College of Engineering Trivandrum

Department of Electronics and Communication Engineering

ECL332 Communication Engineering Laboratory

Experiment No. 5

Simulation of QPSK Modulation and Demodulation through an AWGN Channel

Name:	____________________	Date:	____________________
Roll No.:	____________________	Batch:	____________________

Learning Objectives

After completing this experiment, you will be able to:

- Use a real dataset (image) to generate a **bitstream** and transmit it using **QPSK**.
- Model a **complex baseband AWGN** channel and set the noise level using E_b/N_0.
- Implement an **ML detector** (nearest-neighbor / quadrant decision) for QPSK and estimate **BER**.
- Plot **BER vs** E_b/N_0 and compare with the **theoretical BER** for Gray-coded QPSK.
- For each E_b/N_0, generate:
  - received symbol **scatter plot** (constellation),
  - **reconstructed image** after demodulation and decoding.

Aim

To study the error performance of QPSK transmission over an AWGN channel using an image-derived bitstream, and to compare simulated BER with the theoretical BER as a function of E_b/N_0.

Resources

- Use Python for Programming
- The image is available in https://jinujayachandran.github.io/ay2025-26_ecl332_commlab/.

System Block Diagram

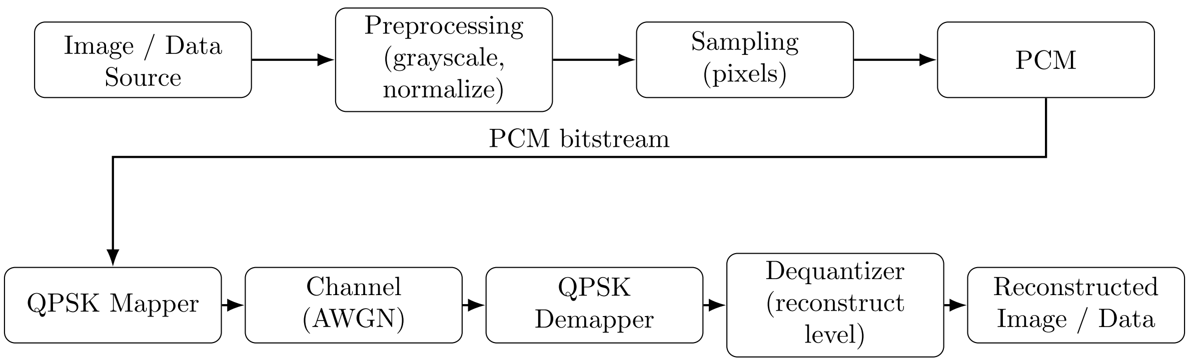

Figure 1: End-to-end PCM + QPSK system over AWGN.

Theoretical Background

1) QPSK signal set and signal space

In Quadrature Phase Shift Keying (QPSK), **2 bits** are transmitted per symbol using four points in the I-Q plane. Using Gray coding and E_b as energy per bit, a convenient normalized constellation is:

$$s \in \left\{ \sqrt{E_b}(+1+j),\ \sqrt{E_b}(-1+j),\ \sqrt{E_b}(-1-j),\ \sqrt{E_b}(+1-j) \right\}.$$

One common Gray mapping is:

$$\begin{aligned} 00 &\to \sqrt{E_b}(+1+j), \\ 01 &\to \sqrt{E_b}(-1+j), \\ 11 &\to \sqrt{E_b}(-1-j), \\ 10 &\to \sqrt{E_b}(+1-j). \end{aligned}$$

Energy relation: Since each symbol carries 2 bits, symbol energy is

$$E_s = 2E_b.$$

(With the above scaling, $|s|^2 = 2E_b$ for every symbol.)

2) Complex baseband AWGN channel model

In complex baseband AWGN:

$$y = s + n,$$

where n is complex Gaussian noise:

$$n = n_I + jn_Q, \qquad n_I \sim \mathcal{N}(0, \sigma^2),\ \ n_Q \sim \mathcal{N}(0, \sigma^2),$$

and the per-dimension variance is related to N_0 by:

$$\sigma^2 = \frac{N_0}{2}.$$

3) What is E_b and E_b/N_0?

- E_b = **energy per bit**.
- E_s = **energy per symbol**. For QPSK, $E_s = 2E_b$.
- N_0 = **noise power spectral density**.
- E_b/N_0 is a standard measure of link quality (bit-level SNR).

In dB:

$$\left(\frac{E_b}{N_0}\right)_{\text{dB}} = 10\log_{10}\left(\frac{E_b}{N_0}\right).$$

4) How to set noise for a desired E_b/N_0 in simulation (Complex AWGN)

Choose E_b (commonly set $E_b = 1$ for convenience). Convert E_b/N_0 from dB to linear:

$$\left(\frac{E_b}{N_0}\right)_{\text{lin}} = 10^{\left(\frac{E_b}{N_0}\right)_{\text{dB}}/10}.$$

Then compute:

$$N_0 = \frac{E_b}{(E_b/N_0)_{\text{lin}}}, \qquad \sigma^2 = \frac{N_0}{2}.$$

Generate complex noise:

$$n = \sqrt{\sigma^2}\,(w_I + jw_Q), \qquad w_I, w_Q \sim \mathcal{N}(0,1),$$

and form:

$$y = s + n.$$

5) ML detection for Gray-coded QPSK (quadrant rule)

For the above Gray mapping, ML detection in AWGN is equivalent to choosing the nearest constellation point. This can be implemented by **sign decisions** on $\Re\{y\}$ and $\Im\{y\}$:

$$\hat{b}_I = \begin{cases} 0, & \Re\{y\} \geq 0, \\ 1, & \Re\{y\} < 0, \end{cases} \qquad \hat{b}_Q = \begin{cases} 0, & \Im\{y\} \geq 0, \\ 1, & \Im\{y\} < 0. \end{cases}$$

Then combine $(\hat{b}_I, \hat{b}_Q)$ into the detected 2-bit symbol using the same Gray mapping used at the transmitter.

Tip

Decision boundaries are the axes: $\Re\{y\} = 0$ and $\Im\{y\} = 0$. At low E_b/N_0, the cloud spreads widely $\Rightarrow$ more quadrant mistakes $\Rightarrow$ higher BER and corrupted image. At high E_b/N_0, points cluster tightly near the 4 ideal symbols $\Rightarrow$ low BER and a clean image.

6) Theoretical BER of Gray-coded QPSK over AWGN

For Gray-coded QPSK in AWGN, the **bit error probability** equals that of BPSK:

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right).$$

You will compare your simulated BER values with this theoretical curve.

Pre-Lab

Assume Gray-coded QPSK with the mapping:

$$00 \to \sqrt{E_b}(+1+j), \;\; 01 \to \sqrt{E_b}(-1+j), \;\; 11 \to \sqrt{E_b}(-1-j), \;\; 10 \to \sqrt{E_b}(+1-j),$$

with $E_b = 1$ unless stated otherwise.

1. **dB to linear conversion:** Convert the following E_b/N_0 values from dB to linear scale:

$$-2 \text{ dB}, \; 0 \text{ dB}, \; 3 \text{ dB}, \; 5 \text{ dB}, \; 10 \text{ dB}.$$

2. **Noise variance calculation (complex AWGN):** For each of the following E_b/N_0 values, compute N_0 and $\sigma^2 = N_0/2$:

$$E_b/N_0 = 0 \text{ dB}, \; 5 \text{ dB}, \; 10 \text{ dB}.$$

3. **Symbol energy relation:** For QPSK, show that $E_s = 2E_b$. If $E_b = 1$, find E_s and list the four ideal constellation points.

4. **Received sample and ML decision (quadrant):** A single transmitted QPSK symbol is $s = (1+j)$ (assume $E_b = 1$). The channel noise sample is $n = (-0.7 + j\,0.4)$.
   (a) Compute the received value $y = s + n$.

   (b) Using ML detection, decide the detected 2-bit symbol.

5. **Decision errors from samples:** The transmitted bit pairs are $\{00, 01, 10, 11\}$ and the received samples are:
$$y = \{0.9 + j0.2,\ -0.2 + j0.6,\ 0.1 - j0.4,\ -0.6 - j0.3\}.$$
   Using quadrant detection, determine the detected bit pairs and compute the number of bit errors and BER.

6. **Theoretical BER evaluation using Q-function:** Theoretical BER for Gray-coded QPSK over AWGN is $P_b = Q\left(\sqrt{2E_b/N_0}\right)$. Using:
$$Q(1.0) \approx 0.1587, \quad Q(1.5) \approx 0.0668, \quad Q(2.0) \approx 0.0228, \quad Q(2.5) \approx 0.0062,$$
   estimate P_b at:
$$E_b/N_0 = 0 \text{ dB},\ 3 \text{ dB},\ 6 \text{ dB}.$$

7. **Expected number of bit errors:** Suppose you transmit $N = 100{,}000$ bits. Using your theoretical BER estimate at $E_b/N_0 = 6$ dB (from the previous question), estimate the expected number of bit errors.

8. **Relating noise variance to scatter spread:** For $E_b/N_0 = 0$ dB and $E_b/N_0 = 10$ dB (with $E_b = 1$), compute σ. Which case will produce a wider scatter cloud in the constellation, and why?

9. **Bitstream size from image (rate calculation):** A 256×256 image is PCM encoded using $b = 8$ bits/pixel.
   (a) Compute the total number of transmitted bits.

   (b) In QPSK, each symbol carries 2 bits. Compute the number of transmitted QPSK symbols.

   (c) If the bit rate is $R_b = 1$ Mbit/s, compute the transmission time for one image (in ms).

Important Setting for this Experiment

- **PCM Quantization:** Use **8-bit PCM** for the image.

$$b = 8, \qquad L = 2^b = 256, \qquad \Delta = \frac{1}{256} \quad \text{(for normalized pixel range } [0,1]\text{)}.$$

- **QPSK:** Use Gray coding and set $E_b = 1$. Then each symbol has energy $E_s = 2$.
- You will vary only the channel quality using E_b/N_0 values.

Procedure / Lab Tasks (8-bit PCM + QPSK over AWGN)

Task 1: Load the image and generate 8-bit PCM bitstream

T1-1 Load the given image, convert to grayscale, and resize to 256×256 (recommended).
T1-2 Normalize pixel values to $[0, 1]$ as $x = X/255$.
T1-3 Perform **8-bit uniform quantization** ($b = 8$, $L = 256$) and generate the PCM bitstream $\mathbf{b}_{tx}$.
T1-4 Record M, N, and the total number of transmitted bits.

PTQ1: What are the values of b, L, and Δ used in this experiment? Show calculation of Δ.

PTQ2: If the image size is $M \times N$, derive the bitstream length for 8-bit PCM and compute it for 256×256.

Task 2: QPSK modulation and constellation (signal space)

T2-1 Choose $E_b = 1$.

T2-2 Group the transmit bitstream into pairs: (b_1, b_2). (If needed, pad one 0 at the end to make the length even.)

T2-3 Map each bit-pair to a QPSK symbol using Gray coding:

$$00 \to (+1 + j),\ 01 \to (-1 + j),\ 11 \to (-1 - j),\ 10 \to (+1 - j) \quad (\text{since } E_b = 1).$$

T2-4 Generate the transmitted symbol sequence $\mathbf{s}$.

T2-5 Plot the ideal QPSK constellation points on the I-Q plane and mark the decision boundaries.

PTQ3: What are the ML decision boundaries for QPSK over AWGN? Explain in 1–2 lines.

PTQ4: **Draw:** QPSK constellation and clearly mark the decision boundaries.

Task 3: AWGN channel simulation (Complex Baseband) for multiple E_b/N_0 values

Use the following E_b/N_0 values (in dB): $[0, 2, 4, 6, 8, 10]$ **dB**. For each value:

T3-1 Convert $(E_b/N_0)_{\text{dB}}$ to linear:

$$(E_b/N_0)_{\text{lin}} = 10^{(E_b/N_0)_{\text{dB}}/10}.$$

T3-2 Compute:

$$N_0 = \frac{E_b}{(E_b/N_0)_{\text{lin}}}, \qquad \sigma^2 = \frac{N_0}{2}.$$

T3-3 Generate complex AWGN and form received symbols:

$$y = s + n, \qquad n = \sqrt{\sigma^2}(w_I + jw_Q),\ w_I, w_Q \sim \mathcal{N}(0, 1).$$

T3-4 For each E_b/N_0, produce a **scatter plot** of received symbols in the I-Q plane:

$$\{(\Re\{y\},\ \Im\{y\})\}.$$

Use a subset of points if required for clarity.

PTQ5: Compute N_0 and σ^2 for $E_b/N_0 = 0$ dB and 10 dB (with $E_b = 1$).

PTQ6: As E_b/N_0 increases, what happens to the scatter plot? (Explain using σ^2.)

PTQ7: **Draw:** Received constellation scatter (rough sketch) for:

- $E_b/N_0 = 0$ dB
- $E_b/N_0 = 10$ dB

Task 4: ML detection, BER calculation, and reconstructed image (QPSK, for each E_b/N_0)

For each E_b/N_0 value:

T4-1 **ML detection (quadrant decision):**

$$\hat{b}_I = \begin{cases} 0, & \Re\{y\} \geq 0, \\ 1, & \Re\{y\} < 0, \end{cases} \qquad \hat{b}_Q = \begin{cases} 0, & \Im\{y\} \geq 0, \\ 1, & \Im\{y\} < 0. \end{cases}$$

Convert $(\hat{b}_I, \hat{b}_Q)$ back into a 2-bit stream using the same Gray mapping.

T4-2 **Compute simulated BER:**

$$\widehat{\text{BER}} = \frac{\#\{b_{tx} \neq b_{rx}\}}{\text{total bits}}.$$

T4-3 Feed the detected bitstream $\mathbf{b}_{rx}$ to the PCM decoder (8-bit) and reconstruct the image.

T4-4 For each E_b/N_0, save/attach:

- measured BER value,
- reconstructed image,
- I-Q scatter plot of received symbols.

PTQ8: Why can a small BER still create visible "salt-and-pepper" artifacts in the reconstructed image?

PTQ9: For $E_b/N_0 = 0$ dB and 10 dB, report your measured BER values (from simulation).

Task 5: Theoretical BER and comparison with simulation

T5-1 Compute theoretical BER for each E_b/N_0:

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right).$$

Use `erfc` function from `scipy.special` in python.

T5-2 Plot on a **semi-log** scale (use `semilogy` in python):

- simulated BER vs E_b/N_0 (markers),
- theoretical BER vs E_b/N_0 (smooth curve).

T5-3 Comment on agreement/differences.

PTQ10: Why is a semi-log plot (log scale on BER axis) preferred for BER curves?

PTQ11: **Draw:** BER vs E_b/N_0 (semi-log) and show both:

- theoretical curve (line)
- simulated points (markers)

PTQ12: From your plot, identify the minimum E_b/N_0 that gives "acceptable" image quality. Justify using BER and the image.

Observations / Results

Table: BER and reconstruction summary

E_b/N_0 (dB)	$(E_b/N_0)_{\text{lin}}$	$\sigma^2 = N_0/2$	Sim. BER	Theory BER	Image quality (short note)
0					
2					
4					
6					
8					
10					

Concluding Remarks

(*Give briefly what you have understood from the experiment.*)

Attach Code: The code should contain the following plots

- Scatter plots of received QPSK symbols for each E_b/N_0.
- Reconstructed images for each E_b/N_0 (label clearly).
- BER vs E_b/N_0 plot: simulated vs theoretical (semi-log).

Student's Signature: ______________________________