College of Engineering Trivandrum

Department of Electronics and Communication Engineering

ECL332 Communication Engineering Laboratory

Experiment No. 3

Simulation of BPSK Modulation and Demodulation through an AWGN Channel

Name:	____________________	Date:	____________________
Roll No.:	____________________	Batch:	____________________

Learning Objectives

After completing this experiment, you will be able to:

- Use a real dataset (image) to generate a **bitstream** and transmit it using **BPSK**.
- Model an **AWGN** channel and set the noise level using E_b/N_0.
- Implement an **ML detector** (threshold detection) for BPSK and estimate **BER**.
- Plot **BER vs** E_b/N_0 and compare with the **theoretical BER**.
- For each E_b/N_0, generate:
  - received symbol **scatter plot** (constellation),
  - **reconstructed image** after demodulation and decoding.

Aim

To study the error performance of BPSK transmission over an AWGN channel using an image-derived bitstream, and to compare simulated BER with the theoretical BER as a function of E_b/N_0.

Resources

- Use Python for Programming
- The image is available in https://jinujayachandran.github.io/ay2025-26_ecl332_commlab/.

System Block Diagram

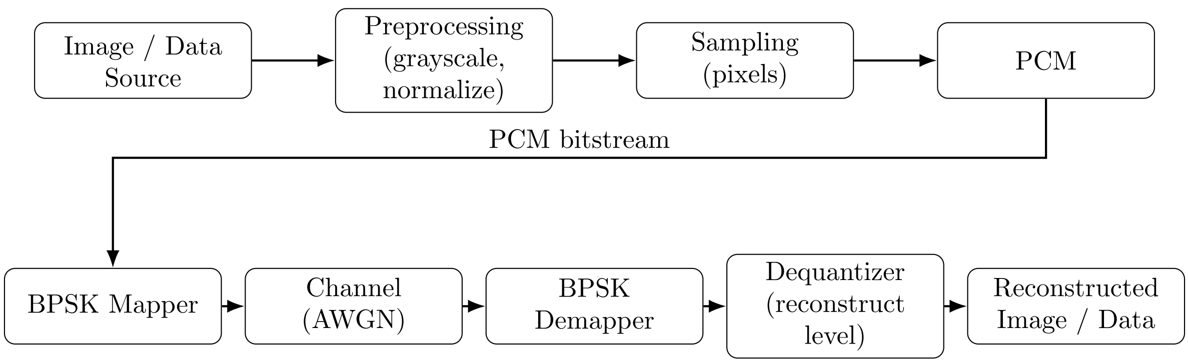

Figure 1: End-to-end PCM system.

Theoretical Background

1) BPSK signal set and signal space

In Binary Phase Shift Keying (BPSK), one bit is transmitted per symbol using two antipodal points:

$$b \in \{0,1\} \quad \Rightarrow \quad s \in \left\{+\sqrt{E_b},\, -\sqrt{E_b}\right\}.$$

A common mapping is:

$$0 \to +\sqrt{E_b}, \qquad 1 \to -\sqrt{E_b}.$$

In 1-D signal space, these are two points on the real axis, separated by distance $2\sqrt{E_b}$.

2) AWGN channel model

In an Additive White Gaussian Noise (AWGN) channel:

$$y = s + n$$

where $n \sim \mathcal{N}(0, \sigma^2)$ is zero-mean Gaussian noise. For baseband (real) BPSK, the variance is related to noise spectral density N_0 by:

$$\sigma^2 = \frac{N_0}{2}.$$

3) What is E_b and E_b/N_0?

- E_b = **energy per bit**. For BPSK with symbol amplitude $\pm\sqrt{E_b}$, each symbol carries 1 bit.
- N_0 = **noise power spectral density**.
- E_b/N_0 is a standard measure of link quality.

In dB:

$$\left(\frac{E_b}{N_0}\right)_{\mathrm{dB}} = 10\log_{10}\left(\frac{E_b}{N_0}\right).$$

4) How to set noise for a desired E_b/N_0 in simulation (Complex AWGN)

Choose E_b (commonly set $E_b = 1$ for convenience). Convert E_b/N_0 from dB to linear:

$$\left(\frac{E_b}{N_0}\right)_{\mathrm{lin}} = 10^{\left(\frac{E_b}{N_0}\right)_{\mathrm{dB}}/10}.$$

Then compute:

$$N_0 = \frac{E_b}{(E_b/N_0)_{\mathrm{lin}}}.$$

For complex baseband AWGN, the noise is generated as

$$n = \frac{1}{\sqrt{2}}(n_I + jn_Q),$$

where the in-phase and quadrature components are independent Gaussians:

$$n_I \sim \mathcal{N}(0, \sigma^2), \qquad n_Q \sim \mathcal{N}(0, \sigma^2).$$

The per-dimension variance is

$$\sigma^2 = \frac{N_0}{2}.$$

Finally, form the received symbol as

$$y = s + n.$$

Note: Even though BPSK symbols $s \in \{\pm\sqrt{E_b}\}$ lie on the real axis, complex AWGN spreads the received samples in both I and Q directions, which is why the scatter plot is shown in the I-Q plane.

5) ML detection for BPSK (threshold rule) with complex AWGN

With complex baseband AWGN, the received symbol is

$$y = s + n,$$

where $s \in \{+\sqrt{E_b}, -\sqrt{E_b}\}$ is real-valued (BPSK lies on the in-phase axis) and $n = n_I + jn_Q$ is complex Gaussian noise. In this case, the Maximum Likelihood (ML) decision depends on the **real (in-phase) component** of y:

$$\hat{s} = \begin{cases} +\sqrt{E_b}, & \Re\{y\} \geq 0, \\ -\sqrt{E_b}, & \Re\{y\} < 0. \end{cases}$$

Equivalently, in terms of bits:

$$\hat{b} = \begin{cases} 0, & \Re\{y\} \geq 0, \\ 1, & \Re\{y\} < 0. \end{cases}$$

Interpretation: the quadrature component $\Im\{y\}$ contains only noise for BPSK and does not affect the optimal decision boundary (which remains a vertical line at $\Re\{y\} = 0$ in the I-Q plane).

6) Theoretical BER of BPSK over AWGN

The theoretical bit error probability is:

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right).$$

You will compare your simulated BER values with this theoretical curve.

Tip

At low E_b/N_0, received symbols spread widely (scatter looks noisy) $\Rightarrow$ high BER and corrupted image. At high E_b/N_0, received symbols cluster near $\pm\sqrt{E_b}$ $\Rightarrow$ low BER and a clean image.

Pre-Lab

Assume BPSK mapping $0 \to +\sqrt{E_b}$ and $1 \to -\sqrt{E_b}$, with $E_b = 1$ unless stated otherwise.

1. **dB to linear conversion:** Convert the following E_b/N_0 values from dB to linear scale:

$$-2 \text{ dB},\ 0 \text{ dB},\ 3 \text{ dB},\ 5 \text{ dB},\ 10 \text{ dB}.$$

2. **Noise variance calculation (AWGN):** For each of the following E_b/N_0 values, compute N_0 and $\sigma^2 = N_0/2$:

$$E_b/N_0 = 0 \text{ dB},\ 5 \text{ dB},\ 10 \text{ dB}.$$

3. **Received sample and ML decision:** A single transmitted BPSK symbol is $s = +1$. The channel noise sample is $n = -0.7$.
   (a) Compute the received value $y = s + n$.

   (b) Using ML detection (threshold at 0), decide the detected bit.

4. **Decision errors from samples:** The transmitted bits are $\{0, 1, 0, 1, 1\}$ and the received samples after AWGN are:
$$y = \{0.9,\ -0.2,\ 0.1,\ 0.6,\ -0.4\}.$$
   Using threshold detection at 0, determine the detected bits and compute the number of bit errors and BER.

5. **Symbol energy and constellation spacing:** For BPSK with $E_b = 1$:
   (a) Write the two constellation points.

   (b) Compute the Euclidean distance between them.

6. **Theoretical BER evaluation using Q-function:** Theoretical BER for BPSK over AWGN is $P_b = Q\left(\sqrt{2E_b/N_0}\right)$. Using the following approximate values:
$$Q(1.0) \approx 0.1587, \quad Q(1.5) \approx 0.0668, \quad Q(2.0) \approx 0.0228, \quad Q(2.5) \approx 0.0062,$$
   estimate P_b at:
$$E_b/N_0 = 0 \text{ dB},\ 3 \text{ dB},\ 6 \text{ dB}.$$

7. **Expected number of bit errors:** Suppose you transmit $N = 100{,}000$ bits. Using your theoretical BER estimate at $E_b/N_0 = 6$ dB (from the previous question), estimate the expected number of bit errors.

8. **Relating noise variance to scatter spread:** For $E_b/N_0 = 0$ dB and $E_b/N_0 = 10$ dB (with $E_b = 1$), compute σ (standard deviation). Which case will produce a wider scatter cloud in the received symbol plot, and why?

9. **Change in E_b:** Now assume $E_b = 4$ (i.e., BPSK symbols are ± 2). For $E_b/N_0 = 5$ dB, compute N_0 and σ^2. Compare σ^2 with the case $E_b = 1$ at the same E_b/N_0.

10. **Bitstream size from image (rate calculation):** A 256×256 image is PCM encoded using $b = 4$ bits/pixel.
    (a) Compute the total number of transmitted bits.

    (b) If the bit rate is $R_b = 1$ Mbit/s, compute the transmission time for one image (in ms).

Important Setting for this Experiment

- **PCM Quantization:** Use **8-bit PCM** for the image.

$$b = 8, \qquad L = 2^b = 256, \qquad \Delta = \frac{1}{256} \quad \text{(for normalized pixel range } [0,1]\text{)}.$$

- You will vary only the channel quality using E_b/N_0 values.

Procedure / Lab Tasks (8-bit PCM + BPSK over AWGN)

Task 1: Load the image and generate 8-bit PCM bitstream

T1-1 Load the given image, convert to grayscale, and resize to 256×256 (recommended).
T1-2 Normalize pixel values to $[0, 1]$ as $x = X/255$.
T1-3 Perform **8-bit uniform quantization** ($b = 8$, $L = 256$) and generate the PCM bitstream $\mathbf{b}_{tx}$.
T1-4 Record M, N, and the total number of transmitted bits.

PTQ1: What are the values of b, L, and Δ used in this experiment? Show calculation of Δ.

PTQ2: If the image size is $M \times N$, derive the bitstream length for 8-bit PCM and compute it for 256×256.

Task 2: BPSK modulation and constellation (signal space)

T2-1 Choose $E_b = 1$.
T2-2 Map bits to BPSK symbols: $0 \to +\sqrt{E_b}, \qquad 1 \to -\sqrt{E_b}$.
T2-3 Generate the transmitted symbol sequence $\mathbf{s}$ from $\mathbf{b}_{tx}$.
T2-4 Plot the ideal constellation points (without noise) on the real axis (or I-Q plane).

PTQ3: What is the ML decision boundary for BPSK over AWGN? Explain in 1–2 lines.

PTQ4: **Draw:** BPSK constellation (signal space) and clearly mark the decision boundary.

Task 3: AWGN channel simulation (Complex Baseband) for multiple E_b/N_0 values

Use the following E_b/N_0 values (in dB): $[0, 2, 4, 6, 8, 10]$ **dB**. For each value:
T3-1 Convert $(E_b/N_0)_{\mathrm{dB}}$ to linear:

$$(E_b/N_0)_{\mathrm{lin}} = 10^{(E_b/N_0)_{\mathrm{dB}}/10}.$$

T3-2 Compute the noise spectral density and per-dimension variance:

$$N_0 = \frac{E_b}{(E_b/N_0)_{\mathrm{lin}}}, \qquad \sigma^2 = \frac{N_0}{2}.$$

Here, σ^2 is the variance of the **real** and **imaginary** components of the complex noise.
T3-3 Generate **complex AWGN**:

$$n = \frac{1}{\sqrt{2}}(n_I + jn_Q), \qquad n_I \sim \mathcal{N}(0, \sigma^2), \;\; n_Q \sim \mathcal{N}(0, \sigma^2),$$

and form the received complex baseband symbols:

$$y = s + n.$$

(Even though BPSK symbols $s \in \{\pm\sqrt{E_b}\}$ lie on the real axis, the channel adds noise in both I and Q.)
T3-4 For each E_b/N_0, produce a **scatter plot** of the received symbols in the I-Q plane:

$$\{(\Re\{y\}, \Im\{y\})\}.$$

Use a subset of points if required for clarity.

PTQ5: Compute N_0 and σ^2 for $E_b/N_0 = 0$ dB and 10 dB (with $E_b = 1$).

PTQ6: As E_b/N_0 increases, what happens to the scatter plot? (Explain using σ^2.)

PTQ7: **Draw:** Received symbol scatter plot (rough sketch) for :

- $E_b/N_0 = 0$ dB
- $E_b/N_0 = 10$ dB

Task 4: ML detection, BER calculation, and reconstructed image (Complex Baseband, for each E_b/N_0)

For each E_b/N_0 value:

T4-1 **ML detection using the in-phase component:** Since BPSK symbols lie on the real axis ($s \in \{\pm\sqrt{E_b}\}$) and the channel adds complex noise $n = n_I + jn_Q$, the received sample is

$$y = s + n.$$

The optimal ML decision depends on the **real part** of y:

$$\hat{b} = \begin{cases} 0, & \Re\{y\} \geq 0, \\ 1, & \Re\{y\} < 0. \end{cases}$$

(Equivalently, decide $+\sqrt{E_b}$ if $\Re\{y\} \geq 0$, else decide $-\sqrt{E_b}$.)

T4-2 **Compute simulated BER:**

$$\widehat{\mathrm{BER}} = \frac{\#\{b_{tx} \neq b_{rx}\}}{\text{total bits}}.$$

T4-3 Feed the detected bitstream $\mathbf{b}_{rx}$ to the PCM decoder (8-bit) and reconstruct the image.

T4-4 For each E_b/N_0, save/attach:

- measured BER value,
- reconstructed image,
- I-Q scatter plot of received symbols: ($\Re\{y\}$, $\Im\{y\}$).

PTQ8: Why does a small BER sometimes create visible "salt-and-pepper" artifacts in the reconstructed image?

PTQ9: For $E_b/N_0 = 0$ dB and 10 dB, report your measured BER values (from simulation).

Task 5: Theoretical BER and comparison with simulation

T5-1 Compute theoretical BER for each E_b/N_0:

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right).$$

Use `erfc` function from `scipy.special` in python

T5-2 Plot on a **semi-log** scale (use `semilogy` function in python):

- simulated BER vs E_b/N_0 (markers),
- theoretical BER vs E_b/N_0 (smooth curve).

T5-3 Comment on the agreement/differences.

PTQ10: Why is a semi-log plot (log scale on BER axis) preferred for BER curves?

PTQ11: At which E_b/N_0 values does the simulated BER deviate most from theory? Give two reasons.

PTQ12: **Draw:** BER vs E_b/N_0 (semi-log) and show both:

- theoretical curve (line)
- simulated points (markers)

PTQ13: From your plot, identify the minimum E_b/N_0 that gives "acceptable" image quality. Justify using BER and the image.

Observations / Results

Table: BER and reconstruction summary

E_b/N_0 (dB)	$(E_b/N_0)_{\text{lin}}$	$\sigma^2 = N_0/2$	Sim. BER	Theory BER	Image quality (short note)
0					
2					
4					
6					
8					
10					

Concluding Remarks

(*Give briefly what you have understood from the experiment.*)

Attach Code: The code should contain the following plots

- Scatter plots of received symbols for each E_b/N_0.
- Reconstructed images for each E_b/N_0 (label clearly).
- BER vs E_b/N_0 plot: simulated vs theoretical (semi-log).

Student's Signature: ______________________________