PHYS 7326: Running Homework and Take-Home Exams

Professor Halverson

February 17, 2018

We are trying something different this semester for the students that wanted more regular homework assignments. Work will still be due biweekly, but problems will be posted on a rolling basis as lectures are developed and given. It is also more natural for me, as assignment ideas come on a rolling basis.

Instructions: Due dates will be listed throughout the document in bold letters. For the first assignment, complete all problems by the first due date. For the n^{th} assignment, complete all problems between the $(n-1)^{\text{th}}$ and n^{th} due date. Two due dates will have the words "TAKE-HOME" on the same line. The first is the take-home midterm, and the second the take-home final. These are your work alone.

I will also organize according to topic.

1 Non-abelian Gauge Theory

- 2 pts. Prove Schwartz equation (25.11).
- 3 pts. Non-abelian gauge invariance. Schwartz problem 25.1.
- 5 pts. Anomaly coefficients. Schwartz problem 25.4.
- 3 points. Peskin 15.1.
- 3 points. Peskin 15.2.

HOMEWORK 1. Due February 1.

- 5 points. Prove Peskin equations 16.48-49 and the second BRST variation of the gauge field discussed in between them.
- 5 pts EXTRA CREDIT. Non-abelian gauge invariance. Consider the Lagrangian

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + A^{abcd}A^a_\mu A^{\mu b}A^c_\nu A^{\nu d} + B^{abc}\partial_\mu A^{\mu a}A^b_\nu A^{\nu c} + C^{abc}\partial_\mu A^a_\nu A^{\mu b}A^{\nu c} \tag{1}$$

  and the gauge transformation

$$A^a_\mu \mapsto A^a_\mu + \frac{1}{g_a}\partial_\mu \alpha^a + T^{ab}A^b_\mu. \tag{2}$$

  By $F^a_{\mu\nu}$, where we mean the usual thing $F^a_{\mu\nu} = (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)$, rather than the non-abelian field strength. The latter comes naturally out of this calculation and will involve the interaction terms.

  - Compute the transformation of each term appearing in the Lagrangian.
  - From the transformed Lagrangian, compute conditions that must be satisfied for the Lagrangian to be gauge invariant. Hint: collecting terms with the same number of derivatives and vector fields is useful.
  - Determine whether or not the solution presented in class

$$g_a = g_b =: g \ \forall a,b \qquad T^{ac} = -f^{abc}\alpha^b A^c_\mu \qquad C^{abc} = -gf^{abc} \qquad A^{abcd} = g^2 f^{kac}f^{kbd} \qquad B^{abc} = 0 \tag{3}$$

    is the only solution.

- 3 points. Zee VI.I.I.
- 3 points. Derive equation VI.10 in Zee, justifying each step. In particular, we did not derive equation VI.8 directly in class, so please do it along the way to VI.10.
- 5 points. Spinors in arbitrary even dimensions from tensor product.

Using the definitions from class, prove that the tensor product is bilinear, i.e.

$$(A_1 v_1 + A_2 v_2) \otimes (B_1 w_1 + B_2 w_2) = A_1 B_1 v_1 \otimes w_1 + A_1 B_2 v_1 \otimes w_2 + A_2 B_1 v_2 \otimes w_1 + A_2 B_2 v_2 \otimes w_2 \quad (4)$$

for scalars A_i, B_i and vectors $v_i \in V$, $w_i \in W$, for $i = 1, 2$. Recall also the identities that we had for how commutators and anticommutators of linear maps get tensored with other linear maps.

Using the Dirac algebra for γ-matrices in $d = 2k$ dimensions,

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}\, 1_{2^k \times 2^k} \quad (5)$$

show that

$$\begin{aligned} \Gamma^\mu &= \gamma^\mu \otimes \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \mu = 0, \dots, d-1 \\ \Gamma^d &= 1_{2^k \times 2^k} \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ \Gamma^{d+1} &= 1_{2^k \times 2^k} \otimes \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \end{aligned} \quad (6)$$

are good Γ-matrices in $d+2$ dimensions, i.e. show that they satisfy the Dirac algebra in $d+2$-dimensions

$$\{\Gamma^M, \Gamma^N\} = 2\eta^{MN}\, 1_{2^{k+1} \times 2^{k+1}}, \quad (7)$$

where the 1's with subscripts are identity matrices of the stated size and $M = 0, \dots d+1$ can be decomposed into μ, d, and $d+1$.

HOMEWORK 2. Due March 1.

- 5 points. Running couplings.

Using the notes and the values of $\alpha_i^{-1}(M_Z)$ therein, (which determines the gauge couplings when measured at the Z mass), compute the beta functions for each in the SM and the MSSM. REMEMBER: this is just group theory, since the beta function has been computed as a function of representations. Critical to make the point I'm trying to make is to normalize the hypercharge with an appropriate 3/5 factor, which can be found in many sources, which comes from grand unification considerations. Plot α_i^{-1} for $i = 1, 2, 3$ as a function of renormalization scale in both the SM and MSSM cases. Comment on the differences between the plots.