

# PHYS 7326: Running Homework and Take-Home Exams

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We are trying something different this semester for the students that wanted more regular homework assignments. Work will still be due biweekly, but problems will be posted on a rolling basis as lectures are developed and given. It is also more natural for me, as assignment ideas come on a rolling basis.

**Instructions:** Due dates will be listed throughout the document in bold letters. For the first assignment, complete all problems by the first due date. For the  $n^{\text{th}}$  assignment, complete all problems between the  $(n-1)^{\text{th}}$  and  $n^{\text{th}}$  due date. Two due dates will have the words “TAKE-HOME” on the same line. The first is the take-home midterm, and the second the take-home final. These are your work alone.

I will also organize according to topic.

## 1 Non-abelian Gauge Theory

- 2 pts. Prove Schwartz equation (25.11).
- 3 pts. Non-abelian gauge invariance. Schwartz problem 25.1.
- 5 pts. Anomaly coefficients. Schwartz problem 25.4.
- 3 points. Peskin 15.1.
- 3 points. Peskin 15.2.

### HOMEWORK 1. Due February 1.

- 5 points. Prove Peskin equations 16.48-49 and the second BRST variation of the gauge field discussed in between them.
- 5 pts EXTRA CREDIT. Non-abelian gauge invariance. Consider the Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^a F^{\mu\nu a} + A^{abcd}A_\mu^a A_\nu^b A_\rho^c A_\sigma^d + B^{abc}\partial_\mu A^{\mu a} A_\nu^b A^{\nu c} + C^{abc}\partial_\mu A_\nu^a A^{\mu b} A^{\nu c} \quad (1)$$

and the gauge transformation

$$A_\mu^a \mapsto A_\mu^a + \frac{1}{g_a}\partial_\mu \alpha^a + T^{ab}A_\mu^b. \quad (2)$$

By  $F_{\mu\nu}^a$ , where we mean the usual thing  $F_{\mu\nu}^a = (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)$ , rather than the non-abelian field strength. The latter comes naturally out of this calculation and will involve the interaction terms.

- Compute the transformation of each term appearing in the Lagrangian.
- From the transformed Lagrangian, compute conditions that must be satisfied for the Lagrangian to be gauge invariant. Hint: collecting terms with the same number of derivatives and vector fields is useful.
- Determine whether or not the solution presented in class

$$g_a = g_b =: g \quad \forall a, b \quad T^{ac} = -f^{abc}\alpha^b A_\mu^c \quad C^{abc} = -gf^{abc} \quad A^{abcd} = g^2 f^{kac} f^{kbd} \quad B^{abc} = 0 \quad (3)$$

is the only solution.

- 3 points. Zee VI.I.I.
- 3 points. Derive equation VI.10 in Zee, justifying each step. In particular, we did not derive equation VI.8 directly in class, so please do it along the way to VI.10.
- 5 points. Running couplings.

Using the notes and the values of  $\alpha_i^{-1}(M_Z)$  therein, (which determines the gauge couplings when measured at the  $Z$  mass), compute the beta functions for each in the SM and the MSSM. REMEMBER: this is just group theory, since the beta function has been computed as a function of representations. Critical to make the point I'm trying to make is to normalize the hypercharge with an appropriate 3/5 factor, which can be found in many sources, which comes from grand unification considerations. Plot  $\alpha_i^{-1}$  for  $i = 1, 2, 3$  as a function of renormalization scale in both the SM and MSSM cases. Comment on the differences between the plots.

We will just be beginning SUSY as you do this homework, and these plots are one of the reasons people find SUSY compelling.

**HOMEWORK 2. Due March 1.**