Data fusion techniques and positioning estimation for land vehicle navigation systems: an overview

Denis Gingras, Dr. Ing.
Department of Electrical and Computer Engineering,
Université de Sherbrooke, Sherbrooke (Québec)
Canada, J1K 2R1

Invited professor, Faculty of Engineering, Universidad
Autonomía de Zacatecas, Zacatecas, Mexico

denis.gingras@usherbrooke.ca

ABSTRACT

In this paper, we will review the problem of estimating in real-time the position of a vehicle for use in land navigation systems. After describing the application context and giving a definition of the problem, we will look at the mathematical framework and technologies involved to design positioning systems. We will compare the performance of some of the most popular data fusion approaches and provide some insights on their limitations and capabilities. We will then look at the case of robustness of the positioning system when one or some of the sensors are faulty. We will describe how the positioning system can be made more robust and adaptive in order to take account the occurrence of faulty sensors. Finally, we will go one step further and explore possible architectures for collaborative positioning systems whereas many vehicles are interacting and exchanging data to improve their own location estimate. We close the paper with some concluding remarks on the future evolution of the field.

INTRODUCTION

An integrated navigation information system is an embedded system installed in a vehicle, which provides useful functionalities to the driver like path planning, guidance, digital map and points of interest directory [1]. The guidance module uses a planned trip to indicate the driver which route to take. To avoid giving wrong indications and impair driving safety, the navigation system relies upon a positioning module to know precisely and continuously the localization of the vehicle. The required performance of the positioning module is achieved by using a cluster of heterogeneous sensors whose measurements are fused. The sensors commonly found in those systems are differential odometer, global positioning system (GPS) and 2 or 3 axis inertial measurement unit respectively. Two or more of these complementary positioning sensing methods must be integrated together to achieve the required performance at low cost. The integration, which implies the fusion of noisy data provided by each sensor, must be performed in some optimal manner. Most positioning system designers choose the Kalman filter as the data fusion method. In particular, one uses the *the extended Kalman filter.* The extended Kalman filter is a variation of the Kalman filter used to cope with nonlinearities of the sensors. Recently, an improvement to the extended Kalman filter has been proposed, the unscented Kalman filter [2]. An other interesting alternative to using Kalman filters is the use of artificial neural network (NN) [3]. In the following section, we will look at those various data fusion approaches and discuss their respective performances.

Next we will look at the robustess required for a real time positioning system. In positioning navigation systems, at any time, any of the sensors can break down or stop sending information, temporarily or permanently. To ensure a practical solution for use in guidance and navigation systems, faulty sensors must be detected and isolated such that their erroneous data will not corrupt the global position estimates. As we just mentioned, Kalman filter is usually popular for data fusion applications. However, an interesting idea is to use it for fault detection architecture as well. In this section, we will evaluate the potential of combining fault detection and data fusion into a single architecture to make a robust positioning navigation system [4].

So far, single vehicle positioning systems have been discussed. In a subsequent section, we will consider the possibility to achieve a distributed collaborative architecture. Proliferation of real time inter-vehicular communications provides new sources of exploitable positioning data. Vehicles can, under numerous situations, have GPS satellite shortages but there will always be vehicles in their vicinity, viewing different GPS satellites line of sights (LOS), to provide them with useful navigation information. We will thus look at a cooperative positioning technique making use of reliable positions of some vehicles to enhance positioning estimates of some others [5]. The approach exploits inter-vehicle data flow to extract good position measurements from vehicles with good GPS satellite LOS, in order to enhance low positioning accuracy of other vehicles, in the neighborhood. The integration of such information is done using geometric data fusion approach.

CONTEXT AND PROBLEM STATEMENT

The principal function of a land navigation system in a vehicle is to guide the driver while minimizing the trip duration and/or the traveled distance. Modern navigation applications require position estimates with a precision of 1 meters or less and at a frequency of 1 Hertz or more. A differential GPS receiver alone could in principle estimate the position with the required performance. It is still too expensive however for automotive applications and the fixed references are not always available. On the other hand, a low cost GPS receiver cannot always achieve the required precision. Furthermore, GPS alone based systems suffer from frequent occlusions of satellite signals by high buildings or heavy foliages, which often prevent computation of a solution by a GPS receiver for several seconds. For all these reasons, a cluster of various sensors, including a GPS receiver, is usually used [6]. Two other popular sensors are the differential odometer and the inertial measurement unit (IMU) [7]. A differential odometer measures the distance traveled by a vehicle and the current azimuth during a sampling period. An inertial measurement unit (IMU) measures the vehicle's inertia characterized by it's acceleration and it's angular velocity. An IMU measures the acceleration and the angular velocity along the axis of a Cartesian coordinate system. With these two sensors, the position of the vehicle is reckoned by applying basic kinematics' equations and using an initial position obtained from another information source. By means of integration, the traveled distance and the azimuth variation can be computed and therefore a new position can be computed with these measurements and the last known position by dead reckoning. The estimated position will eventually drift from the real position because of the accumulation of errors. Indeed, the recursive nature of the positioning computation with the IMU causes the positioning error to grow proportionally with time. A periodic reset is needed. A trivial way to fuse the data from these two sensors is therefore to reset periodically the IMU position estimate with an absolute position estimate from the GPS.

A more complex fusion method than the reckon/reset positioning system described above is required to improve the precision of the estimation. These methods fuse continuously the available measurements in some optimal sense, as depicted in figure 1 in a centralized architecture.

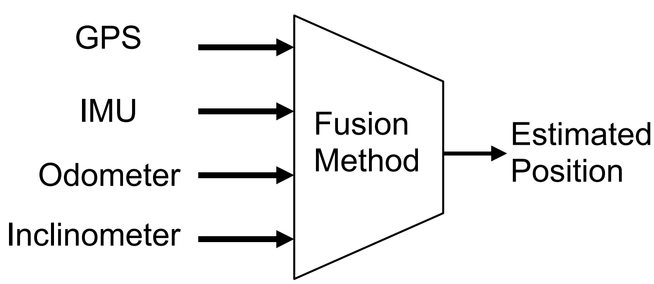

Figure 1 : Centralized fusion

The sensors' measurements are distorted by deterministic and random errors. Those sources of random errors are usually described using stochastic models in a statistical framework. The estimate ideally maximizes the a posteriori probability of the random variable position, resulting from the mathematical transformation of the stochastic processes, which model the sensors' imperfect measurements. The fusion method then defines the mathematical transformation. As we will see later on, the most popular method is the Kalman filter [8]. The Kalman filter is an optimal linear estimator, which uses the a priori information on the sensor noises, the vehicle dynamic and the kinematics equations to compute recursively an optimal position, which minimizes the mean square error [9], [10].

SENSORS MODELS

In a positioning navigation system, various sensors are used. Generally speaking, a sensor is a device that responds to or detects a physical quantity and transmits the resulting signal to a controller. Position sensors can be designed to detect various parameters (coordinate, distance, direction, or angular velocity) of the position of vehicular mechanical systems. Sensors can provide absolute or relative navigation information. GPS and magnetic compass provide absolute position and angular direction (azimuth) respectively, whereas all other sensors provide relative navigation information using dead reckoning. Detailed relationships between the position and typical sensors are shown in Table 1.

Sensor	Output	Relation to position
GPS	Rover position	Directly output position coordinates
IMU	Accelerations and angular rate	Outputs can be integrated by an INS to obtain the vehicle position
Odometer	Distance or increment of distance	Position coordinates are determined by dead reckoning from the distance and direction relative to a known location
Inclinometer	Inclination	
Magnetic compass	Azimuth	

Table 1. Relationship of Vehicle Position and Sensor Outputs

<u>GPS model</u>

The GPS model can be described by equation (1.1) [1].

$$\hat{\rho}[t] = \rho[t] + \delta_{trop}[t] + \delta_{iono}[t] + \delta_{white}[t] + \delta_{mult}[t] \quad (1.1)$$

where $\hat{\rho}[t]$ is the pseudorange measured by the emulated receiver, $\rho[t]$ is the real pseudorange, $\delta_{trop}[t]$ is the tropospheric delay, $\delta_{iono}[t]$ is the ionospheric delay, $\delta_{white}[t]$ is the white noise generated by the receiver's electronic components and $\delta_{mult}[t]$ is the multipath problem.

<u>IMU model</u>

The 3-axis inertial measurement unit has a single model for every gyroscope and accelerometer described by equation (1.2), [6].

$$
\begin{aligned}
\hat{d}_i[t] &= d_i[t] + s_i(d_i[t] + d_j[t]*\sin(\delta_{ij}) \\
&+ d_k[t]*\sin(\delta_{ik})) + b_i[0] + \sum_{n=1}^{t} w_{bi}[n] \\
&+ d_j[t]*\sin(\delta_{ij}) + d_k[t]*\sin(\delta_{ik}) \\
&+ c_i(d_i[t] + d_j[t]*\sin(\delta_{ij}) + d_k[t]*\sin(\delta_{ik}))^2 \\
&+ w_i[t],
\end{aligned}
\tag{1.2}
$$

where $\hat{d}_i[t]$ is the measurement, $d_i[t]$ is the real value, s_i is the scale factor error, $d_j[t]$ is the real value relative to the axis j , $d_k[t]$ is the real value relative to the axis k, δ_{ij} is the misalignment angle between the axis i and j, δ_{ik} is the misalignment angle between the axis i and k, $b_i[0]$ is the turn-on bias, $w_{bi}[n]$ is the random walk white noise characterizing the bias drift, c_i is the non-linear scale factor and $w_i[t]$ is the additive white noise component.

The scale factor error, the bias parameters, the non-linear scale factor and the additive white noise component have different value depending on the sensor type.

<u>Differential odometer model</u>

A differential odometer is constituted of two sensors measuring the number of rotation of each wheel situated on the same axle. The total traveled distance and the azimuth of the vehicle can be computed with the equation (1.3) and (1.4) respectively.

$$
\begin{aligned}
\hat{d}[t] &= \hat{d}[t-1] + \\
&\frac{(1+v_d*v[t]+s_d)*c_d[t]+r[t]}{2} + \\
&\frac{(1+v_g*v[t]+s_g)*c_g[t]+r[t]}{2}
\end{aligned}
\tag{1.3}
$$

$$
\begin{aligned}
\hat{\theta}[t] &= \hat{\theta}[t-1] + \\
&\frac{(1+v_g*v[t]+s_g)*c_g[t]+r[t]}{l} - \\
&\frac{(1+v_d*v[t]+s_d)*c_d[t]+r[t]}{l},
\end{aligned}
\tag{1.4}
$$

where $\hat{d}[t]$ is the total traveled distance at time t, $\hat{d}[t-1]$ is the total traveled distance at time t-1, $v[t]$ is the car's velocity, v_d and v_g are the gains characterizing the tire dilatation, s_d and s_g are the scale factors for the right and the left wheel respectively, $c_d[t]$ and $c_g[t]$ are the numbers of rotations measured within the interval [t-1,t] for the right and left wheel respectively, $r[t]$ is a uniform random variable describing the resolution error, $\hat{\theta}[t]$ is the azimuth estimated at time t, $\hat{\theta}[t-1]$ is the azimuth estimated at time t-1 and l is the axle length.

Data fusion using Kalman Filter

The architecture of a positioning system can be decentralized or centralized. In the centralized architecture, all the sensor measurements are fused by one fusion method only. So it is easy to compare the performance of two different fusion methods when the cluster of sensors is the same. The Kalman filter is an optimal linear estimator introduced in 1960 [8]. The filter is optimal when the process noise and the measurement noise can be modelized by white Gaussian noise. However, it behaves poorly in the presence of nonlinearities. Improvement can be achieved with the extended Kalman filter (EKF) [10], [11]. This filter is based upon the principle of linearizing the state transition matrix and the observation matrix with Taylor series expansions. The extended Kalman filter has been very popular for land navigation system [7, 25, 26]. The equations of a centralized data fusion architecture based on an extended Kalman filter for land navigation positioning system are described in [7]. As expected, the linearization can lead to poor performance and divergence of the filter for highly non-linear problems. In addition, the performance analysis of the extended Kalman filter presents some difficulties due to the recurrence of the measure sequence into the states of the filter [6]. Finally, implementation of the extended filter can be quite laborious depending on the number of states required to model the system. For all these reasons, a recent improvement to the EKF, named the “unscented” Kalman filter (UKF) has been proposed [14]. The UKF approximates the probability density resulting from the non-linear transformation of a random variable instead of approximating the nonlinear functions with a Taylor series expansion. The approximation is done by evaluating the nonlinear function with a minimal set of carefully chosen sample points. The posterior mean and covariance estimated from the sample points are accurate to the second order for any nonlinearity [12]. If the priori random variable is Gaussian, the posterior mean and covariance are accurate to the third order for any nonlinearity [13]. The first use of an unscented Kalman filter for land navigation positioning system is described in [15]. To our knowledge, apart from our work, only one paper has been recently written on the use of the unscented Kalman filter as the fusion method in an integrated navigation information system [16]. An unscented Kalman filter has also been used for GPS positioning [17]. In the following sections, we will look into more details of the use of unscented Kalman filter over the extended Kalman filter for an integrated navigation information system.

Extended Kalman Filter

The extended Kalman filter predicts the states of the random process with equation (2.1). The predicted states are updated with the measurements in equation (2.2) [2].

$$x_{k+1|k} = \Phi_{k+1|k}[x_{k|k}] + w_k \qquad (2.1)$$

$$z_{k+1} = H_{k+1|k}[x_{k+1|k}] + v_{k+1} \qquad (2.2)$$

where $x_{k+1|k}$ is the predicted process state vector, $x_{k|k}$ is the estimated process state vector, $\Phi_{k+1|k}$ is the discrete state transition matrix from k to k+1, w_k is the process noise vector, z_{k+1} is the measurement vector, $H_{k+1|k}$ is the observation matrix and v_{k+1} is the measurement noise vector.

In our study, we have 13 states to describe the random process. A position-velocity-acceleration model is used for each component of the position [11]. The last four states include the slope, the pitch, the azimuth and the yaw velocities. The state transition matrix $\Phi_{k+1|k}$ is linear. Only the observation matrix $H_{k+1|k}$ contains nonlinear equations, the most relevant for horizontal positioning is described by equation (2.3).

$$\begin{bmatrix} a_R \\ a_P \\ a_Y \end{bmatrix} = \begin{bmatrix} \cos(\Phi_y)*\cos(\Phi_p) & -\sin(\Phi_y) & \cos(\Phi_y)*\sin(\Phi_p) \\ \sin(\Phi_y)*\cos(\Phi_p) & \cos(\Phi_y) & \sin(\Phi_y)*\sin(\Phi_p) \\ -\sin(\Phi_p) & 0 & \cos(\Phi_p) \end{bmatrix} * \begin{bmatrix} a_N \\ a_E \\ a_D \end{bmatrix} \qquad (2.3)$$

where a_R, a_P, a_Y are the acceleration vector components along the roll, the pitch and the yaw axis respectively, Φ_p and Φ_y are the euler angles for the pitch and the yaw axis respectively, a_N, a_E, a_D are the acceleration along the north axis, the east axis and the down axis respectively.

The extended Kalman filter approximates the non-linear matrix H based on the Taylor series expanded about the estimated state vector with

$$H[\hat{x}_{k+1|k}] \approx H[\hat{x}_{k|k}] + \frac{\partial H[\hat{x}_{k|k}]}{\partial \hat{x}_{k|k}}(\hat{x}_{k+1|k} - \hat{x}_{k|k}) \qquad (2.4)$$

The linear approximation often introduces large errors in the estimated state vector and can lead to the divergence of the filter.

The Unscented Kalman Filter

The unscented Kalman filter is based on the unscented transformation, which is a method for reckoning the statistics of a random variable undergoing a non-linear transformation. A set of $2*n_\chi + 1$ weighted samples are deterministically chosen to capture the true mean and variance of the prior random variable.

$$n_\chi = n_x + n_w + n_v \qquad (2.5)$$

where n_x is the number of process states, n_w is the dimension of w_k and n_v is the dimension of v_k. The unscented Kalman filter approximates the non-linear observation matrix by

$$H[\hat{x}_{k+1|k}] \approx \sum_{i=0}^{2*n_\chi} W_i * H[\chi^x_{i,k+1|k}] + \chi^v_{i,k+1} \qquad (2.6)$$

where Wi are the weights, $\chi^x_{i,k+1|k}$ are the sigma points describing the prior predicted states and $\chi^v_{i,k+1}$ are the sigma points describing the measurement noise.

In order to obtain statistically reliable data on the performance of both algorithms, one hundred Monte Carlo simulations have been run for each sensor fusion method [20]: For each sampling time, the estimated positions from the Monte Carlo simulations form the sampling distribution. There were 26639 measurement vectors for each Monte Carlo simulation. These sampling distributions approximate the truth continuous distributions of the posteriori random variables describing the estimated positions. The first moment of each sampling distribution has been computed and used for the computation of the performance metrics. The two performance metrics usually encountered in data fusion systems analysis are the accuracy/precision of the fusion and the computational time to perform the fusion. The accuracy is evaluated by taking the Euclidian distance between the estimated position and the true position. The mean and the variance of the Euclidian distances for the whole simulation are reckoned. The variance describes the precision of the fusion method. The horizontal position is described by the tangential plane located at the real vehicle position whose coordinates are given by the latitude and the longitude.

In [2] we give detailed results showing that the unscented Kalman filter has slightly better results for horizontal positioning than the extended Kalman filter. The estimated position is less biased for the unscented Kalman filter than for the extended Kalman filter. It shows also that the unscented Kalman filter is more precise than the extended Kalman filter. However, contrary to the claim in [12, 13], we found that the computational cost of the unscented Kalman filter is significantly greater, by a factor larger than 20, than the computational cost of the extended Kalman filter. The significant execution time difference is related to the number of times equations (2.1) and (2.2) are evaluated for each fusion algorithm. With the unscented Kalman filter, these equations are evaluated 75 times, once for each sigma point. With the extended Kalman filter, the Taylor series expansion of these equations are only evaluated once at each iteration. Furthermore, the Jacobian of the matrix H used in the Taylor series expansion is calculated only once because the observation equations are static. Thus the multiple computations of equations (2.1) and (2.2) by the unscented Kalman filter at each iteration is responsible for the larger computational cost.

Surprisingly, we found that the unscented Kalman filter is less performant than the extended Kalman filter when there is no GPS solution available. In that situation, the acceleration of the vehicle measured by the IMU is used to estimate the vehicle's position described by equation (2.3).

This equation represents the nonlinear transformation of the estimated states which are assumed to be Gaussian random variable in order to predict the IMU measurement. The performance of both filters depends on their capacity to estimate the mean of the resulting random variable. An empirical analysis has been made to evaluate this capacity. In this experiment, each state has been modelized by a discrete Gaussian random variable with 100 realizations distributed uniformely in the range of possible values with a 99% probability of realization. Each realization is present a number of times proportional to its probability of realization in the statistical data representing the probability function. Thus, 24060 samples modeled each random variable. The nonlinear function described by equation (2.3) is then applied to these random variables and the means of the resulting random variables are computed. The same discrete random variables have been used with the Taylor series expansion of equation (2.3). In the extended Kalman filter, the linearization occurs around the states estimated at the previous iteration. The linearized equation is applied to the predicted states at the current time. The linearization error is directly proportional to the difference between the estimated states and the predicted states. For the empirical analysis, the mean variation between the estimated value and the predicted value obtained with the extended Kalman filter for one Monte Carlo simulation has been taken. Table 2 shows the variation between the real mean of the a posteriori probability density and the estimated mean of the a posteriori probability density obtained with the Taylor series expansion and the unscented Kalman filter respectively. As can be seen, the unscented Kalman filter provides no significant improvement over the extended Kalman filter and even brings a degradation in performance for two acceleration components (roll and pitch).

Estimated state	EKF	UKF
Roll acceleration	0.0064 %	0.8070 %
Pitch aceleration	0.0218 %	1.2876 %
Yaw acceleration	0.2482 %	0.0754 %

Table 2: Difference between the real mean and the estimated mean of the a posteriori density

The superiority of the unscented Kalman filter happens only when the variation between the predicted states and the estimated states is important. However, due to the low dynamics of the vehicle, most of this variation is not important enough to generate a significant linearization error for the EKF.

THE USE OF NEURAL NETWORKS IN POSITIONING SYSTEMS

A centralized fusion implied the use of only one algorithm which fuses all the measurements provided by the sensors. The engineering labour required to construct a centralized Kalman fillter can be very high. For this reason, most of the GPS/INS systems have decentralized filters, which gain in simplicity. However, the price to pay is a loss of precision compared with a centralized architecture [21]. An attractive alternative is to use a neural network (NN) for the data fusion. Indeed, a centralized NN is no more difficult to realize than a decentralized one. NNs have already been applied to data fusion related to positioning problem in robotic with success [18], [22]. The major advantage of an NN compared to a Kalman filter resides in the fact that it doesn't need any a priori statistical and mathematical model to find a function which maps optimally the inputs with the outputs, in our case the absolute position. The most difficult task is to gather some sensor data which can cover adequately the different manoeuvres to be encountered by a road vehicle and its dynamic. The manoeuvres are a function of the road geometry and the vehicle's performance. For example, a NN trained with measured data coming only from a straight road segment may not be able to give a good position when the vehicle meets a curve.

A NEURAL NETWORK FOR DATA FUSION

As described in more details in [3, 19], we used a feed-forward backpropagation neural network that was trained with 14939 training data set for 2000 epochs. The NN has 4 layers composed of 20, 20, 20 and 3 neurons respectively. The corresponding transfer functions are linear, log-sigmoid, tan-sigmoid and linear. This architecture was the most promising of the 12 architectures investigated with various number of layers, transfer functions and number of neurons trained initially for 100 epochs. Batch training was prefered over iterative training for it's computation efficiency. The mean square error gave the training performance at each epoch. The second order training method was the scaled conjugate gradient. This method requires only O(N) operations per epoch compared to others methods like Gauss-Newton $O(N^3)$ and Levenberg-Marquardt $O(N^3)$ [24, 19]. When considering the network size, the other training methods were impractical. Table 3 contains the NN's inputs.

Data	**Sensor**
Latitude	GPS
Longitude	GPS
Altitude	GPS
Percent Dilution Of Precision (PDOP)	GPS
Linear acceleration	INS
Horizontal centripetal acceleration	INS
Vertical centripetal acceleration	INS
Yaw angular velocity	INS
Pitch angular velocity	INS
Total traveled distance	Differential odometer
Azimuth	Differential odometer
GPS solution availability	GPS
Sampling time	IMU

Table 3: Neural network inputs

The GPS receiver sampling frequency is usually lower than those of the inertial measurement unit or the differential odometer. For synchronization purpose, a boolean entry specifies to the NN if a GPS solution is available. A centralized Kalman filter has been realized to has a reference for the evaluation of the NN's performances. The centralized Kalman filter has 13 states and 10 measurements. The simulation generates 26430 data samples.

DATA FUSION RESULTS USING NEURAL NETS

The mean and variance of the positioning error during the simulation was computed. Table 4 indicates that the positions estimated by the NN are less biased for the longitude and altitude but more biased for the latitude than the same positions estimated by the Kalman filter. The NN’s estimation biases don’t exceed 10 meters. So even if the NN is a biased estimator, it still meets the required performance.

As shown in Table 5, the variances of the latitude errors and the altitude errors for the NN are less than those of the Kalman filter. The variance of the longitude errors is 5 percent more for the NN than for the Kalman filter. The performance of the NN is generally better than the peformance of the Kalman filter in a mean square error sense. The important gain for the altitude is caused by the NN's ability to estimate the bias of the GPS altitude.

Position	**Kalman (m)**	**ANN (m)**	**Gain (%)**
Latitude	2.73	9.34	-242.90
Longitude	12.44	3.56	71.38
Altitude	49.21	0.38	100.78

Table 4: Mean of the positioning errors by the Kalman Filter and the NN

Position	**Kalman**	**ANN**	**Gain (%)**
Latitude	1073.2	606.1	43.52
Longitude	994.8	1029.7	-3.51
Altitude	12952.0	2.4	99.98

Table 5: Variance of the positioning errors by the Kalman filter and the NN

It can be seen here that NN’s can be used as a centralized fusion method. The results show that neural networks are an attractive alternative to the Kalman filter as a centralized fusion method. In [3], we show also how NNs can also be used as nonlinear pre-processing filters for the land navigation positioning problem. In that paper, we show the NN's capability to successfully learn nonlinear functions when applied to GPS and differential odometer measurement pre-filtering. The major difficulty with an NN is to have access to ground truth data for the supervised training. Usually, the only solution is to have access to some data coming from a reference usually given by high precision, high cost sensors. Some further researches on this topic include the evaluation of various NN’s with real sensors and the replacement of the feed-forward backpropagation NN with a recurrent NN.

ROBUST AND ADAPTIVE POSITION ESTIMATE IN PRESENCE OF SENSORS FAULTS

So far, we considered the case of continuously operating noisy sensors. But what happen to the data fusion process and the position estimate if one of the sensors fails? Either from a user’s safety point of view or a designer’s perspective, all automotive navigation systems should be fully reliable and prevent faults or failures. In all but the most trivial cases the existence of a fault may lead to situations with safety, health, environmental, financial or legal implications. Although good design practice tries to minimize the occurrence of faults and failures, it is recognized that such events do occur. In such cases, faulty sensors must be detected and the system must be able to reconfigure itself so as to overcome the deficiency caused by the fault. In brief, a navigation system must be robust and adaptive.

Faults can cause the loss of the overall performance of a system, which may present hazards to personnel or lead to unacceptable economic loss. In order to minimize the impact, fault detection schemes must be developed. Actually, several fruitful research efforts in the field of fault detection and filter based adaptive architectures, combining fault detection and data fusion, have been proposed to improve the reliability and adaptability of various control systems [32, 33, 34]. However, little has been published in the area of automotive navigation systems.

FAULT DETECTION ARCHITECTURE

A fault is usually defined as an undesired change in system estimated parameters that degrade partial or overall performance. Fault detection is a binary decision making process. Either the system is functioning properly, or there is a fault present.

Generally speaking, fault detection consists of two processes: *residual generation* and *decision making*, as shown in Figure 2 [28, 29].

Residual Generation

Residuals are defined as the resulting differences between analytically redundant quantities in the system model. These are similar to innovations generated by a Kalman filter, which are the differences between the measured and estimated outputs. Under normal conditions, residuals are small or zero mean; while the occurrence of a fault causes the residuals to go to non-zero or unusually large values.

Decision Making

The decision making process, which acts as an arbitrator, involves assessing the residuals and identifying when and where any abnormalities occur. This is done through threshold testing both static and dynamic residual behaviors, and various statistical tests, where the thresholds are typically based on signal/residual variance.

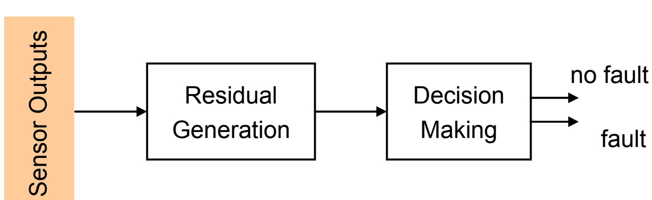

Figure 2: Fault Detection Architecture

SENSOR FAULTY MODELS

So far, although the precision and reliability of sensors are improved significantly with the development of the technology, various sensor faults driven by different situations do exist. In the following, several faulty scenarios of sensors are investigated and discussed.

GLOBAL POSITIONING SYSTEM (GPS) FAULTY MODEL

A low-cost GPS receiver can output the vehicle position and driving speed. However, the measurement is likely to be corrupted by time-correlated noise and the GPS signal is susceptible to jamming. However, the position and velocity measurements do not drift over long periods of time.

A GPS faulty model can be based on four particular parts: typical error budget, environmental interferences, signal loss, and hardware malfunction.

Typical Error Budget

The main error sources in GPS are listed in Table 6. These errors can be divided into two categories [1]: common and non-common. Common errors are approximately the same for receivers operating within a limited geographic region. Non-common errors are unique to each receiver and depend on the receiver type and multipath mitigation technique being used (if any). The point of this classification is that DGPS can effectively remove the common errors.

Source of errors	Standard deviation (m)
Common	
Ionosphere	7.0
Clock and ephemeris	3.6
Troposphere	0.7
Non-common	
Receiver noise	0.1~0.7
Multipath	0.1~5.0

Table 6. GPS Error Sources and their Approximate Deviation [27]

Environmental Interferences

GPS satellite signals, as with any other radio signals, are subject to some form of interference and jamming. It is known that GPS satellite currently transmit position information in the 1,500-MHz frequency band with a typical accuracy under 100 meters to anyone in the world who has a simple receiver costing as little as $100. Any electronic systems generating radio signals in this frequency band, main lobe or side lobe, will tend to be a source of inference to the GPS receiver. With the popularization of personal radio and Wi-Fi devices, electromagnetic interferences, intentional or unintentional, are more and more serious. As an example, the proliferation of ultra-wideband (UWB) devices intended to be mass-marketed to the public could cause harmful interference to GPS.

Signal loss

GPS is a line-of-sight sensor, and therefore GPS measurements are subject to signal outages. If it cannot "see" four satellites, then it will not produce the expected output. This case is called signal loss. It may include the following scenarios:

- Urban environments with all buildings (the so-called urban canyons).
- Inside parking structures.
- In a long tunnel without any relay station.
- Under heavy foliage.
- Under bridges.

Hardware Malfunction

A GPS receiver hardware malfunction can be caused by any abnormality of its components, such as antenna, amplifier, reference oscillator, frequency synthesizer, wire disconnection, and power lost, resulting to no output, or provide an unstable or incorrect signal. Compared to the other sources of GPS faults, the probability of a hardware malfunction of the GPS receiver is rather low and can be considered negligible. Taking all the above into account, the GPS faulty model can be described as in Figure 3.

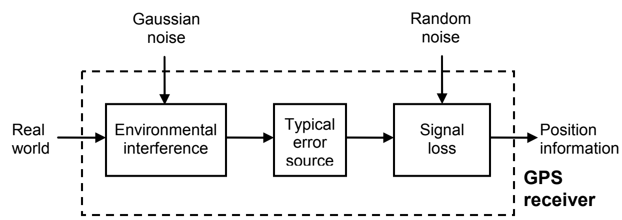

Figure 3: GPS Faulty Model

INERTIAL MEASUREMENT UNIT (IMU) FAULTY MODEL

A low-cost IMU can output the vehicle accelerations and angular rate which can then be integrated by an Inertial Navigation System (INS) to obtain the vehicle position, velocity, and attitude. The advantage of an INS is low sensitivity to high-frequency noise and external conditions. But the measurement error of INS will accumulate if it is not calibrated on-line. The faulty model is presented in figure 4 and the scenarios driving the IMU to a faulty state are discussed below:

IMU Error Sources and Faulty Scenarios

1. Bias due to bearing torques (for momentum wheel types), drive excitation feedthrough, electronics offsets and environmental temperature fluctuations. Intuitively, bias is any nonzero sensor output when the input is zero.
2. Scale factor error, often resulting from aging or manufacturing tolerances.
3. Alignment errors: Most stand-alone IMU implementations include an initial transient period for alignment of the gimbals (for gimbaled systems) or attitude direction cosines (for strapdown systems) with respect to the navigation axes. Errors remaining at the end of this period are the alignment errors. These include tilts and azimuth reference errors. Tilt errors introduce acceleration errors through the miscalculation of gravitational acceleration, and these propagate primarily as Schuler oscillations plus a non-zero-mean position error approximately equal to the tilt error in radians times the radius from the earth center. Initial azimuth errors primarily rotate the system trajectory about the starting point, but there are secondary effects due to Coriolis accelerations and excitation of Schuler oscillations.
4. Cross coupling error (non-linearity).
5. Quantization error, which is inherent in all digitized systems.
6. Fault due to one or multiple of the moving parts wear out or jam, or gimbals lock.

IMU Faulty Model Diagram

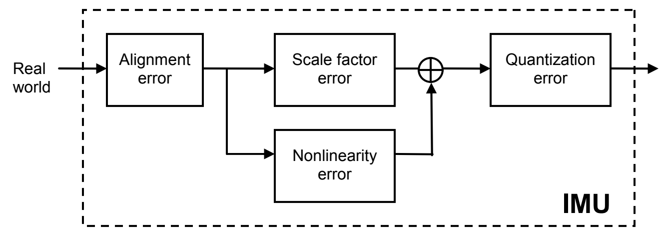

Figure 4: IMU Faulty Model

ODOMETER FAULTY MODEL

An odometer is one of the most common devices used for tracking and relative positioning of vehicles. In the transmission-based odometer, the distance to be determined is based on the number of counts for the wheel and calibration constants which are proportional to the radius of the tire. Thus any potential trends that change the radius and the number of counts can drive the odometer to a faulty output.

Tire radius change

The major sources of tire radius variation are listed in the following reference [30]:

1. Tire radius tends to increase as vehicle velocity increases because of increasing centrifugal force on the tire.
2. Tire radius tends to increase as air pressure within the tire increases due to increase tire temperature or other factors.
3. Tire radius tends to increase as tread is worn off during the lifetime of the tire.

Road Situation

These kinds of error sources depend on the road situation, including:

- Running over objects on the road, slips or skids involving one or more wheels when the vehicle accelerates or decelerates too rapidly or travels on a snowy, icy, or wet road.

- In sharp turns, the contact point between each wheel and the road can change, so that the actual distance between the left and right wheels will be different from the one used to derive the heading.

Gears Tooth Lost

An odometer can operate by counting the pass of teeth or tabs of the ferrous wheel mounted on the rotating shaft of the vehicle. If one or more teeth are lost, then the value will abate *n/t*, where *n* is the number of the lost teeth, *t* is the total teeth of the wheel. In the real situation, the occurrences of losing three or more teeth are so puny that they can be omitted.

In brief, the wear out of the tire, the pressure of the tire, the velocity of the vehicle, the slippage of the tire, and the gear teeth lost will contribute to the odometer fault.

INCLINOMETER FAULTY MODEL

The error sources of the inclinometer may consist of:

1. Error caused by thermal expansion or temperature changes. A normally distribution band-limited white noise is used to demonstrate the thermal noise.
2. Drift, calibration error or quantization error due to analog to digital converter resolution.
3. Electromagnetic interference (the major component of the inclinometer faulty model). It can be a uniform or a Gaussian distribution, or a combination of both. The variance and amplitude depend on the traveling environment.
4. Power lost or hardware malfunction: A permanent fault, but since it is only in a very low possibility, it is omitted in this simulation.

MAGNETIC COMPASS FAULTY MODEL (FLUXGATE COMPASS)

A magnetic compass is an inexpensive absolute direction sensor. The main drawback with this device is that the quantity measured, i.e. the intensity and direction of the magnetic field, can be distorted in the presence of metals and other electrical or magnetic fields, such as power lines, transformers and cars' powertrain system.

Compass operations include the following error sources:

1. Hilly road error [31]: When the vehicle is traveling over a hilly road, the compass plan will not be parallel to the plane of the Earth surface. The compass measures only the projection of the vector components. This is a short-term magnetic anomaly.
2. Random noise error [31]: a) In the situation of traveling nearby power lines, big trucks, steel structures (such as freeway underpasses and tunnels), reinforced concrete buildings, or bridges (short-term magnetic anomalies); b) In an environment of electrical or magnetic noise, or magnetization of the vehicle body (long-term magnetic anomalies).
3. Calibration error: Misalignment of the compass with respect to the vehicle frame simply results in a constant error. This type of error can also be attributed to an inaccurate estimation of the current declination.
4. Permanent fault: power lost or interface cable disconnected (very low possibility, they are being omitted in the simulation).

ADAPTIVE AND ROBUST DATA FUSION ARCHITECTURE

In positioning navigation systems, high precision and reliability with low cost are always pursued. Actually, for road navigation, the benefits of the information obtained by the fusion process make it possible to use multiple less powerful, lower cost sensors to achieve as good a performance as those much more expensive ones. Kalman filter and its derivatives, the most popular data fusion methods, have been used extensively in autonomous or assisted navigation system for several years. But almost all of these applications are based on the assumption that all sensed data are complete and reliable. If one or more sensors are faulty, then the fusion filters will tend to choke. In order to ensure a reliable positioning estimate in the case of faulty sensors, an adaptive approach is proposed as in Figure 5.

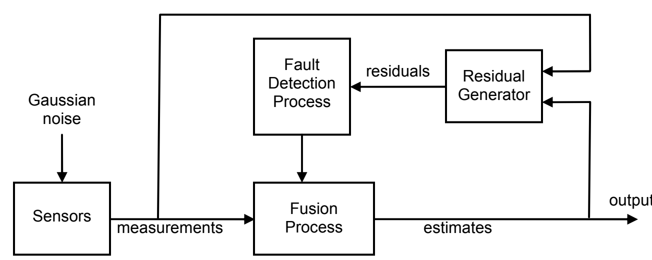

Figure 5: Adaptive Sensor Fusion System

As most of the navigation systems, the objective of the above system is to make the position estimate of the vehicle as accurate and reliable as possible. Sensors (GPS, IMU, odometer, inclinometer and compass) data which are used to compute the position and attitude of the vehicle, often involve sources of uncertainties. Meanwhile, a state space model can be constructed from the vehicle dynamic to perform the function of sensor fusion. Both their outputs (measurements and estimates) can be combined together through a particular function so as to generate a residual signal. Passing this signal through a detection process, a decision is made: either the system is running properly, or there is a fault occurring, which leads to the fusion process rerunning to optimize the position estimates.

From the diagram above, we can see how a residual signal generator and a fault detector are embedded into the conventional data fusion architecture. A Kalman filter approach is used, since it is simpler and more effective to attain the residual signal via state estimation. The key idea is to reconstruct the outputs of the process with the aid of Kalman filter and to use the estimation error, or some particular statistical functions of them to assess the residual signal.

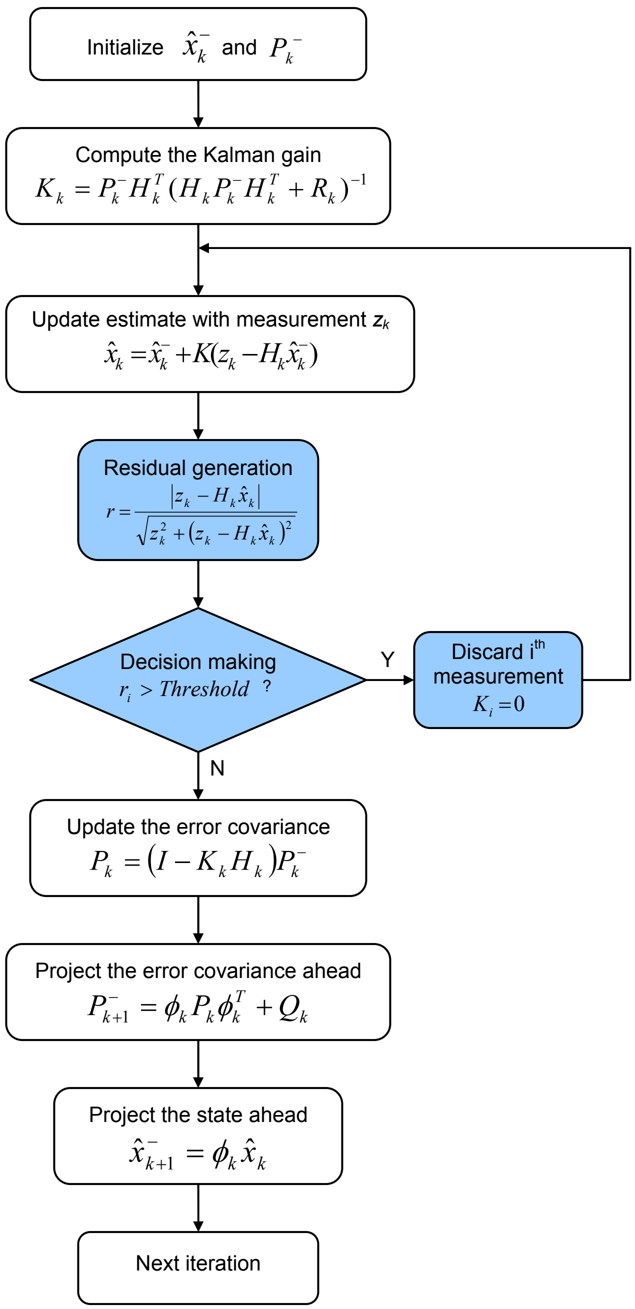

Figure 6: Adaptive Sensor Fusion Architecture Flow Chart

Then the faultiness can be detected by considering the residual properties against some threshold. If any information from the sensors is detected to be faulty (residual properties go over a given threshold), then the corresponding measurements are discarded. In that case, the position estimate needs to be re-updated. In this approach, fault detection and data fusion are combined into a single Kalman architecture to construct a fault tolerance, robust and adaptable vehicle positioning system.

The detailed flow chart is shown in Figure 6. Note that the shaded blocks are part of the fault detection process. Tests results given in [4] show that the proposed Kalman filter based state estimation scheme ensures that the position estimate is always optimal and brings significant benefit to the data fusion system comparing with the conventional fusion architecture without fault detection, in particular for the frequent GPS signal loss case. Performance analysis and more details on this architecture can be found in [4].

COLLABORATIVE DATA FUSION SYSTEMS

So far we have considered the case of a single vehicle only. In real life however, cars are not alone on the road We may therefore asks ourselves the following questions: can we take advantage of the other vehicles proximity and positioning data to improve our own position, and if yes, how? The intelligent vehicle systems (IVS) envisioned here would be able to communicate their position and navigation information through Inter-vehicle communications (IVC). IVS will be evolving in mobile Ad hoc networks called (MANET) that give access to valuable real time data, especially high precision positioning information. However, in terms of global navigation systems (GNS), individual vehicles in a given MANET would not have access to the same constellation of satellites. And certainly vehicles with good lines of sight (LOS) have more precise positioning estimates than vehicles with a poor LOS. Moreover, some vehicles may possess high precision DGPS or beacon-based positioning information to share in the network.

In this section, we will investigate a collaborative positioning architecture (CPA) that uses some of the above IVC features, along with additional range measurement capabilities, to ameliorate positioning estimates of neighboring vehicles in a MANET [5]. Two vehicles at different locations can have different sets of visible satellites, and by collaboration the satellite information can be shared between the vehicles [35]. Many research activities are being conducted in IVS collaborations. See [38] for a more basic, yet good introduction to IVS research and development. For a little more technical, but quite outdated now, [39] would be worth reading. Research in the domain of collaborative navigation takes many forms for instance Collaborative Driving Systems (CDS) are studied with applications in what is called car platoons [40] and [41]. However these kinds of collaborations necessitate either costly vehicle-to-vehicle relative dependence or vehicle-to-infrastructure dependence, whereas our approach leads to a more inexpensive independent navigation. We will treat here only the case of three vehicles, as shown in figure 7, which, although limited, is yet important towards a more general case.

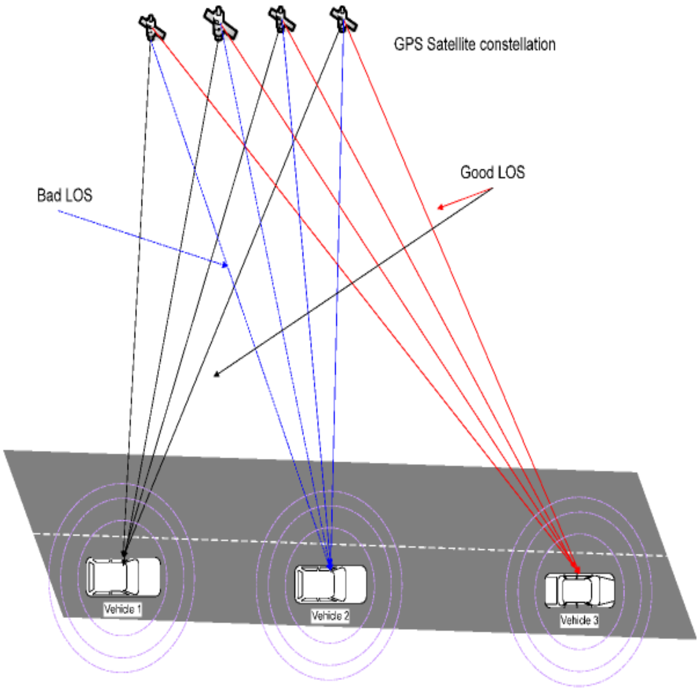

Figure 7: Collaborative positioning architecture

Designing a system solution for accurate estimation of relative positions of neighboring vehicles based on real-time exchange of individual GPS coordinates using vehicle-to-vehicle radio communications is a challenging task [36]. Prior to performing any detailed analysis of our CPA we ought to define the scope of our work with conditions under which our technique could be applied. We thus suppose the following conditions to be valid:

- All vehicles in the MANET are equipped with necessary navigation items (GPS, DR, IVC sensors…etc.)
- All vehicles have range measurement radars, to provide precise inter-vehicle distance. Millimeter wave radars MMW for automotive, studied in [3], would be appropriate solution to our application for they have a high Doppler sensitivity and a 200 meters range with a good precision which is very suitable for our case.
- No vehicle dynamics are considered in the CPA, we instead used a simple motion model.
- Error covariance matrix on position, heading, and inter-vehicle distance contains the global errors of vehicle systems.
- No special vehicle frame is considered
- Coordinate reference system is geodetic altitude, latitude and azimuth.
- No altitude difference is considered, this corresponds to the case where all three vehicles are located on a relatively flat plane with a constant altitude.
- Inter-vehicle communications are real time and safe.

Collaborative Uncertainty Minimization

Therefore, in this distributive, collaborative approach, each vehicles has its own set of position estimates as well as other information such as range information from other neighboring vehicles. To achieve a collective improvement of position estimates, we consider the geometric data fusion approach. This method is based on the geometric analysis of the sensing uncertainty and is motivated by the geometric idea that the volume of the uncertainty ellipsoid should be minimized, as illustrated in figure 8. The resultant fusing equation coincides with those obtained by Bayesian inference, by Kalman filter theory, and by weighted least-squares estimation [42]. The uncertainty ellipsoid encloses a region in space where the true value most likely exists. The center of the ellipsoid is the mean of the measurement and the ellipsoid boundary represents a distance of one standard deviation from the mean [43].

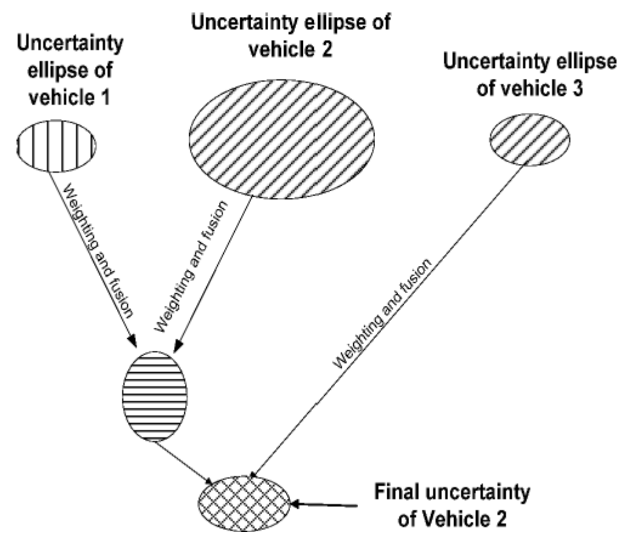

Figure 8: Principle of geometric data fusion approach in collaborative positioning of vehicles

Geometric data fusion has been used in many research applications; [43] and [44] are few examples; and had proven to be a powerful uncertainty management data fusion technique. In our paper [5], we show that a collaborative approach can further improve the position estimate over a conventional data fusion approach. More mathematical details and performance analysis can be found in the paper [5].

CONCLUDING REMARKS

In this paper, we have surveyed the positioning estimation problem applied to land navigation systems and reviewed some of the various sensor fusion techniques usually encountered in such systems. We have discussed their relative performance and limitations. The extended Kalman filter (EKF) in a centralized data fusion architecture remains a design of choice for most applications. We have explained how to make such systems more robust by detecting and identifying sensor faults. Finally we looked at the possibility to exploit the presence of several vehicles in the vicinity, in order to improve one's own position estimate using a collaborative and geometric data fusion approach. With the current evolution of the technologies, positioning sensors will become more and more easily available and at lower cost, thus allowing all vehicles to be equipped with such technologies. In addition, all vehicles are becoming networked and equipped with wireless communication capabilities, thus allowing the use of distributed and collaborative techniques for navigation and positioning. More embedded and distributed intelligence is likely to be encountered in future positioning and navigation systems.

ACKNOWLEDGMENTS

I would like to thank the Canadian NCE AUTO21 for funding this research project, the University of Calgary for providing the acceleration data, Geomatic Canada for the topographical information and E. A. Wan, R. v. d. Merwe and OGI School of Sciences for the Rebel Toolkit. I would like also to thank my former students Mathieu St-Pierre, Guopei Liu and Rabah Abdessamie for their original contributions in this area. Finally, I would like to express my thanks to the Universidad Autonomia de Zacatecas, Mexico and my colleagues at the PDS Lab at UAZ, headed by Dr Gerardo Miramontes, for their kind support and advices during the writing of this paper.

REFERENCES

1. Jay Farrell and Matthew Barth. The Global Positioning System and Inertial Navigation. McGraw-Hill, 1999

2. St-Pierre M., Gingras D., *Comparison between the unscented Kalman filter and the extended Kalman filter for the position estimation module of an integrated navigation information system*, IEEE Intelligent vehicles Conference, Parma, Italy, June 2005.

3. St-Pierre M., Gingras D., *Neural network based data fusion for vehicle positioning in land navigation system,* SAE-2004 conference, paper no. 345 Conference, Detroit, 2004.

4. Guopei L., Gingras D., *Adaptive and Reconfigurable Data Fusion Architectures in Vehicle Positioning Navigation Systems*, SAE-2007 conference, paper no. 1108 Conference, Detroit, 2004.

5. Abdessamie R., Gingras D., *A collaborative navigation approach in intelligent vehicles,* SAE conference 2008, paper no 1249 Detroit, 2008

6. M. S. Grewal, L. R. Weill, and A. P. Andrews, "*Global Positioning Systems, Inertial Navigation, and Integration*". John Wiley & Sons, New York, 2001.

7. E. Abbott and D. Powell, "*Land-Vehicle Navigation Using GPS*", Proceedings of the IEEE, 1999.

8. R. E. Kalman, "*A new approach to linear filtering and prediction problems*", Transactions of the ASME, Journal of Basic Engineering, 1960.

9. A. Gelb, "*Applied Optimal Estimation*". The MIT Press, Cambridge, Massachusetts, 1974.

10. B. Anderson and J. Moore, "*Optimal Filtering*". Prentice Hall, Englewood, NJ, 1979.

11. P. Y. H. Robert Grover Brown. Introduction to Random Signals and Applied Kalman Filtering. John Wiley and Sons, New York, 1997.

12. A. D. R. Merwe, N. Freitas, E. Wan, "*The Unscented Particle Filter, Tech report CUED/F-INFENG/TR-380*", Cambridge University Engineering Department, Cambridge, England August 2000.

13. E. A. Wan and R. v. d. Merwe, "*The Unscented Kalman Filter for Nonlinear Estimation*", Proceedings of Symposium 2000 on Adaptive Systems for Signal Processing, Communication and Control (AS-SPCC), Lake Louise, Alberta, Canada, 2000.

14. S. J. Julier and J. K. Uhlmann, "*A New Extension of the Kalman Filter to Nonlinear Systems*", In Proc. of AeroSense: The 11th Int. Symp. on Aerospace/Defence Sensing, Simulation and Controls, 1997.

15. S. Julier, "*Process Models for the Navigation of High-Speed Land Vehicles*", Wadham College, Oxford, 1997, pp. 184.

16. W. Li and H. Leung, "*Constrained Unscented Kalman Filter Based Fusion of GPS/INS/Digital Map for Vehicle Localization*", IEEE INTERNATIONAL CONFERENCE ON INTELLIGENT TRANSPORTATION SYSTEMS, Shangai, 2003.

17. X. Mao, M. Wada, H. Hashimoto, M. Saito, and I. Suzuki, "*EM/Unscented Smoothing Based Parameter Learning for Nonlinear Models for GPS Positioning*", 10th World Congress on Intelligent Transport Systems, Madrid, 2003.

18. T. F. Futoshi Kobayashi, Fumihito Arai. Sensor fusion system using recurrent fuzzy inference. Journal of Intelligent and Robotic Systems, 23, October 1998.

19. M. T. Hagan and M. Menhaj. Training feedforward networks with the Marquardt algorithm. IEEE Transactions on Neural Networks, 1994.

20. K. T. Keith A. Redmill, John I. Martin. Sensor and data fusion design and evaluation with a virtual environment simulator. Proceedings of the IEEE Intelligent Vehicles Symposium, 2000.

21. D. T. Knight. Rapid development of tightly-coupled gps/ins systems. IEEE Aerospace and Electronic Systems Magazine, 12(2), February 1997.

22. J. B. P.D. Beattie. Self-localisation in the senario autonomous wheelchair. Journal of Intelligent and Robotic Systems, 22:255_267, 1998.

23. O. R. H. Teong Chee Chuah, Bayan S. Sharif. Robust adaptive spread-spectrum receiver with neural-net preprocessing in non-gaussian noise. IEEE Transaction on Neural Networks, 12(3), May 2001.

24. Y. B. P. H. Yann LeCunn, Léon Bottou. Gradient-Based Learning Applied to Document Recognition, chapter 9. IEEE press, 2001.

25. Q. Wu, Z. Gao, and D. Wan, "*An adaptive information fusion method to vehicle*", IEEE Position location and navigation symposium, 2002.

26. K. A. Redmill, T. Kitajima, and U. Özgüner, "*DGPS/INS Integrated Positioning for Control of Automated Vehicle*", IEEE Intelligent Transportation Systems, Oakland, 2001.

27. Jon Kronander. Robust Automotive Positioning: Integration of GPS and Relative Motion Sensors. December 2004

28. Masafumi Hashimoto, Hiroyuki Kawashima, Takashi Nakagami, and Fuminori Oba. *Sensor Fault detection and Identification in Dead-Reckoning System of Mobile Robot: Interacting Multiple Model Approach*. Proceedings of the 2001 IEEE/RSJ International Conference on Intelligent Robot and Systems, Mani, Hawaii, USA, Oct. 29 – Nov. 03, 2001.

29. *S.M. Magrabi and P.W. Gibbens. Decentralized Fault Detection and Diagnosis in Navigation Systems for Unmanned Aerial Vehicles.* IEEE Position Location and Navigation Symposium, *San Diego, 2000.*

30. Thomas W.Lezniak, Richard W.Lewis, and Robert A.Mcmillen. *A Dead Reckoning/Map Correlation System for Automatic Vehicle Tracking*. IEEE Transactions on Vehicular Technology, Vol. VT-26, No.1 February 1977.

31. Yilin Zhao. (1997). Vehicle Location and Navigation Systems. Norwood, Artech House, 345 p

32. R. Patton, P. Frank and R. Clarck, Fault Diagnosis in Dynamic System: Theory and Applications, Prentice Hall, 1989

33. A. D. Pouliezos and G.S. Stavrakakis, Real Time Fault Monitoring of Industrial Processes, Kluwer Academic Publishers, 1994

34. R. Isermann, Fault-Diagnosis Systems: An Introduction From Fault Detection To Fault Tolerance, Springer Verlag, 2005

35. Berefelt F., Boberg B, Nygards J., Stromback P.,Wirkander S.-L., *Collaborative GPS/INS Navigation in Urban Environment*, Swedish Defence Research Agency (FOI)

36. Kukshya, V.; Krishnan, H.; Kellum, C. Design of a System Solution for Relative Positioning of Vehicles *Using Vehicle-to-Vehicle Radio Communications During GPS Outages*, IEEE

37. William D., FMCW MMW Radar for Automotive Longitudinal Control, California PATH Research Report UCB-ITS-PRR-97-19

38. Nirupama B., Wireless Sensor Networks, A Systems Perspective, 2005 Artech House, Boston

39. Jurgen, R. K., Navigation and Intelligent Transportation Systems, 1998, Progress In Technology, Automotive Electronics Series

40. Gaubert, N.; Beauregard, M.; Michaud, F.; de Lafontaine, J.; *Emulation of Collaborative Driving Systems Using Mobile Robots;* Systems, Man and Cybernetics, 2003. IEEE International Conference on Volume 1, 5-8 Oct. 2003 Page(s):856 - 861 vol.1

41. Crawford S.; Cannon M.E., Performance Evaluation of Sensor Combinations for Mobile Platoon Control; Positioning, Location, and Navigation (PLAN) Group, University of Calgary, Calgary Alberta, Canada

42. Abidi, M.A., Gonzalez, R.C., Data Fusion In Robotics And Machine Intelligence, Academic Press 1992, pp. 457-479.

43. Elliott, J. D., Langlois, D., and Croft, E. A., Sensor Uncertainty Management for an Encapsulated *Logical Device Architecture: Part I* - Fusion of Uncertain Sensor Data Industrial Automation Laboratory University of British Columbia Vancouver.

44. Lee, J., and Ha, I., Real-Time Motion Capture for a Human Body using Accelerometers, Robotica (2001) volume 19, pp. 601–610. Cambridge University Press.