PHYS 7326: Running Homework and Take-Home Exams

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We are trying something different this semester for the students that wanted more regular homework assignments. Work will still be due biweekly, but problems will be posted on a rolling basis as lectures are developed and given. It is also more natural for me, as assignment ideas come on a rolling basis.

Instructions: Due dates will be listed throughout the document in bold letters. For the first assignment, complete all problems by the first due date. For the n^{th} assignment, complete all problems between the $(n-1)^{\text{th}}$ and n^{th} due date. Two due dates will have the words "TAKE-HOME" on the same line. The first is the take-home midterm, and the second the take-home final. These are your work alone.

I will also organize according to topic.

1 Non-abelian Gauge Theory

- 2 pts. Prove Schwartz equation (25.11).
- 3 pts. Non-abelian gauge invariance. Schwartz problem 25.1.
- 5 pts. Anomaly coefficients. Schwartz problem 25.4.
- 3 points. Peskin 15.1.
- 3 points. Peskin 15.2.

HOMEWORK 1. Due February 1.

- 5 points. Prove Peskin equations 16.48-49 and the second BRST variation of the gauge field discussed in between them.
- 5 pts EXTRA CREDIT. Non-abelian gauge invariance. Consider the Lagrangian

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + A^{abcd}A^a_\mu A^{\mu b}A^c_\nu A^{\nu d} + B^{abc}\partial_\mu A^{\mu a}A^b_\nu A^{\nu c} + C^{abc}\partial_\mu A^a_\nu A^{\mu b}A^{\nu c} \tag{1}$$

  and the gauge transformation

$$A^a_\mu \mapsto A^a_\mu + \frac{1}{g_a}\partial_\mu \alpha^a + T^{ab}A^b_\mu. \tag{2}$$

  By $F^a_{\mu\nu}$, where we mean the usual thing $F^a_{\mu\nu} = (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)$, rather than the non-abelian field strength. The latter comes naturally out of this calculation and will involve the interaction terms.

  - Compute the transformation of each term appearing in the Lagrangian.
  - From the transformed Lagrangian, compute conditions that must be satisfied for the Lagrangian to be gauge invariant. Hint: collecting terms with the same number of derivatives and vector fields is useful.
  - Determine whether or not the solution presented in class

$$g_a = g_b =: g \ \ \forall a,b \qquad T^{ac} = -f^{abc}\alpha^b A^c_\mu \qquad C^{abc} = -gf^{abc} \qquad A^{abcd} = g^2 f^{kac}f^{kbd} \qquad B^{abc} = 0 \tag{3}$$

    is the only solution.

- 3 points. Zee VI.I.I.
- 3 points. Derive equation VI.3.10 in Zee, justifying each step. In particular, we did not derive equation VI.3.8 directly in class, so please do it along the way to VI.3.10.
- 5 points. Spinors in arbitrary even dimensions from tensor product.

  Using the definitions from class, prove that the tensor product is bilinear, i.e.

$$(A_1 v_1 + A_2 v_2) \otimes (B_1 w_1 + B_2 w_2) = A_1 B_1 v_1 \otimes w_1 + A_1 B_2 v_1 \otimes w_2 + A_2 B_1 v_2 \otimes w_1 + A_2 B_2 v_2 \otimes w_2 \quad (4)$$

  for scalars A_i, B_i and vectors $v_i \in V$, $w_i \in W$, for $i = 1, 2$. Recall also the identities that we had for how commutators and anticommutators of linear maps get tensored with other linear maps.

  Using the Dirac algebra for γ-matrices in $d = 2k$ dimensions,

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}\, 1_{2^k \times 2^k} \quad (5)$$

  show that

$$\begin{aligned} \Gamma^\mu &= \gamma^\mu \otimes \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \mu = 0, \ldots, d-1 \\ \Gamma^d &= 1_{2^k \times 2^k} \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ \Gamma^{d+1} &= 1_{2^k \times 2^k} \otimes \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \end{aligned} \quad (6)$$

  are good Γ-matrices in $d+2$ dimensions, i.e. show that they satisfy the Dirac algebra in $d+2$-dimensions

$$\{\Gamma^M, \Gamma^N\} = 2\eta^{MN}\, 1_{2^{k+1} \times 2^{k+1}}, \quad (7)$$

  where the 1's with subscripts are identity matrices of the stated size and $M = 0, \ldots d+1$ can be decomposed into μ, d, and $d+1$.

HOMEWORK 2. Due March 1.

- 5 points. Running couplings.

  Using the notes and the values of $\alpha_i^{-1}(M_Z)$ therein, (which determines the gauge couplings when measured at the Z mass), compute the beta functions for each in the SM and the MSSM. REMEMBER: this is just group theory, since the beta function has been computed as a function of representations. Critical to make the point I'm trying to make is to normalize the hypercharge with an appropriate 3/5 factor, which can be found in many sources, which comes from grand unification considerations. Plot α_i^{-1} for $i = 1, 2, 3$ as a function of renormalization scale in both the SM and MSSM cases. Comment on the differences between the plots.