

$$\tilde{\mathbf{x}}_{1:n}^k = \tilde{\mathbf{x}}_{1:n-1}^{a_{n-1}^k} \times \tilde{\mathbf{x}}_n^k$$