QUALITATIVE STUDY ON WORTHINGTON JETS

Term Paper by

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INTRODUCTION:

We got the inspiration to do this study from watching a video on social media where a man jumps into a pool with a ball in hand from a considerable height and releases the ball when his body starts forming a cavity in the pool. The ball, when it leaves the hand, is shot up into the air at an unbelievable speed. We gathered some snaps from the video to be shown here and the link to the YouTube video (https://www.youtube.com/watch?v=JWUyEl9qnh8).

The reason for the above phenomenon is due to the formation of water jets known as Worthington or Rayleigh jets. We intend to reproduce the same on a smaller scale using a sphere and study the jets formed due to the interaction of the sphere with water.

In this study, the impact and the resulting Worthington jets of spheres with hydrophobic and hydrophilic surfaces are recorded with a high-speed camera.

Worthington Jets:

Worthington jets have the ability to attain noteworthy altitudes, surpassing the height at which a solid sphere was initially released. The properties of the jet fluctuate significantly based on various factors, such as the surface of the sphere, size of the sphere and the velocity of impact. The Worthington jet is readily observable and recordable, making it an excellent resource for obtaining crucial information about the conditions of an impact post-occurrence if a correlation between the jet characteristics and the impact can be established.

The formation of the jet will vary for hydrophilic and hydrophobic bodies due to their respective surface characteristics influencing the process. It will vary with the size of the body and the velocity of impact as well.

When a hydrophobic body falls into water at a certain velocity, it can create an air cavity or wake. As the object sinks to a particular depth, the air cavity pinches off, collapses, causing the surrounding water to rush in and forming a jet that shoots upwards. This upward jet is known as the Worthington Jet and it can propel objects into the air. The kinetic energy of the object is converted to potential energy of the cavity by the drag force on the object. This potential energy is then transferred to the jet as its kinetic energy. Thus, a momentum transfer takes place from the falling object to the jet and as a result the object is shot upwards. The cavity formation and the pinch off is visible in the two following images. We used a blue light to lit up the water for producing the following images.

In case of hydrophilic bodies, this cavity is formed only if the impact velocity is greater than a threshold value. In case of the impact velocity being lesser than the threshold which is so in most cases, no such cavity is formed. The body's hydrophilic nature enables water to move upward along its surface during impact and propel upwards due to the transfer of momentum, even when it's not fully submerged.

In our investigation, we will examine instances of hydrophobic bodies mainly. This is because the case we want to investigate is that of the human body falling into the water, and the human skin is hydrophobic.

Experimental Setup:

The apparatus used in the experiment are:

- Transparent tumbler
- Bucket
- Tennis ball
- Ping pong ball
- Spherical marbles of 2 sizes
- Canon R10 camera
- Fluorescent Light – Godox LC500R

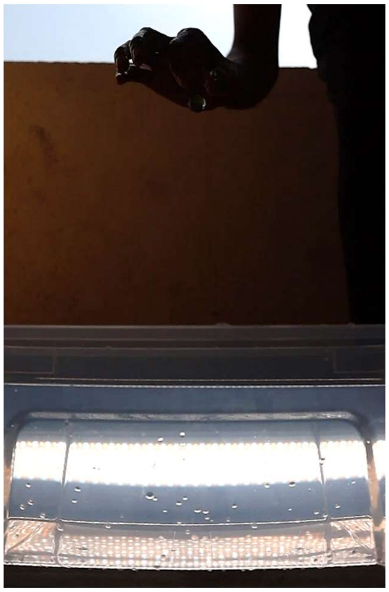
Marbles used are usually hydrophilic in nature. Hence, these are coated using oil to make them hydrophobic in nature and observe the cavity formation clearly.

Images were acquired using a Canon R10 camera with an RF-S 18-150mm Canon lens. The frame rate was set to 100 frames per second which was the maximum for our camera. We took the pictures such that the field of view of each image includes both above and below the free surface, hence both are captured simultaneously by the single camera. Fluorescent light behind the tank is used to accomplish proper lighting and better quality so that the exposure is not compromised when lowering the ISO of the camera.

Observations and Discussion:

Study of Worthington jets due to water entry of hydrophobic sphere:

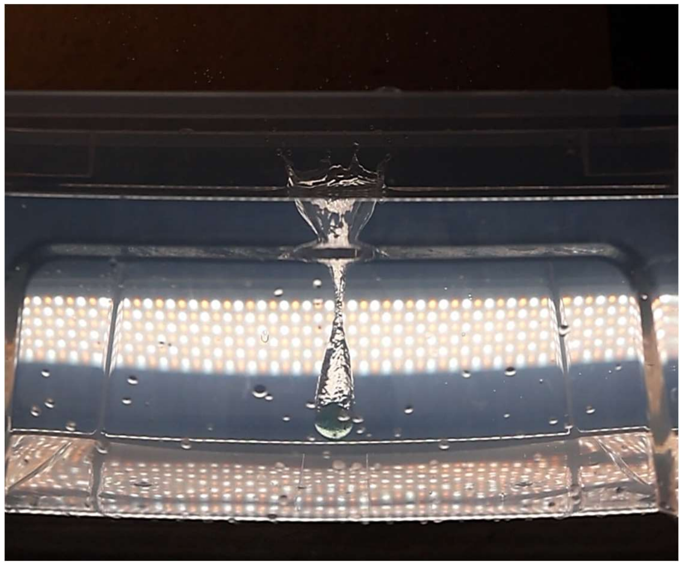
This demonstration is carried out by dropping a small oil-coated spherical marble from a small height into the tumbler filled with water. The drive link to the video of the study is given here:
(https://drive.google.com/drive/folders/1Ich46RyphDRoEP_GLyYuOIbXBX8mlU5F?usp=share_link)

(1)  (2) 

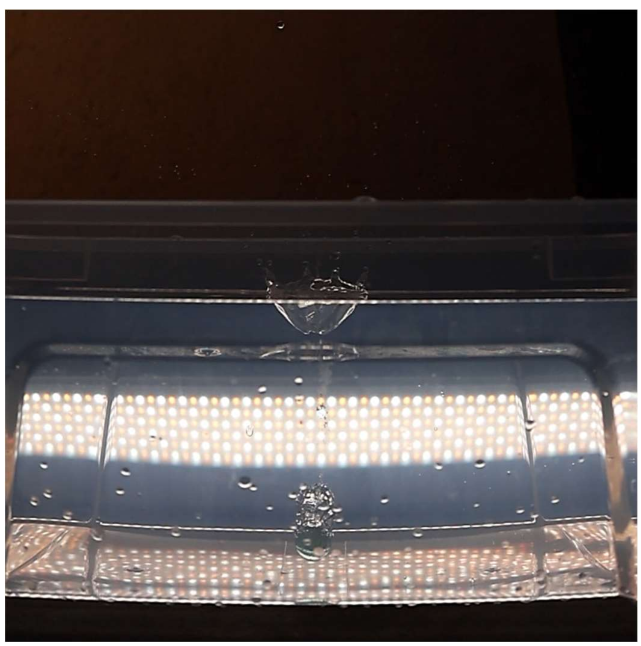
The marble is dropped from a small height as seen in the first image


(3)

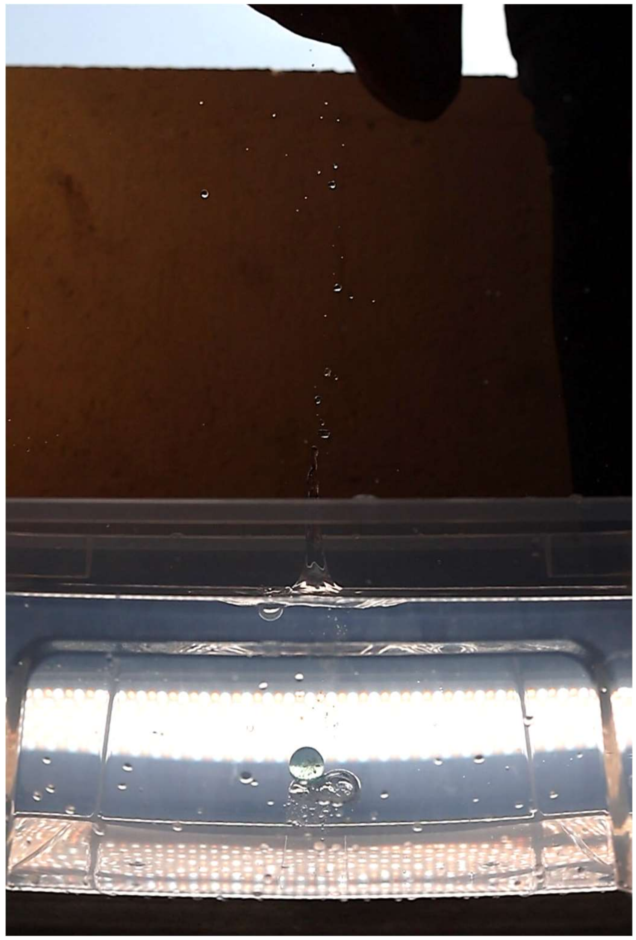
The surface entry of the hydrophobic sphere as seen in the second image is followed by the formation of an air cavity below the free surface as seen in the third image. The fluid flows outwards in a radial direction from the cavity, and there is no vertical velocity component at any point along the cavity wall.

(4) 

In the next stage, the flow turns to be radially inward and pinch off occurs at some depth below the free surface. When the walls of the cavity meet, fluid is forced both upward and downward as we can see in the fourth image. The pinch off results in the split up of the cavity into the Worthington jet and a bubble that stays attached to the sphere.

(5) 

The fifth image, the one shown above, shows the collapse of the cavity after the pinch off. This is followed by the formation of Worthington jet which is seen in the sixth image shown below.


(6)

When the jet reaches its maximum height, droplets are ejected from the tip of the jet such that the continuous portion of the jet maintains an average maximum height and the rest of the jet is formed by the droplets and isn't continuous.


(7)

Finally the Worthington jet breaks up into droplets totally and falls back into the water. The Worthington jets' breakup are significantly influenced by the Rayleigh-Plateau instability's development, which ultimately causes the breakup. Droplets start breaking off the tip of the jet from the moment it forms at the cavity's pinch-off, and this process continues throughout the jet's existence, resulting in the formation of more droplets from the tip. As a result, the continuous fluid column's maximum height is restricted.

The growth rate of the canonical Rayleigh-Plateau instability, ω, is related to the wavenumber of the disturbance, k, through the dispersion relationship

$$U^2 = \frac{\omega^2}{k^2} = \frac{\sigma}{\rho k R_0^2}\frac{I_1(kR_0)}{I_0(kR_0)}(1-(kR_0)^2)$$

where U is the local velocity of the jet, R_0 is the radius of the fluid column, I_1 and I_0 are Bessel functions of the first kind, σ is the surface tension and ρ is the density of fluid.

Due to experimental limitations, we avoid quantitative studies which involves the above mentioned formula and those which follow. The primary observation derived from Rayleigh-Plateau pertains to the correlation between the radius and velocity of a jet and the duration until its disintegration. Consequently, the behaviour of the Worthington jet is a consequence of the interplay between the velocity of the jet and the breakup mechanism influenced by Rayleigh-Plateau. While faster jets can ascend to greater heights in less time, they also tend to disintegrate more rapidly. In contrast, slower jets take longer to form droplets, but their travel distance within the same timeframe is also shorter.

The energy required to power the Worthington jet is derived from the potential energy stored in the cavity. The potential energy within the cavity is a result of the hydrostatic force exerted on the walls by the surrounding fluid. The energy can be related to the equivalent buoyant force, F_b of the cavity,

$$F_b = \rho V$$

where V is the volume of the cavity at pinch-off and ρ is the density of water

The potential energy of the cavity is then approximated as

$$E = F_b v$$

where v is the velocity of the fluid as it enters the Worthington jet. It is very difficult to determine this velocity v. However, the velocity of the tip of the jet (and the droplets it expels) is a readily measurable quantity. Assuming that the fluid particles in the jet tip move at the same velocity as the tip and the droplets, Bernoulli's equation for steady flow can be used to determine the fluid velocity at the jet entrance.

$$\tfrac{1}{2}\rho u_{drop}^2 + \rho g h_{avg} = \tfrac{1}{2}\rho v^2$$

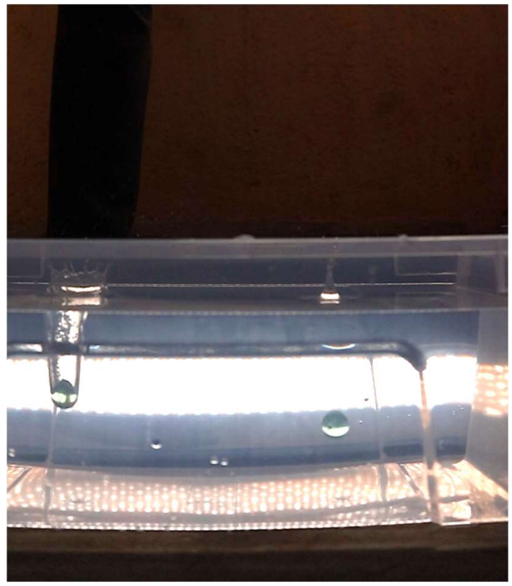
where u_{drop} is the velocity of the tip of the jet, and h_{avg} is the average height of the Worthington jet.

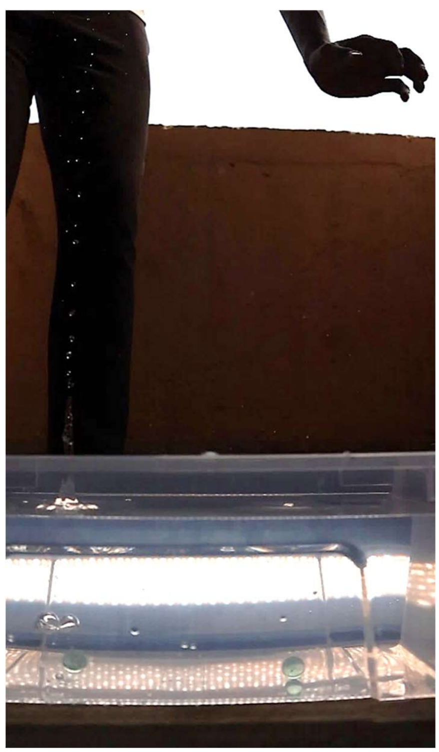
The average maximum height of the Worthington jet increases with increase in potential energy of the cavity.

Comparison of Hydrophilic vs Hydrophobic body water entry:

The difference between jets formed due to water entry of hydrophilic and hydrophobic spheres is observed by dropping an ordinary and an oil-coated marble from the same height simultaneously in the tumbler filled with water. The drive link to the video of the comparison is given here:

(https://drive.google.com/drive/folders/1btLGDRiHEez6mrerF79rxSnGXDomArIx?usp=share_link)





As explained earlier and as visible, there is no cavity formation in case of water entry of a hydrophilic sphere. As a result, the splash and height of the Worthington jet are much smaller in the case of the hydrophilic sphere.

Effect of Variation of Impact Velocity:

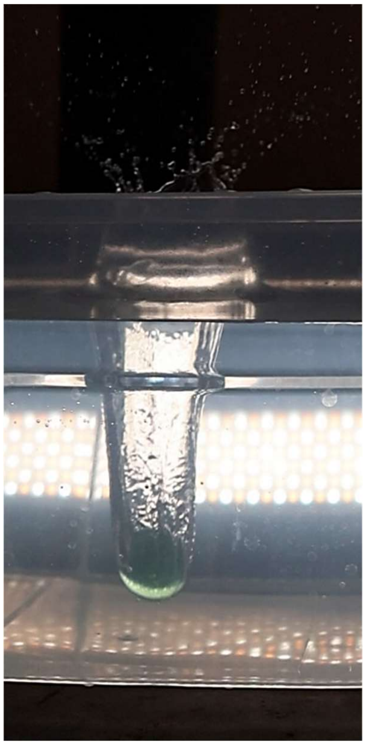
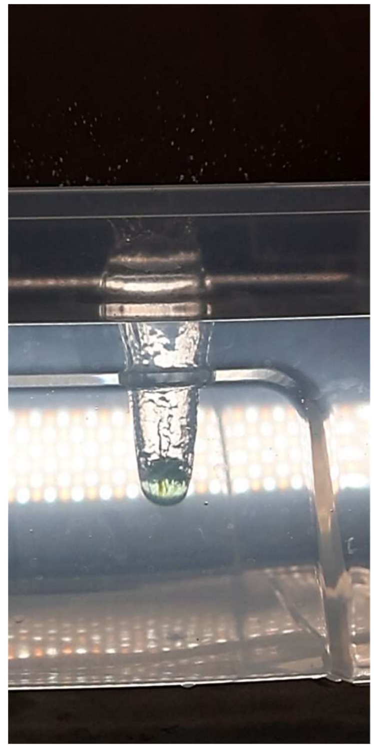
The oil-coated marbles are dropped into the tumbler filled with water from two different heights to attain different impact velocities in order to analyse the difference in properties of Worthington jet formation. The drive link to the video is given here:

(https://drive.google.com/drive/folders/1eD0_GTssdZ9Y6T7hDK-wksISl-wnCZcK?usp=share_link)

The maximum volume of the cavity formed is greater and the pinch-off point is deeper in case of higher impact velocity which is quite obvious and was visible in the experiment. But if the impact velocity is high enough it can lead to surface closure that can impede the jet formed.

<u>Effect of Variation of Size:</u>

Two oil-coated marbles of different diameters are dropped from the same height and the properties of jet formation in both cases are analysed.

It can be observed that the volume of cavity formed before pinch off for the marble with greater diameter is more than that in the case of marble with the smaller diameter, also causing a higher jet in its case. The images on the left are of the larger marble and those on the right are of the smaller marble. The drive link for the video is given here:

(https://drive.google.com/drive/folders/1HbQGlKsnszH3XJ7e2xkNR2NzYRd1Sgvw?usp=share_link)

Surface Closure:

A sphere impacting the free surface with a high enough value of impact velocity can cause closure of the surface splash. This surface closure occurs before the time of pinch off of the cavity and can impede the jet that is formed due to collapsing of the cavity as seen in the image below. The drive link to the same is given here:

(https://drive.google.com/drive/folders/1tx6lBb2yoLyRc0kej5i7aOzV_n1KxphV?usp=share_link)

Demonstration of the upward Shooting of a ping pong ball by the Worthington Jet formed by water entry of a tennis ball:

The ball pop due to Worthington jet is demonstrated using a tennis ball, ping pong ball, and a bucket of water. A tennis ball is used instead of the marbles as it is larger in size and can transfer enough momentum to the ping pong ball to shoot it up. The ping pong ball is placed on top of the tennis ball and held at a height from the surface of the water so that the ping pong ball can enter into the cavity formed due to the water entry of the tennis ball and can be shot up by the emerging Worthington jet hence recreating the phenomenon in the video that we mentioned in the introduction. The drive link to the video is given here:

(https://drive.google.com/drive/folders/1FsulPpRj1_zk7Ag37rOOdW7uZnoUd6gF?usp=share_link)

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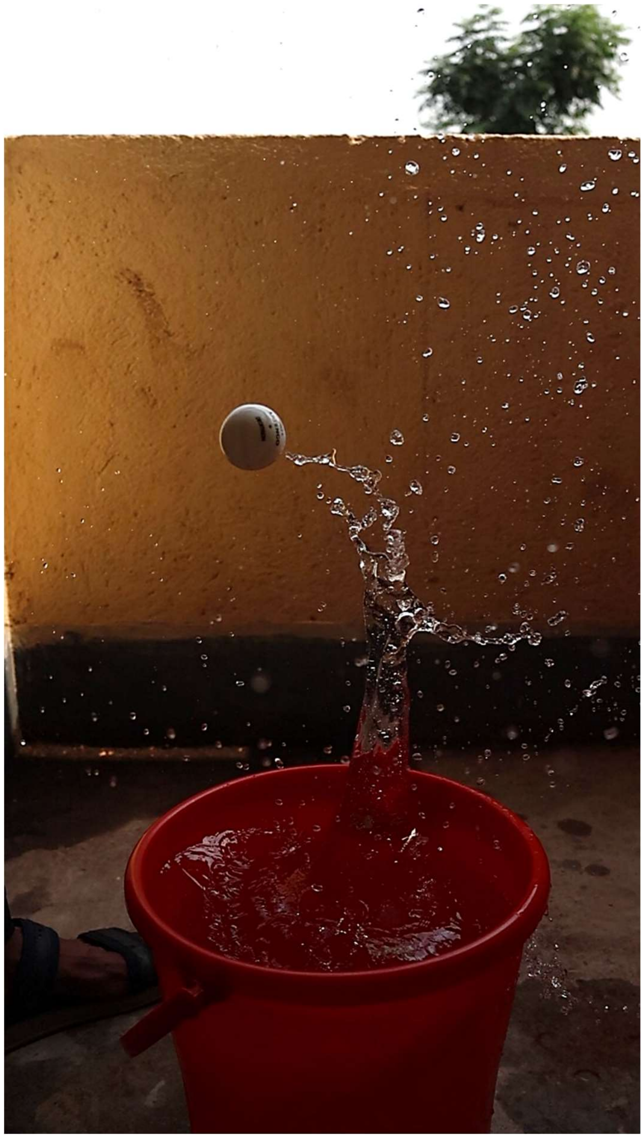

The surface entry of the tennis ball, as seen in the first frame above, is followed by the creation of a big air cavity. The ping pong ball which was on top the tennis proceeds to enter this cavity as seen in the second frame above. When the cavity collapses, a Worthington jet emerges which impacts the ping pong ball. The momentum of the decelerating tennis ball is transferred into the emerging jet which then transfers some of it to the ping pong ball resulting in its shoot up as seen in the final image.

This was the miniatured recreation of the phenomenon we mentioned in the introduction and we succeeded in it. Initially driven by curiosity as to why a ball shoots up into the air when a person jumps into a pool from a height while holding it, we eventually gained an understanding of the underlying mechanics and successfully replicated the phenomenon.

The drive link to all the videos that were taken during the conduction of this study is given here:

(https://drive.google.com/drive/folders/1C_9lKxp6Xoci4-UTjrQUhLYWg2C6b8br?usp=share_link)