Parallel and Distributed Computing
HW6 - Shear and Bitonic Sort

Group No: 18

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Consider an n-element integer number sequence S we wish to sort in non-decreasing order such that si ≤ si+1. For the purposes of this exercise, n is assumed to be a multiple of k. Show how the sequence S can be sorted in this cluster based on:

1. Shear Sort Algorithm.
2. Bitonic Sort Algorithm

Shear Sort Algorithm and Working :

Let's consider the following NxN matrix (N=4) below:

1	4	9	13
2	6	10	14
3	7	11	15
5	8	12	16

We first sort the matrix row wise with each alternate row in same direction

1	4	9	13
2	6	10	14
3	7	11	15
5	8	12	16

1	4	9	13
14	10	6	2
3	7	11	15
16	12	8	5

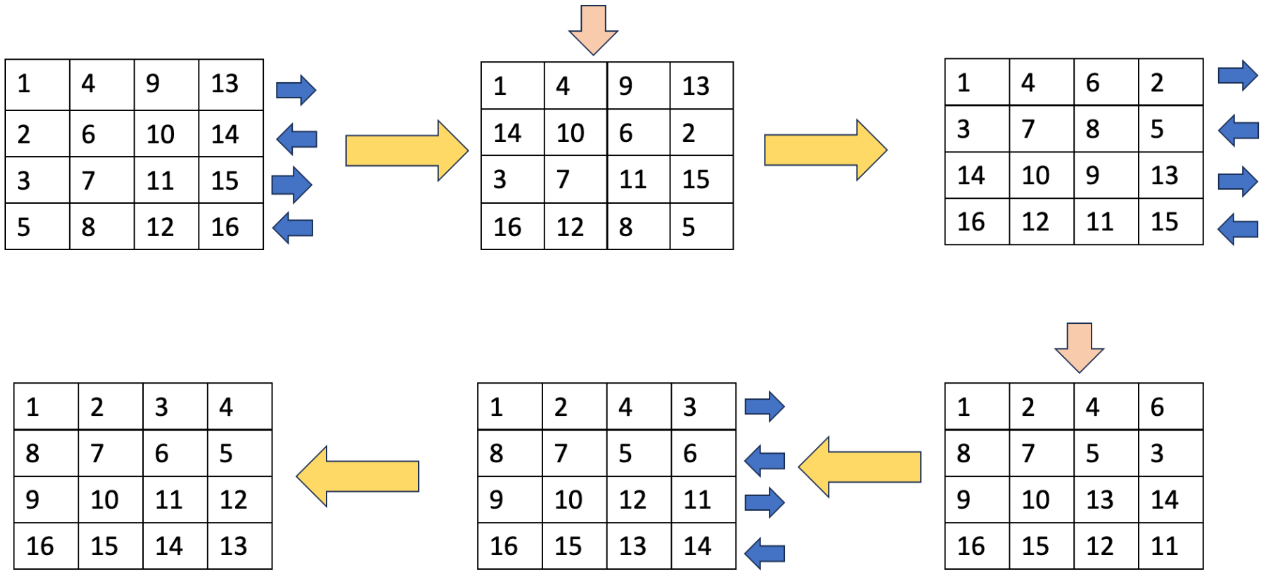
We then sort the matrix column wise , so the initial matrix now becomes :

1	4	9	13
2	6	10	14
3	7	11	15
5	8	12	16

1	4	9	13
14	10	6	2
3	7	11	15
16	12	8	5

1	4	6	2
3	7	8	5
14	10	9	13
16	12	11	15

We repeat the process to finally get the sorted matrix



In the Shear Sort algorithm, each processor in a system with k processors is assigned specific rows to sort based on a predetermined formula. For instance, with n integers to sort and k processors (forming a grid for n=16 integers and k=4 processors), each processor, denoted as Pi, is responsible for sorting the ith row.

Size of memory required by each processor is n//k

The number of compare-exchange operations during each phase of the sort is determined based on whether k is odd or even.

If k is odd, during the odd phase of sorting, k/2 operations occur, and the same number of operations occur during the even phase.

However, if k is even, k/2 operations occur during the odd phase, while $k/2-1$ operations occur during the even phase.

Let ri represent the number of rows allocated to a processor Pi (i ranges from 0 to $k-1$), o denote the number of odd operations, and e denote the number of even operations needed to fully sort a row. Consequently, the total number of compare-exchange operations executed by a processor can be calculated as follows:

When k is odd: $ri \times ((o+e) \times [k/2])$

When k is even: $ri \times ((o+e) \times [k/2] - e)$

For the given example of n=16 integers and k=4 processors, the number of compare-exchange operations a processor performs during the row phase would be $2 \times o + e$

During the column phase of the Shear Sort algorithm, each of the k processors is allocated specific columns from 0 to $k-1$ in a sequential manner. For instance, in a scenario with n=16 integers and k=4 processors, Processor 0 handles Column 0, Processor 1 handles Column 1, and so on.

The number of compare-exchange operations in each phase of sorting depends on whether n/k results in an odd or even number.

If n/k is odd, both the odd and even phases of sorting require $n/(2 \times k)$ operations.

However, if n/k is even, the odd phase requires n/(2×k) operations, while the even phase requires [n/(2×k)]−1 operations.

Let o represent the count of odd swaps and e represent the count of even swaps necessary to sort a column. As a result, the total number of compare-exchange operations per processor is computed as follows:

When n/k is odd: (o+e)×[n2×k]
When n/k is even: (o+e)×[n2×k]−e

For the given example with n=16 and k=4 each processor executes 2×o+2×e2×o+2×e compare-exchange operations during the column phase.

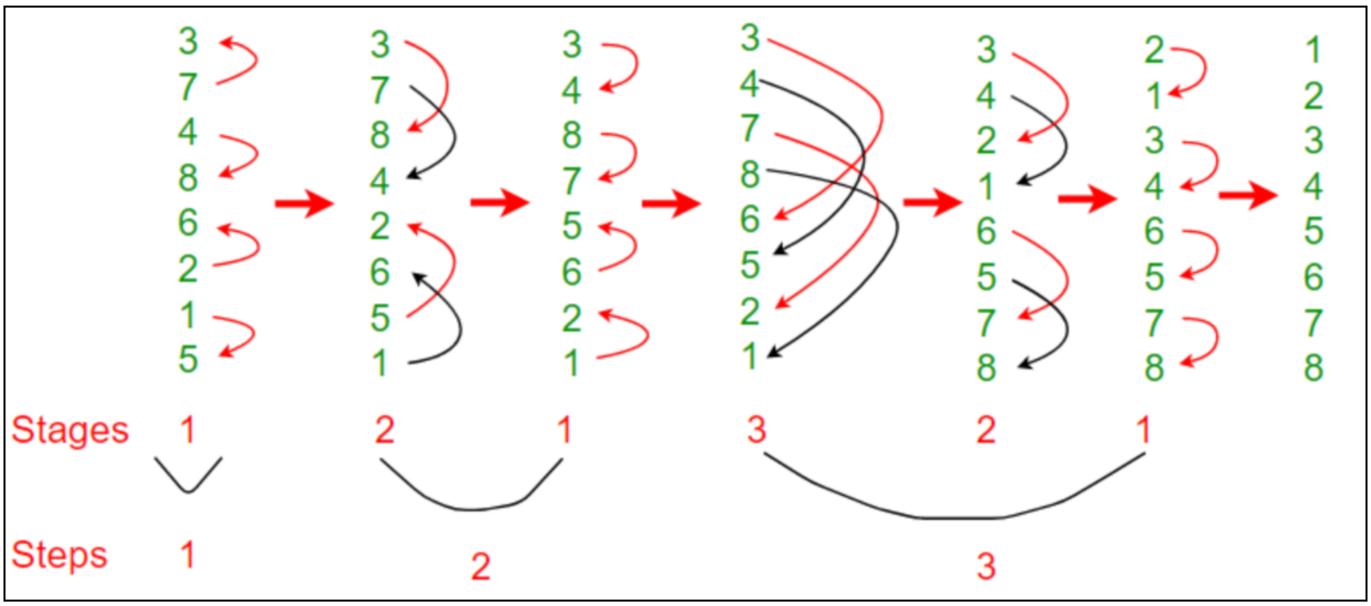
Time Complexity Analysis:

- Comparisons and Exchanges: Each processor performs a certain number of compare-exchange operations during the row and column phases. The number of these operations per processor is based on the size of the allocated rows and columns.
- Iterations: Shear Sort's convergence pattern involves multiple iterations to position elements correctly. For n elements distributed among k processors, Shear Sort typically takes O(log(n/k)) iterations to converge.
- Total Time Complexity: Combining the iterations and the operations within each iteration, the time complexity of Shear Sort can be expressed as **O(k×(log(n/k)+1))**, where: k is the number of processors., n is the number of elements to be sorted.

Bitonic Sort Algorithm and Working :

Bitonic sort is better for parallel implementation because we always compare elements in a predefined sequence and the sequence of comparison doesn't depend on data. Therefore it is suitable for implementation in hardware and parallel processor array.
A sequence is called Bitonic if it is first increasing, then decreasing. Bitonic Sort can only be done if the number of elements to sort is 2^n. A group is a contiguous part of the array that is monotonic. Subgroups are sections within a group with equal length. A 2n-sized array can be partitioned into 2m groups, each with 2n-m elements. For instance, an 8-element array can be subdivided into:

- 1 group of size 8
- 2 groups of size 4 each
- 4 groups of size 2 each
- 8 groups of size 1 each



Algorithm:

1. Bitonic sort sorts small subsequences in parallel.
2. These are called the bitonic sequences.
3. These are then recursively merged for a sorted array.
4. We divide S into n/k subsequences of length k, one per computer.
5. Each computer sorts locally using any O(nlogn) sorting algorithm like merge sort or heapsort.
6. This results in n/k sorted subsequences of length k.
7. We then merge the sorted sequences in log(k) merge steps.
8. In each step, we merge subsequences i and i+n/2k for i from 0 to n/2k-1 in parallel across computers.
9. This results in n/2k merged sorted sequences of length 2k.
10. After log(k) merge steps, we get a sorted array

Let's consider the following array S with **N=8** and **k=4** below:

4	2	70	1	88	3	6	5

S is initially divided evenly:

Processor	Block	
P0	4	2
P1	70	1
P2	88	3
P3	6	5

Each computer sorts its elements locally:

Processor	Block	
P0	2	4
P1	1	70
P2	3	88
P3	5	6

We then merge P0 with P2 and P1 with P3:

Processor	Block			
P0	2	3	4	88
P1	1	5	6	70

Finally, we merge P0 and P1 to get full sorted sequence on P1:

Processor	Result							
P0	1	2	3	4	5	6	70	88

Complexity :

We conclude the iterations when 2^i equals n, resulting in log^2n iterations. During each iteration i, we conduct i subgroupings, ranging from 2^(i-1) to 1.

This leads to a maximum of k * i potential compare-and-swap operations for each iteration, considering that each subgrouping involves, at most, 4 compare-and-swap operations for an array of length 8.

By applying the same formula, the final iteration executes at most k * log2n compare-and-swaps.

- Best Case: $O(log^2n)$
- Average Case: $O(log^2n)$
- Worst Case: $O(log^2n)$

Space Complexity: $O(n.log^2n)$

Analysis:

- **Comparisons:**
  - $O((log_2n)_2)$
- **Memory:**
  - 2 * sizeof(int)
- **Communications:**
  - Within a given subgroup, processors operate independently as each one handles a distinct task, corresponding to a different index pair.

- Communication is not necessary among processors in this scenario.
- However, between subgroups, processors may exchange index values.

In summary, both algorithms achieve O(nlogk) complexity for computation, memory, and communication when implemented on a crossbar interconnect cluster of k computers. Shear sort is a simpler approach while bitonic sort can have better constant factors.

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