-1}g)$
 $\text{② } \gamma(h^{-1})\gamma(h^{-1})(h^{-1}g)$
 $\text{③ } \gamma(\alpha)\gamma(h^{-1})\gamma(h^{-1})(h^{-1}g)$
 $= \gamma(h^{-1}h^{-1}\alpha)$
 $= id_V$

$\text{④ } \phi(a^{-1})(X_{h^{-1}g^{-1}})$

$\omega^2 = \frac{1}{\dim m_h \dim m_k}$

$\text{① } \phi(h)$

$\text{② } \phi(h)\phi(h^{-1})(X_{h^{-1}g^{-1}})$
 $= \phi(h^{-1}h)$
 $= id_V$

$\text{③ } \phi(h)\phi(\alpha^{-1})\phi(\alpha^{-1})(h)$
 $= \phi(h^{-1}\alpha^{-1}h)$
 $= id_V$

$$\text{④ } g_{h^{-1}ka} = \frac{1}{|G|} (m_{ah^{-1}})^{-\frac{1}{2}} (m_{hk})^{-\frac{1}{2}}$$

The diagram illustrates the relationship between the left and right actions of a group G on a set X . It shows two parallel horizontal arrows:

- Left Action:** $x \mapsto g \cdot x = gxg^{-1}$ (indicated by a blue arrow pointing left).
- Right Action:** $x \mapsto x \cdot g = x(g^{-1}g)$ (indicated by a red arrow pointing right).

Below these arrows, a commutative square is shown:

$$\begin{array}{ccc} & f(y) & \\ \text{---} & \downarrow & \text{---} \\ g^* & y_{ba^{-1}} a^{1/2} & (a^{1/2}g)^* X_{ab^{-1}g^{-1}} \\ \text{---} & \downarrow & \text{---} \\ g & X_{ba^{-1}} b & g(y) \\ \text{---} & \downarrow & \text{---} \\ & b^{1/2} y_{ba^{-1}} & \end{array}$$

where $f(y)$ is the image of y under a map f , and g^* is the image of g under a map g^* .

$$\text{braiding on } B^G: \quad c^{B^G}: (x_{(1)})_{\#} (y_{(1)}) = (\underset{g}{\otimes} x \otimes y, \, r_g)(x \otimes y) \xrightarrow{\tau_2^{-1}} r_g(x) \otimes r_g(y) \xrightarrow{\text{braiding}} x \otimes y)$$

$$c_{g,h}^{B^G} := \textcircled{1} c_{g,h}^{B^G}, \quad c_{g,h}^{B^G} : X_g \otimes Y_h \xrightarrow{\epsilon_X \otimes \gamma} Y_h \otimes \gamma(h)(X_g) \xrightarrow{id \otimes \eta_h} Y_h \otimes X_g$$

$$E(c^{b^g}) = \frac{(Xg)_{g^{-1}}}{(Xg)_{g^{-1}} + c} = \frac{(Xg)_{g^{-1}}}{(Xg)_{g^{-1}} + c^{-g}}$$

$$\text{bracketed: } E(x) \underset{E^2}{\underset{\wedge}{\otimes}} E(y) \xrightarrow{c} E(y) \underset{E^2}{\underset{\wedge}{\otimes}} E(x)$$

$$\left(\lim_{n \rightarrow \infty} m_n^{-1} a_n^{-1} g \right)^{-\frac{1}{2}} \left(\lim_{n \rightarrow \infty} m_n \right)^{-\frac{1}{2}}$$

The diagram illustrates a deep learning architecture with two parallel paths. The top path consists of four layers: $J(4x)$, $J(4)$, $J(4)$, and $J(4)$. The bottom path also consists of four layers: $J(4)$, $J(4)$, $J(4)$, and $J(4)$. Activation functions like \tanh and sigmoid are applied at various stages. A red circle labeled ① points to a node in the second layer of the bottom path. A blue box labeled $(q, g)^{-1}$ is present in the third layer of the bottom path. A green arrow indicates a skip connection from the first layer of the bottom path to the fourth layer. Red arrows labeled ② and ③ point to specific nodes in the bottom path's layers.

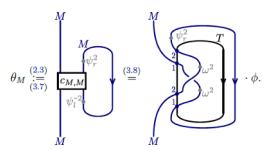
$$= (\dim_{\mathbb{R}} \text{im } ah^{-1})^{-\frac{1}{2}} (\dim_{\mathbb{R}} m_1)^{-\frac{1}{2}}$$

$$= \quad \quad \quad)^{-1}$$

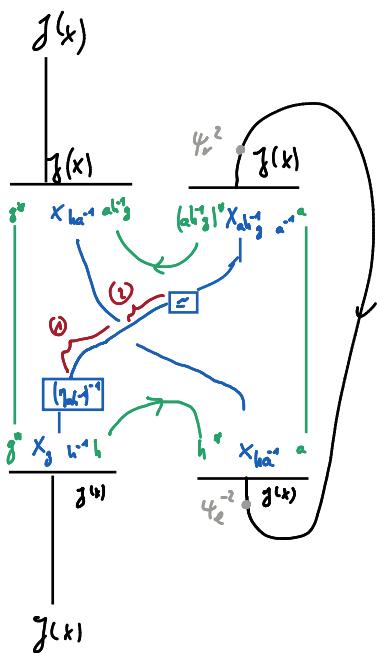
$$\textcircled{④} \quad p(hm^{-1}) \left(X_{al^{-1}} g^{-1} \right) \frac{J(hz)}{g' \gamma_{hm^{-1}} X_{al^{-1}} g^{-1} a^{-1}}$$

$$\begin{aligned}
 & \left(\frac{\varphi(b^{-1})}{\varphi(bc)} \right) \left(X_{b^{-1}ab} \right) = \left(\frac{\varphi(bc)\varphi(b^{-1})}{\varphi(bc)} \right) \left(X_{bab} \right) \\
 & \left(\frac{\varphi(b^{-1})}{\varphi(b)} \right)^{-1} \left(X_a \right) = \left(\frac{\varphi(b)}{\varphi(b)} \right)^{-1} \left(X_a \right) \\
 & \left(\frac{\varphi(b^{-1})}{\varphi(b)} \right)^{-1} \left(X_a \right) = \left(\frac{\varphi(b)}{\varphi(b)} \right)^{-1} \left(X_a \right) \\
 & \text{this step uses the } \frac{\varphi(b)}{\varphi(b)} = 1 \text{ property}
 \end{aligned}$$

Notes

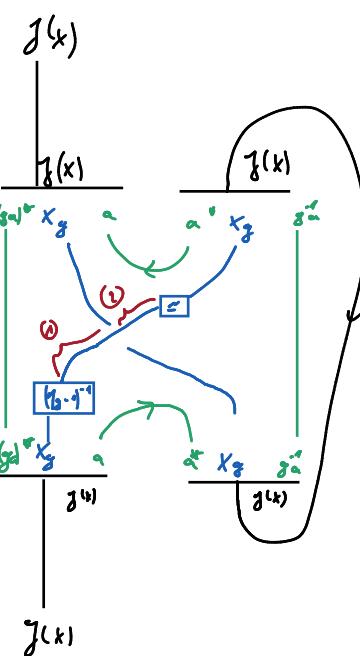


$$\begin{aligned} \Theta_{E(x)}^B &= \\ &(\dim m_{\alpha} h^{-1} g)^{-\frac{1}{2}} (\dim m_{\alpha} h)^{-\frac{1}{2}} \\ (1) \quad &\varphi(h^{-1})(X_{h^{-1}} g^{-1}) \\ (2) \quad &\varphi(h^{-1}) \varphi(h^{-1})(X_{h^{-1}} g^{-1}) \\ (3) \quad &\cong \varphi(h^{-1} h^{-1})(X_{h^{-1}} g^{-1} h^{-1}) \\ &= \varphi(h^{-1})(X_{h^{-1}} g^{-1} h^{-1}) \end{aligned}$$

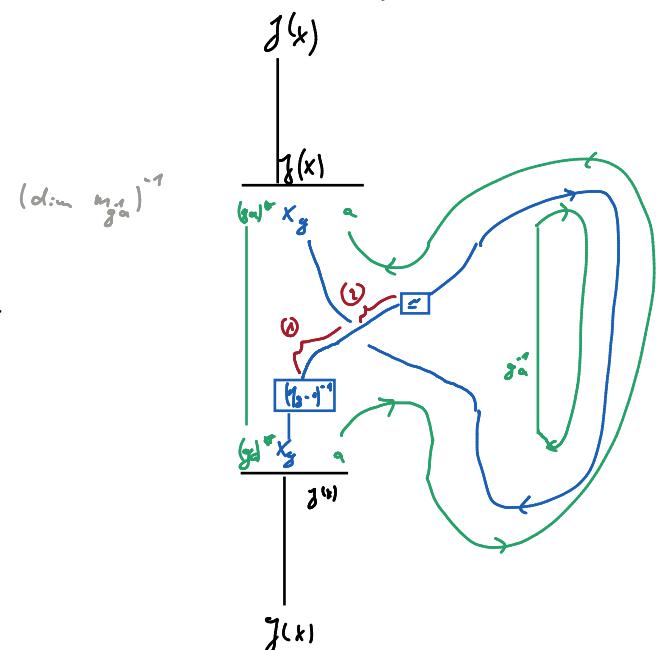


$$= (\dim m_{\alpha} h^{-1} g)^{-\frac{1}{2}} (\dim m_{\alpha} h)^{-\frac{1}{2}}$$

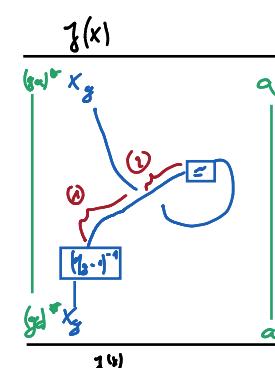
only works for $g^{-1} h = h^{-1} g$ $\dim m_{\alpha}$



$$(1) \varphi(g^{-1})(X_g) \\ (2) \varphi(g) \varphi(g^{-1})(X_g) \\ \cong id$$



$$=$$



$$E(\Theta_{(X_M)}) =$$

(1) $\varphi(g)(X_g)$

Exceptional surjectivity

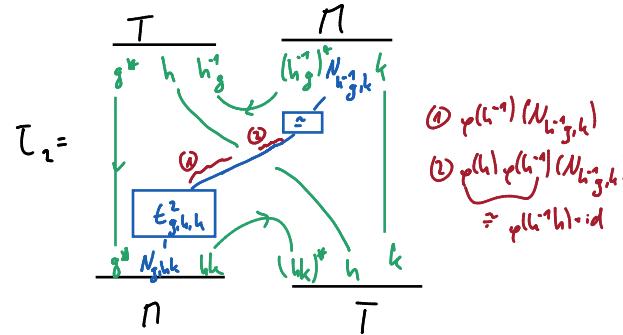
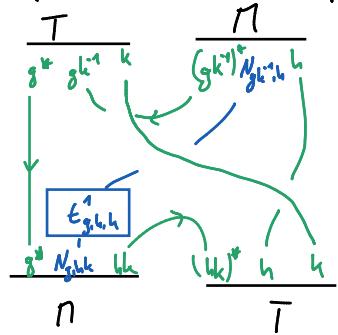
$$\textcircled{1} \quad X := \bigoplus_{b \in \mathcal{C}} N_{b,c}$$

$$\eta_c : \varphi(h)(x) \xrightarrow{\cong} x$$

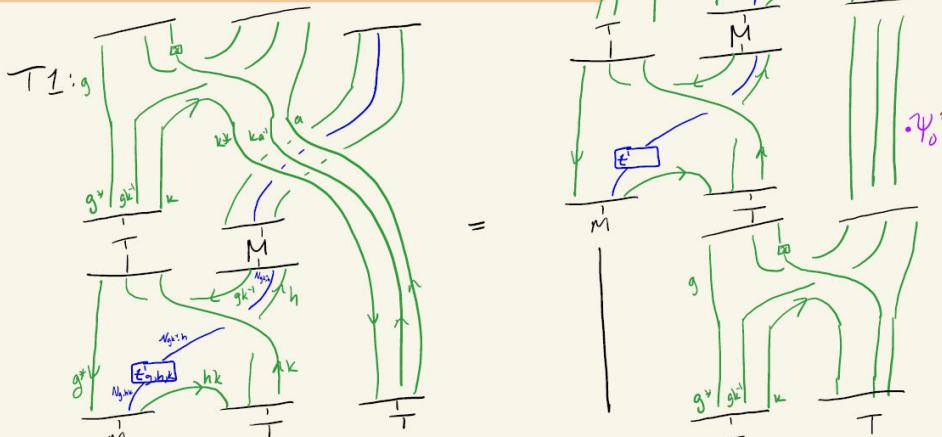
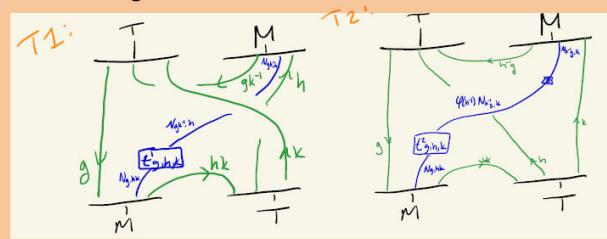
defined via

$$\eta_{c,a^{-1}ba} = \left[\varphi(h)(N_{b,c}) \xrightarrow{(t^1_{g,h,a^{-1}})^{-1}} N_{a^{-1}b,a^{-1}} \xrightarrow{(t^1_{g,h,a^{-1}})^{-1}} N_{a^{-1}b,c} \right]$$

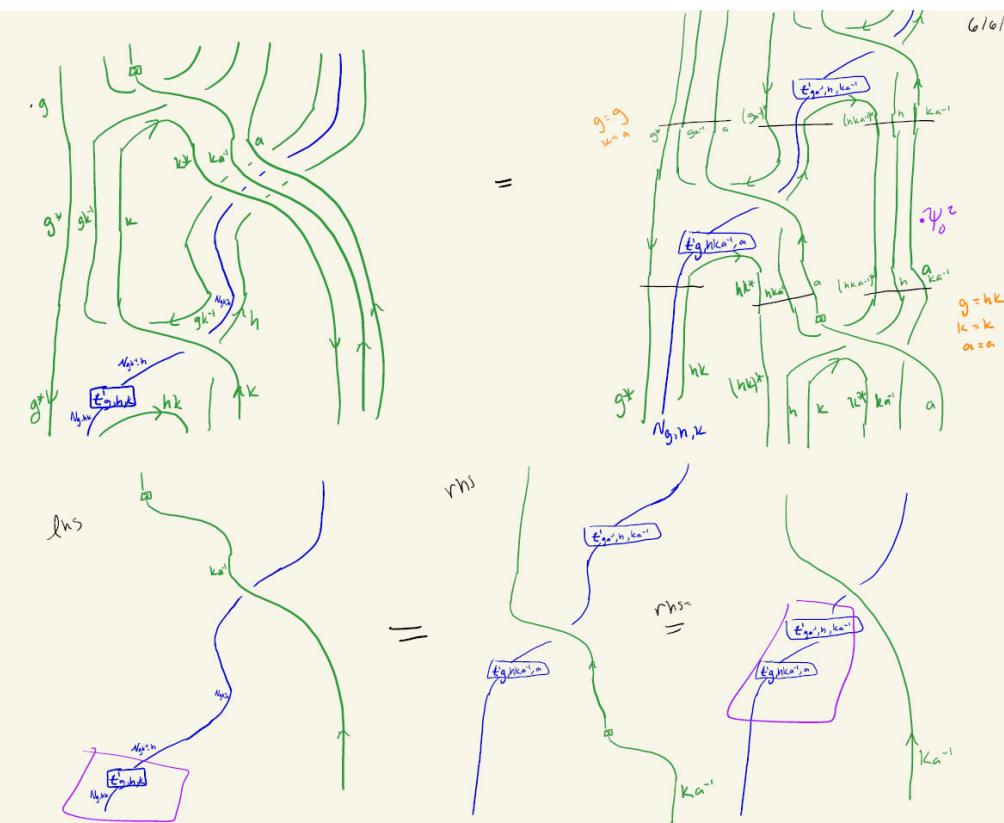
Remark (T1)-(T3) in terms of



Rewriting T1-T3 with t^1 diagrams

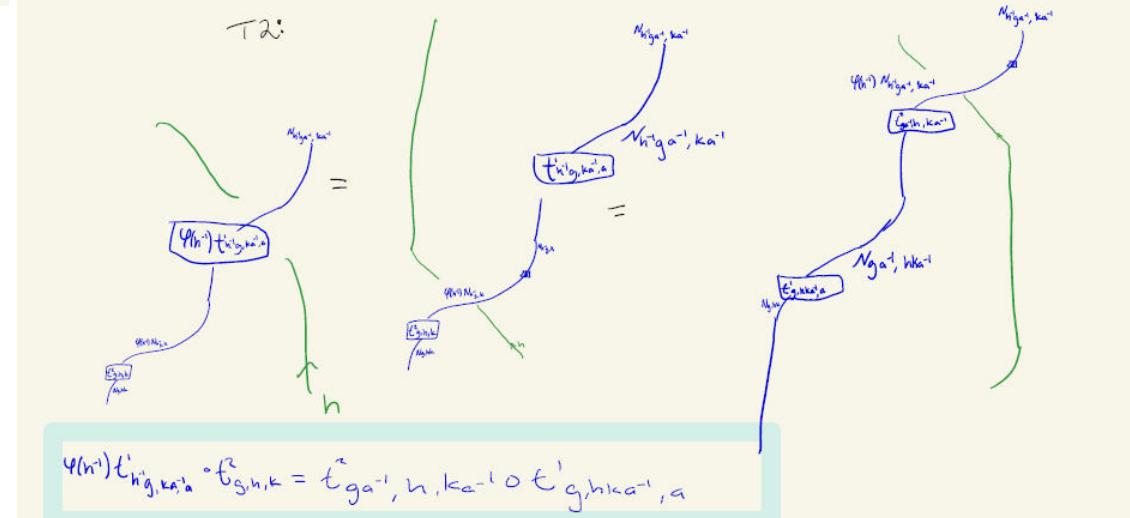
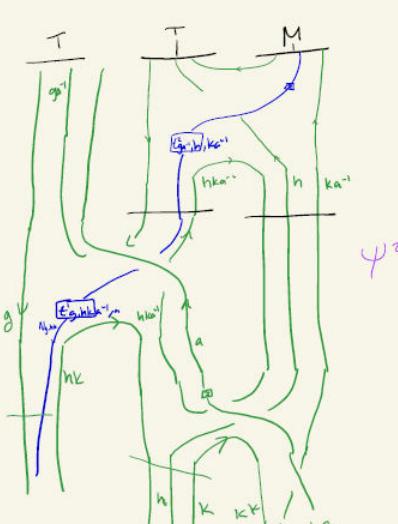
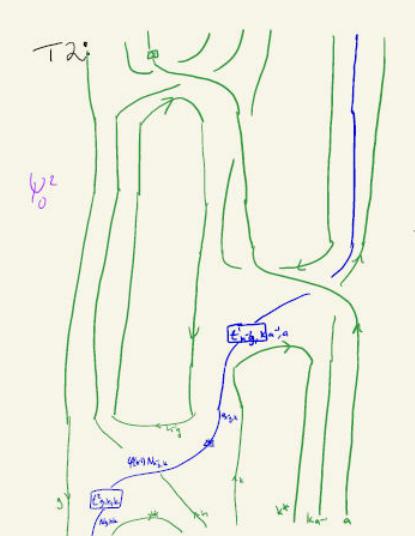
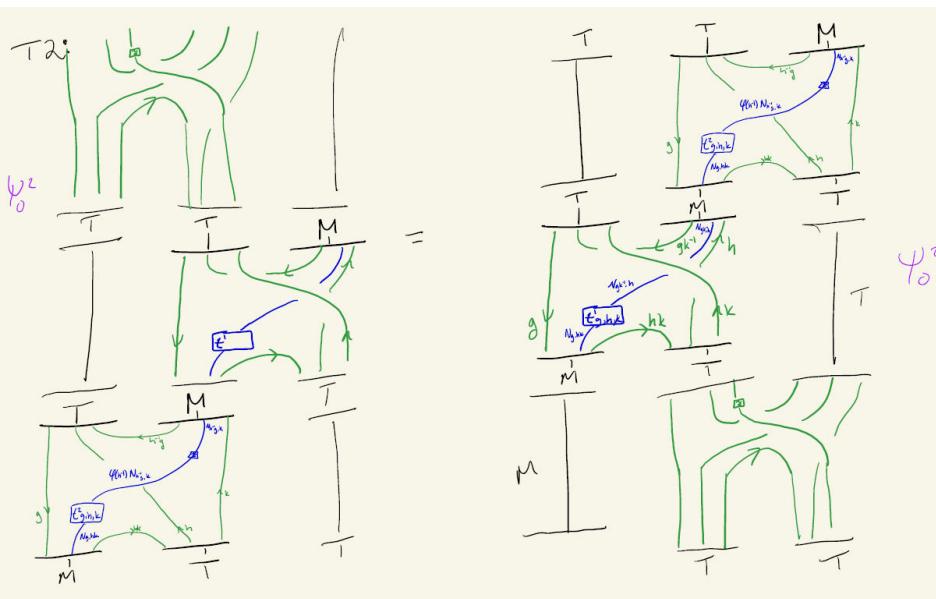


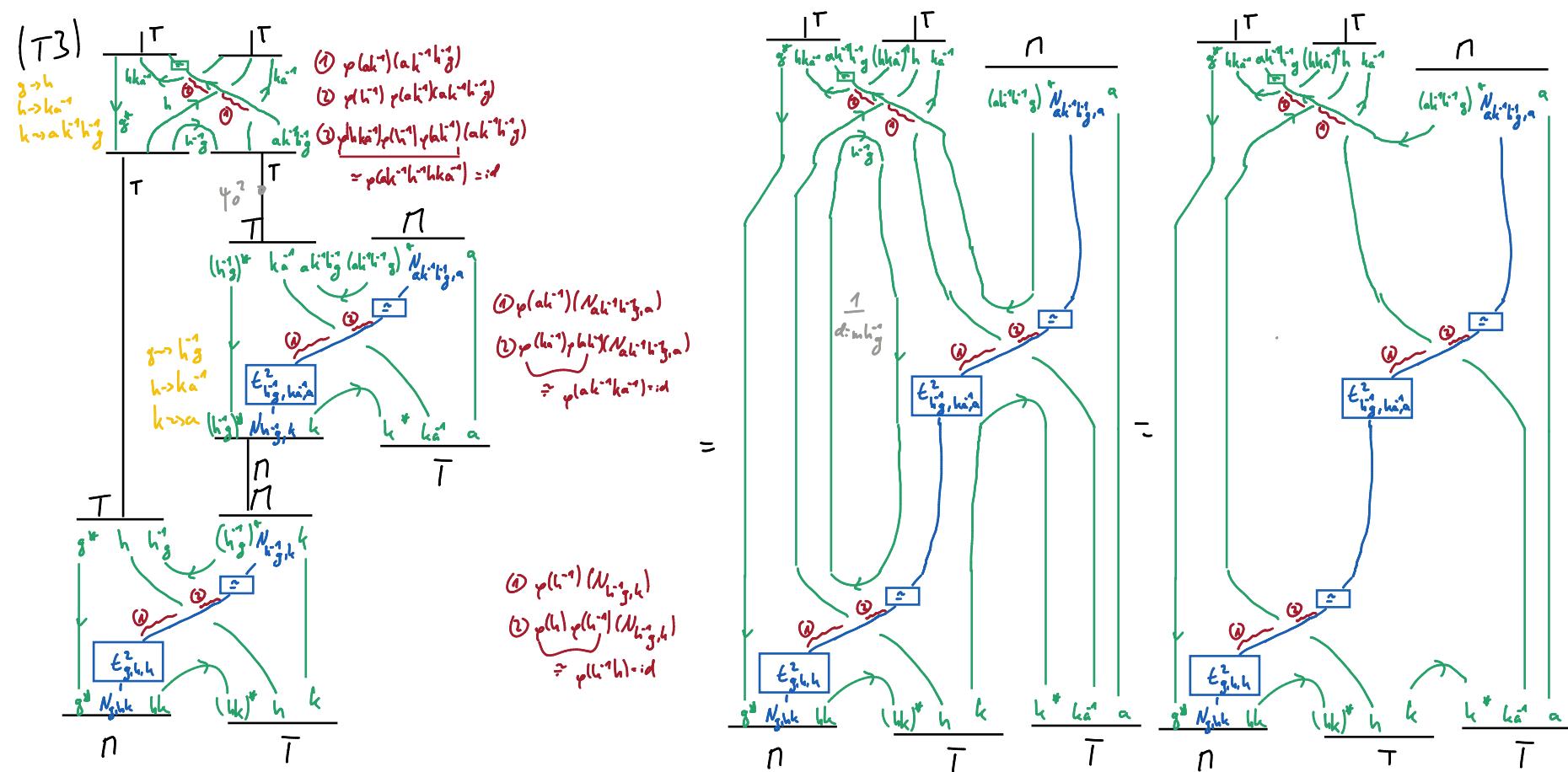
6/6/24



So, (T1) yields

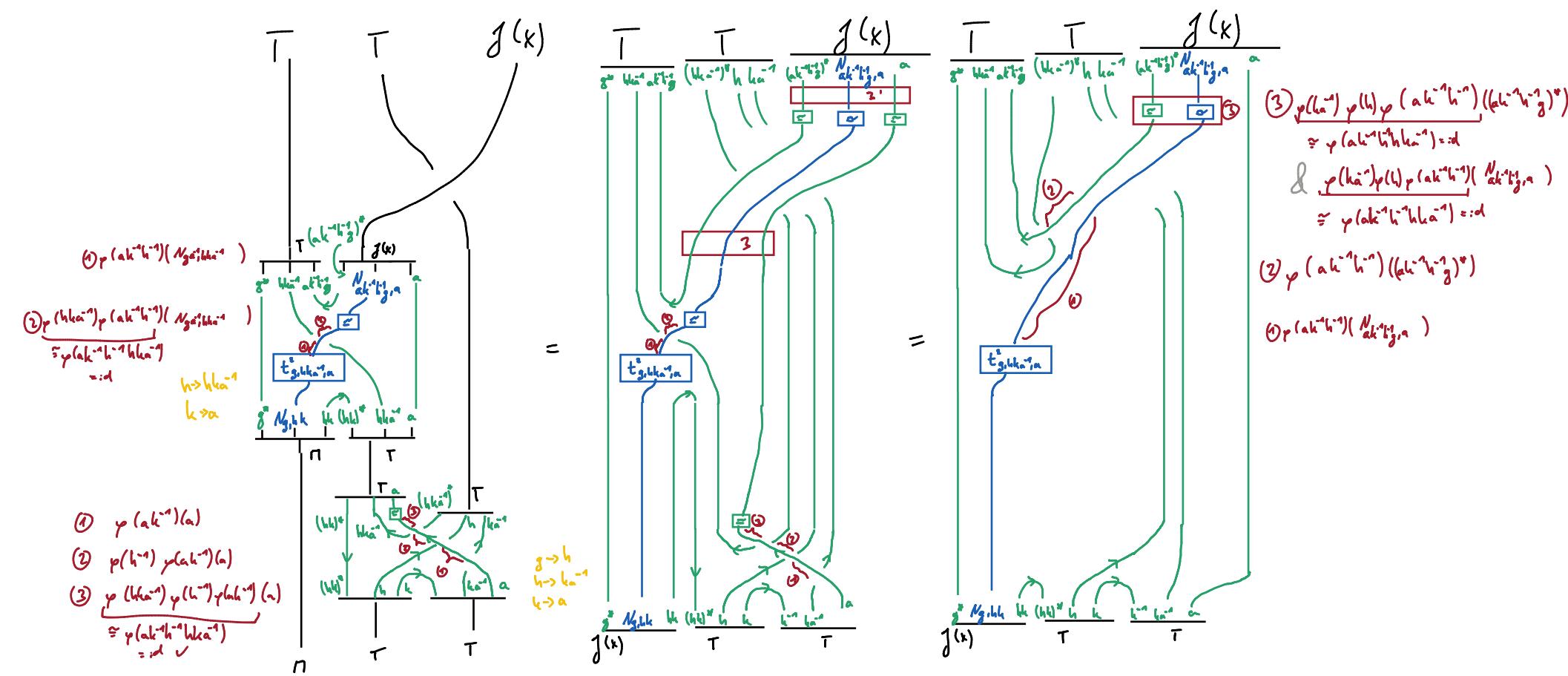
$$t^1_{g,h,k} = t^1_{g,a^{-1}h,ka^{-1}}(t^1_{g,h,k}a^{-1}, a)$$



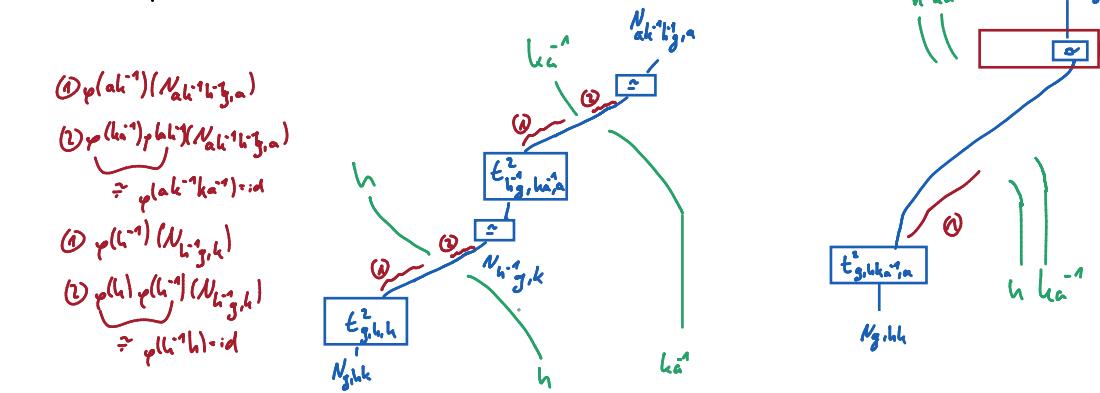


(rhs)

$$\begin{aligned} & 3=3' \\ & \varphi(h^{-1})\varphi(h)\varphi(ak^{-1}h^{-1}) \\ & \approx \varphi(ak^{-1}h^{-1}hka^{-1}) = id \end{aligned}$$



S₁(T3) implies



TODO: Check with isomorphism

$$(\varphi(h^{-1})t^2_{h^{-1}, h^{-1}, a}) \circ (t^2_{g, h, k, a}) = t^2_{g, h, k, a-1}$$

Chak:

$$\begin{array}{ccc} \varphi(g) \varphi(h)(x) & \xrightarrow{\varphi(g)(\gamma_h)} & \varphi(g)(x) \\ \downarrow s & & \downarrow \gamma_g \\ \varphi(hg)(x) & \xrightarrow{\gamma_{hg}} & x \end{array}$$

"
↓

$$\Psi(g) \circ \Psi(h)(N_{b,e}) \xrightarrow{\Psi(g) \left(t_{g^{-1}h^{-1}bh, g, e}^2 \right)^{-1}} \Psi(g) N_{h^{-1}b, h} \xrightarrow{\Psi(g) \left(t_{g^{-1}h^{-1}bh, g^{-1}, g}^1 \right)^{-1}} \Psi(g) N_{g^{-1}h^{-1}bh, e}$$

$$\begin{aligned}
 \text{Eqn. from } & (T^2) \\
 h \rightarrow & g^{-1} \\
 k \rightarrow & e \\
 g \rightarrow & g^{-1} h^{-1} b h \\
 \alpha \rightarrow & h
 \end{aligned}$$

$$\varphi(g) = \underbrace{t_{g^{-1}h^{-1}bh, h^{-1}, h}^1}_{t_{h^{-1}bh, h^{-1}, h}^1} \circ t_{g^{-1}h^{-1}bh, g^{-1}, e}^2$$

|| (via T2)

$$\psi(g) \psi(h) (\lambda_{gh}) \xrightarrow{\psi(g) \left(t_{g^{-1}h^{-1}bh, g^{-1}, h}^1 \right)^{-1}} \psi(g) N_{g^{-1}h^{-1}bh, h} \xrightarrow{\left(t_{g^{-1}h^{-1}bh, g^{-1}, h}^1 \right)^{-1}} N_{g^{-1}h^{-1}bh, g^{-1}h}$$

$$V_{1,a}(T^1) \cdot t'_{g,h,k} = t'_{g^{-1},h^{-1},k^{-1}}(t'_{gh,k^{-1},a}) \quad \begin{matrix} g \mapsto g^{-1} \\ h \mapsto h^{-1} \\ k \mapsto k^{-1} \end{matrix} \quad \begin{matrix} a \mapsto a^{-1} \\ a \mapsto g \end{matrix}$$

$$\varphi(g) \varphi(h) (\lambda_{be}) \xrightarrow{\psi(g) \left(t_{h^{-1}b, h^{-1}e}^2 \right)^{-1}} \varphi(g) N_{h^{-1}b, h} \xrightarrow{\left(t_{g^{-1}h^{-1}b, g^{-1}h}^2 \right)^{-1}} N_{g^{-1}h^{-1}b, g^{-1}h}$$

Via (T3)

$$\begin{aligned} \text{1a (T3)} \\ \psi(g) \psi(h) (N_{\beta e}) &\xrightarrow{\cong} (\psi(gh)) N_{\beta, e} \xrightarrow{?} N_{g^{-1}h^{-1}b, g^{-1}h} \\ (\psi(h) t^2_{g^{-1}h^{-1}g, k^{-1}, a}) t^2_{g, h, k} &= t^2_{g, h k^{-1}, a} (\approx) \\ h \mapsto g^{-1} \end{aligned}$$

1

$$(\eta_{\log})_{g^{-1}t^{-1}bL}$$

$$\varphi(g) \varphi(h)(N_{b,c}) \xrightarrow{\quad} \varphi(hg)(N_{b,c}) \xrightarrow{\left(\frac{t^2}{g}, \frac{t^2}{h}, \frac{t^2}{c} \right)^{-1}} N_{\frac{t^2}{g}b, \frac{t^2}{h}b, \frac{t^2}{c}} \xrightarrow{\left(\frac{t^2}{g}b, \frac{t^2}{h}b, \frac{t^2}{c} \right)^{-1}} N_{\frac{t^2}{g}b, \frac{t^2}{h}b, \frac{t^2}{c}}$$

= Sebastian's leg of the commutative diagram!

So (X, η) is an object in \mathcal{C}^{G_1} (1) ✓

Essential Surjectivity (Q2)

Given τ_1, τ_2

$$t'_{g,e,k}: N_{g,k} \xrightarrow{\sim} N_{gk^{-1},e}$$

Applying E (i.e., $\eta_a|_{B_a^{(1)}} = t^{-1} \circ \epsilon^{-1}$)

Check that the iso ϵ lifts to an iso in the cat. $B_{\mathbb{Z}}$

I.e. Check

$$\text{for } i=1, \dots$$

rhs $i=1$

$N_{g^{-1}, e}$

$t'_{g^{-1}, e, k^{-1}}$

$N_{g^{-1}, k^{-1}}$

$N_{g^{-1}, k^{-1}}$

k

k^{-1}

a

$t_i \approx$

$\sim T_i(M)$

(pg. 21 Ingo's notes)

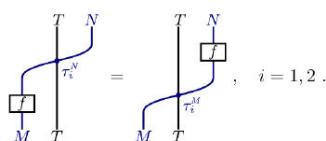
$$\text{For } i=1, \text{ clearly } \text{rhs}=\text{lins} \quad \text{if} \quad \boxed{\frac{t_{ijk}}{t_{ijk} - a}} = \boxed{\frac{1}{t_{ijk} - a}}$$

this should be a consequence of T1, and, indeed, T1 states

$$t_{g, m, k}^i = t_{g, m, k}^{(k) \text{ max}} \left(t_{g, m, k}^{(k) \text{ max}} \right)^{\alpha} \quad \text{So taking} \quad g \mapsto g \quad k \mapsto k \quad \text{in this eqn.}$$

$$\text{gives: } t_{q,e,k}^i = t_{q,a,e,ka^{-1}}^i(t_{q,k,a^{-1},a}) \quad \checkmark$$

i=2

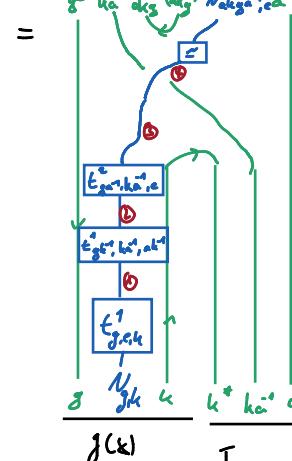
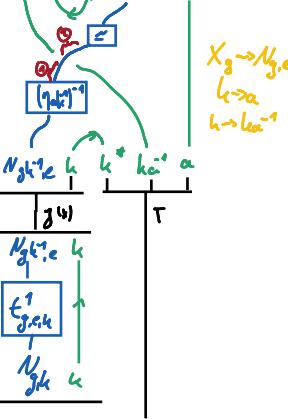
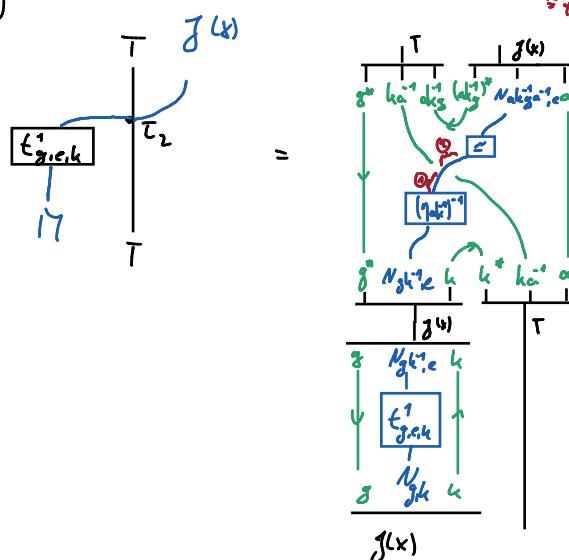


(M)

$$\begin{array}{l} \textcircled{1} \varphi(\text{aki}^{-1})(\text{Nakigae}) \\ \textcircled{2} \varphi(\text{aki}^{-1})\varphi(\text{aki}^{-1})(\text{Nakigae}) \\ \textcircled{3} \varphi(\text{aki}^{-1}\text{aki}^{-1}) = \text{id} \end{array}$$

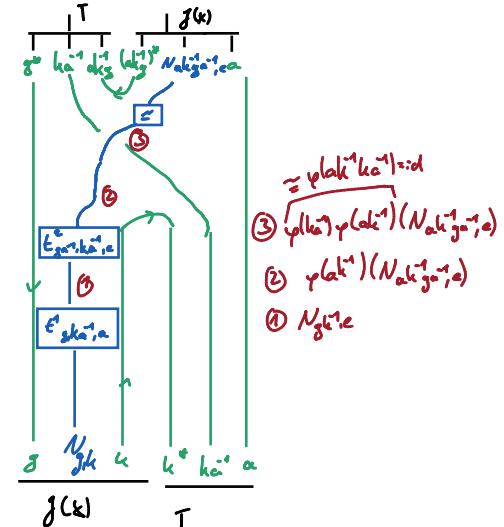
$$\left[\varphi(a)(N_{b,c}) \xrightarrow{(t_a^b)_{b,c}} N_{a^{-1}b,a^{-1}} \xrightarrow{(t_a^b)_{b,c}^{-1}} N_{a^{-1}b,a^{-1}} \right]$$

(LHS)

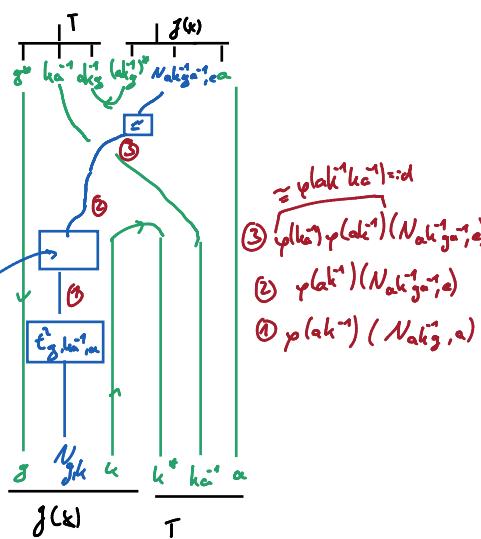


$$\begin{array}{l} \textcircled{1} \varphi(\text{aki}^{-1}\text{aki}^{-1}) = \text{id} \\ \textcircled{2} \varphi(\text{aki}^{-1})\varphi(\text{aki}^{-1})(\text{Nakigae}) \\ \textcircled{3} \varphi(\text{aki}^{-1})(\text{Nakigae}) \\ \textcircled{4} N_{g^{-1},ka^{-1}} \\ \textcircled{5} N_{g^{-1}e} \end{array}$$

$$t'_{g,h,k} = t'_{g^{-1},h,ka^{-1}}(t_{g,h,k^{-1},a})$$



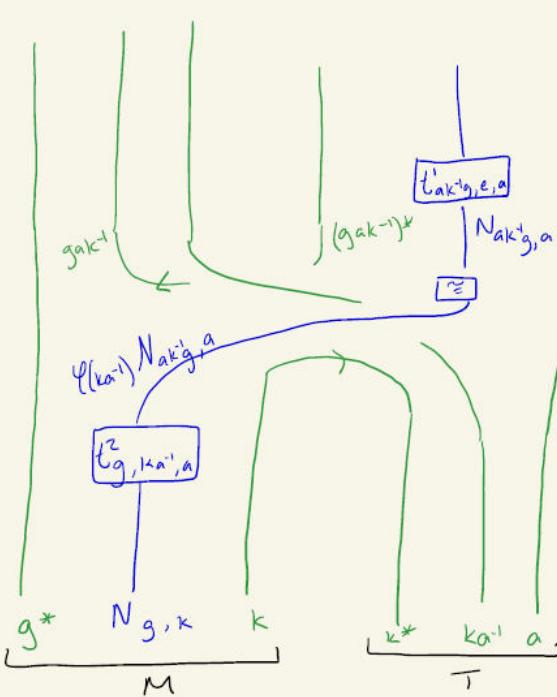
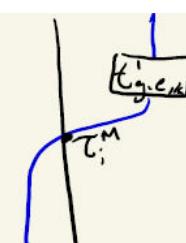
$$\begin{array}{l} \textcircled{1} \varphi(\text{aki}^{-1}\text{aki}^{-1}) = \text{id} \\ \textcircled{2} \varphi(\text{aki}^{-1})\varphi(\text{aki}^{-1})(\text{Nakigae}) \\ \textcircled{3} \varphi(\text{aki}^{-1})(\text{Nakigae}) \\ \textcircled{4} N_{g^{-1}e} \end{array}$$



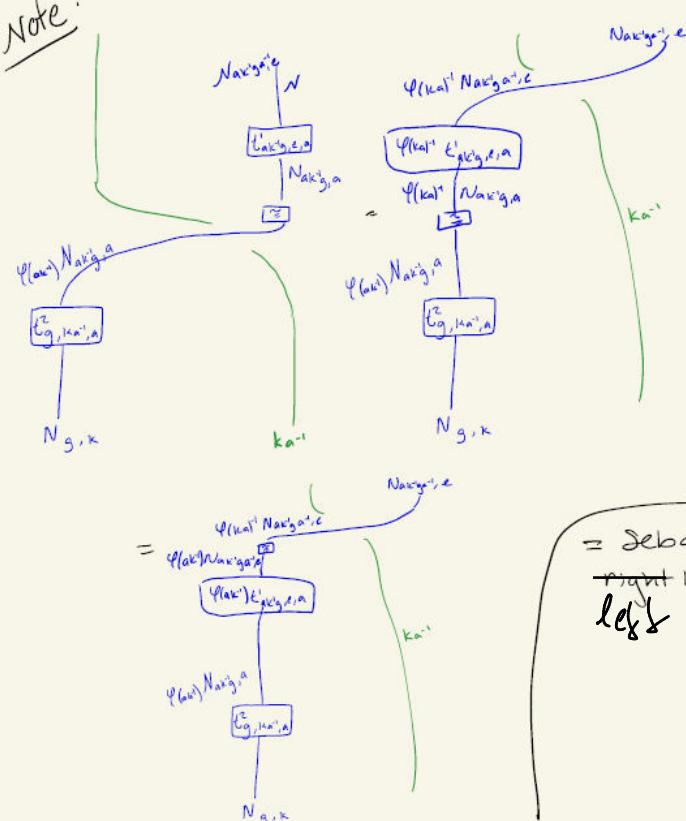
$$\begin{array}{l} \textcircled{1} \varphi(\text{aki}^{-1}\text{aki}^{-1}) = \text{id} \\ \textcircled{2} \varphi(\text{aki}^{-1})\varphi(\text{aki}^{-1})(\text{Nakigae}) \\ \textcircled{3} \varphi(\text{aki}^{-1})(\text{Nakigae}) \\ \textcircled{4} \varphi(\text{aki}^{-1})(\text{Nakigae}) \\ \textcircled{5} \varphi(\text{aki}^{-1}) \end{array}$$

$$\psi(n^i)t'_{h_g,ka^{-1},a} \circ t'_{g,h,k} = t'_{g,a^{-1},h,ka^{-1}} \circ t'_{g,h,k^{-1},a}$$

rhs i=2



Note:



= Sebastian's
right hand side!
leg