BTP Project Report

3D Missile Guidance with Physical Constraints and Smooth Acceleration Profile

By

Aswin D Menon

(21AE10044)

Under the Guidance of

Prof. Sikha Hota

Department of Aerospace Engineering
Indian Institute of Technology, Kharagpur

Abstract

This report presents an investigation into missile guidance laws for achieving smooth acceleration profiles in a 3D interception scenario to achieve all impact angles on a stationary target. Various 2D guidance laws, including Biased Proportional Navigation Guidance (BPNG), Two-Stage Proportional Navigation (PNG), and Time-to-Go Polynomial Guidance (TPG), that met acceleration and FOV (Field of View) constraints, are analyzed and implemented. Simulation results highlights the strengths and limitations of each method, particularly regarding smoothness and terminal performance. Insights from these schemes are discussed, with proposed modifications to achieve the needed 3D interception.

DECLARATION

I certify that

(a) The work contained in this report has been done by me under the guidance of my supervisor.

(b) The work has not been submitted to any other Institute for any degree or diploma.

(c) I have conformed to the norms and guidelines given in the Ethical Code of Conduct of the Institute.

(d) Whenever I have used materials (data, theoretical analysis, figures, and text) from other sources, I have given due credit to them by citing them in the text of the thesis and giving their details in the references. Further, I have taken permission from the copyright owners of the sources, whenever necessary.

Date: November 13, 2024 (Aswin D Menon)
Place: Kharagpur (21AE10044)

DEPARTMENT OF AEROSPACE ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

KHARAGPUR - 721302, INDIA

CERTIFICATE

This is to certify that the project report entitled "**3D Missile Guidance with Physical Constraints and Smooth Acceleration Profile**" submitted by **Aswin D Menon** (Roll No. 21AE10044) to Indian Institute of Technology Kharagpur towards partial fulfillment of requirements for the award of degree of Bachelor of Technology in Aerospace Engineering is a record of bona fide work carried out by him under my supervision and guidance during Autumn Semester, 2024-25.

Prof. Sikha Hota
Department of Aerospace Engineering
Indian Institute of Technology Kharagpur
Kharagpur - 721302, India

Date: November 13, 2024
Place: Kharagpur

Contents

1 Introduction **2**
1.1 Pure Proportional Navigational Guidance Law 3
1.2 Biased Proportional Navigational Guidance Law 3

2 3D Guidance Problem **3**

3 2D missile guidance schemes **4**
3.1 Biased PNG with Teriminal Angle and Physical Constraints 4
3.1.1 Simulations and Inferences 6
3.2 Two-Stage Proportional Navigation with Heading Constraints 8
3.2.1 Simulation and Inferences 10
3.3 Polynomial Guidance Laws with Terminal Angle and Acceleration constraints 11
3.3.1 Simulations and Inferences 14

4 Conclusions and Future Work **16**

References **17**

1 Introduction

Missile guidance, primarily, is the usage of guidance commands to direct a missile toward a target with acceptable accuracy. Guidance commands are usually in the form of lateral acceleration that adjusts the missile trajectory. Passive guidance relies on the missile detecting energy naturally emitted by the target, like infrared radiation from a heat source, while active guidance involves an onboard radar that actively tracks and closes in on the target. The targets are of two types; maneuvering and non maneuvering. In this project, targets are non-maneuvering, but moving at a constant velocity, in some and stationary in others. The geometry of a 2D missile guidance of a ground to ground missile engagement, as in [1], in an X-Y plane is as shown in figure 1.

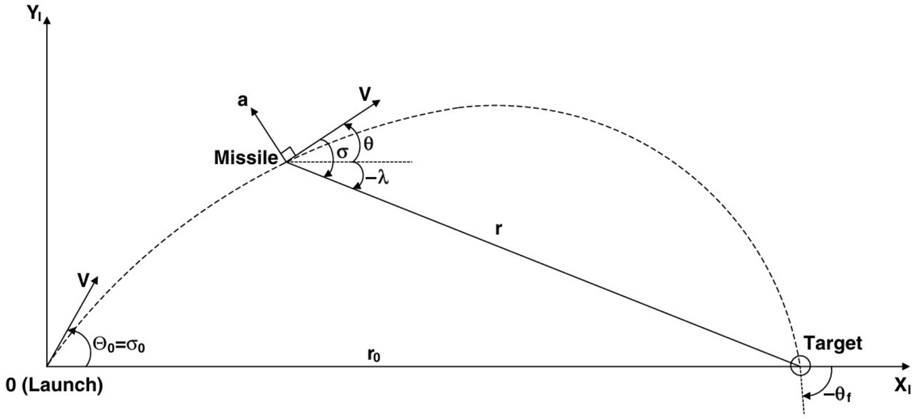


Figure 1: 2D ground to ground missile guidance geometry. (from [1])

Here, the subscripts M and T refer to the missile and target, respectively. The relative range between the missile and target is denoted by r, while λ represents the line-of-sight (LOS) angle. The flight path angle is given by θ or θ_M, and the terminal-impact angle by θ_f. The variable V denotes speed, and a represents the acceleration applied perpendicularly to the velocity vector. Furthermore, assuming a small angle of attack (AOA), the seeker's look angle, indicated by σ, is defined as the angle between the velocity vector and the LOS. Small AOA assumption is taken throughout this project. If at all the target moves, the target path angle is given by θ_T. The kinematic engagement equations between the missile and the target, expressed in a polar coordinate system, are given by:

$$\dot{r} = V_T \cos(\theta_T - \lambda) - V_M \cos(\theta_M - \lambda)$$

$$\dot{\lambda} = \frac{V_T \sin(\theta_T - \lambda) - V_M \sin(\theta_M - \lambda)}{r}$$

$$\sigma = \theta_M - \lambda$$

$$\dot{\theta}_M = \frac{a_M}{V_M}$$

The primary objective of a missile guidance law is that it should guide the missile to hit the target at a specified impact angle with little to no error. And ideally, a guidance law should be able to achieve all angles of impact angle.

The objective of this project is to achieve a 3D missile interception of a stationary target at a specified impact angle such that the guidance command profile is smooth and has no discontinuity. It is also required that the whole process is under a constrained look angle and guidance command.

1.1 Pure Proportional Navigational Guidance Law

Pure Proportional Navigation Guidance is a missile guidance law where the missile's lateral acceleration is proportional to the rate of change of the line-of-sight (LOS) angle. In other words, the missile adjusts its trajectory by applying a lateral acceleration that is a fixed multiple (the navigation constant) of the LOS rate. In Pure PNG, the missile's lateral acceleration a_m is given by the equation:

$$a_m = NV\dot{\lambda}$$

where N is the navigation constant (typically between 2 and 4 in most applications), V is the missile velocity, and $\dot{\lambda}$ is the LOS angle rate, representing the rate of change of the angle between the missile and target as observed from the missile.

1.2 Biased Proportional Navigational Guidance Law

Biased Proportional Navigation Guidance (Biased PNG) is an extension of Pure Proportional Navigation Guidance (PNG) that includes an additional bias term to improve interception accuracy, especially against maneuvering targets. In this guidance law, a constant or time-varying bias acceleration is added to the missile's acceleration command. The missile's lateral acceleration a_m is given by the equation:

$$a_m = NV\dot{\lambda} + b$$

where N is the navigation constant, V is the missile velocity and $\dot{\lambda}$ is the LOS angle rate, and b is the bias term.

2 3D Guidance Problem

The missile interception in the three-dimensional space is solved, as in [2], by initially converting the missile trajectory into 2 planes, as shown in figure 2.

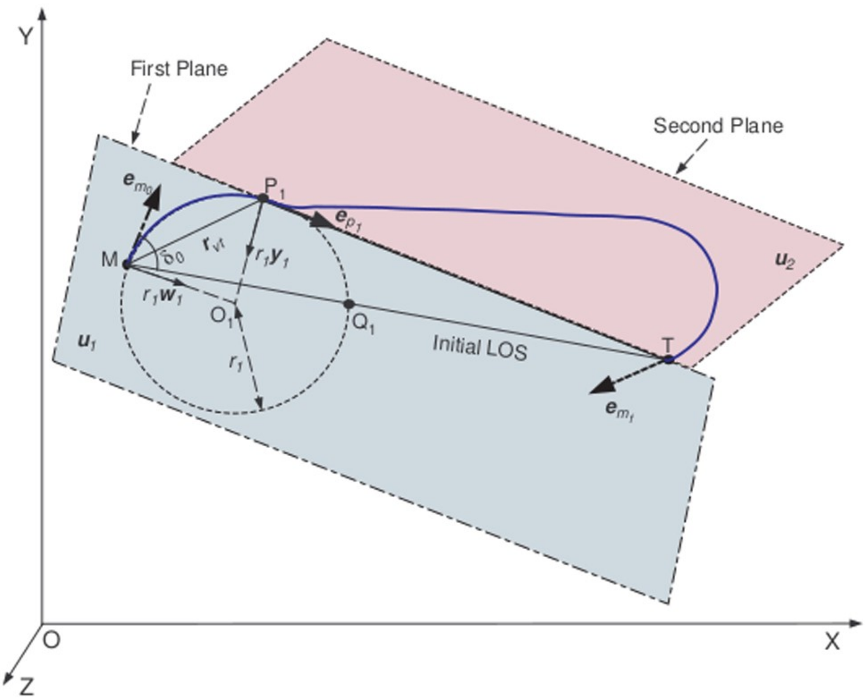


Figure 2: missile interception in two planar phases. (from [2])

The first plane contains the initial missile velocity vector, e_{m0}, and the initial LOS vector. The first stage, in the first plane, involves guiding the missile such that the missile velocity vector aligns with the

LOS vector which would imply that missile is in the collision course to the target. To achieve this, a pure PNG law of N=2 is used, which causes the missile to follow a circular path. And as the $N \geq 2$ here, the lateral acceleration will be bounded. To elaborate, the PNG law is used on a target that is assumed to be at some point, Q_1, on the initial LOS (MT), until the missile reaches the point P_1 where the missile velocity vector, e_{p1}, coincides with the LOS (P_1T). As a result the missile's lateral acceleration a_m in the first phase would be given by:

$$a_m = 2 * V\dot{\lambda}$$

Now, the second plane is made by missile velocity, e_{p1}, and the final missile heading vector, e_{mf}. Here, a suitable 2D guidance law is to be used that guides the missile to hit the stationary target at T with the required impact angle, also obeying the physical constraints. The goal would be to reduce the number of phases taken in this second plane because as the number of phases increases, the number of points of discontinuity in the guidance command profile increases, as each instance of phase change involves a sudden change in the guidance command.

The goal of this project is to find a proper 2D guidance law that can be used for the second plane in this problem. A thorough review of literature was necessary for designing an apt guidance law for the second plane.

3 2D missile guidance schemes

3.1 Biased PNG with Teriminal Angle and Physical Constraints

Initially, a biased PNG law by Park *et al.*[3] was studied. In this study, the target is non-maneuvering and has constant velocity. The proposed law consisted of two time-varying biases: One is to satisfy the impact angle constraint within the acceleration limit; the other is to maintain the constant look angle when reaching the FOV (Field Of View) limit. Figure 1 can be viewed for reference. For the biased PNG, the guidance command $\dot{\theta}_M$ can be given by:

$$\dot{\theta}_M = N\dot{\lambda} + b$$

where N is the navigation gain and when $N \geq 2(1 + V_T/V_M)$ the guidance command is bounded. Integration of the above equation results in:

$$\int_0^{t_f} b\,dt = \theta_{M_f} - \theta_{M_0} - N\lambda_f$$

where t_f and λ_f denote the total flight time and the final LOS angle, respectively. Hence, it is clear that the bias term b should vary accordingly for the specified terminal angle to be achieved. Further calculations result in:

$$B_{ref} = \int_0^{t_f} b\,dt = \begin{cases} \theta_{M_f} - \theta_{M_0} - N\tan^{-1}\left(\frac{V_M \sin\theta_{M_f}}{V_M\cos\theta_{M_f} - V_T}\right) & \text{for } \theta_T = 0 \\ \theta_{M_f} - \theta_{M_0} - N\tan^{-1}\left(\frac{V_M \sin\theta_{M_f}}{V_M\cos\theta_{M_f} + V_T}\right) & \text{for } \theta_T = 180° \end{cases}$$

B_{ref} is the integral value of bias to achieve the required terminal angle. To satisfy the angular constraint through the integral value of the bias, as shown in figure 3, the proposed BPNG law employs two types of time-varying biases based on a switching logic. The first bias, b_1, is designed to control the impact angle within the acceleration limits $|a_M(t)| \leq a_{\max}$; the second bias, b_2, aims to keep the look angle constant when $|\sigma(t)| \geq \sigma_{\max}$. For a comprehensive explanation of the proposed law, two cases can be considered: first, when the look angle is unrestricted during the homing phase; second, when the look angle exceeds the maximum allowable value at a certain point.

In the first case where $0 \leq \sigma(t) < \sigma_{\max}$ for $t = [0, t_f]$ or only a low impact angle constraint is required, only bias b_1 is applied during the homing phase. The bias b_1 is determined by multiplying the gain $\frac{1}{\tau}$ with the bias error integral $E = B_{\text{ref}} - B$, which is updated at each guidance cycle. As a result, we get:

$$b(t) = b_1 = \frac{B_{\text{ref}}}{\tau}e^{-t/\tau}$$

$$B(t) = \int_0^t b\,dt = \left(1 - e^{-t/\tau}\right)B_{\text{ref}}$$

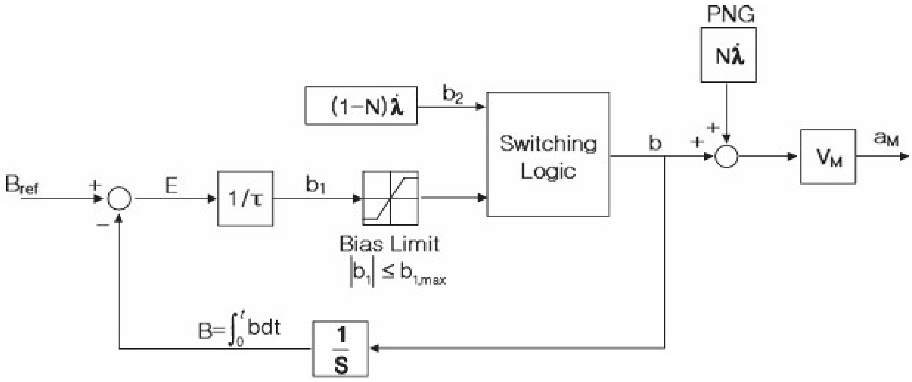


Figure 3: bias switching for impact angle control. (from [3])

where the bias b_1 goes to zero and the integral value B_{ref} goes to infinity. The design parameter τ should be chosen to be sufficiently small so that $B \to B_{\text{ref}}$ before reaching the target. In the initial stage when the E is large, the bias term dominates over the LOS rate and the acceleration may exceed the constraint. To prevent this, a bias limiter is used.

In the other case, if a higher impact angle than in the first case is required, then $|\sigma(t_c)| \geq \sigma_{\max}$ will occur at a certain time t_c. Here, b_1 is initially applied after launch, and then b_2 is switched from b_1 to maintain σ constant when $\sigma = \sigma_{\max}$. b_2 is given by:

$$b_2 = (1 - N)\dot{\lambda}$$

As a result, a command of $a = V_M\dot{\lambda}$ is generated. After that, the bias returns to the original b_1 for interception with the desired impact angle when $|b_1| \leq |b_2|$. The guidance command a_M can, hence, be written as:

$$a_M = \begin{cases} V_M\left(N\dot{\lambda} + b_1\right) & \text{for initial homing phase} \\ V_M\left(N\dot{\lambda} + b_2\right) = V_M\dot{\lambda} & \text{if } |\sigma(t)| \geq \sigma_{\max} \\ V_M\left(N\dot{\lambda} + b_1\right) & \text{if } |b_1| \leq |b_2| \text{ and until interception} \end{cases}$$

This study [3] also goes on to explain an iterative calculation procedure for finding the maximum possible impact angle that you can achieve for a given set of initial conditions and physical constraints. For impact angles beyond this, the missile wasn't able to hit the target with accuracy. This calculative process was implemented in MATLAB as well. The flow diagram of the same is shown in figure 4.

The algorithm starts by initializing essential parameters, including relative range, missile speed, target speed, bias limits, acceleration limits, look angle limits, bias gain, and navigation gain. It then selects a random final impact angle θ_{M_f} and calculates the initial bias b_1, which is limited by the maximum allowable bias $b_{1,\max}$. Next, it updates the line-of-sight angle and range, checking if the calculated bias b_2 falls within acceptable limits. If so, the algorithm recalculates the range and time-to-go, verifying that the final bias matches the reference B_{ref} and that the missile's acceleration a_M remains within the acceleration limit $a_{\max}$. If any of these conditions aren't met, it is required to reenter another θ_{M_f} and the process continues. This way, the algorithm uses an iterative process to determine the maximum achievable impact angle $\theta_{M_f,\max}$.

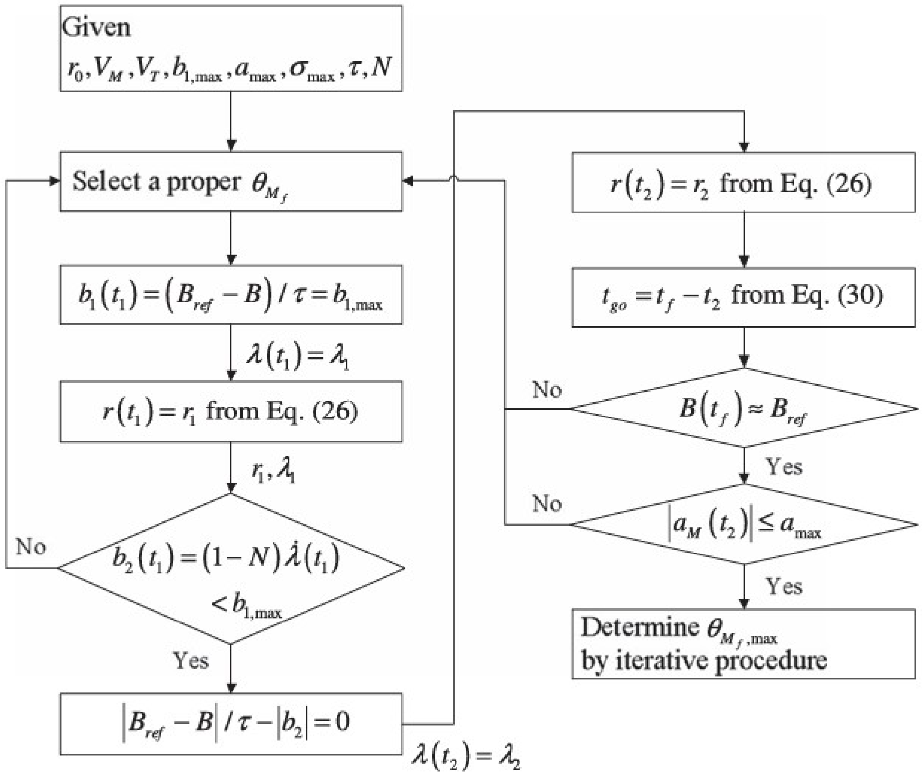


Figure 4: Maximum achievable impact angle calculation. (from [3])

3.1.1 Simulations and Inferences

The Biased PNG law was implemented on MATLAB for a constant speed missile model. The following conditions were used: the initial relative range is $r_0 = 3000\,\mathrm{m}$; missile speed $V_M = 200\,\mathrm{m/s}$; initial flight path angle $\theta_{M_0} = \sigma_0 = 30°$; navigation gain $N = 3$; acceleration limit $a_{\max} = 70\,\mathrm{m/s^2}$; look angle limit $\sigma_{\max} = 45°$; target speed $V_T = 30\,\mathrm{m/s}$ with $\theta_T = 0$. A bias gain of $\tau = 1.0\,\mathrm{s}$ and a bias limiter of $b_{1,\max} = 0.35\,\mathrm{rad/s}$ were used. The plots generated by the MATLAB code are presented in figure 5.

The maximum possible impact angle for these set of initial conditions were found to be $\theta_{M_f,\max} = -84°$ using the calculation process. The simulations show that the impact angles up to $-90°$ are achievable with almost no errors. But in the case of $-100°$ the acceleration is saturated after switching to the terminal phase producing an impact angle error of about a degree. It can also be noted that the acceleration profile is bounded, as was made sure by the bias limiter. The constraint for look angle too has been achieved as can be seen from the plot (e) in figure 5.

This method does consider physical constraints and can achieve an impact angle that is allowable for a given conditions. Even though this method works for the case when the initial look angle is zero, all impact angle cannot be achieved. Hence, other methods had to be studied.

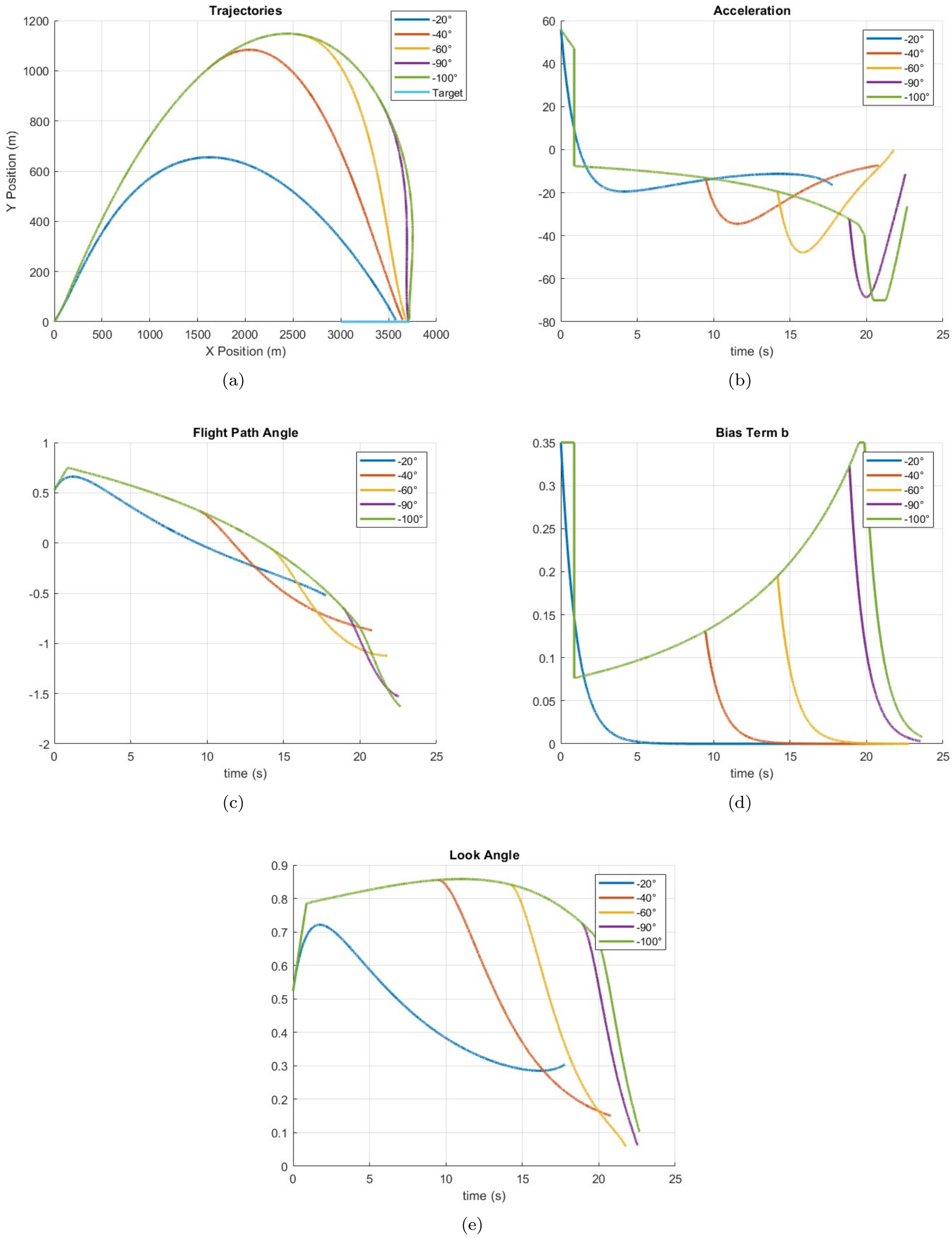


Figure 5: Simulation results of Biased PNG

3.2 Two-Stage Proportional Navigation with Heading Constraints

Literature of guidance laws which can achieve all impact angles were studied. Ratnoo, in [6], discusses a two-staged proportional navigation on a stationary target, which can achieve any impact angle from 0° to −180° on a ground to ground basis. Inferences were taken from [4] and [5] as well. For PNG law, rate of change of look angle can be given by:

$$\dot{\sigma} = \dot{\theta}_m - \dot{\lambda} = (N-1)\dot{\lambda} == -\frac{(N-1)v_m \sin\sigma}{R}$$

where θ is the flight path angle, λ the LOS angle, σ the Look Angle and R is the LOS. It can then be deduced that:

$$\dot{\sigma} = \begin{cases} > 0 & \forall -\pi < \sigma < 0,\, N > 1 \\ < 0 & \forall\, 0 < \sigma < \pi,\, N > 1 \\ = 0 & \text{if } \sigma = 0,\, \pi \\ = 0 & \text{if } N = 1 \end{cases}$$

which would mean that the look angle would be bounded for $N \geq 1$. And we know that the guidance command is bounded for $N \geq 2$ (because the target is at rest). If N_{min} is the minimum value of navigation gain, the maximum achievable impact angle, as also illustrated in figure 6, would be:

$$\theta_{mf} = \begin{cases} \lambda_0 - \frac{\sigma_0}{(N_{\min}-1)} & \text{if } N = N_{\min} \\ \lambda_0 & \text{if } N \to \infty \end{cases}$$

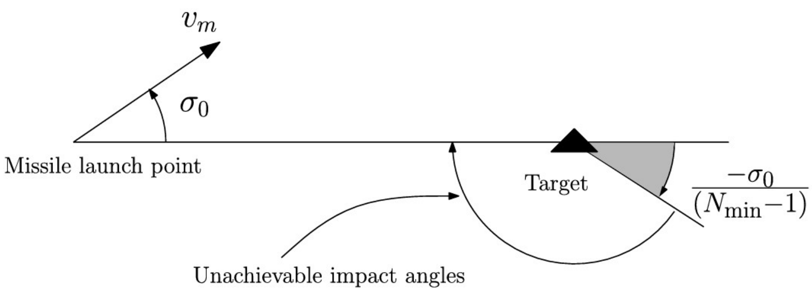


Figure 6: Impact Angle Zone. (from [6])

which means:

$$\theta_{mf} \in \left[\lambda_0 - \frac{\sigma_0}{(N_{\min}-1)},\, \lambda_0\right]$$

and here, the λ_0 is zero since it is ground to ground engagement. The first stage in this guidance scheme uses a PNG Law of gain $N = 1$. This guides the missile in a logarithmic spiral path. The resulting kinematics can be governed by:

$$\dot{R} = -v_m \cos\sigma_0$$

$$\dot{\lambda} = -\frac{v_m \sin\sigma_0}{R}$$

and the R can be deduced as:

$$R = R_0 e^{\cot(\sigma_0)\lambda}$$

The R varies with time as:

$$\dot{R} = -v_m \cos\sigma_0 = \begin{cases} < 0 & \sigma_0 \in \left(0, \frac{\pi}{2}\right) \\ = 0 & \sigma_0 = \frac{\pi}{2} \\ > 0 & \sigma_0 \in \left(\frac{\pi}{2}, \pi\right) \end{cases}$$

meaning that the LOS reduces as the initial flight path angle is acute. The variation of θ is given by:

$$\dot{\lambda} = -\frac{v_m \sin\sigma_0}{R_0 e^{\cot(\sigma_0)\lambda}} < 0 \quad \forall t \geq 0, \quad \sigma_0 \in (0, \pi)$$

meaning that the Los angle reduces monotonically with time.

The second phase begins when the λ reaches λ_s such that the desired impact angle can be achieved from there. Let N_s be the gain of the second phase. The relation is given by:

$$\theta_{mf} = \lambda_s - \frac{\sigma_0}{(N_s - 1)}$$

or

$$\lambda_s = \theta_{mf} + \frac{\sigma_0}{(N_s - 1)}$$

Since an impact angle of $-180°$ is to be achieved, the equation then becomes:

$$\lambda_s = -\pi + \frac{\sigma_0}{(N_s - 1)}$$

This phase involves a gain of $N_s \geq 2$ until the missile intercepts the target. The final phase ensures that the desired impact angle is achieved.

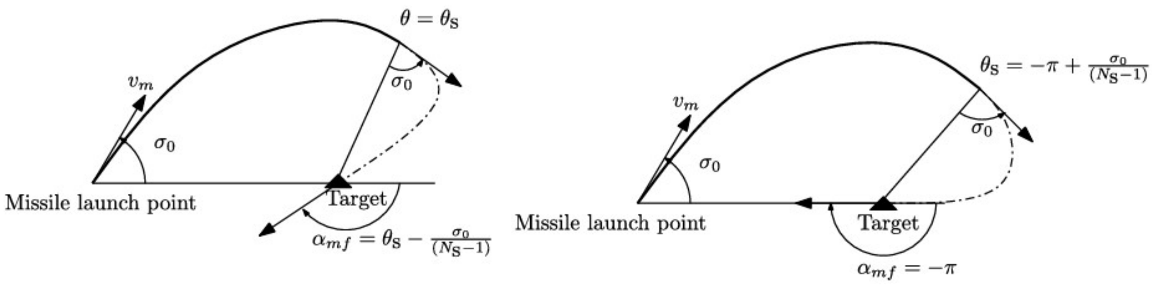


Figure 7: Switching cases. (from [6])

The figure 7 illustrates the switching. In the figure, θ is the LOS angle and guidance command is given as α and the notations in the figure are to be neglected. The θ_{mf} (the flight path angle) then can be written as:

$$\theta_{mf} = \begin{cases} -\frac{\sigma_0}{(N_s-1)} & \lambda_s = 0 \\ -\pi & \lambda_s = -\pi + \frac{\sigma_0}{(N_s-1)} \end{cases}$$

Finally, the maximum acceleration requirement is at the switching instance and it is derived to be (for $N_s \geq 2$):

$$\max\{|a_m|\} = \frac{N_s v_m^2 \sin\sigma_0}{R_0 e^{\cot(\sigma_0)\lambda_s}}$$

3.2.1 Simulation and Inferences

MATLAB implementation for the above guidance law was done and simulations were conducted for a constant speed missile for a range of impact angles having the initial conditions: $v_m = 300\,\mathrm{m/s}$, $(x_{m0}, z_{m0}) = (0, 0)$, $(x_{t0}, z_{t0}) = (10000\,\mathrm{m}, 0)$, $\theta_{mf} = 45°$ and $N_s = 2$, where x is the position coordinate on ground and z is the position coordinate fron the ground upwards. The plots generated are presented in the figure 8. Thm is for flight path angle.

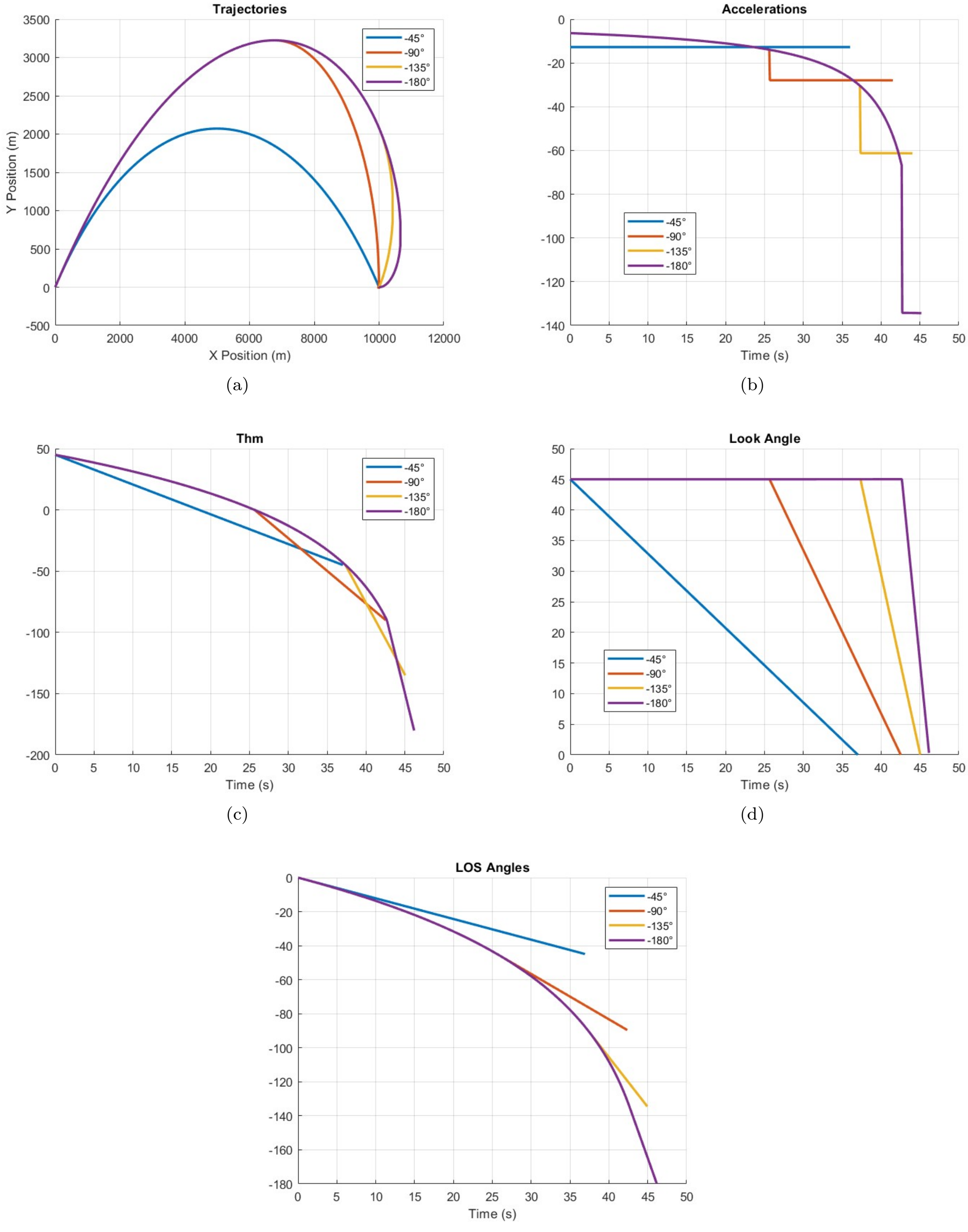


Figure 8: Simulation results of Two-Staged PNG

The two-staged PNG scheme was successful in achieving all impact angles in the negative spectrum. It made sure that the look angle was bounded and formulated the equation for the maximum required guidance command, which would be at the switching instance. Higher acceleration limit is required for a smaller FOV (Field of View) limit.

Even though all negative impact angles can be achieved, since there is a switch in the gain of PNG law, the acceleration command comes out to be discontinuous as can be seen as a jump in (b) of figure 8. Such discontinuities cannot be tolerated by realistic missiles. As a result, this guidance command cannot be used if a smooth profile for acceleration is required. Hence, other guidance schemes had to be explored.

3.3 Polynomial Guidance Laws with Terminal Angle and Acceleration constraints

With the goal of achieving a smooth acceleration profile, a polynomial guidance law was implemented. The time-to-go polynomial guidance law, as in [7] and [8], which assumes the guidance command as a polynomial function of the time-to-go. This scheme of missile guidance can handle the miss distance, the impact angle, and terminal acceleration efficiently. Inferences were taken from [9] as well. The time-to-go, t_{go} can be represented as:

$$t_{go} = t_f - t$$

where t_f is the total time taken to intercept the target and t is the time taken already at a particular stage in the trajectory. In other words, the time-to-go is the time remaining for the missile to make interception with the target. The target is stationary in this study. Figure 9 depicts the engagement geometry of the TPG law.

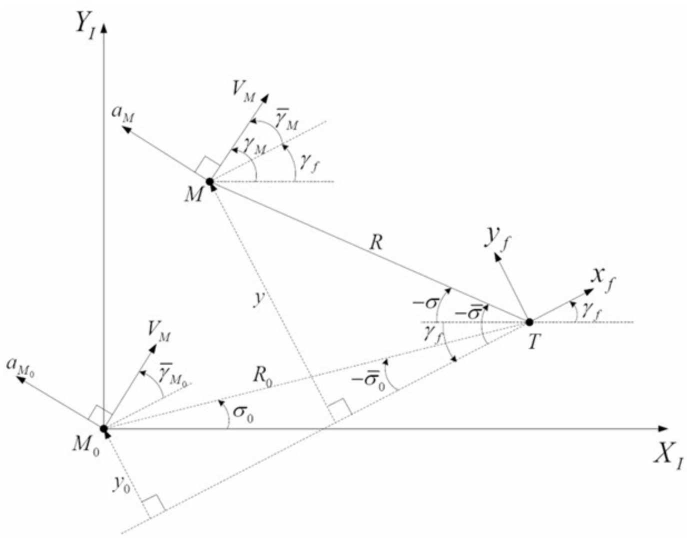


Figure 9: Engagement Geometry of TPG law. (from [7])

In figure 9, the γ represents flight path angle and the σ represents LOS angle. But they will be referred as θ and λ respectively as so were they defined from the beginning of the report. So, denotations in the figure can be neglected. The (X_1, Y_1) represents the inertial frame of reference and (x_f, y_f) the impact angle frame. The impact angle frame is obtained by rotating the inertial frame by the desired impact angle θ_f. Subscripts M and T denotes missile and trajectory and R and V_M denotes LOS (or relative distance) and missile velocity respectively. The variables in the impact angle frame can be represented as:

$$\bar{\theta}_M \triangleq \theta_M - \theta_f, \quad \bar{\lambda} \triangleq \lambda - \theta_f.$$

Additional kinematic equation would be:

$$\dot{y} = v = V_M \sin \bar{\theta}_M$$

where y and v denote the position and velocity perpendicular to the impact course, respectively, as shown in figure 9. The guidance command is initially assumed as:

$$a_M(t) = c_0 + c_1 t_{go} + c_2 t_{go}^2 + c_3 t_{go}^3 + \cdots$$

where c_0, c_1,... are coefficients.To achieve the desired impact angle and for improving the terminal performances, two constraints are considered:

$$a_M(t_f) = 0$$
$$\dot{a}_M(t_f) = 0$$

Here, f denotes 'final', which means that the guidance command and its time derivative is zero at interception. As a result, two coefficients are sufficient. The command, therefore, becomes:

$$a_M(t) = c_m t_{go}^m + c_n t_{go}^n \quad \text{where } n > m \geq 0.$$

If $n > m > 1$, then additional terminal constraints too are satisfied. The terminal lateral miss distance and velocity can be written as:

$$y(t_f) = y(0) + v(0)t_f + \frac{c_m}{m+2} t_f^{m+2} + \frac{c_n}{n+2} t_f^{n+2}$$

$$v(t_f) = v(0) + \frac{c_m}{m+1} t_f^{m+1} + \frac{c_n}{n+1} t_f^{n+1}$$

After extensive calculations, we get:

$$a_M(t) = -\frac{(m+2)(n+2)}{t_{go}^2} y(t) - \frac{(m+n+3)}{t_{go}} v(t)$$

Flight path angle and LOS angle can be written as:

$$\bar{\theta}_M = \frac{v}{V_M}, \quad \bar{\lambda} = -\frac{y}{R} = -\frac{y}{V_M t_{go}}.$$

The same can be rearranged as:

$$y = V_M t_{go}(\theta_f - \lambda)$$

$$v = V_M(\theta_M - \theta_f).$$

Substituting in the guidance command equation, we get:

$$a_M(t) = -\frac{V_M}{t_{go}} \left[-(m+2)(n+2)\lambda(t) + (m+n+3)\theta_M(t) + (m+1)(n+1)\theta_f \right].$$

where the λ and θ can be measured directly from the built in seeker and internal navigation system respectively. But the time-to-go t_{go} has to be estimated.

A TPG-mn guidance law means that guidance gains are m and n. For example, TPG-23 means that $m = 2$ and $n = 3$. After solving a second order differential order in y, we get:

$$y(t) = C_1 t_{go}^{n+2} + C_2 t_{go}^{m+2}$$

where,

$$C_1 = \frac{y(0)(m+2) + v(0)t_f}{(m-n)t_f^{n+2}}$$

$$C_2 = \frac{y(0)(n+2) + v(0)t_f}{(n-m)t_f^{m+2}}.$$

Now, substituting y and its derivative v in the guidance command, we get:

$$a_M(t) = C_1(n+1)(n+2)t_{go}^n + C_2(m+1)(m+2)t_{go}^m$$

The guidance command tends to zero at interception for $n > m > 0$. Large homing errors cause huge initial guidance commands and the analysis of guidance commands is done for worst scenarios only. There are two boundary points; one is the initial acceleration and the other is the acceleration at which $\dot{a}_M(t) = 0$. The worst case is when the highest magnitude of the acceleration is at the initial point.

The initial guidance command is proportional to the guidance gain m. A systematic method is discussed that can be used to find the guidance gains that accede to practical limits of look angle/FOV and guidance command. For this analysis, we consider $n = m + 1$ for the ease of calculation. And we obtain:

$$0 < m \leq \frac{-\left(5y_{0,\max} + 2V_M\bar{\theta}_{M_0,\max}t_f\right) + \sqrt{\left(y_{0,\max} + 2V_M\bar{\theta}_{M_0,\max}t_f\right)^2 + 4a_{M,\lim}y_{0,\max}t_f^2}}{2y_{0,\max}}$$

and

$$0 < m \leq \frac{1}{y_{0,\max}}\left((\sigma_{\lim} - \alpha_{M,\max})\frac{V_M t_f}{K_m} - 3y_{0,\max} - V_M\bar{\theta}_{M_0,\max}t_f\right)$$

where λ_{lim} is the missile's FOV limit and $\alpha_{M,max}$ is the maximum possible value of angle of attack (AOA) of the missile. m is selected from the intersection of these two boundaries so that both command limit and FOV limit are satisfied.

The most important value to be estimated is the Time-to-go t_{go}, which cannot be measured using sensors. The formula for t_{go} is given by:

$$t_{go} = \frac{S(t)}{V_M} = \frac{R}{V_M}\left\{1 + P_1\left[\left(\frac{1}{2}(\theta_M - \theta_f) - P_2(\lambda - \theta_f)\right)^2 + P_3(\theta_M - \theta_f)^2\right] - \frac{1}{2}(\lambda - \theta_f)^2\right\}$$

where $S(t)$ is the estimated distance to cover between t and t_f, and is given by:

$$S(t) = R\left\{1 + P_1\left[\left(\frac{1}{2}\bar{\theta}_M - P_2\bar{\lambda}\right)^2 + P_3\bar{\theta}_M^2\right] - \frac{1}{2}\bar{\lambda}^2\right\}$$

and:

$$P_1 = \frac{1}{(2m+3)(2n+3)(m+n+3)}$$

$$P_2 = (m+2)(n+2)$$

$$P_3 = \left(m + \frac{3}{2}\right)\left(n + \frac{3}{2}\right)$$

This time-to-go estimate depends on the guidance gain, the LOS angle, the flight path angle, and the desired impact angle. Errors in seeker and INS measurements introduce a time-to-go estimation error that significantly impacts terminal performance. Either way, with TPG where $n > m > 1$, the missile reaches the target along the impact course without executing a maneuver at the terminal time. This characteristic helps improve terminal performance in the presence of time-to-go estimation errors.

As the guidance gain increases, additional higher-order zero time-derivatives of terminal accelerations can be accommodated, further enhancing performance at terminal conditions.

3.3.1 Simulations and Inferences

For initial conditions: Missile position $X_M(0), Y_M(0) = (0, 1000)\,\mathrm{m}$; Target position $X_T(0), Y_T(0) = (4000, 0)\,\mathrm{m}$; Missile velocity $V_M = 200\,\mathrm{m/s}$; Initial flight path angle $= 0°$, MATLAB simulations were done on a model made on the basis of the TPG-23 scheme, where $m = 2$ and $n = 3$. Plots are shown in figure 10 with appropriate titles.

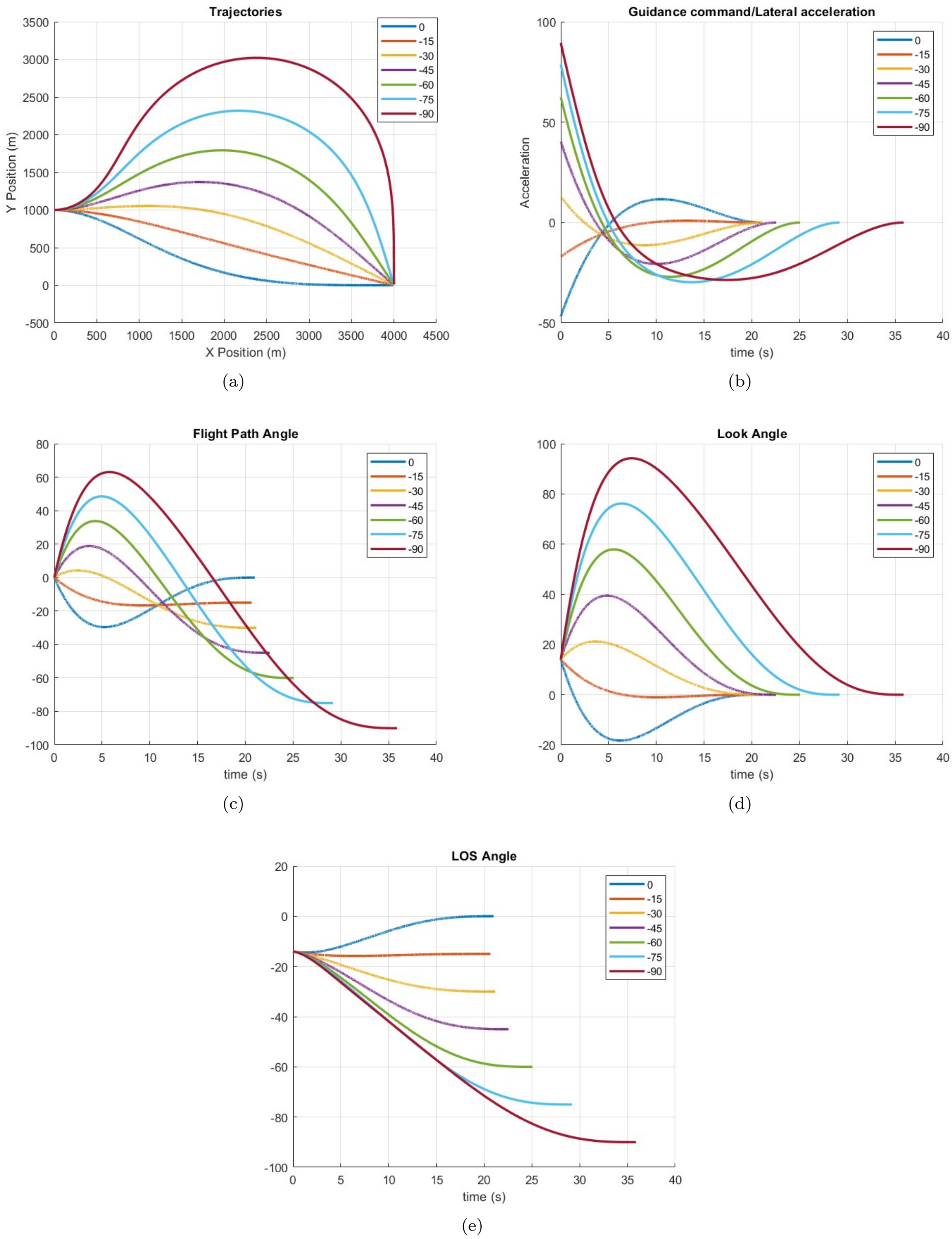
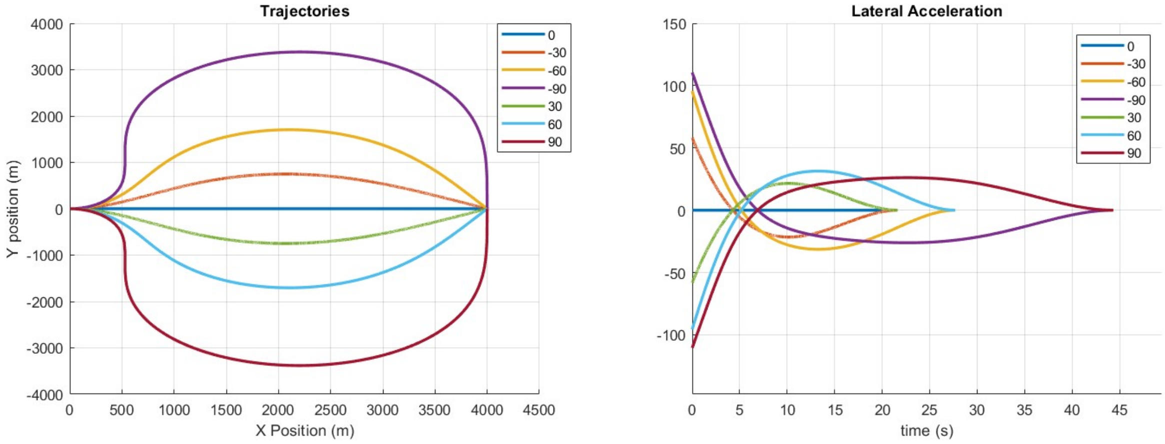


Figure 10: Simulation results of TPG-23 - 1st case

Another simulations for initial conditions: Missile position $X_M(0), Y_M(0) = (0,0)\,\mathrm{m}$; Target position $X_T(0), Y_T(0) = (4000,0)\,\mathrm{m}$; Missile velocity $V_M = 200\,\mathrm{m/s}$; Initial flight path angle $= 0°$, were conducted and the resultant plots are shown in figure 11. This simulation involved target interception in both positive and negative spectra.



(a) (b)

Figure 11: Simulation results of TPG-23 - 2nd case

The TPG resulted provided guidance commands that were continuous and had no points of discontinuity. The constraints of acceleration and FOV were realized. But the spectrum of impact angles that can be achieved via this TPG scheme is limited. For the simulation done, impact angle of only till $-120°$ was achieved. Even though both positive and negative spectra were covered, the whole spectrum couldn't be achieved. Hence it would not be enough for using as a 2D guidance law for the second plane in the 3D guidance problem.

4 Conclusions and Future Work

A variety of 2D guidance laws were implemented, of three different kinds. The characteristics required for the second plane 2D guidance was achieved by different schemes. But no schemes were able to achieve all the requirements. As a result, these schemes cannot be used as such. But, insights can be taken from these implemented guidance laws.

The major issue was with achieving a smooth acceleration profile, which can be mitigated by using a polynomial guidance law. But such guidance law lack in the ability to achieve any desired impact angle, or has a constraint on impact angle. Polynomial guidance schemes can also have a problem of having unsustainably high initial guidance commands if the guidance gains are high, which is not favorable.

One approach would be to use multi-staged guidance laws, like biased PNG or two-stage PNG, and then try to eliminate the discontinuities in the acceleration profile. So can be done by making sure that the acceleration values are not allowed to change instantly when there is transition between two stages. The acceleration value should be gradually modified to the required value, thus making sure that the guidance command profile is smooth. This procedure is being studied on and no literature on such procedures were found.

Another approach would be try different combinations of polynomial guidance laws. The work is being done currently, on developing such a scheme. Since, the problem with polynomial guidance laws were with the inability to cover the whole spectrum, it should be possible in a 3D guidance perspective to initially align the missile, in the first phase, in such a way that interception with the required impact angle is possible from that point, in second phase. To attain this, the initial phase can be another polynomial guidance law that can guide the missile from the initial point (plane 1) to another point from where the final impact angle is achievable (phase 2). This approach has to implemented and simulated in the future for testing feasibility.

References

[1] Tae-Hun Kim, Bong-Gyun Park, and Min-Jea Tahk. Bias-shaping method for biased proportional navigation with terminal-angle constraint. *Journal of Guidance, Control, and Dynamics*, 36(6):1810–1816, 2013.

[2] Amit Kumar, Nikhil K Singh, and Sikha Hota. Impact angle constrained time-optimal guidance law for stationary target interception in 3d. In *AIAA SCITECH 2022 Forum*, page 1391, 2022.

[3] Bong-Gyun Park, Tae-Hun Kim, and Min-Jea Tahk. Biased png with terminal-angle constraint for intercepting nonmaneuvering targets under physical constraints. *IEEE Transactions on Aerospace and Electronic systems*, 53(3):1562–1572, 2017.

[4] Ashwini Ratnoo and Debasish Ghose. Impact angle constrained interception of stationary targets. *Journal of Guidance, Control, and Dynamics*, 31(6):1817–1822, 2008.

[5] Ashwini Ratnoo and Debasish Ghose. Impact angle constrained guidance against nonstationary nonmaneuvering targets. *Journal of Guidance, Control, and Dynamics*, 33(1):269–275, 2010.

[6] Ashwini Ratnoo. Analysis of two-stage proportional navigation with heading constraints. *Journal of Guidance, Control, and Dynamics*, 39(1):156–164, 2016.

[7] Chang-Hun Lee, Tae-Hun Kim, Min-Jea Tahk, and Ick-Ho Whang. Polynomial guidance laws considering terminal impact angle and acceleration constraints. *IEEE Transactions on Aerospace and Electronic Systems*, 49(1):74–92, 2013.

[8] Tae-Hun Kim, Chang-Hun Lee, and Min-Jea Tahk. Time-to-go polynomial guidance with trajectory modulation for observability enhancement. *IEEE Transactions on Aerospace and Electronic Systems*, 49(1):55–73, 2013.

[9] Chang-Kyung Ryoo, Hangju Cho, and Min-Jea Tahk. Time-to-go weighted optimal guidance with impact angle constraints. *IEEE Transactions on control systems technology*, 14(3):483–492, 2006.