Robotic SLAM

1 Introduction

Terms

- State estimation - find out the pose
- Localisation - pose w.r.to landmark or map
- Mapping
- navigation and motion planning - a star, wave front dijkstra

1.1 What is SLAM

Computing robot's poses and the map of the environment at the same time.
Localisation : estimating robots location
Mapping : building a MAP

Given

- Robots control inputs

$$u_{1:T} = \{u_1, u_2, u_3....u_T\}$$

- Observations

$$z_{1:T} = \{z_1, z_2, z_3, ..., z_T\}$$

Wanted

- Map of the environment

$$m$$

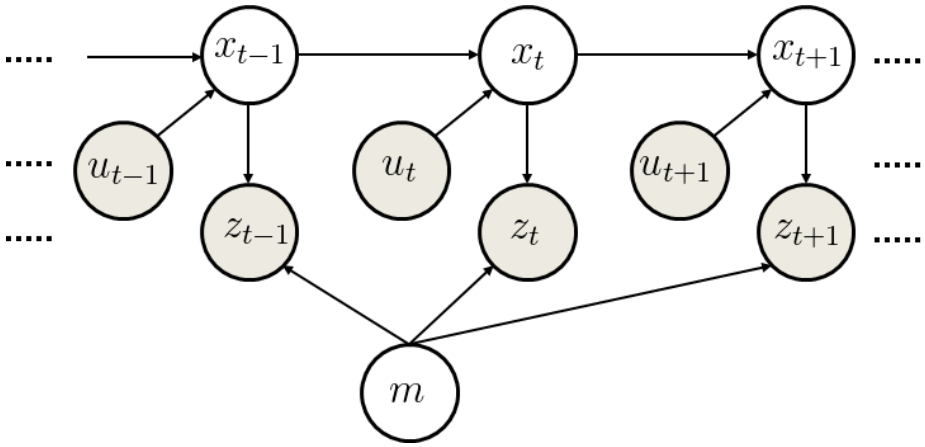
- path of the Robot

$$x_{0:T} = \{x_0, x_1, x_2, ..., x_T\}$$

Using the robots control inputs we can predict the position of the robot. From the observations $z_{1:T}$, we can calculate the position of the robot. Both the steps have some error associcated with it . Lets call the first one the model noise and second one the sensor noise. So we have to associate a probability with both of them. The error accumulates over time(even if the error in individual measurements is really small)

So in the probalistic terms our problem minimises to

$$p(x_{0:T}, m|z_{1:T}, u_{1:T})$$



1.2 Full Slam vs online SLAM

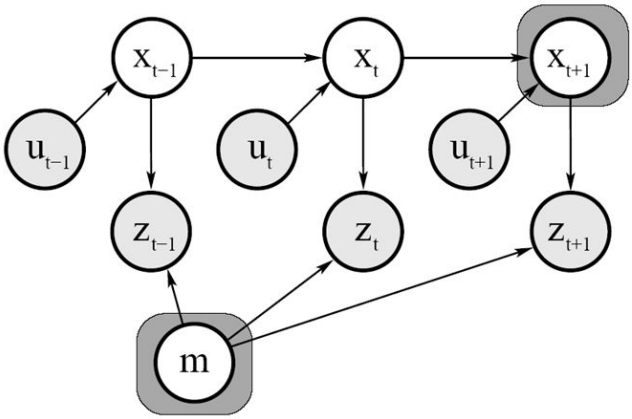
- Full SLAM estimates the entire path

$$p(x_{0:T}, m | z_{1:T}, u_{1:T})$$

- Online SLAM estimates only the most recent pose

$$p(x_t, m | z_{1:T}, u_{1:T})$$

Graphical Model of Online SLAM:



$$p(x_t, m \mid z_{1:t}, u_{1:t}) = \int \int \ldots \int p(x_{1:t}, m \mid z_{1:t}, u_{1:t})\, dx_1 dx_2 \ldots dx_{t-1}$$

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1.3 Types of SLAM

occupancy maps created from lidars, sonars etc. - volumetric SLAM feature based approach - store features and localise based on that volumetric SLAM maybe better for navigation applications . Topological representations vs geometric representations. Static vs dynamic features. Active - robot decides the path so as to build a map vs passive slam - may follow a fixed path i.e. path not optimised for mapping/ exploration

2 Bayes Filter

2.1 State Estimation

Goal $p(x|z,u)$
Recursive Bayes Filter

$$\begin{aligned} bel(x_t) &= p(x_t|z_{1:t}, u_{1:t}) \\ &= \eta p(z_t|x_t, z_{1:t-1}, u_{1:t}) * p(x_t|z_{1:t-1}, u1:t) \\ &= \eta p(z_t|x_t) * p(x_t|z_{1:t-1}, u1:t) \\ &= \eta p(z_t|x_t) \int_{x_{t-1}} p(x_t|x_{t-1}, z_{1:t-1}, u_{1:t}) * p(x_{t-1}|z_{1:t-1}u_{1:t-1}) dx_{t-1} \\ &= \eta p(z_t|x_t) \int_{x_{t-1}} p(x_t|x_{t-1}, u_t) * bel(x_{t-1}) dx_{t-1} \end{aligned}$$

we can split this into predict and update steps where
Predict Step

$$\overline{bel(x_t)} = \int_{x_{t-1}} p(x_t|x_{t-1}, u_t) * bel(x_{t-1}) dx_{t-1}$$

Update Step

$$bel(x_t) = \eta * p(z_t|x_t) * \overline{bel(x_t)}$$

Bayes filter gives a framework for recursive state estimation using the above equations. The actual realisation may be kalman filtering , EKF or particle filter (Linear non linear motion models) (distributions) *Kalman Filter* - Gaussians , requires linear or linearised model *Particle filter* - Non-parametric , Arbitrary models

2.2 Probability motion models

$p(x_t|u_t, x_{t-1})$ we can model this in two ways

- odometry models - measurement of velocity (tends to be more accurate)
- velocity models - we know the input commands, but no measurement of vel

2.3 Model for laser scanners

scan z consisits of k beams $z_t \epsilon \Re^k$ i.e. $z_t = \{z_t^1, z_t^2, ...z_t^k\}$,
Assuming beams are independednt, then

$$p(z_t|x_t, m) = \prod_{i=1}^{k} p(z_t^i|x_t, m)$$

- Beam endpoint model (likelihood calculated as gaussian blur on occupancy map)
- Ray cast model (occlusion, sensor accuray, satuaration , random)
- model for range bearing sensors $z_t^i = (r_t^i, \phi_t^i)^T$

$$r_t^i = ||m - x|| + gaussian$$

$$\phi_t^i = \angle(m - x) - \theta + gaussian$$

3 Kalman filter Equations

$$\begin{aligned}
\bar{\mu_t} &= g(u_t, \mu_{t-1}) \\
\bar{\Sigma_t} &= G_t \Sigma_{t-1} G_t^T + R_t \\
K_t &= \bar{\Sigma_t} H_t^T (H_t \bar{\Sigma_t} H_t^T + Q_t)^{-1} \\
\mu_t &= \bar{\mu_t} + K_t(z_t - h(\bar{\mu_t})) \\
\Sigma_t &= (I - K_t H_t)\bar{\Sigma_t}
\end{aligned}$$

3.1 Summary

- Diverge for large non linearities
- Can deal only single modes
- Successful in medium scale scenes with good data associations
- Approximations exists to reduce the computational complexity

commonly used Datasets:

- Victoria Park Data sets - Trees are the landmark -Data assosciation -girth and height
- Tennis Court Dataset - for mapping precescion

4 Extended Kalman Filter vs Unscented Kalman Filter

EKF works by linearising the state transfer equations thus making sure that all the conditional and marginal distributions will remain gaussian. Gaussians are closed space. **UKF** uses the nonlinear state transmission equation, then tries to sample a gaussian distribution from the non-gaussian resulted from the non-linear operation.

4.1 Strategy for chosing sampling points and weights for UKF

$$
\begin{aligned}
X^{[0]} &= \mu \\
X^{[i]} &= \mu + (\sqrt{(n+\lambda)\Sigma})_i \; for \; i = 1, ..., n \\
X^{[i]} &= \mu - (\sqrt{(n+\lambda)\Sigma})_i \; for \; i = n+1, ..., 2n \\
w_m^{[0]} &= \frac{\lambda}{n+\lambda} \\
w_c^{[0]} &= w_m^{[0]} + (a - \alpha^2 + \beta) \\
w_m^{[i]} = w_c^{[i]} &= \frac{1}{2(n+\lambda)} \; for \; i = 1, ..., 2n
\end{aligned}
$$

where $\alpha \, \epsilon \, (0, 1]$; $k >= 0$; $\lambda = \alpha^2(n+k) - n$;
too small value of K will lead to UKF EKF . Too large - diverge
$\sqrt{\Sigma} = VD^{1/2}V^{-1}$, we are sampling the gaussian along the eigen vectors of the covariance matrix.

$$\bar{\mu}_t = \sum_{1=0}^{2n} w^{[i]} g(X^{[i]})$$

$$\bar{\Sigma}_t = \sum_{1=0}^{2n} w^{[i]} (g(X^{[i]}) - \bar{\mu}_t)(g(X^{[i]}) - \bar{\mu}_t)^T + R_t$$

5 Grid Maps

SLAM problems can make one of the types of maps

- **Feature Based Maps:** Maps is repsented by a few lanndmarks. Advantage of this type of maps is the space time complexity can be smaller compared to Volumetric Maps. On the downside, we have to implement a feature detector - i.e. observations are not directly used in this method.

- **Volumetric Maps:** Volumetric Maps on the other hand uses all observations and represent the map using a 3D(lidar point cloud) or 2D(range sensor) grid. Occupancy is often shown with a black pixel and free areas are white. Unseen areas are marked with grey. Feature based maps are used in kalman filter implementations, Particle filter based approaches use Grid mapping

6 Links and references

Webpage from Syrill Stachniss can be found here
pythonrobotics