$$\tilde{\mathbf{X}}_{1:n-1}^{a_{n-1}^k} \sim \hat{\pi}_{n-1}\big(\tilde{\mathbf{X}}_{1:n-1}\big)$$