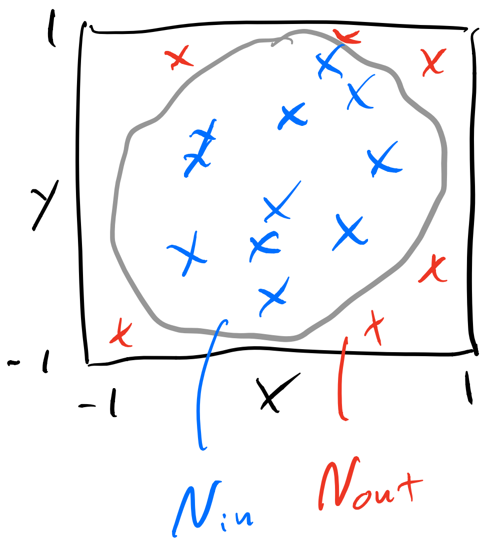
$\int$ntegration via MC methods

Lets' measure the area of circle.



$x^2 + y^2 \le r^2 \quad (r=1)$

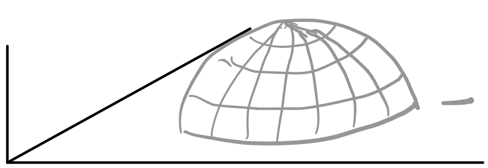
• Total area: $\int_{-1}^{1} dx \int_{-1}^{1} dy = 4$

$$\frac{N_{in}}{N_{in} + N_{out}} \simeq \frac{A_{in}}{A_{total}} \Rightarrow \frac{N_{in}}{N_{in} + N_{out}} A_{total}$$

Redo, this time with

$$f(x,y) = x^2 + y^2$$

same limits



– Half sphere

Mean Value Theorem

$$I = \int_a^b f(x)\,dx = (a-b)\langle f\rangle$$

$$\simeq (b-a)\frac{1}{N}\sum_{i=1}^{N} \underbrace{f(x_i)}$$

Higher dimensions: was 0 or 1 before

$$\int_a^b dx \int_c^d dy\, f(x,y) \simeq (b-a)(d-c)\frac{1}{N}\sum_i^N f(x_i)$$

$$= (b-a)(d-c)\langle f\rangle$$

Error in XD integrals

no D

- Statistical error → $\epsilon_{MC} = \frac{1}{\sqrt{N}}$

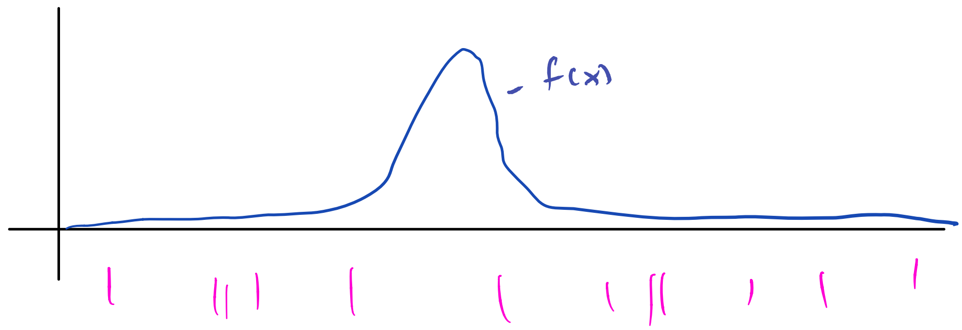
↳ starts beating grid based integration around 3D-4D!

10D integration example:

$$I = \int_0^1 dx_1 \cdots \int_0^1 dx_{10}\,(x_1 + \cdots + x_{10})^2$$

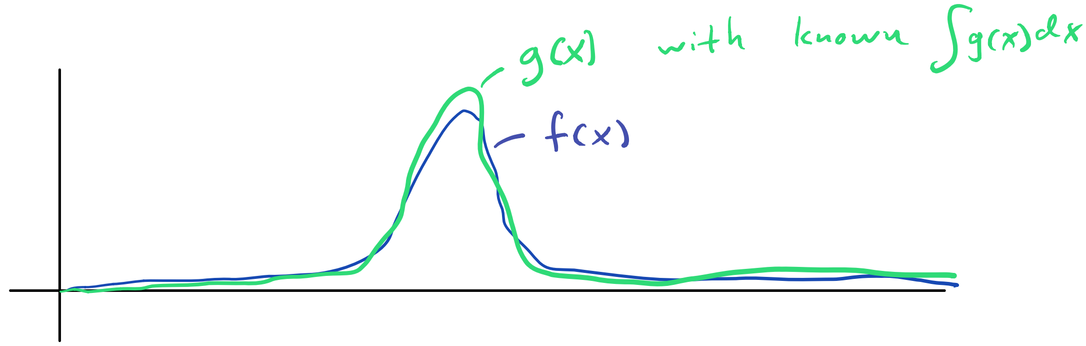
$$= \frac{155}{6}$$

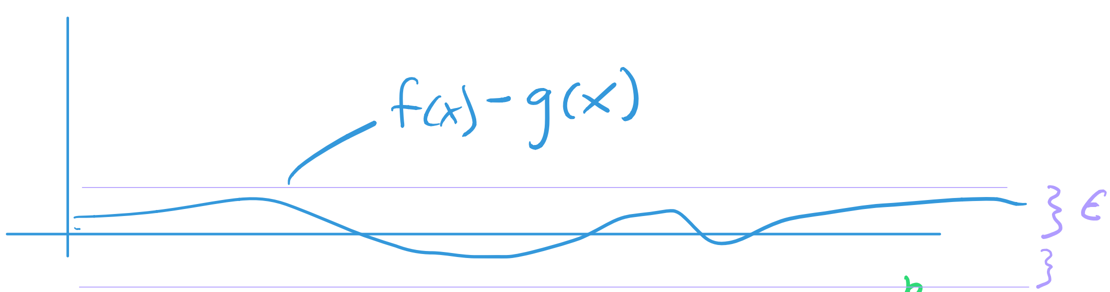
Problem:



Most MC values have little effect on result → wasted evaluations

Solution: Variance Reduction





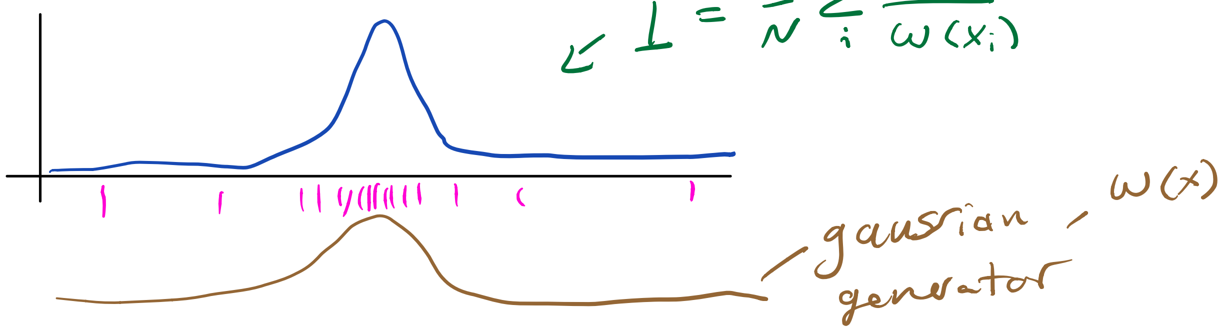
$$\int_a^b f(x)\,dx = \int_a^b \left[f(x) - g(x)\right] dx + \int_a^b g(x)\,dx$$

Solution: Importance Sampling

$$I = \int_a^b w(x)\frac{f(x)}{w(x)}\,dx$$

sample from this PDF!

$$I = \frac{1}{N}\sum_i^N \frac{f(x_i)}{w(x_i)}$$



Von Neumann Rejection

- will talk about this when we talk about PDFs