Can Conventional Measures Identify Geographically Varying Mixed Regression Relationships? A Simulation-based Analysis of Locally Weighted Regression

Aaron Swoboda

Carleton College

aswoboda@carleton.edu

$$Y = \beta_0 + \beta_1 * X_1 + \beta_2 * X_2 + \epsilon$$

$$Y = \beta_0 + \beta_1 * X_1 + \beta_2 * X_2 + \epsilon$$

Locally Weighted Regression (LWR) to the Rescue?

OLS

$$\hat{\beta} = (X'X)^{-1}(X'Y)$$

LWR

$$\hat{\beta}(location_i) = (X'W(location_i)X)^{-1}(X'W(location_i)Y)$$

OLS

$$\hat{\beta} = (X'X)^{-1}(X'Y)$$

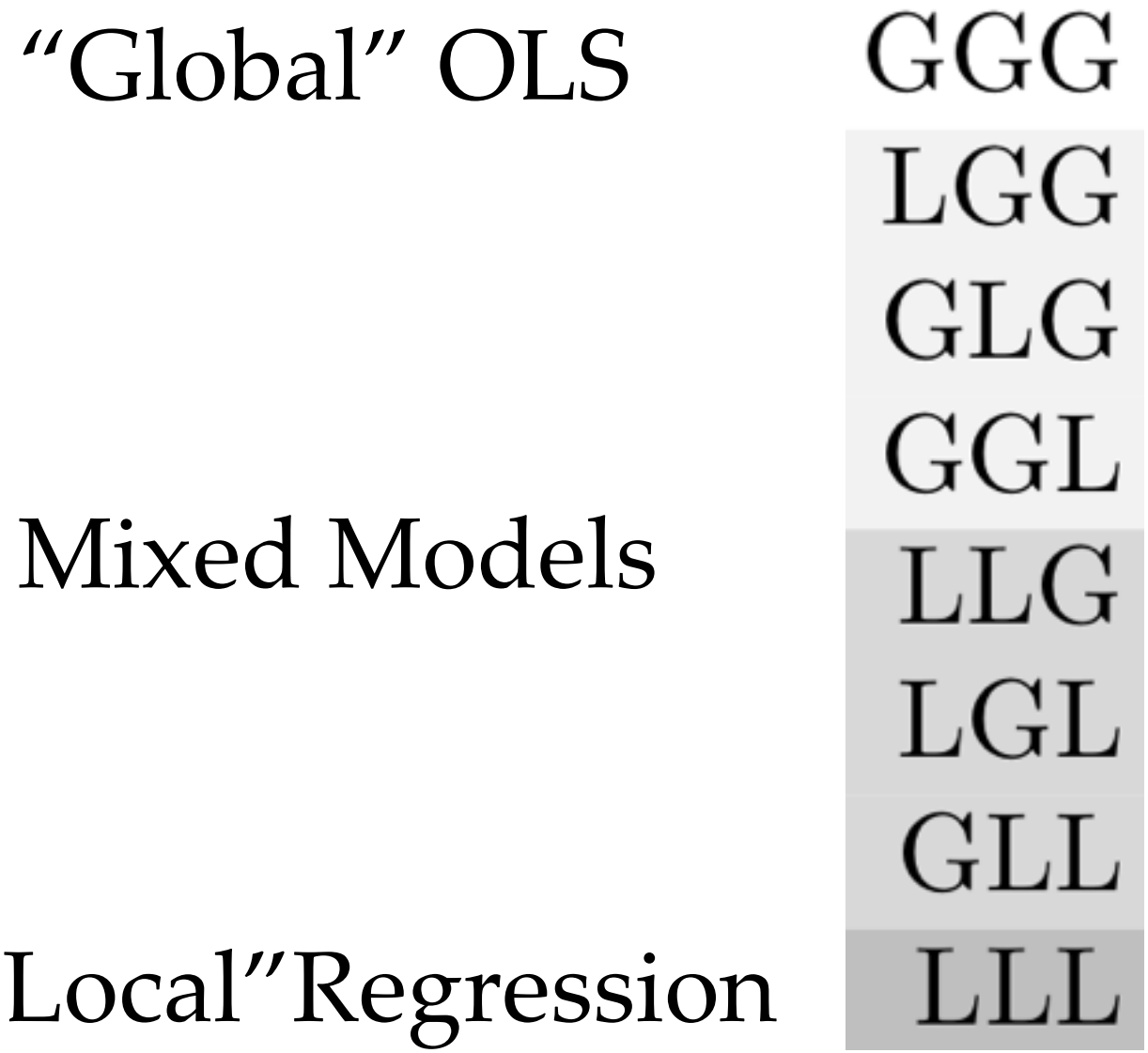
LWR

$$\hat{\beta}(location_i) = (X'W(location_i)X)^{-1}(X'W(location_i)Y)$$

$$w_{jj} = \left[1 - \left(\frac{d_{ij}}{d_{ik}}\right)^2\right]^2 \text{ if } d_{ij} < d_{ik}, \text{ otherwise} = 0,$$

“Global” OLS GGG

“Local”Regression LLL


"Global" OLS
Mixed Models
"Local"Regression
GGG
LGG
GLG
GGL
LLG
LGL
GLL
LLL

The Researcher's Problem

Choose a model and a bandwidth

Bandwidths are commonly selected with…

Leave One Out Cross Validation

Akaike Information Criterion

Generalized Cross Validation

Standardized Cross Validation

$$LOOCV = \frac{1}{N}\sqrt{\sum_{i=1}^{N}(y - \hat{y}_{\neq i})^2},$$

$$SCV_i(k) = \frac{(y_i - \hat{y}_{\neq i}(k))^2}{\sum_k (y_i - \hat{y}_{\neq i})^2}$$

$$SCV(k) = \sum_i SCV_i(k)$$

S Farber and A Páez. A systematic investigation of cross-validation in GWR model estimation: empirical analysis and Monte Carlo simulations. *Journal of Geographical Systems*, 9(4):371–396, 2007.

$$GCV = n * \sum_{i=1}^{n} \frac{(y_i - \hat{y}_i)^2}{(n - v_1)^2},$$

$$AIC = 2 * n * ln(\hat{\sigma}) + n * ln(2 * \pi) + n * \frac{n + v_1}{n - 2 - v_1}$$

Experiment Data Generation Process

$$Y_i = \beta_0(East_i, North_i) + \beta_1(East_i, North_i) * X_{1i} + \beta_2(East_i, North_i) * X_{2i} + \epsilon_i$$

$$n \in \{50, 100, 200, 400, 800\}$$

$$\sigma^2 \in \{0.25, .5, 1, 2, 3\}$$

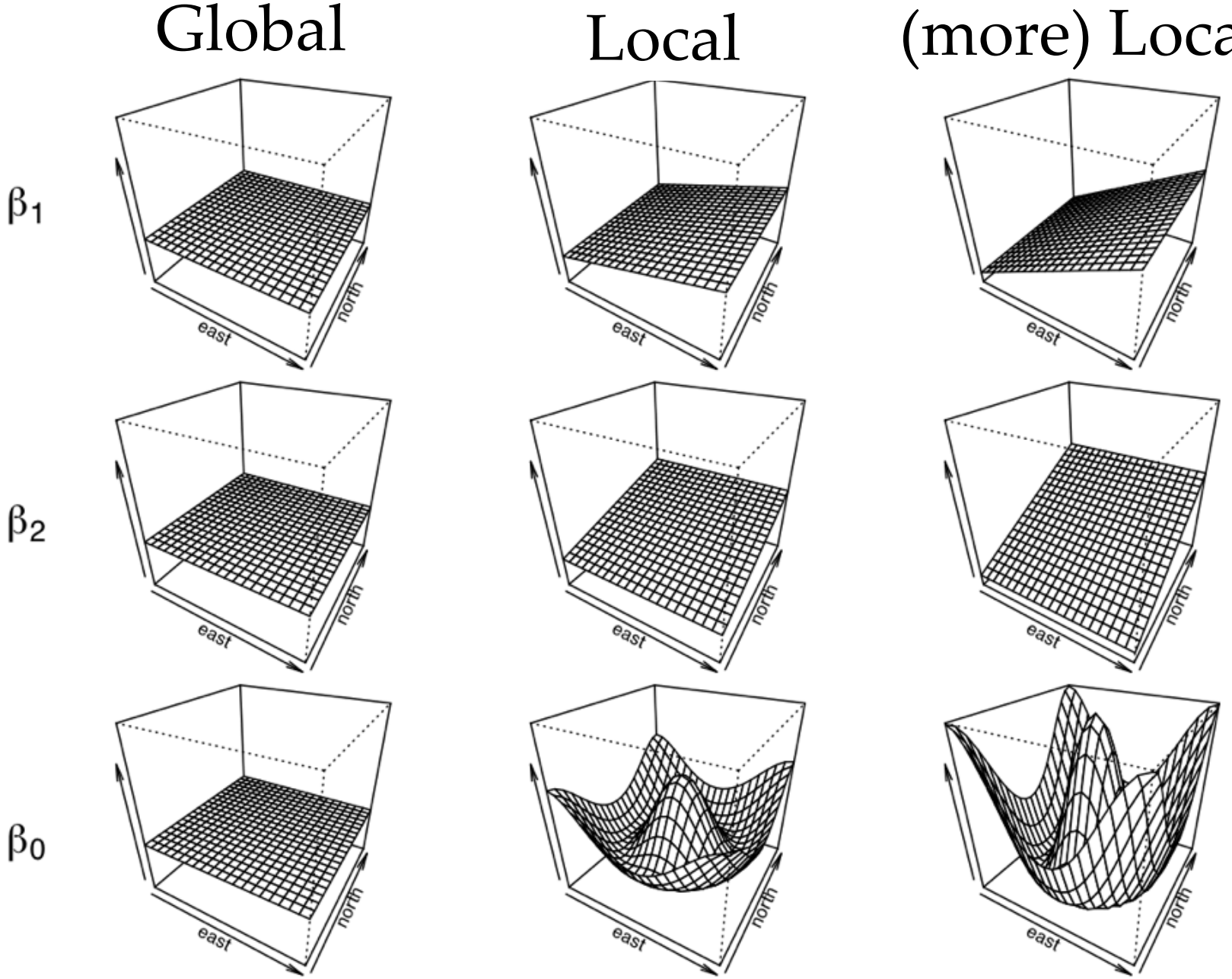
$$X_1 \sim u[0, 1]$$

$$X_2 \sim u[0, 1]$$

$$East \sim u[0, 1]$$

$$North \sim u[0, 1]$$

Coefficient Spatial Variation



With Our Data…

$$[Y, X_1, X_2, East, North]$$

Estimate all models

GGG
LGG
GLG
GGL
LLG
LGL
GLL
LLL

7 bandwidths each

50 combinations total

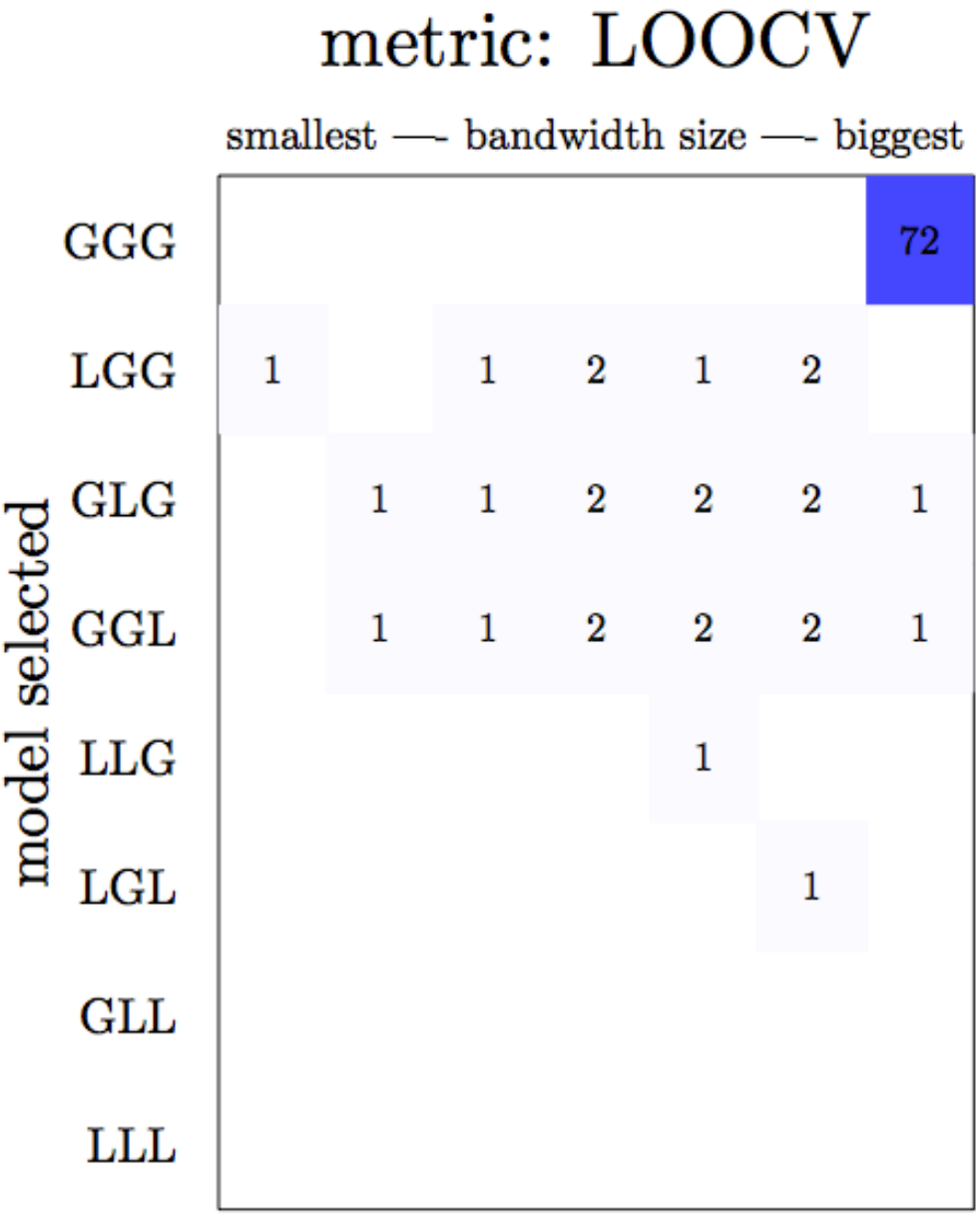
Calculate the values of the four metrics (LOOCV, GCV, SCV, AIC)

Is the model with the optimized metric value the correct model?

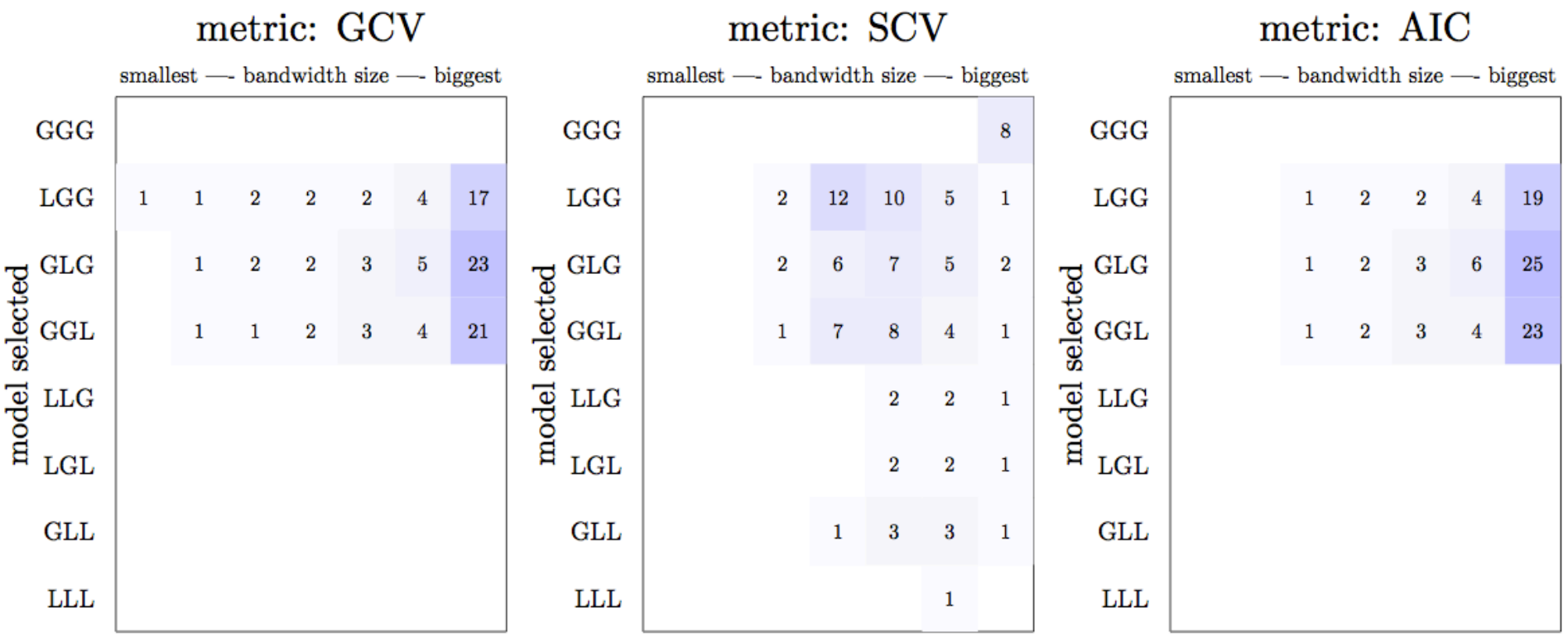
Start Simple… True Model: GGG

Model Selected	Metric				
	LOOCV	GCV	SCV	AIC	
GGG	72	0	8	0	3/3 Correct
LGG	7	28	29	28	
GLG	8	36	22	37	2/3 Correct
GGL	8	33	22	34	
LLG	1	1	5	0	
LGL	2	1	5	1	1/3 Correct
GLL	1	1	8	0	
LLL	0	0	1	0	0/3 Correct
	100	100	100	100	

True Model: GGG



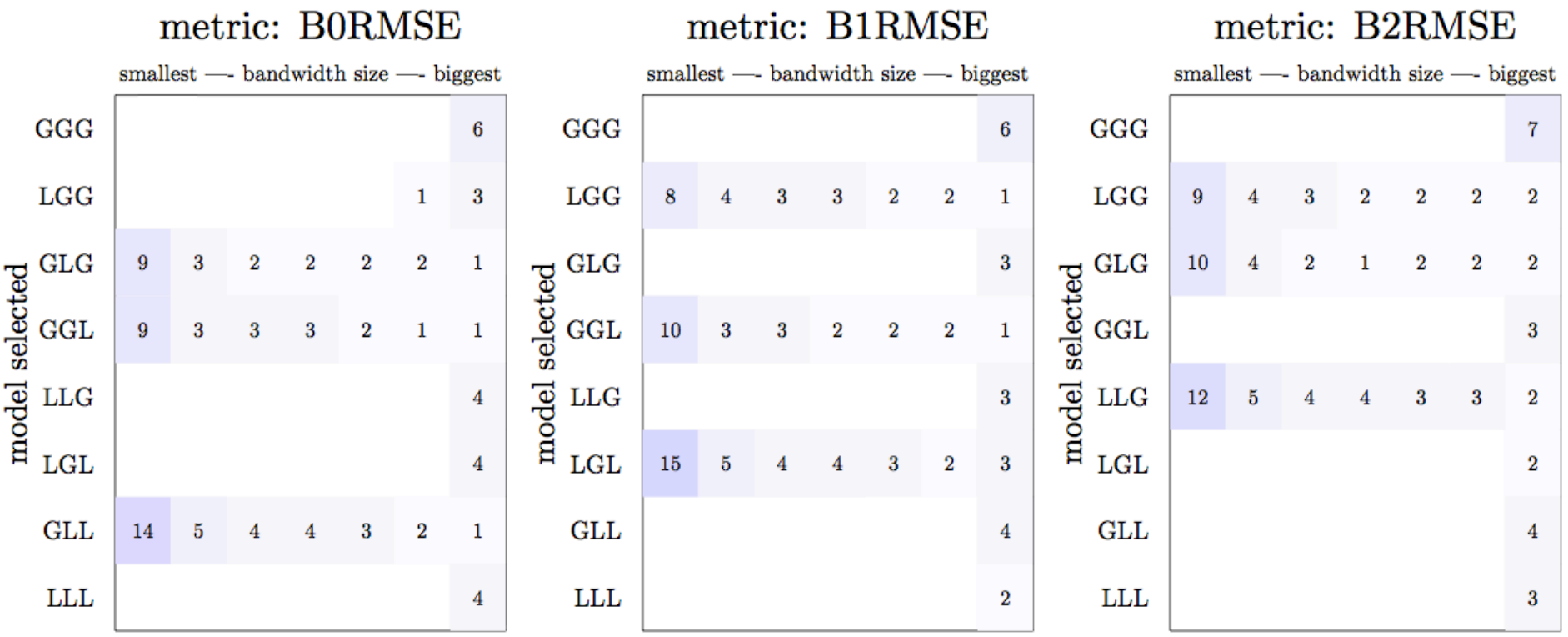
True Model: GGG



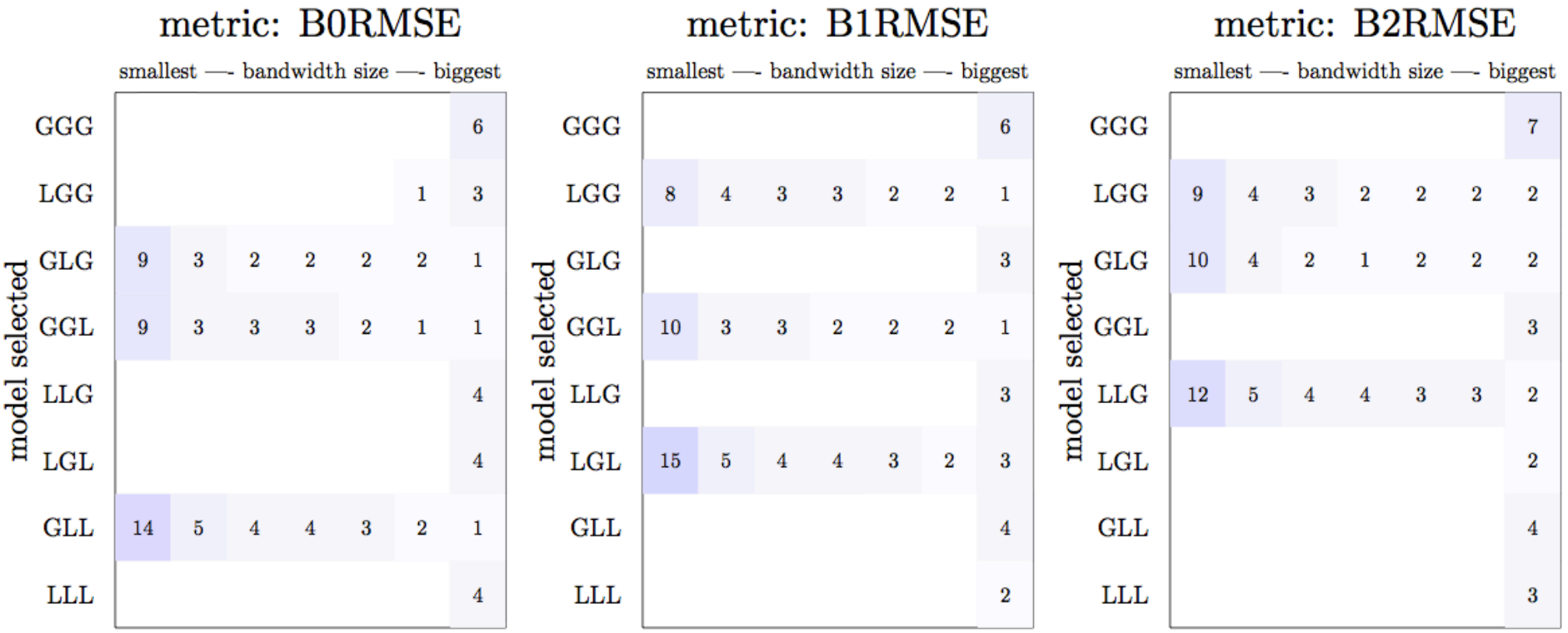
True Model: GGG

Model Selected	Coefficient RMSE β_0	β_1	β_2
GGG	6	6	7
LGG	3	23	24
GLG	22	4	24
GGL	23	23	3
LLG	5	3	33
LGL	5	36	2
GLL	32	4	4
LLL	4	2	3
	100	100	100

True Model: GGG

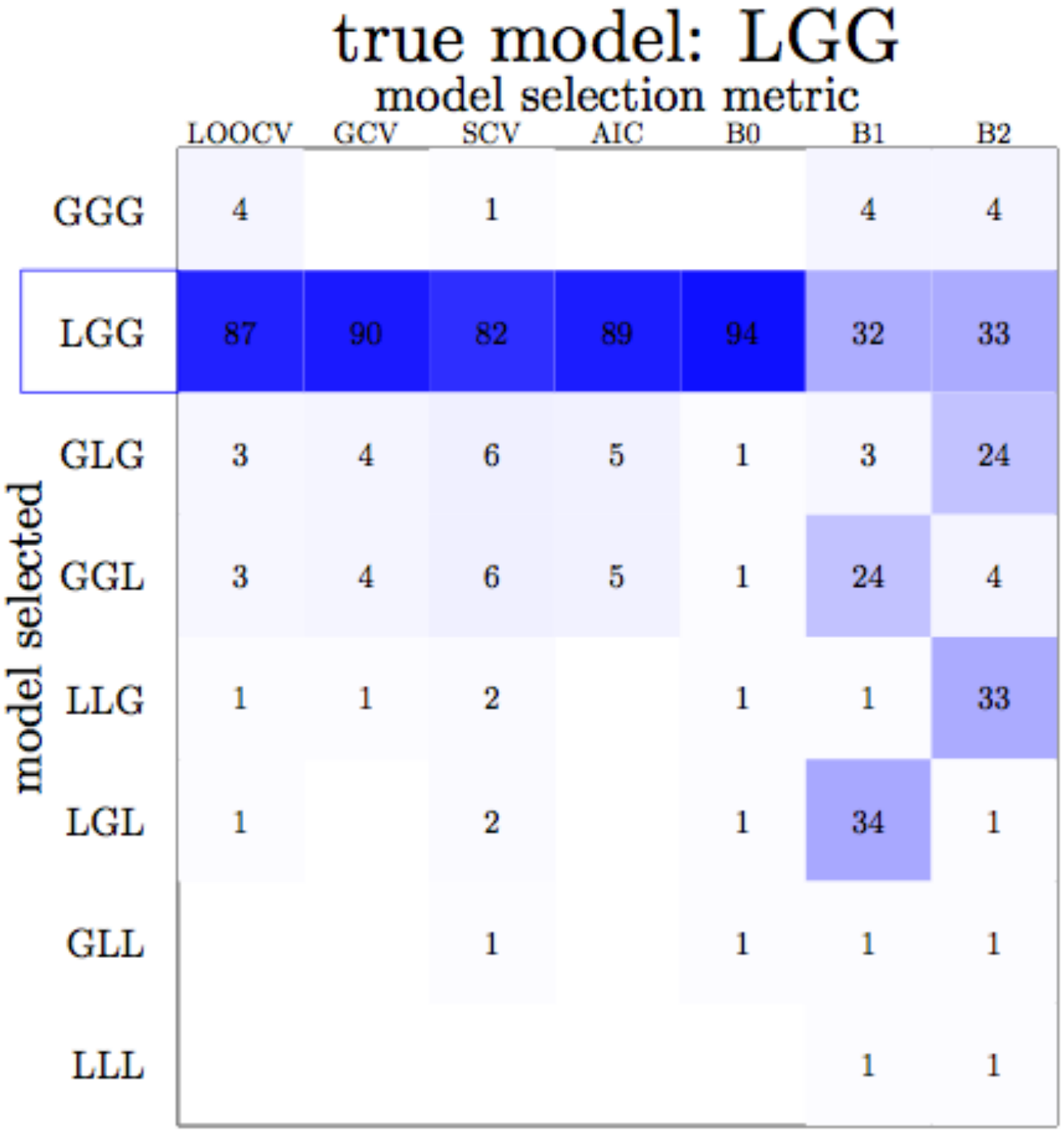


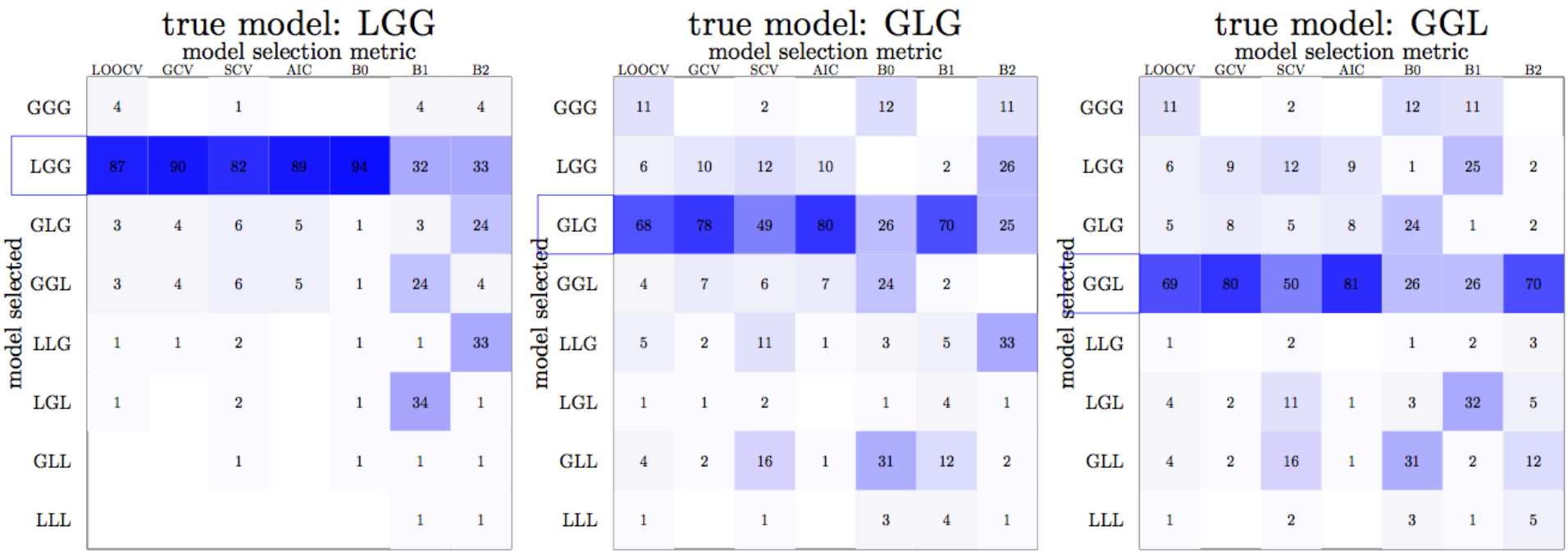
True Model: GGG

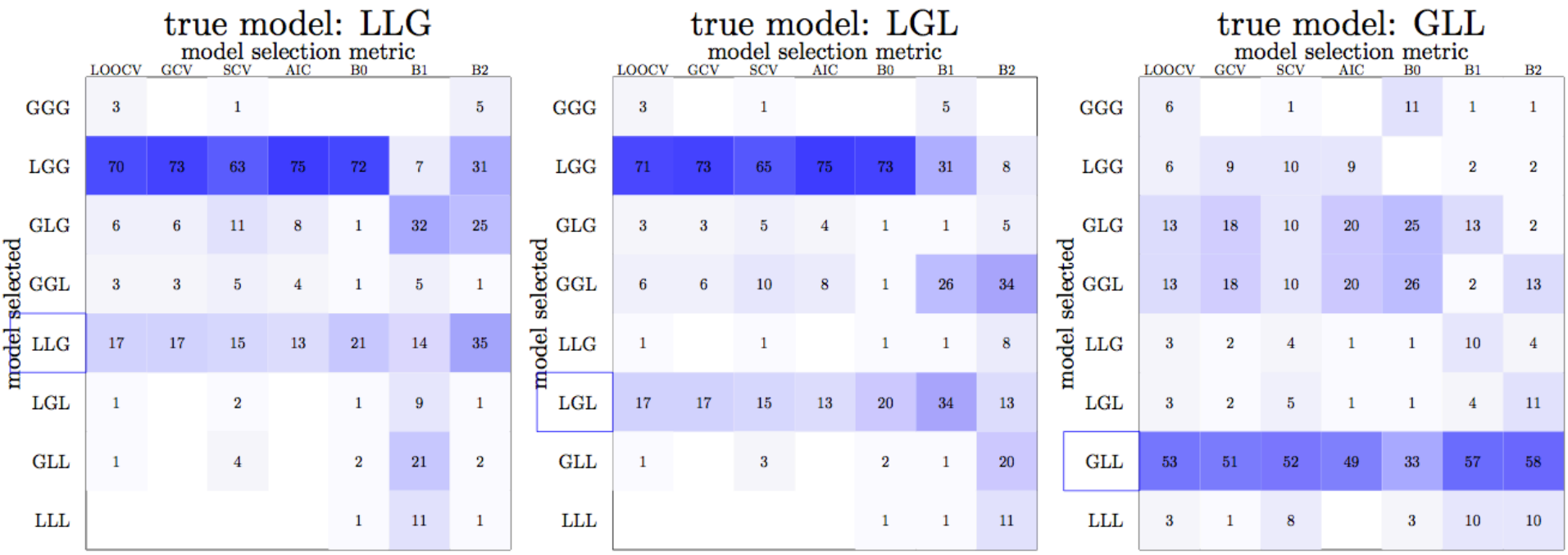


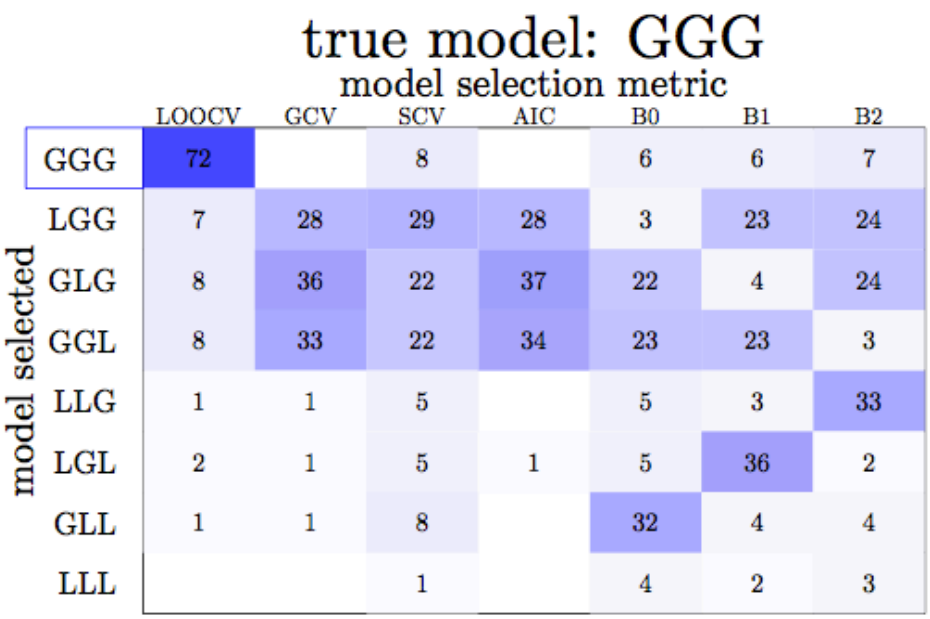
Most accurate coefficient estimates:

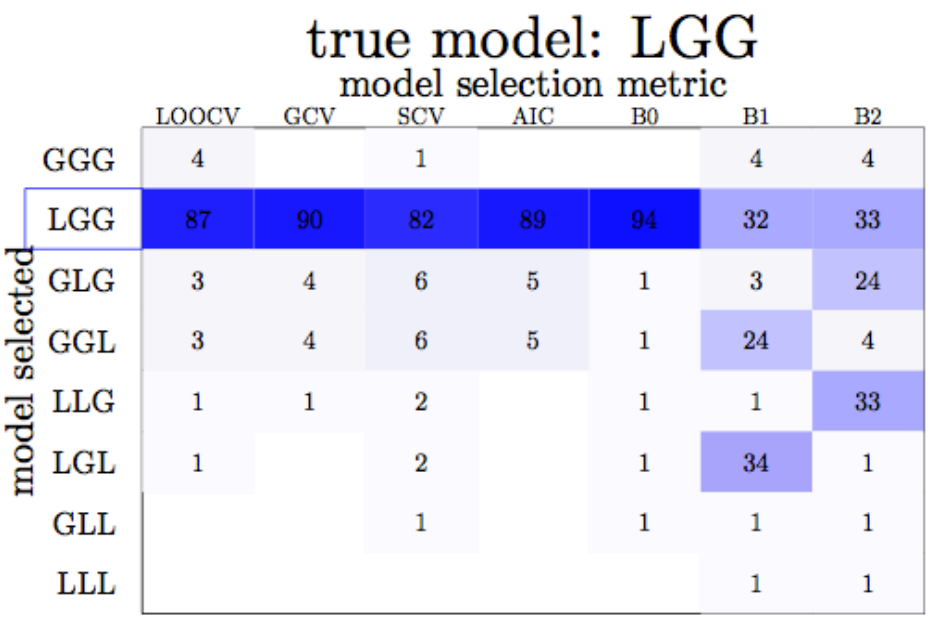
- Tend not to be the correct model
- Allow other coefficients to vary


true model: LGG
model selection metric
LOOCV GCV SCV AIC B0 B1 B2
model selected
GGG 4 1 4 4
LGG 87 90 82 89 94 32 33
GLG 3 4 6 5 1 3 24
GGL 3 4 6 5 1 24 4
LLG 1 1 2 1 1 33
LGL 1 2 1 34 1
GLL 1 1 1 1
LLL 1 1


true model: LGG
model selection metric
LOOCV GCV SCV AIC B0 B1 B2
model selected
GGG 4 1 4 4
LGG 87 90 82 89 94 32 33
GLG 3 4 6 5 1 3 24
GGL 3 4 6 5 1 24 4
LLG 1 1 2 1 1 33
LGL 1 2 1 34 1
GLL 1 1 1 1
LLL 1 1
true model: GLG
model selection metric
LOOCV GCV SCV AIC B0 B1 B2
model selected
GGG 11 2 12 11
LGG 6 10 12 10 2 26
GLG 68 78 49 80 26 70 25
GGL 4 7 6 7 24 2
LLG 5 2 11 1 3 5 33
LGL 1 1 2 1 4 1
GLL 4 2 16 1 31 12 2
LLL 1 1 3 4 1
true model: GGL
model selection metric
LOOCV GCV SCV AIC B0 B1 B2
model selected
GGG 11 2 12 11
LGG 6 9 12 9 1 25 2
GLG 5 8 5 8 24 1 2
GGL 69 80 50 81 26 26 70
LLG 1 2 1 2 3
LGL 4 2 11 1 3 32 5
GLL 4 2 16 1 31 2 12
LLL 1 2 3 1 5


true model: LLG
model selection metric
LOOCV GCV SCV AIC B0 B1 B2
model selected
GGG 3 1 5
LGG 70 73 63 75 72 7 31
GLG 6 6 11 8 1 32 25
GGL 3 3 5 4 1 5 1
LLG 17 17 15 13 21 14 35
LGL 1 2 1 9 1
GLL 1 4 2 21 2
LLL 1 11 1
true model: LGL
model selected
GGG 3 1 5
LGG 71 73 65 75 73 31 8
GLG 3 3 5 4 1 1 5
GGL 6 6 10 8 1 26 34
LLG 1 1 1 1 8
LGL 17 17 15 13 20 34 13
GLL 1 3 2 1 20
LLL 1 1 11
true model: GLL
model selected
GGG 6 1 11 1 1
LGG 6 9 10 9 2 2
GLG 13 18 10 20 25 13 2
GGL 13 18 10 20 26 2 13
LLG 3 2 4 1 1 10 4
LGL 3 2 5 1 1 4 11
GLL 53 51 52 49 33 57 58
LLL 3 1 8 3 10 10


true model: GGG
model selection metric
LOOCV GCV SCV AIC B0 B1 B2
model selected
GGG 72 8 6 6 7
LGG 7 28 29 28 3 23 24
GLG 8 36 22 37 22 4 24
GGL 8 33 22 34 23 23 3
LLG 1 1 5 5 3 33
LGL 2 1 5 1 5 36 2
GLL 1 1 8 32 4 4
LLL 1 4 2 3


true model: LGG
model selection metric
LOOCV GCV SCV AIC B0 B1 B2
model selected
GGG 4 1 4 4
LGG 87 90 82 89 94 32 33
GLG 3 4 6 5 1 3 24
GGL 3 4 6 5 1 24 4
LLG 1 1 2 1 1 33
LGL 1 2 1 34 1
GLL 1 1 1 1
LLL 1 1

true model: GLG

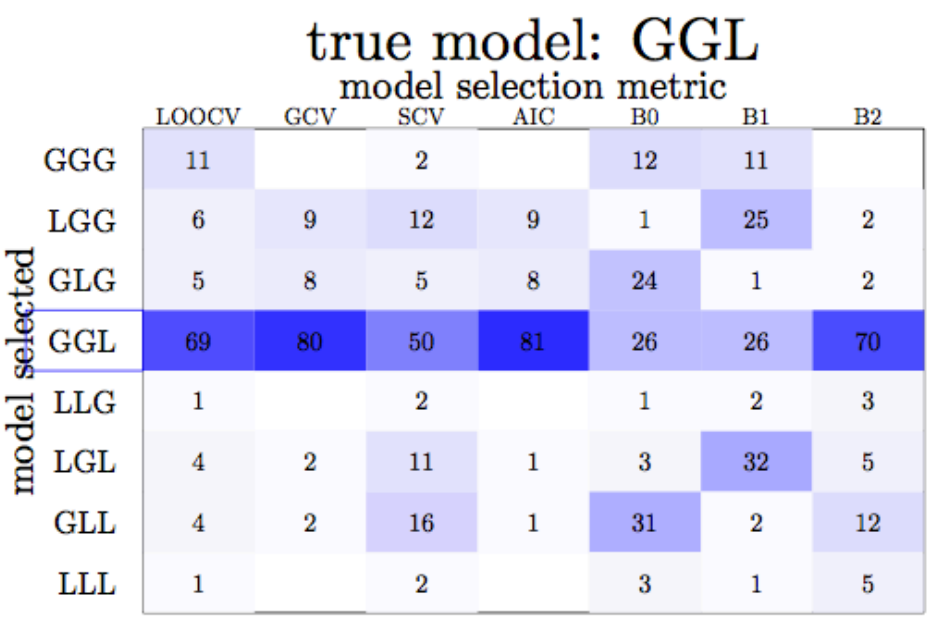
model selection metric

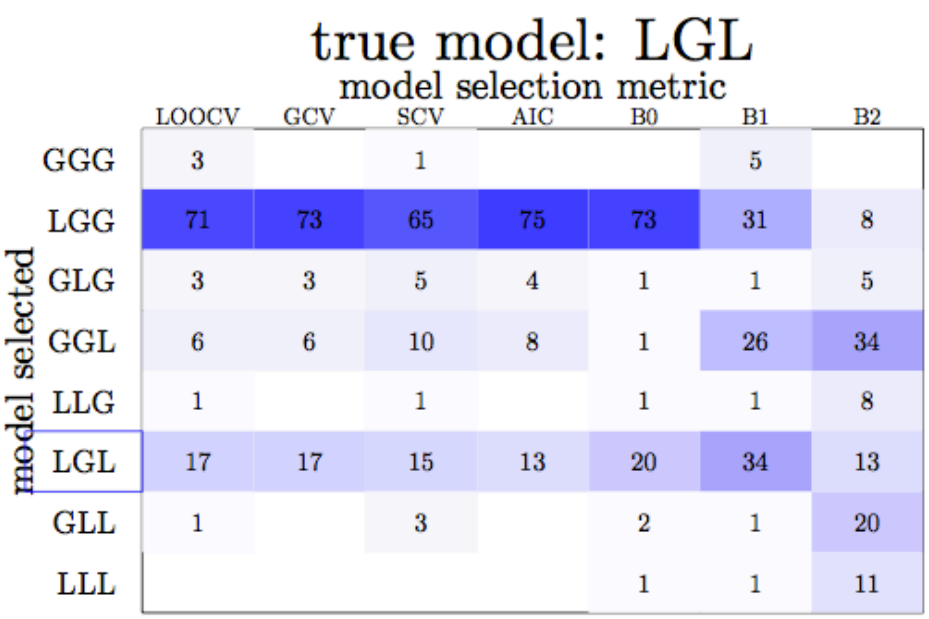
model selected	LOOCV	GCV	SCV	AIC	B0	B1	B2
GGG	11		2		12		11
LGG	6	10	12	10		2	26
GLG	68	78	49	80	26	70	25
GGL	4	7	6	7	24	2	
LLG	5	2	11	1	3	5	33
LGL	1	1	2		1	4	1
GLL	4	2	16	1	31	12	2
LLL	1		1		3	4	1

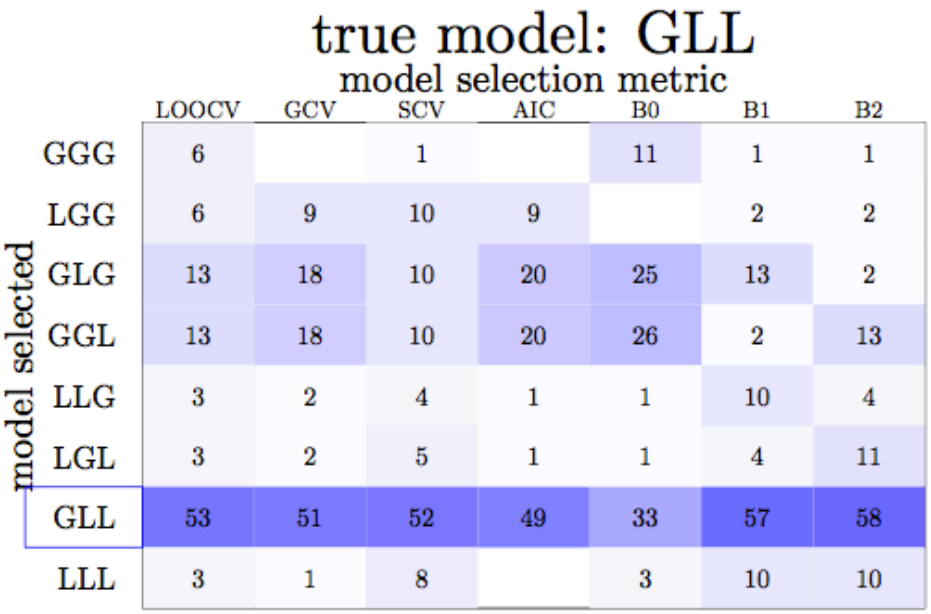
true model: LLG

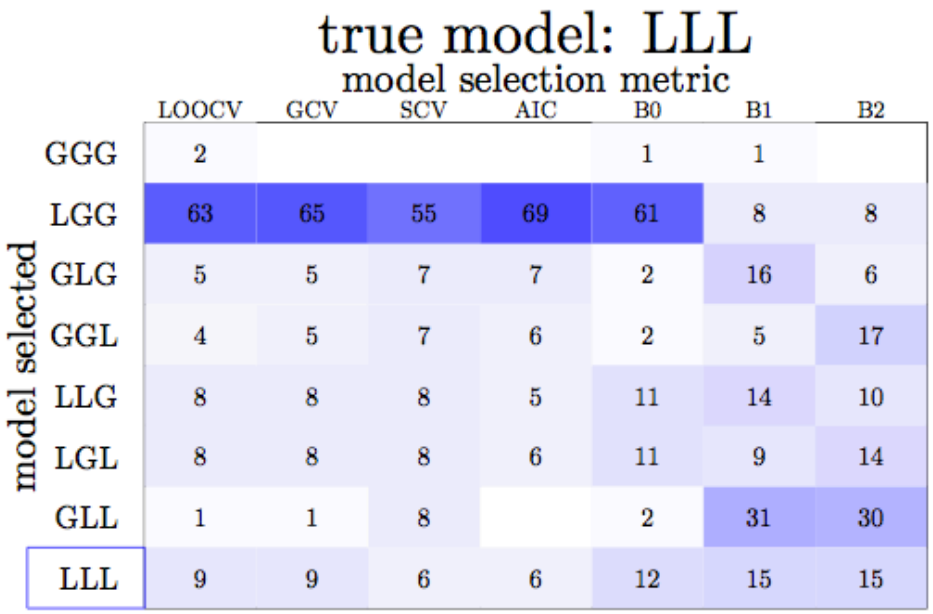
model selection metric

model selected	LOOCV	GCV	SCV	AIC	B0	B1	B2
GGG	3		1				5
LGG	70	73	63	75	72	7	31
GLG	6	6	11	8	1	32	25
GGL	3	3	5	4	1	5	1
LLG	17	17	15	13	21	14	35
LGL	1		2		1	9	1
GLL	1		4		2	21	2
LLL					1	11	1


true model: GGL
model selection metric
LOOCV GCV SCV AIC B0 B1 B2
model selected
GGG 11 2 12 11
LGG 6 9 12 9 1 25 2
GLG 5 8 5 8 24 1 2
GGL 69 80 50 81 26 26 70
LLG 1 2 1 2 3
LGL 4 2 11 1 3 32 5
GLL 4 2 16 1 31 2 12
LLL 1 2 3 1 5


true model: LGL
model selection metric
LOOCV GCV SCV AIC B0 B1 B2
model selected
GGG 3 1 5
LGG 71 73 65 75 73 31 8
GLG 3 3 5 4 1 1 5
GGL 6 6 10 8 1 26 34
LLG 1 1 1 1 8
LGL 17 17 15 13 20 34 13
GLL 1 3 2 1 20
LLL 1 1 11


true model: GLL
model selection metric
LOOCV GCV SCV AIC B0 B1 B2
model selected
GGG 6 1 11 1 1
LGG 6 9 10 9 2 2
GLG 13 18 10 20 25 13 2
GGL 13 18 10 20 26 2 13
LLG 3 2 4 1 1 10 4
LGL 3 2 5 1 1 4 11
GLL 53 51 52 49 33 57 58
LLL 3 1 8 3 10 10


true model: LLL
model selection metric
LOOCV GCV SCV AIC B0 B1 B2
model selected
GGG 2 1 1
LGG 63 65 55 69 61 8 8
GLG 5 5 7 7 2 16 6
GGL 4 5 7 6 2 5 17
LLG 8 8 8 5 11 14 10
LGL 8 8 8 6 11 9 14
GLL 1 1 8 2 31 30
LLL 9 9 6 6 12 15 15

Some Conclusions and Questions

LOOCV is pretty good!

Some Conclusions and Questions

LOOCV is pretty good!

Most accurate coefficient estimates are not necessarily from correct models.

Some Conclusions and Questions

LOOCV is pretty good!

Most accurate coefficient estimates are not necessarily from correct models.

Are larger variances in some coefficients driving the results?

Some Conclusions and Questions

LOOCV is pretty good!

Most accurate coefficient estimates are not necessarily from correct models.

Are larger variances in some coefficients driving the results?

Are coefficient estimates equally accurate across metrics?

Some Conclusions and Questions

LOOCV is pretty good!

Most accurate coefficient estimates are not necessarily from correct models.

Are larger variances in some coefficients driving the results?

Are coefficient estimates equally accurate across metrics?

What happens, ceteris paribus, with greater error variance?